

# RMC Breach Toolbox

RMC Internal Erosion Suite

RMC-CPD-2023-09

November 2023



**US Army Corps  
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Prepared by the Risk Management Center

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<b>14. ABSTRACT</b> The spreadsheet tools contained in this toolbox help assess the likelihood of breach due to gross enlargement of a concentrated leak pipe using the excess shear stress equation, unraveling of the downstream slope due to flows through the rockfill, and sinkhole development, in addition to providing general guidance for assessing slope instability.					
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## **PREPARED**

The results, findings, and recommendations provided in this document are technically sound and consistent with current Corps of Engineers practice.

Damon Amlung, Risk Management Center

## **REVIEWED**

This report has been checked and reviewed and is believed to be in accordance with the standards of the profession.

Tim O'Leary, Risk Management Center

## **APPROVED**

Nate Snorteland, Risk Management Center

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# 1. Introduction

The Risk Management Center (RMC) of the U.S. Army Corps of Engineers (USACE) has developed a suite of Microsoft Excel spreadsheets to support risk assessments for dam and levee safety. Each analysis suite is composed of multiple toolboxes (Microsoft Excel workbooks), and each toolbox contains multiple spreadsheet tools or calculation worksheets (Microsoft Excel worksheets). The RMC Breach Toolbox is part of the RMC Internal Erosion Suite.

The information from these spreadsheet tools, along with other pertinent information, informs judgment when developing a list of more and less likely factors and estimating probabilities. USACE best practice for estimating probabilities is to use the best available and multiple methods, but all final probabilities are estimated using team elicitation based on the totality and strength of the evidence.

The RMC continuously works to improve the performance of RMC software; report possible bugs directly to the RMC at the address listed below. Ideally, report suspected errors in written form with a description of the problem and the steps that lead to its occurrence. Suggestions for improvement are also welcomed.

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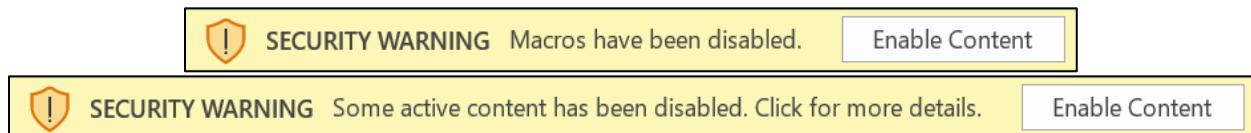
## 3. General Overview

### 3.1. Getting Started

Copy or download the toolbox file to the computer. To open the toolbox file, either:

- Find the file on the computer and double-click it. This opens the file in Microsoft Excel.
- Open Microsoft Excel and use the application to open the file: Once Microsoft Excel is open, go to the File menu at the top of the window and select Open.

The toolbox is an Excel binary workbook (.xslb) that uses macros. You may need to enable the macros, either before opening the file or by clicking “Enable Content” in the yellow Security Warning message bar with a shield icon that appears after the file is opened. The actual message in the message bar will vary depending on the computer’s settings and installed add-ins. Figure 1 displays examples of different wordings that may appear in the message bar.



**Figure 1. Security warning message bars with the “Enable Content” option to enable macros.**

### 3.2. Organization

Although the toolbox does not provide a calculation cover sheet, adding one is strongly recommended. A calculation cover sheet captures project information, a description and purpose of the calculation, the assumptions for critical input parameters, a summary of the major conclusion and results, and a revision history.

Each toolbox has a similar appearance and organizational structure:

- The first worksheet, About, summarizes the purpose of the toolbox and gives contact information for the RMC software development team.
- The second worksheet, Terms and Conditions, contains the terms and conditions for use of the toolbox (IWR software).
- The third worksheet, Version History, contains the revision history. Semantic versioning is used in the format of MAJOR.MINOR.PATCH:
  - MAJOR – significant worksheet changes not compatible with previous versions.
  - MINOR – additional features or enhancements that do not fundamentally change the calculations.
  - PATCH – backward-compatible bug fixes.

- The fourth worksheet, References, lists the references cited for each calculation worksheet.

The workbook and worksheets are not protected to prevent unwanted changes. However, because the toolbox has user-defined functions (UDFs) and subroutines in Visual Basic, you cannot directly copy worksheets to another workbook without potentially losing functionality. A note in a bold red font at the upper right margin indicates if the selected worksheet includes such features.

At the top of each calculation worksheet, input information for the preparer and checker for quality control (QC) documentation and the calculation title in case multiple copies of the worksheet are created for different analysis scenarios (Figure 2). The footer of each calculation worksheet contains the version number, which can be cross-referenced with the revision history on the third worksheet.

Prepared by:	Office:	Date:
Checked by:	Office:	Date:
Calculation Title:		

**Figure 2. Calculation worksheet heading.**

User-specified input includes values and selections from drop-down lists. User input cells are light yellow, and these cells are unprotected. When cells use drop-down lists, a note in blue font in the right margin of the row alerts the user to use the drop-down list. Conditional formatting applies a gray background to cells that are not based on a user selection. When a user-specified value or calculated value is outside of acceptable ranges, the cell is orange to indicate caution to the user.

All units for user-specified input values are clearly labeled. Most user-specified input values use English units. However, values may be in metric where metric units are more common in practice (e.g., particle size in millimeters or permeability in centimeters per second). The toolbox may convert English units to metric units to perform some calculations or if required for a specific formula based on the reference material for the equation.

If the calculation worksheet is a function of headwater level, up to seven headwater and tailwater levels may be specified at the top of the worksheet. Tailwater may be required to calculate the net hydraulic head and hydraulic gradient. Specify the elevation datum by selecting one of three options from the drop-down list: ft-NAVD88, ft-NGVD29, and Other. The two datum selections include English units of length (feet). If Other is selected, provide a user-specified datum along with feet (e.g., ft-MSL [Mean Sea Level]). Figure 3 through Figure 5 illustrate the three possible scenarios.

Elevation datum	ft-NAVD88	Specify datum	◀ Use drop-down list.
HW (ft)	195.5	201.6	213.5
TW (ft)	184.0	184.0	184.0

**Figure 3. Headwater and tailwater input: NAVD88.**

Elevation datum	ft-NGVD29	Specify datum	◀ Use drop-down list.
HW (ft)	195.5	201.6	213.5
TW (ft)	184.0	184.0	184.0

**Figure 4. Headwater and tailwater input: NGVD29.**

Elevation datum	Other	Specify datum	ft-MSL		◀ Use drop-down list.	
HW (ft)	195.5	201.6	213.5	218.9	223.0	234.0
TW (ft)	184.0	184.0	184.0	184.0	184.0	184.0

◀ Headwater level, HW (ft-MSL)  
◀ Tailwater level, TW (ft-MSL)

**Figure 5. Headwater and tailwater input: User-specified datum.**

Most calculation worksheets break down complex analysis into computational steps following a logical sequence (Figure 6). Some simpler worksheets do not have steps. Generally, different methodologies are unique worksheets. Some worksheets may include multiple methodologies, which are labeled as options (Figure 7).

**Step 1: Select the method of analysis**

**Figure 6. Example of step banner.**

**Option 1: Riverside blanket (top stratum) for Cases 5, 7, and 8**

**Figure 7. Example of option banner.**

Some calculation worksheets can perform either a deterministic or probabilistic analysis. Although not required to perform a probabilistic analysis, Palisade @RISK software (standalone version or as part of the Palisade DecisionTools Suite) can customize the probabilistic analysis. A note appears in a bold red font at the upper right-hand margin of a calculation worksheet indicating if this feature is included with the toolbox.

User notes generally appear in the right margin of each calculation worksheet. Some notes are in blue or red font for heightened awareness. These notes include references to source materials for equations, figures, tables, pages, etc. If the RMC modified the source material, the reference citation says “adapted from” instead of “from.”

Tabular and/or graphical summaries are generally the primary output of the toolbox. The UDFs in the PlotScale module change the minimum and maximum values of the x-axis and y-axis for charts. If the calculation worksheet is a function of headwater level, you can define up to five headwater levels of interest and plot them as vertical reference lines. By selecting the chart and then selecting the Filter icon to display the filter pane, you can choose which data series to display. This is useful when computing the results from multiple methodologies, but not all are applicable or desired to display.

## 4. Background

Breach is the final phase of internal erosion and is defined as a catastrophic failure characterized by the sudden, rapid, and uncontrolled release of impounded water or liquid-borne solids (Federal Emergency Management Agency P-1025, 2015). Fell et al. (2008) lists four breach mechanisms caused by internal erosion and typically considered for earthen embankments.

- **Gross enlargement of a pipe or concentrated leak:** When an erosion pathway or pipe connects to the impounded water, the earthen sidewalls rapidly erode. This continues until the embankment collapses unless the impounded water level drops below the pipe entrance or the impounded water level drops enough so that the hydraulic shear stress on the pipe sidewalls becomes less than the critical shear stress of the soil. If the amount of crest drop is greater than the available freeboard, embankment overtopping could quickly lead to full breach formation. If overtopping does not occur, the embankment could be severely damaged, and breach may still occur by concentrated leak erosion through cracks.
- **Downstream face sloughing or unraveling:** Increased seepage into the downstream or landside zone of an embankment can over-steepen the exit face, leading to progressive sloughing or unraveling. Sloughing requires a cohesionless material and progresses more rapidly with steep slopes. In some cases, as soil particles erode, a void may grow near the exit face until a roof can no longer be supported, and the void collapses (mass masting), like sapping involving seepage erosion undercutting. Unraveling refers to progressive removal of individual rocks by large seepage flows through a downstream rockfill zone. The sloughing and unraveling repeats and progresses until freeboard is lost.
- **Overtopping due to sinkhole development:** Internal migration can create a sinkhole or depression in the embankment that must be large enough to lead to overtopping. If the sinkhole is on the downstream slope away from the crest, progressive instability leads to breach. Internal erosion can also lead to excessive crest settlement and overtopping due to embankment loss or foundation materials.
- **Downstream slope instability:** Internal erosion may cause high pore pressures in the foundation or embankment, resulting in reduced shear strength and slope failure. Breach could occur if the failure surface intersects the impounded water level, or if the slope deformations are significant enough that the remnant embankment cannot resist the water load.

All four breach mechanisms ultimately lead to crest settlement and embankment overtopping. The most likely breach mechanism depends on the internal erosion mechanism, embankment zonation, and the specific failure mode evaluated. Table 1, adapted from Fell et al. (2008), lists the breach mechanisms based on the embankment zoning. Although one or more of the mechanisms may occur during the breach, risk assessment usually only considers the critical breach mechanism. The breach mechanism also informs time for intervention, warning issuance, and evacuation, and breach parameters.

This toolbox informs the likelihood of breach due to gross enlargement of a concentrated leak pipe using the excess shear stress equation, downstream slope unraveling due to flows through the rockfill, and sinkhole development and provides general guidance for assessing breach due to slope instability.

**Table 1**  
**Breach mechanism screening by zoning type (adapted from Fell et al. 2008).**

Dam Zoning Type	Breach Mechanisms			
	Gross Enlargement	Slope Instability	Sloughing or Unravelling	Sinkhole Development
Homogeneous earthfill	✓*	✓	Exclude, except if downstream fill is cohesionless	✓
Earthfill with filters	✓*	✓	Exclude, except if downstream fill is cohesionless	✓
Earthfill with rockfill toe	✓*	✓	Exclude, except if downstream fill is cohesionless	✓
Zoned earthfill	Exclude, except if downstream fill can support a roof	✓	Exclude, except if downstream fill is cohesionless	✓
Zoned earthfill and rockfill	Exclude, except if downstream fill can support a roof	✓	✓*	✓
Central core earth and rockfill (or gravel shells)	Exclude, except if downstream fill can support a roof	Exclude, except if existing dam has marginal stability	✓*	✓
Concrete face earthfill	✓	✓*	Exclude, except if downstream fill is cohesionless	✓
Concrete face rockfill (including gravel fill)	Exclude	Exclude, except if dam is gravel or low permeability	✓*	Exclude
Puddle core earthfill	✓*	✓	Exclude, except if downstream fill is cohesionless	✓
Earthfill with core wall	Exclude	✓*	Exclude, except if downstream fill is cohesionless	✓
Rockfill with core wall	Exclude	Exclude, except if existing dam has marginal stability	✓*	✓
Hydraulic fill	Exclude, except if downstream fill can support a roof	✓	✓*	✓

Key: ✓ Breach mechanism can occur.

✓\* Breach mechanism can occur and is usually the more critical mechanism.

## 5. Gross Enlargement

For concentrated leak erosion, the resistance to initiation is characterized by the critical shear stress ( $\tau$ ). When the applied hydraulic shear stress exceeds the critical value ( $\tau_c$ ), concentrated leak erosion will initiate. The rate of pipe enlargement in the progression phase is characterized by the erodibility coefficient ( $k_d$ ). The rate of erosion ( $\dot{\varepsilon}_t$ ) is the rate of volume of material removed per unit surface area per unit time and is calculated using the excess shear stress equation shown in Equation 1.

$$\dot{\varepsilon}_t = k_d (\tau - \tau_c) \quad (1)$$

where:

$k_d$  = erodibility coefficient  
 $\tau$  = hydraulic shear stress  
 $\tau_c$  = critical shear stress

The hydraulic shear stress ( $\tau$ ) on the surface of a circular pipe can be estimated using Equation 2 (see RMC-CPD-2023-08 for more details).

$$\tau = \gamma_w i \frac{D}{4} \quad (2)$$

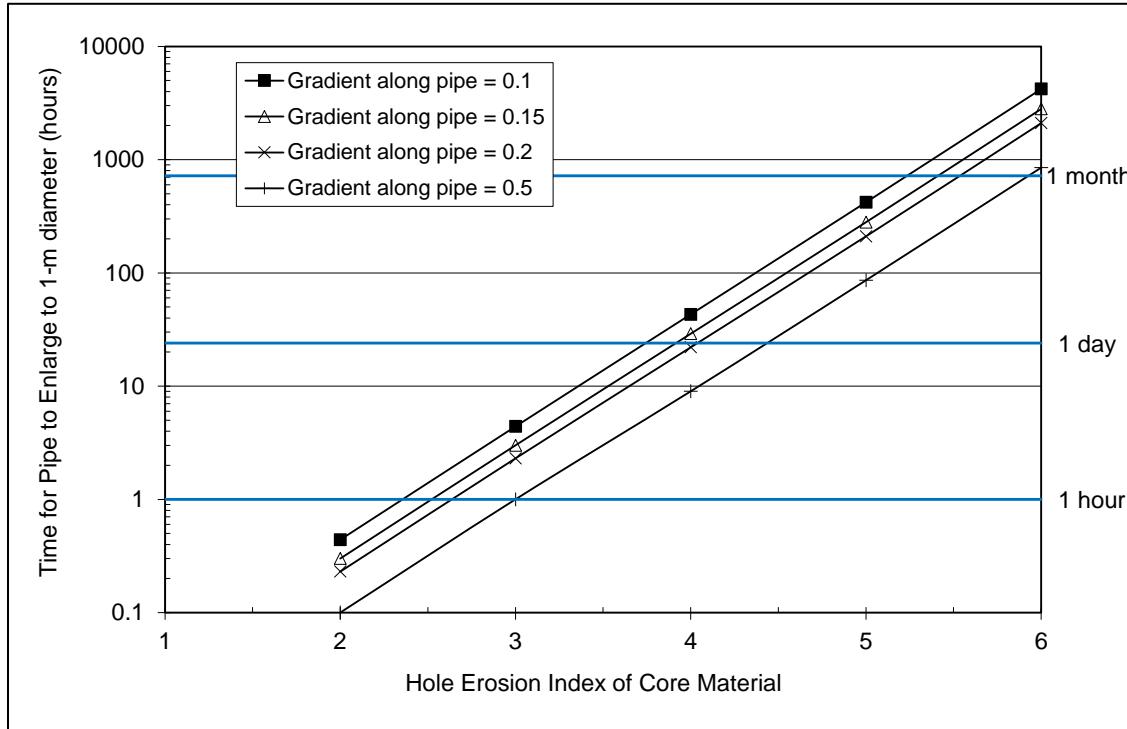
where:

$\gamma_w$  = unit weight of water  
 $i$  = hydraulic gradient across the embankment core  
 $D$  = diameter of the pipe

Figure 8 shows the approximate time for a concentrated leak pipe in an embankment to enlarge from 1 inch (25 millimeters) to 3 feet (1 meter) in diameter as a function of erosion resistance and hydraulic gradient based on Hole Erosion Test (HET) results and the following assumptions.

- Cross-section of the pipe is uniform from upstream to downstream (waterside to landside).
- Headwater remains constant.
- Steady uniform flow occurs through the pipe.
- Head loss is linear from upstream to downstream (waterside to landside).
- Frictional resistance is uniform along the surface of the pipe.
- Frictional resistance is equal to the driving force.

Erosion resistance increases from left to right in Figure 8. Even in the most resistant of soils, enlargement occurs in only 100 to 500 hours (4 days to 3 weeks). The approximate time to erode to 6 feet (2 meters) is about 20 percent greater.



**Figure 8. Approximate time for pipe to enlarge to 3-foot diameter (adapted from Fell et al. 2008).**

This worksheet estimates the diameter of a cylindrical concentrated leak pipe through an earthen embankment as a function of time during erosion using Equations 1 and 2 and the method described in Wan and Fell (2002). In addition to the previously listed assumptions, the enlarging pipe is assumed to sustain a roof up until collapse at failure. Equation 3 calculates the rate of erosion per unit surface area at a given time.

$$\dot{\varepsilon}_t = \frac{1}{\psi_t} \frac{dV_t}{dt} = k_d (\tau - \tau_c) \quad (3)$$

where:

- $\psi_t$  = surface area of the pipe at time  $t$
- $\frac{dV_t}{dt}$  = rate of soil volume removal during erosion at time  $t$
- $k_d$  = erodibility coefficient
- $\tau_c$  = critical hydraulic shear stress

The surface area of the pipe at any given time is equal to the wetted perimeter ( $P_w$ ) multiplied by the length of the pipe ( $L$ ), as shown in Equation 4.

$$\psi_t = P_w L = \pi D_t L \quad (4)$$

where:

- $D_t$  = pipe diameter at time  $t$

Combining Equations 3 and 4 and dividing by  $L$  obtains the erosion loss per unit length, as shown in Equations 5 and 6.

$$dV_t = k_d (\tau - \tau_c) P_w dt \quad (5)$$

$$dV_t = k_d (\tau - \tau_c) (\pi D_t) dt \quad (6)$$

Assuming the pipe enlarges the same amount radially in all directions and is not limited by a non-erodible boundary, Equations 7 to 12 derive the change in pipe diameter at a given time ( $d\phi_t$ ) from the change in pipe volume per unit length  $\left(\frac{dV_t}{dt}\right)$ .

$$\frac{dV_t}{dt} = \frac{\pi \left( D_t + \frac{d\phi_t}{dt} \right)^2}{4} - \frac{\pi D_t^2}{4} \quad (7)$$

$$\frac{dV_t}{dt} = \frac{\pi}{4} \left( D_t^2 + 2D_t \frac{d\phi_t}{dt} + \frac{d\phi_t^2}{dt} - D_t^2 \right) \quad (8)$$

$$\left(\frac{4}{\pi}\right) \frac{dV_t}{dt} = 2D_t \frac{d\phi_t}{dt} + \frac{d\phi_t^2}{dt} \quad (9)$$

$$\left(\frac{4}{\pi}\right) dV_t = 2D_t d\phi_t + d\phi_t^2 \quad (10)$$

$$d\phi_t^2 + 2D_t d\phi_t - \left(\frac{4}{\pi}\right) dV_t = 0 \quad (11)$$

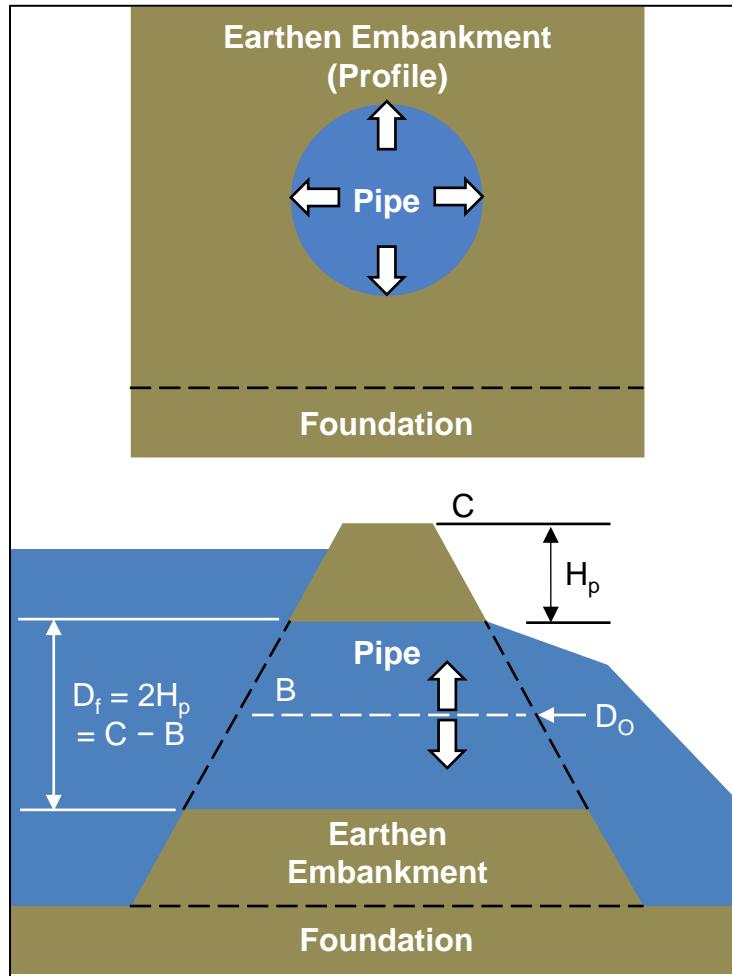
$$d\phi_t = \frac{-2D_t \pm \sqrt{(2D_t)^2 - 4\left(\left(\frac{4}{\pi}\right)dV_t\right)}}{2} \quad (12)$$

Because the change in pipe diameter cannot be negative, the negative root from the solution to the quadratic equation is neglected. The resulting solution is Equation 13 to calculate the diameter of a cylindrical pipe as a function of time during erosion.

$$d\phi_t = -D_t + \sqrt{D_t^2 + \frac{4dV_t}{\pi}} \quad (13)$$

## 5.1. Pipe Diameter Characterization

Input the embankment crest elevation ( $C$ ), initial pipe elevation ( $B$ ), and length of pipe ( $L$ ) at the beginning of step 1. Dam break analysis typically assumes the pipe diameter at failure ( $D_f$ ) occurs when the pipe diameter reaches twice the vertical distance from the crest to the roof of the pipe ( $H_p$ ) (Visser et al. 2013). Based on this assumption, the pipe diameter at failure is equal to the difference between the crest elevation ( $C$ ) and the initial elevation of the pipe ( $B$ ), as shown in Figure 9.



**Figure 9. Failure criterion for gross enlargement of a pipe (adapted from Visser et al. 2013).**

The initial estimate of pipe diameter at failure provides an anchoring value to help inform judgment for the user-specified input. Input the minimum, most likely (mode), and maximum values for the initial pipe diameter and the pipe diameter at failure at the end of step 1. Although these values represent a triangular distribution, a probabilistic analysis is not performed. Instead, the values are used to perform a sensitivity analysis. Figure 10 illustrates the pipe diameter characterization.

Step 1: Estimate the initial pipe diameter and pipe diameter at failure				
Embankment crest elevation, C		239.0	ft-NGVD29	
Initial pipe elevation, B		194.0	ft-NGVD29	
Length of pipe, L		216.7	ft	
Pipe diameter at failure assumed to be twice the vertical distance from the crest to the roof of the pipe, $D_f = C - B$				45.0 ft
Parameter	Units	Minimum	Most Likely	Maximum
Initial pipe diameter, $D_o$	mm	5	10	25
Use pipe diameter at failure, $D_f$	ft	40.0	45.0	50.0
Note: Assumes the pipe can enlarge radially in all directions and is not limited by a non-erodible boundary.				

Figure 10. Step 1 of Gross Enlargement worksheet: Pipe diameter characterization.

## 5.2. Erodibility Parameters

In step 2, the erodibility parameters are defined. Input includes the minimum, most likely (mode), and maximum estimates for the critical shear stress and the erodibility coefficient of the embankment material, as shown Figure 11. Although these values represent a triangular distribution, a probabilistic analysis is not performed. Instead, the values are used to perform a sensitivity analysis. The RMC Erodibility Parameters Toolbox contains empirical relationships and published values based on field and laboratory testing.

Step 2: Assess the erodibility parameters				
Parameter	Units	Minimum	Most Likely	Maximum
Critical shear stress, $\tau_c$	psf	0.080	0.100	0.180
Erodibility coefficient, $k_d$	(ft/hr)/psf	0.008	0.300	1.250

Figure 11. Step 2 of Gross Enlargement worksheet: Erodibility parameters.

## 5.3. Hydraulic Gradient

Step 3 calculates the average hydraulic gradient through the pipe by dividing the net hydraulic head across the pipe by the length of the pipe for each defined headwater-tailwater combination, as shown in Figure 12. If the headwater elevation is below the pipe elevation, the upstream hydraulic head ( $H_1$ ) is zero; if the tailwater is below the pipe elevation, the downstream hydraulic head ( $H_2$ ) is zero.

### Step 3: Estimate the hydraulic gradient

Hydraulic gradient across the core,  $i = (H_1 - H_2)/L$

where Hydraulic head at US end of pipe,  $H_1$  (ft) = HW – B      ( $H_1 = 0$  if HW is below base of pipe)

Hydraulic head at DS end of pipe,  $H_2$  (ft) = TW – B      ( $H_2 = 0$  if TW is below base of pipe)

HW (ft)	201.6	213.5	221.0	228.5	231.0	235.0	239.0
TW (ft)	184.0	184.0	184.0	184.0	184.0	188.0	190.0
$H_1$ (ft)	7.6	19.5	27.0	34.5	37.0	41.0	45.0
$H_2$ (ft)	0.0	0.0	0.0	0.0	0.0	0.0	0.0
i	0.035	0.090	0.125	0.159	0.171	0.189	0.208

Figure 12. Step 3 of Gross Enlargement worksheet: Hydraulic gradient.

## 5.4. Estimated Time to Failure

In step 4, select the initial pipe diameter used in the subsequent plots from the drop-down list. The minimum, most likely (mode), and maximum values of initial pipe diameter from step 1 can be evaluated.

Select the time increment for the pipe diameter calculations from the drop-down list. Time increments of 5, 10, 15, 30, and 60 minutes can be evaluated. Because gross enlargement is typically rapid, use care when selecting this time increment. The results of the analysis can be very sensitive to the time increment. Choose the smallest time increment possible to provide adequate resolution on how the pipe diameter changes with time, but it also must be large enough to allow enough time in the analysis for the pipe diameter to enlarge to failure. If the average hydraulic shear stress on the sidewalls does not exceed the critical shear stress for a given headwater level, concentrated leak erosion initiation is not predicted, and the pipe diameter remains constant with time. When the critical shear stress is exceeded, concentrated leak erosion initiation is predicted, and the pipe diameter increases with time. The rate of pipe enlargement also accelerates with time.

Figure 13 illustrates the input for time to failure calculations.

### Step 4: Estimate the time to failure

Initial pipe diameter,  $D_0$  25 mm for plotting

Rate of volume erosion (per unit surface area of the pipe at time t),  $\varepsilon_{r,t} = (1/\Psi_t)(dV_t/dt) = k_d(\tau - \tau_c)$

where Surface area of the pipe at time t,  $\Psi_t = P_w L$

and Rate of soil volume removal due to erosion at time t,  $dV_t/dt$

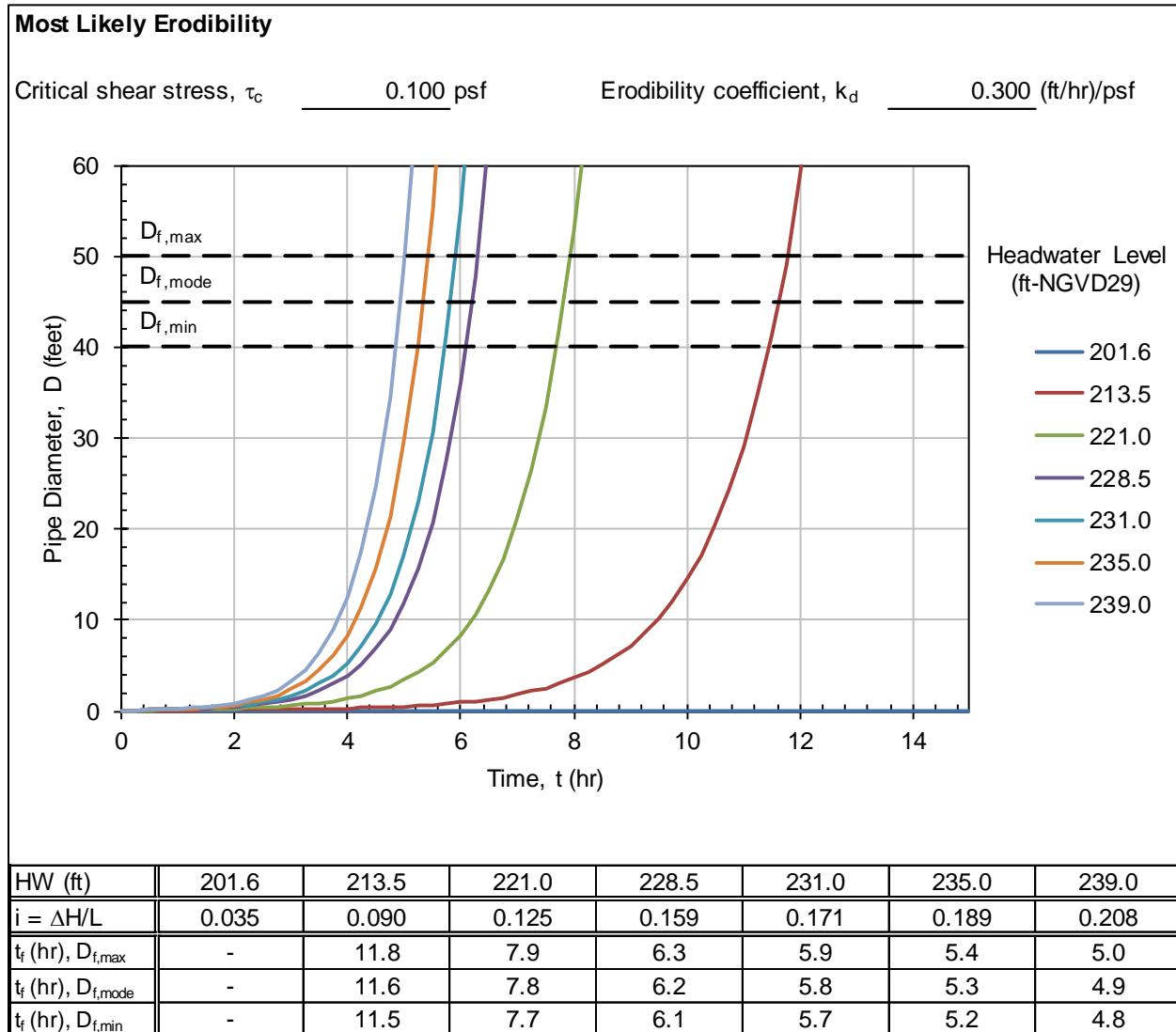
Thus, volume erosion loss (per unit length),  $dV_t = k_d(\tau - \tau_c)(P_w) dt = k_d(\tau - \tau_c)(\pi D_t) dt$

Change in pipe diameter,  $d\phi$  (at time t) =  $-D_t + [(D_t)^2 + 4(dV_t)/\pi]^{0.5}$

Time increment for pipe diameter calculations,  $dt$  15 min

Figure 13. Step 4 of Gross Enlargement worksheet: Time to failure input.

Summary plots portraying the change in pipe diameter as a function of time are provided for each of the erodibility estimates in step 2. Figure 14 provides an example plot for the most likely erodibility parameters. The results for the most likely erodibility estimate use the most likely values for both the critical shear stress and the erodibility coefficient. The results for the low erodibility estimate use the maximum critical shear stress and the minimum erodibility coefficient. The results for the high erodibility estimate use the minimum critical shear stress and the maximum erodibility coefficient. Horizontal reference lines for the minimum, most likely, and maximum estimates of the pipe diameter at failure display on all three plots. A table summarizing the number of hours for the pipe to exceed each of the estimated pipe diameters at failure from step 1 is provided under each plot.



**Figure 14. Step 4 of Gross Enlargement worksheet: Graphical output.**

Figure 15 illustrates the plot options for this chart. The maximum value for the x-axis (time) is user-specified.

Worksheet	Gross Enlargement					
y-axis bounds						
minimum	0				Value Primary Min: 0	
maximum	60				Value Primary Max: 60	
x-axis bounds						
minimum	0				Category Primary Min: 0	
maximum	15.0	◀ Enter maximum time.			Category Primary Max: 15	

**Figure 15. Step 4 of Gross Enlargement worksheet: Plot options.**

## 6. Unraveling

This worksheet evaluates the stable rock size for flow through a rockfill as a function of the unit discharge for a given downstream slope

At the beginning of the worksheet, input the slope cotangent, estimates of the unit discharge for each headwater level being considered, and the median rock size. Input the minimum, most likely (mode), and maximum values for both the unit discharge through the rockfill and the median rock size. The mean estimate of the median rock side is calculated as the average of the minimum, most likely (mode), and maximum values. Although these values represent a triangular distribution, a probabilistic analysis is not performed. Instead, the values are used to perform a sensitivity analysis. The slope, unit discharge, and median rock size characterization input are illustrated in Figure 16.

Slope cotangent		1.00 H:1V		Slope, $S_o$		1.00 (V/H)	
Reasonable estimates of unit discharge through the rockfill, $q_t$							
HW (ft)	201.6	213.5	221.0	228.5	231.0	235.0	239.0
$q_{t,min}$ (cfs/ft)	0.2	1.0	1.3	2.0	2.2	3.6	4.1
$q_{t,mode}$ (cfs/ft)	1.2	2.0	2.5	3.0	3.5	4.6	5.1
$q_{t,max}$ (cfs/ft)	2.2	3.0	3.3	4.0	4.2	5.6	6.1
$q_{t,mean}$ (cfs/ft)	1.2	2.0	2.4	3.0	3.3	4.6	5.1
Note: The foundation topography will likely concentrate flow into low points.							
Median Rock Size	Minimum	Most Likely	Maximum	Mean			
$d_{50}$ (in)	3.0	6.0	12.0	7.0			
$d_{50}$ (mm)	76.2	152.4	304.8	177.8			

Figure 16. Unraveling worksheet: Slope, unit discharge, and median rock size characterization.

### 6.1. Solvik (1991) and Olivier (1967)

Option 1 evaluates the minimum stable rock size needed to withstand a given unit discharge through rockfill using Equation 14 developed by Solvik (1991) based on sloping flume test results. It is the same method presented in Olivier (1967) but converted to metric units.

$$d_s = 1.46 q_t^{0.66} S_o^{0.78} \quad (14)$$

where:

$d_s$  = minimum stable rock size (m) for the rockfill, assumed to be the median rock size ( $d_{50}$ )

$q_t$  = unit discharge through the rockfill ( $\text{m}^3/\text{s}/\text{m}$ )

$S_o$  = downstream slope (measured as V/H)

The minimum stable rock size is calculated as a function of unit discharge and the downstream slope for the minimum, most likely (mode), maximum, and mean unit discharges. The critical unit discharge is also calculated for the mean  $d_{50}$ , and the headwater elevation where that critical unit discharge is first exceeded

is linearly interpolated from the user-specified headwater-mean unit discharge relationship. Figure 17 illustrates the calculations for this option.

Option 1: Assess rock size to withstand flows through the rockfill (Solvik 1991, Olivier 1967)							
$d_s = 1.46q_t^{0.66}S_o^{0.78}$ in SI units				where $d_s$ is assumed to $d_{50}$			
HW (ft)	201.6	213.5	221.0	228.5	231.0	235.0	239.0
$d_{s,min}$ (in)	4.1	12.0	14.2	18.9	20.2	27.9	30.4
$d_{s,mode}$ (in)	13.5	18.9	21.9	24.7	27.4	32.8	35.1
$d_{s,max}$ (in)	20.2	24.7	26.3	29.9	30.9	37.3	39.5
$d_{s,mean}$ (in)	13.5	18.9	21.2	24.7	26.3	32.8	35.1
Critical unit discharge for mean $d_{50}$ , $q_{t,crit}$				0.4 cfs/ft			
Headwater where $q_{t,crit}$ is exceeded for mean $d_{50}$ and $q_t$ , HW <sub>crit</sub>				< 201.6 ft			
<ul style="list-style-type: none"> <li>◀ based on minimum <math>q_t</math></li> <li>◀ based on most likely (mode) <math>q_t</math></li> <li>◀ based on maximum <math>q_t</math></li> <li>◀ based on mean <math>q_t</math></li> </ul>							

Figure 17. Option 1 of Unraveling worksheet: Solvik (1991) and Olivier (1967).

## 6.2. EBL (2005) Method

Option 2 evaluates the minimum stable rock size needed to withstand a given unit discharge through rockfill using Equation 15 from EBL (2005) based on model tests of flow through rockfill dams up to 20 feet tall. The calculations are the same as in Option 1, except for the equation to calculate rock size. Figure 18 illustrates the calculations for this option.

$$d_{50} = 0.43q_t^{0.43}S_o^{0.78} \quad (15)$$

where:

$d_{50}$  = median rock size (m) needed for stability

Option 2: Assess rock size to withstand flows through the rockfill (EBL 2005)							
$d_{50} = 0.43S_o^{0.43}q_t^{0.78}$ in SI units							
HW (ft)	201.6	213.5	221.0	228.5	231.0	235.0	239.0
$d_{50,min}$ (in)	0.8	2.7	3.3	4.6	4.9	7.2	8.0
$d_{50,mode}$ (in)	3.1	4.6	5.4	6.2	7.0	8.7	9.5
$d_{50,max}$ (in)	4.9	6.2	6.7	7.8	8.1	10.2	10.9
$d_{50,mean}$ (in)	3.1	4.6	5.2	6.2	6.7	8.7	9.5
Critical unit discharge for mean $d_{50}$ , $q_{t,crit}$				3.5 cfs/ft			
Headwater where $q_{t,crit}$ is exceeded for mean $d_{50}$ and $q_t$ , HW <sub>crit</sub>				< 231.5 ft			
<ul style="list-style-type: none"> <li>◀ based on minimum <math>q_t</math></li> <li>◀ based on most likely (mode) <math>q_t</math></li> <li>◀ based on maximum <math>q_t</math></li> <li>◀ based on mean <math>q_t</math></li> </ul>							

Figure 18. Option 2 of Unraveling worksheet: EBL (2005).

## 6.3. Summary

The mean unit discharge through the rockfill is plotted as a function of headwater level and mean value of  $d_{50}$  at the bottom of the worksheet, as shown in Figure 19. Reference lines for the critical  $d_{50}$  (median rock

size needed for stability) for Solvik (1991) and Olivier (1967) (black dashed line) and EBL (2005) (black solid line) are also plotted as a function of unit discharge through the rockfill. When a unit discharge through the rockfill at a given headwater level plots right of a line of critical  $d_{50}$ , the rockfill is predicted to be unstable. A blue box is also plotted showing the parameter limits for unit discharge through the rockfill and median rock size.

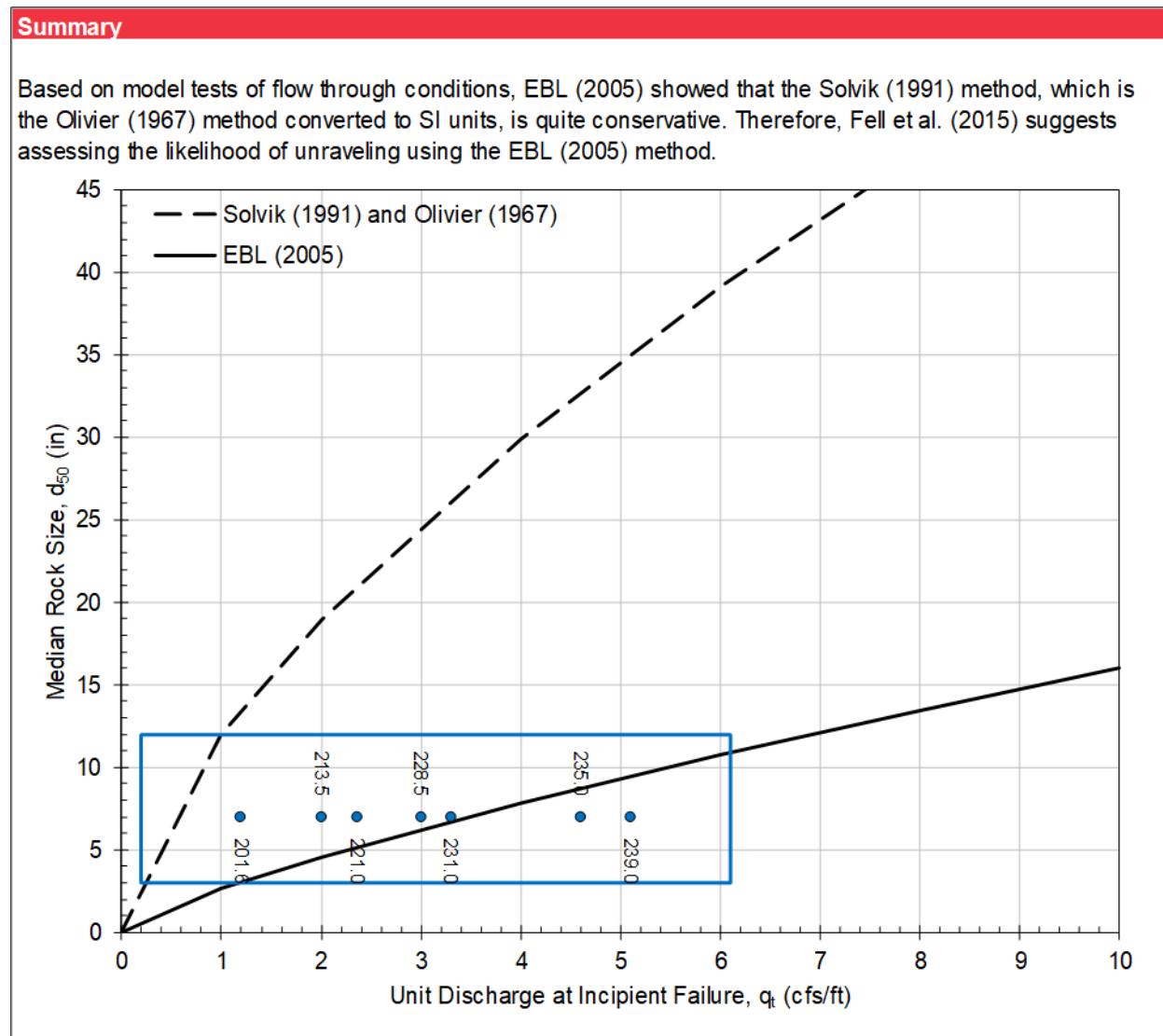


Figure 19. Unraveling worksheet: Graphical output.

Figure 20 illustrates the plot options for this chart. The maximum values for the y-axis (median rock size) and x-axis (unit discharge) are user-specified.

Worksheet	Unraveling					
y-axis bounds						
minimum	0				Value Primary Min: 0	
maximum	45	◀ Enter maximum median rock size.			Value Primary Max: 45	
x-axis bounds						
minimum	0				Category Primary Min: 0	
maximum	10	◀ Enter maximum unit discharge.			Category Primary Max: 10	

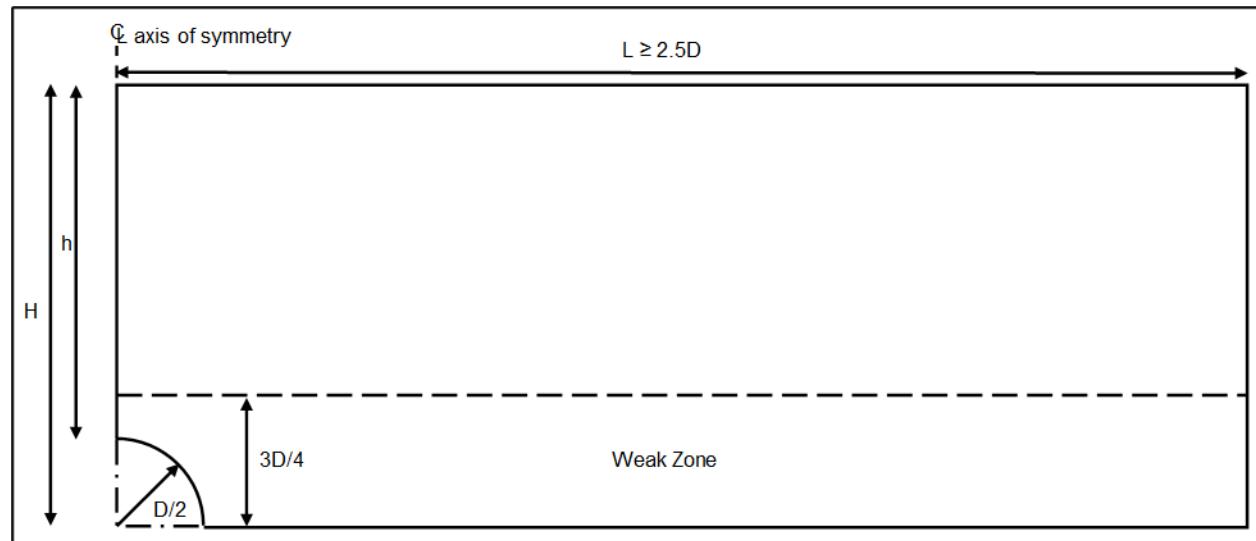
Figure 20. Unraveling worksheet: Plot options.

## 7. Sinkhole

This worksheet assesses the stability of residual soil as a function of potential soil void diameter to assess the impact of a sinkhole developing at a given location on the likelihood of breach. Drumm et al. (2009) developed a dimensionless stability chart to evaluate the stability of residual soils in karst where subsurface voids may exist near the rock contact and collapse of these voids may result in a sinkhole.

The sinkhole size and location and impounded water level at the time of the sinkhole are critical to evaluating the likelihood of breach due to sinkhole development. If the sinkhole occurs near the embankment crest with an elevated water level, it may lower the crest quickly and lead to overtopping with breach. If the sinkhole occurs downstream of the embankment centerline, progressive instability is needed to eventually cause loss of freeboard, but for this scenario, there is usually sufficient time for intervention and corrective action.

Figure 21 shows the idealized profile from Drumm et al. (2009) with residual soil thickness ( $h$ ) above a subsurface void of diameter ( $D$ ) overlying bedrock. A weak zone of thickness  $3D/4$  overlying the rock surface is also shown in the figure, which is discussed in section 7.2.



**Figure 21. Axisymmetric idealization of residual soil with a void overlying bedrock.**

The thickness ( $h$ ) of the residual soil above the void at the rock surface is calculated using Equation 16.

$$h = H - \frac{D}{2} \quad (16)$$

where:

$H$  = embankment height above the rock contact

$D$  = void diameter

Assuming no internal cavity pressure, the methodology compares a dimensionless stability number for a spherical void in residual soil overlying the rock surface for undrained (short-term) and drained (long-term) conditions to a critical dimensionless stability number. From the results of the numerical analyses,

critical stability numbers were developed for different strength and geometric conditions as shown in Figure 22.

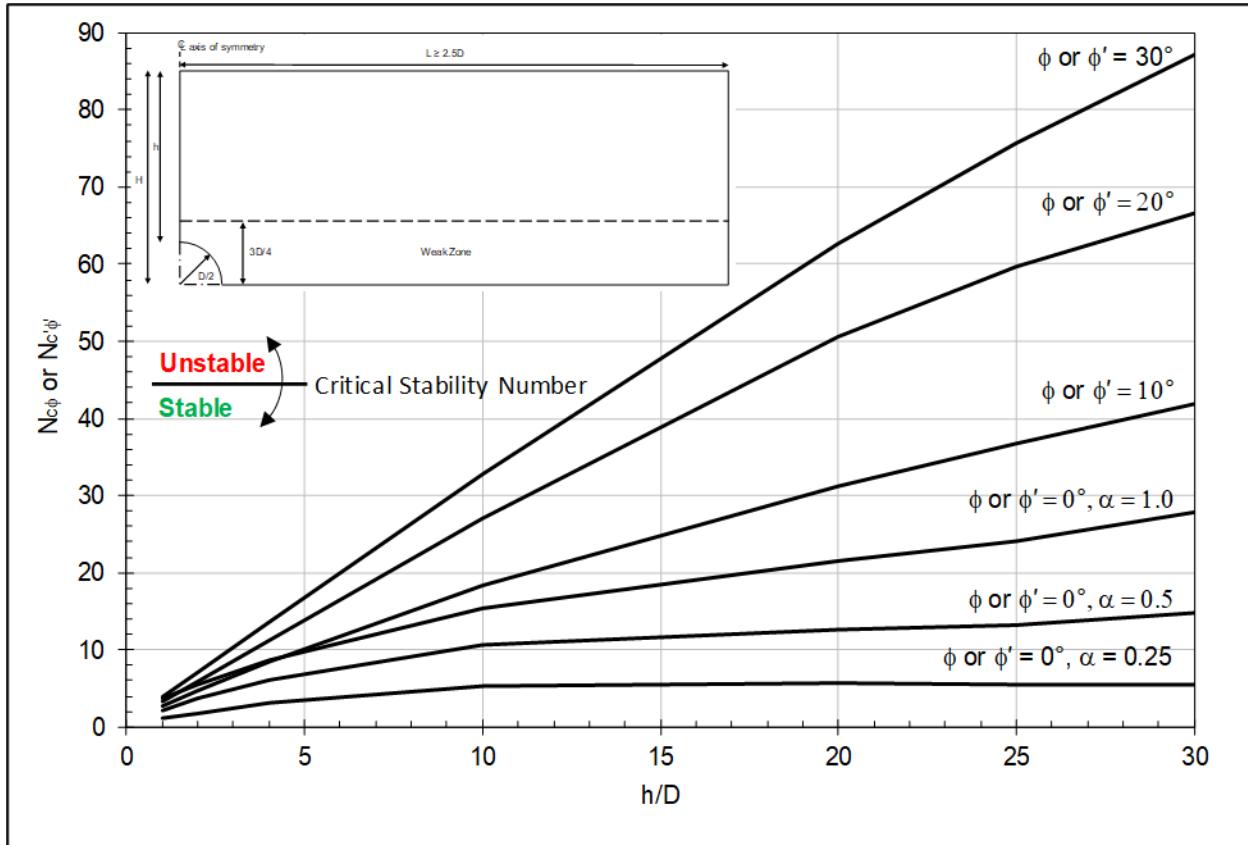


Figure 22. Stability chart,  $N_{c\phi}$  with effect of  $\phi$  and inverted soil profile.

## 7.1. Embankment Characterization

Step 1 characterizes the embankment. Input the unit weight of the residual clay soil and the height of the embankment as shown in Figure 23.

Step 1: Assess the embankment characteristics	
Unit weight of residual clay soil, $\gamma$	114.6 pcf
Embankment height, $H$	29.5 ft
Void diameter at rock surface, $D$	
Thickness of residual soil above void , $h = H - D/2$	

Diagram illustrating embankment geometry: A cross-section shows an embankment of height  $H$  resting on a void of diameter  $D$ . The embankment has an axis of symmetry. The thickness of residual soil above the void is  $h = H - D/2$ . A dimension  $3D/4$  is also indicated.

Figure 23. Step 1 of Sinkhole worksheet: Embankment characteristics.

## 7.2. Undrained Stability (Short-Term Conditions)

Step 2 assesses the stability of residual soils in karst, where subsurface voids may exist near the rock surface, under undrained (short-term) conditions. Soils in karst often show a shear strength profile that decreases at depths near the rock contact, commonly known as an inverted residual soil strength profile. To account for the reduction in shear strength, the evaluation of undrained conditions includes a reduced cohesion value ( $c^*$ ) for a weak zone overlying rock with a thickness of  $3D/4$ .

Input the undrained cohesion ( $c$ ) of the residual soil above the weak zone and the reduced cohesion of the weak zone above the rock surface ( $c^*$ ) with thickness  $3D/4$ , as shown in Figure 24.

The dimensionless stability number ( $N_{c\phi}$ ) for undrained (short-term) conditions is calculated using Equation 17.

$$N_{c\phi} = \frac{\gamma h}{c} \quad (17)$$

where:

$\gamma$  = unit weight of the residual clay soil

$h$  = soil thickness above the void

$c$  = undrained cohesion

The critical dimensionless stability number for undrained (short-term) conditions as a function of  $h/D$  is portrayed as a stability chart in Figure 22 and is calculated using Equation 18.

$$N_{c\phi,cr} = a(h/D)^3 - b\left(\frac{h}{D}\right)^2 + c\left(\frac{h}{D}\right) + d \quad (18)$$

where:

$a, b, c$ , and  $d$  = constants (see Table 2)

$h/D$  = ratio of the thickness of the residual soil above the void at the rock surface to void diameter

To account for the inverted residual soil strength profile in the stability chart, Drumm et al. (2009) also evaluated a weak zone of thickness  $3D/4$  overlying the rock surface with a reduced cohesion value ( $c^*$ ). The inverted strength factor ( $\alpha$ ) relating to the two cohesion values is calculated using Equation 19.

$$\alpha = \frac{c^*}{c} \quad (19)$$

where:

$c$  = undrained cohesion for the soil above the weak zone

$c^*$  = reduced cohesion for the soil in the weak zone

The values of the constants for undrained (short-term) conditions are shown in Table 2 for values of  $\alpha$  equal to 0.25, 0.5, and 1.0 and undrained angle of internal friction ( $\phi$ ) equal to zero. Intermediate values are linearly interpolated.

**Table 2**

Coefficients for stability number  $N_{c\phi,cr}$  and corresponding  $R^2$  values for undrained conditions.

$\alpha (\phi = 0^\circ)$	$a$	$B$	$C$	$D$	$R^2$
1.0	0.0013	0.0766	1.9944	1.8914	0.9982
0.5	0.0014	0.0826	1.6923	0.6220	0.9959
0.25	0.0006	0.0400	0.8339	0.3145	0.9954

When  $N_{c\phi}$  is greater than  $N_{c\phi,cr}$ , the void is predicted to be unstable, and a sinkhole is likely to form. The factor of safety ( $FS_{c\phi}$ ) against undrained instability is calculated using Equation 20.

$$FS_{c\phi} = \frac{N_{c\phi,cr}}{N_{c\phi}} \quad (20)$$

Input (in ascending order) up to seven different void diameters as shown in Figure 24. If the void radius ( $D/2$ ) exceeds the embankment height ( $H$ ) input in step 1, the void diameter cells have an orange background. Cells with FS less than 1 have an orange background. The critical thickness of residual soil to void diameter ratio ( $h/D)_{cr}$  is linearly interpolated for a FS of 1. If possible, provide a sufficient range of void diameters to result in at least one computed FS greater than 1 and one less than 1, to calculate  $(h/D)_{cr}$ .

Solving Equation 16 for the void diameter ( $D$ ) yields Equation 21.

$$D = \frac{H}{\frac{h}{D} + 0.5} \quad (21)$$

Substituting  $(h/D)_{cr}$  for  $h/D$  in the denominator of Equation 21, the critical void diameter ( $D_{cr}$ ) is calculated using Equation 22.

$$D_{cr} = \frac{H}{\left(\frac{h}{D}\right)_{cr} + 0.5} \quad (22)$$

The volume of eroded soil prior to void collapse ( $V$ ), assuming a spherical void, is calculated using Equation 23.

$$V = \frac{4}{3}\pi r^3 \quad (23)$$

The radius ( $r$ ) in Equation 23 is calculated using Equation 24:

$$r = \frac{D_{cr}}{2} \quad (24)$$

If the range of user-specified diameters of the void above the rock contact is not sufficient to interpolate a FS of 1, N/A is displayed for  $(h/D)_{cr}$ ,  $D_{cr}$ , and  $V$ . The undrained (short-term) stability evaluation is illustrated in Figure 24.

Step 2: Estimate the stability under undrained (short-term) conditions							
Undrained cohesion for most of the soil profile, $c$				2,088 psf			
Reduced undrained cohesion of bottom 3D/4 part of soil, $c^*$				1,044 psf, where $c^* < c$			
Inverted strength factor, $\alpha = c^*/c$				0.50			
 Undrained critical stability number, $N_{c\phi,cr} = a(h/D)^3 - b(h/D)^2 + c(h/D) + d$							
Undrained stability number, $N_{c\phi} = \gamma h/c$							
Factor of safety, $FS = N_{c\phi,cr} / N_{c\phi}$							
$\alpha (\phi = 0^\circ)$	a	b	c	d	$R^2$		
1.0	0.0013	0.0766	1.9944	1.8914	0.9982		
0.5	0.0014	0.0826	1.6923	0.6220	0.9959		
0.25	0.0006	0.0400	0.8339	0.3145	0.9954		
D (ft)	3.0	4.0	5.0	6.6	7.0	8.0	10.0
3D/4	2.3	3.0	3.8	5.0	5.3	6.0	7.5
h (ft)	28.0	27.5	27.0	26.2	26.0	25.5	24.5
h/D	9.33	6.88	5.40	3.97	3.71	3.19	2.45
$N_{c\phi}$	1.54	1.51	1.48	1.44	1.43	1.40	1.34
$N_{c\phi,cr}$	10.36	8.81	7.57	6.13	5.84	5.22	4.29
FS	6.74	5.84	5.11	4.26	4.09	3.73	3.19
Critical thickness of residual soil to void diameter ratio, $(h/D)_{cr}$					N/A		
Critical void diameter, $D_{cr} = H / [(h/D)_{cr} + 0.5]$					N/A ft		
Volume of eroded material prior to void collapse, $V = 4/3(\pi r^3)$					N/A ft <sup>3</sup>		

Figure 24. Step 2 of Sinkhole worksheet: Undrained (short-term) stability.

### 7.3. Drained Stability (Long-Term Conditions)

Step 3 assesses the stability of residual soils in karst, where subsurface voids may exist near the rock surface, under drained (long-term) conditions. Input the effective cohesion ( $c'$ ) and the effective angle of internal friction ( $\phi'$ ) of the residual clay soil as shown in Figure 25.

The dimensionless stability number ( $N_{c'\phi'}$ ) for drained (long-term) conditions is calculated using Equation 25.

$$N_{c'\phi'} = \frac{\gamma h}{c'} \quad (25)$$

where:

$\gamma$  = unit weight of the residual clay soil

$h$  = soil thickness above the void

$c'$  = effective (drained) cohesion

The critical dimensionless stability number for drained (long-term) conditions as a function of  $h/D$  is portrayed as a stability chart in Figure 22 and is calculated using Equation 26.

$$N_{c'\phi',cr} = a(h/D)^3 - b\left(\frac{h}{D}\right)^2 + c(h/D) + d \quad (26)$$

where:

$a, b, c$ , and  $d$  = constants (see Table 3)

$h/D$  = ratio of the thickness of the residual soil above the void at the rock surface to void diameter

The values of the constants for drained (long-term) conditions are shown in Table 3 for various values of effective (drained) angle of internal angle ( $\phi'$ ). Intermediate values are linearly interpolated.

**Table 3**  
**Coefficients for stability number  $N_{c'\phi',cr}$  and corresponding  $R^2$  values for drained conditions.**

$\phi' \text{ (deg)}$	<b>A</b>	<b>B</b>	<b>C</b>	<b>d</b>	<b><math>R^2</math></b>
0	0.0013	0.0766	1.9944	1.8914	0.9982
10	0.0004	0.0353	2.0744	0.6521	0.9990
20	-0.0008	-0.0101	2.6131	0.6484	0.9994
30	-0.0005	-0.0033	3.2346	0.6168	0.9987

When  $N_{c'\phi'}$  is greater than  $N_{c'\phi',cr}$ , the void is predicted to be unstable, and a sinkhole is likely to form. The FS against drained instability ( $FS_{c'\phi'}$ ) is calculated using Equation 27.

$$FS_{c'\phi'} = \frac{N_{c'\phi',cr}}{N_{c'\phi'}} \quad (27)$$

Input (in ascending order) up to seven different void diameters as shown in Figure 25. If the void radius ( $D/2$ ) exceeds the embankment height ( $H$ ) input in step 1, the void diameter cells have an orange background. Cells with FS less than 1 have an orange background. The critical thickness of residual soil to void diameter ratio ( $h/D$ )<sub>cr</sub> is linearly interpolated for a FS of 1. If possible, provide a sufficient range of void diameters to result in at least one computed FS greater than 1 and one less than 1, to calculate ( $h/D$ )<sub>cr</sub>. The volume of eroded soil prior to void collapse ( $V$ ) is calculated the same as in undrained (short-term) conditions. The drained (long-term) stability evaluation is illustrated in Figure 25.

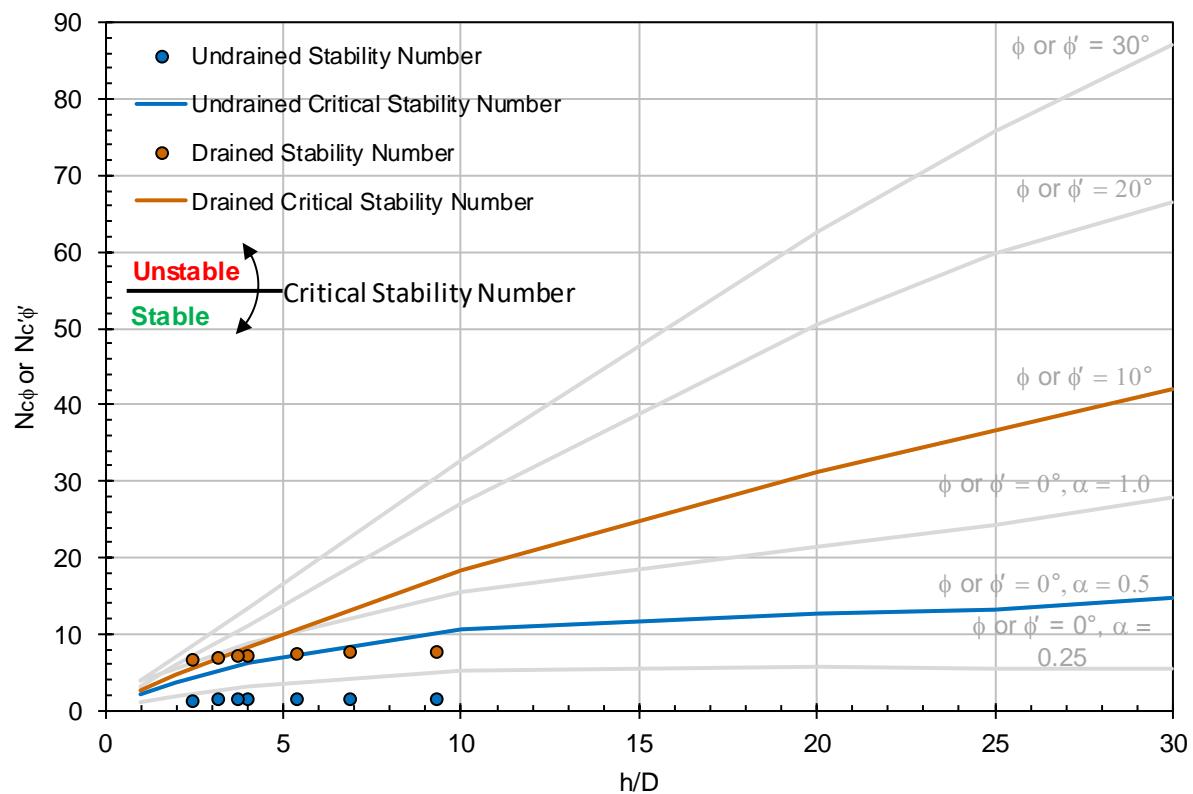
Step 3: Estimate the stability under drained (long-term) conditions														
Effective cohesion, $c'$	418 psf													
Effective friction angle, $\phi'$	10 deg													
Drained critical stability number, $N_{c'\phi',cr} = a(h/D)^3 - b(h/D)^2 + c(h/D) + d$														
Drained stability number, $N_{c'\phi'} = \gamma h/c'$														
Factor of safety, FS = $N_{c'\phi',cr} / N_{c'\phi'}$														
$\phi'$ (deg)	a	b	c	d	$R^2$									
0	0.0013	0.0766	1.9944	1.8914	0.9982									
10	0.0004	0.0353	2.0744	0.6521	0.9990									
20	-0.0008	-0.0101	2.6131	0.6484	0.9994									
30	-0.0005	-0.0033	3.2346	0.6168	0.9987									
D (ft)	3.0	4.0	5.0	6.6	7.0	8.0	10.0							
h (ft)	28.0	27.5	27.0	26.2	26.0	25.5	24.5							
h/D	9.33	6.88	5.40	3.97	3.71	3.19	2.45							
$N_{c'\phi'}$	7.68	7.54	7.40	7.18	7.13	6.99	6.72							
$N_{c'\phi',cr}$	17.26	13.38	10.89	8.36	7.89	6.92	5.53							
FS	2.25	1.77	1.47	1.16	1.11	0.99	0.82							
Critical thickness of residual soil to void diameter ratio, $(h/D)_{cr}$	3.23													
Critical void diameter, $D_{cr} = H / [(h/D)_{cr} + 0.5]$	7.90 ft													
Volume of eroded material prior to void collapse, $V = 4/3(\pi r^3)$	258.28 ft <sup>3</sup>													

Figure 25. Step 3 of Sinkhole worksheet: Drained (long-term) stability.

## 7.4. Summary

In step 4, the stability numbers for undrained (short-term) conditions (blue circles) and drained (long-term) conditions (red circle) are plotted as a function of  $h/D$  at the bottom of the worksheet, as shown in Figure 26. Reference lines for the critical stability number for undrained (short-term) conditions (blue line) and drained (long-term) conditions (red line) are also plotted as a function of  $h/D$ . When a stability number plots above the reference line, the void is predicted to be unstable, and a sinkhole is likely to form. The light gray lines provide the critical stability numbers for the undrained and drained conditions modeled by Drumm et al. (2009) for reference.

**Step 4: Summarize the results for undrained (short-term) and drained (long-term) conditions**



**Figure 26. Step 4 of Sinkhole worksheet: Graphical output.**

## 8. Slope Instability

This worksheet does not assess the probability of breach by slope instability due to internal erosion. The recommended approach from Fell et al. (2008) is to assess whether internal drainage measures in the dam or levee prevent pore pressures from rising in the embankment and/or its foundation and use the estimated pore pressures to assess embankment stability using slope stability software.

## 9. References

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## Appendix A. Acronym List

3D	Three-Dimensional
CPD	Computer Program Document
FS	Factor of Safety
HEC	Hydrologic Engineering Center
HET	Hole Erosion Test
IWR	Institute for Water Resources
MSL	Mean Sea Level
NAVD 88	North American Vertical Datum of 1988
NGVD 29	National Geodetic Vertical Datum of 1929
ORD	Ohio River Datum
QC	Quality Control
RMC	Risk Management Center
UDF	User-Defined Function
U.S.	United States
USACE	United States Army Corps of Engineers