# RMC Backward Erosion Piping (Initiation) Toolbox

## RMC Internal Erosion Suite

RMC-CPD-2023-05

November 2023

US Army Corps of Engineers®
Dam and Levee Safety Programs

US Army Corps of Engineers®
Dam and Levee Safety Programs

Cover Photo: Sand boil during 2011 Mississippi River flooding (USACE).

US Army Corps of Engineers®
Dam and Levee Safety Programs

# RMC Backward Erosion Piping (Initiation) Toolbox

RMC Internal Erosion Suite

RMC-CPD-2023-05

November 2023

Prepared by the Risk Management Center

# REPORT DOCUMENTATION PAGE

<table>
<tr><td>1. REPORT DATE</td><td colspan="2">2. REPORT TYPE</td><td colspan="2">3. DATES COVERED<br>START DATE / END DATE</td></tr>
<tr><td>Nov 2023</td><td colspan="2">Computer Program Document</td><td>N/A</td><td>N/A</td></tr>
<tr><td colspan="5">4. TITLE AND SUBTITLE<br>RMC Backward Erosion Piping (Initiation) Toolbox: RMC Internal Erosion Suite</td></tr>
<tr><td colspan="2">5a. CONTRACT NUMBER<br>N/A</td><td>5b. GRANT NUMBER<br>N/A</td><td colspan="2">5c. PROGRAM ELEMENT NUMBER<br>N/A</td></tr>
<tr><td colspan="2">5d. PROJECT NUMBER<br>N/A</td><td>5e. TASK NUMBER<br>N/A</td><td colspan="2">5f. WORK UNIT NUMBER<br>N/A</td></tr>
<tr><td colspan="5">6. AUTHOR(S)<br>Tim O’Leary, Risk Management Center</td></tr>
<tr><td colspan="4">7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)<br>Risk Management Center<br>12596 West Bayaud Ave. Suite 400<br>Lakewood, CO 80228</td><td>8. PERFORMING ORGANIZATION REPORT NUMBER<br>RMC-CPD-2023-05</td></tr>
<tr><td colspan="3">9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)<br>Risk Management Center<br>12596 West Bayaud Ave. Suite 400<br>Lakewood, CO 80228</td><td>10. SPONSOR/MONITOR’S ACRONYM(S)<br>CEIWR-RMC</td><td>11. SPONSOR/MONITOR’S REPORT NUMBER(S)<br>RMC-CPD-2023-05</td></tr>
<tr><td colspan="5">12. DISTRIBUTION/AVAILABILITY STATEMENT<br>Approved for public release; distribution is unlimited.</td></tr>
<tr><td colspan="5">13. SUPPLEMENTARY NOTES<br>N/A</td></tr>
<tr><td colspan="5">14. ABSTRACT<br>The spreadsheet tools contained in this toolbox deterministically and probabilistically assess the likelihood of initiation of backward erosion piping using Blanket Theory as described in TM 3-424 (1956) and EM 1110-2-1913 (2000), with corrections from ERDC/GSL TR-18-24 (Brandon et al. 2018), and first-order, second-moment method of reliability analysis as described in ETL 1110-2-561 (2006).</td></tr>
<tr><td colspan="5">15. SUBJECT TERMS<br>Internal erosion, backward erosion piping, initiation, hydraulic condition, heave, blowout, Blanket Theory, seepage analysis</td></tr>
<tr><td colspan="3">16. SECURITY CLASSIFICATION OF:</td><td rowspan="2">17. LIMITATION OF ABSTRACT<br>UU</td><td rowspan="2">18. NUMBER OF PAGES<br>95</td></tr>
<tr><td>a. REPORT<br>U</td><td>b. ABSTRACT<br>U</td><td>c. THIS PAGE<br>U</td></tr>
<tr><td colspan="3">19a. NAME OF RESPONSIBLE PERSON<br>Tim O’Leary</td><td colspan="2">19b. PHONE NUMBER (Include area code)<br>(502) 315-6599</td></tr>
</table>



**STANDARD FORM 298 (REF 5/2020)**
*Prescribed by ANSI Std Z39.18*

PREVIOUS EDITION IS OBSOLETE

# PREPARED

The results, findings, and recommendations provided in this document are technically sound and consistent with current Corps of Engineers practice.

Tim O'Leary, Risk Management Center

# REVIEWED

This report has been checked and reviewed and is believed to be in accordance with the standards of the profession.

Mike Navin, Levee Safety Center

# APPROVED

Nate Snorteland, Risk Management Center

# TABLE OF CONTENTS

**1. INTRODUCTION** .......... **1**

**2. TERMS AND CONDITIONS FOR USE** .......... **2**

2.1. Terms and Conditions for Use of Institute for Water Resources Software .......... 2
2.2. Waiver of Warranty .......... 2
2.3. Limitation of Liability .......... 3
2.4. Indemnity .......... 3

**3. GENERAL OVERVIEW** .......... **4**

3.1. Getting Started .......... 4
3.2. Organization .......... 4

**4. BACKGROUND** .......... **7**

4.1. Blanket Theory .......... 7
4.2. Mississippi Valley Division Design Guidance .......... 9
4.3. Finite Element Analysis .......... 11

**5. BLANKET THEORY TRANSFORMATION** .......... **13**

5.1. Transformed Blanket Thickness for Length to Effective Seepage Exit .......... 13
5.2. Transformed Blanket Thickness for Allowable Uplift Pressure .......... 14

**6. BLANKET THEORY CASE 1** .......... **16**

6.1. Method of Analysis .......... 16
6.2. Levee Geometry .......... 16
6.3. Pervious Substratum Characterization .......... 17
6.4. Blanket Theory Assumptions .......... 18
6.5. Seepage Characterization .......... 19
6.6. Likelihood of Heave/Blowout at Landside Toe .......... 20
6.7. Likelihood of Heave/Blowout at Given Distance from Landside Toe .......... 20

**7. BLANKET THEORY CASE 2** .......... **21**

7.1. Method of Analysis .......... 21
7.2. Levee Geometry .......... 21
7.3. Pervious Substratum Characterization .......... 21
7.4. Landside Blanket (Top Stratum) Characterization .......... 21
7.5. Blanket Theory Assumptions .......... 23
7.6. Seepage Characterization .......... 24

7.7. Likelihood of Heave/Blowout at Landside Toe ..... 24
7.8. Likelihood of Heave/Blowout at Given Distance from Landside Toe ..... 28

**8. BLANKET THEORY CASE 3 ..... 33**
8.1. Method Analysis ..... 33
8.2. Levee Geometry ..... 33
8.3. Pervious Substratum Characterization ..... 33
8.4. Blanket Theory Assumptions ..... 33
8.5. Seepage Characterization ..... 34
8.6. Likelihood of Heave/Blowout at Landside Toe ..... 34
8.7. Likelihood of Heave/Blowout at Given Distance from Landside Toe ..... 34

**9. BLANKET THEORY CASE 4 ..... 35**
9.1. Method of Analysis ..... 35
9.2. Levee Geometry ..... 35
9.3. Pervious Substratum Characterization ..... 35
9.4. Landside Blanket (Top Stratum) Characterization ..... 35
9.5. Blanket Theory Assumptions ..... 36
9.6. Seepage Characterization ..... 36
9.7. Likelihood of Heave/Blowout at Landside Toe ..... 36
9.8. Likelihood of Heave/Blowout at Given Distance from Landside Toe ..... 37

**10. BLANKET THEORY CASE 5 ..... 38**
10.1. Method Analysis ..... 38
10.2. Levee Geometry ..... 38
10.3. Pervious Substratum Characterization ..... 39
10.4. Riverside Blanket (Top Stratum) Characterization ..... 39
10.5. Blanket Theory Assumptions ..... 43
10.6. Seepage Characterization ..... 44
10.7. Likelihood of Heave/Blowout at Landside Toe ..... 44
10.8. Likelihood of Heave/Blowout at Given Distance from Landside Toe ..... 44

**11. BLANKET THEORY CASE 6 ..... 45**
11.1. Method of Analysis ..... 45
11.2. Levee Geometry ..... 45
11.3. Pervious Substratum Characterization ..... 46
11.4. Landside Blanket (Top Stratum) Characterization ..... 46

11.5. Blanket Theory Assumptions ........ 54
11.6. Seepage Characterization........ 54
11.7. Likelihood of Heave/Blowout at Landside Toe........ 55
11.8. Likelihood of Heave/Blowout at Given Distance from Landside Toe ........ 55

**12. BLANKET THEORY CASE 7 ........ 57**

12.1. Method of Analysis ........ 57
12.2. Levee Geometry ........ 57
12.3. Pervious Substratum Characterization........ 60
12.4. Riverside Blanket (Top Stratum) Characterization ........ 60
12.5. Landside Blanket (Top Stratum) Characterization ........ 60
12.6. Blanket Theory Assumptions ........ 60
12.7. Seepage Characterization........ 61
12.8. Likelihood of Heave/Blowout at Landside Toe........ 61
12.9. Likelihood of Heave/Blowout at Given Distance from Landside Toe ........ 62

**13. BLANKET THEORY CASE 8 ........ 63**

13.1. Method of Analysis ........ 63
13.2. Levee Geometry ........ 63
13.3. Pervious Substratum Characterization........ 65
13.4. Riverside Blanket (Top Stratum) Characterization ........ 65
13.5. Landside Blanket (Top Stratum) Characterization ........ 65
13.6. Blanket Theory Assumptions ........ 65
13.7. Seepage Characterization........ 66
13.8. Likelihood of Heave/Blowout at Landside Toe........ 67
13.9. Likelihood of Heave/Blowout at Given Distance from Landside Toe ........ 68

**14. FIRST-ORDER, SECOND-MOMENT RELIABILITY ........ 70**

14.1. Foundation Characterization........ 71
14.2. Reliability Analysis ........ 72
14.3. Likelihood of Heave/Blowout at Landside Toe........ 77

**15. REFERENCES ........ 81**

# LIST OF FIGURES

Figure 1. Security warning message bars with the “Enable Content” option to enable macros. ........ 4
Figure 2. Calculation worksheet heading........ 5

Figure 3. Headwater and tailwater input: NAVD88. ..... 5

Figure 4. Headwater and tailwater input: NGVD29. ..... 5

Figure 5. Headwater and tailwater input: User-specified datum. ..... 6

Figure 6. Example of step banner. ..... 6

Figure 7. Example of option banner. ..... 6

Figure 8. Equipotential lines for levee cross section having $L_2/d > 1$ for Blanket Theory Case 1 (ERDC/GSL TR-18-24). ..... 8

Figure 9. BT parameters for BT Case 8 (adapted from ERDC/GSL TR-18-24). ..... 9

Figure 10. Riverside blanket thickness and vertical permeability transformation for Cases 5, 7, and 8. ..... 14

Figure 11. Landside blanket thickness and vertical permeability transformation for Cases 6, 7, and 8. ..... 15

Figure 12. Step 1 of BT Case 1 worksheet: Deterministic analysis. ..... 16

Figure 13. Step 1 of BT Case 1 worksheet: Probabilistic analysis without using @RISK. ..... 16

Figure 14. Step 1 of BT Case 1 worksheet: Probabilistic analysis using @RISK. ..... 16

Figure 15. Step 2 of BT Case 1 worksheet: Levee geometry. ..... 17

Figure 16. Step 3 of BT Case 1 worksheet: Deterministic input. ..... 17

Figure 17. Step 3 of BT Case 1 worksheet: Probabilistic input without using @RISK. ..... 18

Figure 18. Step 3 of BT Case 1 worksheet: Probabilistic input using @RISK. ..... 18

Figure 19. Step 4 of BT Case 1 worksheet: BT assumptions. ..... 18

Figure 20. Step 5 of BT Case 1 worksheet: Deterministic output. ..... 19

Figure 21. Step 5 of BT Case 1 worksheet: Probabilistic output without using @RISK. ..... 20

Figure 22. Step 2 of BT Case 2 worksheet: Levee geometry. ..... 21

Figure 23. Step 4 of BT Case 2 worksheet: Deterministic input. ..... 22

Figure 24. Step 4 of BT Case 2 worksheet: Probabilistic input without using @RISK. ..... 23

Figure 25. Step 4 of BT Case 2 worksheet: Probabilistic input using @RISK. ..... 23

Figure 26. Step 5 of BT Case 2 worksheet: BT assumptions. ..... 24

Figure 27. Step 7 of BT Case 2 worksheet: Deterministic tabular output. ..... 25

Figure 28. Step 7 of BT Case 2 worksheet: Probabilistic tabular output without using @RISK. ..... 26

Figure 29. Step 7 of BT Case 2 worksheet: Deterministic graphical output. ..... 26

Figure 30. Step 7 of BT Case 2 worksheet: Plot options for deterministic graphical output. ..... 27

Figure 31. Step 7 of BT Case 2 worksheet: Probabilistic graphical output. ..... 27

Figure 32. Step 7 of BT Case 2 worksheet: Plot options for probabilistic graphical output. ..... 28

Figure 33. Step 8 of BT Case 2 worksheet: Deterministic tabular output. ..... 29

Figure 34. Step 8 of BT Case 2 worksheet: Probabilistic tabular output without using @RISK. ..... 30

Figure 35. Step 8 of BT Case 2 worksheet: Deterministic graphical output. ..... 30

Figure 36. Step 8 of BT Case 2 worksheet: Plot options for deterministic graphical output. ..... 31

Figure 37. Step 7 of BT Case 2 worksheet: Probabilistic graphical output. .......... 31
Figure 38. Step 7 of BT Case 2 worksheet: Plot options for probabilistic graphical output. .......... 32
Figure 39. Step 2 of BT Case 3 worksheet: Levee geometry .......... 33
Figure 40. Step 4 of BT Case 3 worksheet: BT assumptions .......... 34
Figure 41. Step 2 of BT Case 4 worksheet: Levee geometry .......... 35
Figure 42. Step 5 of BT Case 4 worksheet: BT assumptions .......... 36
Figure 43. Step 2 of BT Case 5 worksheet: Levee geometry .......... 39
Figure 44. Step 4 of BT Case 5 worksheet: Deterministic input. .......... 40
Figure 45. Step 4 of BT Case 5 worksheet: Deterministic input for permeability ratio as input. .......... 40
Figure 46. Step 4 of BT Case 5 worksheet: Probabilistic input without using @RISK. .......... 41
Figure 47. Step 4 of BT Case 5 worksheet: Probabilistic input without using @RISK for permeability ratio as input .......... 41
Figure 48. Step 4 of BT Case 5 worksheet: Probabilistic input using @RISK. .......... 42
Figure 49. Step 4 of BT Case 5 worksheet: Probabilistic input using @RISK for permeability ratio as input. .......... 42
Figure 50. Step 5 of BT Case 5 worksheet: BT assumptions .......... 44
Figure 51. Step 2 of BT Case 6 worksheet: Levee geometry .......... 46
Figure 52. Step 4 of BT Case 6 worksheet: Deterministic input. .......... 47
Figure 53. Step 4 of BT Case 6 worksheet: Deterministic input for permeability ratio as input. .......... 48
Figure 54. Step 4 of BT Case 6 worksheet: Probabilistic input without using @RISK. .......... 49
Figure 55. Step 4 of BT Case 6 worksheet: Probabilistic input without using @RISK for permeability ratio as input .......... 50
Figure 56. Step 4 of BT Case 6 worksheet: Probabilistic input using @RISK. .......... 51
Figure 57. Step 4 of BT Case 6 worksheet: Probabilistic input using @RISK for permeability ratio as input. .......... 52
Figure 58. Example warning message when performing probabilistic analysis using @RISK. .......... 53
Figure 59. Step 5 of BT Case 6 worksheet: BT assumptions .......... 54
Figure 60. BT Case 7a: Basic levee geometry for infinitely long landside blanket. .......... 58
Figure 61. BT Case 7b: Basic levee geometry for open seepage exit. .......... 58
Figure 62. BT Case 7c: Basic levee geometry for seepage block. .......... 59
Figure 63. Step 2 of BT Case 7 worksheet: Levee geometry .......... 60
Figure 64. Step 6 of BT Case 7 worksheet: BT assumptions .......... 61
Figure 65. BT Case 8a: Basic levee geometry for infinitely long landside blanket. .......... 63
Figure 66. BT Case 8b: Basic levee geometry for open seepage exit. .......... 64
Figure 67. BT Case 8c: Basic levee geometry for seepage block. .......... 64
Figure 68. Step 2 of BT Case 8 worksheet: Levee geometry .......... 65

Figure 69. Seepage analysis run cases for first-order, second-moment reliability analysis. 70
Figure 70. Step 1 of FOSM reliability worksheet: Input for standard deviation using 3-sigma rule. 72
Figure 71. Step 1 of FOSM Reliability Analysis worksheet: Input for standard deviation without using 3-sigma rule. 72
Figure 72. Lognormal probability density function of factor of safety (ETL 1110-2-561). 74
Figure 73. Transformed probability density function of factor of safety (ETL 1110-2-561). 74
Figure 74. Step 2 of FOSM Reliability Analysis worksheet: Calculations based on seepage analysis results. 76
Figure 75. Step 2 of FOSM Reliability Analysis worksheet: Tabular output based on seepage analysis results. 77
Figure 76. Step 3 of FOSM Reliability Analysis worksheet: Tabular output based on selected headwater levels. 77
Figure 77. Step 3 of FOSM Reliability Analysis worksheet: Graphical output. 78
Figure 78. Step 3 of FOSM Reliability Analysis worksheet: Plot options for deterministic graphical output. 79
Figure 79. Step 3 of FOSM Reliability Analysis worksheet: Probabilistic graphical output. 80
Figure 80. Step 3 of FOSM Reliability Analysis worksheet: Plot options for probabilistic graphical output. 80

# LIST OF TABLES

Table 1. Suggested design values for riverside blanket permeability. 10
Table 2. Suggested design values for landside blanket permeability. 10
Table 3. Suggested vertical permeability of landside blanket. 11
Table 4. Complete elliptical integrals of the first kind. 67

# APPENDIXES

**APPENDIX A. ACRONYM LIST 83**

# 1. Introduction

The Risk Management Center (RMC) of the U.S. Army Corps of Engineers (USACE) has developed a suite of Microsoft Excel spreadsheets to support risk assessments for dam and levee safety. Each analysis suite is composed of multiple toolboxes (Microsoft Excel workbooks), and each toolbox contains multiple spreadsheet tools or calculation worksheets (Microsoft Excel worksheets). The RMC Backward Erosion Piping (Initiation) Toolbox is part of the RMC Internal Erosion Suite.

The information from these spreadsheet tools, along with other pertinent information, informs judgment when developing a list of more and less likely factors and estimating probabilities. USACE best practice for estimating probabilities is to use the best available and multiple methods, but all final probabilities are estimated using team elicitation based on the totality and strength of the evidence.

The RMC continuously works to improve the performance of RMC software; report possible bugs directly to the RMC at the address listed below. Ideally, report suspected errors in written form with a description of the problem and the steps that lead to its occurrence. Suggestions for improvement are also welcomed.

U.S. Army Corps of Engineers
Institute for Water Resources
Risk Management Center
RMC.software@usace.army.mil

# 2. Terms and Conditions for Use

By using Institute for Water Resources (IWR) software, users voluntarily accept the following terms and conditions. Users that do not agree to these terms and conditions should uninstall the IWR software and return any program materials to IWR and its technical centers. If users downloaded the software and do not have disk media, delete all copies and cease using the software.

## 2.1. Terms and Conditions for Use of Institute for Water Resources Software

The United States Government, U.S. Army Corps of Engineers, Institute for Water Resources ("IWR"), and IWR's technical centers including the Risk Management Center ("RMC") and Hydrologic Engineering Center ("HEC") grant to the user the rights to install "IWR Software" (either from a disk copy obtained from IWR and IWR's technical centers, a distributor or another user or by downloading it from a network) and to use, copy and/or distribute copies of the IWR Software to other users, subject to the following Terms and Conditions of Use:

- All copies of the IWR Software received or reproduced by or for user pursuant to the authority of this Terms and Conditions of Use will be and remain the property of IWR.

- User may reproduce and distribute the IWR Software provided that the recipient agrees to the Terms and Conditions for Use noted herein.

- IWR and IWR's technical centers are solely responsible for the content of the IWR Software. The IWR Software may not be modified, abridged, decompiled, disassembled, unobfuscated or reverse engineered. The user is solely responsible for the content, interactions, and effects of any and all amendments, if present, whether they be extension modules, language resource bundles, scripts, or any other amendment.

- The name of the IWR Software must not be used to endorse or promote products derived from the IWR Software. Products derived from the IWR Software may not be called the IWR Software nor may any part of the IWR Software name appear within the name of derived products.

- No part of this Terms and Conditions for Use may be modified, deleted or obliterated from the IWR Software.

- No part of the IWR Software may be exported or re-exported in contravention of U.S. export laws or regulations.

## 2.2. Waiver of Warranty

THE UNITED STATES GOVERNMENT AND ITS AGENCIES, OFFICIALS, REPRESENTATIVES, AND EMPLOYEES, INCLUDING ITS CONTRACTORS AND SUPPLIERS PROVIDE THE IWR SOFTWARE "AS IS," WITHOUT ANY WARRANTY OR CONDITION, EXPRESS, IMPLIED OR STATUTORY, AND SPECIFICALLY DISCLAIM ANY IMPLIED WARRANTIES OF TITLE, MERCHANTABILITY, FITNESS FOR A PARTICULAR PURPOSE AND NON-INFRINGEMENT. Depending on state law, the foregoing disclaimer may not apply to you, and you may also have other legal rights that vary from state to state.

## 2.3. Limitation of Liability

IN NO EVENT SHALL THE UNITED STATES GOVERNMENT AND ITS AGENCIES, OFFICIALS, REPRESENTATIVES, AND EMPLOYEES, INCLUDING ITS CONTRACTORS AND SUPPLIERS, BE LIABLE FOR LOST PROFITS OR ANY SPECIAL, INCIDENTAL OR CONSEQUENTIAL DAMAGES ARISING OUT OF OR IN CONNECTION WITH USE OF THE IWR SOFTWARE REGARDLESS OF CAUSE, INCLUDING NEGLIGENCE. THE UNITED STATES GOVERNMENT'S LIABILITY, AND THE LIABILITY OF ITS AGENCIES, OFFICIALS, REPRESENTATIVES, AND EMPLOYEES, INCLUDING ITS CONTRACTORS AND SUPPLIERS, TO YOU OR ANY THIRD PARTIES IN ANY CIRCUMSTANCE IS LIMITED TO THE REPLACEMENT OF CERTIFIED COPIES OF THE IWR SOFTWARE WITH IDENTIFIED ERRORS CORRECTED. Depending on state law, the above limitation or exclusion may not apply to you.

## 2.4. Indemnity

As a voluntary user of the IWR Software you agree to indemnify and hold the United States Government, and its agencies, officials, representatives, and employees, including its contractors and suppliers, harmless from any claim or demand, including reasonable attorneys' fees, made by any third party due to or arising out of your use of the IWR Software or breach of this Agreement or your violation of any law or the rights of a third party.

# 3. General Overview

## 3.1. Getting Started

Copy or download the toolbox file to the computer. To open the toolbox file, either:

- Find the file on the computer and double-click it. This opens the file in Microsoft Excel.
- Open Microsoft Excel and use the application to open the file: Once Microsoft Excel is open, go to the File menu at the top of the window and select Open.

The toolbox is an Excel binary workbook (.xlsb) that uses macros. You may need to enable the macros, either before opening the file or by clicking “Enable Content” in the yellow Security Warning message bar with a shield icon that appears after the file is opened. The actual message in the message bar will vary depending on the computer’s settings and installed add-ins. Figure 1 displays examples of different wordings that may appear in the message bar.

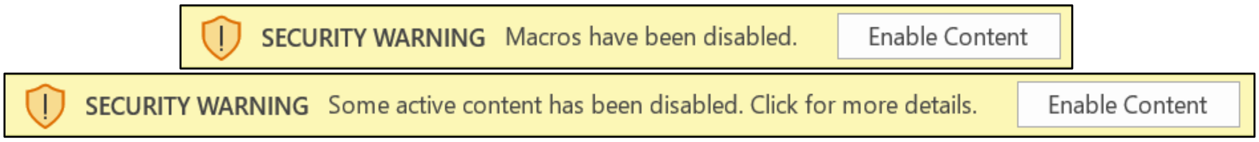



**Figure 1. Security warning message bars with the “Enable Content” option to enable macros.**

## 3.2. Organization

Although the toolbox does not provide a calculation cover sheet, adding one is strongly recommended. A calculation cover sheet captures project information, a description and purpose of the calculation, the assumptions for critical input parameters, a summary of the major conclusion and results, and a revision history.

Each toolbox has a similar appearance and organizational structure:

- The first worksheet, About, summarizes the purpose of the toolbox and gives contact information for the RMC software development team.
- The second worksheet, Terms and Conditions, contains the terms and conditions for use of the toolbox (IWR software).
- The third worksheet, Version History, contains the revision history. Semantic versioning is used in the format of MAJOR.MINOR.PATCH:
  - MAJOR – significant worksheet changes not compatible with previous versions.
  - MINOR – additional features or enhancements that do not fundamentally change the calculations.
  - PATCH – backward-compatible bug fixes.

- The fourth worksheet, References, lists the references cited for each calculation worksheet.

The workbook and worksheets are not protected to prevent unwanted changes. However, because the toolbox has user-defined functions (UDFs) and subroutines in Visual Basic, you cannot directly copy worksheets to another workbook without potentially losing functionality. A note in a bold red font at the upper right margin indicates if the selected worksheet includes such features.

At the top of each calculation worksheet, input information for the preparer and checker for quality control (QC) documentation and the calculation title in case multiple copies of the worksheet are created for different analysis scenarios (Figure 2). The footer of each calculation worksheet contains the version number, which can be cross-referenced with the revision history on the third worksheet.

| | | | | | |
|---|---|---|---|---|---|
| Prepared by: | | Office: | | Date: | |
| Checked by: | | Office: | | Date: | |
| Calculation Title: | | | | | |

**Figure 2. Calculation worksheet heading.**

User-specified input includes values and selections from drop-down lists. User input cells are light yellow, and these cells are unprotected. When cells use drop-down lists, a note in blue font in the right margin of the row alerts the user to use the drop-down list. Conditional formatting applies a gray background to cells that are not based on a user selection. When a user-specified value or calculated value is outside of acceptable ranges, the cell is orange to indicate caution to the user.

All units for user-specified input values are clearly labeled. Most user-specified input values use English units. However, values may be in metric where metric units are more common in practice (e.g., particle size in millimeters or permeability in centimeters per second). The toolbox may convert English units to metric units to perform some calculations or if required for a specific formula based on the reference material for the equation.

If the calculation worksheet is a function of headwater level, up to seven headwater and tailwater levels may be specified at the top of the worksheet. Tailwater may be required to calculate the net hydraulic head and hydraulic gradient. Specify the elevation datum by selecting one of three options from the drop-down list: ft-NAVD88, ft-NGVD29, and Other. The two datum selections include English units of length (feet). If Other is selected, provide a user-specified datum along with feet (e.g., ft-MSL [Mean Sea Level]). Figure 3 through Figure 5 illustrate the three possible scenarios.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Elevation datum | | ft-NAVD88 | | Specify datum | | | | ◄ Use drop-down list. |
| | | | | | | | | |
| HW (ft) | 195.5 | 201.6 | 213.5 | 218.9 | 223.0 | 234.0 | 239.0 | ◄ Headwater level, HW (ft-NAVD88) |
| TW (ft) | 184.0 | 184.0 | 184.0 | 184.0 | 184.0 | 184.0 | 184.0 | ◄ Tailwater level, TW (ft-NAVD88) |

**Figure 3. Headwater and tailwater input: NAVD88.**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Elevation datum | | ft-NGVD29 | | Specify datum | | | | ◄ Use drop-down list. |
| | | | | | | | | |
| HW (ft) | 195.5 | 201.6 | 213.5 | 218.9 | 223.0 | 234.0 | 239.0 | ◄ Headwater level, HW (ft-NGVD29) |
| TW (ft) | 184.0 | 184.0 | 184.0 | 184.0 | 184.0 | 184.0 | 184.0 | ◄ Tailwater level, TW (ft-NGVD29) |

**Figure 4. Headwater and tailwater input: NGVD29.**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Elevation datum | Other | | Specify datum | | ft-MSL | | ◄ Use drop-down list. | |
| HW (ft) | 195.5 | 201.6 | 213.5 | 218.9 | 223.0 | 234.0 | 239.0 | ◄ Headwater level, HW (ft-MSL) |
| TW (ft) | 184.0 | 184.0 | 184.0 | 184.0 | 184.0 | 184.0 | 184.0 | ◄ Tailwater level, TW (ft-MSL) |

**Figure 5. Headwater and tailwater input: User-specified datum.**

Most calculation worksheets break down complex analysis into computational steps following a logical sequence (Figure 6). Some simpler worksheets do not have steps. Generally, different methodologies are unique worksheets. Some worksheets may include multiple methodologies, which are labeled as options (Figure 7).

**Step 1: Select the method of analysis**

**Figure 6. Example of step banner.**

**Option 1: Riverside blanket (top stratum) for Cases 5, 7, and 8**

**Figure 7. Example of option banner.**

Some calculation worksheets can perform either a deterministic or probabilistic analysis. Although not required to perform a probabilistic analysis, Palisade @RISK software (standalone version or as part of the Palisade DecisionTools Suite) can customize the probabilistic analysis. A note appears in a bold red font at the upper right-hand margin of a calculation worksheet indicating if this feature is included with the toolbox.

User notes generally appear in the right margin of each calculation worksheet. Some notes are in blue or red font for heightened awareness. These notes include references to source materials for equations, figures, tables, pages, etc. If the RMC modified the source material, the reference citation says “adapted from” instead of “from.”

Tabular and/or graphical summaries are generally the primary output of the toolbox. The UDFs in the PlotScale module change the minimum and maximum values of the x-axis and y-axis for charts. If the calculation worksheet is a function of headwater level, you can define up to five headwater levels of interest and plot them as vertical reference lines. By selecting the chart and then selecting the Filter icon to display the filter pane, you can choose which data series to display. This is useful when computing the results from multiple methodologies, but not all are applicable or desired to display.

# 4. Background

Backward erosion piping (BEP) is the detachment of soil particles that occurs at a free, unfiltered surface in which the process gradually works its way toward the upstream or riverside of the embankment or its foundation until a continuous pipe is formed. Erosion initiates at the landside of a levee or the downstream side of a dam through unfiltered seepage exits that may exist due to penetrations or weaknesses in the overlying blanket, such as ditches, animal burrows (such as rodent or crawfish holes), root holes, former sand boils, cracks, or other thin or weak spots. Once erosion initiates, the pipe may progress horizontally through the foundation toward the impounded water if hydraulic gradients in the foundation are sufficiently high. The hydraulic gradients and flow must remain high near the upstream or riverside tip of the progressing pipe for particles to continue eroding. (Note: Floodside, waterside, and riverside are used interchangeably for levees.)

Initiation of BEP involves heave or blowout of the foundation materials at the landside toe of a levee or the downstream toe of a dam. Fluidization or liquefaction occurs near the seepage exit (that is, a zero effective stress condition) in which the sand expands and turns into a fluid state. Underseepage analysis methods are used to inform the likelihood of heave/blowout and include Blanket Theory (BT) and finite-element seepage analysis.

This toolbox deterministically and probabilistically assesses the likelihood of initiation of BEP (heave/blowout) using BT and the first-order, second-moment (FOSM) method of reliability analysis. Factors leading to initiation of BEP may not be addressed by traditional seepage analysis. This is particularly true when dealing with the movement of fine-grained material through the embankment or foundation of a dam or levee. This toolbox also assesses the quantity of flow or seepage using BT. These quantities from BT or finite-element analysis can be used to inform judgment for other nodes in the BEP event tree, such as whether the flow in a pipe is sufficiently high to keep the pipe open for progression, the ability to flood-fight for unsuccessful intervention, and the likelihood of breach.

## 4.1. Blanket Theory

Engineer Manual (EM) 1110-2-1913 (USACE 2000) provides closed-form BT solutions for seepage pressures and flows beneath levees. The solutions were developed for the geology along the Mississippi River, where a pervious stratum is overlaid by a less pervious top stratum or blanket. If the top stratum is composed of different soil layers of less pervious materials, the thickness and vertical permeabilities of these layers must be transformed into a uniform thickness and vertical permeability.

These solutions were originally developed in Technical Manual (TM) 3-424 (USACE 1956) and have been used by USACE since then in various forms. The solutions use a compilation of existing theories and methods, such as the method of fragments (Forchheimer 1917, Muskat 1937, Pavlovsky 1956, and Harr 1962), investigation of Mississippi River levee underseepage (Turnbull and Mansur 1959), and the effects of semi-pervious blankets (Bennett 1946 and Barron 1948). Engineer Research and Development Center (ERDC)/Geotechnical Structures Laboratory (GSL) Technical Report (TR)-18-24 (Brandon et al. 2018) highlighted errors in the BT equations in EM 1110-2-1913 (USACE 2000).

The fundamental assumption of this method is that equipotential lines at various critical parts of the flow region can be approximated by vertical lines that divide the region into sections or fragments. Equipotential lines produced from a finite-element analysis (FEA) are nearly vertical when the base width of the impervious levee ($L_2$) is greater than the thickness of the pervious substratum ($d$). Figure 8 illustrates the method of fragments, and BT solutions are limited to the case where $L_2/d \geq 1$ (Batool

2013). BT includes closed-form solutions for various blanket conditions to determine an equivalent upstream entrance distance ($x_1$) and/or equivalent downstream exit distance ($x_3$). Where the blanket is impervious, these distances are equal to the length of the blanket ($x_1 = L_1$ and/or $x_3 = L_3$, discussed in the following cases). Where there is no blanket, as in Figure 8, $x_1$ and/or $x_3$ for a homogeneous, isotropic pervious foundation is equal to 0.43$d$ from the method of fragments.

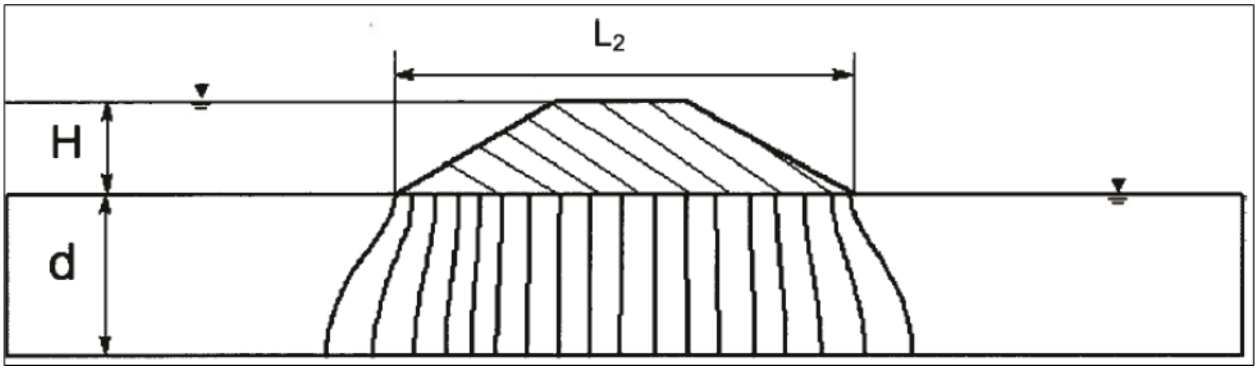



**Figure 8. Equipotential lines for levee cross section having $L_2/d$ > 1 for Blanket Theory Case 1 (ERDC/GSL TR-18-24).**

EM 1110-2-1913 (2000) includes seven standard BT cases. Case 1 is for no top stratum. Cases 2, 3, and 4 are for impervious top stratum conditions. Case 2 considers an impervious top stratum on both the riverside and landside of the levee; Case 3 considers an impervious top stratum on only the riverside; and Case 4 considers an impervious top stratum on only the landside. Cases 5, 6, and 7 are for semi-pervious top stratum conditions. Case 5 considers the presence of a semi-pervious top stratum on only the riverside; Case 6 considers the presence of a semi-pervious top stratum on only the landside; and Case 7 considers a semi-pervious top stratum on both the riverside and landside. Case 8 was reintroduced by ERDC/GSL TR-18-24 (Brandon et al. 2018), which adds a partially penetrating seepage barrier to Case 7. ERDC/GSL TR-18-24 (Brandon et al. 2018) provides derivations of the BT equations. Some key findings about the applicability of the BT solutions follow:

- The width of the impermeable boundary, which depends on the BT case and boundary condition, must be greater than or equal to the depth or thickness of the pervious substratum to ensure horizontal flow in the confined aquifer.

- The ratio of horizontal permeability of the pervious substratum to the vertical permeability of the blanket must be greater than or equal to 10 to maintain confined horizontal flow in the pervious substratum and vertical seepage or leakage through the blanket.

- The transition between semi-pervious and impervious blanket behavior occurs at a ratio of horizontal permeability of the pervious stratum to vertical permeability of the blanket between 1,000 and 4,000. At permeability ratios in the range of these values, the semi-pervious solutions (Cases 5, 6, and 7) produce the same values of heads and flows as the impervious solutions (Cases 3, 4, and 2, respectively), and the results of BT agree closely with FEA.

- The transformation from a fully pervious to a semi-pervious blanket occurs at a ratio of horizontal permeability of the pervious stratum to vertical permeability of the blanket of about 2. In other words, using the semi-pervious equations produces a more accurate determination of the flow and the excess hydraulic head for permeability ratios equal to or greater than 2 as compared to solutions considering the blanket as fully pervious (nonexistent). For permeability ratios less than 2, the presence of the blanket may be ignored, and the solutions for cases having no blanket provide more accurate results than the solutions for the semi-pervious cases.

To simplify the calculations, the semi-pervious top stratum is replaced by an equivalent impervious top stratum, such that the seepage beneath the levee is the same. For Cases 5, 7, and 8, this is accomplished by calculating an equivalent length ($x_1$) of an impervious top stratum for a semi-pervious stratum of length ($L_1$) on the riverside. For Cases 6, 7, and 8, this is accomplished by calculating an equivalent length ($x_3$) of an impervious top stratum for a semi-pervious stratum of length ($L_3$) on the landside. The semi-pervious stratum lengths are included with $L_2$ for the appropriate cases when evaluating the fundamental assumption of this method (Batool et al. 2015). ERDC/GSL TR-18-24 (Brandon et al. 2018) provides additional details.

For Cases 1 through 7, there are three zones for the pervious substratum corresponding to the riverside of the riverside levee toe, base width of the levee, and landside of the landside levee toe. Due to the presence of the partially penetrating seepage barrier, Case 8 subdivides the middle zone (base width of the levee) into two zones: riverside and landside of the seepage barrier. Figure 9 illustrates these zones and the parameters used in BT for Case 8.

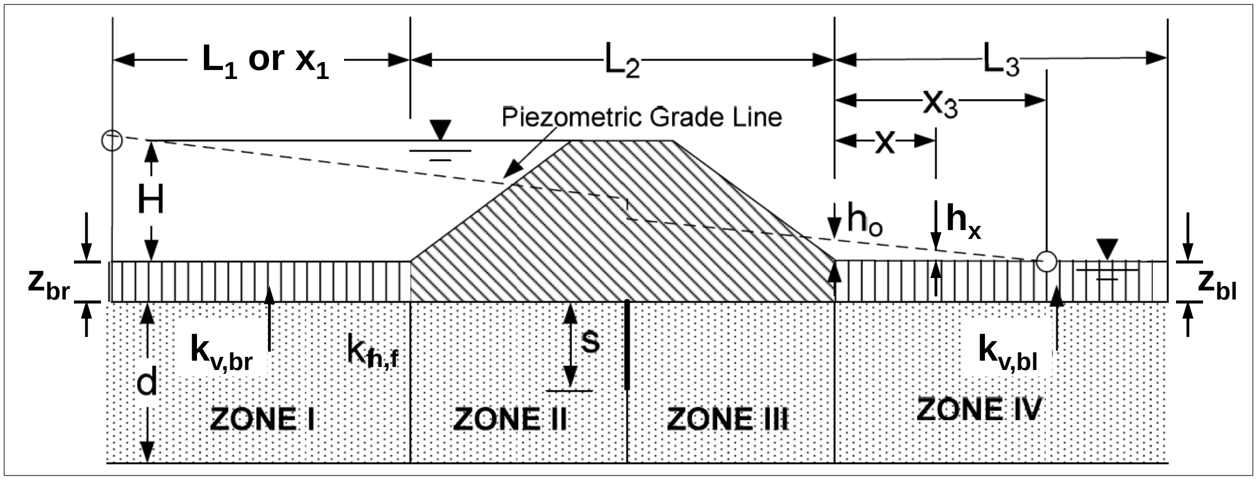



**Figure 9. BT parameters for BT Case 8 (adapted from ERDC/GSL TR-18-24).**

Since the flow in the pervious substratum is assumed to be horizontal, the hydraulic head loss is linear with horizontal distance, and the excess hydraulic head at the landside levee toe ($h_o$) and at a distance $x$ from the landside levee toe ($h_x$) are obtained by solving similar triangles. The distances used on the riverside ($L_1$ or $x_1$) and landside ($L_3$ or $x_3$) of the levee depend on the BT case and boundary condition and are discussed for each relevant case in subsequent chapters of this document.

## 4.2. Mississippi Valley Division Design Guidance

**The USACE Mississippi Valley Division (MVD) developed design guidance for levee underseepage protection for the Mississippi River and major tributary levees. Table 1, Table 2, and**

Table 3, from Change 2 of MVD's Division Regulation (DIVR) 1110-1-400 (USACE 1998), are guides for assessing the riverside and landside blanket vertical permeabilities if site-specific permeability data is not available. However, these are suggested design values developed for the Mississippi River alluvial valley. Use care when applying design values to risk assessments or in different geologic conditions.

In these tables, the landside blanket vertical permeabilities are less than the riverside blanket vertical permeabilities for the same soil type primarily due to cracking in the riverside blanket being repeatedly filled with silt from flood inundation and, to a lesser extent, the impounded water pressure on the riverside and cracking in the landside blanket due to seepage uplift.

**Table 1.**
**Suggested design values for riverside blanket permeability.**

| Soil Type | Riverside Blanket Thickness, $z_{br}$ (feet) | Vertical Permeability of Riverside Blanket, $k_{v,br}$ (centimeters/second) |
|---|---|---|
| Silty sand | <5<br>5 to 10 | 7.0E-04<br>2.5E-04 |
| Silt and sandy silt | <5<br>5 to 10<br>>10 | 2.0E-04<br>1.5E-04<br>1.0E-04 |
| Clay | <5<br>5 to 10<br>10 to 15<br>>15 | 0.8E-04<br>0.5E-04<br>0.2E-04<br>0.05E-04 |

**Table 2.**
**Suggested design values for landside blanket permeability.**

| Soil Type | Landside Blanket Thickness, $z_{bl}$ (feet) | Vertical Permeability of Landside Blanket, $k_{v,bl}$ (centimeters/second) | Permeability Ratio, $k_{h,f}$ / $k_{v,bl}$ |
|---|---|---|---|
| Silty sand | <5<br>5 to 10<br>>10 | 10.0E-0<br>8.0E-04<br>6.0E-04 | 125<br>150<br>200 |
| Silt and sandy silt | <5<br>5 to 10<br>10 to 15<br>>15 | 5.0E-04<br>4.0E-04<br>3.0E-04<br>2.0E-04 | 250<br>300<br>400<br>600 |
| Clay and silty clay | <5<br>5 to 10<br>10 to 15<br>15 to 20<br>>20 | 4.0E-04<br>3.0E-04<br>1.5E-04<br>0.5E-04<br>0.08E-04 | 250<br>400<br>800<br>2,500<br>15,000 |

**Table 3.**
**Suggested vertical permeability of landside blanket.**

| Landside Blanket Thickness, $z_{bl}$ (feet) | Vertical Permeability of Clay Landside Blanket, $k_{v,bl}$ (centimeter/second) | Vertical Permeability of Silt Landside Blanket, $k_{v,bl}$ (centimeter/second) | Vertical Permeability of Silty Sand Landside Blanket, $k_{v,bl}$ (centimeter/second) |
|---|---|---|---|
| 1 | 4.56E-04 | 5.70E-04 | 10.50E-04 |
| 2 | 4.30E-04 | 5.40E-04 | 10.10E-04 |
| 3 | 4.06E-04 | 5.10E-04 | 9.80E-04 |
| 4 | 3.80E-04 | 4.80E-04 | 9.40E-04 |
| 5 | 3.57E-04 | 4.50E-04 | 9.00E-04 |
| 6 | 3.30E-04 | 4.25E-04 | 8.60E-04 |
| 7 | 3.10E-04 | 4.00E-04 | 8.30E-04 |
| 8 | 2.88E-04 | 3.80E-04 | 8.00E-04 |
| 9 | 2.66E-04 | 3.60E-04 | 7.80E-04 |
| 10 | 2.45E-04 | 3.40E-04 | 7.55E-04 |
| 11 | 2.25E-04 | 3.23E-04 | 7.30E-04 |
| 12 | 2.08E-04 | 3.10E-04 | 7.10E-04 |
| 13 | 1.89E-04 | 2.95E-04 | 6.95E-04 |
| 14 | 1.72E-04 | 2.81E-04 | 6.80E-04 |
| 15 | 1.57E-04 | 2.70E-04 | 6.60E-04 |
| 16 | 1.41E-04 | 2.60E-04 | 6.50E-04 |
| 17 | 1.28E-04 | 2.50E-04 | 6.40E-04 |
| 18 | 1.14E-04 | 2.40E-04 | 6.2E-04 |
| 19 | 1.01E-04 | 2.30E-04 | 6.10E-04 |
| 20 | 0.90E-04 | 2.22E-04 | 6.00E-04 |
| 21 | 0.78E-04 | 2.14E-04 | 5.85E-04 |
| 22 | 0.69E-04 | 2.05E-04 | 5.80E-04 |
| 23 | 0.60E-04 | 1.98E-04 | 5.65E-04 |
| 24 | 0.52E-04 | 1.90E-04 | 5.60E-04 |
| 25 | 0.45E-04 | 1.83E-04 | 5.45E-04 |
| 26 | 0.38E-04 | 1.78E-04 | 5.40E-04 |
| 27 | 0.31E-04 | 1.71E-04 | 5.30E-04 |
| 28 | 0.25E-04 | 1.66E-04 | 5.20E-04 |
| 29 | 0.20E-04 | 1.60E-04 | 5.15E-04 |
| 30 | 0.15E-04 | 1.56E-04 | 5.10E-04 |

## 4.3. Finite Element Analysis

FEA is commonly used to perform seepage analysis using commercial off-the-shelf software like Geostudio’s SEEP/W, provided the model and boundary conditions are properly defined. FEA estimates seepage or flows and excess hydraulic heads (and thus, factors of safety against heave/blowout) for complex foundation profiles and levee geometries.

Unlike slope stability analysis, there is currently no probabilistic analysis method for finite-element seepage analysis. Any deterministic two-dimensional (2D) analytical model can be evaluated probabilistically using first-order, second-moment method of reliability analysis as described in Engineer Technical Letter (ETL) 1110-2-561 (2006). USACE has performed probabilistic analysis using this method for many years.

The Taylor series method evaluates the coefficient of variation of the factor of safety (FS) against heave/blowout and computes the lognormal reliability index. The reliability index is directly and uniquely related to the probability of unsatisfactory performance. The probability of unsatisfactory performance merely indicates the likelihood that an adverse event or condition; in this case, a factor of safety against heave/blowout less than 1 occurs. The outcome of these reliability analyses is the probability of the factor of safety against heave/blowout being less than 1. This is undesirable, but it does not automatically result in breach of the dam or levee.

# 5. Blanket Theory Transformation

The top stratum is usually composed of several layers of different soils instead of one uniform material. This worksheet transforms the thickness and vertical permeabilities of these layers into a blanket of uniform thickness and vertical permeability to use in the BT solutions. This worksheet is not used in any of the calculations. It may be used to inform selecting distributions for the transformed thickness and vertical permeability of the riverside and landside blankets (top strata) and effective thickness of the landside blanket (top stratum) on other calculation worksheets.

The concept of a transformed thickness of the top stratum is applicable only to semi-pervious top stratum cases and not for impervious top stratum cases. Cases 5, 6, 7, and 8 are for semi-pervious top stratum conditions. Case 5 considers the presence of a semi-pervious top stratum on only the riverside; Case 6 considers the presence of a semi-pervious top stratum on only the landside; Case 7 considers a semi-pervious top stratum on both the riverside and landside; and Case 8 is the same as in Case 7, except a partially penetrating seepage barrier is present at the levee centerline. The semi-pervious riverside blanket thickness and vertical permeability must be transformed for Cases 5, 7, and 8 (Option 1), and the semi-pervious landside blanket thickness and vertical permeability must be transformed for Cases 6, 7, and 8 (Option 2). Use the drop-down list to select the case. Cells that do not apply have a gray background.

## 5.1. Transformed Blanket Thickness for Length to Effective Seepage Exit

If the vertical permeability of each soil layer ($k_{vi}$) is known, the top stratum can be transformed into a blanket characterized by an overall thickness and vertical permeability. The transformation procedure in EM 1110-2-1913 (2000) consists of assuming a uniform vertical permeability for the generalized top stratum equal to the vertical permeability of the most impervious soil layer ($k_{v,b}$) and using a transformation factor ($F_t$) to determine the corresponding thickness for the entire stratum ($z_b$). This generalized top stratum is used in the seepage analysis to compute of the length to the effective seepage exit. This transformation procedure can be applied to the riverside and/or landside blankets, depending on the case. The thickness transformation factor is calculated in Equation 1 as the ratio of the vertical permeability of the blanket, after it was transformed ($k_{v,b}$) to the vertical permeability of the material being transformed ($k_{vi}$).

$$F_{ti} = \frac{k_{v,b}}{k_{vi}} \tag{1}$$

where:

$k_{v,b}$ = minimum vertical permeability of the soil layers comprising the top stratum
$k_{vi}$ = vertical permeability of soil layer $i$

If the in situ thickness of each soil layer ($z_i$) is known, the corresponding transformed thickness ($z_{bi}$) of each soil layer can be expressed using Equation 2.

$$z_{bi} = z_i F_{ti} \tag{2}$$

where:

$z_i$ = in situ thickness of soil layer $i$

The total transformed thickness of the blanket ($z_b$) is calculated using Equation 3 as the sum of the transformed thicknesses.

$$z_b = \sum_{i=1}^{n} z_{bi} \tag{3}$$

where:

$n$ = total number of soil layers comprising the blanket (top stratum)

Figure 10 illustrates the calculations for transformed riverside blanket (top stratum) thickness for length to effective seepage exit. The process is the same for the landside blanket (top stratum) as shown in Figure 11.

**Option 1: Riverside blanket (top stratum) for Cases 5, 7, and 8**

| Layer, i | Soil Description | $z_i$ (ft) | $k_{vi}$ (cm/sec) | $F_{ti}$ | $z_{br,i}$ (ft) |
|---|---|---|---|---|---|
| 1 | Sandy silt | 5.0 | 2.00E-04 | 0.5 | 2.5 |
| 2 | Clay | 8.0 | 1.00E-04 | 1.0 | 8.0 |
| 3 | Silty sand | 5.0 | 1.00E-03 | 0.1 | 0.5 |
| - | | | | - | - |
| - | | | | - | - |
| 3 | Total | 18.0 | | | 11.0 |

Note: $z_i$ and $k_{vi}$ are the actual in situ thickness and vertical permeability, respectively.

| | | |
|---|---|---|
| Total in situ thickness of blanket, $z = \Sigma z_i$ | 18.0 | ft |
| Transformed vertical permeability of blanket, $k_{v,br} = \mathrm{MIN}(k_{vi})$ | 1.00E-04 | cm/sec |
| Transformation factor for layer i, $F_{ti} = k_{v,br}/k_{vi}$ | | |
| Transformed thickness of layer i, $z_{br,i} = z_i F_{ti}$ | | |
| Transformed thickness of blanket, $z_{br} = \Sigma z_{br,i}$ | 11.0 | ft |

**Figure 10. Riverside blanket thickness and vertical permeability transformation for Cases 5, 7, and 8.**

## 5.2. Transformed Blanket Thickness for Allowable Uplift Pressure

The transformed thickness of the landside blanket (top stratum) used to compute the length to the effective seepage exit ($z_{bl}$) may or may not be the same as the effective thickness of the landside top stratum ($z_t$) used to compute the allowable uplift pressure beneath the top stratum. The effective thickness of the landside blanket (top stratum) used to compute the allowable uplift pressure beneath the top stratum ($z_t$) equals the in situ thicknesses of all strata above the base of the least pervious stratum plus the transformed thicknesses of the underlying more pervious top strata (as described in the previous section). Therefore, $z_b$ equals $z_t$ only when the least pervious stratum is at the ground surface. Figure 11 illustrates the additional calculation for transformed thickness of the landside blanket (top stratum) used to compute the length to the effective seepage exit.

| Option 2: Landside blanket (top stratum) for Cases 6, 7, and 8 | | | | | | |
|---|---|---|---|---|---|---|
| Layer, i | Soil Description | $z_i$ (ft) | $k_{vi}$ (cm/sec) | $F_{ti}$ | $z_{bl,i}$ (ft) | $z_{ti}$ (ft) |
| 1 | Sandy silt | 5.0 | 2.00E-04 | 0.5 | 2.5 | 5.0 |
| 2 | Clay | 8.0 | 1.00E-04 | 1.0 | 8.0 | 8.0 |
| 3 | Silty sand | 5.0 | 1.00E-03 | 0.1 | 0.5 | 0.5 |
| - | | | | - | - | - |
| - | | | | - | - | - |
| 3 | Total | 18.0 | | | 11.0 | 13.5 |

Note: $z_i$ and $k_{vi}$ are the actual in situ thickness and vertical permeability, respectively.

| | | |
|---|---|---|
| Total in situ thickness of blanket, $z = \Sigma z_i$ | 18.0 | ft |
| Transformed vertical permeability of blanket, $k_{v,bl} = \mathrm{MIN}(k_{vi})$ | 1.00E-04 | cm/sec |
| Transformation factor for layer i, $F_{ti} = k_{v,bl}/k_{vi}$ | | |
| Transformed thickness of layer i, $z_{ti} = z_i F_{ti}$ | | |
| Transformed thickness of blanket for computing length to effective seepage exit, $z_{bl} = \Sigma z_{bi,i}$ | 11.0 | ft |
| Effective thickness of blanket for computing allowable uplift pressure beneath the top stratum, $z_t = \Sigma z_{ti}$ | 13.5 | ft |

**Figure 11. Landside blanket thickness and vertical permeability transformation for Cases 6, 7, and 8.**

# 6. Blanket Theory Case 1

Case 1 has no landside and riverside top stratum (no blankets). The pervious substratum is divided into three zones to apply the method of fragments as shown in step 2 (Figure 15).

## 6.1. Method of Analysis

In step 1, use the drop-down list to select the method of analysis (probabilistic or deterministic). There are two options for probabilistic analysis. The first performs 1,000 iterations (judged adequate for most applications) without using Palisade's @RISK software. This provides flexibility if an @RISK software license is not available. The second uses @RISK to customize the probabilistic analysis. Use the drop-down list to select Yes if @RISK is used and No if @RISK is not used. Figure 12 through Figure 14 illustrate the three possible scenarios.

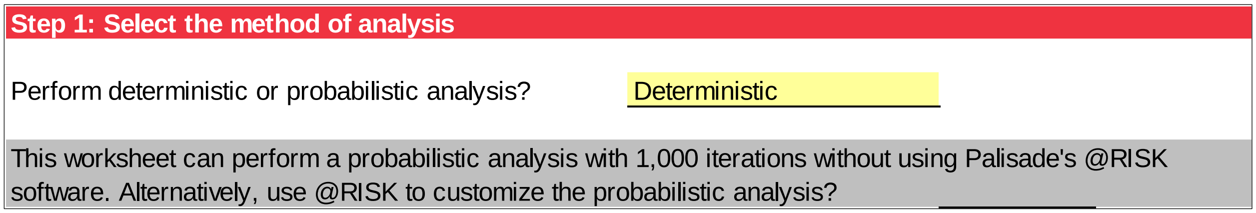



**Figure 12. Step 1 of BT Case 1 worksheet: Deterministic analysis.**

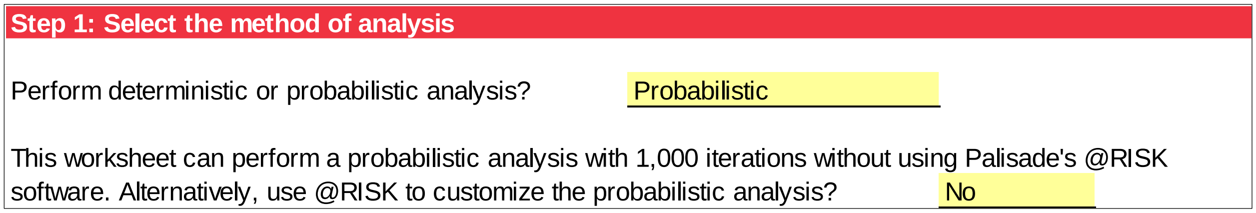



**Figure 13. Step 1 of BT Case 1 worksheet: Probabilistic analysis without using @RISK.**

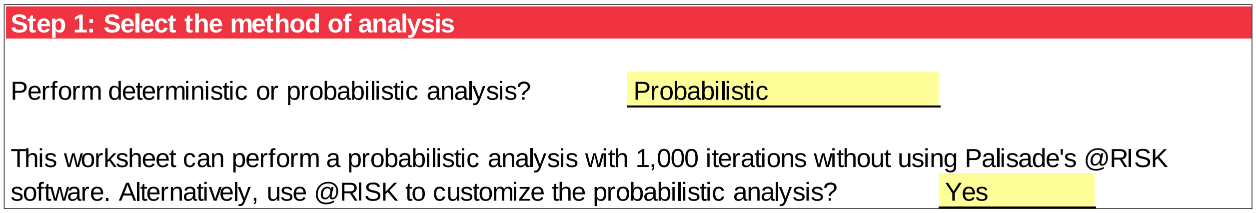
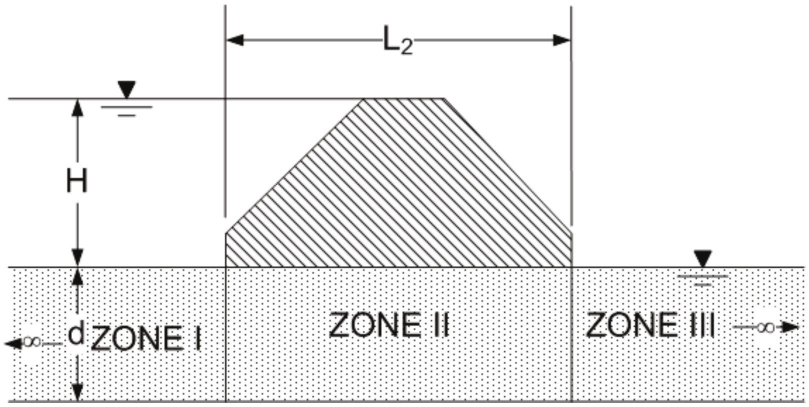



**Figure 14. Step 1 of BT Case 1 worksheet: Probabilistic analysis using @RISK.**

## 6.2. Levee Geometry

Step 2 characterizes the levee geometry. The input includes the levee crest elevation, landside levee toe elevation, and base width of levee ($L_2$) as illustrated in Figure 15.

**Step 2: Assess the levee geometry**




| | | |
|---|---|---|
| Levee crest elevation | 45.0 | ft-NAVD88 |
| Landside toe elevation | 20.0 | ft-NAVD88 |
| Base width of levee, $L_2$ | 110.0 | ft |

**Figure 15. Step 2 of BT Case 1 worksheet: Levee geometry.**

## 6.3. Pervious Substratum Characterization

Step 3 characterizes the pervious stratum. The selections in step 1 affect the input for step 3, and cells that do not apply have a gray background. These cells are not used in subsequent calculations even if data is present. The input includes the thickness (*d*) and horizontal permeability ($k_{h,f}$) of the pervious substratum.

For deterministic analysis, input only the most likely values. The mean values used for subsequent calculations are the most likely (or mode) values. Figure 16 illustrates the deterministic input.

**Step 3: Characterize the pervious substratum**

| Parameter | Units | Minimum | Most Likely | Maximum |
|---|---|---|---|---|
| Thickness, d | ft | 10.0 | 20.0 | 40.0 |
| Horizontal permeability, $k_{h,f}$ | cm/sec | 1.00E-02 | 4.00E-02 | 9.00E-02 |

| Parameter | Units | Mean | @RISK Formula | Mean |
|---|---|---|---|---|
| d | ft | #NAME? | | 20.0 |
| $k_{h,f}$ | cm/sec | #NAME? | | 4.00E-02 |

**Figure 16. Step 3 of BT Case 1 worksheet: Deterministic input.**

For probabilistic analysis without using @RISK, input the minimum and maximum values in addition to the most likely value, and triangular distributions represent the random variables. The mean values used in subsequent calculations are the average of the minimum, most likely, and maximum values. Figure 17 illustrates the probabilistic input without using @RISK.

| Step 3: Characterize the pervious substratum | | | | |
|---|---|---|---|---|
| Parameter | Units | Minimum | Most Likely | Maximum |
| Thickness, d | ft | 10.0 | 20.0 | 40.0 |
| Horizontal permeability, $k_{h,f}$ | cm/sec | 1.00E-02 | 4.00E-02 | 9.00E-02 |

| Parameter | Units | Mean | @RISK Formula | Mean |
|---|---|---|---|---|
| d | ft | #NAME? | | 23.3 |
| $k_{h,f}$ | cm/sec | #NAME? | | 4.67E-02 |

**Figure 17. Step 3 of BT Case 1 worksheet: Probabilistic input without using @RISK.**

For probabilistic analysis using @RISK, input the minimum, most likely, and maximum values, and use an @RISK formula for a triangular distribution in the third column as a default. Alternatively, input a valid @RISK distribution in lieu of this default formula, and the user-specified input displays in the fourth column. The mean values used for subsequent calculations are the mean for the @RISK distribution entered in the third column. Figure 18 illustrates the probabilistic input using @RISK.

| Step 3: Characterize the pervious substratum | | | | |
|---|---|---|---|---|
| Parameter | Units | Minimum | Most Likely | Maximum |
| Thickness, d | ft | 10.0 | 20.0 | 40.0 |
| Horizontal permeability, $k_{h,f}$ | cm/sec | 1.00E-02 | 4.00E-02 | 9.00E-02 |

| Parameter | Units | Mean | @RISK Formula | Mean |
|---|---|---|---|---|
| d | ft | 23.3 | =@RiskTriang(F42,G42,H42) | 23.3 |
| $k_{h,f}$ | cm/sec | 4.67E-02 | =@RiskTriang(F43,G43,H43) | 4.67E-02 |

**Figure 18. Step 3 of BT Case 1 worksheet: Probabilistic input using @RISK.**

If using @RISK to perform probabilistic analysis, delete unnecessary calculation worksheets because the simulation is performed for all worksheets in the workbook, which this is time consuming. If cycling through iterations using @RISK, the displayed results are no longer mean values of the random variables; they are the selected iteration.

## 6.4. Blanket Theory Assumptions

Step 4 checks the BT (method of fragments) assumptions against the input parameters to ensure essentially vertical equipotential lines. For deterministic analysis, the assumptions are checked for the most likely values of the random variables. For probabilistic analysis, the assumptions are checked for the mean values of the random variables. Values outside of the model assumptions have an orange background. Figure 19 illustrates the check of BT assumptions.

| Step 4: Check blanket theory (method of fragments) assumptions | |
|---|---|
| Assumption | Mean |
| Essentially vertical equipotential lines, $L_2/d \geq 1$ | 4.7 |

**Figure 19. Step 4 of BT Case 1 worksheet: BT assumptions.**

## 6.5. Seepage Characterization

Step 5 calculates the net hydraulic head on the levee ($H$) using Equation 4.

$$H = HW - TW \tag{4}$$

where:

$HW$ = headwater level
$TW$ = tailwater level

The flow or seepage per unit length of the levee ($Q_s$) is calculated using Equation 5.

$$Q_s = k_{h,f} H \frac{d}{0.86d + L_2} \tag{5}$$

where:

$k_{h,f}$ = horizontal permeability of the pervious substratum
$H$ = net hydraulic head on the levee
$L_2$ = base width of the levee
$d$ = thickness of the pervious substratum

For deterministic analysis, $Q_s$ is calculated for each headwater level using the most likely values of the random variables and is summarized in a table. Cells that do not apply have a gray background. For probabilistic analysis, $Q_s$ is calculated as described for the deterministic analysis but for the mean values of the random variables, and multiple iterations are performed by sampling the distributions in step 3. For probabilistic analysis without using @RISK, 1,000 iterations are used. For probabilistic analysis using @RISK, the number of iterations is user-specified. If cycling through iterations using @RISK, the displayed results are no longer mean values; they are the selected iteration. Figure 20 illustrates the deterministic output, and Figure 21 illustrates the probabilistic output without using @RISK.

**Step 5: Assess the flow or seepage**

Net hydraulic head, H = MAX[(HW − Landside Toe Elevation), 0)
Flow or seepage (per unit length), $Q_s = k_{h,f}H\ [d/(0.86d + L_2)]$

Iterations: #N/A

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| HW (ft) | 15.0 | 20.0 | 25.0 | 30.0 | 35.0 | 40.0 | 45.0 |
| TW (ft) | 20.0 | 20.0 | 20.0 | 20.0 | 20.0 | 20.0 | 20.0 |
| H (ft) | 0.0 | 0.0 | 5.0 | 10.0 | 15.0 | 20.0 | 25.0 |
| $h_o$ (ft) | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| $Q_S$ (cfs/ft) | 0.00E+00 | 0.00E+00 | 1.03E-03 | 2.06E-03 | 3.10E-03 | 4.13E-03 | 5.16E-03 |
| $Q_S$ (gpm/ft) | 0.00E+00 | 0.00E+00 | 4.63E-01 | 9.26E-01 | 1.39E+00 | 1.85E+00 | 2.32E+00 |

**Figure 20. Step 5 of BT Case 1 worksheet: Deterministic output.**

**Step 5: Assess the flow or seepage**

Net hydraulic head, H = MAX[(HW − Landside Toe Elevation), 0)

Flow or seepage (per unit length), $Q_s = k_{h,f}H\ [d/(0.86d + L_2)]$ Iterations: 1000

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| HW (ft) | 15.0 | 20.0 | 25.0 | 30.0 | 35.0 | 40.0 | 45.0 |
| TW (ft) | 20.0 | 20.0 | 20.0 | 20.0 | 20.0 | 20.0 | 20.0 |
| H (ft) | 0.0 | 0.0 | 5.0 | 10.0 | 15.0 | 20.0 | 25.0 |
| $h_o$ (ft) | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| $Q_S$ (cfs/ft) | 0.00E+00 | 0.00E+00 | 1.37E-03 | 2.75E-03 | 4.12E-03 | 5.49E-03 | 6.87E-03 |
| $Q_S$ (gpm/ft) | 0.00E+00 | 0.00E+00 | 6.16E-01 | 1.23E+00 | 1.85E+00 | 2.47E+00 | 3.08E+00 |

**Figure 21. Step 5 of BT Case 1 worksheet: Probabilistic output without using @RISK.**

## 6.6. Likelihood of Heave/Blowout at Landside Toe

Since the excess hydraulic head at the landside levee toe is zero, vertical seepage exit gradients are not computed for this case.

## 6.7. Likelihood of Heave/Blowout at Given Distance from Landside Toe

Since the excess hydraulic head at the landside levee toe and at any distance *x* from the landside levee toe are zero, vertical seepage exit gradients at any distance *x* from the landside levee toe are not computed for this case.

# 7. Blanket Theory Case 2

Case 2 has an impervious top stratum on both the riverside and landside of the levee. The pervious substratum is divided into three zones to apply the method of fragments as shown in Figure 22.

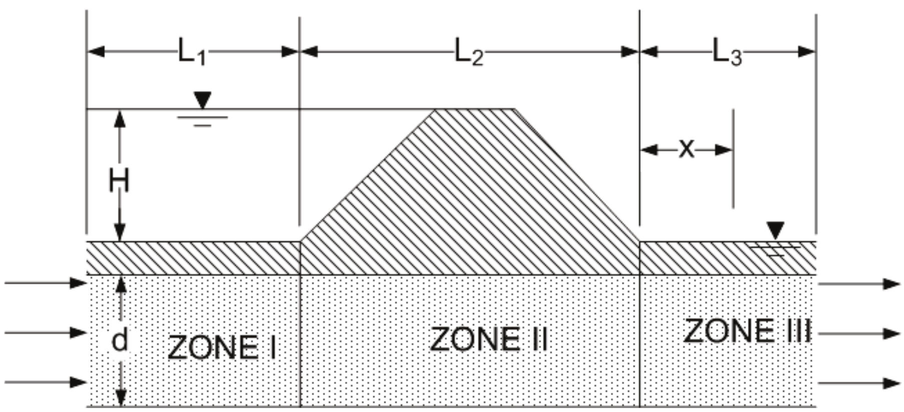
## 7.1. Method of Analysis

The method of analysis is the same as in Case 1.

## 7.2. Levee Geometry

Step 2 characterizes the levee geometry. The input includes the levee crest elevation, landside toe elevation, distance from the riverside levee toe to the river ($L_1$), base width of the levee ($L_2$), and length of the foundation and top stratum beyond the landside levee toe ($L_3$), as illustrated in Figure 22.

**Step 2: Assess the levee geometry**




| | | |
|---|---|---|
| Levee crest elevation | 45.0 | ft-NAVD88 |
| Landside toe elevation | 20.0 | ft-NAVD88 |
| Base width of levee, $L_2$ | 110.0 | ft |
| Distance from river to riverside toe, $L_1$ | 100.0 | ft |
| Length of foundation and top stratum beyond landside toe, $L_3$ | 250.0 | ft |

**Figure 22. Step 2 of BT Case 2 worksheet: Levee geometry.**

## 7.3. Pervious Substratum Characterization

The pervious stratum characterization is the same as in Case 1.

## 7.4. Landside Blanket (Top Stratum) Characterization

Step 4 characterizes the landside (top stratum). The selections in step 1 affect the input for step 4, and cells that do not apply have a gray background. These cells are not used in subsequent calculations even if data is present. The input includes thickness ($z_{bl}$) and saturated unit weight ($\gamma_{sat}$) of the landside blanket. For deterministic analysis, input only the most likely values. The mean values used for subsequent calculations are the most likely (or mode) values.

The critical vertical seepage exit gradient for heave/blowout ($i_{cv}$), as defined in Terzaghi, Peck, and Mesri (1996), is calculated using Equation 6.

$$i_{cv} = \frac{\gamma'}{\gamma_w} = \frac{\gamma_{sat} - \gamma_w}{\gamma_w} \tag{6}$$

where:

$\gamma'$ = submerged unit weight of the landside blanket
$\gamma_{sat}$ = saturated unit weight of the landside blanket
$\gamma_w$ = unit weight of water

Figure 23 illustrates the deterministic input.

| Step 4: Characterize the landside blanket (top stratum) | | | | |
|---|---|---|---|---|
| Parameter | Units | Minimum | Most Likely | Maximum |
| Thickness, $z_{bl}$ | ft | 5.0 | 10.0 | 18.0 |
| Saturated unit weight, $\gamma_{sat}$ | pcf | 110.0 | 115.0 | 120.0 |
| Critical vertical exit gradient, $i_{cv} = (\gamma_{sat} - \gamma_w) / \gamma_w$ | - | 0.763 | 0.843 | 0.923 |

| Parameter | Units | Mean | @RISK Formula | Mean |
|---|---|---|---|---|
| $z_{bl}$ | ft | #NAME? | | 10.0 |
| $\gamma_{sat}$ | pcf | #NAME? | | 115.0 |
| $i_{cv}$ | - | | | 0.843 |

**Figure 23. Step 4 of BT Case 2 worksheet: Deterministic input.**

For probabilistic analysis without using @RISK, input the minimum and maximum values in addition to the most likely value, and triangular distributions represent the random variables. The mean values used in subsequent calculations are the average of the minimum, most likely, and maximum values. The triangular distribution for $i_{cv}$ is calculated using the minimum, most likely, and maximum values for $\gamma_{sat}$. Since $i_{cv}$ is a function of the random variable $\gamma_{sat}$, it is correlated with $\gamma_{sat}$, and is also a random variable. Figure 24 is an example of the probabilistic input without using @RISK.

**Step 4: Characterize the landside blanket (top stratum)**

| Parameter | Units | Minimum | Most Likely | Maximum |
|---|---|---|---|---|
| Thickness, $z_{bl}$ | ft | 5.0 | 10.0 | 18.0 |
| Saturated unit weight, $\gamma_{sat}$ | pcf | 110.0 | 115.0 | 120.0 |
| Critical vertical exit gradient, $i_{cv} = (\gamma_{sat} - \gamma_w) / \gamma_w$ | - | 0.763 | 0.843 | 0.923 |

| Parameter | Units | Mean | @RISK Formula | Mean |
|---|---|---|---|---|
| $z_{bl}$ | ft | #NAME? | | 11.0 |
| $\gamma_{sat}$ | pcf | #NAME? | | 115.0 |
| $i_{cv}$ | - | | | 0.843 |

**Figure 24. Step 4 of BT Case 2 worksheet: Probabilistic input without using @RISK.**

For probabilistic analysis using @RISK, input the minimum, most likely, and maximum values, and use an @RISK formula for a triangular distribution in the third column as a default. Alternatively, input a valid @RISK distribution in lieu of this default formula, and the user-specified input displays in the fourth column. The mean values used for subsequent calculations are the mean for the @RISK distribution in the third column. Figure 25 illustrates the probabilistic input using @RISK.

**Step 4: Characterize the landside blanket (top stratum)**

| Parameter | Units | Minimum | Most Likely | Maximum |
|---|---|---|---|---|
| Thickness, $z_{bl}$ | ft | 5.0 | 10.0 | 18.0 |
| Saturated unit weight, $\gamma_{sat}$ | pcf | 110.0 | 115.0 | 120.0 |
| Critical vertical exit gradient, $i_{cv} = (\gamma_{sat} - \gamma_w) / \gamma_w$ | - | 0.763 | 0.843 | 0.923 |

| Parameter | Units | Mean | @RISK Formula | Mean |
|---|---|---|---|---|
| $z_{bl}$ | ft | 11.0 | =@RiskTriang(F55,G55,H55) | 11.0 |
| $\gamma_{sat}$ | pcf | 115.0 | =@RiskTriang(F56,G56,H56) | 115.0 |
| $i_{cv}$ | - | | | 0.843 |

**Figure 25. Step 4 of BT Case 2 worksheet: Probabilistic input using @RISK.**

If using @RISK to perform probabilistic analysis, delete unnecessary calculation worksheets because the simulation is performed for all worksheets in the workbook, and this is time consuming. If cycling through iterations using @RISK, the displayed results are no longer mean values of the random variables; they are the selected iteration's values.

## 7.5. Blanket Theory Assumptions

Step 5 checks the BT (method of fragments) assumptions against the input parameters to ensure essentially vertical equipotential lines. For deterministic analysis, the assumptions are checked for the most likely values of the random variables. For probabilistic analysis, the assumptions are checked for the mean values of the random variables. Values outside of the model assumptions have an orange background. Figure 26 illustrates the check of BT assumptions.

| Step 5: Check blanket theory (method of fragments) assumptions | |
|---|---|
| Assumption | Mean |
| Essentially vertical equipotential lines, $(L_1 + L_2 + L_3) / d \geq 1$ | 19.7 |

**Figure 26. Step 5 of BT Case 2 worksheet: BT assumptions.**

## 7.6. Seepage Characterization

Step 6 calculates the net hydraulic head on the levee ($H$) is calculated the same as in Case 1. The flow or seepage per unit length of the levee ($Q_s$) is calculated using Equation 7.

$$Q_s = k_{h,f} H \frac{d}{L_1 + L_2 + L_3} \tag{7}$$

where:

$k_{h,f}$ = horizontal permeability of the pervious substratum
$H$ = net hydraulic head on the levee
$L_1$ = distance from the riverside levee toe to the river
$L_2$ = base width of the levee
$L_3$ = length of the foundation and top stratum beyond the landside levee toe
$d$ = thickness of the pervious substratum

The format for the tabular output is the same as in Case 1.

## 7.7. Likelihood of Heave/Blowout at Landside Toe

Step 7 calculates the excess hydraulic head at the landside levee toe ($h_o$) using Equation 8.

$$h_o = H \frac{L_3}{L_1 + L_2 + L_3} \tag{8}$$

where:

$H$ = net hydraulic head on the levee
$L_1$ = distance from riverside levee toe to the river
$L_2$ = base width of the levee
$L_3$ = length of the foundation and top stratum beyond the landside levee toe

The vertical seepage exit gradient at the landside levee toe ($i_v$) is calculated using Equation 9.

$$i_v = \frac{h_o}{z_{bl}} \tag{9}$$

where:

$h_o$ = excess hydraulic head at the landside levee toe
$z_{bl}$ = thickness of the landside blanket (top stratum)

The factor of safety against heave/blowout (based on vertical seepage gradients) at the landside levee toe ($FS_{vg}$) is calculated using Equation 10.

$$FS_{vg} = \frac{i_{cv}}{i_v} \tag{10}$$

where:

$i_{cv}$ = critical vertical seepage exit gradient for heave/blowout
$i_v$ = vertical seepage exit gradient at the landside levee toe

For deterministic analysis, $FS_{vg}$ is calculated for each headwater level using the most likely values of the random variables and is summarized in a table. Cells that do not apply have a gray background. For probabilistic analysis, $FS_{vg}$ is calculated as described for the deterministic analysis but for the mean values of the random variables, and multiple iterations are performed by sampling the distributions in steps 3 and 4. The probability of heave/blowout is equal to the percentage of iterations that resulted in a $FS_{vg}$ less than 1 [P($FS_{vg}$ < 1)]. For probabilistic analysis performed without using @RISK, 1,000 iterations are used. For probabilistic analysis using @RISK, the number of iterations is user-specified, and "@RISK" displays in parentheses after the number of iterations for this scenario. If cycling through iterations using @RISK, the displayed results are no longer mean values; they are the selected iteration's values. For deterministic and probabilistic analyses, cells with $FS_{vg}$ less than 1 have an orange background. Figure 27 illustrates the deterministic tabular output, and Figure 28 illustrates the probabilistic tabular output without using @RISK.

**Step 7: Assess the likelihood of heave/blowout at the landside toe**

Excess hydraulic head at landside toe, $h_o = H [L_3/(L_1 + L_2 + L_3)]$
Vertical exit gradient at landside toe, $i_v = h_o/(z_t = z_{bl})$
Factor of safety against heave/blowout at landside toe, $FS_{vg} = i_{cv}/i_v$
Probability of a FS against heave/blowout less than 1 at landside toe, P($FS_{vg}$ < 1)
Iterations: #N/A

Likelihood of initiation of backward erosion piping at landside toe

| HW (ft) | 15.0 | 20.0 | 25.0 | 30.0 | 35.0 | 40.0 | 45.0 |
|---|---|---|---|---|---|---|---|
| TW (ft) | 20.0 | 20.0 | 20.0 | 20.0 | 20.0 | 20.0 | 20.0 |
| H (ft) | 0.0 | 0.0 | 5.0 | 10.0 | 15.0 | 20.0 | 25.0 |
| $h_o$ (ft) | 0.0 | 0.0 | 2.7 | 5.4 | 8.2 | 10.9 | 13.6 |
| $i_v$ | 0.000 | 0.000 | 0.272 | 0.543 | 0.815 | 1.087 | 1.359 |
| $FS_{vg}$ | ∞ | ∞ | 3.10 | 1.55 | 1.03 | 0.78 | 0.62 |
| P($FS_{vg}$ < 1) | #N/A | #N/A | #N/A | #N/A | #N/A | #N/A | #N/A |

**Figure 27. Step 7 of BT Case 2 worksheet: Deterministic tabular output.**

**Step 7: Assess the likelihood of heave/blowout at the landside toe**

Excess hydraulic head at landside toe, $h_o = H\,[L_3/(L_1 + L_2 + L_3)]$

Vertical exit gradient at landside toe, $i_v = h_o/(z_t = z_{bl})$

Factor of safety against heave/blowout at landside toe, $FS_{vg} = i_{cv}/i_v$

Probability of a FS against heave/blowout less than 1 at landside toe, $P(FS_{vg} < 1)$

Iterations: 1000

Likelihood of initiation of backward erosion piping at landside toe

| HW (ft) | 15.0 | 20.0 | 25.0 | 30.0 | 35.0 | 40.0 | 45.0 |
|---|---|---|---|---|---|---|---|
| TW (ft) | 20.0 | 20.0 | 20.0 | 20.0 | 20.0 | 20.0 | 20.0 |
| H (ft) | 0.0 | 0.0 | 5.0 | 10.0 | 15.0 | 20.0 | 25.0 |
| $h_o$ (ft) | 0.0 | 0.0 | 2.7 | 5.4 | 8.2 | 10.9 | 13.6 |
| $i_v$ | 0.000 | 0.000 | 0.247 | 0.494 | 0.741 | 0.988 | 1.235 |
| $FS_{vg}$ | ∞ | ∞ | 3.41 | 1.71 | 1.14 | 0.85 | 0.68 |
| $P(FS_{vg} < 1)$ | 0.00E+00 | 0.00E+00 | 0.00E+00 | 3.20E-02 | 3.36E-01 | 7.54E-01 | 9.55E-01 |

**Figure 28. Step 7 of BT Case 2 worksheet: Probabilistic tabular output without using @RISK.**

At the end of step 7, summary plots are generated. The first plot is the mean FS against heave/blowout (red solid line) and vertical seepage exit gradient at the landside levee toe (green solid line) as functions of headwater level. $FS_{vg}$ is plotted on the primary y-axis, and $i_v$ is plotted on the secondary y-axis. Horizontal reference lines display for the mean critical vertical seepage exit gradient (green dashed line) and $FS_{vg}$ of 1 (red dashed line). If cycling through iterations using @RISK, the displayed results are no longer mean values; they are the selected iteration. Figure 29 illustrates the deterministic graphical output.

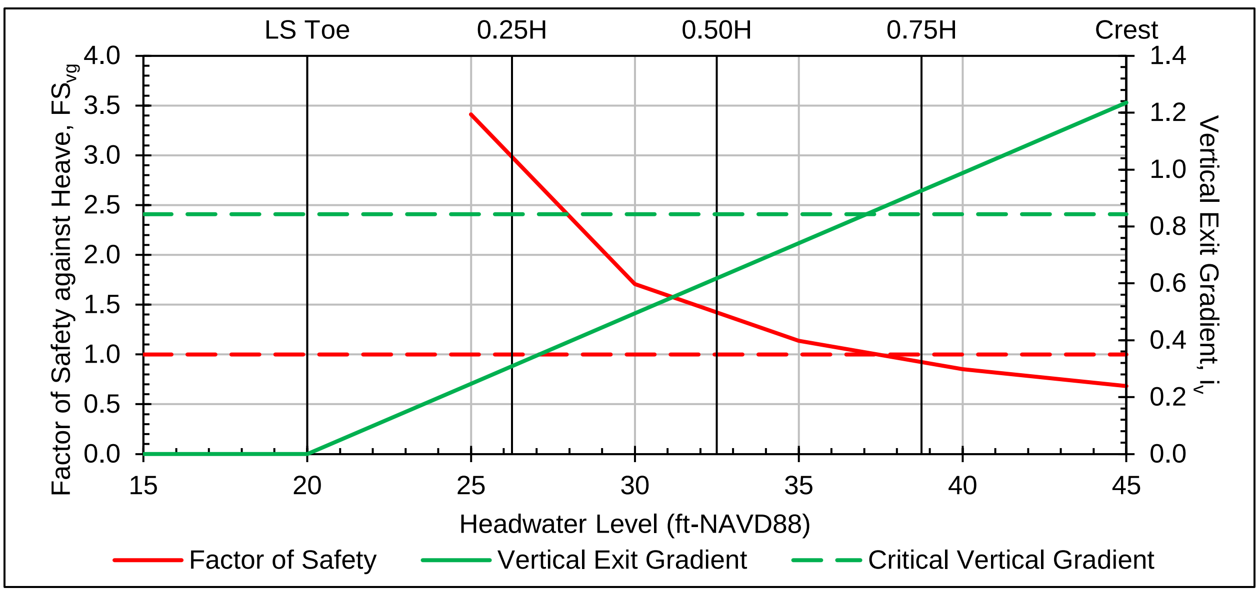


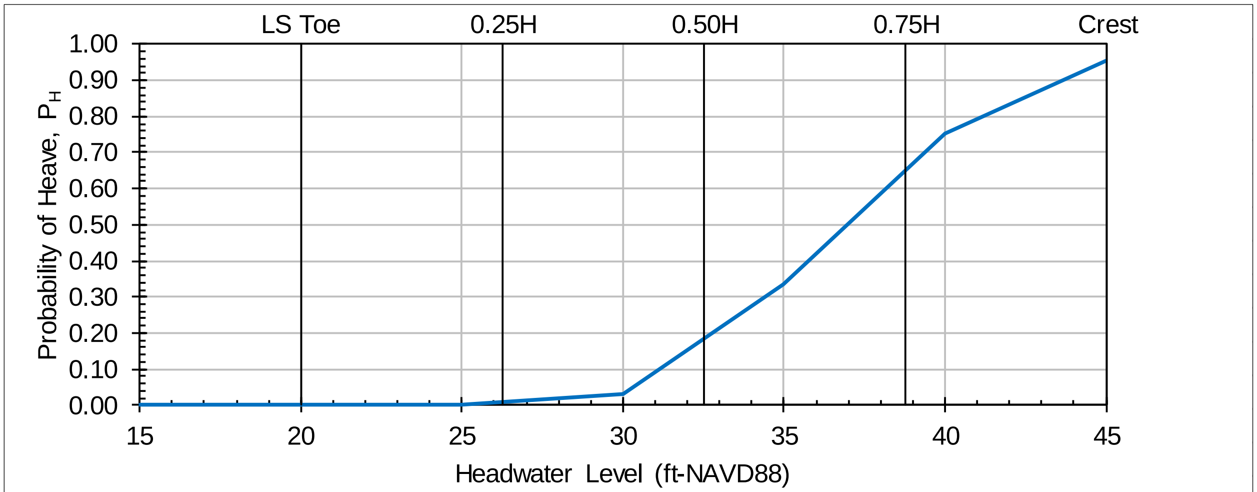

**Figure 29. Step 7 of BT Case 2 worksheet: Deterministic graphical output.**

Figure 30 illustrates the plot options for this chart. The maximum value for the primary y-axis ($FS_{vg}$), maximum value for the secondary y-axis ($i_v$), and minimum and maximum values for the x-axis (headwater level) are user-specified. The five vertical reference elevations displayed at the top of the chart correspond to 25-percent increments of the levee height (levee crest elevation minus levee landside toe elevation) and cannot be changed.

| | | | |
|---|---|---|---|
| Worksheet | BT Case 2 | | |
| | | | |
| y-axis bounds (primary) | | | |
| minimum | 0 | | Value Primary Min: 0 |
| maximum | 4.0 | ◄ Enter maximum factor of safety. | Value Primary Max: 4 |
| | | | |
| y-axis bounds (secondary) | | | |
| minimum | 0 | | Value Secondary Min: 0 |
| maximum | 1.4 | ◄ Enter maximum exit gradient. | Value Secondary Max: 1.4 |
| | | | |
| x-axis bounds | | | |
| minimum | 15.0 | ◄ Enter minimum headwater level. | Category Primary Min: 15 |
| maximum | 45.0 | ◄ Enter maximum headwater level. | Category Primary Max: 45 |

**Figure 30. Step 7 of BT Case 2 worksheet: Plot options for deterministic graphical output.**

For probabilistic analysis, the mean probability of heave/blowout at the landside levee toe is plotted as a function of headwater level. If cycling through iterations using @RISK, this plot has a gray background because the probability of initiation cannot be calculated from a single iteration. Similarly, this plot has a gray background for deterministic analysis. Figure 31 illustrates the graphical output for probabilistic analysis.




**Note: Probabilities obtained from this method should not be used directly in risk analyses. Rather, the values should be used to help develop a list of more and less likely factors.**

**Figure 31. Step 7 of BT Case 2 worksheet: Probabilistic graphical output.**

Figure 32 illustrates the plot options for this chart. The vertical reference elevations and minimum and maximum values for the x-axis (headwater level) are the same as the previous chart. Only the maximum value for the y-axis (probability of heave/blowout) is user-specified.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Worksheet | BT Case 2 | | | | | | |
| | | | | | | | |
| y-axis bounds | | | | | | | |
| minimum | 0 | | | | Value Primary Min: 0 | | |
| maximum | 1.0 | ◄ Enter maximum probability. | | | Value Primary Max: 1 | | |
| | | | | | | | |
| x-axis bounds | | | | | | | |
| minimum | 15.0 | ◄ Enter minimum headwater level. | | | Category Primary Min: 15 | | |
| maximum | 45.0 | ◄ Enter maximum headwater level. | | | Category Primary Max: 45 | | |

**Figure 32. Step 7 of BT Case 2 worksheet: Plot options for probabilistic graphical output.**

## 7.8. Likelihood of Heave/Blowout at Given Distance from Landside Toe

Step 8 calculates the excess hydraulic head at a distance x from the landside levee toe ($h_x$) using Equation 11.

$$h_x = h_o \frac{L_3 - x}{L_3} \tag{11}$$

where:

$h_o$ = excess hydraulic head at the landside levee toe
$L_3$ = length of the foundation and top stratum beyond the landside levee toe
$x$ = distance from the landside levee toe

The vertical seepage exit gradient at a distance $x$ from the landside levee toe ($i_{v,x}$) is calculated using Equation 12.

$$i_{v,x} = \frac{h_x}{z_{bl}} \tag{12}$$

where:

$h_x$ = excess hydraulic head at a distance $x$ from the landside levee toe
$z_{bl}$ = thickness of the landside blanket (top stratum)

The factor of safety against heave/blowout (based on vertical seepage gradients) at a distance $x$ from the landside levee toe ($FS_{vg,x}$) is calculated using Equation 13.

$$FS_{vg,x} = \frac{i_{cv}}{i_{v,x}} \tag{13}$$

where:

$i_{cv}$ = critical vertical seepage exit gradient for heave/blowout

$i_{v,x}$ = vertical seepage exit gradient at a distance $x$ from the landside levee toe

For deterministic analysis, $FS_{vg,x}$ is calculated for each headwater level using the most likely values of the random variables and is summarized in a table. Cells that do not apply have a gray background. For probabilistic analysis, $FS_{vg,x}$ is calculated as described for the deterministic analysis but for the mean values of the random variables, and multiple iterations are performed by sampling the distributions in steps 3 and 4. The probability of heave/blowout at a distance $x$ from the landside levee is equal to the percentage of iterations that resulted in a $FS_{vg,x}$ less than 1 [P($FS_{vg,x}$ < 1)]. For probabilistic analysis performed without using @RISK, 1,000 iterations are used. For probabilistic analysis using @RISK, the number of iterations is user-specified, and "@RISK" displays in parentheses after the number of iterations for this scenario. If cycling through iterations using @RISK, the displayed results are no longer mean values; they are the selected iteration's values. For deterministic and probabilistic analyses, cells with $FS_{vg,x}$ less than 1 have an orange background. Figure 33 illustrates the deterministic tabular output, and Figure 34 illustrates the probabilistic tabular output without using @RISK.

**Step 8: Assess the likelihood of heave/blowout at a given distance from the landside toe**

Distance from landside toe, $x \le L_3$ 15.0 ft

Hydraulic head at distance x from landside toe, $h_x = h_o(L_3 - x)/L_3$ for $x \le L_3$ and $h_x = 0$ for $x \ge L_3$

Vertical exit gradient at distance x from landside toe, $i_{v,x} = h_x/(z_t = z_{bl})$

Factor of safety against heave/blowout at distance x from landside toe, $FS_{vg,x} = i_{cv}/i_{v,x}$

Probability of a FS against heave/blowout less than 1 at distance x from landside toe, $P(FS_{vg,x} < 1)$

Iterations: #N/A

Likelihood of initiation of backward erosion piping at distance x from landside toe

| HW (ft) | 15.0 | 20.0 | 25.0 | 30.0 | 35.0 | 40.0 | 45.0 |
|---|---|---|---|---|---|---|---|
| TW (ft) | 20.0 | 20.0 | 20.0 | 20.0 | 20.0 | 20.0 | 20.0 |
| $h_o$ (ft) | 0.0 | 0.0 | 2.7 | 5.4 | 8.2 | 10.9 | 13.6 |
| $h_x$ (ft) | 0.0 | 0.0 | 2.6 | 5.1 | 7.7 | 10.2 | 12.8 |
| $i_{v,x}$ | 0.000 | 0.000 | 0.255 | 0.511 | 0.766 | 1.022 | 1.277 |
| $FS_{vg,x}$ | ∞ | ∞ | 3.30 | 1.65 | 1.10 | 0.83 | 0.66 |
| $P(FS_{vg,x} < 1)$ | #N/A | #N/A | #N/A | #N/A | #N/A | #N/A | #N/A |

**Figure 33. Step 8 of BT Case 2 worksheet: Deterministic tabular output.**

**Step 8: Assess the likelihood of heave/blowout at a given distance from the landside toe**

Distance from landside toe, $x \leq L_3$ 15.0 ft

Hydraulic head at distance x from landside toe, $h_x = h_o(L_3 - x)/L_3$ for $x \leq L_3$ and $h_x = 0$ for $x \geq L_3$

Vertical exit gradient at distance x from landside toe, $i_{v,x} = h_x/(z_t = z_{bl})$

Factor of safety against heave/blowout at distance x from landside toe, $FS_{vg,x} = i_{cv}/i_{v,x}$

Probability of a FS against heave/blowout less than 1 at distance x from landside toe, $P(FS_{vg,x} < 1)$

Iterations: 1000

Likelihood of initiation of backward erosion piping at distance x from landside toe

| HW (ft) | 15.0 | 20.0 | 25.0 | 30.0 | 35.0 | 40.0 | 45.0 |
|---|---|---|---|---|---|---|---|
| TW (ft) | 20.0 | 20.0 | 20.0 | 20.0 | 20.0 | 20.0 | 20.0 |
| $h_o$ (ft) | 0.0 | 0.0 | 2.7 | 5.4 | 8.2 | 10.9 | 13.6 |
| $h_x$ (ft) | 0.0 | 0.0 | 2.6 | 5.1 | 7.7 | 10.2 | 12.8 |
| $i_{v,x}$ | 0.000 | 0.000 | 0.232 | 0.464 | 0.697 | 0.929 | 1.161 |
| $FS_{vg,x}$ | ∞ | ∞ | 3.63 | 1.82 | 1.21 | 0.91 | 0.73 |
| $P(FS_{vg,x} < 1)$ | 0.00E+00 | 0.00E+00 | 0.00E+00 | 1.60E-02 | 2.59E-01 | 6.68E-01 | 9.19E-01 |

**Figure 34. Step 8 of BT Case 2 worksheet: Probabilistic tabular output without using @RISK.**

At the end of Step 8, summary plots are generated after the tabular output. The first plot is the mean FS against heave/blowout at a distance *x* from the landside levee (red solid line) and vertical seepage exit gradient at a distance *x* from the landside levee toe (green solid line) as functions of headwater level. $FS_{vg,x}$ is plotted on the primary y-axis, and $i_{v,x}$ is plotted on the secondary y-axis. Horizontal reference lines display for the mean critical vertical seepage exit gradient (green dashed line) and $FS_{vg,x}$ of 1 (red dashed line). If cycling through iterations using @RISK, the displayed results are no longer mean values; they are the selected iteration. Figure 35 illustrates the deterministic graphical output.

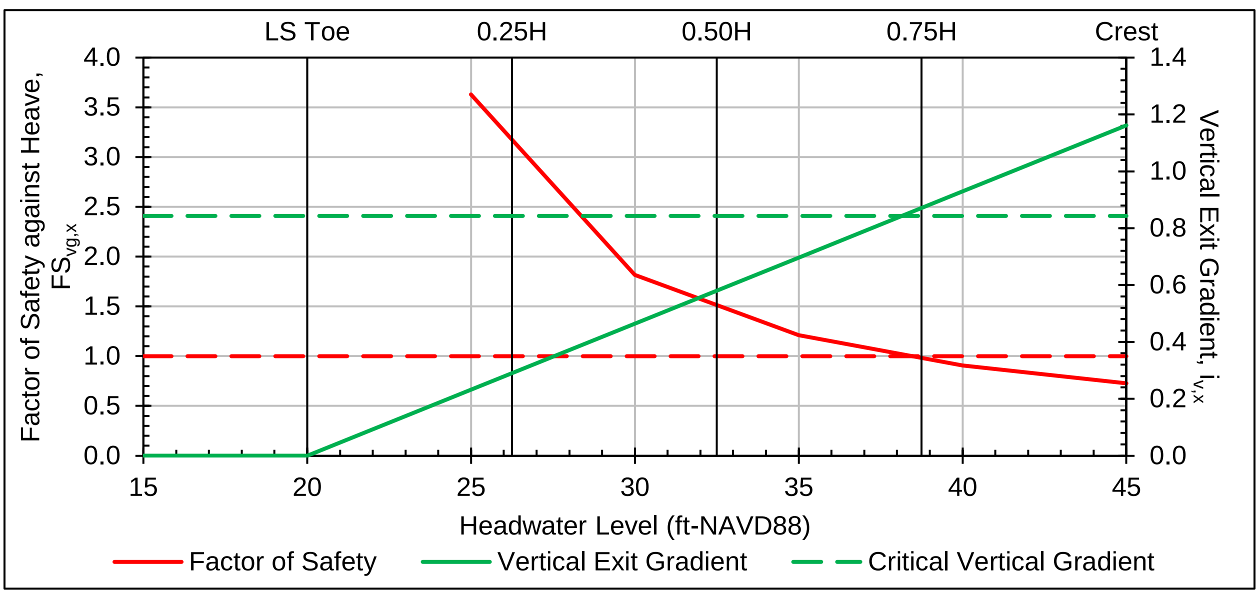


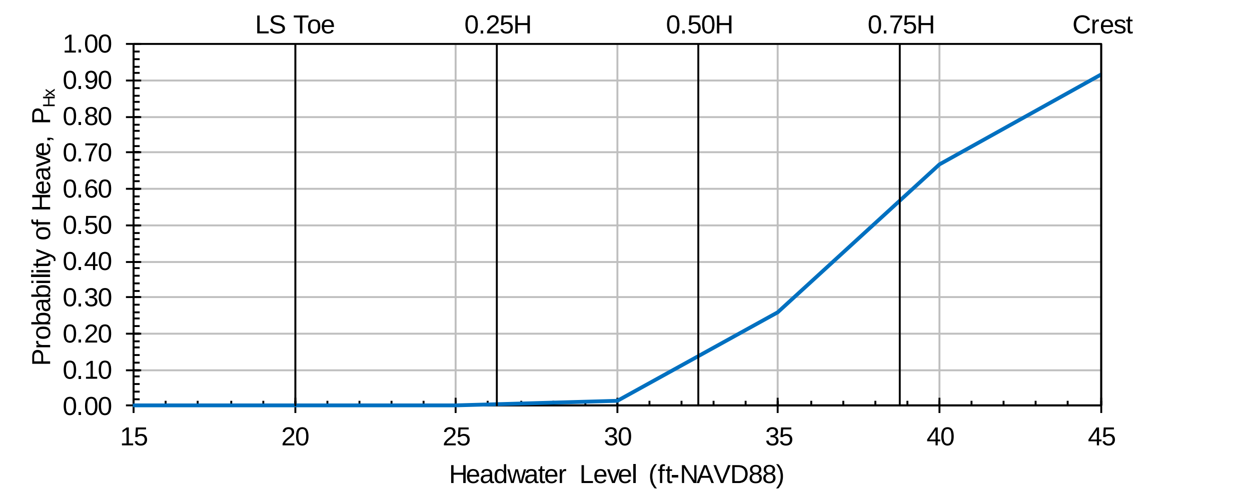

**Figure 35. Step 8 of BT Case 2 worksheet: Deterministic graphical output.**

Figure 36 illustrates the plot options for this chart. The maximum value for the primary y-axis ($FS_{vg,x}$), maximum value for the secondary y-axis ($i_{v,x}$), and minimum and maximum values for the x-axis (headwater level) are user-specified. The five vertical reference elevations displayed at the top of the chart correspond to 25-percent increments of the levee height (levee crest elevation minus levee landside toe elevation) and cannot be changed.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Worksheet | BT Case 2 | | | | | | |
| | | | | | | | |
| y-axis bounds (primary) | | | | | | | |
| minimum | 0 | | | | Value Primary Min: 0 | | |
| maximum | 4.0 | ◄ Enter maximum factor of safety. | | | Value Primary Max: 4 | | |
| | | | | | | | |
| y-axis bounds (secondary) | | | | | | | |
| minimum | 0 | | | | Value Secondary Min: 0 | | |
| maximum | 1.4 | ◄ Enter maximum exit gradient. | | | Value Secondary Max: 1.4 | | |
| | | | | | | | |
| x-axis bounds | | | | | | | |
| minimum | 15.0 | ◄ Enter minimum headwater level. | | | Category Primary Min: 15 | | |
| maximum | 45.0 | ◄ Enter maximum headwater level. | | | Category Primary Max: 45 | | |

**Figure 36. Step 8 of BT Case 2 worksheet: Plot options for deterministic graphical output.**

For probabilistic analysis, the mean probability of heave/blowout at a distance *x* from the landside levee is plotted as a function of headwater level. If cycling through iterations using @RISK, this plot has a gray background because the probability of initiation cannot be calculated from a single iteration. Similarly, this plot has a gray background for deterministic analysis. Figure 37 illustrates the probabilistic graphical output.




**Note: Probabilities obtained from this method should not be used directly in risk analyses. Rather, the values should be used to help develop a list of more and less likely factors.**

**Figure 37. Step 7 of BT Case 2 worksheet: Probabilistic graphical output.**

Figure 38 illustrates the plot options for this chart. The vertical reference elevations and minimum and maximum values for the x-axis (headwater level) are the same as the previous. Only the maximum value for the y-axis (probability of heave/blowout) is user-specified.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Worksheet | BT Case 2 | | | | | | |
| | | | | | | | |
| y-axis bounds | | | | | | | |
| minimum | 0 | | | | Value Primary Min: 0 | | |
| maximum | 1.0 | ◄ Enter maximum probability. | | | Value Primary Max: 1 | | |
| | | | | | | | |
| x-axis bounds | | | | | | | |
| minimum | 15.0 | ◄ Enter minimum headwater level. | | | Category Primary Min: 15 | | |
| maximum | 45.0 | ◄ Enter maximum headwater level. | | | Category Primary Max: 45 | | |

**Figure 38. Step 7 of BT Case 2 worksheet: Plot options for probabilistic graphical output.**

# 8. Blanket Theory Case 3

Case 3 has an impervious top stratum only on the river side of the levee. The pervious substratum is divided into three zones to apply the method of fragments as shown in Figure 39.

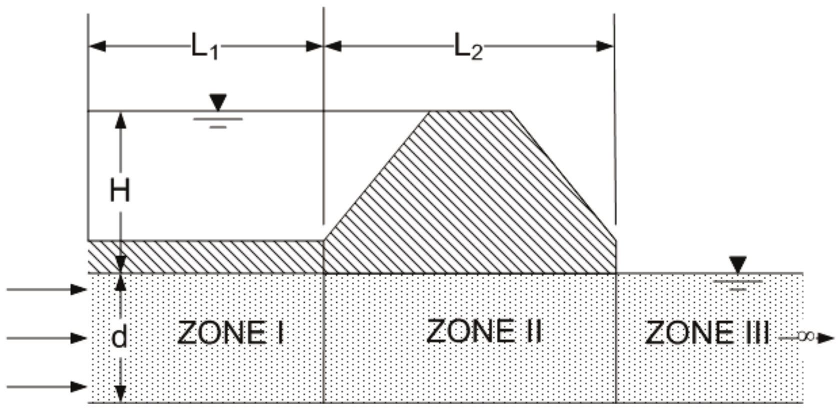
## 8.1. Method Analysis

The method of analysis is the same as in Case 1.

## 8.2. Levee Geometry

Step 2 characterizes the levee geometry. The input includes the levee crest elevation, landside toe elevation, distance from riverside levee toe to the river ($L_1$), and base width of levee ($L_2$), as illustrated in Figure 39.

**Step 2: Assess the levee geometry**




| | | |
|---|---|---|
| Levee crest elevation | 45.0 | ft-NAVD88 |
| Landside toe elevation | 20.0 | ft-NAVD88 |
| Base width of levee, $L_2$ | 110.0 | ft |
| Distance from river to riverside toe, $L_1$ | 100.0 | ft |

**Figure 39. Step 2 of BT Case 3 worksheet: Levee geometry.**

## 8.3. Pervious Substratum Characterization

The pervious stratum characterization is the same as in Case 1.

## 8.4. Blanket Theory Assumptions

Step 4 checks the BT (method of fragments) assumptions against the input parameters to ensure essentially vertical equipotential lines. For deterministic analysis, the assumptions are checked for the most likely values of the random variables. For probabilistic analysis, the assumptions are checked for the mean values of the random variables. Values outside of the model assumptions have an orange background. Figure 40 illustrates the check of BT assumptions.

| Step 4: Check blanket theory (method of fragments) assumptions | |
|---|---|
| Assumption | Mean |
| Essentially vertical equipotential lines, $(L_1 + L_2) / d \geq 1$ | 9.0 |

**Figure 40. Step 4 of BT Case 3 worksheet: BT assumptions.**

## 8.5. Seepage Characterization

Step 5 calculates the net hydraulic head on the levee ($H$) the same as in Case 1. The flow or seepage per unit length of the levee ($Q_s$) using Equation 14.

$$Q_s = k_{h,f} H \frac{d}{L_1 + L_2 + 0.43d} \tag{14}$$

where:

$k_{h,f}$ = horizontal permeability of pervious substratum
$H$ = net hydraulic head on the levee
$L_1$ = distance from the riverside levee toe to the river
$L_2$ = base width of the levee
$d$ = thickness of the pervious substratum

The format for the tabular output is the same as in Case 1.

## 8.6. Likelihood of Heave/Blowout at Landside Toe

Since the excess hydraulic head at the landside levee toe is zero, vertical seepage exit gradients are not computed for this case.

## 8.7. Likelihood of Heave/Blowout at Given Distance from Landside Toe

Since the excess hydraulic head at the landside levee toe and at any distance x from the landside levee toe are zero, vertical seepage exit gradients at any distance $x$ from the landside levee toe are not computed for this case.

# 9. Blanket Theory Case 4

Case 4 has an impervious top stratum on only the landside of the levee. The pervious substratum is divided into three zones to apply the method of fragments as shown in Figure 41.

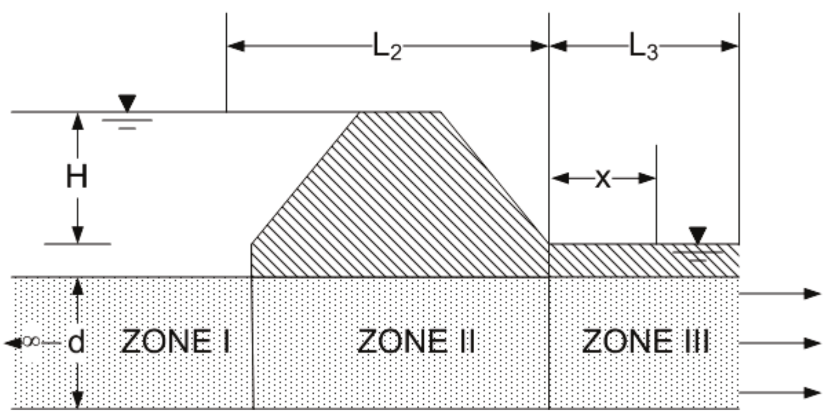
## 9.1. Method of Analysis

The method of analysis is the same as in Case 1.

## 9.2. Levee Geometry

Step 2 characterizes the levee geometry. The input includes the levee crest elevation, landside toe elevation, base width of levee ($L_2$), and length of the foundation and top stratum beyond the landside levee toe ($L_3$), as illustrated in Figure 41.

**Step 2: Assess the levee geometry**




| | | |
|---|---|---|
| Levee crest elevation | 45.0 | ft-NAVD88 |
| Landside toe elevation | 20.0 | ft-NAVD88 |
| Base width of levee, $L_2$ | 110.0 | ft |
| Length of foundation and top stratum beyond landside toe, $L_3$ | 250.0 | ft |

**Figure 41. Step 2 of BT Case 4 worksheet: Levee geometry.**

## 9.3. Pervious Substratum Characterization

The pervious stratum characterization is the same as in Case 1.

## 9.4. Landside Blanket (Top Stratum) Characterization

The landside blanket (top stratum) characterization is the same as in Case 2.

## 9.5. Blanket Theory Assumptions

Step 5 checks the BT (method of fragments) assumptions against the input parameters to ensure essentially vertical equipotential lines. For deterministic analysis, the assumptions are checked for the most likely values of the random variables. For probabilistic analysis, the assumptions are checked for the mean values of the random variables. Values outside of the model assumptions have an orange background. Figure 42 illustrates the check of BT assumptions.

| Step 5: Check blanket theory (method of fragments) assumptions | |
|---|---|
| | |
| Assumption | Mean |
| Essentially vertical equipotential lines, $(L_2 + L_3) / d \geq 1$ | 15.4 |

**Figure 42. Step 5 of BT Case 4 worksheet: BT assumptions.**

## 9.6. Seepage Characterization

Step 6 calculates the net hydraulic head on the levee ($H$) the same as in Case 1. The flow or seepage per unit length of the levee ($Q_s$) is calculated using Equation 15.

$$Q_s = k_{h,f} H \frac{d}{0.43d + L_2 + L_3} \tag{15}$$

where:

$k_{h,f}$ = horizontal permeability of pervious substratum
$H$ = net hydraulic head on the levee
$L_2$ = base width of the levee
$L_3$ = length of the foundation and top stratum beyond the landside levee toe
$d$ = thickness of the pervious substratum

The format for the tabular output is the same as in Case 1.

## 9.7. Likelihood of Heave/Blowout at Landside Toe

Step 7 calculates the excess hydraulic head at the landside levee toe ($h_o$) using Equation 16.

$$h_o = H \frac{L_3}{0.43d + L_2 + L_3} \tag{16}$$

where:

$H$ = net hydraulic head on the levee
$L_2$ = base width of the levee
$L_3$ = length of the foundation and top stratum beyond the landside levee toe
$d$ = thickness of the pervious substratum

The vertical seepage exit gradient at the landside levee toe ($i_v$) and factor of safety against heave/blowout (based on vertical seepage gradients) at the landside levee toe ($FS_{vg}$) are calculated the same as in Case 2. The format for the tabular and graphical output is the same as in Case 2.

## 9.8. Likelihood of Heave/Blowout at Given Distance from Landside Toe

Step 8 calculates the excess hydraulic head at a distance $x$ from the landside levee toe ($h_x$) using Equation 17.

$$h_x = h_o \frac{L_3 - x}{L_3} \tag{17}$$

where:

$h_o$ = excess hydraulic head at the landside levee toe
$L_3$ = length of the foundation and top stratum beyond the landside levee toe
$x$ = distance from the landside levee toe

The vertical seepage exit gradient at a distance $x$ from the landside levee toe ($i_{v,x}$) and factor of safety against heave/blowout (based on vertical seepage gradients) at a distance $x$ from the landside levee toe ($FS_{vg,x}$) are calculated the same as in Case 2. The format for the tabular and graphical output is the same as in Case 2.

# 10. Blanket Theory Case 5

Case 5 has a semi-pervious top stratum on only the riverside of the levee. The pervious substratum is divided into three zones to apply the method of fragments as shown in Figure 43. The difference between Cases 3 and 5 is that there is an impervious top stratum on the riverside for Case 3 while the top stratum is semi-pervious for Case 5. Therefore, the effective seepage entry distance from the riverside levee toe must be calculated based on the type of seepage entrance (riverside boundary condition).

## 10.1. Method Analysis

The method of analysis is the same as in Case 1.

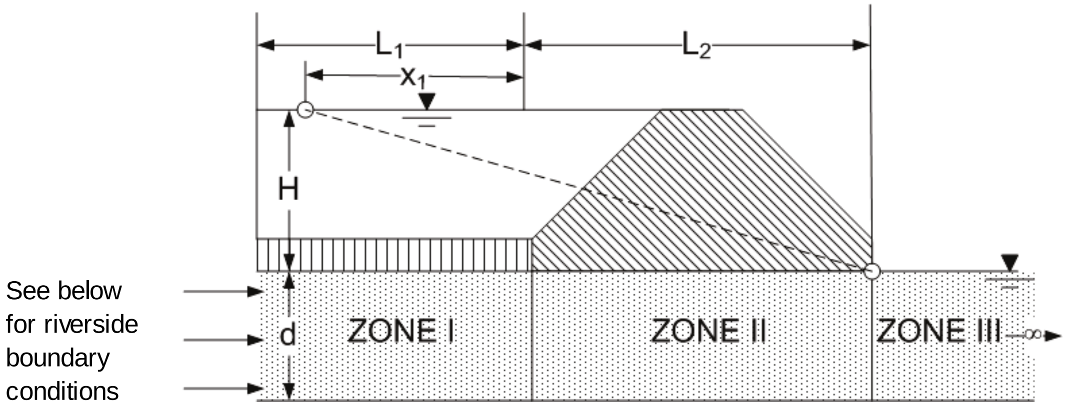
## 10.2. Levee Geometry

Step 2 characterizes the levee geometry. The input includes the levee crest elevation, landside levee toe elevation, and base width of levee ($L_2$). Use the drop-down list to select the riverside boundary condition using a drop-down list. The options for the type of seepage entrance include:

- No riverside borrow pits or seepage blocks
- Riverside borrow pit (open seepage entrance) that penetrates the blanket and extends to the pervious substratum
- A seepage block (impervious boundary) between the riverside levee toe and the river that prevents any seepage entrance into the pervious foundation riverside of the seepage block

For "no riverside borrow pits or seepage blocks," the input for $L_1$ is the distance from the riverside levee toe to the river. For "riverside borrow pit (open seepage entrance)," the input for $L_1$ is the distance from the riverside levee toe to the borrow pit. For "seepage block (impervious boundary)," the input for $L_1$ is the distance from the riverside levee toe to the seepage block. The input is illustrated in Figure 43.

**Step 2: Assess the levee geometry**




| | | |
|---|---|---|
| Riverside boundary condition | no riverside borrow pits or seepage blocks | |
| Levee crest elevation | 45.0 | ft-NAVD88 |
| Landside toe elevation | 20.0 | ft-NAVD88 |
| Base width of levee, $L_2$ | 110.0 | ft |
| Distance from riverside toe to river, $L_1$ | 100.0 | ft |

**Figure 43. Step 2 of BT Case 5 worksheet: Levee geometry.**

## 10.3. Pervious Substratum Characterization

The pervious stratum characterization is the same as in Case 1.

## 10.4. Riverside Blanket (Top Stratum) Characterization

The selection in step 1 affects the input for step 4, and cells that do not apply have a gray background. These cells are not used in subsequent calculations even if data is present. The input includes the transformed thickness ($z_{br}$) and transformed vertical permeability ($k_{v,br}$) of the riverside blanket.

Use the drop-down list to select the method of estimating the transformed vertical permeability of the riverside blanket. Input the transformed vertical permeability directly or calculate it using an input permeability ratio ($k_{h,f}/k_{v,br}$).

For deterministic analysis, input only the most likely values. The mean values used for subsequent calculations are the most likely (or mode) values. Figure 44 illustrates the deterministic input. Figure 45 illustrates the deterministic input for permeability ratio as input.

| Step 4: Characterize the riverside blanket (top stratum) | | | | |
|---|---|---|---|---|
| Estimate vertical permeability of riverside blanket using permeability ratio for pervious stratum ($k_{h,f}/k_{v,br}$)? | | | No | |
| Parameter | Units | Minimum | Most Likely | Maximum |
| Transformed thickness, $z_{br}$ | ft | 5.0 | 10.0 | 18.0 |
| Permeability ratio, $k_{h,f}/k_{v,br}$ | - | 125.0 | 250.0 | 300.0 |
| Transformed vertical permeability, $k_{v,br}$ | cm/sec | - | - | - |
| Transformed vertical permeability, $k_{v,br}$ | cm/sec | 8.00E-05 | 1.60E-04 | 3.00E-04 |

| Parameter | Units | Mean | @RISK Formula | Mean |
|---|---|---|---|---|
| $z_{br}$ | ft | #NAME? | | 10.0 |
| $k_{h,f}/k_{v,br}$ | - | #NAME? | | 250.0 |
| $k_{v,br}$ | cm/sec | #NAME? | | 1.60E-04 |

**Figure 44. Step 4 of BT Case 5 worksheet: Deterministic input.**

| Step 4: Characterize the riverside blanket (top stratum) | | | | |
|---|---|---|---|---|
| Estimate vertical permeability of riverside blanket using permeability ratio for pervious stratum ($k_{h,f}/k_{v,br}$)? | | | Yes | |
| Parameter | Units | Minimum | Most Likely | Maximum |
| Transformed thickness, $z_{br}$ | ft | 5.0 | 10.0 | 18.0 |
| Permeability ratio, $k_{h,f}/k_{v,br}$ | - | 125.0 | 250.0 | 300.0 |
| Transformed vertical permeability, $k_{v,br}$ | cm/sec | 8.00E-05 | 1.60E-04 | 3.00E-04 |
| Transformed vertical permeability, $k_{v,br}$ | cm/sec | 8.00E-05 | 1.60E-04 | 3.00E-04 |

| Parameter | Units | Mean | @RISK Formula | Mean |
|---|---|---|---|---|
| $z_{br}$ | ft | #NAME? | | 10.0 |
| $k_{h,f}/k_{v,br}$ | - | #NAME? | | 250.0 |
| $k_{v,br}$ | cm/sec | #NAME? | | 1.60E-04 |

**Figure 45. Step 4 of BT Case 5 worksheet: Deterministic input for permeability ratio as input.**

For probabilistic analysis without using @RISK, input the minimum and maximum values in addition to the most likely value, and triangular distributions represent the random variables. The mean values used in subsequent calculations are the average of the minimum, most likely, and maximum values. Figure 46 illustrates the probabilistic input without using @RISK. Figure 47 illustrates the probabilistic input without using @RISK for permeability ratio as input.

| Step 4: Characterize the riverside blanket (top stratum) | | | | |
|---|---|---|---|---|
| Estimate vertical permeability of riverside blanket using permeability ratio for pervious stratum ($k_{h,f}/k_{v,br}$)? | | | No | |
| Parameter | Units | Minimum | Most Likely | Maximum |
| Transformed thickness, $z_{br}$ | ft | 5.0 | 10.0 | 18.0 |
| Permeability ratio, $k_{h,f}/k_{v.br}$ | - | 125.0 | 250.0 | 300.0 |
| Transformed vertical permeability, $k_{v,br}$ | cm/sec | - | - | - |
| Transformed vertical permeability, $k_{v,br}$ | cm/sec | 8.00E-05 | 1.60E-04 | 3.00E-04 |

| Parameter | Units | Mean | @RISK Formula | Mean |
|---|---|---|---|---|
| $z_{br}$ | ft | #NAME? | | 11.0 |
| $k_{h,f}/k_{v,br}$ | - | #NAME? | | 225.0 |
| $k_{v,br}$ | cm/sec | #NAME? | | 1.80E-04 |

**Figure 46. Step 4 of BT Case 5 worksheet: Probabilistic input without using @RISK.**

| Step 4: Characterize the riverside blanket (top stratum) | | | | |
|---|---|---|---|---|
| Estimate vertical permeability of riverside blanket using permeability ratio for pervious stratum ($k_{h,f}/k_{v,br}$)? | | | Yes | |
| Parameter | Units | Minimum | Most Likely | Maximum |
| Transformed thickness, $z_{br}$ | ft | 5.0 | 10.0 | 18.0 |
| Permeability ratio, $k_{h,f}/k_{v.br}$ | - | 125.0 | 250.0 | 300.0 |
| Transformed vertical permeability, $k_{v,br}$ | cm/sec | 8.00E-05 | 1.60E-04 | 3.00E-04 |
| Transformed vertical permeability, $k_{v,br}$ | cm/sec | 8.00E-05 | 1.60E-04 | 3.00E-04 |

| Parameter | Units | Mean | @RISK Formula | Mean |
|---|---|---|---|---|
| $z_{br}$ | ft | #NAME? | | 11.0 |
| $k_{h,f}/k_{v,br}$ | - | #NAME? | | 225.0 |
| $k_{v,br}$ | cm/sec | #NAME? | | 1.80E-04 |

**Figure 47. Step 4 of BT Case 5 worksheet: Probabilistic input without using @RISK for permeability ratio as input.**

For probabilistic analysis using @RISK, input the minimum, most likely, and maximum values of the random variables, and use an @RISK formula for a triangular distribution in the third column as a default. Alternatively, input a valid @RISK distribution in lieu of this default formula, and the user-specified input displays in the fourth column. The mean values used in subsequent calculations are the mean for the @RISK distribution entered in the third column. Figure 48 illustrates the probabilistic input using @RISK. Figure 49 illustrates the probabilistic input without using @RISK for permeability ratio as input.

| Step 4: Characterize the riverside blanket (top stratum) | | | | | |
|---|---|---|---|---|---|
| Estimate vertical permeability of riverside blanket using permeability ratio for pervious stratum ($k_{h,f}/k_{v,br}$)? | | | No | | |
| Parameter | Units | Minimum | Most Likely | Maximum | |
| Transformed thickness, $z_{br}$ | ft | 5.0 | 10.0 | 18.0 | |
| Permeability ratio, $k_{h,f}/k_{v.br}$ | - | 125 | 250 | 300 | |
| Transformed vertical permeability, $k_{v,br}$ | cm/sec | - | - | - | |
| Transformed vertical permeability, $k_{v,br}$ | cm/sec | 8.00E-05 | 1.60E-04 | 3.00E-04 | |

| Parameter | Units | Mean | @RISK Formula | Mean |
|---|---|---|---|---|
| $z_{br}$ | ft | 11.0 | =@RiskTriang(F59,G59,H59) | 11.0 |
| $k_{h,f}/k_{v,br}$ | - | 225 | | 225 |
| $k_{v,br}$ | cm/sec | | | 1.80E-04 |
| | | 1.80E-04 | =@RiskTriang(F62,G62,H62) | |

**Figure 48. Step 4 of BT Case 5 worksheet: Probabilistic input using @RISK.**

| Step 4: Characterize the riverside blanket (top stratum) | | | | |
|---|---|---|---|---|
| Estimate vertical permeability of riverside blanket using permeability ratio for pervious stratum ($k_{h,f}/k_{v,br}$)? | | | Yes | |
| Parameter | Units | Minimum | Most Likely | Maximum |
| Transformed thickness, $z_{br}$ | ft | 5.0 | 10.0 | 18.0 |
| Permeability ratio, $k_{h,f}/k_{v.br}$ | - | 125 | 250 | 300 |
| Transformed vertical permeability, $k_{v,br}$ | cm/sec | 8.00E-05 | 1.60E-04 | 3.00E-04 |
| Transformed vertical permeability, $k_{v,br}$ | cm/sec | 8.00E-05 | 1.60E-04 | 3.00E-04 |

| Parameter | Units | Mean | @RISK Formula | Mean |
|---|---|---|---|---|
| $z_{br}$ | ft | 11.0 | =@RiskTriang(F59,G59,H59) | 11.0 |
| $k_{h,f}/k_{v,br}$ | - | 225 | =@RiskTriang(F60,G60,H60) | 225 |
| $k_{v,br}$ | cm/sec | | | 2.07E-04 |
| | | 1.80E-04 | | |

**Figure 49. Step 4 of BT Case 5 worksheet: Probabilistic input using @RISK for permeability ratio as input.**

If no riverside seepage block (impervious boundary) exists, the effective seepage entry distance from the riverside levee toe ($x_1$) is calculated using Equations 18 and 19.

$$x_1 = \frac{\tanh(c_{br}L_1)}{c_{br}} \tag{18}$$

where:

$L_1$ = distance from the riverside levee toe to the river for the “no riverside borrow pits or seepage blocks” boundary condition or the distance from the riverside levee toe to the borrow pit for “riverside borrow pit (open seepage entrance)” boundary condition
$c_{br}$ = constant for the riverside blanket

$$c_{br} = \sqrt{\frac{k_{v,br}}{k_{h,f} z_{br} d}} \tag{19}$$

where:

$k_{h,f}$ = horizontal permeability of pervious substratum
$d$ = thickness of the pervious substratum
$k_{v,br}$ = transformed vertical permeability of the riverside blanket (top substratum)
$z_{br}$ = transformed thickness of the riverside blanket (top substratum)

If a seepage block exists between the riverside levee toe and the river that prevents any seepage entrance into the pervious foundation riverside of the seepage block, the effective seepage entry distance from the riverside levee toe ($x_1$) is calculated using Equation 20.

$$x_1 = \frac{1}{c_{br} \tanh(c_{br} L_1)} \tag{20}$$

where:

$L_1$ = distance from the riverside levee toe to the seepage block for the “seepage block (impervious boundary)” boundary condition

The effective seepage length plus width of the levee is calculated using Equation 21.

$$S = x_1 + L_2 \tag{21}$$

where:

$L_2$ = base width of the levee
$x_1$ = effective seepage entry distance from the riverside levee toe for selected boundary condition

The riverside boundary condition selection in step 2 affects the equation for $x_1$ used in step 4, and cells that do not apply have a gray background.

## 10.5. Blanket Theory Assumptions

Step 5 checks the BT (method of fragments) assumptions against the input parameters to ensure essentially vertical equipotential lines, vertical flow through the blanket and horizontal flow through the pervious foundation, and semi-pervious blanket behavior. For permeability ratios greater than about 1,000 to 4,000, the semi-pervious blanket is effectively impervious, and Case 3 is more appropriate than Case 5. For deterministic analysis, the assumptions are checked for the most likely values of the random variables. For probabilistic analysis, the assumptions are checked for the mean values of the random variables. Values outside of the model assumptions have an orange background. Figure 50 illustrates the check of BT assumptions.

| Step 5: Check blanket theory (method of fragments) assumptions | |
|---|---|
| Assumption | Mean |
| Essentially vertical equipotential lines, $(x_1 + L_2) / d \geq 1$ | 8.8 |
| Vertical riverside blanket flow and horizontal foundation flow, $k_{h,f}/k_{v,br} \geq 10$ | 259 |
| Semi-pervious riverside blanket behavior, $k_{h,f}/k_{v,br}$ < 1,000 to 4,000 | 259 |

**Figure 50. Step 5 of BT Case 5 worksheet: BT assumptions.**

## 10.6. Seepage Characterization

Step 6 calculates the net hydraulic head on the levee ($H$) the same as in Case 1. The flow or seepage per unit length of the levee ($Q_s$) is calculated using Equation 22.

$$Q_s = k_{h,f} H \frac{d}{x_1 + L_2 + 0.43d} \tag{22}$$

where:

$k_{h,f}$ = horizontal permeability of pervious substratum
$H$ = net hydraulic head on the levee
$L_2$ = base width of the levee
$d$ = thickness of the pervious substratum
$x_1$ = effective seepage entrance from the riverside levee toe

The format for the tabular output is the same as in Case 1.

## 10.7. Likelihood of Heave/Blowout at Landside Toe

Since the excess hydraulic head at the landside levee toe is zero, vertical seepage exit gradients are not computed for this case.

## 10.8. Likelihood of Heave/Blowout at Given Distance from Landside Toe

Since the excess hydraulic head at the landside levee toe and at any distance $x$ from the landside levee toe are zero, vertical seepage exit gradients at any distance $x$ from the landside levee toe are not computed for this case.

# 11. Blanket Theory Case 6

Case 6 has a semi-pervious top stratum on only the landside of the levee. The pervious substratum is divided into three zones to apply the method of fragments as shown in Figure 51. The difference between Cases 4 and 6 is that there is an impervious top stratum on the landside for Case 4 while the top stratum is semi-pervious for Case 6. Therefore, the effective seepage exit distance from the landside levee toe must be calculated based on the type of seepage exit (landside boundary condition).

## 11.1. Method of Analysis

The method of analysis is the same as in Case 1.

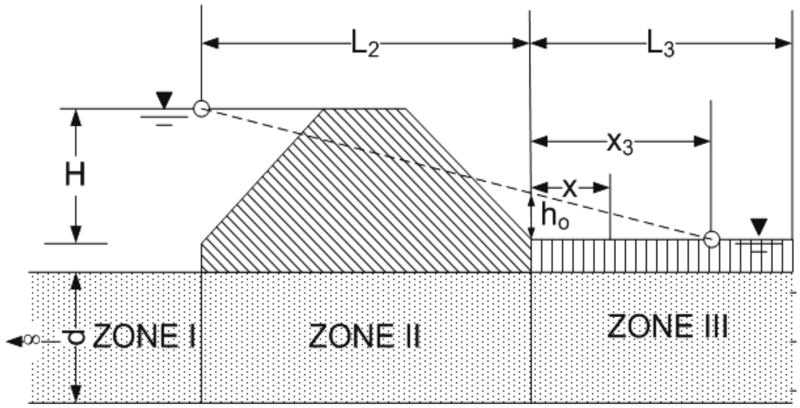
## 11.2. Levee Geometry

Step 2 characterizes the levee geometry. The input includes the levee crest elevation, landside levee toe elevation, and base width of levee ($L_2$). Use the drop-down list to select the landside boundary condition. The options for the type of seepage exit include:

- A blanket extending infinitely landward
- An open seepage exit
- A seepage block (impervious boundary) beyond the landside levee toe that prevents any seepage exit into the pervious foundation landside of the seepage block.

For “blanket extending infinitely landward,” no input is needed for $L_3$ since it is infinitely long. For “open seepage exit,” the input for $L_3$ is the distance from the landside levee toe to the borrow pit. For “seepage block (impervious boundary),” the input for $L_3$ is the distance from the landside toe to the seepage block. The input is illustrated in Figure 51.

**Step 2: Assess the levee geometry**




See below for landside boundary conditions

| | | |
|---|---|---|
| Landside boundary condition | open seepage exit | |
| Levee crest elevation | 45.0 | ft-NAVD88 |
| Landside toe elevation | 20.0 | ft-NAVD88 |
| Base width of levee, $L_2$ | 110.0 | ft |
| Length of foundation and blanket beyond landside toe, $L_3$ | - | ft |
| Distance from landside levee toe to open seepage exit, $L_3$ | 15.0 | ft |

**Figure 51. Step 2 of BT Case 6 worksheet: Levee geometry.**

## 11.3. Pervious Substratum Characterization

The pervious stratum characterization is the same as in Case 1.

## 11.4. Landside Blanket (Top Stratum) Characterization

The selection in step 1 affects the input for step 4, and cells that do not apply have a gray background. These cells are not used in subsequent calculations even if data is present. The input includes the transformed thickness for effective seepage length ($z_{bl}$), effective thickness for allowable uplift pressure ($z_t$), saturated unit weight of the landside blanket ($\gamma_{sat}$), and transformed vertical permeability of the landside blanket ($k_{v,bl}$).

Use the drop-down list to select the method of estimating the transformed vertical permeability of the landside blanket. Input the transformed vertical permeability directly or calculate it using an input permeability ratio ($k_{h,f}/k_{v,bl}$).

For a deterministic analysis, input only the most likely values. The mean values used for subsequent calculations are the most likely (or mode) values. Figure 52 illustrates the deterministic input. Figure 53 illustrates the deterministic input for permeability ratio as input.

**Step 4: Characterize the landside blanket (top stratum)**

Estimate vertical permeability of landside blanket using permeability ratio for pervious stratum ($k_{h,f}/k_{v,bl}$)? No

| Parameter | Units | Minimum | Most Likely | Maximum |
|---|---|---|---|---|
| Transformed thickness for effective seepage length, $z_{bl}$ | ft | 5.0 | 10.0 | 18.0 |
| Effective thickness for allowable uplift pressure, $z_t$ | ft | 5.0 | 10.0 | 18.0 |
| Saturated unit weight, $\gamma_{sat}$ | pcf | 110.0 | 115.0 | 120.0 |
| Critical vertical exit gradient, $i_{cv} = (\gamma_{sat} - \gamma_w) / \gamma_w$ | - | 0.763 | 0.843 | 0.923 |
| Permeability ratio, $k_{h,f}/k_{v.bl}$ | - | 125 | 250 | 300 |
| Transformed vertical permeability, $k_{v,bl}$ | cm/sec | - | - | - |
| Transformed vertical permeability, $k_{v,bl}$ | cm/sec | 8.00E-05 | 1.60E-04 | 3.00E-04 |

| Parameter | Units | Mean | @RISK Formula | Mean |
|---|---|---|---|---|
| $z_{bl}$ | ft | #NAME? | | 10.0 |
| $z_t$ | ft | #NAME? | | 10.0 |
| $\gamma_{sat}$ | pcf | #NAME? | | 115.0 |
| $i_{cv}$ | - | | | 0.843 |
| $k_{h,f}/k_{v,bl}$ | - | #NAME? | | 250 |
| $k_{v,bl}$ | cm/sec | #NAME? | | 1.60E-04 |

**Figure 52. Step 4 of BT Case 6 worksheet: Deterministic input.**

| Step 4: Characterize the landside blanket (top stratum) | | | | |
|---|---|---|---|---|
| Estimate vertical permeability of landside blanket using permeability ratio for pervious stratum ($k_{h,f}/k_{v,bl}$)? | | | Yes | |
| Parameter | Units | Minimum | Most Likely | Maximum |
| Transformed thickness for effective seepage length, $z_{bl}$ | ft | 5.0 | 10.0 | 18.0 |
| Effective thickness for allowable uplift pressure, $z_t$ | ft | 5.0 | 10.0 | 18.0 |
| Saturated unit weight, $\gamma_{sat}$ | pcf | 110.0 | 115.0 | 120.0 |
| Critical vertical exit gradient, $i_{cv} = (\gamma_{sat} - \gamma_w) / \gamma_w$ | - | 0.763 | 0.843 | 0.923 |
| Permeability ratio, $k_{h,f}/k_{v.bl}$ | - | 125 | 250 | 300 |
| Transformed vertical permeability, $k_{v,bl}$ | cm/sec | 8.00E-05 | 1.60E-04 | 3.00E-04 |
| Transformed vertical permeability, $k_{v,bl}$ | cm/sec | 8.00E-05 | 1.60E-04 | 3.00E-04 |

| Parameter | Units | Mean | @RISK Formula | Mean |
|---|---|---|---|---|
| $z_{bl}$ | ft | #NAME? | | 10.0 |
| $z_t$ | ft | #NAME? | | 10.0 |
| $\gamma_{sat}$ | pcf | #NAME? | | 115.0 |
| $i_{cv}$ | - | | | 0.843 |
| $k_{h,f}/k_{v,bl}$ | - | #NAME? | | 250 |
| $k_{v,bl}$ | cm/sec | #NAME? | | 1.60E-04 |

**Figure 53. Step 4 of BT Case 6 worksheet: Deterministic input for permeability ratio as input.**

For probabilistic analysis without using @RISK, input the minimum and maximum values in addition to the most likely value, and triangular distributions represent the random variables. The mean values used in subsequent calculations are the average of the minimum, most likely, and maximum values. Figure 54 illustrates the probabilistic input without using @RISK. Figure 55 illustrates the probabilistic input without using @RISK for permeability ratio as input.

**Step 4: Characterize the landside blanket (top stratum)**

Estimate vertical permeability of landside blanket using permeability ratio for pervious stratum ($k_{h,f}/k_{v,bl}$)? No

| Parameter | Units | Minimum | Most Likely | Maximum |
|---|---|---|---|---|
| Transformed thickness for effective seepage length, $z_{bl}$ | ft | 5.0 | 10.0 | 18.0 |
| Effective thickness for allowable uplift pressure, $z_t$ | ft | 5.0 | 10.0 | 18.0 |
| Saturated unit weight, $\gamma_{sat}$ | pcf | 110.0 | 115.0 | 120.0 |
| Critical vertical exit gradient, $i_{cv} = (\gamma_{sat} - \gamma_w) / \gamma_w$ | - | 0.763 | 0.843 | 0.923 |
| Permeability ratio, $k_{h,f}/k_{v.bl}$ | - | 125.0 | 250.0 | 300.0 |
| Transformed vertical permeability, $k_{v,bl}$ | cm/sec | - | - | - |
| Transformed vertical permeability, $k_{v,bl}$ | cm/sec | 8.00E-05 | 1.60E-04 | 3.00E-04 |

| Parameter | Units | Mean | @RISK Formula | Mean |
|---|---|---|---|---|
| $z_{bl}$ | ft | #NAME? | | 11.0 |
| $z_t$ | ft | #NAME? | | 11.0 |
| $\gamma_{sat}$ | pcf | #NAME? | | 115.0 |
| $i_{cv}$ | - | | | 0.843 |
| $k_{h,f}/k_{v,bl}$ | - | #NAME? | | 225.0 |
| $k_{v,bl}$ | cm/sec | #NAME? | | 1.80E-04 |

**Figure 54. Step 4 of BT Case 6 worksheet: Probabilistic input without using @RISK.**

**Step 4: Characterize the landside blanket (top stratum)**

Estimate vertical permeability of landside blanket using permeability ratio for pervious stratum ($k_{h,f}/k_{v,bl}$)? Yes

| Parameter | Units | Minimum | Most Likely | Maximum |
|---|---|---|---|---|
| Transformed thickness for effective seepage length, $z_{bl}$ | ft | 5.0 | 10.0 | 18.0 |
| Effective thickness for allowable uplift pressure, $z_t$ | ft | 5.0 | 10.0 | 18.0 |
| Saturated unit weight, $\gamma_{sat}$ | pcf | 110.0 | 115.0 | 120.0 |
| Critical vertical exit gradient, $i_{cv} = (\gamma_{sat} - \gamma_w) / \gamma_w$ | - | 0.763 | 0.843 | 0.923 |
| Permeability ratio, $k_{h,f}/k_{v.bl}$ | - | 125.0 | 250.0 | 300.0 |
| Transformed vertical permeability, $k_{v,bl}$ | cm/sec | 8.00E-05 | 1.60E-04 | 3.00E-04 |
| Transformed vertical permeability, $k_{v,bl}$ | cm/sec | 8.00E-05 | 1.60E-04 | 3.00E-04 |

| Parameter | Units | Mean | @RISK Formula | Mean |
|---|---|---|---|---|
| $z_{bl}$ | ft | #NAME? | | 11.0 |
| $z_t$ | ft | #NAME? | | 11.0 |
| $\gamma_{sat}$ | pcf | #NAME? | | 115.0 |
| $i_{cv}$ | - | | | 0.843 |
| $k_{h,f}/k_{v,bl}$ | - | #NAME? | | 225.0 |
| $k_{v,bl}$ | cm/sec | <br>#NAME? | | 1.80E-04 |

**Figure 55. Step 4 of BT Case 6 worksheet: Probabilistic input without using @RISK for permeability ratio as input.**

For probabilistic analysis using @RISK, input the minimum, most likely, and maximum values of the random variables, and use an @RISK formula for a triangular distribution in the third column as a default. Alternatively, input a valid @RISK distribution in lieu of this default formula, and the user-specified input displays in the fourth column. The mean values used in subsequent calculations are the mean for the @RISK distribution entered in the third column. Figure 56 illustrates the probabilistic input using @RISK. Figure 57 illustrates the probabilistic input without using @RISK for permeability ratio as input.

| Step 4: Characterize the landside blanket (top stratum) | | | | |
|---|---|---|---|---|
| Estimate vertical permeability of landside blanket using permeability ratio for pervious stratum ($k_{h,f}/k_{v,bl}$)? | | No | | |
| Parameter | Units | Minimum | Most Likely | Maximum |
| Transformed thickness for effective seepage length, $z_{bl}$ | ft | 5.0 | 10.0 | 18.0 |
| Effective thickness for allowable uplift pressure, $z_t$ | ft | 5.0 | 10.0 | 18.0 |
| Saturated unit weight, $\gamma_{sat}$ | pcf | 110.0 | 115.0 | 120.0 |
| Critical vertical exit gradient, $i_{cv} = (\gamma_{sat} - \gamma_w) / \gamma_w$ | - | 0.763 | 0.843 | 0.923 |
| Permeability ratio, $k_{h,f}/k_{v.bl}$ | - | 125 | 250 | 300 |
| Transformed vertical permeability, $k_{v,bl}$ | cm/sec | - | - | - |
| Transformed vertical permeability, $k_{v,bl}$ | cm/sec | 8.00E-05 | 1.60E-04 | 3.00E-04 |

| Parameter | Units | Mean | @RISK Formula | Mean |
|---|---|---|---|---|
| $z_{bl}$ | ft | 11.0 | =@RiskTriang(F60,G60,H60,RiskCorrmat(zCorr,1)) | 11.0 |
| $z_t$ | ft | 11.0 | =@RiskTriang(F61,G61,H61,RiskCorrmat(zCorr,2)) | 11.0 |
| $\gamma_{sat}$ | pcf | 115.0 | =@RiskTriang(F62,G62,H62) | 115.0 |
| $i_{cv}$ | - | | | 0.843 |
| $k_{h,f}/k_{v,bl}$ | - | 225 | | 225 |
| $k_{v,bl}$ | cm/sec | 1.80E-04 | =@RiskTriang(F66,G66,H66) | 1.80E-04 |

**Figure 56. Step 4 of BT Case 6 worksheet: Probabilistic input using @RISK.**

| Step 4: Characterize the landside blanket (top stratum) | | | | | |
|---|---|---|---|---|---|
| Estimate vertical permeability of landside blanket using permeability ratio for pervious stratum ($k_{h,f}/k_{v,bl}$)? | | | | Yes | |
| Parameter | Units | Minimum | Most Likely | Maximum | |
| Transformed thickness for effective seepage length, $z_{bl}$ | ft | 5.0 | 10.0 | 18.0 | |
| Effective thickness for allowable uplift pressure, $z_t$ | ft | 5.0 | 10.0 | 18.0 | |
| Saturated unit weight, $\gamma_{sat}$ | pcf | 110.0 | 115.0 | 120.0 | |
| Critical vertical exit gradient, $i_{cv} = (\gamma_{sat} - \gamma_w) / \gamma_w$ | - | 0.763 | 0.843 | 0.923 | |
| Permeability ratio, $k_{h,f}/k_{v.bl}$ | - | 125 | 250 | 300 | |
| Transformed vertical permeability, $k_{v,bl}$ | cm/sec | 8.00E-05 | 1.60E-04 | 3.00E-04 | |
| Transformed vertical permeability, $k_{v,bl}$ | cm/sec | 8.00E-05 | 1.60E-04 | 3.00E-04 | |

| Parameter | Units | Mean | @RISK Formula | Mean |
|---|---|---|---|---|
| $z_{bl}$ | ft | 11.0 | =@RiskTriang(F60,G60,H60,RiskCorrmat(zCorr,1)) | 11.0 |
| $z_t$ | ft | 11.0 | =@RiskTriang(F61,G61,H61,RiskCorrmat(zCorr,2)) | 11.0 |
| $\gamma_{sat}$ | pcf | 115.0 | =@RiskTriang(F62,G62,H62) | 115.0 |
| $i_{cv}$ | - | | | 0.843 |
| $k_{h,f}/k_{v,bl}$ | - | 225 | =@RiskTriang(F64,G64,H64) | 225 |
| $k_{v,bl}$ | cm/sec | 1.80E-04 | | 2.07E-04 |

**Figure 57. Step 4 of BT Case 6 worksheet: Probabilistic input using @RISK for permeability ratio as input.**

The thicknesses $z_{bl}$ and $z_t$ are positively correlated. For probabilistic analysis, values for the distributions for $z_{bl}$ and $z_t$ must be sampled in the same direction. If a high value of $z_{bl}$ is sampled, a high value of $z_t$ must be sampled; if a low value of $z_{bl}$ is sampled, a low value of $z_t$ must be sampled. For probabilistic analysis without using @RISK, this correlation is accounted for by sampling the same percentile from each distribution. For probabilistic analysis using @RISK, the correlation matrix named "zCorr" defined on the BT Transformation worksheet must be added to the end of the user-specified input distributions for $z_{bl}$ and $z_t$ using the @RISK RiskCorrmat property function. Use RiskCorrmat(zCorr,1) for $z_{bl}$ and RiskCorrmat(zCorr,2) for $z_t$. For example, the triangular input distributions of RiskTriang(F60,G60,H60) for $z_{bl}$ and RiskTriang(F61,G61,H61) for $z_t$ become RiskTriang(F60,G60,H60,RiskCorrmat(zCorr,1)) and RiskTriang(F61,G61,H61,RiskCorrmat(zCorr,2)), respectively.

For probabilistic analysis using @RISK, if more than one of the BT Case 6, BT Case 7, and BT Case 8 worksheets exist in the active workbook, @RISK returns a warning message when the simulate button in the @RISK ribbon is selected as shown in Figure 58. (The workbook name in brackets matches the user's active workbook name.) Select "Yes" to continue the simulation because the same correlation matrix (named range of "zCorr" on the BT Transformation worksheet) is used for the input distributions for $z_{bl}$ and $z_t$ on the BT Case 6, BT Case 7, and BT Case 8 worksheets.

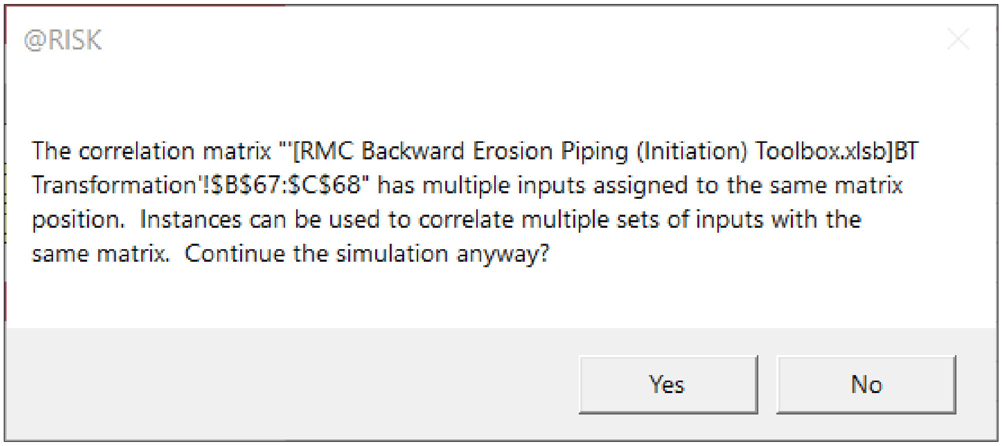



**Figure 58. Example warning message when performing probabilistic analysis using @RISK.**

If the blanket extends infinitely landward, the effective seepage exit distance from the landside levee toe ($x_3$) is calculated using Equations 23 and 24.

$$x_3 = \frac{1}{c_{bl}} \tag{23}$$

where:

$c_{bl}$ = constant for the landside blanket

$$c_{bl} = \sqrt{\frac{k_{v,bl}}{k_{h,f} z_{bl} d}} \tag{24}$$

where:

$k_{h,f}$ = horizontal permeability of pervious substratum
$d$ = thickness of the pervious substratum
$k_{v,bl}$ = transformed vertical permeability of the landside blanket (top substratum)
$z_{br}$ = transformed thickness of the riverside blanket (top substratum)

If a seepage block exists landward of the landside levee toe that prevents any seepage exit into the pervious foundation landside of the seepage block, the effective seepage exit distance from the landside levee toe ($x_3$) is calculated using Equation 25:

$$x_3 = \frac{1}{c_{bl} \tanh(c_{bl} L_3)} \tag{25}$$

where:

$L_3$ = distance from the landside levee toe to the seepage block for the “seepage block (impervious boundary)” boundary condition

If an open seepage exit exists landward of the landside levee toe, the effective seepage exit distance from the landside levee toe ($x_3$) is calculated using Equation 26:

$$x_3 = \frac{\tanh(c_{bl}L_3)}{c_{bl}} \tag{26}$$

where:

$L_3$ = distance from the landside levee toe to the open seepage exit for the "open seepage exit" boundary condition

The landside boundary condition selection in step 2 affects the equation for $x_3$ used in step 4, and cells that do not apply have a gray background.

## 11.5. Blanket Theory Assumptions

Step 5 checks the BT (method of fragments) assumptions against the input parameters to ensure essentially vertical equipotential lines, vertical flow through the blanket and horizontal flow through the pervious foundation, and semi-pervious blanket behavior. For permeability ratios greater than about 1,000 to 4,000, the semi-pervious blanket is effectively impervious, and Case 4 is more appropriate than Case 6. For deterministic analysis, the assumptions are checked for the most likely values of the random variables. For probabilistic analysis, the assumptions are checked for the mean values of the random variables. Values outside of the model assumptions have an orange background. For a blanket extending infinitely landward ($L_3 = \infty$), $(L_2 + x_3)/d = \infty$ for all values of $d$. Figure 59 illustrates the check of BT assumptions.

| Step 5: Check blanket theory (method of fragments) assumptions | |
|---|---|
| Assumption | Mean |
| Essentially vertical equipotential lines, $(L_2 + x_3) / d \geq 1$ | ∞ |
| Vertical landside blanket flow and horizontal foundation flow, $k_{h,f}/k_{v,bl} \geq 10$ | 259 |
| Semi-pervious landside blanket behavior, $k_{h,f}/k_{v,bl} < 1{,}000$ to $4{,}000$ | 259 |

**Figure 59. Step 5 of BT Case 6 worksheet: BT assumptions.**

## 11.6. Seepage Characterization

Step 6 calculates the net hydraulic head on the levee ($H$) the same as in Case 1. The flow or seepage per unit length of the levee ($Q_s$) is calculated using Equation 27.

$$Q_s = k_{h,f} H \frac{d}{0.43d + L_2 + x_3} \tag{27}$$

where:

$k_{h,f}$ = horizontal permeability of pervious substratum
$H$ = net hydraulic head on the levee
$L_2$ = base width of the levee
$d$ = thickness of the pervious substratum
$x_3$ = effective seepage exit from the landside levee toe

The format for the tabular output is the same as in Case 1.

## 11.7. Likelihood of Heave/Blowout at Landside Toe

Step 7 calculates the excess hydraulic head at the landside levee toe ($h_o$) using Equation 28.

$$h_o = H\frac{x_3}{0.43d + L_2 + x_3} \tag{28}$$

where:

$H$ = net hydraulic head on the levee
$L_2$ = base width of the levee
$d$ = thickness of the pervious substratum
$x_3$ = effective seepage exit from the landside levee toe

The vertical seepage exit gradient at the landside levee toe ($i_v$) is calculated using Equation 29.

$$v = \frac{h_o}{z_t} \tag{29}$$

where:

$h_o$ = excess hydraulic head at the landside levee toe
$z_t$ = effective thickness of the landside blanket (top stratum)

The factor of safety against heave/blowout (based on vertical seepage gradients) at the landside levee toe ($FS_{vg}$) is calculated the same as in Case 2. The format for the tabular and graphical output is the same as in Case 2.

## 11.8. Likelihood of Heave/Blowout at Given Distance from Landside Toe

If the blanket extends infinitely landward, Step 8 calculates the excess hydraulic head at a distance $x$ from the landside levee toe ($h_x$) using Equation 30.

$$h_x = h_o e^{-c_{bl}x} \tag{30}$$

where:

$h_o$ = excess hydraulic head at the landside levee toe
$c_{bl}$ = constant for the landside blanket
$x$ = distance from the landside levee toe

If a seepage block exists landward of the landside levee toe that prevents any seepage exit into the pervious foundation landside of the seepage block, Step 8 calculates the excess hydraulic head at a distance $x$ from the landside levee toe ($h_x$) using Equation 31.

$$h_x = h_o \frac{\cosh\left(c_{bl}(L_3 - x)\right)}{\cosh(c_{bl}L_3)} \tag{31}$$

where:

$h_o$ = excess hydraulic head at the landside levee toe
$c_{bl}$ = constant for the landside blanket
$L_3$ = distance from the landside levee toe to the seepage block for the "seepage block (impervious boundary)" boundary condition
$x$ = distance from the landside levee toe

If an open seepage exit exists landward of the landside levee toe, Step 8 calculates the excess hydraulic head at a distance *x* from the landside levee toe ($h_x$) using Equation 32.

$$h_x = h_o \frac{\sinh(c_{bl}(L_3 - x))}{\sinh(c_{bl}L_3)} \tag{32}$$

where:

$h_o$ = excess hydraulic head at the landside levee toe
$c_{bl}$ = constant for the landside blanket
$L_3$ = distance from the landside levee toe to the open seepage exit for the "open seepage exit" boundary condition
$x$ = distance from the landside levee toe

The vertical seepage exit gradient at a distance *x* from the landside levee toe ($i_{v,x}$) is calculated using Equation 33.

$$i_{v,x} = \frac{h_x}{z_t} \tag{33}$$

where:

$h_x$ = excess hydraulic head at a distance x from the landside levee toe
$z_t$ = effective thickness of the landside blanket (top stratum)

The factor of safety against heave/blowout (based on vertical seepage gradients) at a distance *x* from the landside levee toe ($FS_{vg,x}$) is calculated the same as in Case 2. The format for the tabular and graphical output is the same as in Case 2.

# 12. Blanket Theory Case 7

Case 7 has a semi-pervious top stratum on both the riverside and landside of the levee. The pervious substratum is divided into three zones to apply the method of fragments as shown in Figure 63. The difference between Cases 2 and 7 is that there is an impervious top stratum on both the riverside and landside of the levee for Case 2, while the top strata are semi-pervious for Case 7. Therefore, the effective seepage entry distance from the riverside levee toe must be calculated based on the type of seepage entrance (riverside boundary condition), and the effective seepage exit distance from the landside levee toe must be calculated based on the type of seepage exit (landside boundary condition).

## 12.1. Method of Analysis

The method of analysis is the same as in Case 1.

## 12.2. Levee Geometry

Step 2 characterizes the levee geometry. The input includes the levee crest elevation, landside levee toe elevation, and base width of levee ($L_2$). Use the drop-down lust to select the riverside boundary condition. The options for the type of seepage entrance include:

- No riverside borrow pits or seepage blocks
- Riverside borrow pit (open seepage entrance) that penetrates the blanket and extends to the pervious substratum
- A seepage block (impervious boundary) between the riverside levee toe and the river that prevents any seepage entrance into the pervious foundation riverside of the seepage block

For "no riverside borrow pits or seepage blocks," the input for $L_1$ is the distance from the riverside levee toe to the river. For "riverside borrow pit (open seepage entrance)," the input for $L_1$ is the distance from the riverside levee toe to the borrow pit. For "seepage block (impervious boundary)," the input for $L_1$ is the distance from the riverside levee toe to the seepage block.

Use the drop-down list to select the landside boundary condition. The options for the type of seepage exit include:

- A blanket extending infinitely landward as illustrated in Figure 60 (Case 7a)
- An open seepage exit as illustrated in Figure 61 (Case 7b)
- A seepage block (impervious boundary) beyond the landside levee toe that prevents any seepage exit into the pervious foundation landside of the seepage block as illustrated in Figure 62 (Case 7c)

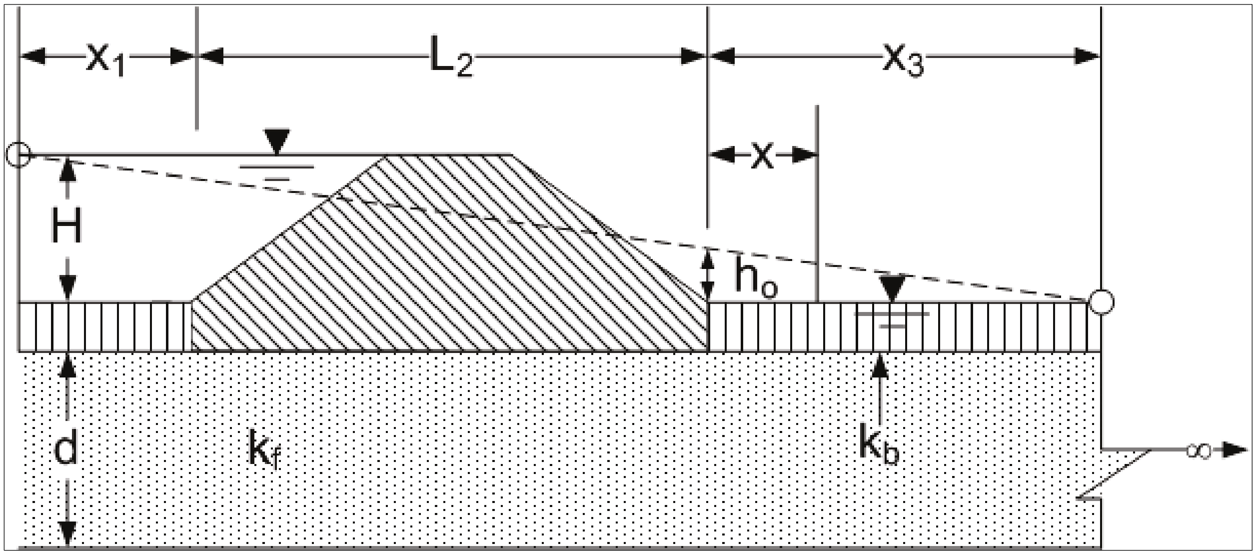



**Figure 60. BT Case 7a: Basic levee geometry for infinitely long landside blanket.**

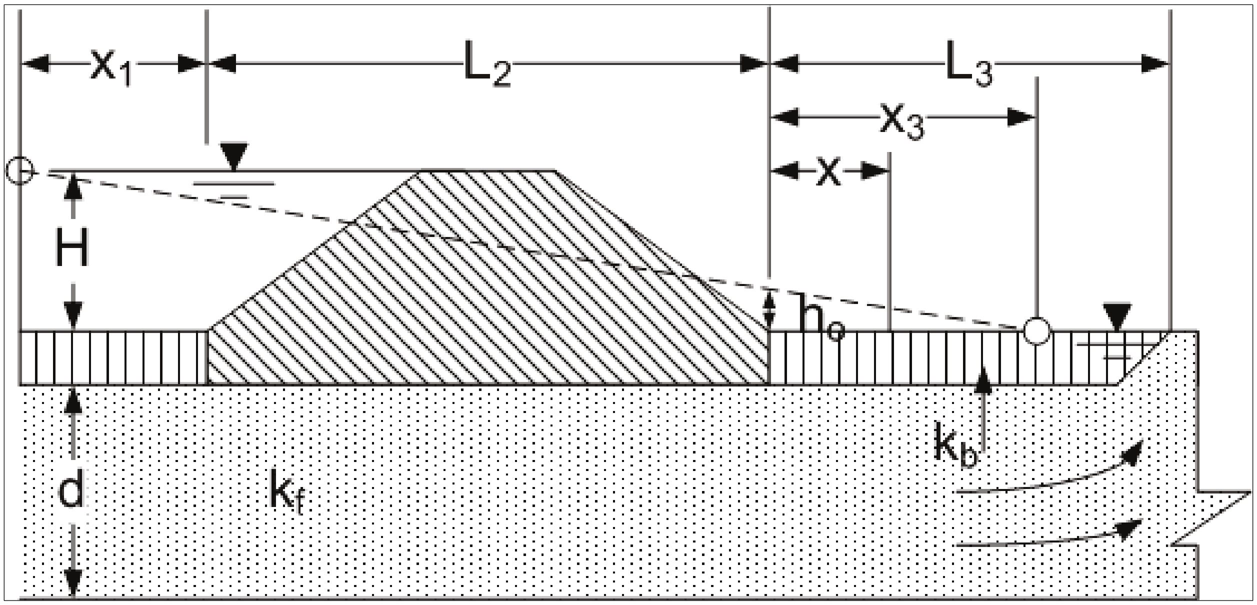



**Figure 61. BT Case 7b: Basic levee geometry for open seepage exit.**

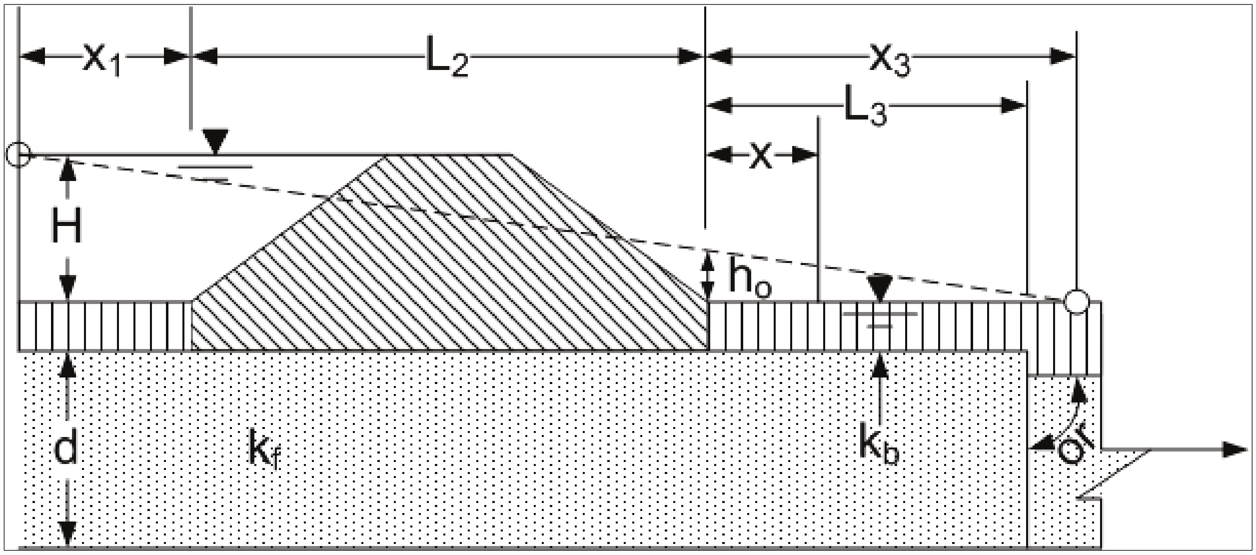
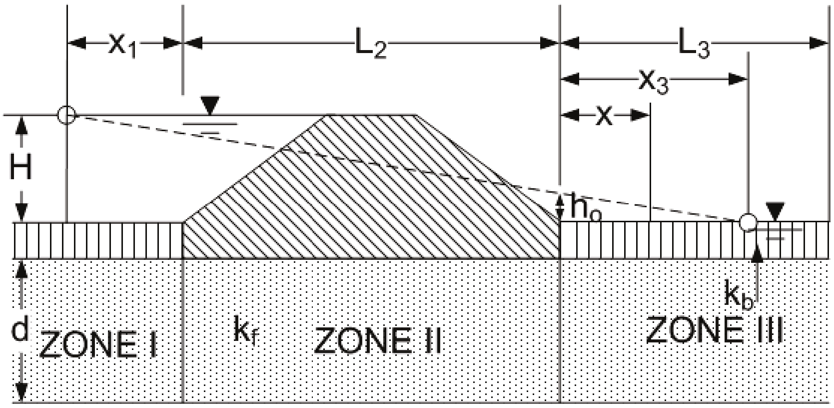



**Figure 62. BT Case 7c: Basic levee geometry for seepage block.**

For “blanket extending infinitely landward,” no input is needed for $L_3$ since it is infinitely long. For “open seepage exit,” the input for $L_3$ is the distance from the landside levee toe to the borrow pit. For “seepage block (impervious boundary),” the input for $L_3$ is the distance from the landside toe to the seepage block. The input is illustrated in Figure 63.

**Step 2: Assess the levee geometry**

See below for riverside boundary conditions




See below for landside boundary conditions

| | |
|---|---|
| Riverside boundary condition | no riverside borrow pits or seepage blocks |
| Landside boundary condition | blanket extending infinitely landward |

| | | |
|---|---|---|
| Levee crest elevation | 45.0 | ft-NAVD88 |
| Landside toe elevation | 20.0 | ft-NAVD88 |
| Base width of levee, $L_2$ | 110.0 | ft |

| | | |
|---|---|---|
| Distance from riverside toe to river, $L_1$ | 100.0 | ft |
| Length of foundation and blanket beyond landside toe, $L_3$ | ∞ | ft |
| Distance from landside levee toe to boundary condition, $L_3$ | 15.0 | ft |

**Figure 63. Step 2 of BT Case 7 worksheet: Levee geometry.**

## 12.3. Pervious Substratum Characterization

The pervious stratum characterization is the same as in Case 1.

## 12.4. Riverside Blanket (Top Stratum) Characterization

The riverside blanket (top stratum) characterization is the same as in Case 5.

## 12.5. Landside Blanket (Top Stratum) Characterization

The landside blanket (top stratum) characterization is the same as in Case 6.

## 12.6. Blanket Theory Assumptions

Step 6 checks the BT (method of fragments) assumptions against the input parameters to ensure essentially vertical equipotential lines, vertical flow through the blanket and horizontal flow through the pervious foundation, and semi-pervious blanket behavior. For permeability ratios greater than about 1,000 to 4,000, the semi-pervious blankets are effectively impervious, and Case 2 is more appropriate than Case 7. For deterministic analysis, the assumptions are checked for the most likely values of the random variables. For probabilistic analysis, the assumptions are checked for the mean values of the random variables. Values outside of the model assumptions have an orange background. For a blanket extending infinitely landward ($L_3 = \infty$), $(x_1 + L_2 + x_3)/d = \infty$ for all values of $d$. Figure 64 illustrates the check of BT assumptions.

| Step 6: Check blanket theory (method of fragments) assumptions | |
|---|---|
| Assumption | Mean |
| Essentially vertical equipotential lines, $(x_1 + L_2 + x_3) / d \geq 1$ | ∞ |
| Vertical riverside blanket flow and horizontal foundation flow, $k_{h,f}/k_{v,br} \geq 10$ | 259 |
| Vertical landside blanket flow and horizontal foundation flow, $k_{h,f}/k_{v,bl} \geq 10$ | 259 |
| Semi-pervious riverside blanket behavior, $k_{h,f}/k_{v,br} <$ 1,000 to 4,000 | 259 |
| Semi-pervious landside blanket behavior, $k_{h,f}/k_{v,bl} <$ 1,000 to 4,000 | 259 |

**Figure 64. Step 6 of BT Case 7 worksheet: BT assumptions.**

## 12.7. Seepage Characterization

Step 7 calculates the net hydraulic head on the levee ($H$) the same as in Case 1. The flow or seepage per unit length of the levee ($Q_s$) is calculated using Equation 34.

$$Q_s = k_{h,f} H \frac{d}{x_1 + L_2 + x_3} \tag{34}$$

where:

$k_{h,f}$ = horizontal permeability of pervious substratum
$H$ = net hydraulic head on the levee
$L_2$ = base width of the levee
$d$ = thickness of the pervious substratum
$x_1$ = effective seepage entrance from the riverside levee toe
$x_3$ = effective seepage exit from the landside levee toe

The format for the tabular output is the same as in Case 1.

## 12.8. Likelihood of Heave/Blowout at Landside Toe

Step 8 calculates the excess hydraulic head at the landside levee toe ($h_o$) using Equation 35.

$$h_o = H \frac{x_3}{x_1 + L_2 + x_3} \tag{35}$$

where:

$H$ = net hydraulic head on the levee
$L_2$ = base width of the levee
$d$ = thickness of the pervious substratum
$x_1$ = effective seepage entrance from the riverside levee toe
$x_3$ = effective seepage exit from the landside levee toe

The vertical seepage exit gradient at the landside levee toe ($i_v$) is calculated the same as in Case 6, and the factor of safety against heave/blowout (based on vertical seepage gradients) at the landside levee toe ($FS_{vg}$) is calculated the same as in Case 2. The format for the tabular and graphical output is the same as in Case 2.

## 12.9. Likelihood of Heave/Blowout at Given Distance from Landside Toe

The vertical seepage exit gradient at a distance x from the landside levee toe ($i_{v,x}$) is calculated the same as in Case 6, and the factor of safety against heave/blowout (based on vertical seepage gradients) at a distance x from the landside levee toe ($FS_{vg,x}$) is calculated the same as in Case 2. The format for the tabular and graphical output is the same as in Case 2.

# 13. Blanket Theory Case 8

Case 8 has a semi-pervious top stratum on both the riverside and landside of the levee, and a partially penetrating seepage barrier (for example, sheet pile wall or slurry wall) into the pervious substratum is at the centerline of the levee. The pervious substratum is divided into four zones to apply the method of fragments as shown in Figure 68 due to the partially penetrating seepage barrier. Since the top strata are semi-pervious, the effective seepage entry distance from the riverside levee toe must be calculated based on the type of seepage entrance (riverside boundary condition), and the effective seepage exit distance from the landside levee toe must be calculated based on the type of seepage exit (landside boundary condition).

## 13.1. Method of Analysis

The method of analysis is the same as in Case 1.

## 13.2. Levee Geometry

This levee geometry is the same as in Case 7, but there is a partially penetrating seepage barrier into the pervious substratum is at the centerline of the levee. Figure 65 illustrates a blanket extending infinitely landward (Case 8a). Figure 66 illustrates an open seepage exit (Case 8b). Figure 67 illustrates a seepage block (impervious boundary) beyond the landside levee toe that prevents any seepage exit into the pervious foundation landside of the seepage block (Case 8c).

There is an additional input for the length of the embedment of the seepage barrier into the pervious substratum (*s*). Values greater than or equal to the thickness of the pervious stratum (*d*) have an orange background. The input is illustrated in Figure 68.

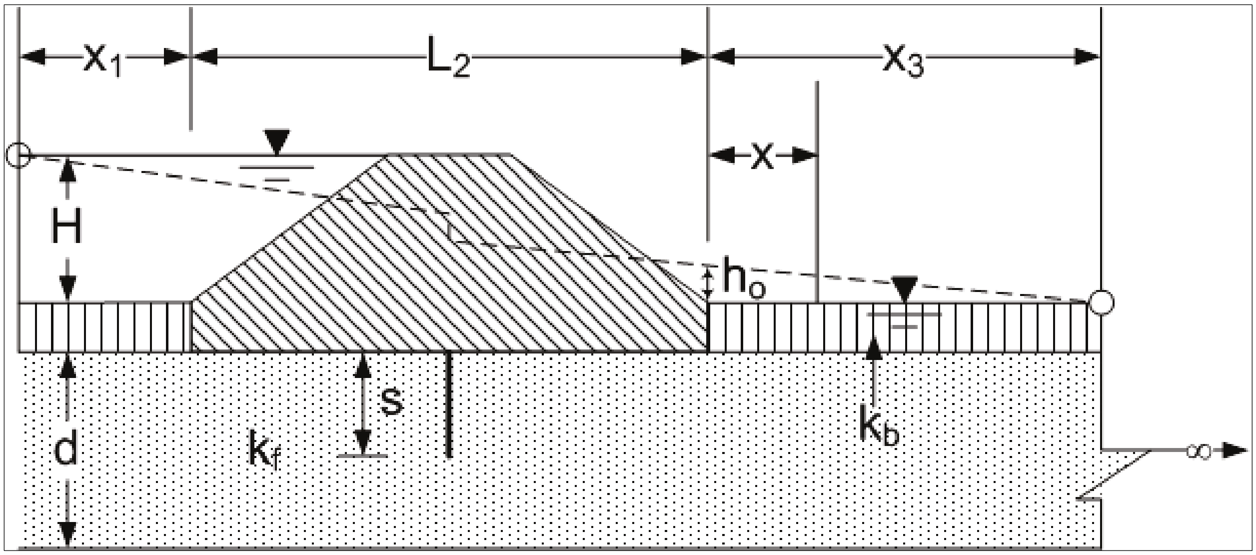



**Figure 65. BT Case 8a: Basic levee geometry for infinitely long landside blanket.**

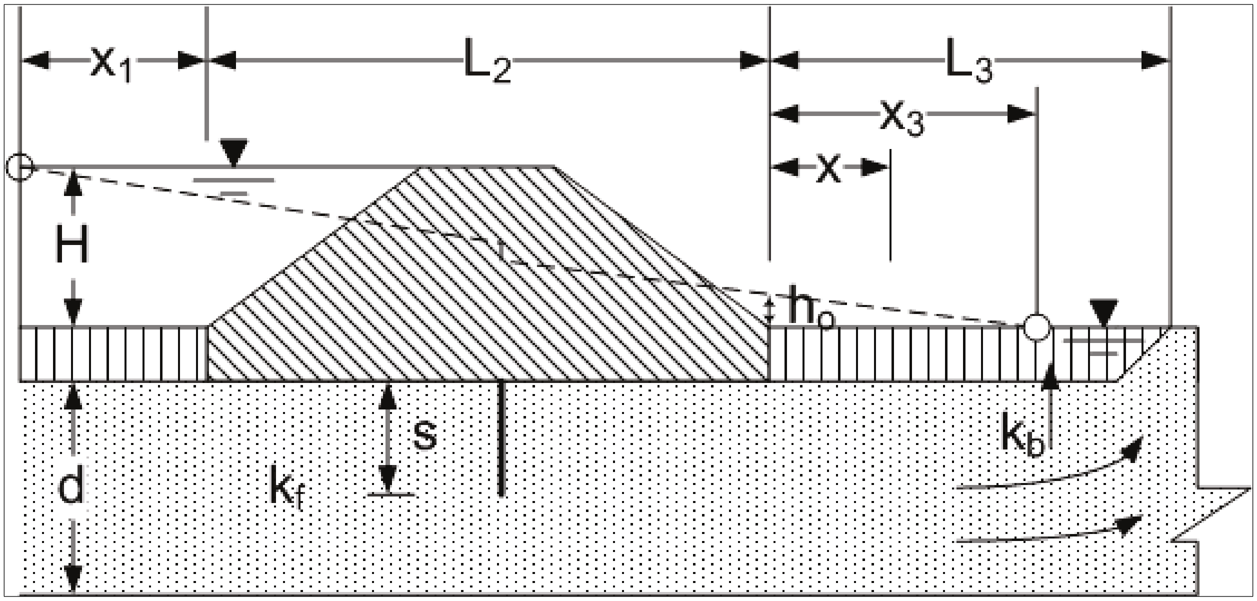



**Figure 66. BT Case 8b: Basic levee geometry for open seepage exit.**

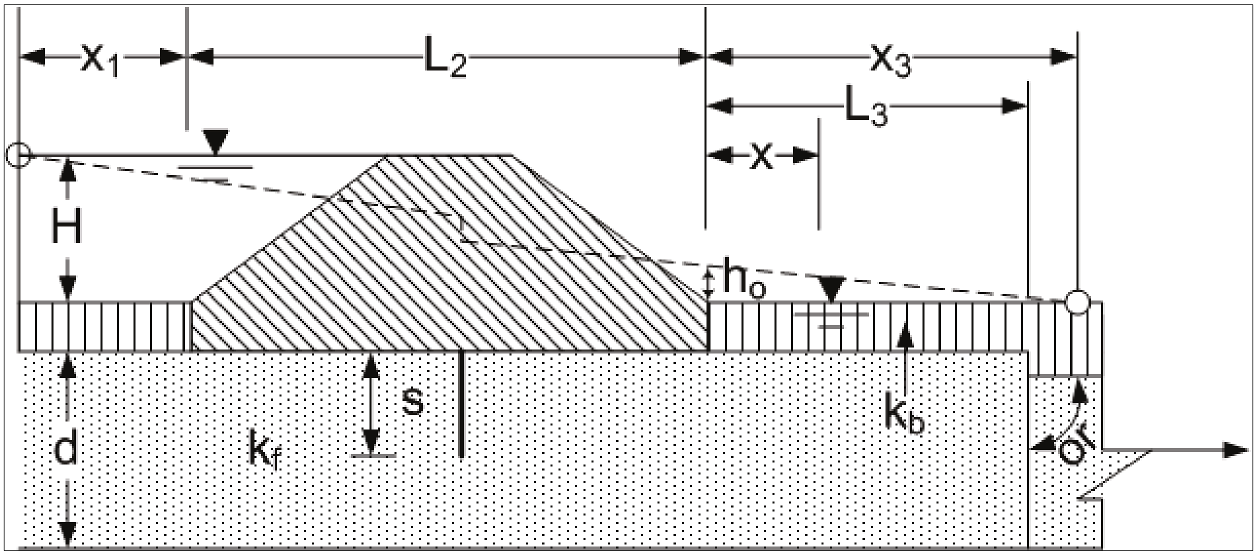
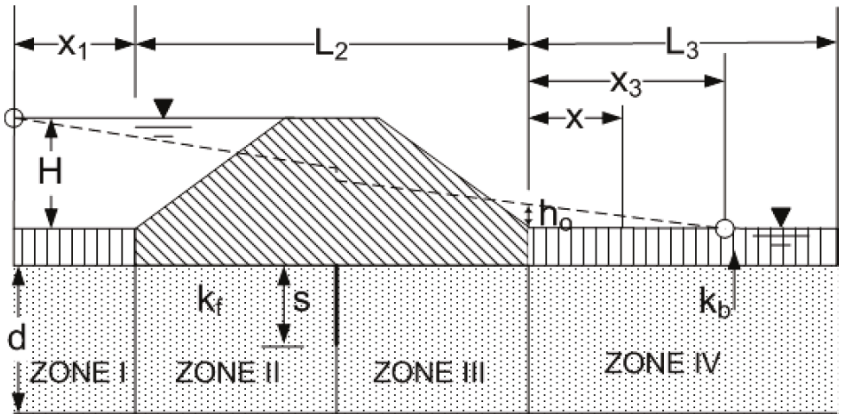



**Figure 67. BT Case 8c: Basic levee geometry for seepage block.**

**Step 2: Assess the levee geometry**

See below for riverside boundary conditions




See below for landside boundary conditions

| | |
|---|---|
| Riverside boundary condition | no riverside borrow pits or seepage blocks |
| Landside boundary condition | blanket extending infinitely landward |

| | | |
|---|---|---|
| Levee crest elevation | 45.0 | ft-NAVD88 |
| Landside toe elevation | 20.0 | ft-NAVD88 |
| Base width of levee, $L_2$ | 110.0 | ft |
| Distance from riverside toe to river, $L_1$ | 100.0 | ft |
| Length of foundation and blanket beyond landside toe, $L_3$ | ∞ | ft |
| Distance from landside levee toe to boundary condition, $L_3$ | 15.0 | ft |
| Length of embedment of seepage barrier into pervious substratum, s < d | 5.0 | ft |

**Figure 68. Step 2 of BT Case 8 worksheet: Levee geometry.**

## 13.3. Pervious Substratum Characterization

The pervious stratum characterization is the same as in Case 1.

## 13.4. Riverside Blanket (Top Stratum) Characterization

The riverside blanket (top stratum) characterization is the same as in Case 5.

## 13.5. Landside Blanket (Top Stratum) Characterization

The landside blanket (top stratum) characterization is the same as in Case 6.

## 13.6. Blanket Theory Assumptions

The check of BT assumptions is the same as in Case 7.

## 13.7. Seepage Characterization

Step 7 calculates the net hydraulic head on the levee ($H$) the same as in Case 1. The flow or seepage per unit length of the levee ($Q_s$) is calculated using Equation 36.

$$Q_s = k_{h,f} H \frac{d}{x_1 + 2d(K/K') + x_3} \tag{36}$$

where:

$k_{h,f}$ = horizontal permeability of pervious substratum
$H$ = net hydraulic head on the levee
$d$ = thickness of the pervious substratum
$x_1$ = effective seepage entrance from the riverside levee toe
$x_3$ = effective seepage exit from the landside levee toe
$K/K'$ = ratio of the complete elliptic integral of the first kind to the complementary integral

The value of $K/K'$ is obtained from a table of complete elliptical integrals of the first kind. In Table 4, the modulus ($m$) is calculated using Equation 37.

$$m = \cos\left(\frac{\pi s}{2d}\right)\sqrt{\tanh^2\left(\frac{\pi b}{2d}\right) + \tan\left(\frac{\pi s}{2d}\right)} \tag{37}$$

where:

$s$ = length of embedment of the seepage barrier into the pervious substratum
$b$ = half base width of the levee = $L_2/2$

**Table 4.**
**Complete elliptical integrals of the first kind.**

| $m^2$ | $K/K'$ | $m^2$ | $K/K'$ | $m^2$ | $K/K'$ | $m^2$ | $K/K'$ | $m^2$ | $K/K'$ |
|---|---|---|---|---|---|---|---|---|---|
| 0.000 | 0.000 | 0.17 | 0.706 | 0.43 | 0.938 | 0.69 | 1.198 | 0.95 | 1.828 |
| 0.001 | 0.325 | 0.18 | 0.716 | 0.44 | 0.946 | 0.70 | 1.211 | 0.96 | 1.901 |
| 0.002 | 0.349 | 0.19 | 0.726 | 0.45 | 0.955 | 0.71 | 1.224 | 0.97 | 1.992 |
| 0.003 | 0.366 | 0.20 | 0.735 | 0.46 | 0.964 | 0.72 | 1.237 | 0.98 | 2.123 |
| 0.004 | 0.379 | 0.21 | 0.745 | 0.47 | 0.973 | 0.73 | 1.251 | 0.990 | 2.347 |
| 0.005 | 0.389 | 0.22 | 0.754 | 0.48 | 0.982 | 0.74 | 1.265 | 0.991 | 2.381 |
| 0.006 | 0.398 | 0.23 | 0.763 | 0.49 | 0.991 | 0.75 | 1.279 | 0.992 | 2.418 |
| 0.007 | 0.406 | 0.24 | 0.773 | 0.50 | 1.000 | 0.76 | 1.294 | 0.993 | 2.461 |
| 0.008 | 0.413 | 0.25 | 0.782 | 0.51 | 1.009 | 0.77 | 1.310 | 0.994 | 2.510 |
| 0.009 | 0.420 | 0.26 | 0.791 | 0.52 | 1.018 | 0.78 | 1.326 | 0.995 | 2.568 |
| 0.01 | 0.426 | 0.27 | 0.800 | 0.53 | 1.028 | 0.79 | 1.343 | 0.996 | 2.639 |
| 0.02 | 0.471 | 0.28 | 0.808 | 0.54 | 1.037 | 0.80 | 1.360 | 0.997 | 2.731 |
| 0.03 | 0.502 | 0.29 | 0.817 | 0.55 | 1.047 | 0.81 | 1.377 | 0.998 | 2.860 |
| 0.04 | 0.526 | 0.30 | 0.826 | 0.56 | 1.057 | 0.82 | 1.397 | 0.999 | 3.081 |
| 0.05 | 0.547 | 0.31 | 0.834 | 0.57 | 1.066 | 0.83 | 1.416 | 0.9991 | 3.115 |
| 0.06 | 0.565 | 0.32 | 0.843 | 0.58 | 1.076 | 0.84 | 1.439 | 0.9992 | 3.152 |
| 0.07 | 0.582 | 0.33 | 0.852 | 0.59 | 1.087 | 0.85 | 1.462 | 0.9993 | 3.195 |
| 0.08 | 0.598 | 0.34 | 0.860 | 0.60 | 1.098 | 0.86 | 1.484 | 0.9994 | 3.244 |
| 0.09 | 0.612 | 0.35 | 0.869 | 0.61 | 1.107 | 0.87 | 1.511 | 0.9995 | 3.302 |
| 0.10 | 0.625 | 0.36 | 0.877 | 0.62 | 1.118 | 0.88 | 1.538 | 0.9996 | 3.373 |
| 0.11 | 0.638 | 0.37 | 0.886 | 0.63 | 1.129 | 0.89 | 1.567 | 0.9997 | 3.465 |
| 0.12 | 0.650 | 0.38 | 0.895 | 0.64 | 1.140 | 0.90 | 1.600 | 0.9998 | 3.594 |
| 0.13 | 0.662 | 0.39 | 0.903 | 0.65 | 1.151 | 0.91 | 1.634 | 0.9999 | 3.814 |
| 0.14 | 0.674 | 0.40 | 0.911 | 0.66 | 1.162 | 0.92 | 1.672 | 1 | ∞ |
| 0.15 | 0.684 | 0.41 | 0.920 | 0.67 | 1.174 | 0.93 | 1.718 | | |
| 0.16 | 0.695 | 0.42 | 0.929 | 0.68 | 1.186 | 0.94 | 1.770 | | |

To interpolate intermediate values of $K/K'$ for $m^2$ between 0.9999 and 1 in Table 4, the maximum number that can be stored by Excel of 1.79769313486232E308 is used for $K/K' = \infty$. If the seepage barrier is fully penetrating ($s \geq d$), $m$ is 1; $K/K' = \infty$; and $Q_s = 0$.

The format for the tabular output is the same as in Case 1.

## 13.8. Likelihood of Heave/Blowout at Landside Toe

Step 8 calculates the excess hydraulic head at the landside levee toe ($h_o$) using Equation 38.

$$h_o = H \frac{x_3}{x_1 + 2d(K/K') + x_3} \tag{38}$$

where:

$H$ = net hydraulic head on the levee
$d$ = thickness of the pervious substratum
$x_1$ = effective seepage entrance from the riverside levee toe
$x_3$ = effective seepage exit from the landside levee toe

The vertical seepage exit gradient at the landside levee toe ($i_x$) is calculated the same as in Case 6, and the factor of safety against heave/blowout (based on vertical seepage gradients) at the landside levee toe ($FS_{vg}$) is calculated the same as in Case 2. The format for the tabular and graphical output is the same as in Case 2.

## 13.9. Likelihood of Heave/Blowout at Given Distance from Landside Toe

If the blanket extends infinitely landward, Step 9 calculates the excess hydraulic head at a distance $x$ from the landside levee toe ($h_x$) using Equation 39.

$$h_x = h_o e^{-c_{bl}x} \tag{39}$$

where:

$h_o$ = excess hydraulic head at the landside levee toe
$x$ = distance from the landside levee toe

If a seepage block exists landward of the landside levee toe that prevents any seepage exit into the pervious foundation landside of the seepage block, Step 9 calculates the excess hydraulic head at a distance $x$ from the landside levee toe ($h_x$) using Equation 40.

$$h_x = h_o \frac{\cosh\left(c_{bl}(L_3 - x)\right)}{\cosh(c_{bl}L_3)} \tag{40}$$

where:

$h_o$ = excess hydraulic head at the landside levee toe
$c_{bl}$ = constant for the landside blanket
$L_3$ = distance from the landside levee toe to the seepage block for the “seepage block (impervious boundary)” boundary condition
$x$ = distance from the landside levee toe

If an open seepage exit exists landward of the landside levee toe, Step 9 calculates the excess hydraulic head at a distance $x$ from the landside levee toe ($h_x$) using Equation 33.

$$h_x = h_o \frac{\sinh\left(c_{bl}(L_3 - x)\right)}{\cosh(c_{bl}L_3)} \tag{33}$$

where:

$h_o$ = excess hydraulic head at the landside levee toe
$c_{bl}$ = constant for the landside blanket
$L_3$ = distance from the landside levee toe to the open seepage exit for the “open seepage exit” boundary condition
$x$ = distance from the landside levee toe

The vertical seepage exit gradient at a distance $x$ from the landside levee toe ($i_{v,x}$) is calculated the same as in Case 6, and the factor of safety against heave/blowout (based on vertical seepage gradients) at a distance $x$ from the landside levee toe ($FS_{vg,x}$) is calculated the same as in Case 2. The format for the tabular and graphical output is the same as in Case 2.

# 14. First-Order, Second-Moment Reliability

The Taylor series is a method to estimate the expected value (mean) and standard deviation of a limit state, given the means and standard deviations of the parameters. This method is termed a first-order, second-moment (FOSM) method, as only first-order (linear) terms of the series are retained and only the first two moments (mean and the standard deviation) are considered. The Taylor series method of reliability analysis has been used by USACE for many years to compute the reliability index and probability of unsatisfactory performance.

Only the first two moments (mean and variance) represent the probability density function in the reliability analysis, and all higher order terms are neglected in the Taylor series expansion used to estimate the mean and variance of the natural logarithm of the factor of safety. When using the reliability index method, no knowledge is needed of the exact probability density function that represents random variable.

Figure 69 shows the seepage analysis run cases for a single headwater level of interest after estimating the mean and standard deviation of each random variable (RV). The first run case uses the mean ($\mu$) of all random variables to obtain the mean value of the vertical seepage exit gradient ($i_v$). In subsequent run cases, each random variable is adjusted up or down by its standard deviation ($\sigma$) while using the mean values of all other random variables. The vertical seepage exit gradient is from seepage analysis for the various permutations. Run Cases 10 and 11 involve the saturated unit weight of the landside blanket, which is used to calculate the critical vertical seepage exit gradient for heave/blowout. Since no changes to the seepage model are required for these two run cases, the mean vertical seepage exit gradient from Run Case 1 is used. After obtaining the vertical seepage exit gradient for Run Cases 1 to 9, the factor of safety against heave/blowout (based on vertical seepage gradients) is calculated for all eleven run cases.

| Run Case | RV 1 | RV 2 | RV 3 | RV 4 | $\gamma_{sat}$ (pcf) | $i_v$ | $FS_{vg}$ |
|---|---|---|---|---|---|---|---|
| 1 | $\mu$ | $\mu$ | $\mu$ | $\mu$ | $\mu$ | | |
| 2 | $\mu - \sigma$ | $\mu$ | $\mu$ | $\mu$ | $\mu$ | | |
| 3 | $\mu + \sigma$ | $\mu$ | $\mu$ | $\mu$ | $\mu$ | | |
| 4 | $\mu$ | $\mu - \sigma$ | $\mu$ | $\mu$ | $\mu$ | | |
| 5 | $\mu$ | $\mu + \sigma$ | $\mu$ | $\mu$ | $\mu$ | | |
| 6 | $\mu$ | $\mu$ | $\mu - \sigma$ | $\mu$ | $\mu$ | | |
| 7 | $\mu$ | $\mu$ | $\mu + \sigma$ | $\mu$ | $\mu$ | | |
| 8 | $\mu$ | $\mu$ | $\mu$ | $\mu - \sigma$ | $\mu$ | | |
| 9 | $\mu$ | $\mu$ | $\mu$ | $\mu + \sigma$ | $\mu$ | | |
| 10 | $\mu$ | $\mu$ | $\mu$ | $\mu$ | $\mu - \sigma$ | $\mu$ | |
| 11 | $\mu$ | $\mu$ | $\mu$ | $\mu$ | $\mu + \sigma$ | $\mu$ | |

**Figure 69. Seepage analysis run cases for first-order, second-moment reliability analysis.**

The mean and standard deviation of the natural logarithm of the factor of safety are obtained using the mean, standard deviation, and correlation coefficients of the random variables in the Taylor series method.

Up to five random variables and seven headwater levels for the seepage analysis can be input. For five random variables, a total of eleven seepage analyses are required for each headwater level. If seven headwater levels are assessed with five random variables, a total of seventy-seven seepage analyses are required. Thus, the number of seepage analysis model runs is a function of the number of random variables and headwater levels, and the Taylor series method is very labor intensive. Usually, only a few random variables control the seepage analysis. Select only those random variables that are expected to significantly drive the results to reduce unnecessary computational effort that does not significantly improve the precision of the results.

## 14.1. Foundation Characterization

Step 1 characterizes the foundation. The input includes the saturated unit weight of the landside top stratum ($\gamma_{sat}$) and up to four additional random variables (e.g., thickness of top stratum, horizontal permeability of pervious foundation, permeability ratio, etc.). Normal distributions represent the random variables, and the mean ($\mu$) and standard deviation ($\sigma$) are input.

The 3-sigma rule is based on the normal distribution probability density function, where the lowest conceivable value is about three standard deviations (3 sigma) below the mean, and the highest conceivable value is about three standard deviations above the mean. For the mean plus or minus three standard deviations ($\mu \pm 3\sigma$), 99.73 percent of the area under the normal distribution is included. Therefore, 99.73 percent of all possible values of the random variable are included in the range of $6\sigma$ (the highest possible value to the lowest possible value of the random variable). Thus, the standard deviation using the 3-sigma rule using Equation 41.

$$\sigma = \frac{HCV - LCV}{6} \tag{41}$$

where:

$HCV$ = the highest conceivable value of the random variable
$LCV$ = the lowest conceivable value of the random variable

Use the drop-down list to select whether to use the 3-sigman rule to calculate the standard deviation of the random variables. For "Yes," input the lowest conceivable value (LCV) and highest conceivable value (HCV). Although the standard deviation is calculated using the 3-sigma rule, it informs the user-specified value in the subsequent row, which is used in the calculations. For "No," the standard deviation of the random variables is user-specified. Cells that do not apply have a gray background. These cells are not used in subsequent calculations even if data is present. The two possible scenarios are illustrated in Figure 70 and Figure 71.

Use 3-sigma rule to estmate standard deviation for a normal distrbution? Yes

| Parameter | Random Variables | | | | |
|---|---|---|---|---|---|
| | Kha (fpd) | Khb (fpd) | Ta (ft) | Tb (ft) | $\gamma_{sat}$ (pcf) |
| Mean, $\mu$ | 40 | 500 | 10 | 80 | 117.1 |
| Lowest conceivable value, LCV | 15 | 300 | 10 | 40 | 108.0 |
| Highest conceivable value, HCV | 60 | 750 | 10 | 160 | 126.2 |
| Standard deviation, $\sigma$ = (HCV − LCV)/6 | 7.5 | 75 | 0 | 20 | 3.0 |
| Use standard deviation, $\sigma$ | 7.5 | 75 | 0 | 20 | 3.0 |

**Figure 70. Step 1 of FOSM reliability worksheet: Input for standard deviation using 3-sigma rule.**

Use 3-sigma rule to estmate standard deviation for a normal distrbution? No

| Parameter | Random Variables | | | | |
|---|---|---|---|---|---|
| | Kha (fpd) | Khb (fpd) | Ta (ft) | Tb (ft) | $\gamma_{sat}$ (pcf) |
| Mean, $\mu$ | 40 | 500 | 10 | 80 | 117.1 |
| Lowest conceivable value, LCV | 15 | 300 | 10 | 40 | 108.0 |
| Highest conceivable value, HCV | 60 | 750 | 10 | 160 | 126.2 |
| Standard deviation, $\sigma$ = (HCV − LCV)/6 | - | - | - | - | - |
| Use standard deviation, $\sigma$ | 7.5 | 75 | 0 | 20 | 3.0 |

**Figure 71. Step 1 of FOSM Reliability Analysis worksheet:
Input for standard deviation without using 3-sigma rule.**

## 14.2. Reliability Analysis

Step 2 calculates the critical vertical seepage exit gradient for heave/blowout ($i_{cv}$) using Equation 42.

$$i_{cv} = \frac{\gamma'}{\gamma_w} = \frac{\gamma_{sat} - \gamma_w}{\gamma_w} \quad (42)$$

where:

$\gamma'$ = submerged unit weight of the landside blanket
$\gamma_{sat}$ = saturated unit weight of the landside blanket
$\gamma_w$ = unit weight of water

The factor of safety against heave/blowout (based on vertical seepage gradients) ($FS_{vg}$) is calculated using Equation 43.

$$FS_{vg} = \frac{i_{cv}}{i_v} \quad (43)$$

where:

$i_v$ = vertical seepage exit gradient

$FS_{vg}$ represents an upper bound for an intact blanket (top stratum), and initiation occurs at significantly lower values in many cases through defects in the blanket (such as the data from Turnbull and Mansur for the lower Mississippi River).

The variance of the factor of safety against heave/blowout ($Var[FS_{vg}]$) is calculated using Equation 44.

$$\mathrm{Var}\left[FS_{vg}\right] = \sigma_{FS_{vg}}^2 = \sum_{i=0}^{n} \left[\frac{FS_{vg}\left(x_i + \sigma_{x_i}\right) - FS_{vg}\left(x_i - \sigma_{x_i}\right)}{2}\right]^2 \tag{44}$$

where:

$n$ = number of random variables

The $\mathrm{Var}[FS_{vg}]$ for each random variable divided by the sum of all the variances gives the percentage that random variable affects the analysis. Thus, the random variable(s) that control(s) the reliability analysis can easily be determined based on the higher percentages. The relative contribution to the analysis is shown as a percentage beneath the $\mathrm{Var}[FS_{vg}]$ for each random variable.

The standard deviation and coefficient of variation of the $FS_{vg}$ are calculated using Equations 45 and 46.

$$\sigma_{FS_{vg}} = \sqrt{\mathrm{Var}[FS_{vg}]} \tag{45}$$

$$V_{FS_{vg}} = \frac{\sigma_{FS_{vg}}}{\mu_{FS_{vg}}} \tag{46}$$

Physical quantities that result from a summation of many independent processes have distributions that are approximately normal. Physical quantities resulting from a product of many independent processes have distributions that are approximately lognormal. Lognormal distributions are used with reliability analyses. The probability density function of a factor of safety can be represented by a lognormal probability density function.

In Figure 72, the hatched area under the curve and to left of a FS of 1 gives the probability of a FS less than 1, $P(FS < 1)$, often referred to as the probability of unsatisfactory performance, $P(u)$, in reliability analysis. When the lognormal distribution is transformed to a normal distribution by taking the natural logarithm of the factor of safety, $\ln(1) = 0$. Thus, $P(FS < 1)$ is represented by the hatched area under the normal distribution curve to the left of zero in Figure 73.

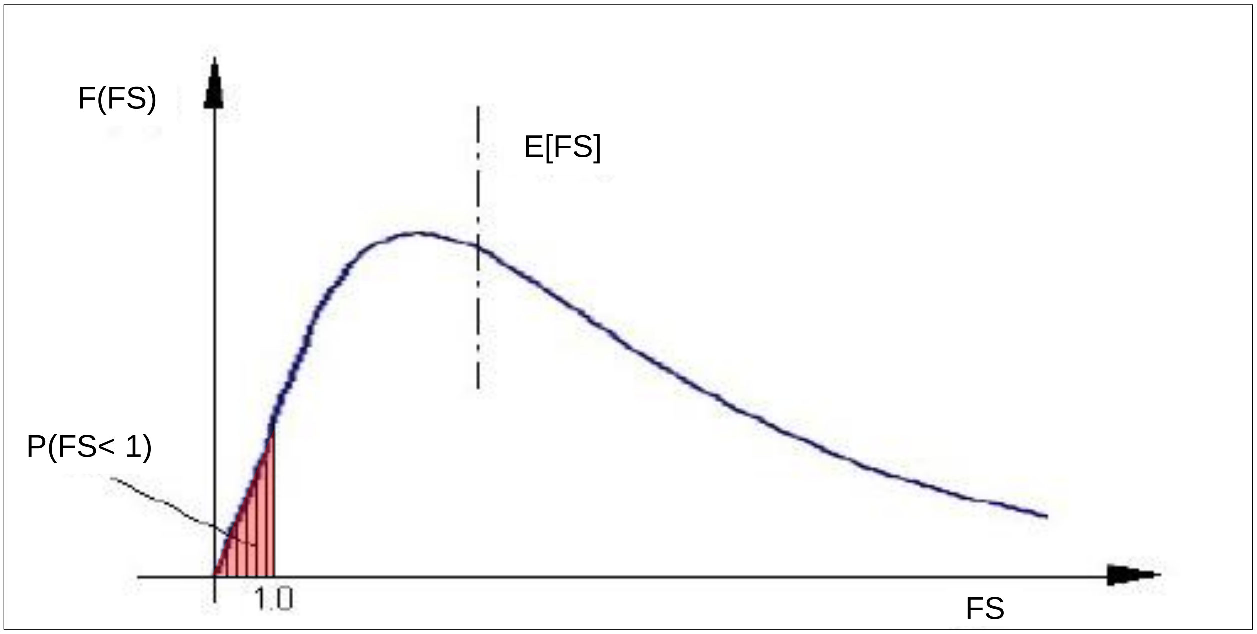



**Figure 72. Lognormal probability density function of factor of safety (ETL 1110-2-561).**

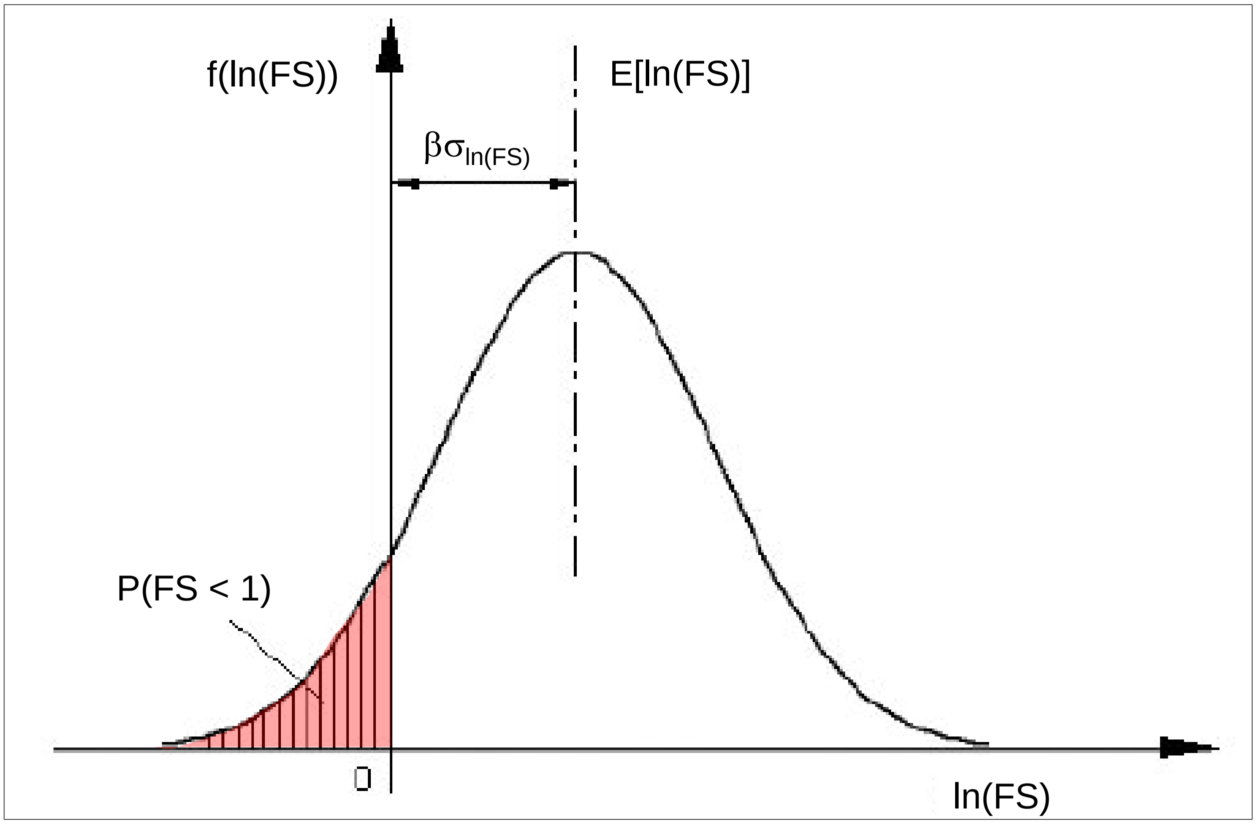



**Figure 73. Transformed probability density function of factor of safety (ETL 1110-2-561).**

Lognormal is favorable for FS for several reasons. First, a lognormal function is never negative, and the FS is never negative. Second, the product of several variables is lognormally distributed.

The reliability index ($\beta$) is a measure of the distance that the mean of the FS is away from 1 or unsatisfactory performance. The larger the value of $\beta$, the more reliable the structure is for the performance mode being assessed (for example, seepage or stability). The smaller the value of $\beta$, the closer the condition is to unsatisfactory performance. The reliability index ($\beta$) is calculated using Equation 47.

$$\beta_{ln(FS_{vg})} = \frac{\ln\left(\frac{\mu_{FS_{vg}}}{\sqrt{1+V_{FS_{vg}}^2}}\right)}{\sqrt{\ln\left(1+V_{FS_{vg}}^2\right)}} \tag{47}$$

The probability of a factor of safety against heave/blowout (based on vertical seepage gradients) less than 1 is calculated using Equation 48.

$$\mathrm{P}(FS_{vg} < 1) = \Phi\left(-\beta_{ln(FS_{vg})}\right) \tag{48}$$

For each headwater level evaluated in the seepage analysis, the probability of a factor of safety against heave/blowout less than 1, $\mathrm{P}(FS_{vg} < 1)$, is calculated as shown in the example in Figure 74. The headwater input for these tables must be the headwater used in the seepage analysis, which can differ from the headwater levels input at the top of the worksheet. Do not include the headwater level where $i_v = 0$ in step 2 to avoid unexpected interpolation errors in step 3.

**Step 2: Estimate the vertical exit gradient and assess the likelihood of heave/blowout**

Critical vertical exit gradient, $i_{cv} = \gamma' / \gamma_w = (\gamma_{sat} - \gamma_w) / \gamma_w$ 0.877 pcf (mean)

Factor of safety against heave/blowout, $FS_{vg} = i_{cv}/i_v$, where $i_v$ = vertical seepage exit gradient

Variance of factor of safety, $Var(FS) = \sigma_{FS}^2 = \Sigma \{[FS(x_i + \sigma_{xi}) - FS(x_i - \sigma_{xi})] / 2\}^2$

Standard deviation of factor of safety, $\sigma_{FS} = [\Sigma Var(FS)]^{0.5}$

Coefficient of variation of factor of safety, $V_{FS} = \sigma_{FS} / \mu_{FS}$

Reliability index, $\beta_{LN} = \ln[\mu_{FS}/(1 + V_{FS}^2)^{0.5}] / [\ln(1 + V_{FS}^2)]^{0.5}$

Probability of factor of safety less than 1, $P(FS_{vg} < 1) = \Phi(-\beta_{LN})$

**Headwater** 201.6 ft-NGVD29 used in seepage analysis

| Kha (fpd) | Khb (fpd) | Ta (ft) | Tb (ft) | $\gamma_{sat}$ (pcf) | $i_v$ | $FS_{vg}$ | $Var(FS_{vg})$ |
|---|---|---|---|---|---|---|---|
| 40 | 500 | 10 | 80 | 117.1 | 0.172 | 5.097 | |
| 32.5 | 500 | 10 | 80 | 117.1 | 0.202 | 4.340 | 0.5658 |
| 47.5 | 500 | 10 | 80 | 117.1 | 0.150 | 5.844 | (24.4%) |
| 40 | 425 | 10 | 80 | 117.1 | 0.145 | 6.046 | 0.6366 |
| 40 | 575 | 10 | 80 | 117.1 | 0.197 | 4.450 | (27.4%) |
| 40 | 500 | 10 | 80 | 117.1 | 0.172 | 5.097 | 0 |
| 40 | 500 | 10 | 80 | 117.1 | 0.172 | 5.097 | (0%) |
| 40 | 500 | 10 | 60 | 117.1 | 0.136 | 6.446 | 1.041 |
| 40 | 500 | 10 | 100 | 117.1 | 0.199 | 4.405 | (44.8%) |
| 40 | 500 | 10 | 80 | 114.1 | 0.172 | 4.817 | 0.0781 |
| 40 | 500 | 10 | 80 | 120.1 | 0.172 | 5.376 | (3.4%) |

| | | | |
|---|---|---|---|
| $\sigma_{FS}$ = 1.5237 | $V_{FS}$ = 0.2990 | $\beta_{LN}$ = 5.42 | $P(FS_{vg} < 1)$ = 2.983E-08 |

**Figure 74. Step 2 of FOSM Reliability Analysis worksheet: Calculations based on seepage analysis results.**

At the end of step 2, the mean factor of safety against heave/blowout and the probability of a factor of safety less than 1 are summarized as illustrated in Figure 75.

| **Summary** | | | | | | | |
|---|---|---|---|---|---|---|---|
| HW (ft) | 201.6 | 213.5 | 222.0 | 231.0 | 235.0 | 239.0 | - |
| $i_v$ | 0.172 | 0.342 | 0.463 | 0.574 | 0.648 | 0.705 | - |
| $FS_{vg}$ | 5.10 | 2.56 | 1.89 | 1.53 | 1.35 | 1.24 | - |
| | | | | | | | |
| HW (ft) | 201.6 | 213.5 | 222.0 | 231.0 | 235.0 | 239.0 | - |
| $P(FS_{vg} < 1)$ | 2.98E-08 | 6.99E-05 | 9.16E-03 | 3.42E-02 | 9.50E-02 | 1.77E-01 | - |

**Note: Probabilities obtained from this method should not be used directly in risk analyses. Rather, the values should be used to help develop a list of more and less likely factors.**

**Figure 75. Step 2 of FOSM Reliability Analysis worksheet: Tabular output based on seepage analysis results.**

## 14.3. Likelihood of Heave/Blowout at Landside Toe

Step 3 interpolates the mean factor of safety against heave/blowout (based on vertical seepage gradients) and the probability of a factor of safety less than 1 from the results of the seepage analysis for the headwater levels input at the top of the worksheet. The mean $FS_{vg}$ values are linearly interpolated, and the $P(FS_{vg} < 1)$ is interpolated using logarithmic scale for probability and linear scale for headwater as illustrated in Figure 76.

| **Step 3: Assess the likelihood of heave/blowout for the stages under consideration** | | | | | | | |
|---|---|---|---|---|---|---|---|
| HW (ft) | 201.6 | 205.0 | 213.5 | 222.0 | 231.0 | 235.0 | 239.0 |
| $i_v$ | 0.172 | 0.221 | 0.342 | 0.463 | 0.574 | 0.648 | 0.705 |
| $FS_{vg}$ | 5.10 | 4.37 | 2.56 | 1.89 | 1.53 | 1.35 | 1.24 |
| | | | | | | | |
| HW (ft) | 201.6 | 205.0 | 213.5 | 222.0 | 231.0 | 235.0 | 239.0 |
| $P(FS_{vg} < 1)$ | 2.98E-08 | 2.74E-07 | 6.99E-05 | 9.16E-03 | 3.42E-02 | 9.50E-02 | 1.77E-01 |

**Note: Probabilities obtained from this method should not be used directly in risk analyses. Rather, the values should be used to help develop a list of more and less likely factors.**

**Figure 76. Step 3 of FOSM Reliability Analysis worksheet: Tabular output based on selected headwater levels.**

Summary plots are generated after the tabular output. The first plot is the mean FS against heave/blowout (red solid line) and vertical seepage exit gradient at the landside levee toe (green solid line) as functions of headwater level. $FS_{vg}$ is plotted on the primary y-axis, and $i_v$ is plotted on the secondary y-axis. Horizontal reference lines display for the mean critical vertical seepage exit gradient (green dashed line) and factor of safety of 1 (red dashed line). Figure 77 illustrates the graphical deterministic output.

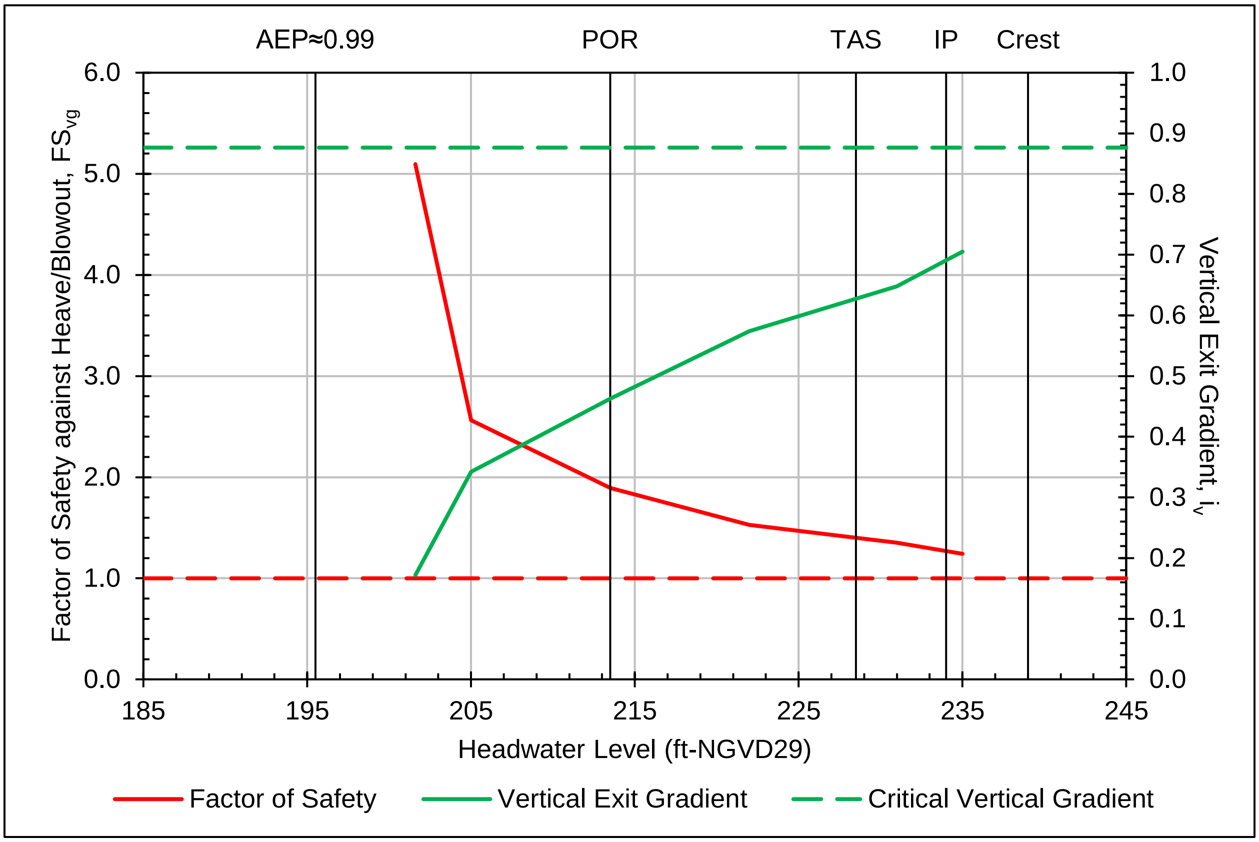


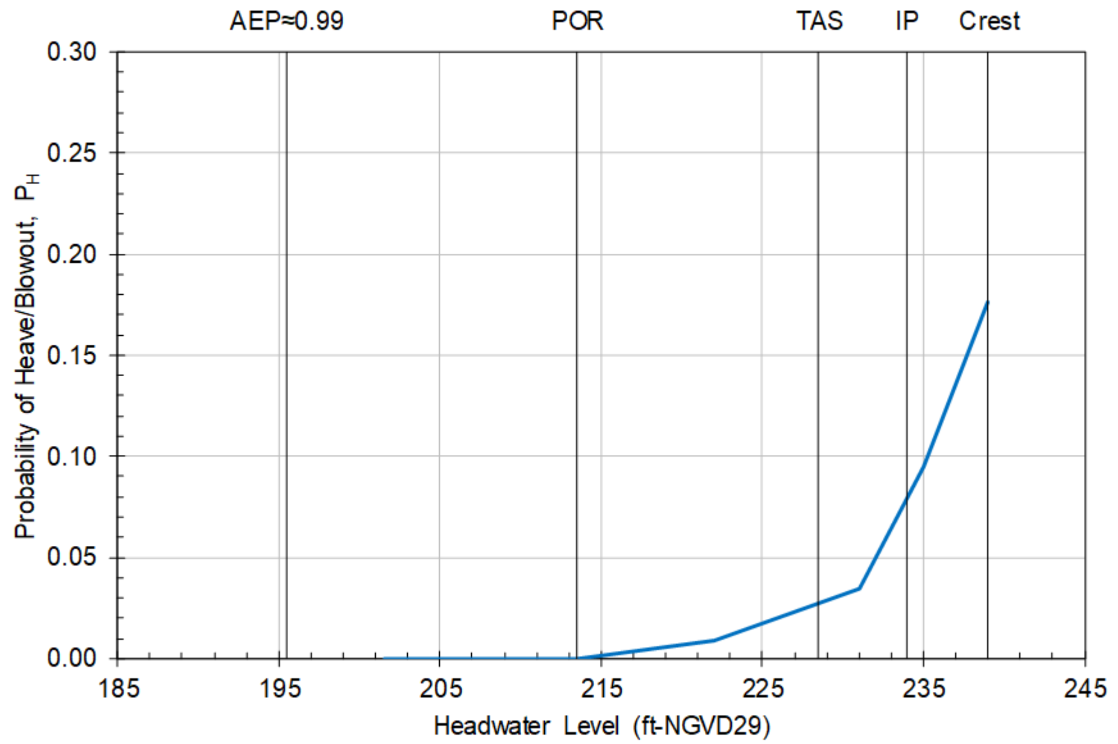

**Figure 77. Step 3 of FOSM Reliability Analysis worksheet: Graphical output.**

Figure 78 illustrates the plot options for this chart. The maximum value for the primary y-axis ($FS_{vg}$), maximum value for the secondary y-axis ($i_v$), and minimum and maximum values for the x-axis (headwater level) are user-specified. Users can input up to five vertical reference elevations, and user-specified labels display at the top of the chart.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Worksheet | FOSM Reliability Analysis | | | | | | |
| | | | | | | | |
| y-axis bounds (primary) | | | | | | | |
| minimum | 0 | | | | Value Primary Min: 0 | | |
| maximum | 6.0 | ◄ Enter maximum factor of safety. | | | Value Primary Max: 6 | | |
| | | | | | | | |
| y-axis bounds (secondary) | | | | | | | |
| minimum | 0 | | | | Value Secondary Min: 0 | | |
| maximum | 1.00 | ◄ Enter maximum exit gradient. | | | Value Secondary Max: 1 | | |
| | | | | | | | |
| x-axis bounds | | | | | | | |
| minimum | 185.0 | ◄ Enter minimum headwater level. | | | Category Primary Min: 185 | | |
| maximum | 245.0 | ◄ Enter maximum headwater level. | | | Category Primary Max: 245 | | |
| | | | | | | | |
| Enter up to 5 vertical reference lines for headwater levels of interest. | | | | | | | |
| AEP≈0.99 | POR | TAS | IP | Crest | ◄ Enter headwater description. | | |
| 195.5 | 213.5 | 228.5 | 234.0 | 239.0 | ◄ Enter headwater level. | | |

**Figure 78. Step 3 of FOSM Reliability Analysis worksheet:**
**Plot options for deterministic graphical output.**

The mean probability of heave/blowout at the landside levee toe is plotted as a function of headwater level. Figure 79 illustrates the graphical output for probabilistic analysis.




**Note: Probabilities obtained from this method should not be used directly in risk analyses. Rather, the values should be used to help develop a list of more and less likely factors.**

**Figure 79. Step 3 of FOSM Reliability Analysis worksheet: Probabilistic graphical output.**

Figure 80 illustrates the plot options for this chart. The vertical reference elevations and minimum and maximum values for the x-axis (headwater level) are the same as the input in Figure 78. Only the maximum value for the y-axis (probability of heave/blowout) is user-specified.

| | | | |
|---|---|---|---|
| Worksheet | FOSM Reliability Analysis | | |
| | | | |
| y-axis bounds | | | |
| minimum | 0 | | Value Primary Min: 0 |
| maximum | 0.3 | ◄ Enter maximum probability. | Value Primary Max: 0.3 |
| | | | |
| x-axis bounds | | | |
| minimum | 185.0 | ◄ Enter minimum headwater level. | Category Primary Min: 185 |
| maximum | 245.0 | ◄ Enter maximum headwater level. | Category Primary Max: 245 |

**Figure 80. Step 3 of FOSM Reliability Analysis worksheet: Plot options for probabilistic graphical output.**

# 15. References

Barron, R. A. (1948). The effect of a slightly pervious top blanket on the performance of relief wells. *Proceedings, 2nd International Conference on Soil Mechanics and Foundation Engineering, Vol. 4,* International Society for Soil Mechanics and Geotechnical Engineering.

Batool, A. (2013). *Advanced underseepage analyses for levees* [Doctoral Dissertation, Virginia Polytechnic Institute and State University]. https://vtechworks.lib.vt.edu/bitstream/handle/10919/52507/Batool_A_D_2013.pdf;sequence=1.

Batool, A., VandenBerge, D.R., Brandon, T.L. (2015). Practical application of Blanket Theory and the finite-element method to levee underseepage analysis. *Journal of Geotechnical and Geoenvironmental Engineering, 141*(4), 1–10. https://doi.org/10.1061/(ASCE)GT.1943-5606.0001269.

Bennett, P. T. (1946). The effect of blankets on the seepage through pervious foundation. *Transactions, American Society of Civil Engineers, 111*(1), 215–228.

Brandon, T.L., Batool, A., Jimenez, M., Vroman, N.D., and Corcoran, M.K. (2018). *Comparison of levee underseepage analysis methods using Blanket Theory and finite-element analysis* (ERDC/GSL TR-18-24). U.S. Army Corps of Engineers, Engineer Research and Development Center. https://usace.contentdm.oclc.org/digital/collection/p266001coll1/id/7861/.

Forchheimer, P. (1917). Zur Grundwasserbewegung nach isothermischen Kurvenscharen [On the movement of groundwater according to sets of isothermal curves]. *Sitzungsber K-K Akad der Wissenschaft, 126*(4), 409–440.

Harr, M. E. (1962). *Groundwater and seepage.* McGraw-Hill.

Muskat, M. (1937). *The flow of homogeneous fluids through porous media*. McGraw-Hill.

Pavlovsky, N. N. (1956). *Collected works.* Leningrad, Russia: Akad, Nauk, USSR.

Terzaghi, K., Peck, R.B., and Mesri, G. (1996). *Soil Mechanics in Engineering Practice.* Third Edition. Wiley. https://www.wiley.com/en-us/Soil+Mechanics+in+Engineering+Practice%2C+3rd+Edition-p-9780471086581.

Turnbull, W.J. and Mansur, C.I. (1959). Investigation of underseepage-Mississippi River levees. *Journal of the Soil Mechanics and Foundations Division, 85*(4), 41–93. https://doi.org/10.1061/JSFEAQ.0000215.

U.S. Army Corps of Engineers (USACE). (1956). *Investigation of underseepage and its control: Lower Mississippi River Levees* (Technical Manual [TM] 3-424). Waterways Experiment Station. https://usace.contentdm.oclc.org/digital/collection/p266001coll1/id/2416/.

USACE. (1998). Soil mechanics data. *Section 8 Ground Water and Seepage* (Division Regulation [DIVR] 1110-1-400, Change 2). Mississippi Valley Division. https://www.mvn.usace.army.mil/Portals/56/docs/engineering/HurrGuide/DIVR_1110-1-400_Section_8_GroundWaterandSeepage.pdf.

USACE. (2000). *Design and construction of levees* (Engineer Manual [EM] 1110-2-1913). https://www.publications.usace.army.mil/Portals/76/Publications/EngineerManuals/EM_1110-2-1913.pdf.

USACE. (2006). *Reliability analysis and risk assessment for seepage and slope stability failure modes for embankment dams* (Engineer Technical Letter [ETL] 1110-2-561).

# Appendix A. Acronym List

| | |
|---|---|
| 2D | Two-Dimensional |
| BEP | Backward Erosion Piping |
| BT | Blanket Theory |
| CPD | Computer Program Document |
| DIVR | Division Regulation |
| EM | Engineer Manual |
| ERDC | U.S. Army Engineer Research and Development Center |
| ETL | Engineer Technical Letter |
| FS | Factor of Safety |
| FEA | Finite-Element Analysis |
| FOSM | First Order, Second Moment |
| GSL | Geotechnical and Structures Laboratory |
| HCV | Highest Conceivable Value |
| HEC | Hydrologic Engineering Center |
| HW | Headwater |
| LCV | Lowest Conceivable Value |
| MVD | Mississippi Valley Division |
| NAVD 88 | North American Vertical Datum of 1988 |
| NGVD 29 | National Geodetic Vertical Datum of 1929 |
| RMC | Risk Management Center |
| RV | Random Variable |
| TW | Tailwater |
| UDF | User-Defined Function |
| U.S. | United States |

| | |
|---|---|
| USACE | United States Army Corps of Engineers |