RMC Backward Erosion Piping (Progression) Toolbox

RMC Internal Erosion Suite

RMC-CPD-2023-06

November 2023

US Army Corps of Engineers®
Dam and Levee Safety Programs

US Army Corps of Engineers®
Dam and Levee Safety Programs

Cover Photo: Missouri River levee unit L-575 (west of Hamburg, IA and just south of the Missouri-Iowa border) showing multiple pipes that progressed towards the river during the 2011 flooding prior to breach.

US Army Corps of Engineers®
Dam and Levee Safety Programs

RMC Backward Erosion Piping (Progression) Toolbox

RMC Internal Erosion Suite

RMC-CPD-2023-06

November 2023

Prepared by the Risk Management Center

REPORT DOCUMENTATION PAGE

1. REPORT DATE	2. REPORT TYPE	3. DATES COVERED START DATE	END DATE
Nov 2023	Computer Program Document	N/A	N/A

4. TITLE AND SUBTITLE
RMC Backward Erosion Piping (Progression) Toolbox: RMC Internal Erosion Suite

5a. CONTRACT NUMBER	5b. GRANT NUMBER	5c. PROGRAM ELEMENT NUMBER
N/A	N/A	N/A
5d. PROJECT NUMBER	**5e. TASK NUMBER**	**5f. WORK UNIT NUMBER**
N/A	N/A	N/A

6. AUTHOR(S)
Tim O'Leary, Risk Management Center

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)	8. PERFORMING ORGANIZATION REPORT NUMBER
Risk Management Center 12596 West Bayaud Ave. Suite 400 Lakewood, CO 80228	RMC-CPD-2023-06

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSOR/MONITOR'S ACRONYM(S)	11. SPONSOR/MONITOR'S REPORT NUMBER(S)
Risk Management Center 12596 West Bayaud Ave. Suite 400 Lakewood, CO 80228	CEIWR-RMC	RMC-CPD-2023-06

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution is unlimited.

13. SUPPLEMENTARY NOTES
N/A

14. ABSTRACT
The spreadsheet tools contained in this toolbox deterministically and probabilistically assess the likelihood of backward erosion piping progression (hydraulic condition) using the adjusted Schmertmann (2000) method and the adjusted calculation rule of Sellmeijer et al. (2011) in addition to creep ratio methods of Bligh (1910) and Lane (1935).

15. SUBJECT TERMS
Internal erosion, backward erosion piping, progression, Schmertmann, Sellmeijer, Bligh, Lane, creep ratio

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES
a. REPORT	**b. ABSTRACT**	**c. THIS PAGE**		
U	U	U	UU	67

19a. NAME OF RESPONSIBLE PERSON	19b. PHONE NUMBER (Include area code)
Tim O'Leary	(502) 315-6599

STANDARD FORM 298 (REF 5/2020)
Prescribed by ANSI Std Z39.18

PREVIOUS EDITION IS OBSOLETE

PREPARED

The results, findings, and recommendations provided in this document are technically sound and consistent with current Corps of Engineers practice.

Tim O'Leary, Risk Management Center

REVIEWED

This report has been checked and reviewed and is believed to be in accordance with the standards of the profession.

Damon Amlung, Risk Management Center

APPROVED

Nate Snorteland, Risk Management Center

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

APPENDIXES

1. Introduction

The Risk Management Center (RMC) of the U.S. Army Corps of Engineers (USACE) has developed a suite of Microsoft Excel spreadsheets to support risk assessments for dam and levee safety. Each analysis suite is composed of multiple toolboxes (Microsoft Excel workbooks), and each toolbox contains multiple spreadsheet tools or calculation worksheets (Microsoft Excel worksheets). The RMC Backward Erosion Piping (Progression) Toolbox is part of the RMC Internal Erosion Suite.

The information from these spreadsheet tools, along with other pertinent information, informs judgment when developing a list of more and less likely factors and estimating probabilities. USACE best practice for estimating probabilities is to use the best available and multiple methods, but all final probabilities are estimated using team elicitation based on the totality and strength of the evidence.

The RMC continuously works to improve the performance of RMC software; report possible bugs directly to the RMC at the address listed below. Ideally, report suspected errors in written form with a description of the problem and the steps that lead to its occurrence. Suggestions for improvement are also welcomed.

U.S. Army Corps of Engineers
Institute for Water Resources
Risk Management Center
RMC.software@usace.army.mil

2. Terms and Conditions for Use

By using Institute for Water Resources (IWR) software, users voluntarily accept the following terms and conditions. Users that do not agree to these terms and conditions should uninstall the IWR software and return any program materials to IWR and its technical centers. If users downloaded the software and do not have disk media, delete all copies and cease using the software.

2.1. Terms and Conditions for Use of Institute for Water Resources Software

The United States Government, U.S. Army Corps of Engineers, Institute for Water Resources ("IWR"), and IWR's technical centers including the Risk Management Center ("RMC") and Hydrologic Engineering Center ("HEC") grant to the user the rights to install "IWR Software" (either from a disk copy obtained from IWR and IWR's technical centers, a distributor or another user or by downloading it from a network) and to use, copy and/or distribute copies of the IWR Software to other users, subject to the following Terms and Conditions of Use:

- All copies of the IWR Software received or reproduced by or for user pursuant to the authority of this Terms and Conditions of Use will be and remain the property of IWR.

- User may reproduce and distribute the IWR Software provided that the recipient agrees to the Terms and Conditions for Use noted herein.

- IWR and IWR's technical centers are solely responsible for the content of the IWR Software. The IWR Software may not be modified, abridged, decompiled, disassembled, unobfuscated or reverse engineered. The user is solely responsible for the content, interactions, and effects of any and all amendments, if present, whether they be extension modules, language resource bundles, scripts, or any other amendment.

- The name of the IWR Software must not be used to endorse or promote products derived from the IWR Software. Products derived from the IWR Software may not be called the IWR Software nor may any part of the IWR Software name appear within the name of derived products.

- No part of this Terms and Conditions for Use may be modified, deleted or obliterated from the IWR Software.

- No part of the IWR Software may be exported or re-exported in contravention of U.S. export laws or regulations.

2.2. Waiver of Warranty

THE UNITED STATES GOVERNMENT AND ITS AGENCIES, OFFICIALS, REPRESENTATIVES, AND EMPLOYEES, INCLUDING ITS CONTRACTORS AND SUPPLIERS PROVIDE THE IWR SOFTWARE "AS IS," WITHOUT ANY WARRANTY OR CONDITION, EXPRESS, IMPLIED OR STATUTORY, AND SPECIFICALLY DISCLAIM ANY IMPLIED WARRANTIES OF TITLE, MERCHANTABILITY, FITNESS FOR A PARTICULAR PURPOSE AND NON-INFRINGEMENT. Depending on state law, the foregoing disclaimer may not apply to you, and you may also have other legal rights that vary from state to state.

2.3. Limitation of Liability

IN NO EVENT SHALL THE UNITED STATES GOVERNMENT AND ITS AGENCIES, OFFICIALS, REPRESENTATIVES, AND EMPLOYEES, INCLUDING ITS CONTRACTORS AND SUPPLIERS, BE LIABLE FOR LOST PROFITS OR ANY SPECIAL, INCIDENTAL OR CONSEQUENTIAL DAMAGES ARISING OUT OF OR IN CONNECTION WITH USE OF THE IWR SOFTWARE REGARDLESS OF CAUSE, INCLUDING NEGLIGENCE. THE UNITED STATES GOVERNMENT'S LIABILITY, AND THE LIABILITY OF ITS AGENCIES, OFFICIALS, REPRESENTATIVES, AND EMPLOYEES, INCLUDING ITS CONTRACTORS AND SUPPLIERS, TO YOU OR ANY THIRD PARTIES IN ANY CIRCUMSTANCE IS LIMITED TO THE REPLACEMENT OF CERTIFIED COPIES OF THE IWR SOFTWARE WITH IDENTIFIED ERRORS CORRECTED. Depending on state law, the above limitation or exclusion may not apply to you.

2.4. Indemnity

As a voluntary user of the IWR Software you agree to indemnify and hold the United States Government, and its agencies, officials, representatives, and employees, including its contractors and suppliers, harmless from any claim or demand, including reasonable attorneys' fees, made by any third party due to or arising out of your use of the IWR Software or breach of this Agreement or your violation of any law or the rights of a third party.

3. General Overview

3.1. Getting Started

Copy or download the toolbox file to the computer. To open the toolbox file, either:

- Find the file on the computer and double-click it. This opens the file in Microsoft Excel.
- Open Microsoft Excel and use the application to open the file: Once Microsoft Excel is open, go to the File menu at the top of the window and select Open.

The toolbox is an Excel binary workbook (.xlsb) that uses macros. You may need to enable the macros, either before opening the file or by clicking “Enable Content” in the yellow Security Warning message bar with a shield icon that appears after the file is opened. The actual message in the message bar will vary depending on the computer’s settings and installed add-ins. Figure 1 displays examples of different wordings that may appear in the message bar.

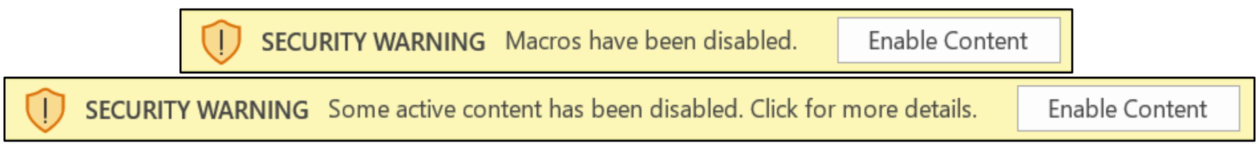


Figure 1. Security warning message bars with the “Enable Content” option to enable macros.

3.2. Organization

Although the toolbox does not provide a calculation cover sheet, adding one is strongly recommended. A calculation cover sheet captures project information, a description and purpose of the calculation, the assumptions for critical input parameters, a summary of the major conclusion and results, and a revision history.

Each toolbox has a similar appearance and organizational structure:

- The first worksheet, About, summarizes the purpose of the toolbox and gives contact information for the RMC software development team.
- The second worksheet, Terms and Conditions, contains the terms and conditions for use of the toolbox (IWR software).
- The third worksheet, Version History, contains the revision history. Semantic versioning is used in the format of MAJOR.MINOR.PATCH:
  - MAJOR – significant worksheet changes not compatible with previous versions.
  - MINOR – additional features or enhancements that do not fundamentally change the calculations.
  - PATCH – backward-compatible bug fixes.

- The fourth worksheet, References, lists the references cited for each calculation worksheet.

The workbook and worksheets are not protected to prevent unwanted changes. However, because the toolbox has user-defined functions (UDFs) and subroutines in Visual Basic, you cannot directly copy worksheets to another workbook without potentially losing functionality. A note in a bold red font at the upper right margin indicates if the selected worksheet includes such features.

At the top of each calculation worksheet, input information for the preparer and checker for quality control (QC) documentation and the calculation title in case multiple copies of the worksheet are created for different analysis scenarios (Figure 2). The footer of each calculation worksheet contains the version number, which can be cross-referenced with the revision history on the third worksheet.

Prepared by:		Office:		Date:	
Checked by:		Office:		Date:	
Calculation Title:					

Figure 2. Calculation worksheet heading.

User-specified input includes values and selections from drop-down lists. User input cells are light yellow, and these cells are unprotected. When cells use drop-down lists, a note in blue font in the right margin of the row alerts the user to use the drop-down list. Conditional formatting applies a gray background to cells that are not based on a user selection. When a user-specified value or calculated value is outside of acceptable ranges, the cell is orange to indicate caution to the user.

All units for user-specified input values are clearly labeled. Most user-specified input values use English units. However, values may be in metric where metric units are more common in practice (e.g., particle size in millimeters or permeability in centimeters per second). The toolbox may convert English units to metric units to perform some calculations or if required for a specific formula based on the reference material for the equation.

If the calculation worksheet is a function of headwater level, up to seven headwater and tailwater levels may be specified at the top of the worksheet. Tailwater may be required to calculate the net hydraulic head and hydraulic gradient. Specify the elevation datum by selecting one of three options from the drop-down list: ft-NAVD88, ft-NGVD29, and Other. The two datum selections include English units of length (feet). If Other is selected, provide a user-specified datum along with feet (e.g., ft-MSL [Mean Sea Level]). Figure 3 through Figure 5 illustrate the three possible scenarios.

Elevation datum		ft-NAVD88		Specify datum				◄ Use drop-down list.
HW (ft)	195.5	201.6	213.5	218.9	223.0	234.0	239.0	◄ Headwater level, HW (ft-NAVD88)
TW (ft)	184.0	184.0	184.0	184.0	184.0	184.0	184.0	◄ Tailwater level, TW (ft-NAVD88)

Figure 3. Headwater and tailwater input: NAVD88.

Elevation datum		ft-NGVD29		Specify datum				◄ Use drop-down list.
HW (ft)	195.5	201.6	213.5	218.9	223.0	234.0	239.0	◄ Headwater level, HW (ft-NGVD29)
TW (ft)	184.0	184.0	184.0	184.0	184.0	184.0	184.0	◄ Tailwater level, TW (ft-NGVD29)

Figure 4. Headwater and tailwater input: NGVD29.

Elevation datum		Other		Specify datum		ft-MSL		◄ Use drop-down list.
HW (ft)	195.5	201.6	213.5	218.9	223.0	234.0	239.0	◄ Headwater level, HW (ft-MSL)
TW (ft)	184.0	184.0	184.0	184.0	184.0	184.0	184.0	◄ Tailwater level, TW (ft-MSL)

Figure 5. Headwater and tailwater input: User-specified datum.

Most calculation worksheets break down complex analysis into computational steps following a logical sequence (Figure 6). Some simpler worksheets do not have steps. Generally, different methodologies are unique worksheets. Some worksheets may include multiple methodologies, which are labeled as options (Figure 7).

Step 1: Select the method of analysis

Figure 6. Example of step banner.

Option 1: Riverside blanket (top stratum) for Cases 5, 7, and 8

Figure 7. Example of option banner.

Some calculation worksheets can perform either a deterministic or probabilistic analysis. Although not required to perform a probabilistic analysis, Palisade @RISK software (standalone version or as part of the Palisade DecisionTools Suite) can customize the probabilistic analysis. A note appears in a bold red font at the upper right-hand margin of a calculation worksheet indicating if this feature is included with the toolbox.

User notes generally appear in the right margin of each calculation worksheet. Some notes are in blue or red font for heightened awareness. These notes include references to source materials for equations, figures, tables, pages, etc. If the RMC modified the source material, the reference citation says “adapted from” instead of “from.”

Tabular and/or graphical summaries are generally the primary output of the toolbox. The UDFs in the PlotScale module change the minimum and maximum values of the x-axis and y-axis for charts. If the calculation worksheet is a function of headwater level, you can define up to five headwater levels of interest and plot them as vertical reference lines. By selecting the chart and then selecting the Filter icon to display the filter pane, you can choose which data series to display. This is useful when computing the results from multiple methodologies, but not all are applicable or desired to display.

4. Background

Backward erosion piping (BEP) is the detachment of soils particles that occurs at a free, unfiltered surface in which the process gradually works its way toward the upstream or floodside of the embankment or its foundation until a continuous pipe is formed. Erosion initiates at the downstream side of a dam or the landside of a levee through unfiltered seepage exits that may exist due to penetrations or weaknesses in the overlying blanket, such as ditches, animal burrows (such as rodent or crawfish holes), root holes, former sand boils, cracks, or other thin or weak spots. Once erosion initiates, the pipe may progress horizontally through the foundation in the direction toward the impounded water if hydraulic gradients in the foundation are sufficiently high. The hydraulic gradients and flow must remain high near the upstream or floodside tip of the progressing pipe for particles to continue eroding.

To assess the likelihood of BEP progression (hydraulic condition), the global or average horizontal gradient in the foundation at the pipe head, $i_{avf} = \Delta H/L$ as illustrated in Figure 8, can be compared to the critical horizontal gradient for BEP progression. There are several methods for assessing the critical horizontal gradient. Creep ratios are an empirical method based on observations of seepage performance for a range of soil types and are among the oldest methods. The inverse of the creep ratio is the average horizontal gradient if there are no vertical structures. More recently, methods based on horizontal gradient have gained more attention for evaluating the potential for BEP of cohesionless soils, specifically based on laboratory research involving laboratory flume tests to study the hydraulic gradient across a structure required to achieve complete pipe formation.

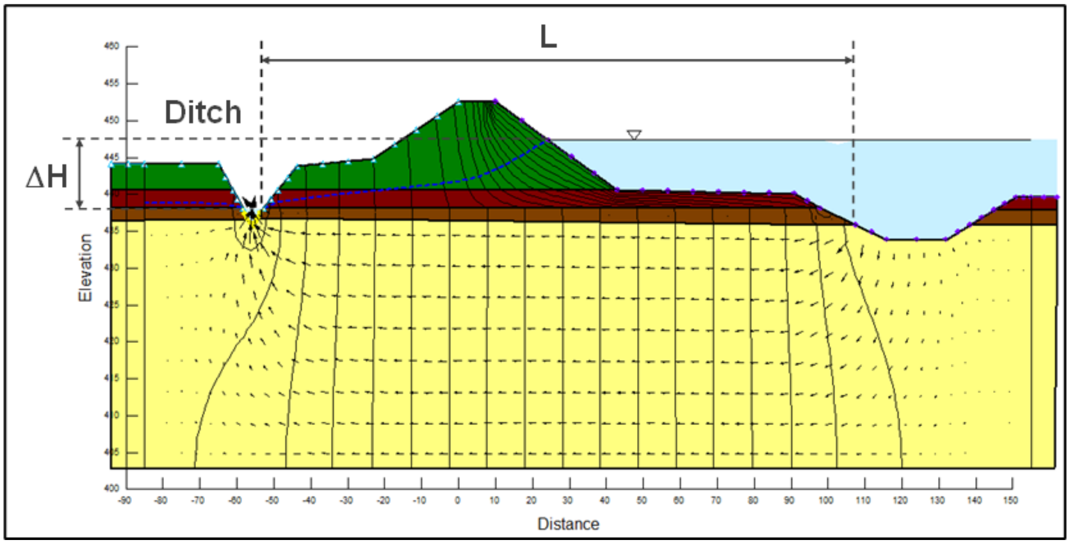


Figure 8. Geometry for average horizontal gradient.

This toolbox deterministically and probabilistically assesses the likelihood of BEP progression using the adjusted Schmertmann (2000) method and the adjusted calculation rule of Sellmeijer et al. (2011), in addition to the creep ratio methods of Bligh (1910) and Lane (1935).

Applying these methods correctly requires an understanding of the context from which each method was developed. Robbins and van Beek (2015) provide a more detailed review of the background, advantages, and disadvantages of each method and the various laboratory test conditions (such as density, exit configuration, soil characteristics, and scale effects) that significantly impact the findings. For example, the adjusted Schmertmann method and adjusted calculation rule of Sellmeijer et al. can be used only for situations that have a purely two-dimensional (2D) seepage regime (applicable only to situations that have uniform boundary conditions parallel to the embankment centerline such as an exposed ditch or no confining layer). Some methods may not apply to the materials under consideration. For example, the adjusted calculation rule of Sellmeijer et al. applies only within the range of soils tested. For soils beyond the suggested ranges and differing exit configurations, the methods are not necessarily applicable, and the actual critical horizontal gradients may be quite different than what is estimated.

5. Schmertmann

Schmertmann (2000) developed an approach for estimating the factor of safety (FS) regarding piping using the concept of ambient, pre-pipe, hydraulic gradients along the potential piping path based on laboratory flume tests. The FS for pipe progression is given in Equation 1.

$$FS = \left(\frac{C_D C_L C_S C_K C_\gamma C_Z C_\alpha i_{pmt}}{C_R i_f} \right) \tag{1}$$

where:

i_{pmt} = maximum pre-pipe hydraulic gradient along the pipe path in the reference laboratory test (that is, laboratory horizontal critical gradient)
i_f = maximum pre-pipe hydraulic gradient along the pipe path in the field
C_D = depth/length factor
C_L = total pipe length factor
C_S = grain-size factor
C_K = anisotropic permeability factor
C_γ = density factor
C_Z = underlayer factor
C_α = adjustment for pipe inclination
C_R = embankment axis curvature factor

Schmertmann applied these correction factors to more than 100 laboratory flume tests to adjust the laboratory results to the reference values shown in Table 1.

Table 1.
Schmertmann reference test values.

Parameter	Minimum
Seepage path length, L	5 feet
Piping layer depth, D	1 foot
Particle size with 10% passing by weight, d_{10}	0.20 millimeters
Anisotropy, $R_k = k_h/k_v$	1.5
Relative density, D_r	60 percent
Pipe path inclination, α	0 degrees

From the corrected values, Schmertmann proposed a no-test default relationship as the default value of i_{pmt} to use in assessments if laboratory flume tests were not conducted, shown in Equation 2 and Figure 9.

$$i_{pmt} = 0.05 + 0.183(C_u - 1) \tag{2}$$

where:

C_u = coefficient of uniformity

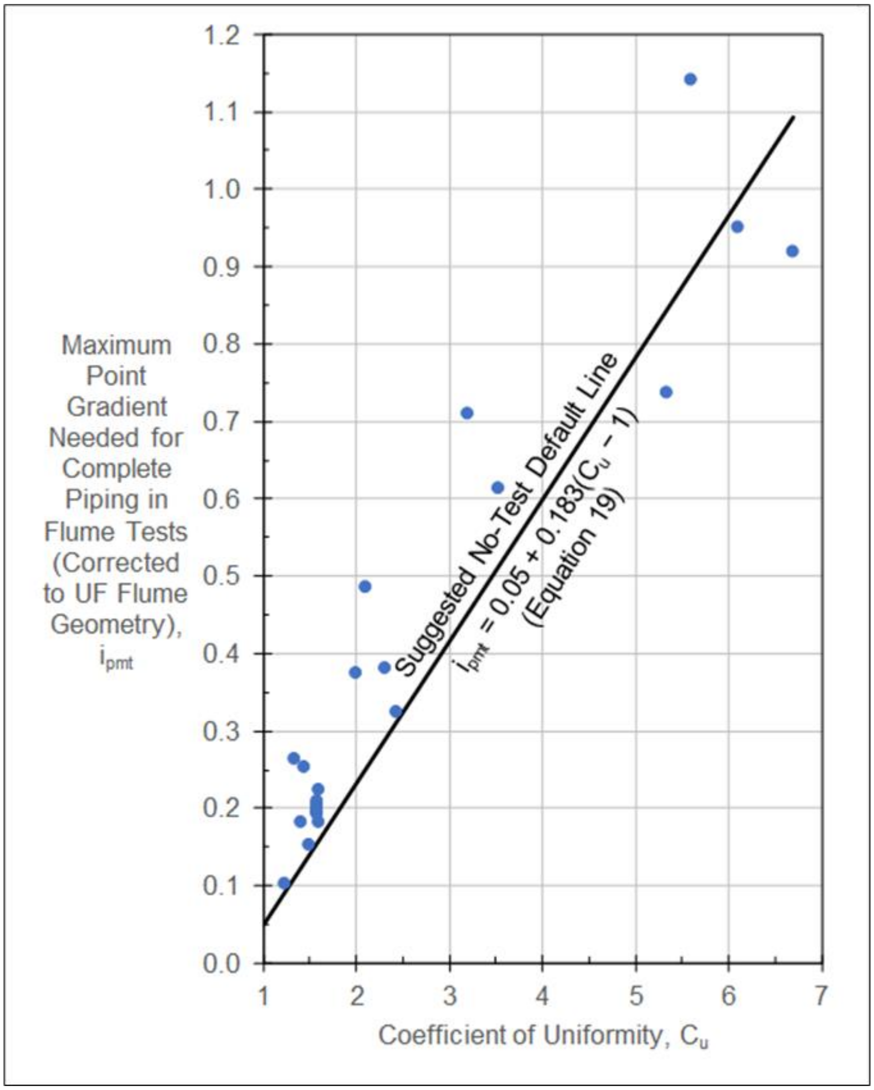


Figure 9. Schmertmann laboratory critical gradients.

Equations 1 and 2 can be used along with the correction factors to assess the FS for BEP progression in the field. While this figure is quite useful, it is difficult to estimate the uncertainty in the critical point gradient because study averages were presented. For example, one of the single data points is the average from 14 individual laboratory tests.

To assess the likelihood of BEP progression, Robbins and Sharp (2016) compiled all the data in Schmertmann (2000) and developed a probabilistic chart for determining i_{pmt}, again relative to the reference values, as shown in Figure 10. Robbins and O'Leary (2020) expanded this chart to include 0 percent and 100 percent probability trend lines based on linear extrapolation of the slopes and intercepts of the quantile regression lines.

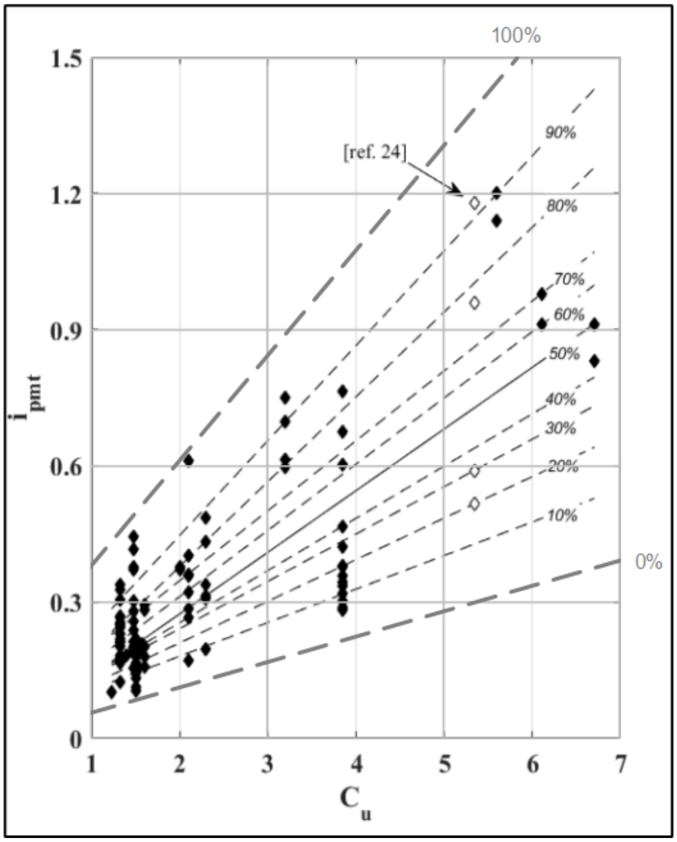


Figure 10. Schmertmann laboratory critical gradients with Robbins and Sharp (2016) estimated probability curves and Robbins and O'Leary (2020) modifications.

To calculate a probability of BEP progression using the lines shown on the probability chart, the value of i_{pmt} must be calculated for a given field scenario as shown in Equation 3.

$$i_{pmt} = \frac{C_R i_f}{C_D C_L C_S C_K C_Z C_\gamma C_\alpha} \tag{3}$$

The probabilistic chart from Robbins and Sharp shows that the uncertainty with the laboratory critical gradient increases with increasing C_u, where there is less data. In addition, at higher C_u values, soils tend to become internally unstable. Therefore, the probabilistic chart is limited to a C_u between 1.1 and about 4.

5.1. Method of Analysis

In step 1, use the drop-down list to select the method of analysis (probabilistic or deterministic). There are two options for probabilistic analysis. The first performs 1,000 iterations (judged adequate for most applications) without using Palisade's @RISK software. This provides flexibility if an @RISK software license is not available. The second uses @RISK to customize the probabilistic analysis. Use the drop-down list to select Yes if @RISK is to be used and No if @RISK is not used. The three possible scenarios are illustrated in Figure 11 through Figure 13.

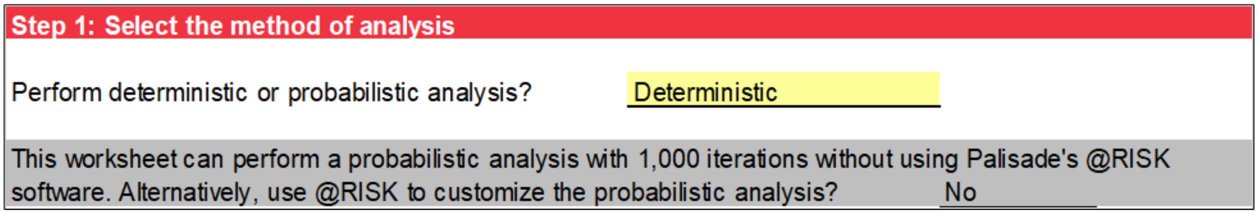


Figure 11. Step 1 of Schmertmann worksheet: Deterministic analysis.

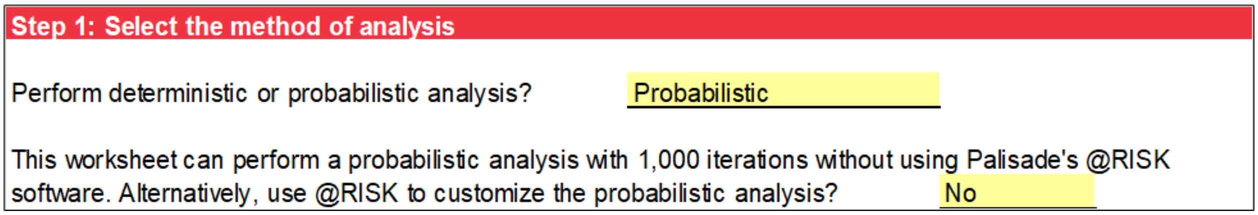


Figure 12. Step 1 of Schmertmann worksheet: Probabilistic analysis without using @RISK.

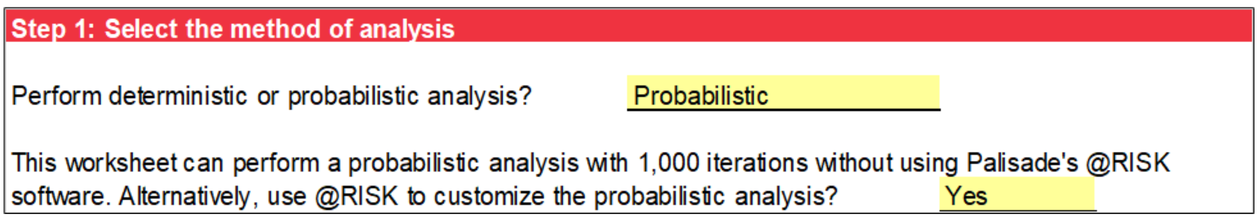


Figure 13. Step 1 of Schmertmann worksheet: Probabilistic analysis using @RISK.

5.2. Piping Layer Characterization

Step 2 characterizes the piping layer. The input includes the depth (thickness) of the piping layer (D), coefficient of uniformity (C_u), grain size in the field with 10 percent finer by weight (d_{10f}), anisotropy in the field of the piping layer (R_{kf}), relative density in the field (D_{rf}), and angle of the pipe path (α). The adjusted Schmertmann method uses the probabilistic chart of Robbins and Sharp (2016), expanded by Robbins and O'Leary (2020), for both the deterministic and probabilistic methods and is limited to C_u between 1.1 and 4. C_u values outside of this range have an orange background. The depth (thickness of the piping layer) is in the direction perpendicular to the angle of the pipe path discussed later in this section.

The selections in step 1 affect the input for step 2, and cells that do not apply have a gray background. These cells are not used in subsequent calculations even if data is present.

For deterministic analysis, input only the most likely values. The mean value used for subsequent calculations is the most likely (or mode) value. Figure 14 illustrates the deterministic input.

Step 2: Characterize the piping layer				
Parameter	Units	Minimum	Most Likely	Maximum
Depth (thickness) of piping layer, D	ft	10.0	15.0	20.0
Coefficient of uniformity, 1.1 ≤ C_u ≤ 4	-	1.50	2.00	3.00
Grain size (field) with 10% finer by weight, d_{10f}	mm	0.100	0.150	0.300
Anisotropy of piping layer (field), $R_{kf} = k_h/k_v$	-	1.0	1.5	2.0
Relative density (field), D_{rf}	percent	25.0	35.0	40.0

Parameter	Units	Mean	@RISK Formula	Mean
D	ft	#NAME?		15.0
C_u	-	#NAME?		2.00
d_{10f}	mm	#NAME?		0.150
R_{kf}	-	#NAME?		1.5
D_{rf}	percent	#NAME?		35.0

Figure 14. Step 2 of Schmertmann worksheet: Deterministic input.

For probabilistic analysis without using @RISK, input the minimum and maximum values in addition to the most likely value, and triangular distributions represent the random variables. The mean values used in subsequent calculations are the average of the minimum, most likely, and maximum values. Figure 15 illustrates probabilistic input without using @RISK.

Step 2: Characterize the piping layer				
Parameter	Units	Minimum	Most Likely	Maximum
Depth (thickness) of piping layer, D	ft	10.0	15.0	20.0
Coefficient of uniformity, 1.1 ≤ C_u ≤ 4	-	1.50	2.00	3.00
Grain size (field) with 10% finer by weight, d_{10f}	mm	0.100	0.150	0.300
Anisotropy of piping layer (field), $R_{kf} = k_h/k_v$	-	1.0	1.5	2.0
Relative density (field), D_{rf}	percent	25.0	35.0	40.0

Parameter	Units	Mean	@RISK Formula	Mean
D	ft	#NAME?		15.0
C_u	-	#NAME?		2.17
d_{10f}	mm	#NAME?		0.183
R_{kf}	-	#NAME?		1.5
D_{rf}	percent	#NAME?		33.3

Figure 15. Step 2 of Schmertmann worksheet: Probabilistic input without using @RISK.

For probabilistic analysis using @RISK, input the minimum, most likely, and maximum values, and use an @RISK formula for a triangular distribution in the third column as a default. Alternatively, input a valid @RISK distribution in lieu of this default formula, and the user-specified input displays in the fourth column. The mean values used for subsequent calculations are the means for the @RISK distributions entered in the third column. Figure 16 illustrates the probabilistic input without using @RISK.

Step 2: Characterize the piping layer				
Parameter	Units	Minimum	Most Likely	Maximum
Depth (thickness) of piping layer, D	ft	10.0	15.0	20.0
Coefficient of uniformity, $1.1 \le C_u \le 4$	-	1.50	2.00	3.00
Grain size (field) with 10% finer by weight, d_{10f}	mm	0.100	0.150	0.300
Anisotropy of piping layer (field), $R_{kf} = k_h/k_v$	-	1.0	1.5	2.0
Relative density (field), D_{rf}	percent	25.0	35.0	40.0

Parameter	Units	Mean	@RISK Formula	Mean
D	ft	15.0	=@RiskTriang(F23,G23,H23)	15.0
C_u	-	2.17	=@RiskTriang(F24,G24,H24)	2.17
d_{10f}	mm	0.183	=@RiskTriang(F25,G25,H25)	0.183
R_{kf}	-	1.5	=@RiskTriang(F26,G26,H26)	1.5
D_{rf}	percent	33.3	=@RiskTriang(F27,G27,H27)	33.3

Figure 16. Step 2 of Schmertmann worksheet: Probabilistic input using @RISK.

If using @RISK to perform probabilistic analysis, delete unnecessary calculation worksheets because the simulation is performed for all worksheets in the workbook, which is time consuming. If cycling through iterations using @RISK, the displayed results are no longer mean values of the random variables; they are the selected iteration's values.

The remaining input for step 2 addresses pipe path geometry as illustrated in Figure 17 and includes the angle of the pipe path (α) with respect to horizontal and the direct (not meandered) length between ends of a completed pipe path (L). A figure is included in this step as a guide.

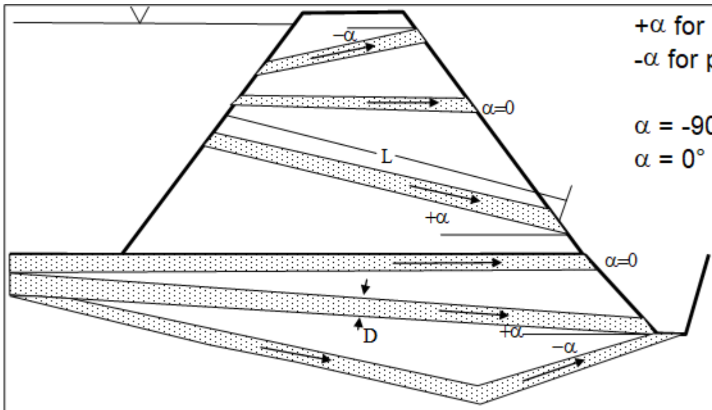
In most cases, the pipe path is horizontal, and $\alpha = 0$ degrees. The angle is positive for pipe paths that progress upwards and negative for pipe paths that progress downwards. Based on Schmertmann (2000), the range for α is between -90 degrees and 40 degrees. Cells where the input parameter is outside these limits have an orange background.

Based on the user-specified anisotropy, the ratio of the depth of the piping layer to the length of the pipe path in the field (D/L_f) is calculated, and the transformed length of the piping path in the field (L_f) is calculated using Equation 4.

$$L_f = \frac{L}{(R_{kf})^{0.5}} \tag{4}$$

where:

R_{kf} = anisotropy in the field of the piping layer (k_h/k_v)



+α for pipe paths that progress upwards
-α for pipe paths that progress downwards

α = -90° for vertical (downward) pipe path
α = 0° for horizontal pipe path

Angle of pipe path with respect to horizontal, -90° ≤ α ≤ 40°	0.0	deg
Direct (not meandered) length between ends of a completed pipe path, L	200	ft
Transformed length of pipe path in the field, $L_f = L / (R_{kf})^{0.5}$	163.3	ft
Ratio of depth of piping layer to length of pipe path in the field, D/L_f	0.09	

Figure 17. Step 2 of Schmertmann worksheet: Pipe path geometry.

5.3. Laboratory Horizontal Critical Gradient

Step 3 estimates the laboratory horizontal critical gradient. Use the drop-down list to indicate if a laboratory flume test was performed. Cells that do not apply have a gray background. For a laboratory flume test, input the measured laboratory horizontal critical gradient as illustrated in Figure 18. In practice, laboratory flume tests are rarely conducted to measure the critical horizontal gradient, and the linear relationship proposed by Schmertmann (2000) is the primary means for estimating critical gradient values as a function of coefficient of uniformity (C_u). In this toolbox, the 50 percent quantile line from Robbins and Sharp (2016) is used in lieu of Schmertmann's no-test default line using Equation 5 as illustrated in Figure 19.

$$i_{pmt} = 0.1358(C_u) + 0.002 \quad (5)$$

Robbins and Sharp found that the best fit, median line of all test results indicates Schmertmann's no-test default line is conservative for $C_u < 3$ and unconservative for $C_u > 3$.

Step 3: Estimate laboratory horizontal critical gradient for backward erosion piping progression	
Laboratory flume test performed?	Yes
Measured laboratory horizontal critical gradient, i_{pmt}	0.250
Estimated laboratory horizontal critical gradient, $i_{pmt} = 0.1358(C_u) + 0.002$	#N/A
from Robbins and Sharp (2016) 50% quantile line in lieu of Schmertmann (2000) Equation 19 for no-test default line	

Figure 18. Step 3 of Schmertmann worksheet: Critical gradient from laboratory flume test.

Step 3: Estimate laboratory horizontal critical gradient for backward erosion piping progression	
Laboratory flume test performed?	No
Measured laboratory horizontal critical gradient, i_{pmt}	
Estimated laboratory horizontal critical gradient, i_{pmt} = 0.1358(C_u) + 0.002	0.296
from Robbins and Sharp (2016) 50% quantile line in lieu of Schmertmann (2000) Equation 19 for no-test default line	

Figure 19. Step 3 of Schmertmann worksheet: Estimated laboratory critical gradient.

5.4. Field Horizontal Critical Gradient

Step 4 calculates the field horizontal critical gradient by applying the various correction factors to the estimated or measured laboratory horizontal critical gradient as illustrated in Figure 20. The correction factors for foundation depth, seepage path length, grain size, anisotropy, and soil density are estimated using Equations 6, 7, 8, 9, and 10. Since a straight axis is assumed for the embankment, there is no correction factor for curvature in the embankment axis, and $C_R = 1$.

$$C_D = \frac{\left(\frac{D}{L_f}\right)^{\frac{0.2}{\left(\frac{D}{L_f}\right)^2 - 1}}}{\left(\frac{1\text{ ft}}{5\text{ ft}}\right)^{\frac{0.2}{\left(\frac{1\text{ ft}}{5\text{ ft}}\right)^2 - 1}}} = \frac{\left(\frac{D}{L_f}\right)^{\frac{0.2}{\left(\frac{D}{L_f}\right)^2 - 1}}}{1.4} \tag{6}$$

$$C_L = \left(\frac{5\text{ ft}}{L_f}\right)^{0.2} \tag{7}$$

$$C_S = \left(\frac{d_{10f}}{0.20\text{ mm}}\right)^{0.2} \tag{8}$$

$$C_K = \left(\frac{1.5}{\frac{k_h}{k_v}}\right)^{0.5} = \left(\frac{1.5}{R_{kf}}\right)^{0.5} \tag{9}$$

$$C_\gamma = 1 + 0.4\left(\frac{D_{rf}}{100} - 0.6\right) \tag{10}$$

where:

D/L_f = ratio of depth of piping layer to length of pipe path in the field
L_f = transformed length (feet) of pipe path in the field
k_h/k_v = anisotropy in the field (R_{kf})
D_{rf} = relative density in the field

Step 4: Estimate the field horizontal critical gradient for backward erosion piping progression		
Correction Factors to Adjust from Laboratory-to-Field Conditions		
Depth/length factor, $C_D = (D/L_f)$^[0.2 / $((D/L_f)^2 - 1)$] / 1.4		1.158
Total pipe length factor, $C_L = (5\text{ ft} / L_f)^{0.2}$		0.498
Grain-size factor, $C_S = (d_{10f} / 0.20\text{ mm})^{0.2}$		0.983
Anisotropic permeability factor, $C_K = (1.5 / R_{kf})^{0.5}$		1.000
Density factor, $C_\gamma = 1 + 0.4(D_{rf} / 100 - 0.6)$		0.893
Embankment axis curvature factor, C_R	Assume straight axis	1.000

Figure 20. Step 4 of Schmertmann worksheet: Corrections for field conditions.

A higher permeability layer beneath the piping layer increases the flow toward the pipe and decreases the critical gradient. Use the drop-down list to indicate the presence of a higher permeability underlayer (for example, fine sand layer over coarse sand layer). If there is no higher permeability underlayer, $C_Z = 1$ as illustrated in Figure 21. Cells that do not apply have a gray background. If a higher permeability underlayer exists, input the permeability of the piping layer (k_p), permeability of the underlayer (k_u), and equivalent radius of the developing pipe cross section (r) to estimate the underlayer correction factor (C_Z).

Schmertmann (2000) provided C_Z as a function of D/r for $k_u/k_p \leq 1$, $k_u/k_p = 5$, and $k_u/k_p \geq 1$ based on very limited data derived from flow nets of the University of Florida flume. C_Z is obtained from two-way linear interpolation using these three curves as illustrated in Figure 22. Fell et al. (2008) suggests using $C_Z = 1$ since r is very small and D/r is typically very large in practice; C_Z is approximately 1 when D/r is greater than about 80. For very thin erodible layers (less than a couple of feet), Fell et al. (2008) suggested using r = 2.5 to 10 mm, where the horizontal critical gradient decreases as r increases. If thin alternating layers of erodible and non-erodible soil are modeled as a homogenous layer with high anisotropy, use $C_Z = 1$.

Underlayer Factor (C_Z)

Does the piping sand have a higher permeability underlayer? No

Permeability of piping layer, k_p ____________ cm/sec
Permeability of underlayer, k_u ____________ cm/sec

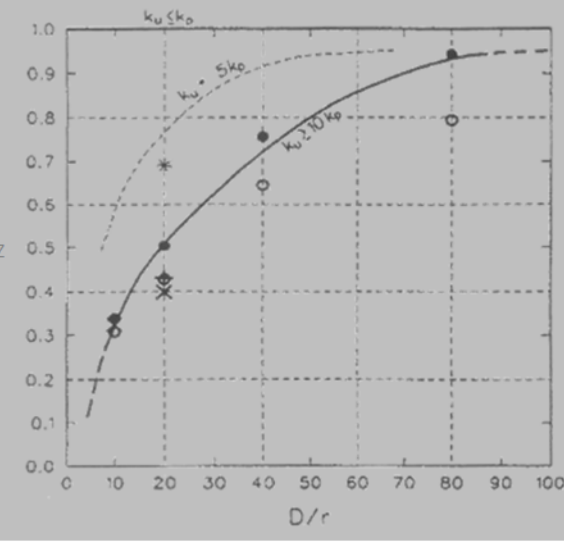
Equivalent radius of developing pipe cross section, r
(Use r = 2.5 to 10 mm for very thin erodible layers)

r ____________ mm

k_u/k_p #N/A

D/r #N/A

For practical purposes, r is very small and D/r is very large; suggest C_Z=1. If thin alternating layers of erodible and non-erodible soil are modelled as a homogenous layer with high anisotropy, use C_Z=1.



C_Z 1.000

Figure 21. Step 4 of Schmertmann worksheet: No underlayer factor.

Underlayer Factor (C_Z)

Does the piping sand have a higher permeability underlayer? Yes

Permeability of piping layer, k_p 5.00E-02 cm/sec
Permeability of underlayer, k_u 1.00E+01 cm/sec

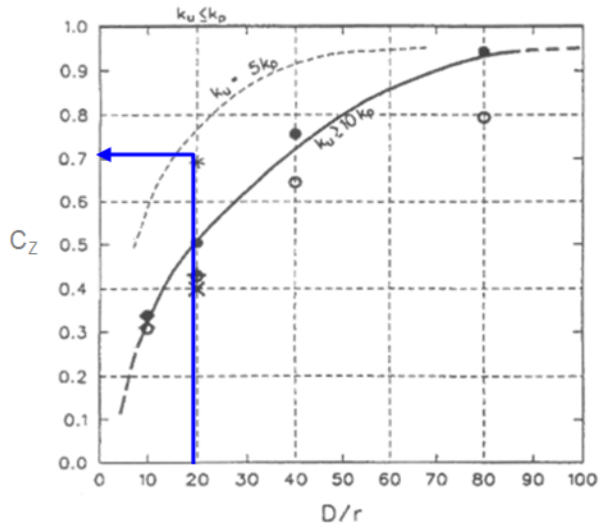
Equivalent radius of developing pipe cross section, r
(Use r = 2.5 to 10 mm for very thin erodible layers)

r 10 mm

k_u/k_p 6.0

D/r 20

For practical purposes, r is very small and D/r is very large; suggest C_Z=1. If thin alternating layers of erodible and non-erodible soil are modelled as a homogenous layer with high anisotropy, use C_Z=1.



C_Z 0.708

Figure 22. Step 4 of Schmertmann worksheet: Underlayer factor.

The field horizontal critical gradient is calculated using the above correction factors and Equation 11 as illustrated in Figure 23.

$$i_{ch} = \left(\frac{C_D C_L C_S C_K C_\gamma C_Z}{C_R}\right) i_{pmt} \tag{11}$$

Field Horizontal Critical Gradient (i_{po})

Field horizontal critical gradient, $i_{ch} = i_{po} = [(C_D)(C_L)(C_S)(C_K)(C_\gamma)(C_Z) / C_R](i_{pmt})$ 0.150

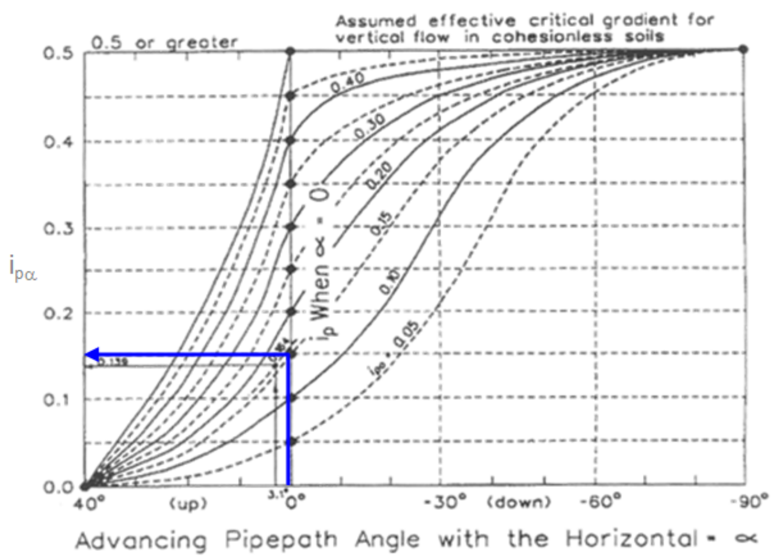
Figure 23. Step 4 of Schmertmann worksheet: Field horizontal critical gradient.

5.5. Pipe Path Inclination

Step 5 determines the adjustment for pipe path inclination. In most cases, the pipe path is horizontal (α = 0 degrees), and C_α = 1. Cells that do not apply have a gray background. If the pipe path inclination is not zero, Schmertmann (2000) provided the field critical gradient adjusted for pipe path inclination ($i_{p\alpha}$) as a function of α for field horizontal critical gradients (i_{po}) of 0.05, 0.10, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40, 0.45, and 0.50 or greater, as illustrated in Figure 24. $i_{p\alpha}$ is obtained from two-way linear interpolation using these ten curves. The back-calculated adjustment for pipe path inclination uses Equation 12.

$$C_\alpha = \frac{i_{p\alpha}}{i_{po}} \tag{12}$$

Step 5: Adjust the field horizontal critical gradient for pipe inclination



Field horizontal critical gradient adjusted for pipe path inclination, $i_{p\alpha}$	0.150
Adjustment for pipe path inclination, $C_\alpha = i_{p\alpha} / i_{po}$	1.000

Figure 24. Step 5 of Schmertmann worksheet: Adjustment for pipe path inclination.

5.6. Field Critical Gradient for BEP Progression

Step 6 adjusts the field horizontal critical gradient for pipe path inclination using the adjustment for pipe path inclination (C_α) from step 4 using Equation 13.

$$i_{p\alpha} = i_{ch} C_\alpha \tag{13}$$

If the pipe path is horizontal, $i_{p\alpha} = i_{ch}$ because $C_\alpha = 1$.

Schmertmann's adjusted model applies to 2D seepage with plane or ditch exits parallel to the embankment centerline. Van Beek et al. (2015) found that three-dimensional (3D) configurations with flow toward a single point (such as a hole in a confining layer) resulted in significantly smaller critical gradients than predicted by the Dutch piping model. In both the small and medium-scale experiments, the model overestimated the critical gradient by a factor of approximately two. Although not developed and calibrated for this method, a similar reduction for a 3D exit condition is appropriate. If the field horizontal critical gradient is further adjusted for 3D flow, input a user-specified gradient reduction factor (GRF). Cells that do not apply have a gray background. Figure 25 and Figure 26 illustrate the two scenarios.

Step 6: Estimate the field critical gradient for backward erosion piping progression		
Note: The adjusted Schmertmann method applies to 2D seepage with plane or ditch exits parallel to the embankment centerline.		
$i_{p\alpha} = (i_{pmt})_{corrected} = (i_{po})(C_\alpha)$	(For $\alpha = 0°$, $i_{p\alpha} = i_{ch}$)	0.150
Note: Van Beek et al. (2015) found that 3D configurations with flow towards a single point (e.g., hole in a confining layer) resulted in significantly smaller critical gradients than predicted by the Sellmeijer model. The model overestimated the critical gradient by a factor of approximately 2. A similar reduction is appropriate for the adjusted Schemrtmann method.		
3D flow towards single point exit instead of 2D exit (ditch or outflow area)?		Yes
Gradient reduction factor to reduce 2D prediction for 3D exit (hole), GRF		2.0
$i_{ch} = (i_{pmt})_{corrected} = (i_{po})(C_\alpha)$ / GRF for 3D exit (hole)		0.075

Figure 25. Step 6 of Schmertmann worksheet: Field critical gradient with gradient reduction factor.

Step 6: Estimate the field critical gradient for backward erosion piping progression		
Note: The adjusted Schmertmann method applies to 2D seepage with plane or ditch exits parallel to the embankment centerline.		
$i_{p\alpha} = (i_{pmt})_{corrected} = (i_{po})(C_\alpha)$	(For $\alpha = 0°$, $i_{p\alpha} = i_{ch}$)	0.150
Note: Van Beek et al. (2015) found that 3D configurations with flow towards a single point (e.g., hole in a confining layer) resulted in significantly smaller critical gradients than predicted by the Sellmeijer model. The model overestimated the critical gradient by a factor of approximately 2. A similar reduction is appropriate for the adjusted Schemrtmann method.		
3D flow towards single point exit instead of 2D exit (ditch or outflow area)?		No
Gradient reduction factor to reduce 2D prediction for 3D exit (hole), GRF		2.0
$i_{ch} = (i_{pmt})_{corrected} = (i_{po})(C_\alpha)$ / GRF for 3D exit (hole)		#N/A

Figure 26. Step 6 of Schmertmann worksheet: Field critical gradient without gradient reduction factor.

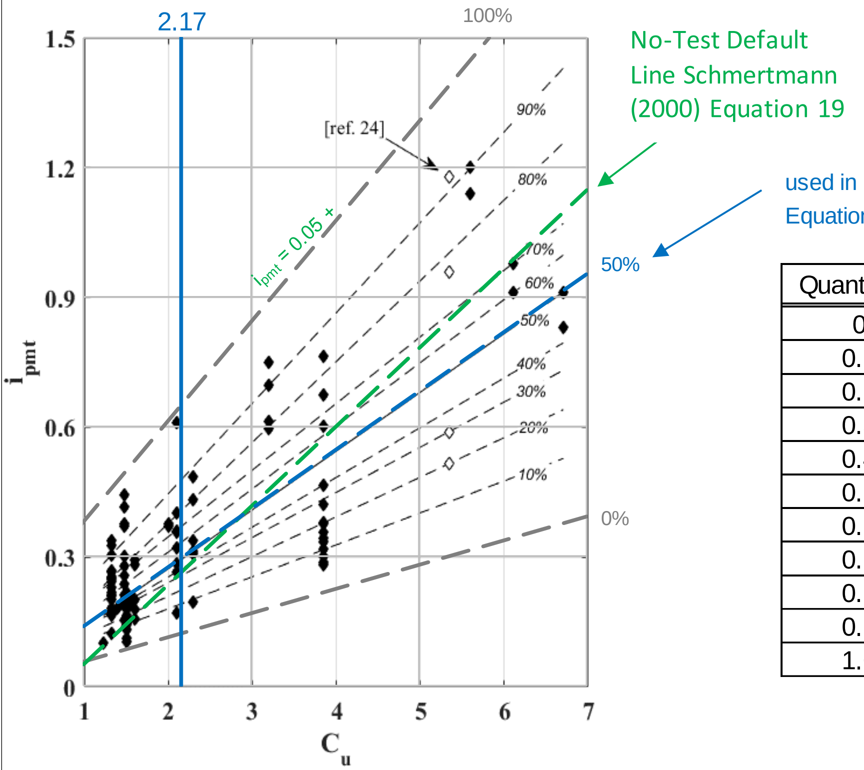
5.7. Likelihood of Backward Erosion Piping Progression

Robbins and Sharp (2016) presented the results of a best-fit quantile regression analysis from the individual laboratory flume tests, which were expanded by Robbins and O'Leary (2020) to include 0 percent and 100 percent probability trend lines based on linear extrapolation of the slopes and intercepts of the quantile regression lines. Step 7 uses this modified chart to develop a cumulative density function to estimate the probability of BEP progression as illustrated in Figure 27.

Step 7: Estimate the likelihood of backward erosion piping progression

Correction factors, $\prod C = (C_D)(C_L)(C_S)(C_K)(C_\gamma)(C_Z)(C_\alpha) / C_R$ 0.506

$i_{p\alpha} = (\prod C)\, i_{po}$



Quantile, P	i_{pmt}	$i_{p\alpha}$
0	0.122	0.062
0.1	0.193	0.098
0.2	0.225	0.114
0.3	0.258	0.131
0.4	0.273	0.138
0.5	0.296	0.150
0.6	0.336	0.170
0.7	0.373	0.189
0.8	0.409	0.207
0.9	0.481	0.243
1.0	0.652	0.330

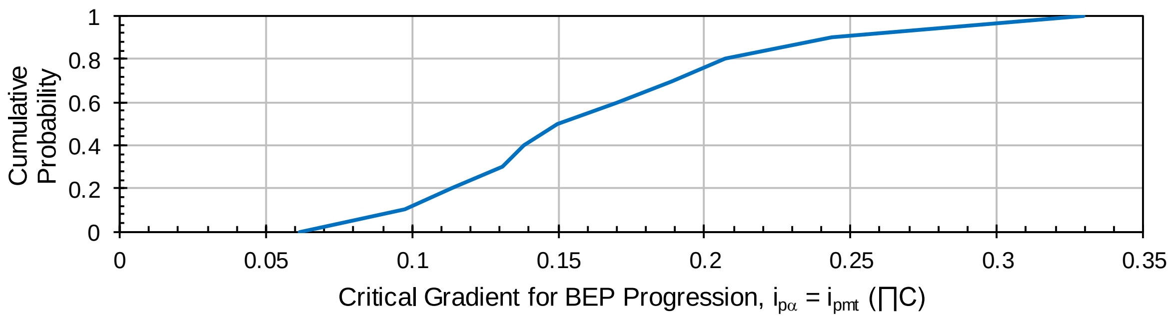


Figure 27. Step 7 of Schmertmann worksheet:
Cumulative density function for backward erosion piping progression.

The average horizontal gradient in the foundation at the pipe head is calculated by dividing the net hydraulic head by the seepage path length using Equation 14.

$$i_{avf} = \frac{\Delta H}{L} = \frac{HW - TW}{L} \tag{14}$$

where:

ΔH = net hydraulic head (feet)
HW = headwater level (feet)

TW = tailwater level (feet).
L = direct (not meandered) seepage path length (feet)

The FS against BEP progression is calculated using Equation 15.

$$FS = \frac{i_{p\alpha}}{i_{avf}} \tag{15}$$

where:

$i_{p\alpha}$ = critical gradient for BEP progression (adjusted for pipe path inclination)
i_{avf} = average horizontal gradient in the foundation at the pipe head

For deterministic analysis, the FS is calculated using the most likely values of the random variables and summarized in a table. Cells that do not apply have a gray background. For probabilistic analysis, the FS is calculated as described for the deterministic analysis but for the mean values of the random variables, and multiple iterations are performed by sampling the distributions in step 2. The probability of BEP progression is equal to the percentage of iterations that resulted in a FS less than 1 [(P(FS < 1)]. For probabilistic analysis performed without using @RISK, 1,000 iterations are used. For probabilistic analysis using @RISK, the number of iterations is user-specified, and "@RISK" displays in parentheses after the number of iterations for this scenario. If cycling through iterations using @RISK, the displayed results are no longer mean values; they are the selected iteration's values. For deterministic and probabilistic analyses, cells with FS less than 1 have an orange background. Figure 28 illustrates the deterministic tabular output, and Figure 29 illustrates the probabilistic tabular output without using @RISK.

Average hydraulic gradient in foundation at pipe head, $i_{avf} = \Delta H/L$
Factor of safety against backward erosion piping progression, $FS = i_{p\alpha}/i_{avf}$
Probability of backward erosion piping progression, P_{adv}
Iterations: #N/A

HW (ft)	195.5	201.6	213.5	218.9	223.0	234.0	239.0
TW (ft)	184.0	184.0	184.0	184.0	184.0	184.0	184.0
i_{avf}	0.058	0.088	0.148	0.175	0.195	0.250	0.275
FS	1.165	0.761	0.454	0.384	0.344	0.268	0.244
P_{adv}	2.80E-01	7.40E-01	9.94E-01	1.00E+00	1.00E+00	1.00E+00	1.00E+00

Figure 28. Step 7 of Schmertmann worksheet: Tabular output for deterministic analysis.

Average hydraulic gradient in foundation at pipe head, $i_{avf} = \Delta H/L$
Factor of safety against backward erosion piping progression, $FS = i_{p\alpha}/i_{avf}$
Probability of backward erosion piping progression, P_{adv}
Iterations: 1000

HW (ft)	195.5	201.6	213.5	218.9	223.0	234.0	239.0
TW (ft)	184.0	184.0	184.0	184.0	184.0	184.0	184.0
i_{avf}	0.058	0.088	0.148	0.175	0.195	0.250	0.275
FS	1.304	0.852	0.508	0.430	0.384	0.300	0.273
P_{adv}	2.45E-01	6.51E-01	9.56E-01	9.90E-01	9.98E-01	1.00E+00	1.00E+00

Figure 29. Step 7 of Schmertmann worksheet:
Tabular output for probabilistic analysis without using @RISK.

5.8. Summary Plots

Step 8 generates the summary plots. The first plot is the mean FS against BEP progression (red solid line) and average horizontal gradient at the pipe head (green solid line) as functions of headwater level. If cycling through iterations using @RISK, the displayed results are no longer mean values; they are the selected iteration. FS against BEP progression is plotted on the primary axis, and average horizontal gradient at the pipe head is plotted on the secondary axis. Horizontal reference lines display for the mean field critical gradient for BEP progression (green dashed line) and FS of 1 (red shaded line). Figure 30 illustrates the deterministic graphical output.

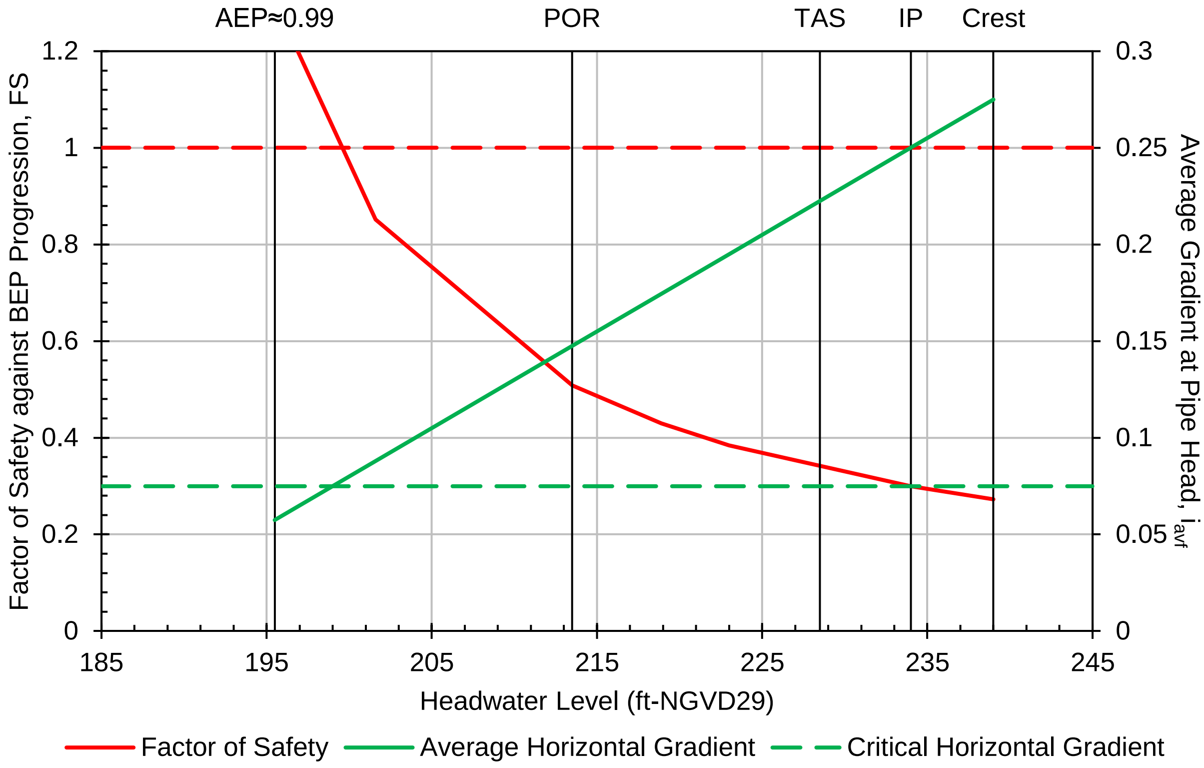


Figure 30. Step 7 of Schmertmann worksheet: Graphical output for deterministic analysis.

Figure 31 illustrates the plot options this chart. The maximum value for the primary y-axis (FS against BEP progression), maximum value for the secondary y-axis (average horizontal gradient at the pipe head), and minimum and maximum values for the x-axis (headwater level) are user-specified. Users can input up to five vertical reference elevations, and user-specified labels display at the top of the chart.

Worksheet	Schmertmann					
y-axis bounds (primary)						
minimum	0				Value Primary Min: 0	
maximum	1.2	◄ Enter maximum FS.			Value Primary Max: 1.2	
y-axis bounds (secondary)						
minimum	0				Value Secondary Min: 0	
maximum	0.30	◄ Enter maximum average gradient.			Value Secondary Max: 0.3	
x-axis bounds						
minimum	185.0	◄ Enter minimum headwater level.			Category Primary Min: 185	
maximum	245.0	◄ Enter maximum headwater level.			Category Primary Max: 245	
Enter up to 5 vertical reference lines for headwater levels of interest.						
AEP≈0.99	POR	TAS	IP	Crest	◄ Enter headwater description.	
195.5	213.5	228.5	234.0	239.0	◄ Enter headwater level.	

Figure 31. Step 7 of Schmertmann worksheet: Plot options for deterministic analysis.

Because the adjusted Schmertmann method uses the probabilistic chart of Robbins and Sharp (2016), expanded by Robbins and O'Leary (2020), a probability of BEP progression is calculated for both the deterministic and probabilistic methods as illustrated in Figure 32 and Figure 33, respectively. The mean probability of BEP progression is plotted as a function headwater level. For probabilistic analysis, the 90 percent uncertainty bounds (5th and 95th percentiles) are also plotted as a function headwater level using dashed lines, as shown in Figure 33.

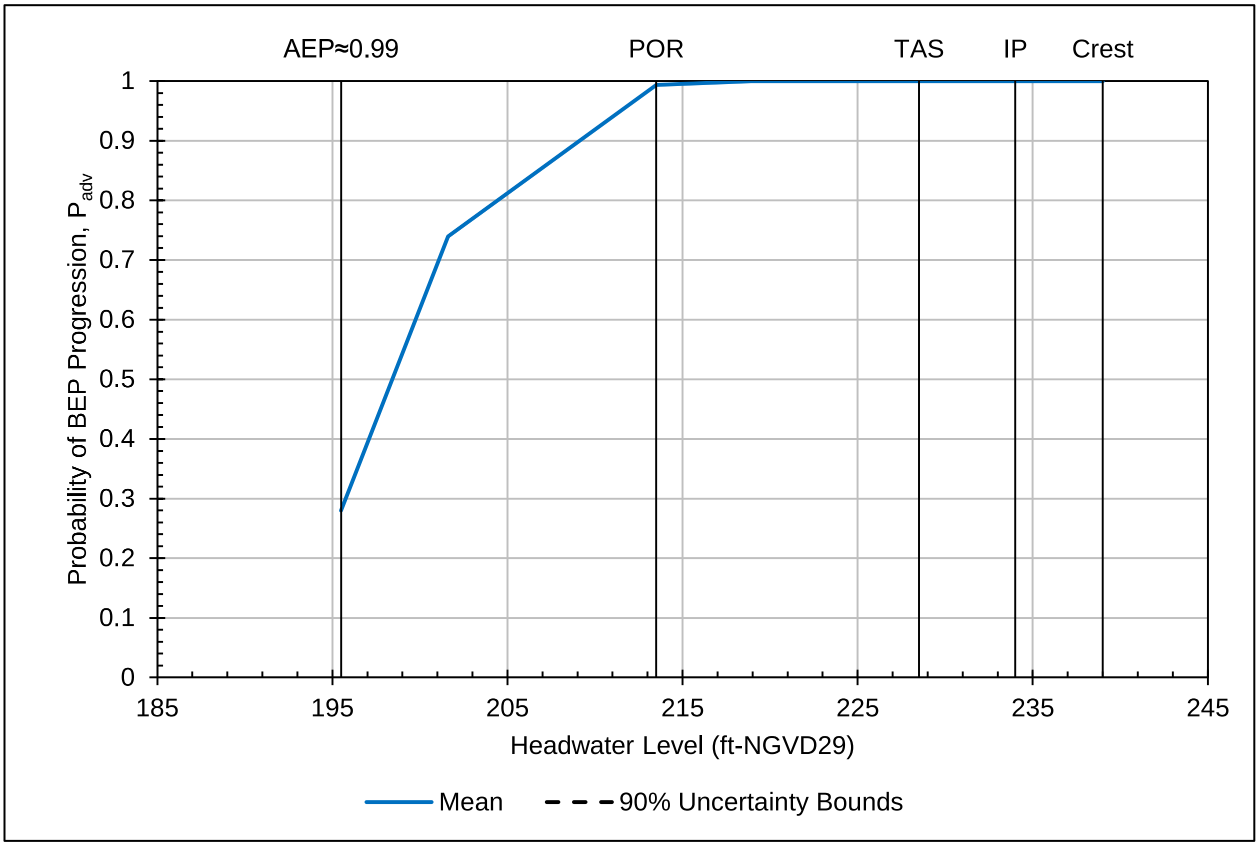


Figure 32. Step 7 of Schmertmann worksheet: Graphical output for deterministic analysis.

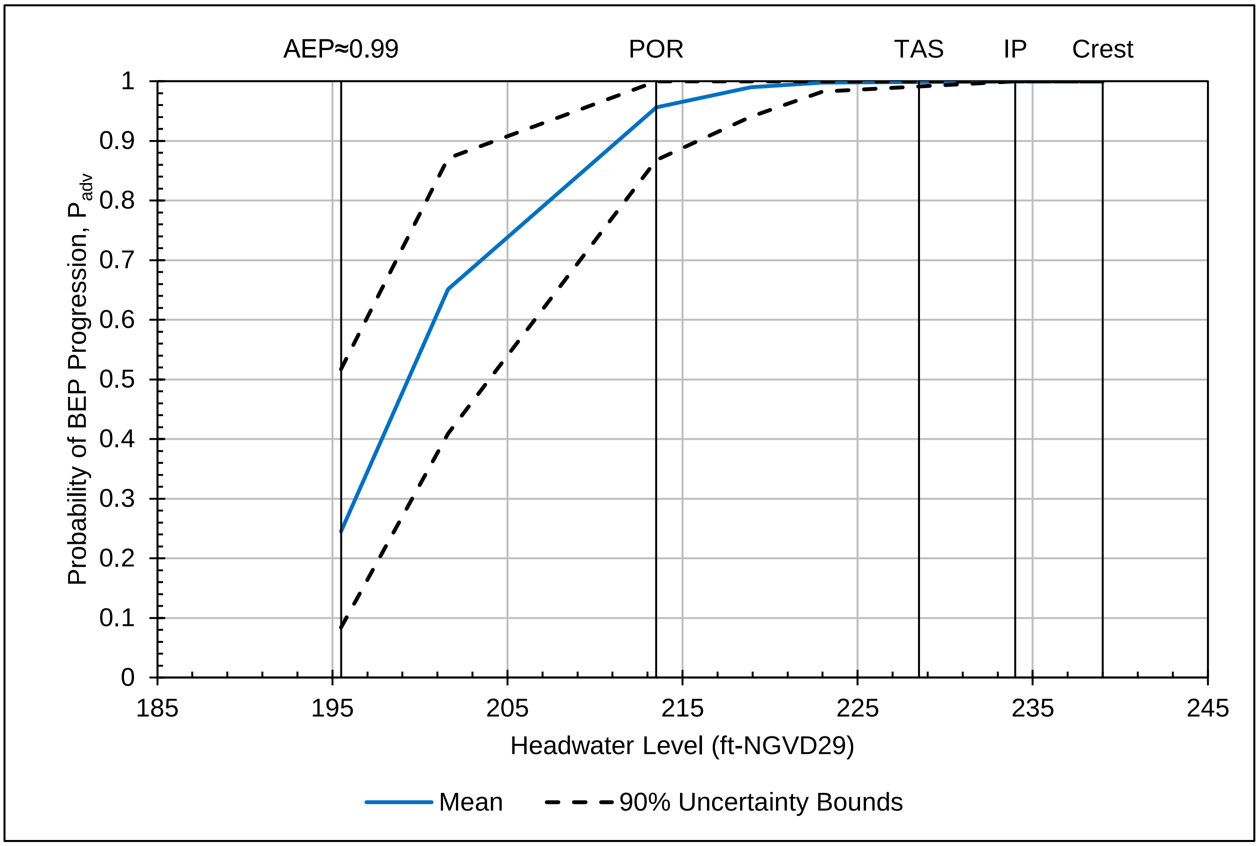


Figure 33. Step 7 of Schmertmann worksheet: Graphical output for probabilistic analysis.

Figure 34 illustrates the plot options for the charts in Figure 32 and Figure 33. The vertical reference elevations and minimum and maximum values for the x-axis (headwater level) are the same as Figure 31. Only the maximum value for the y-axis (probability of BEP progression) is user-specified.

Worksheet	Schmertmann		
y-axis bounds			
minimum	0		Value Primary Min: 0
maximum	1.0	◄ Enter maximum probability.	Value Primary Max: 1
x-axis bounds			
minimum	185.0	◄ Enter minimum headwater level.	Category Primary Min: 185
maximum	245.0	◄ Enter maximum headwater level.	Category Primary Max: 245

Figure 34. Step 7 of Schmertmann worksheet: Plot options for graphical output.

6. Sellmeijer

Between 1988 and 1993, Hans Sellmeijer and his coworkers developed a mathematical model for BEP progression. Curve-fitting numerical solutions in the process yielded a simplified calculation rule used in the Netherlands. The Sellmeijer model provided a relationship between pipe length and hydraulic head at which sand grains are in equilibrium. For any head less than the critical head *(H_c)*, the development of the piping channels stops. If the head is increased, erosion begins again. The critical head occurs when the length of the channel (l) is about 0.3 to 0.5 of flow path length (L). For net hydraulic head (H) less than H_c, progression of the pipe reaches a stable condition. For H greater than H_c, the piping channel extends upstream and breaks through to the impounded water.

Sellmeijer's model was implemented in a 2D numerical groundwater model to account for different configurations and used to derive a calculation rule for a standard levee located on top of a homogeneous confined aquifer. The original and adjusted calculation rules are described in Sellmeijer et al. (2011). The adjusted calculation rule extended and updated the piping model based on results of a wide range of tests (several small-scale, seven medium-scale, and four large-scale field IJkdijk experiments). Thus, the Sellmeijer model is known to result in good predictions for large-scale experiments with 2D configurations. The method is applicable only within the limits of the piping model testing parameters shown in Table 2.

Table 2.
Sellmeijer parameter limits.

Parameter	Minimum	Maximum
Particle size with 70 percent finer by weight, d_{70} (millimeters)	0.150	0.430
Coefficient of uniformity, *U*	1.3	2.6
Roundness of particles, *KAS* (percent)	35	70
Relative density, *RD* (percent)	34	100

6.1. Method of Analysis

The method of analysis is the same as the Schmertmann worksheet.

6.2. Soil Particle Characterization

Step 2 characterizes the soil particles in the piping layer. The input includes the specific gravity of sand particles (G_s*)*, bedding angle of sand (θ), and White's constant (η) as illustrated in Figure 35. The bedding angle and White's constant are set to 37 degrees and 0.25, respectively, due to model calibration and are not changed in Dutch practice.

Step 2: Characterize the soil particles in the piping layer		
Specific gravity of sand particles, G_s	2.65	
Bedding angle of sand, θ	37	deg
White's constant, η	0.25	

Figure 35. Step 2 of Sellmeijer worksheet: Discrete input.

The selections in step 1 affect the remaining input for step 2. Cells that do not apply have a gray background. These cells are not used in subsequent calculations even if data is present. The input includes the particle size with 70 percent finer by weight (*d_{70}*), coefficient of uniformity (*U*), roundness of particles (*KAS*), and relative density (*RD*). The method is applicable only within the limits of the piping model testing parameters. Cells where the input parameter is outside the model parameter limits have an orange background, and a caution displays for those parameters. Mean values are used to normalize project-specific values.

The roundness of particles can be analyzed using optical images. Software such as ImageJ approximates an ellipse for each individual particle and calculates the roundness of each ellipse. For example, a perfect circle has a resulting roundness of 100 percent. Each image contains a variable number of particles, and multiple images of each soil sample should be analyzed. In practice, optical images of soil samples are rarely conducted to obtain KAS. Van Beek et al. (2009) provides guidance for visual estimation of *KAS* as illustrated in Figure 36.

Roundness of particles, KAS

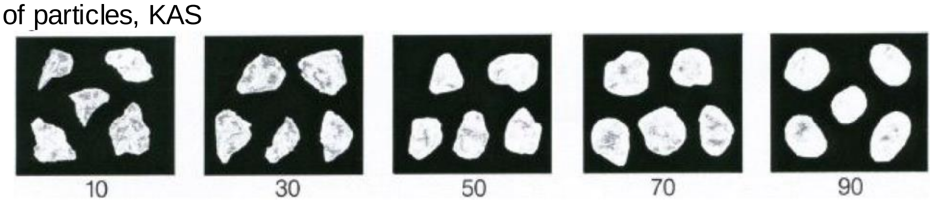


Figure 36. Step 2 of Sellmeijer worksheet: Visual estimation of KAS.

For deterministic analysis, input only the most likely values. The mean values used for subsequent calculations are the most likely (or mode) values. Figure 37 illustrates the deterministic input.

Parameter	Units	Minimum	Most Likely	Maximum
Particle size with 70% finer by weight, d_{70}	mm	0.150	0.500	1.900
Coefficient of uniformity, $U = C_u$	-	1.00	3.00	5.00
Roundness of particles, KAS	percent	35.0	44.4	70.0
Relative density, $RD = D_r$	percent	10.0	35.0	60.0

Model Parameter Limits

Parameter	Minimum	Maximum	Mean	
d_{70} (mm)	0.150	0.430	0.208	Calculation rule is N/A.
U (-)	1.3	2.6	1.81	Calculation rule is N/A.
KAS (%)	35	70	49.8	
RD (%)	34	100	72.5	Calculation rule is N/A.

Parameter	Units	Mean	@RISK Formula	Mean
d_{70}	mm	#NAME?		0.500
U	-	#NAME?		3.00
KAS	percent	#NAME?		44.4
RD	percent	#NAME?		35.0

Figure 37. Step 2 of Sellmeijer worksheet: Deterministic input.

For probabilistic analysis without using @RISK, input the minimum and maximum values in addition to the most likely value, and triangular distributions represent the random variables. The mean values used in subsequent calculations are the average of the minimum, most likely, and maximum values. Figure 38 illustrates the probabilistic input without using @RISK.

Parameter	Units	Minimum	Most Likely	Maximum
Particle size with 70% finer by weight, d_{70}	mm	0.150	0.500	1.900
Coefficient of uniformity, $U = C_u$	-	1.00	3.00	5.00
Roundness of particles, KAS	percent	35.0	44.4	70.0
Relative density, $RD = D_r$	percent	10.0	35.0	60.0

Model Parameter Limits

Parameter	Minimum	Maximum	Mean	
d_{70} (mm)	0.150	0.430	0.208	Calculation rule is N/A.
U (-)	1.3	2.6	1.81	Calculation rule is N/A.
KAS (%)	35	70	49.8	
RD (%)	34	100	72.5	Calculation rule is N/A.

Parameter	Units	Mean	@RISK Formula	Mean
d_{70}	mm	#NAME?		0.850
U	-	#NAME?		3.00
KAS	percent	#NAME?		49.8
RD	percent	#NAME?		35.0

Figure 38. Step 2 of Sellmeijer worksheet: Probabilistic input without using @RISK.

For probabilistic analysis using @RISK, input the minimum, most likely, and maximum values, and use an @RISK formula for a triangular distribution in the third column as a default. Alternatively, input a valid @RISK distribution in lieu of this default formula, and the user-specified input displays in the fourth column. The mean values used for subsequent calculations are the means for the @RISK distributions entered in the third column. Figure 39 illustrates the probabilistic input using @RISK.

Parameter	Units	Minimum	Most Likely	Maximum
Particle size with 70% finer by weight, d_{70}	mm	0.150	0.500	1.900
Coefficient of uniformity, $U = C_u$	-	1.00	3.00	5.00
Roundness of particles, KAS	percent	35.0	44.4	70.0
Relative density, $RD = D_r$	percent	10.0	35.0	60.0

Model Parameter Limits

Parameter	Minimum	Maximum	Mean	
d_{70} (mm)	0.150	0.430	0.208	Calculation rule is N/A.
U (-)	1.3	2.6	1.81	Calculation rule is N/A.
KAS (%)	35	70	49.8	
RD (%)	34	100	72.5	Calculation rule is N/A.

Parameter	Units	Mean	@RISK Formula	Mean
d_{70}	mm	0.850	=@RiskTriang(F34,G34,H34)	0.850
U	-	3.00	=@RiskTriang(F35,G35,H35)	3.00
KAS	percent	49.8	=@RiskTriang(F36,G36,H36)	49.8
RD	percent	35.0	=@RiskTriang(F37,G37,H37)	35.0

Figure 39. Step 2 of Sellmeijer worksheet: Probabilistic input using @RISK.

If using @RISK to perform probabilistic analysis, delete unnecessary calculation worksheets because the simulation is performed for all worksheets in the workbook, which this is time consuming. If cycling through iterations using @RISK, the displayed results are no longer mean values of the random variables; they are the selected iteration.

6.3. Piping Layer Characterization

The selections in step 1 affect the input for step 3, and cells that do not apply have a gray background. These cells are not used in subsequent calculations even if data is present. Step 3 characterizes the piping layer. The input includes the horizontal permeability ($k_{h,f}$) and thickness (D_f) of the fine sand (piping) layer, seepage water temperature (T), and seepage path length (L). A lookup table is used to obtain the dynamic viscosity of water (μ) as a function of seepage water temperature.

The intrinsic permeability of the fine sand (piping) layer is calculated using Equation 16.

$$\kappa_f = k_{h,f}\left(\frac{\mu}{\gamma_w}\right) \tag{16}$$

where:

k_h = permeability of the fine sand (piping) layer in the horizontal direction (m/s)
μ = dynamic viscosity of water (N·s/m^2)
γ_w = unit weight of water (N/m^3)

Use the drop-down list to indicate the presence of a multi-layer foundation (fine sand layer over coarse sand layer). Cells that do not apply have a gray background. Van Beek et al. (2013) conducted piping experiments on multi-layer sand samples to validate Sellmeijer's model, and the calculation rule was extended for multi-layer aquifers. A coarse layer beneath the piping layer increases the flow toward the pipe and decreases the critical head (and critical gradient). If a multi-layer foundation exists, the input also includes the horizontal permeability ($k_{h,c}$) and thickness (D_c) of the underlying coarse sand layer.

For a multi-layer aquifer, when flow toward the pipe is predominantly horizontal (for elongated aquifers), the increase of flow toward the pipe is reflected by the average horizontal permeability of a multilayer aquifer. A layer-weighted average of horizontal permeability ($k_{h,avg}$) is used in lieu of the horizontal permeability of the fine sand (piping) layer, shown in Figure 40, using Equation 17.

Figure 40. Step 3 of Sellmeijer worksheet: Multi-layer foundation input.

$$k_{h,avg} = \frac{k_{h,f}D_f + k_{h,c}D_c}{D} \tag{17}$$

where:

$k_{h,f}$ = permeability of the fine (piping) layer in the horizontal direction (centimeter/second)
D_f = thickness of fine (piping) layer (feet)
$k_{h,c}$ = permeability of the coarse underlayer in the horizontal direction (centimeter/second)
D_c = thickness of coarse underlayer (feet)
D = total aquifer thickness (feet)

When the contrast of permeability is large and the aquifer thickness is relatively large compared to the seepage path length, assuming predominant horizontal flow is no longer valid. Hence, the extension of Sellmeijer's calculation rule for multi-layer foundations can be applied to elongated aquifers ($D/L < 0.3$) with low contrasts of permeability ($k_{h,c}/k_{h,f} < 10$). D/L and $k_{h,c}/k_{h,f}$ values outside of these ranges have an orange background.

The average intrinsic permeability is calculated using Equation 18.

$$\kappa_{avg} = k_{h,avg}\left(\frac{\mu}{\gamma_w}\right) \tag{18}$$

For deterministic analysis of single-layer aquifer, input only the most likely value for the fine sand (piping) layer. The mean value used for subsequent calculations is the most likely (or mode) value. Figure 41 illustrates the deterministic input for single-layer aquifer.

Step 3: Characterize the piping layer

Does the piping sand have a higher permeability underlayer? No

Parameter	Units	Minimum	Most Likely	Maximum
Horizontal permeability of fine sand, $k_{h,f}$	cm/sec	3.53E-03	1.06E-02	2.82E-02
Horizontal permeability of coarse sand, $k_{h,c}$	cm/sec	1.41E-02	2.12E-02	3.88E-02

Sand Layer	Thickness, D (ft)	Mean k_h (cm/sec)	@RISK Formula for k_h (cm/sec)	Mean k_h (m/sec)
Fine	10.0	#NAME?		1.06E-04
Coarse	60.0	#NAME?		2.12E-04

Seepage water temperature, T	66.0	°F	18.9	°C
Dynamic viscosity of water, μ	from White (2011) Table A.1		1.033E-03	N s/m^2
Intrinsic permeability, $\kappa_f = k_{h,f}\ (\mu/\gamma_w)$			1.11E-11	m^2

Note: Adjusted calculation rule for multi-layer aquifers can be applied to elongated aquifers (D/L < 0.3) with low permeability contrasts ($k_{h,c}/k_{h,f}$ < 10).

Thickness of piping layer, D_f	10.0	ft	3.0	m
Total aquifer thickness, $D = D_f + D_c$	#N/A	ft	#N/A	m
Seepage path length, L	200.0	ft	61.0	m
D/L (where $D = D_f$ for a single-layer aquifer)	0.05			

Permeability contrast, $k_{h,c}/k_{h,f}$	#N/A	
Average horizontal permeability, $k_{avg} = [(k_{h,f})(D_f) + (k_{h,c})(D_c)] / D$	#N/A	m/sec
Average intrinsic permeability, $\kappa_{avg} = k_{h,avg}(\mu/\gamma_w)$	#N/A	m^2

Figure 41. Step 3 of Sellmeijer worksheet: Deterministic input for single-layer aquifer.

For deterministic analysis of multi-layer aquifer, input the most likely values for the fine sand (piping) layer and underlying coarse sand layer. The mean values used for subsequent calculations are the most likely (or mode) values. Figure 42 illustrates the deterministic input for multi-layer aquifer.

Step 3: Characterize the piping layer

Does the piping sand have a higher permeability underlayer? Yes

Parameter	Units	Minimum	Most Likely	Maximum
Horizontal permeability of fine sand, $k_{h,f}$	cm/sec	3.53E-03	1.06E-02	2.82E-02
Horizontal permeability of coarse sand, $k_{h,c}$	cm/sec	1.41E-02	2.12E-02	3.88E-02

Sand Layer	Thickness, D (ft)	Mean k_h (cm/sec)	@RISK Formula for k_h (cm/sec)	Mean k_h (m/sec)
Fine	10.0	#NAME?		1.06E-04
Coarse	60.0	#NAME?		2.12E-04

Seepage water temperature, T	66.0	°F	18.9	°C
Dynamic viscosity of water, μ	from White (2011) Table A.1		1.033E-03	N s/m^2
Intrinsic permeability, $\kappa_f = k_{h,f}\ (\mu/\gamma_w)$			#N/A	m^2

Note: Adjusted calculation rule for multi-layer aquifers can be applied to elongated aquifers (D/L < 0.3) with low permeability contrasts ($k_{h,c}/k_{h,f}$ < 10).

Thickness of piping layer, D_f	10.0	ft	3.0	m
Total aquifer thickness, $D = D_f + D_c$	70.0	ft	21.3	m
Seepage path length, L	200.0	ft	61.0	m
D/L (where $D = D_f$ for a single-layer aquifer)	0.35	Calculation rule is N/A.		
Permeability contrast, $k_{h,c}/k_{h,f}$			2.00	
Average horizontal permeability, $k_{avg} = [(k_{h,f})(D_f) + (k_{h,c})(D_c)] / D$			1.97E-04	m/sec
Average intrinsic permeability, $\kappa_{avg} = k_{h,avg}(\mu/\gamma_w)$			2.07E-11	m^2

Figure 42. Step 3 of Sellmeijer worksheet: Deterministic input for multi-layer aquifer.

For probabilistic analysis of single-layer aquifer without using @RISK, input the minimum and maximum values in addition to the most likely value, and a triangular distribution represents the random variable for the fine sand (piping) layer. The mean value used in subsequent calculations is the average of the minimum, most likely, and maximum values. Figure 43 illustrates the probabilistic input for single-layer aquifer without using @RISK.

Step 3: Characterize the piping layer

Does the piping sand have a higher permeability underlayer? No

Parameter	Units	Minimum	Most Likely	Maximum
Horizontal permeability of fine sand, $k_{h,f}$	cm/sec	3.53E-03	1.06E-02	2.82E-02
Horizontal permeability of coarse sand, $k_{h,c}$	cm/sec	1.41E-02	2.12E-02	3.88E-02

Sand Layer	Thickness, D (ft)	Mean k_h (cm/sec)	@RISK Formula for k_h (cm/sec)	Mean k_h (m/sec)
Fine	10.0	#NAME?		1.41E-04
Coarse	60.0	#NAME?		2.47E-04

Seepage water temperature, T — 66.0 °F — 18.9 °C

Dynamic viscosity of water, μ — from White (2011) Table A.1 — 1.033E-03 N s/m^2

Intrinsic permeability, $\kappa_f = k_{h,f}\ (\mu/\gamma_w)$ — 1.49E-11 m^2

Note: Adjusted calculation rule for multi-layer aquifers can be applied to elongated aquifers (D/L < 0.3) with low permeability contrasts ($k_{h,c}/k_{h,f}$ < 10).

Thickness of piping layer, D_f	10.0	ft	3.0	m
Total aquifer thickness, $D = D_f + D_c$	#N/A	ft	#N/A	m
Seepage path length, L	200.0	ft	61.0	m
D/L (where $D = D_f$ for a single-layer aquifer)	0.05			

Permeability contrast, $k_{h,c}/k_{h,f}$	#N/A	
Average horizontal permeability, $k_{avg} = [(k_{h,f})(D_f) + (k_{h,c})(D_c)] / D$	#N/A	m/sec
Average intrinsic permeability, $\kappa_{avg} = k_{h,avg}(\mu/\gamma_w)$	#N/A	m^2

Figure 43. Step 3 of Sellmeijer worksheet:
Probabilistic input for single-layer aquifer without using @RISK.

For probabilistic analysis of multi-layer aquifer without using @RISK, input the most likely values for the fine sand (piping) layer and underlying coarse sand layer. The mean values used for subsequent calculations are the average of the minimum, most likely, and maximum values. Figure 44 illustrates the probabilistic input for multi-layer aquifer without using @RISK.

Step 3: Characterize the piping layer

Does the piping sand have a higher permeability underlayer? Yes

Parameter	Units	Minimum	Most Likely	Maximum
Horizontal permeability of fine sand, $k_{h,f}$	cm/sec	3.53E-03	1.06E-02	2.82E-02
Horizontal permeability of coarse sand, $k_{h,c}$	cm/sec	1.41E-02	2.12E-02	3.88E-02

Sand Layer	Thickness, D (ft)	Mean k_h (cm/sec)	@RISK Formula for k_h (cm/sec)	Mean k_h (m/sec)
Fine	10.0	#NAME?		1.41E-04
Coarse	60.0	#NAME?		2.47E-04

Seepage water temperature, T	66.0	°F	18.9	°C
Dynamic viscosity of water, μ	from White (2011) Table A.1		1.033E-03	N s/m^2
Intrinsic permeability, $\kappa_f = k_{h,f}\ (\mu/\gamma_w)$			#N/A	m^2

Note: Adjusted calculation rule for multi-layer aquifers can be applied to elongated aquifers (D/L < 0.3) with low permeability contrasts ($k_{h,c}/k_{h,f}$ < 10).

Thickness of piping layer, D_f	10.0	ft	3.0	m
Total aquifer thickness, $D = D_f + D_c$	70.0	ft	21.3	m
Seepage path length, L	200.0	ft	61.0	m
D/L (where $D = D_f$ for a single-layer aquifer)	0.35	Calculation rule is N/A.		
Permeability contrast, $k_{h,c}/k_{h,f}$			1.75	
Average horizontal permeability, $k_{avg} = [(k_{h,f})(D_f) + (k_{h,c})(D_c)] / D$			2.32E-04	m/sec
Average intrinsic permeability, $\kappa_{avg} = k_{h,avg}(\mu/\gamma_w)$			2.44E-11	m^2

Figure 44. Step 3 of Sellmeijer worksheet:
Probabilistic input for multi-layer aquifer without using @RISK.

For probabilistic analysis of single-layer aquifer using @RISK, input the minimum, most likely, and maximum values for the fine sand (piping) layer, and use an @RISK formula for a triangular distribution in the third column as a default. Alternatively, input a valid @RISK distribution in lieu of this default formula, and the user-specified input displays in the fourth column. The mean value used for subsequent calculations is the mean for the @RISK distribution entered in the third column. Figure 45 illustrates the probabilistic input for single-layer aquifer using @RISK.

Step 3: Characterize the piping layer

Does the piping sand have a higher permeability underlayer? No

Parameter	Units	Minimum	Most Likely	Maximum
Horizontal permeability of fine sand, $k_{h,f}$	cm/sec	3.53E-03	1.06E-02	2.82E-02
Horizontal permeability of coarse sand, $k_{h,c}$	cm/sec	1.41E-02	2.12E-02	3.88E-02

Sand Layer	Thickness, D (ft)	Mean k_h (cm/sec)	@RISK Formula for k_h (cm/sec)	Mean k_h (m/sec)
Fine	10.0	1.41E-02	=@RiskTriang(F57,G57,H57)	1.41E-04
Coarse	60.0	2.47E-02	=@RiskTriang(F58,G58,H58)	2.47E-04

Seepage water temperature, T	66.0	°F	18.9	°C
Dynamic viscosity of water, μ	from White (2011) Table A.1		1.033E-03	N s/m^2
Intrinsic permeability, $\kappa_f = k_{h,f}\ (\mu/\gamma_w)$			1.49E-11	m^2

Note: Adjusted calculation rule for multi-layer aquifers can be applied to elongated aquifers (D/L < 0.3) with low permeability contrasts ($k_{h,c}/k_{h,f}$ < 10).

Thickness of piping layer, D_f	10.0	ft	3.0	m
Total aquifer thickness, $D = D_f + D_c$	#N/A	ft	#N/A	m
Seepage path length, L	200.0	ft	61.0	m
D/L (where $D = D_f$ for a single-layer aquifer)	0.05			
Permeability contrast, $k_{h,c}/k_{h,f}$			#N/A	
Average horizontal permeability, $k_{avg} = [(k_{h,f})(D_f) + (k_{h,c})(D_c)] / D$			#N/A	m/sec
Average intrinsic permeability, $\kappa_{avg} = k_{h,avg}(\mu/\gamma_w)$			#N/A	m^2

Figure 45. Step 3 of Sellmeijer worksheet:
Probabilistic input for single-layer aquifer using @RISK.

For probabilistic analysis of multi-layer aquifer using @RISK, input the minimum, most likely, and maximum values for the fine sand (piping) layer and underlying coarse sand layer, and use an @RISK formula for a triangular distribution in the third column as a default. Alternatively, input a valid @RISK distribution in lieu of this default formula, and the user-specified input displays in the fourth column. The mean values used for subsequent calculations are the mean for the @RISK distribution entered in the third column. Figure 46 the probabilistic input for multi-layer aquifer using @RISK.

Step 3: Characterize the piping layer				
Does the piping sand have a higher permeability underlayer?			Yes	
Parameter	Units	Minimum	Most Likely	Maximum
Horizontal permeability of fine sand, $k_{h,f}$	cm/sec	3.53E-03	1.06E-02	2.82E-02
Horizontal permeability of coarse sand, $k_{h,c}$	cm/sec	1.41E-02	2.12E-02	3.88E-02

Sand Layer	Thickness, D (ft)	Mean k_h (cm/sec)	@RISK Formula for k_h (cm/sec)	Mean k_h (m/sec)
Fine	10.0	1.41E-02	=@RiskTriang(F57,G57,H57)	1.41E-04
Coarse	60.0	2.47E-02	=@RiskTriang(F58,G58,H58)	2.47E-04

Seepage water temperature, T	66.0	°F	18.9	°C
Dynamic viscosity of water, μ	from White (2011) Table A.1		1.033E-03	N s/m^2
Intrinsic permeability, $\kappa_f = k_{h,f}\,(\mu/\gamma_w)$			#N/A	m^2

Note: Adjusted calculation rule for multi-layer aquifers can be applied to elongated aquifers (D/L < 0.3) with low permeability contrasts ($k_{h,c}/k_{h,f}$ < 10).

Thickness of piping layer, D_f	10.0	ft	3.0	m
Total aquifer thickness, $D = D_f + D_c$	70.0	ft	21.3	m
Seepage path length, L	200.0	ft	61.0	m
D/L (where $D = D_f$ for a single-layer aquifer)	0.35	Calculation rule is N/A.		
Permeability contrast, $k_{h,c}/k_{h,f}$			1.75	
Average horizontal permeability, $k_{avg} = [(k_{h,f})(D_f) + (k_{h,c})(D_c)] / D$			2.32E-04	m/sec
Average intrinsic permeability, $\kappa_{avg} = k_{h,avg}(\mu/\gamma_w)$			2.44E-11	m^2

Figure 46. Step 3 of Sellmeijer worksheet: Probabilistic input for multi-layer aquifer using @RISK.

6.4. Field Critical Horizontal Gradient for BEP Progression

Step 4 calculates the field critical horizontal gradient for BEP progression as illustrated in Figure 47. Equation 19, for the critical horizontal gradient for BEP progression (i_{ch}), is composed of three terms: a resistance factor that accounts for the strength of the layer subject to BEP; a scale factor that relates pore size and seepage size; and a geometrical shape factor.

$$i_{ch} = (F_R)(F_S)(F_G) \tag{19}$$

where:

F_R = resistance factor (strength of the layer subject to BEP)
F_S = scale factor (relating pore size and seepage size)
F_G = geometrical shape factor

The resistance factor is a function of White's constant (η), bedding angle (θ), relative density (RD), roundness of the particles (KAS), and coefficient of uniformity (U) and is calculated using Equation 20.

$$F_R = \eta \left(\frac{\gamma_p'}{\gamma_w}\right) \tan\theta \left(\frac{RD}{72.5}\right)^{0.35} \left(\frac{U}{1.81}\right)^{0.13} \left(\frac{KAS}{49.2}\right)^{-0.02} \tag{20}$$

where:

KAS = roundness of the particles (percent)
RD = relative density (percent)
U = coefficient of uniformity
γ'_p = submerged unit weight of soil particles (N/m^3)
γ_w = unit weight of water (N/m^3)
θ = bedding angle (degrees)
η = White's constant

Since it is easier to estimate the specific gravity of soil particles than the submerged unit weight of soil particles, the substitution in Equation 21 is made to the equation for F_R.

$$\frac{\gamma_p'}{\gamma_w} = G_s - 1 \tag{21}$$

The bedding angle and White's constant have been set to 37 degrees and 0.25, respectively, due to model calibration and are not changed in Dutch practice. Since the values were fixed in the multi-variate regression, any influence they had is accounted for in related variables. It is, therefore, not appropriate to vary these parameters.

Van Beek et al. (2010) indicate that KAS and U appear less important than other sand characteristics, and the results of multivariate regression analysis show that they have a weak influence on critical gradient. Therefore, U and KAS terms in equation for F_R are sometimes ignored in practice. The toolbox provides an option to neglect the expressions containing these terms from the equation for F_R as illustrated in Figure 48.

The scale factor is a function of the particle size for which 70 percent is finer (by weight) (d_{70}), the horizontal seepage path length (L), and the intrinsic permeability of the piping layer (κ) and is calculated using Equation 22.

$$F_S = \left(\frac{d_{70}}{\kappa L}\right)^{1/3} \left(\frac{0.000208}{d_{70}}\right)^{0.6} \tag{22}$$

where:

d_{70} = particle size (m) for which 70 percent is finer (by weight)
κ = intrinsic permeability (m^2) of the piping layer

The intrinsic permeability of the piping layer in Equation 20 is either κ_f for a single-layer aquifer or $k_{h,avg}$ for a multi-layer aquifer.

The value of the horizontal permeability used to calculate the intrinsic permeability depends on the selection in step 3. For single-layer aquifers, it is the value for the fine sand (piping) layer. For multi-layer aquifers, it is the layer-weighted average.

The geometrical factor is a function of the ratio of the depth of the piping layer to the horizontal seepage path length (D/L) and is calculated using Equation 23.

$$F_G = 0.91\left(\frac{D}{L}\right)^{\frac{0.28}{\left(\frac{D}{L}\right)^{2.8}-1}+0.04} \tag{23}$$

where:

D = thickness of the piping layer (m) where $D = D_f$ for a single-layer aquifer
L = seepage path length (m) through the piping layer (measured horizontally)

Step 4: Estimate the field horizontal critical gradient for backward erosion piping progression

Resistance Factor (F_R)

Ignore U and KAS since they do not contribute significantly? No

$F_R = \eta\ (\gamma'_p/\gamma_w)\ \tan(\vartheta)\ (RD/RD_m)^{0.35}\ (U/U_m)^{0.13}\ (KAS/KAS_m)^{-0.02}$ 0.257

$F_R = \eta\ (\gamma'_p/\gamma_w)\ \tan(\vartheta)\ (RD/RD_m)^{0.35}$

Note: $\gamma'_p/\gamma_w = G_s - 1$

Scale Factor (F_S)

$F_S = [d_{70}/(\kappa L)^{1/3}]\ (d_{70m}/d_{70})^{0.6}$ 0.378

Geometrical Shape Factor (F_G)

$F_G = 0.91(D/L)$ ^ $\{(0.28 / [(D/L)^{2.8} - 1]) + 0.04\}$ 1.868

Note: The geometrical shape factor in the piping rule is valid only for a sand layer of constant thickness.

Figure 47. Step 4 of Sellmeijer worksheet: Critical gradient terms with U and KAS.

Step 4: Estimate the field horizontal critical gradient for backward erosion piping progression

Resistance Factor (F_R)

Ignore U and KAS since they do not contribute significantly?	Yes
$F_R = \eta\ (\gamma'_p/\gamma_w)\ \tan(\vartheta)\ (RD/RD_m)^{0.35}\ (U/U_m)^{0.13}\ (KAS/KAS_m)^{-0.02}$	0.241
$F_R = \eta\ (\gamma'_p/\gamma_w)\ \tan(\vartheta)\ (RD/RD_m)^{0.35}$	
Note: $\gamma'_p/\gamma_w = G_s - 1$	

Scale Factor (F_S)

$F_S = [d_{70}/(\kappa L)^{1/3}]\ (d_{70m}/d_{70})^{0.6}$	0.378

Geometrical Shape Factor (F_G)

$F_G = 0.91(D/L)$ ^ $\{(0.28 / [(D/L)^{2.8} - 1]) + 0.04\}$	1.868
Note: The geometrical shape factor in the piping rule is valid only for a sand layer of constant thickness.	

Figure 48. Step 4 of Sellmeijer worksheet: Critical gradient terms without U and KAS.

Sellmeijer's model applies to 2D seepage with plane or ditch exits parallel to the embankment centerline for fine to medium sands within the limits of the model test parameters. Van Beek et al. (2015) found that 3D configurations with flow toward a single point (such as a hole in a confining layer) resulted in significantly smaller critical gradients than predicted by the adjusted calculation model. In both the small and medium-scale experiments, the model overestimated the critical gradient by a factor of approximately two. If the field horizontal critical gradient is further adjusted for 3D flow, input a user-specified GRF. Cells that do not apply have a gray background. The two scenarios are illustrated in Figure 49 and Figure 50.

Critical Horizontal Gradient for Progression of the Pipe

Note: The calculation rule applies to 2D seepage with plane or ditch exits parallel to the embankment centerline for fine to medium sands within the limits of the test parameters shown above.

$i_{ch} = (F_R)(F_S)(F_G)$ for 2D exit (ditch or outflow area)	0.170

Note: 3D configurations with flow towards a single point (e.g., hole in a confining layer) resulted in significantly smaller critical gradients than predicted by the Sellmeijer model. The model overestimated the critical gradient by a factor of approximately 2.

3D flow towards single point exit instead of 2D exit (ditch or outflow area)?	Yes
Gradient reduction factor to reduce 2D prediction for 3D exit (hole), GRF	2.0
$i_{ch} = (F_R)(F_S)(F_G)$ / GRF for 3D exit (hole)	0.085

Figure 49. Step 4 of Sellmeijer worksheet: Field critical gradient with gradient reduction factor.

Critical Hydraulic Gradient for Progression of the Pipe	
Note: The calculation rule applies to 2D seepage with plane or ditch exits parallel to the embankment centerline for fine to medium sands within the limits of the test parameters shown above.	
i_{ch} = (F_R)(F_S)(F_G) for 2D exit (ditch or outflow area)	0.170
Note: 3D configurations with flow towards a single point (e.g., hole in a confining layer) resulted in significantly smaller critical gradients than predicted by the Sellmeijer model. The model overestimated the critical gradient by a factor of approximately 2.	
3D flow towards single point exit instead of 2D exit (ditch or outflow area)?	No
Gradient reduction factor to reduce 2D prediction for 3D exit (hole), GRF	2.0
i_{ch} = (F_R)(F_S)(F_G) / GRF for 3D exit (hole)	#N/A

Figure 50. Step 4 of Sellmeijer worksheet: Field critical gradient without gradient reduction factor.

6.5. Likelihood of Backward Erosion Piping Progression

Step 5 calculates the average hydraulic gradient in the foundation by dividing the net hydraulic head by the seepage path length using Equation 24.

$$i_{avf} = \frac{\Delta H}{L} = \frac{HW - TW}{L} \tag{24}$$

where:

ΔH = net hydraulic head (feet)
HW = headwater level (feet)
TW = tailwater level (feet)
L = seepage path length (feet)

The FS against BEP progression is calculated using Equation 25.

$$FS = \frac{i_{ch}}{i_{avf}} \tag{25}$$

where:

i_{ch} = critical horizontal gradient for BEP progression
i_{avf} = average horizontal gradient in the foundation at the pipe head

For deterministic analysis, the FS is calculated using the most likely values of the random variables and is summarized in a table. Cells that do not apply have a gray background. For probabilistic analysis, the FS is calculated as described for the deterministic analysis but for the mean values of the random variables, and multiple iterations are performed by sampling the distributions in step 2. The probability of BEP progression is equal to the percentage of iterations that resulted in a FS less than 1 [(P(FS < 1)]. For probabilistic analysis performed without using @RISK, 1,000 iterations are used. For probabilistic analysis using @RISK, the number of iterations is user-specified, and "@RISK" displays in parentheses after the number of iterations for this scenario. If cycling through iterations using @RISK, the displayed results are no longer mean values; they are the selected iteration's values. For deterministic and probabilistic analyses, cells with FS less than 1 have an orange background. Figure 51 illustrates the deterministic tabular output, and Figure 52 illustrates the probabilistic tabular output without using @RISK.

Step 5: Estimate the likelihood of backward erosion piping progression

Average hydraulic gradient in foundation at pipe head, $i_{avf} = \Delta H/L$

Factor of safety against backward erosion piping progression, $FS = i_{ch}/i_{avf}$

Probability of a factor of safety against backward erosion piping progression less than 1, P(FS < 1)

Iterations: #N/A

HW (ft)	195.5	201.6	213.5	218.9	223.0	234.0	239.0
TW (ft)	184.0	184.0	184.0	184.0	184.0	184.0	184.0
i_{avf}	0.058	0.088	0.148	0.175	0.195	0.250	0.275
FS	1.31	0.86	0.51	0.43	0.39	0.30	0.27
P(FS < 1)	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A

Note: Probabilities obtained from this method should not be used directly in risk analyses. Rather, the values should be used to help develop a list of more and less likely factors.

Figure 51. Step 5 of Sellmeijer worksheet: Deterministic tabular output.

Step 5: Estimate the likelihood of backward erosion piping progression

Average hydraulic gradient in foundation at pipe head, $i_{avf} = \Delta H/L$

Factor of safety against backward erosion piping progression, $FS = i_{ch}/i_{avf}$

Probability of a factor of safety against backward erosion piping progression less than 1, P(FS < 1)

Iterations: 1000

HW (ft)	195.5	201.6	213.5	218.9	223.0	234.0	239.0
TW (ft)	184.0	184.0	184.0	184.0	184.0	184.0	184.0
i_{avf}	0.058	0.088	0.148	0.175	0.195	0.250	0.275
FS	1.48	0.97	0.58	0.49	0.44	0.34	0.31
P(FS < 1)	9.30E-02	5.99E-01	9.95E-01	1.00E+00	1.00E+00	1.00E+00	1.00E+00

Note: Probabilities obtained from this method should not be used directly in risk analyses. Rather, the values should be used to help develop a list of more and less likely factors.

Figure 52. Step 5 of Sellmeijer worksheet: Probabilistic tabular output without using @RISK.

6.6. Summary Plots

Step 6 generates the summary plots. The first plot is the mean FS against BEP progression (red solid line) and average horizontal gradient at the pipe head (green solid line) as functions of headwater level. If cycling through iterations using @RISK, the displayed results are no longer mean values; they are the selected iteration. FS against BEP progression is plotted on the primary axis, and average horizontal gradient at the pipe head is plotted on the secondary axis. Horizontal reference lines display for the mean field critical gradient for BEP progression (green dashed line) and FS of 1 (red shaded line). Figure 53 illustrates the deterministic graphical output.

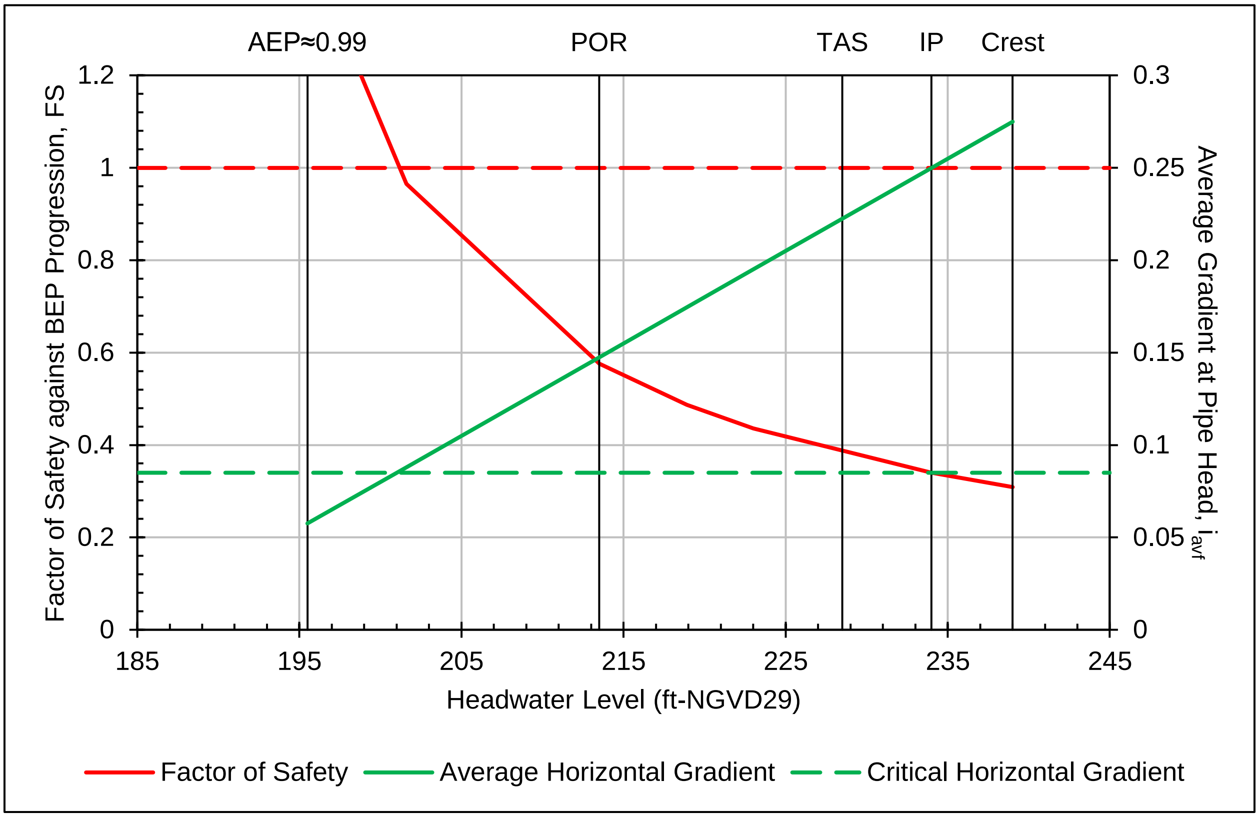


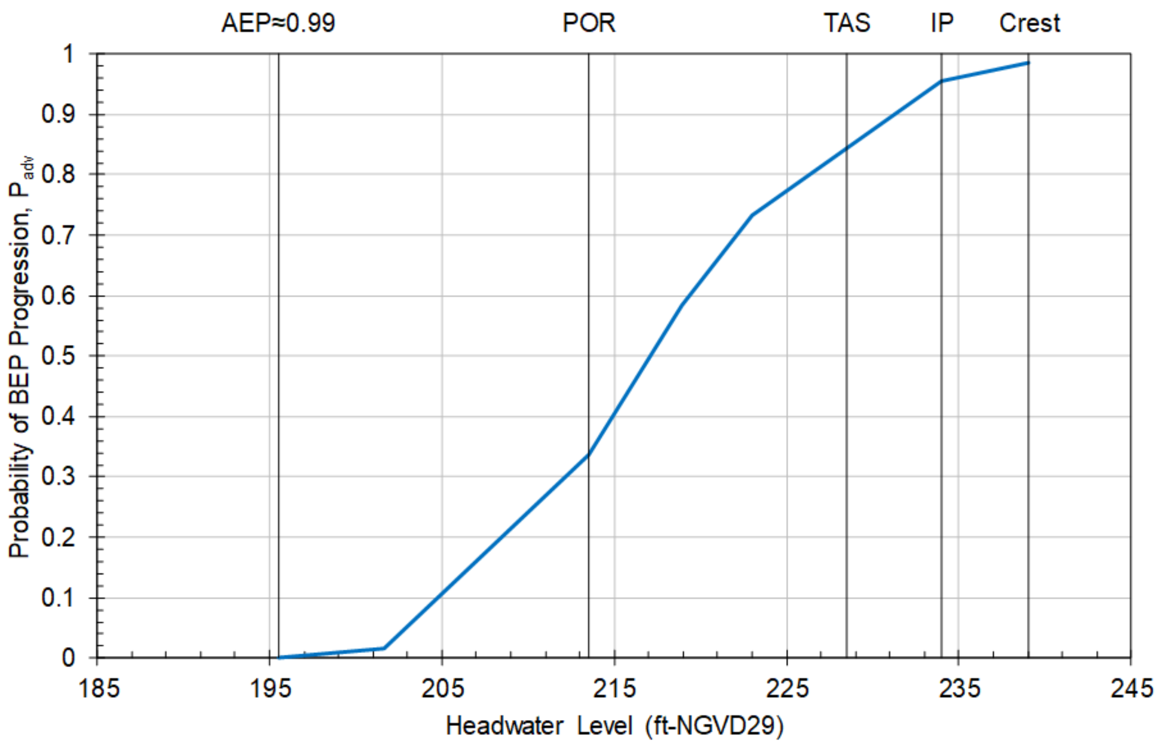
Figure 53. Step 6 of Sellmeijer worksheet: Deterministic graphical output.

Figure 54 illustrates the plot options for this chart. The maximum value for the primary y-axis (FS against BEP progression), maximum value for the secondary y-axis (average horizontal gradient at the pipe head), and minimum and maximum values for the x-axis (headwater level) are user-specified. Users can input up to five vertical reference elevations, and user-specified labels display at the top of the chart

Worksheet	Sellmeijer						
y-axis bounds (primary)							
minimum	0				Value Primary Min: 0		
maximum	1.2	◄ Enter maximum FS.			Value Primary Max: 1.2		
y-axis bounds (secondary)							
minimum	0				Value Secondary Min: 0		
maximum	0.30	◄ Enter maximum average gradient.			Value Secondary Max: 0.3		
x-axis bounds							
minimum	185.0	◄ Enter minimum headwater level.			Category Primary Min: 185		
maximum	245.0	◄ Enter maximum headwater level.			Category Primary Max: 245		
Enter up to 5 vertical reference lines for headwater levels of interest.							
AEP≈0.99	POR	TAS	IP	Crest	◄ Enter headwater description.		
195.5	213.5	228.5	234.0	239.0	◄ Enter headwater level.		

Figure 54. Step 6 of Sellmeijer worksheet: Plot options for deterministic graphical output.

For probabilistic analysis, the mean probability of BEP progression is plotted as a function headwater level. If cycling through iterations using @RISK, this plot has a gray background because the probability of initiation cannot be calculated from a single iteration. Similarly, this plot has a gray background for deterministic analysis. Figure 55 illustrates the graphical output for probabilistic analysis.



Note: Probabilities obtained from this method should not be used directly in risk analyses. Rather, the values should be used to help develop a list of more and less likely factors.

Figure 55. Step 6 of Sellmeijer worksheet: Probabilistic graphical output.

Figure 56 illustrates the plot options for this chart. The vertical reference elevations and minimum and maximum values for the x-axis (headwater level) are the same as the input for the previous chart. Only the maximum value for the y-axis (probability of BEP progression) is user-specified.

Worksheet	Sellmeijer		
y-axis bounds			
minimum	0		Value Primary Min: 0
maximum	1.0	◄ Enter maximum probability.	Value Primary Max: 1
x-axis bounds			
minimum	185.0	◄ Enter minimum headwater level.	Category Primary Min: 185
maximum	245.0	◄ Enter maximum headwater level.	Category Primary Max: 245

Figure 56. Step 6 of Sellmeijer worksheet: Plot options for probabilistic graphical output.

7. Creep Ratios

Creep ratio methods such as Bligh (1910) and Lane (1935) are empirical methods still used by some practitioners to assess the likelihood of BEP progression (hydraulic condition) based on observations of seepage performance for a range of soil types. Both empirical methods involve estimating the line of creep (or seepage path length) beneath concrete structures (such as weirs), including seepage barrier walls. For embankment dams and levees, the seepage path length includes the impervious embankment, as well as any upstream/floodside and downstream/landside impervious blankets or berms, seepage barrier walls, impervious foundation trenches, etc. as illustrated in Figure 57.

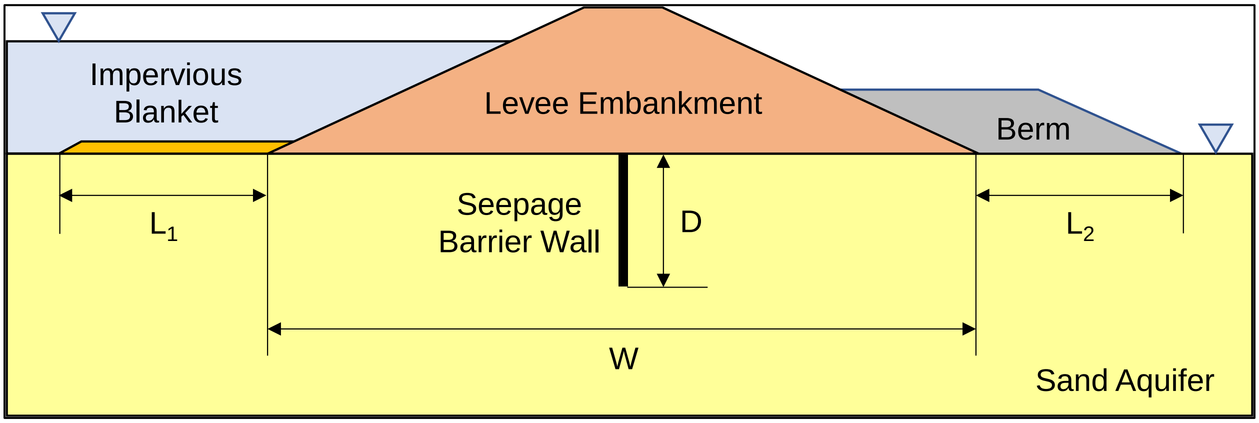


Figure 57. Levee geometry for line-of-creep methods.

To assess the likelihood of BEP progression (hydraulic condition), the creep ratio for the headwater level under consideration is compared to the minimum (or safe) creep ratio for the piping material. The creep ratio is calculated as the line of creep (seepage path length) divided by the net hydraulic head. For Lane's method, horizontal seepage path lengths are weighted three times less than the vertical seepage path lengths to account for the greater resistance to pipe development when a vertical structure (such as a seepage barrier wall) is present in the path of the pipe. Hence, it is often referred to as a weighted creep method. BEP progression is expected if the creep ratio is less than the minimum creep ratio.

Evaluating creep ratio is informative where the levee consists of fine-grained material and no landside blanket exists. The method is not appropriate for a compromised confining layer overlying a confined aquifer. Duncan et al. (2011) state that while informative, the creep ratio is considered a "quick-and-dirty check." The state of practice is to use rational methods based on blanket theory, flow nets, or finite-element method analysis, and the greatest remaining value of creep ratios lies in indicating the relative erosion potential of various soil types.

7.1. Line of Creep

Step 1 calculates the line of creep (seepage path length) as illustrated in Figure 58. The input includes the horizontal impervious dimensions as well as the depth of any vertical structures (such as a seepage barrier walls). The depth of vertical structures is doubled in the line-of-creep calculation because seepage must pass down, around, and up the embedded structure. If a blanket, berm, or vertical structure does not exist, input zero for those values.

The line of creep (L) for Bligh's method is calculated using Equation 26. The weighted line of creep for Lane's method (L_W) is calculated using Equation 27.

$$L = L_1 \ + W \ + \ L_2 \ + \ 2D \tag{26}$$

$$L_W = (L_1 \ + \ W \ + \ L_2)/3 \ + \ 2D \tag{27}$$

where:

D = depth (feet) of embedded vertical structure
L_1 = length (feet) of upstream/floodside impervious blanket or berm
L_2 = length (feet) of downstream/landside impervious blanket or berm
W = width (feet) of base of concrete weir or impervious embankment

Step 1: Estimate the line of creep		
Length of upstream/floodside blanket or berm, L_1	1,000	ft
Width of base of impervious embankment, W	500	ft
Length of downstream/floodside blanket or berm, L_2	0	ft
Depth of vertical structure (e.g., cutoff or weir), D	15	ft
Bligh's line of creep, $L = \Sigma H_i + \Sigma V_i = L_1 + W + L_2 + 2D$	1,530	ft
Lane's weighted line of creep, $L_w = \Sigma H_i / 3 + \Sigma V_i = (L_1 + W + L_2) / 3 + 2D$	530	ft

where
H_i = length of the i-th horizontal distance measured along the base of the structure
V_i = length of the i-th vertical distance measured along the base of the structure

Figure 58. Step 1 of Creep Ratios worksheet: Line-of-creep input.

7.2. Minimum Creep Ratio

Step 2 characterizes the foundation to obtain the minimum (or safe) creep ratio for both empirical methods from Lane (1935) based on soil type as illustrated in Figure 59. Use the drop-down list to select the material that best represents the material along the piping path.

Step 2: Estimate the minimum creep ratio		
Material	Minimum Creep Ratio	
	Bligh (1910)	Lane (1935)
Very fine sand or silt	18	8.5
Fine sand	15	7.0
Medium sand	#N/A	6.0
Coarse sand	12	5.0
Fine gravel	#N/A	4.0
Medium gravel	#N/A	3.5
Gravel and sand	9	#N/A
Coarse gravel, including cobbles	#N/A	3.0

Material	Fine sand

Creep Ratio Method	Bligh (1910)	Lane (1935)
Minimum (or safe) creep ratio, C or C_w	15	7.0
Critical gradient for progression of the pipe, i_{ch} = 1/C or i_{ch} = 1/C_w	0.067	0.143

Figure 59. Step 2 of Creep Ratios worksheet: Material input.

When seepage exits horizontally, piping can progress with an average horizontal gradient in the foundation as low as 0.05. This critical horizontal gradient is approximately the same as the inverse of the Bligh's creep ratio for fine sand. The field horizontal critical gradient for BEP progression (i_{ch}) is the inverse of the minimum (or safe) creep ratio if there are no vertical structures (D = 0). The field horizontal critical gradient is calculated using Equations 28 and 29 for Bligh's method and Lane's method, respectively. For D greater than zero, "N/A" displays for i_{ch}.

$$i_{ch} = \frac{1}{C_{min}} \tag{28}$$

$$i_{ch} = \frac{1}{C_{W,min}} \tag{29}$$

where:

C_{min} = minimum (or safe) creep ratio for Bligh's method
$C_{W,min}$ = minimum (or safe) weighted creep ratio for Lane's method

7.3. Creep Ratio

Step 3 calculates the creep ratio by dividing the line of creep from step 1 by the net hydraulic head (ΔH) for the headwater level under consideration. The creep ratio for Bligh's method (C) is calculated using Equation 29. The weighted creep ratio for Lane's method (C_W) is calculated using Equation 30.

$$C = \frac{L}{\Delta H} = \frac{L}{HW-TW} \tag{29}$$

$$C_W = \frac{L_W}{\Delta H} = \frac{L_W}{HW-TW} \tag{30}$$

where:

ΔH = net hydraulic head (feet)
HW = headwater level (feet)
TW = tailwater level (feet)
L = line of creep for Bligh's method (feet)
L_W = line of creep for Lane's method (feet)

The calculated creep ratio is compared to the minimum creep ratio from step 2, and cells where the calculated creep ratio exceeds the minimum value have an orange background. Figure 60 illustrates the tabular summary.

Step 3: Estimate the creep ratio

Net hydraulic head, ΔH = HW − TW

Bligh's creep ratio, C = L / H

HW (ft)	195.5	201.6	213.5	218.9	223.0	234.0	239.0
TW (ft)	184.0	184.0	184.0	184.0	184.0	184.0	184.0
ΔH (ft)	11.5	17.6	29.5	34.9	39.0	50.0	55.0
C	133.0	86.9	51.9	43.8	39.2	30.6	27.8

Lane's weighted creep ratio, $C_w = L_w / H$

HW (ft)	195.5	201.6	213.5	218.9	223.0	234.0	239.0
TW (ft)	184.0	184.0	184.0	184.0	184.0	184.0	184.0
ΔH (ft)	11.5	17.6	29.5	34.9	39.0	50.0	55.0
C_w	46.1	30.1	18.0	15.2	13.6	10.6	9.6

Figure 60. Step 3 of Creep Ratios worksheet: Deterministic tabular output.

The calculated creep ratio is plotted as a function of headwater level for both methods. Horizontal reference lines display for the minimum (or safe) creep ratios for each method. Figure 61 illustrates the graphical output. The legend can be moved within the plot area as necessary for clarity.

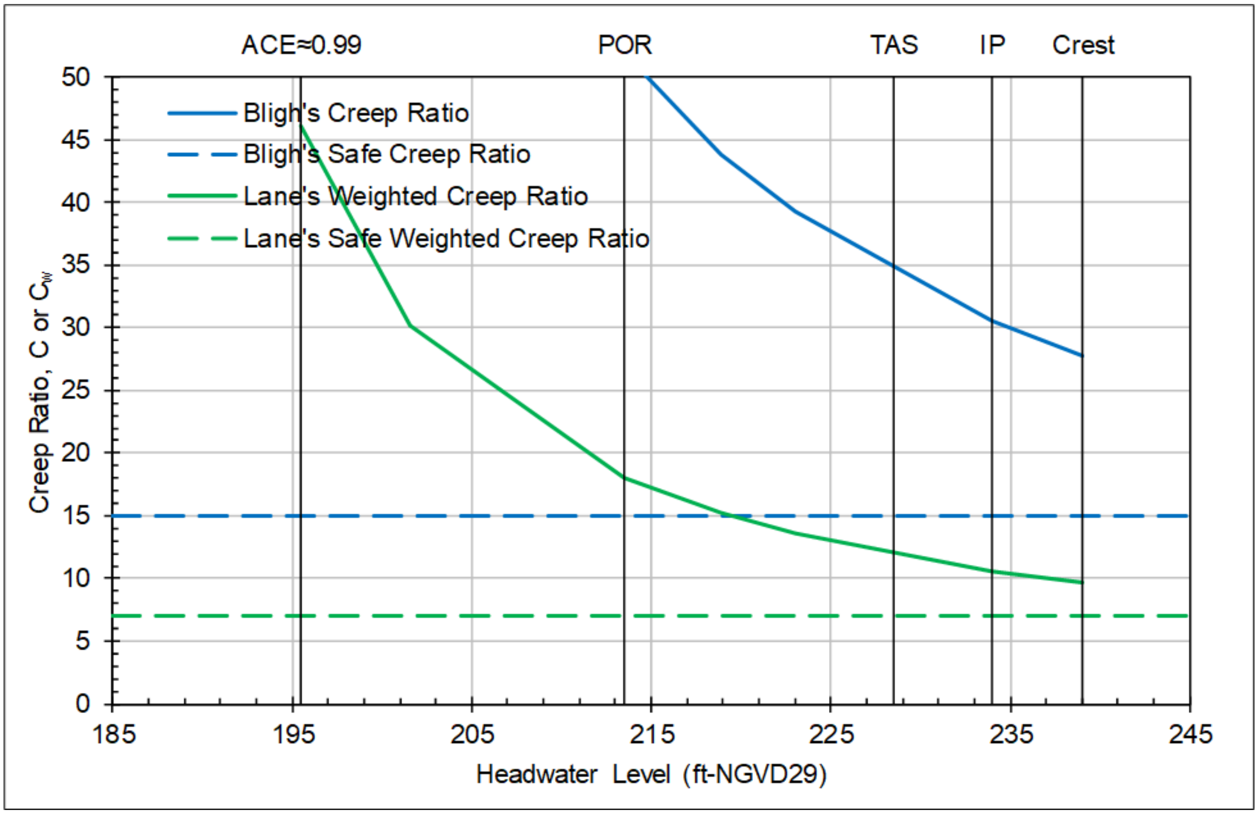


Figure 61. Step 3 of Creep Ratios worksheet: Deterministic graphical output.

Figure 62 illustrates the plot options for this chart. The maximum value for the y-axis (creep ratio) and minimum and maximum values for the x-axis (headwater level) are user-specified. Users can input up to five vertical reference elevations, and user-specified labels displayed at the top of the chart.

◄ Adjust legend position to minimize overlapping of data series if necessary.

Worksheet	Creep Ratios		
y-axis bounds			
minimum	0		Value Primary Min: 0
maximum	50	◄ Enter maximum creep ratio.	Value Primary Max: 50
x-axis bounds			
minimum	185.0	◄ Enter minimum headwater level.	Category Primary Min: 185
maximum	245.0	◄ Enter maximum headwater level.	Category Primary Max: 245

Enter up to 5 vertical reference lines for headwater levels of interest.

ACE≈0.99	POR	TAS	IP	Crest	◄ Enter headwater description.
195.5	213.5	228.5	234.0	239.0	◄ Enter headwater level.

◄ Select the chart with a left-click of mouse in plot area.
Select the Filters icon to toggle which series (critical gradient) to plot; then hit Apply.

Figure 62. Step 3 of Creep Ratios worksheet: Plot options for deterministic graphical output.

By selecting the chart and then selecting the filter icon to display the filter pane, the data series for display can be selected as illustrated in Figure 63. For example, if only the results from the Bligh's method are judged applicable, the data series for "Lane's Weighted Creep Ratio" and "Lane's Safe Weighted Creep Ratio" can be deselected so that they do not plot or display in the legend.

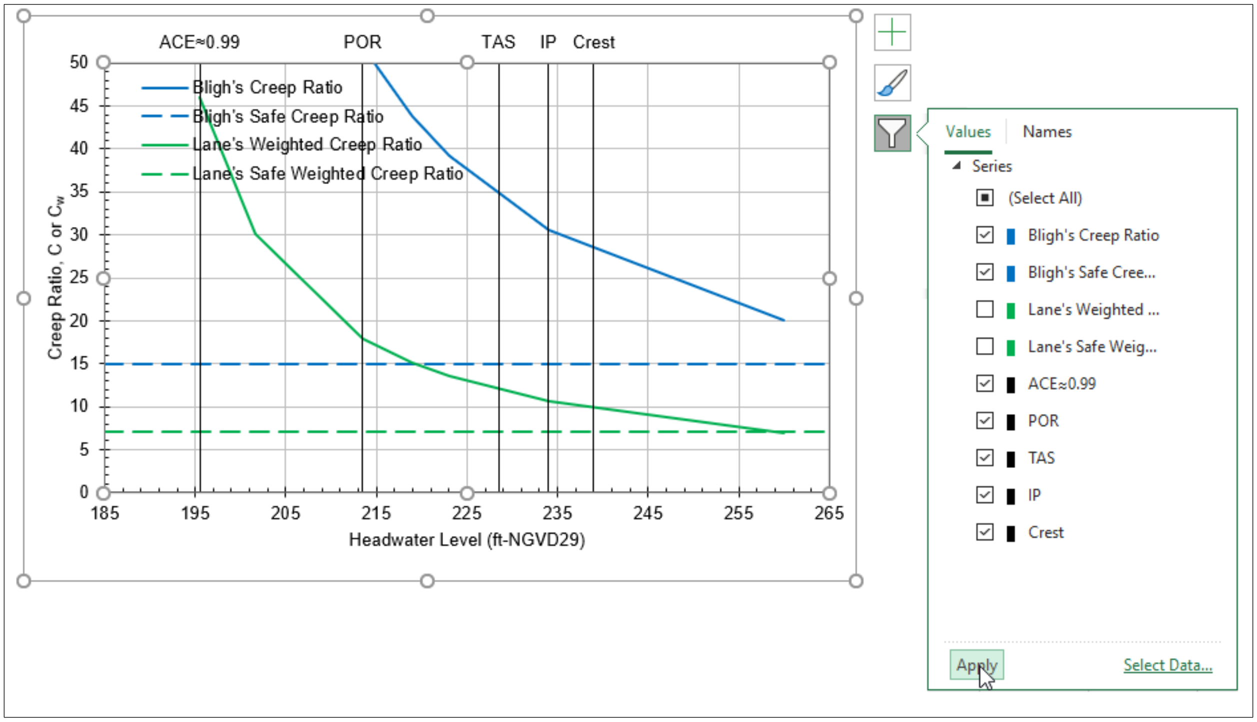


Figure 63. Step 3 of Creep Ratios worksheet: Filtering data.

8. Summary

This worksheet provides summary plots of the various methods used to assess the likelihood of BEP progression. The first plot is the average hydraulic gradient at the pipe head (black solid line) as a function of headwater level. If cycling through iterations using @RISK, the displayed results are no longer mean values; they are the selected iteration. Horizontal reference lines display for the mean field critical gradient for the adjusted Schmertmann method (blue dashed line), mean field horizontal critical gradient for the adjusted calculation rule of Sellmeijer et al. (green dashed line), inverse of Bligh's minimum creep ratio (red dashed line), and inverse of Lane's minimum weighted creep ratio (orange dashed line). If cycling through iterations using @RISK, the displayed results are no longer mean values; they are the selected iteration. Figure 64 illustrates the deterministic graphical output.

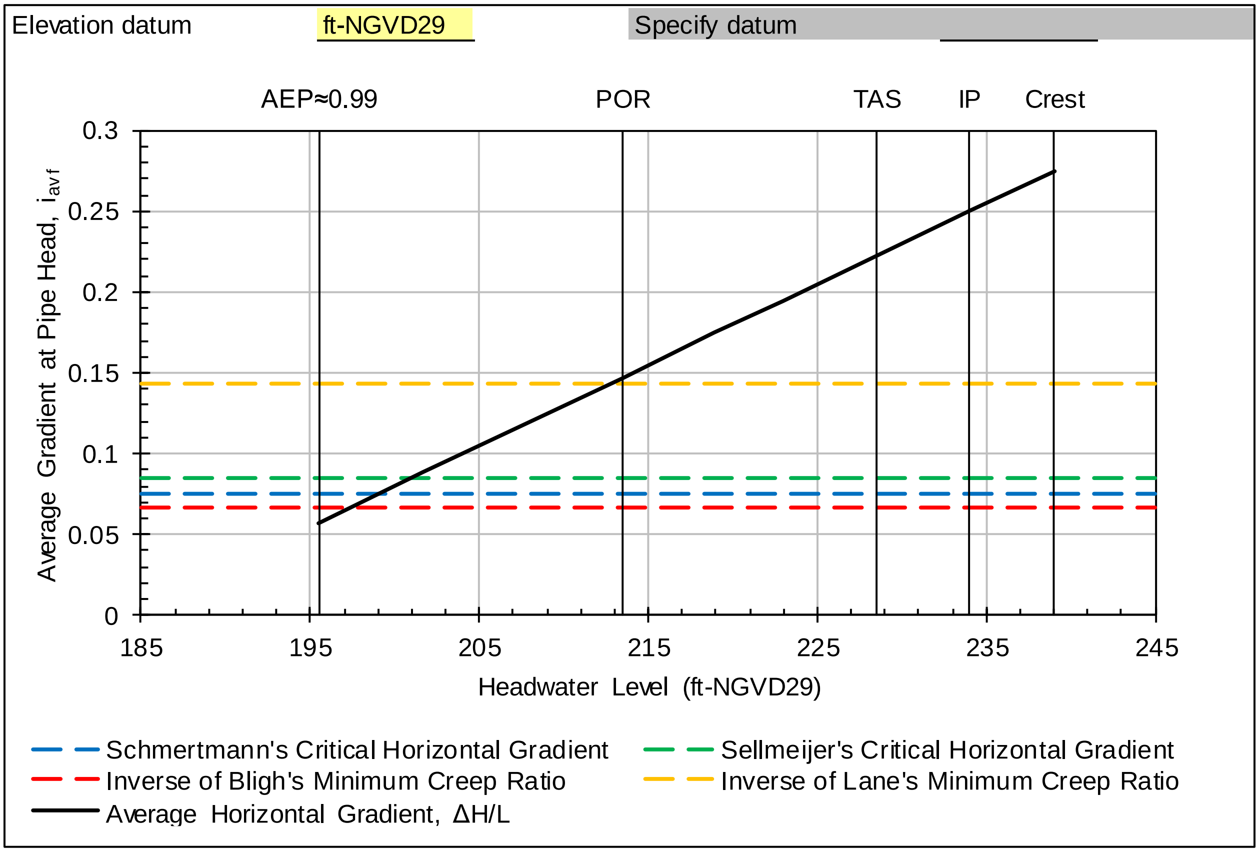


Figure 64. Summary worksheet: Deterministic graphical output.

Figure 65 illustrates the plot options for this chart. The maximum value for the y-axis (average horizontal gradient at the pipe head) and minimum and maximum values for the x-axis (headwater level) are user-specified. Users can input up to five vertical reference elevations, and user-specified labels display at the top of the chart.

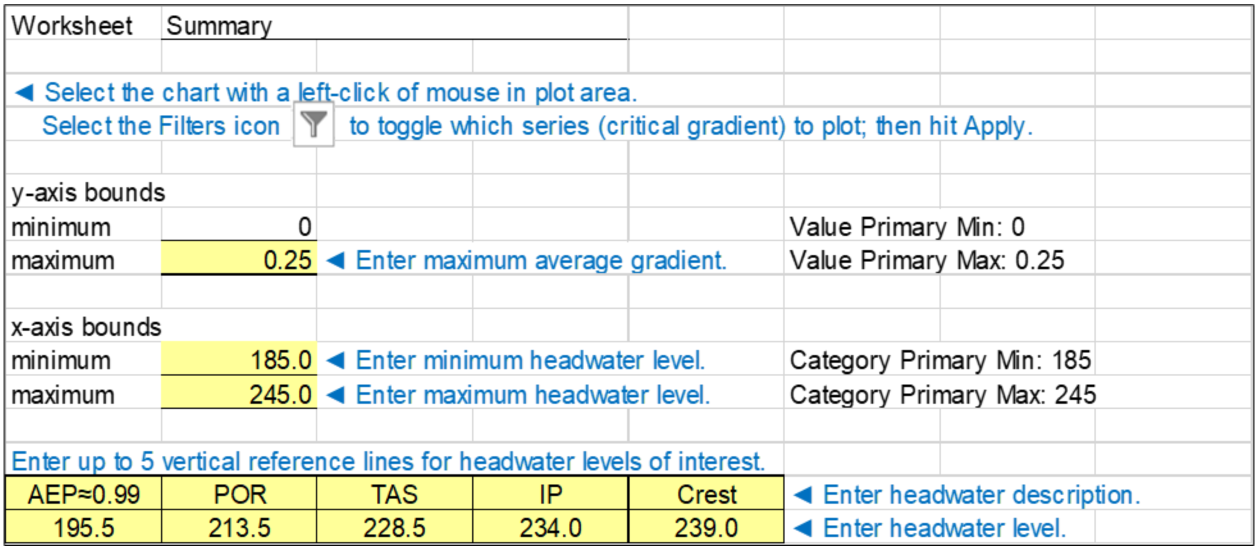


Figure 65. Summary worksheet: Plot options for deterministic graphical output.

Select the chart and filter icon to display the filter pane to select or deselect data series, as shown in Figure 66. For example, if only the results from the adjusted Schmertmann method and inverse of Bligh's creep ratio are judged applicable, deselecting the other data series removes them from the plot and the legend.

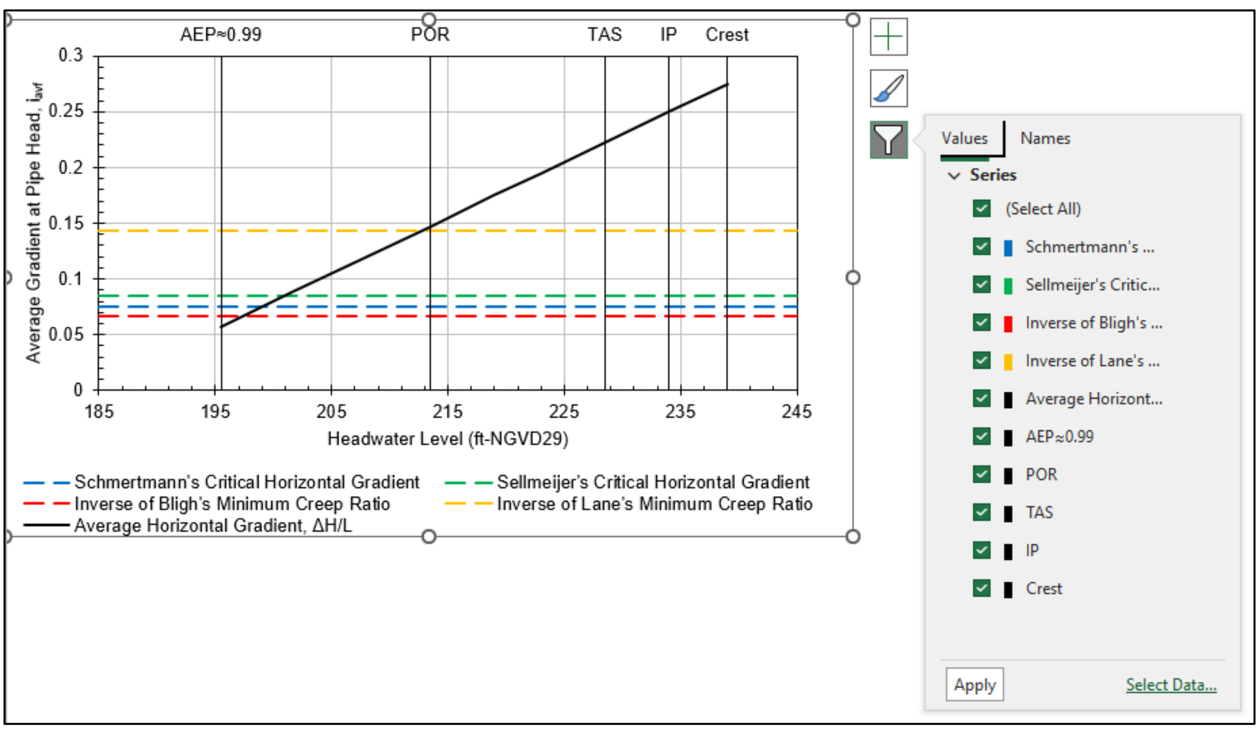


Figure 66. Summary worksheet: Example of filtering data.

The second plot of the worksheet is the probability of BEP progression for the adjusted Schmertmann method (blue solid line) and adjusted calculation rule of Sellmeijer et al. (green solid line) as functions of headwater level. Because the adjusted Schmertmann method uses the probabilistic chart of Robbins and Sharp (2016), expanded by Robbins and O'Leary (2020), a probability of BEP progression displays for both deterministic and probabilistic analysis. However, the adjusted calculation rule of Sellmeijer et al. displays for probabilistic analysis only. Figure 67 illustrates the probabilistic graphical output.

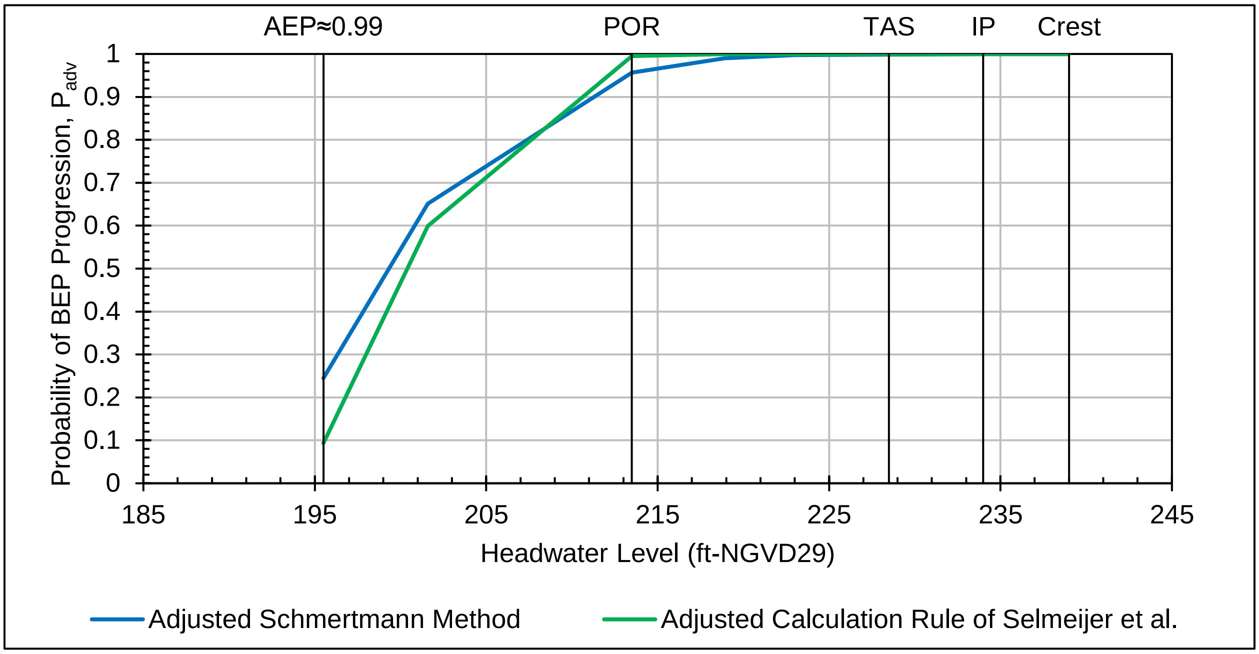


Figure 67. Summary worksheet: Probabilistic graphical output.

Figure 68 illustrates the plot options for this chart. The vertical reference elevations and minimum and maximum values for the x-axis (headwater level) are the same as the input for Figure 65. Only the maximum value for the y-axis (probability of BEP progression) is user-specified.

Worksheet	Summary		
◄ Select the chart with a left-click of mouse in plot area.			
Select the Filters icon		to toggle which series (critical gradient) to plot; then hit Apply.	
y-axis bounds			
minimum	0		Value Primary Min: 0
maximum	1.00	◄ Enter maximum probability.	Value Primary Max: 1
x-axis bounds			
minimum	185.0	◄ Enter minimum headwater level.	Category Primary Min: 185
maximum	245.0	◄ Enter maximum headwater level.	Category Primary Max: 245

Figure 68. Summary worksheet: Plot options for probabilistic graphical output.

9. References

Bligh, W.G. (1910). Dams, barrages and weirs on porous foundations. *Engineering News Record, 64*(20), 708–710.

Duncan, J.M., O'Neil, B., and Brandon, T. (2011). *Evaluation of potential for erosion in levees and levee foundations* (CPGR 64). Virginia Polytechnic Institute and State University, Center for Geotechnical Practice and Research.

Fell, R., Foster, M.A., Cyganiewicz, J., Sills, G.L., Vroman, N.D., and Davidson, R.R. (2008). *Risk analysis for dam safety: A unified method for estimating probabilities of failure of embankment dams by internal erosion and piping*. URS Australia. http://vm.civeng.unsw.edu.au/uniciv/R-446.pdf.

Lane, E.W. (1935). Security from underseepage. *Transactions of the American Society of Civil Engineers*, Volume 100, 1235–1351. https://doi.org/10.1061/TACEAT.0004655.

Robbins, B.A. and O'Leary, T.M. (2020). Charts for estimating probabilities of backward erosion piping progression in deep, pervious foundations. *Proceedings, Dam Safety 2020*. American Association of State Dam Safety Officials. 1–8. https://damsafety.org/reference/charts-estimating-probabilities-backward-erosion-piping-progression-deep-pervious.

Robbins, B.A. and Sharp, M.K. (2016). Incorporating uncertainty into backward erosion piping risk assessments. *Proceedings, 3rd European Conference on Flood Risk Management*. FLOODrisk 2016. Article 03007, 1–6. https://doi.org/10.1051/e3sconf/20160703007.

Schmertmann, J.H. (2000). The no-filter factor of safety against piping through sands. *Judgment and Innovation: The Heritage and Future of the Geotechnical Engineering Profession* (Geotechnical Special Publication No. 111). American Society of Civil Engineers. 65–132. https://doi.org/10.1061/9780784405376.006.

Sellmeijer, J.B., López de la Cruz, H., van Beek, V.M., and Knoeff, J.G. (2011). Fine-tuning of the backward erosion piping model through small-scale, medium-scale, and IJkdijk experiments. *European Journal of Environmental and Civil Engineering*, *15*(8), 1139–1154. https://doi.org/10.1080/19648189.2011.9714845.

van Beek, V.M., Knoeff, J.G., de Bruijn, H.T.J., and Sellmeijer, J.B. (2009). Influence of sand characteristics on the piping process – Small-scale experiments. *International Workshop on Internal Erosion in Dams and Foundations*. St. Petersburg, Russia.

van Beek, V.M., Knoeff, J.G., Rietdijk, J., Sellmeijer, J.B., and Lopez De La Cruz, H. (2010). Influence of sand and scale on the piping process – Experiments and multivariate analysis. In. Springman, S., Laue, J., and Seward, L. (Eds.), *Physical Modeling in Modelling in Geotechnics (Proceedings, of the 7th International Conference on Physical in Modelling in Geotechnics, Zurich)*. (pp. 1221–1226). CRC Press. https://www.taylorfrancis.com/chapters/edit/10.1201/b10554-206.

van Beek, V.M., Van Essen, H.M., Vandenboer, K., and Bezuijen, A. (2015). Developments in modelling of backward erosion piping. *Géotechnique*, *65*(9), 740–754. https://doi.org/10.1680/geot.14.P.119.

van Beek, V.M., Yao, Q., Van, M., and Barends, F. (2012). Validation of Sellmeijer's model for backward erosion piping under dikes on multiple sand layers. *Proceedings, 6th International Conference on Scour and Erosion*. Paris. 543–550.

Appendix A. Acronym List

2D	Two-Dimensional
3D	Three-Dimensional
BEP	Backward Erosion Piping
CPD	Computer Program Document
FS	Factor of Safety
GRF	Gradient Reduction Factor
HEC	Hydrologic Engineering Center
HW	Headwater
IWR	Institute for Water Resources
NAVD 88	North American Vertical Datum of 1988
NGVD 29	National Geodetic Vertical Datum of 1929
QC	Quality Control
RMC	Risk Management Center
TW	Tailwater
UDF	User-Defined Function
U.S.	United States
USACE	United States Army Corps of Engineers