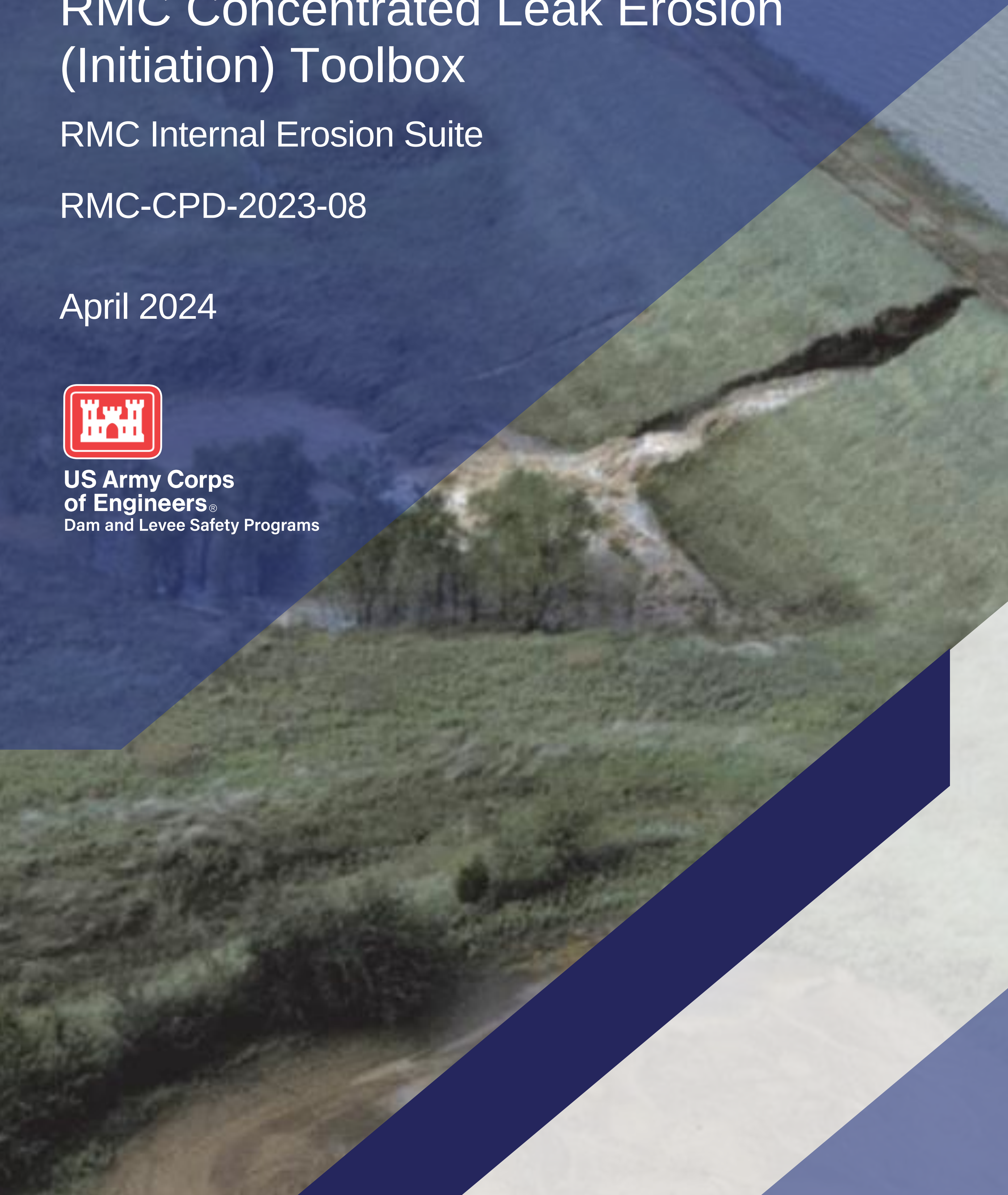
RMC Concentrated Leak Erosion (Initiation) Toolbox

RMC Internal Erosion Suite

RMC-CPD-2023-08

April 2024



US Army Corps
of Engineers®
Dam and Levee Safety Programs

US Army Corps
of Engineers®
Dam and Levee Safety Programs

Cover Photo: Upper Red Rock Site 20 in Oklahoma.
Courtesy U.S. Department of Agriculture, Agricultural Research Service.

US Army Corps of Engineers®
Dam and Levee Safety Programs

RMC Concentrated Leak Erosion (Initiation) Toolbox

RMC Internal Erosion Suite

RMC-CPD-2023-08

April 2024

Prepared by the Risk Management Center

REPORT DOCUMENTATION PAGE

1. REPORT DATE	2. REPORT TYPE	3. DATES COVERED START DATE	END DATE
Apr 2024	Computer Program Document	N/A	N/A

4. TITLE AND SUBTITLE
RMC Concentrated Leak Erosion (Initiation) Toolbox: RMC Internal Erosion Suite

5a. CONTRACT NUMBER	5b. GRANT NUMBER	5c. PROGRAM ELEMENT NUMBER
N/A	N/A	N/A
5d. PROJECT NUMBER	**5e. TASK NUMBER**	**5f. WORK UNIT NUMBER**
N/A	N/A	N/A

6. AUTHOR(S)
Damon Amlung, Risk Management Center

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)	8. PERFORMING ORGANIZATION REPORT NUMBER
Risk Management Center 12596 West Bayaud Ave. Suite 400 Lakewood, CO 80228	RMC-CPD-2023-08

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSOR/MONITOR'S ACRONYM(S)	11. SPONSOR/MONITOR'S REPORT NUMBER(S)
Risk Management Center 12596 West Bayaud Ave. Suite 400 Lakewood, CO 80228	CEIWR-RMC	RMC-CPD-2023-08

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution is unlimited.

13. SUPPLEMENTARY NOTES
N/A

14. ABSTRACT
The spreadsheet tools contained in this toolbox deterministically and probabilistically assess the likelihood of initiation of concentrated leak erosion by estimating the hydraulic shear stress in continuous, transverse cracks, gaps, or pipes using the average hydraulic gradient.

15. SUBJECT TERMS
Internal erosion, concentrated leak erosion, initiation

16. SECURITY CLASSIFICATION OF: a. REPORT	b. ABSTRACT	c. THIS PAGE	17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES
U	U	U	UU	67

19a. NAME OF RESPONSIBLE PERSON	19b. PHONE NUMBER (Include area code)
Tim O'Leary	(502) 315-6599

Page 1 of 1

STANDARD FORM 298 (REF 5/2020)
Prescribed by ANSI Std Z39.18

PREVIOUS EDITION IS OBSOLETE

PREPARED

The results, findings, and recommendations provided in this document are technically sound and consistent with current Corps of Engineers practice.

Damon Amlung, Risk Management Center

REVIEWED

This report has been checked and reviewed and is believed to be in accordance with the standards of the profession.

Tim O'Leary, Risk Management Center

APPROVED

Nate Snorteland, Risk Management Center

TABLE OF CONTENTS

1. INTRODUCTION **1**

2. TERMS AND CONDITIONS FOR USE **2**

2.1. Terms and Conditions for Use of Institute for Water Resources Software 2

2.2. Waiver of Warranty 2

2.3. Limitation of Liability 3

2.4. Indemnity 3

3. GENERAL OVERVIEW **4**

3.1. Getting Started 4

3.2. Organization 4

4. BACKGROUND **7**

5. CYLINDRICAL PIPE **9**

5.1. Method of Analysis 9

5.2. Critical Shear Stress and Initial Pipe Diameter 9

5.3. Core Geometry 11

5.4. Hydraulic Shear Stress 12

5.5. Likelihood of Initiation of Concentrated Leak Erosion 13

5.6. Headwater Level for Initiation of Concentrated Leak Erosion 19

5.7. Critical Pipe Diameter for Initiation of Concentrated Leak Erosion 21

6. HORIZONTAL CRACK **22**

6.1. Method of Analysis 22

6.2. Critical Shear Stress and Initial Crack Dimensions 22

6.3. Core Geometry 22

6.4. Hydraulic Shear Stress 22

6.5. Likelihood of Initiation of Concentrated Leak Erosion 22

6.6. Headwater Level for Initiation of Concentrated Leak Erosion 23

6.7. Critical Crack Width for Initiation of Concentrated Leak Erosion 23

7. VERTICAL RECTANGULAR CRACK **24**

7.1. Method of Analysis 24

7.2. Critical Shear Stress and Initial Crack Dimensions 24

7.3. Core Geometry 26

7.4. Hydraulic Shear Stress 26

7.5. Likelihood of Initiation of Concentrated Leak Erosion 27
7.6. Headwater Level for Initiation of Concentrated Leak Erosion 27
7.7. Critical Crack Width for Initiation of Concentrated Leak Erosion 27

8. VERTICAL TRIANGULAR CRACK 29
8.1. Method of Analysis 29
8.2. Critical Shear Stress and Initial Crack Dimensions 29
8.3. Core Geometry 29
8.4. Hydraulic Shear Stress 29
8.5. Likelihood of Initiation of Concentrated Leak Erosion 29
8.6. Headwater Level for Initiation of Concentrated Leak Erosion 29
8.7. Critical Crack Width for Initiation of Concentrated Leak Erosion 29

9. PROBABILITY TABLES 31
9.1. Flaw and Embankment Core Characterization 31
9.2. Core Geometry and Initial Pipe or Crack Dimensions 31
9.3. Hydraulic Gradient in the Pipe or Crack 33
9.4. Likelihood of Initiation of Concentrated Leak Erosion 34
9.5. Likelihood of Initiation of Concentrated Leak Erosion Considering Uncertainty 37

10. REFERENCES 39

LIST OF FIGURES

Figure 1. Security warning message bars with the “Enable Content” option to enable macros. 4
Figure 2. Calculation worksheet heading. 5
Figure 3. Headwater and tailwater input: NAVD88. 5
Figure 4. Headwater and tailwater input: NGVD29. 5
Figure 5. Headwater and tailwater input: user-specified datum. 6
Figure 6. Example of step banner. 6
Figure 7. Example of option banner. 6
Figure 8. Pipe and crack geometries evaluated by the toolbox (adapted from Fell et al. 2015). 8
Figure 9. Step 1 of Cylindrical Pipe worksheet: Deterministic analysis. 9
Figure 10. Step 1 of Cylindrical Pipe worksheet: Probabilistic analysis without using @RISK. 9
Figure 11. Step 1 of Cylindrical Pipe worksheet: Probabilistic analysis using @RISK. 9
Figure 12. Step 2 of Cylindrical Pipe worksheet: Deterministic analysis. 10
Figure 13. Step 2 of Cylindrical Pipe worksheet: Probabilistic analysis without using @RISK. 10

Figure 14. Step 2 of Cylindrical Pipe worksheet: Probabilistic analysis using @RISK. 10
Figure 15. Step 3 of Cylindrical Pipe worksheet: Core geometry. 11
Figure 16. Step 4 of Cylindrical Pipe worksheet: Hydraulic shear stress. 13
Figure 17. Step 5 of Cylindrical Pipe worksheet: Graphical output of hydraulic shear stress for deterministic analysis. 14
Figure 18. Step 5 of Cylindrical Pipe worksheet: Graphical output of hydraulic shear stress for probabilistic analysis. 15
Figure 19. Step 5 of Cylindrical Pipe worksheet: Plot options for hydraulic shear stress. 15
Figure 20. Step 5 of Cylindrical Pipe worksheet: Tabular output of hydraulic shear stress for deterministic analysis. 16
Figure 21. Step 5 of Cylindrical Pipe worksheet: Tabular output of hydraulic shear stress for probabilistic analysis without using @RISK. 16
Figure 22. Step 5 of Cylindrical Pipe worksheet: Graphical output of factor of safety for deterministic analysis. 17
Figure 23. Step 5 of Cylindrical Pipe worksheet: Graphical output of factor of safety for probabilistic analysis. 18
Figure 24. Step 5 of Cylindrical Pipe worksheet: Plot options for factor of safety. 18
Figure 25. Step 5 of Cylindrical Pipe worksheet: Graphical output of probability of initiation for probabilistic analysis. 19
Figure 26. Step 5 of Cylindrical Pipe worksheet: Plot options for probability of initiation. 19
Figure 27. Step 6 of Cylindrical Pipe worksheet: Critical headwater level for deterministic analysis. 20
Figure 28. Step 6 of Cylindrical Pipe worksheet: Critical headwater level for probabilistic analysis. 20
Figure 29. Step 7 of Cylindrical Pipe worksheet: Deterministic analysis. 21
Figure 30. Step 7 of Cylindrical Pipe worksheet: Probabilistic analysis. 21
Figure 31. Horizontal crack geometry. 22
Figure 32. Step 2 of Vertical Rectangular Crack worksheet: Deterministic analysis. 24
Figure 33. Step 2 of Vertical Rectangular Crack worksheet: Probabilistic analysis without using @RISK. 25
Figure 34. Step 2 of Vertical Rectangular Crack worksheet: Probabilistic analysis using @RISK. 25
Figure 35. Equivalent crack length without tailwater (top) and with tailwater (bottom) (adapted from Foster et al. 2002). 27
Figure 36. Step 1 of Probability Tables worksheet: Flaw type and embankment core soil properties input. 31
Figure 37. Step 2 of Probability Tables worksheet: Cylindrical pipe input. 32
Figure 38. Step 2 of Probability Tables worksheet: Vertical crack input. 33
Figure 39. Step 3 of Probability Tables worksheet: Average hydraulic gradient for cylindrical pipe. 34
Figure 40. Step 3 of Probability Tables worksheet: Average hydraulic gradient for vertical cracks. 34
Figure 41. Step 4 of Probability Tables worksheet: Crack width at HW_{mid} for a vertical triangular crack. 35

Figure 42. Step 4 of Probability Tables worksheet: Best estimate probabilities from Fell et al. (2008). ... 35
Figure 43. Step 4 of Probability Tables worksheet: Tabular and graphical output of best estimate probabilities. ... 36
Figure 44. Step 4 of Probability Tables worksheet: Plot options for best estimate probability. ... 36
Figure 45. Step 5 of Probability Tables worksheet: Graphical output of probability estimates. ... 37
Figure 46. Step 5 of Probability Tables worksheet: Plot options for probability estimates. ... 38
Figure A-1. Flow through a cylindrical pipe. ... 40
Figure A-2. Free-body diagram for flow through a cylindrical pipe. ... 41
Figure A-3. Flow through a horizontal crack. ... 43
Figure A-4. Free-body diagram for flow through a horizontal crack. ... 44
Figure A-5. Flow through a vertical rectangular crack. ... 46
Figure A-6. Free-body diagram for flow through a vertical rectangular crack. ... 47
Figure A-7. Flow through a vertical triangular crack. ... 49
Figure A-8. Free-body diagram for flow through a vertical triangular crack. ... 51
Figure A-9. Hydrostatic force on a triangular face. ... 52

LIST OF TABLES

Table B-1 Hydraulic shear stress in pipe or crack. ... 56
Table B-2 Critical pipe diameter or crack width. ... 57

APPENDIXES

APPENDIX A. DERIVATION OF HYDRAULIC SHEAR STRESS EQUATIONS ... 40

APPENDIX B. SUMMARY OF HYDRAULIC SHEAR STRESS EQUATIONS ... 56

APPENDIX C. ACRONYM LIST ... 58

1. Introduction

The Risk Management Center (RMC) of the U.S. Army Corps of Engineers (USACE) has developed a suite of Microsoft Excel spreadsheets to support risk assessments for dam and levee safety. Each analysis suite is composed of multiple toolboxes (Microsoft Excel workbooks), and each toolbox contains multiple spreadsheet tools or calculation worksheets (Microsoft Excel worksheets). The RMC Concentrated Leak Erosion (Initiation) Toolbox is part of the RMC Internal Erosion Suite.

The information from these spreadsheet tools, along with other pertinent information, informs judgment when developing a list of more and less likely factors and estimating probabilities. USACE best practice for estimating probabilities is to use the best available and multiple methods, but all final probabilities are estimated using team elicitation based on the totality and strength of the evidence.

The RMC continuously works to improve the performance of RMC software; report possible bugs directly to the RMC at the address listed below. Ideally, report suspected errors in written form with a description of the problem and the steps that lead to its occurrence. Suggestions for improvement are also welcomed.

U.S. Army Corps of Engineers
Institute for Water Resources
Risk Management Center
RMC.software@usace.army.mil

2. Terms and Conditions for Use

By using Institute for Water Resources (IWR) software, users voluntarily accept the following terms and conditions. Users that do not agree to these terms and conditions should uninstall the IWR software and return any program materials to IWR and its technical centers. If users downloaded the software and do not have disk media, delete all copies and cease using the software.

2.1. Terms and Conditions for Use of Institute for Water Resources Software

The United States Government, U.S. Army Corps of Engineers, Institute for Water Resources ("IWR"), and IWR's technical centers including the Risk Management Center ("RMC") and Hydrologic Engineering Center ("HEC") grant to the user the rights to install "IWR Software" (either from a disk copy obtained from IWR and IWR's technical centers, a distributor or another user or by downloading it from a network) and to use, copy and/or distribute copies of the IWR Software to other users, subject to the following Terms and Conditions of Use:

- All copies of the IWR Software received or reproduced by or for user pursuant to the authority of this Terms and Conditions of Use will be and remain the property of IWR.

- User may reproduce and distribute the IWR Software provided that the recipient agrees to the Terms and Conditions for Use noted herein.

- IWR and IWR's technical centers are solely responsible for the content of the IWR Software. The IWR Software may not be modified, abridged, decompiled, disassembled, unobfuscated or reverse engineered. The user is solely responsible for the content, interactions, and effects of any and all amendments, if present, whether they be extension modules, language resource bundles, scripts, or any other amendment.

- The name of the IWR Software must not be used to endorse or promote products derived from the IWR Software. Products derived from the IWR Software may not be called the IWR Software nor may any part of the IWR Software name appear within the name of derived products.

- No part of this Terms and Conditions for Use may be modified, deleted or obliterated from the IWR Software.

- No part of the IWR Software may be exported or re-exported in contravention of U.S. export laws or regulations.

2.2. Waiver of Warranty

THE UNITED STATES GOVERNMENT AND ITS AGENCIES, OFFICIALS, REPRESENTATIVES, AND EMPLOYEES, INCLUDING ITS CONTRACTORS AND SUPPLIERS PROVIDE THE IWR SOFTWARE "AS IS," WITHOUT ANY WARRANTY OR CONDITION, EXPRESS, IMPLIED OR STATUTORY, AND SPECIFICALLY DISCLAIM ANY IMPLIED WARRANTIES OF TITLE, MERCHANTABILITY, FITNESS FOR A PARTICULAR PURPOSE AND NON-INFRINGEMENT. Depending on state law, the foregoing disclaimer may not apply to you, and you may also have other legal rights that vary from state to state.

2.3. Limitation of Liability

IN NO EVENT SHALL THE UNITED STATES GOVERNMENT AND ITS AGENCIES, OFFICIALS, REPRESENTATIVES, AND EMPLOYEES, INCLUDING ITS CONTRACTORS AND SUPPLIERS, BE LIABLE FOR LOST PROFITS OR ANY SPECIAL, INCIDENTAL OR CONSEQUENTIAL DAMAGES ARISING OUT OF OR IN CONNECTION WITH USE OF THE IWR SOFTWARE REGARDLESS OF CAUSE, INCLUDING NEGLIGENCE. THE UNITED STATES GOVERNMENT'S LIABILITY, AND THE LIABILITY OF ITS AGENCIES, OFFICIALS, REPRESENTATIVES, AND EMPLOYEES, INCLUDING ITS CONTRACTORS AND SUPPLIERS, TO YOU OR ANY THIRD PARTIES IN ANY CIRCUMSTANCE IS LIMITED TO THE REPLACEMENT OF CERTIFIED COPIES OF THE IWR SOFTWARE WITH IDENTIFIED ERRORS CORRECTED. Depending on state law, the above limitation or exclusion may not apply to you.

2.4. Indemnity

As a voluntary user of the IWR Software you agree to indemnify and hold the United States Government, and its agencies, officials, representatives, and employees, including its contractors and suppliers, harmless from any claim or demand, including reasonable attorneys' fees, made by any third party due to or arising out of your use of the IWR Software or breach of this Agreement or your violation of any law or the rights of a third party.

3. General Overview

3.1. Getting Started

Copy or download the toolbox file to the computer. To open the toolbox file, either:

- Find the file on the computer and double-click it. This opens the file in Microsoft Excel.
- Open Microsoft Excel and use the application to open the file: Once Microsoft Excel is open, go to the File menu at the top of the window and select Open.

The toolbox is an Excel binary workbook (.xlsb) that uses macros. You may need to enable the macros, either before opening the file or by clicking "Enable Content" in the yellow Security Warning message bar with a shield icon that appears after the file is opened. The actual message in the message bar will vary depending on the computer's settings and installed add-ins. Figure 1 displays examples of different wordings that may appear in the message bar.

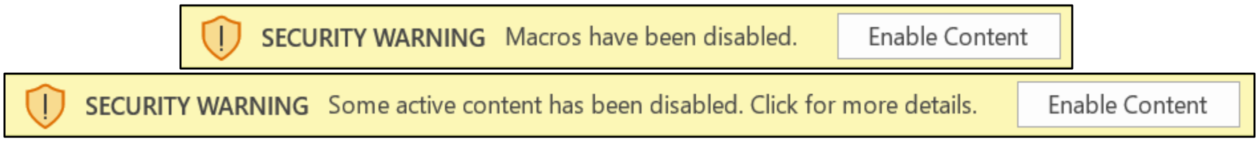


Figure 1. Security warning message bars with the "Enable Content" option to enable macros.

3.2. Organization

Although the toolbox does not provide a calculation cover sheet, adding one is strongly recommended. A calculation cover sheet captures project information, a description and purpose of the calculation, the assumptions for critical input parameters, a summary of the major conclusion and results, and a revision history.

Each toolbox has a similar appearance and organizational structure:

- The first worksheet, About, summarizes the purpose of the toolbox and gives contact information for the RMC software development team.
- The second worksheet, Terms and Conditions, contains the terms and conditions for use of the toolbox (IWR software).
- The third worksheet, Version History, contains the revision history. Semantic versioning is used in the format of MAJOR.MINOR.PATCH:
  - MAJOR – significant worksheet changes not compatible with previous versions.
  - MINOR – additional features or enhancements that do not fundamentally change the calculations.
  - PATCH – backward-compatible bug fixes.

- The fourth worksheet, References, lists the references cited for each calculation worksheet.

The workbook and worksheets are not protected to prevent unwanted changes. However, because the toolbox has user-defined functions (UDFs) and subroutines in Visual Basic, you cannot directly copy worksheets to another workbook without potentially losing functionality. A note in a bold red font at the upper right margin indicates if the selected worksheet includes such features.

At the top of each calculation worksheet, input information for the preparer and checker for quality control (QC) documentation and the calculation title in case multiple copies of the worksheet are created for different analysis scenarios (Figure 2). The footer of each calculation worksheet contains the version number, which can be cross-referenced with the revision history on the third worksheet.

Prepared by:		Office:		Date:	
Checked by:		Office:		Date:	
Calculation Title:					

Figure 2. Calculation worksheet heading.

User-specified input includes values and selections from drop-down lists. User input cells are light yellow, and these cells are unprotected. When cells use drop-down lists, a note in blue font in the right margin of the row alerts the user to use the drop-down list. Conditional formatting applies a gray background to cells that are not based on a user selection. When a user-specified value or calculated value is outside of acceptable ranges, the cell is orange to indicate caution to the user.

All units for user-specified input values are clearly labeled. Most user-specified input values use English units. However, values may be in metric where metric units are more common in practice (e.g., particle size in millimeters or permeability in centimeters per second). The toolbox may convert English units to metric units to perform some calculations or if required for a specific formula based on the reference material for the equation.

If the calculation worksheet is a function of headwater level, up to seven headwater and tailwater levels may be specified at the top of the worksheet. Tailwater may be required to calculate the net hydraulic head and hydraulic gradient. Specify the elevation datum by selecting one of three options from the drop-down list: ft-NAVD88, ft-NGVD29, and Other. The two datum selections include English units of length (feet). If Other is selected, provide a user-specified datum along with feet (e.g., ft-MSL [Mean Sea Level]). Figure 3 through Figure 5 illustrate the three possible scenarios.

Elevation datum		ft-NAVD88		Specify datum				◄ Use drop-down list.
HW (ft)	195.5	201.6	213.5	218.9	223.0	234.0	239.0	◄ Headwater level, HW (ft-NAVD88)
TW (ft)	184.0	184.0	184.0	184.0	184.0	184.0	184.0	◄ Tailwater level, TW (ft-NAVD88)

Figure 3. Headwater and tailwater input: NAVD88.

Elevation datum		ft-NGVD29		Specify datum				◄ Use drop-down list.
HW (ft)	195.5	201.6	213.5	218.9	223.0	234.0	239.0	◄ Headwater level, HW (ft-NGVD29)
TW (ft)	184.0	184.0	184.0	184.0	184.0	184.0	184.0	◄ Tailwater level, TW (ft-NGVD29)

Figure 4. Headwater and tailwater input: NGVD29.

Elevation datum	Other		Specify datum		ft-MSL		◄ Use drop-down list.	
HW (ft)	195.5	201.6	213.5	218.9	223.0	234.0	239.0	◄ Headwater level, HW (ft-MSL)
TW (ft)	184.0	184.0	184.0	184.0	184.0	184.0	184.0	◄ Tailwater level, TW (ft-MSL)

Figure 5. Headwater and tailwater input: user-specified datum.

Most calculation worksheets break down complex analysis into computational steps following a logical sequence (Figure 6). Some simpler worksheets do not have steps. Generally, different methodologies are unique worksheets. Some worksheets may include multiple methodologies, which are labeled as options (Figure 7).

Step 1: Select the method of analysis

Figure 6. Example of step banner.

Option 1: Riverside blanket (top stratum) for Cases 5, 7, and 8

Figure 7. Example of option banner.

Some calculation worksheets can perform either a deterministic or probabilistic analysis. Although not required to perform a probabilistic analysis, Palisade @RISK software (standalone version or as part of the Palisade DecisionTools Suite) can customize the probabilistic analysis. A note appears in a bold red font at the upper right-hand margin of a calculation worksheet indicating if this feature is included with the toolbox.

User notes generally appear in the right margin of each calculation worksheet. Some notes are in blue or red font for heightened awareness. These notes include references to source materials for equations, figures, tables, pages, etc. If the RMC modified the source material, the reference citation says “adapted from” instead of “from.”

Tabular and/or graphical summaries are generally the primary output of the toolbox. The UDFs in the PlotScale module change the minimum and maximum values of the x-axis and y-axis for charts. If the calculation worksheet is a function of headwater level, you can define up to five headwater levels of interest and plot them as vertical reference lines. By selecting the chart and then selecting the Filter icon to display the filter pane, you can choose which data series to display. This is useful when computing the results from multiple methodologies, but not all are applicable or desired to display.

4. Background

Concentrated leak erosion is a form of scour, and the process involves leakage flow through a continuous, transverse flaw (crack, gap, or pipe). Leakage flow through the flaw applies hydraulic shear stresses or tractive forces onto the surface of the flaw, leading to particle detachment from the surface. To assess the likelihood of initiation of concentrated leak erosion in a flaw, the hydraulic shear stress on the surface of a flaw from flow of water in the flaw can be compared to the critical shear stress of the embankment core material.

The hydraulic shear stress in a flaw for a given headwater (HW) level is based on the geometry of the embankment core, the estimated flaw dimensions, and the average hydraulic gradient through the flaw. According to Wan (2006), the hydraulic shear stress can be estimated using Equation 1.

$$\tau = \rho_w g \left(\frac{\Delta H}{L}\right)\left(\frac{A}{P_w}\right) \tag{1}$$

where:

ρ_w = density of water
g = acceleration due to gravity
ΔH = net hydraulic head
L = length of the flaw over which the hydraulic head difference occurs
A = average cross-sectional area of the flaw
P_w = average wetted perimeter of the flaw

Substituting $\gamma_\omega = \rho_\omega g$ for the unit weight of water and $i = \Delta H/L$ for the average hydraulic gradient into Equation 1, Equation 1 can be simplified to Equation 2.

$$\tau = \gamma_w i \frac{A}{P_w} \tag{2}$$

Equations to approximate the hydraulic shear stress for flow through a cylindrical pipe, horizontal crack, vertical rectangular crack (or gap), and a vertical triangular crack (or gap) were derived using this basic equation. Figure 8 illustrates each flaw type. These derivations are based on the following simplifying assumptions:

- Cross-section of the flaw is uniform from upstream to downstream (waterside to landside).
- Steady uniform flow occurs through the flaw.
- Head loss is linear from upstream to downstream (waterside to landside).
- Frictional resistance is uniform along the surface of the flaw.
- Frictional resistance is equal to the driving force.

This toolbox calculates the hydraulic shear stress in the flaw and compares it to the critical shear stress of the soil for the selected flaw geometry. For parameter combinations where the hydraulic shear stress exceeds the critical shear stress, the toolbox assumes initiation. The toolbox also calculates the critical crack width or pipe diameter as a function of critical shear stress and headwater level. Use the probability tables developed by Fell et al. (2008) to perform screening estimates of the probability of initiation of concentrated leak erosion based on the soil properties of the embankment core, the flaw size and geometry, and the average hydraulic gradient of flow through the flaw.

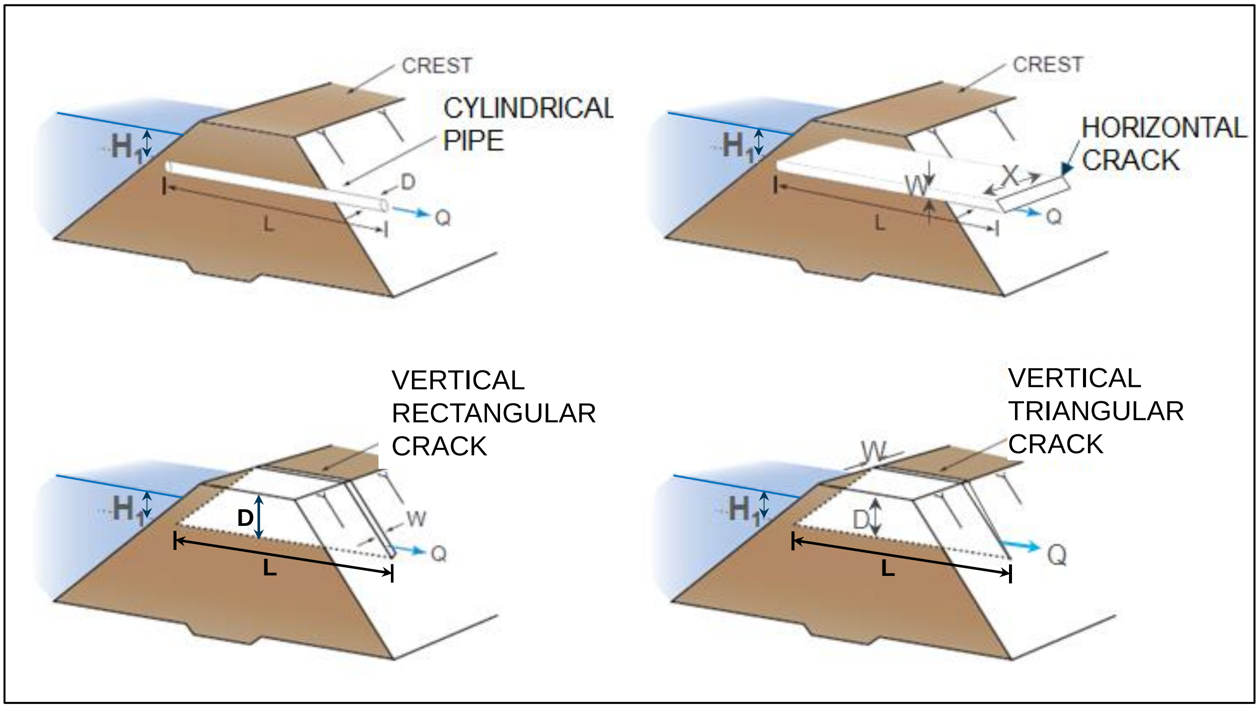


Figure 8. Pipe and crack geometries evaluated by the toolbox (adapted from Fell et al. 2015).

5. Cylindrical Pipe

5.1. Method of Analysis

In step 1, use the drop-down list to select the method of analysis (probabilistic or deterministic). There are two options for probabilistic analysis. The first performs 1,000 iterations (judged adequate for most applications) without using Palisade's @RISK software. This provides flexibility if an @RISK software license is not available. The second uses @RISK to customize the probabilistic analysis. Use the drop-down list to select Yes if @RISK is used and No if @RISK is not used. Figure 9 through Figure 11 illustrate the three possible scenarios.

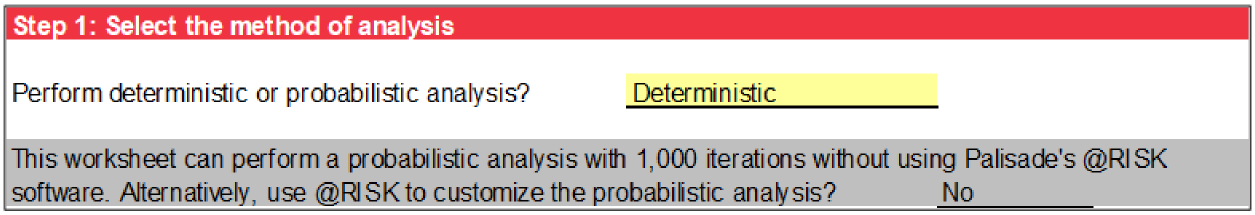


Figure 9. Step 1 of Cylindrical Pipe worksheet: Deterministic analysis.

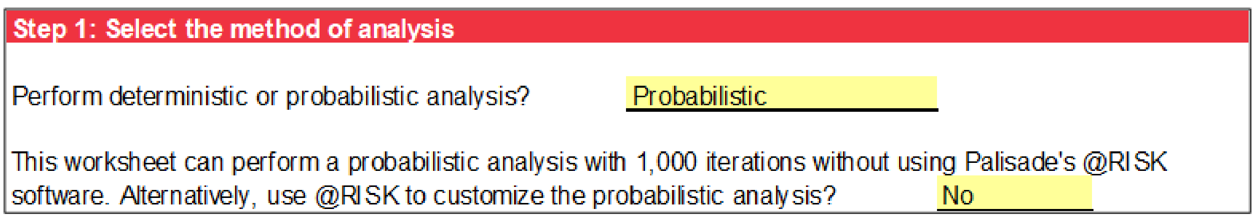


Figure 10. Step 1 of Cylindrical Pipe worksheet: Probabilistic analysis without using @RISK.

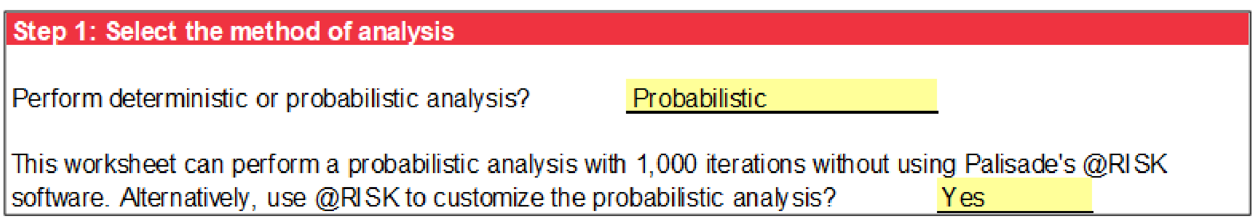


Figure 11. Step 1 of Cylindrical Pipe worksheet: Probabilistic analysis using @RISK.

5.2. Critical Shear Stress and Initial Pipe Diameter

In step 2, input the critical shear stress for the embankment core and initial pipe diameter. The selections in step 1 affect the input for step 2, and cells that do not apply have a gray background. These cells are not used in subsequent calculations, even if data is present.

For deterministic analysis, input only the most likely values. The mean values used for subsequent calculations are the most likely (or mode) value. Figure 12 illustrates the deterministic input.

Step 2: Estimate the critical shear stress and initial pipe diameter

Parameter	Units	Minimum	Most Likely	Maximum
Critical shear stress, τ_c	psf	0.021	0.042	0.100
Initial pipe diameter, D	mm	2	5	10

Parameter	Units	Mean	@RISK Formula	Mean
τ_c	psf	#NAME?		0.042
D	mm	#NAME?		5.0

Figure 12. Step 2 of Cylindrical Pipe worksheet: Deterministic analysis.

For probabilistic analysis without using @RISK, input the minimum and maximum values in addition to the most likely value, and triangular distributions represent the random variables. The mean values used in subsequent calculations are the average of the minimum, most likely, and maximum values. Figure 13 illustrates the probabilistic input without @RISK.

Step 2: Estimate the critical shear stress and initial pipe diameter

Parameter	Units	Minimum	Most Likely	Maximum
Critical shear stress, τ_c	psf	0.021	0.042	0.100
Initial pipe diameter, D	mm	2	5	10

Parameter	Units	Mean	@RISK Formula	Mean
τ_c	psf	#NAME?		0.054
D	mm	#NAME?		5.7

Figure 13. Step 2 of Cylindrical Pipe worksheet: Probabilistic analysis without using @RISK.

For probabilistic analysis using @RISK, input the minimum, most likely, and maximum values, and use an @RISK formula for a triangular distribution in the third column as a default. Alternatively, input a valid @RISK distribution in lieu of this default formula, and the user-specified input displays in the fourth column. The mean values used for subsequent calculations are the means for the @RISK distributions entered in the third column. Figure 14 illustrates the probabilistic input using @RISK.

Step 2: Estimate the critical shear stress and initial pipe diameter

Parameter	Units	Minimum	Most Likely	Maximum
Critical shear stress, τ_c	psf	0.021	0.042	0.100
Initial pipe diameter, D	mm	2	5	10

Parameter	Units	Mean	@RISK Formula	Mean
τ_c	psf	0.054	=@RiskTriang(F23,G23,H23)	0.054
D	mm	5.7	=@RiskTriang(F24,G24,H24)	5.7

Figure 14. Step 2 of Cylindrical Pipe worksheet: Probabilistic analysis using @RISK.

The RMC Erodibility Parameters Toolbox helps estimate the critical shear stress of the embankment core. The toolbox contains empirical relationships and published values based on field and laboratory testing.

The RMC Concentrated Leak Erosion (Cracking) Toolbox helps estimate the initial pipe diameter. Because it is very difficult to predict the initial pipe diameter, sensitivity analysis is recommended.

If using @RISK to perform probabilistic analysis, delete unnecessary calculation worksheets because the simulation is performed for all worksheets in the workbook, which is time consuming. If cycling through iterations using @RISK, the displayed results are no longer mean values of the random variables; they are the selected iteration's values.

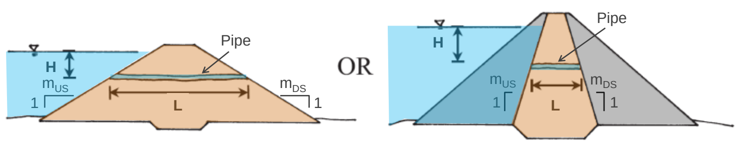
The remaining input for step 2 is the base of pipe elevation. When setting up this worksheet, include a headwater partition for the base of pipe elevation. The probability of initiation of concentrated leak erosion is zero at and below this elevation.

5.3. Core Geometry

In step 3, the embankment core geometry is defined. Input includes the top of the core elevation, width at the top of the core, and slope data for the core. Slopes are defined by elevation, starting at the top of the core and ending at the base of the embankment. Figure 15 illustrates the core geometry input. Step 4 uses the slopes of the embankment core to calculate the pipe length over which the hydraulic head difference occurs. Therefore, the lowest core elevation (minimum z_{bot}) must be less than or equal to the base of pipe elevation to compute the length. A caution displays with an orange background if the minimum user-specified z_{bot} for the upstream or downstream slope of the core is greater than base of pipe elevation.

Step 3: Assess the core geometry

Definition of Equivalent Length at the Level of the Pipe for the Hydraulic Gradient across the Core



Top of core elevation, C 239.0 ft-NGVD29
Width at top of core 30.0 ft

Upstream Slope of Core		
z_{top} (ft)	z_{bot} (ft)	m_{US} = H/V
239.0	228.5	2.00
228.5	221.0	2.50
221.0	194.0	3.00
-		

Downstream Slope of Core at Exit		
z_{top} (ft)	z_{bot} (ft)	m_{DS} = H/V
239.0	228.5	3.00
228.5	221.0	3.50
221.0	194.0	4.00
-		

Note: z_{bot} must extend below lowest base of pipe elevation in Step 2 of 194.0 ft-NGVD29

Figure 15. Step 3 of Cylindrical Pipe worksheet: Core geometry.

5.4. Hydraulic Shear Stress

Step 4 calculates the hydraulic shear stress on the surface of the cylindrical pipe from flow of water in the pipe as shown in Equation 3.

$$\tau = \gamma_w i \frac{D}{4} \tag{3}$$

where:

D = initial diameter of the pipe

The average hydraulic gradient used in Equation 3 is calculated by dividing the net hydraulic head across the pipe by the length at the base of the pipe for each defined headwater-tailwater combination.

For a homogenous embankment, the pipe length is measured at the base elevation of the pipe as a straight line from the upstream slope of the embankment to the downstream slope. For a zoned embankment where the upstream and downstream zones are very permeable (e.g., rockfill), the pipe length is measured as a straight line at the base elevation of the pipe from the upstream slope of the impervious core to the downstream slope of the impervious core. Override the formula for pipe length with a user-specified length for embankments with sloping cores, pipe orientations that are not perpendicular to the embankment centerline, etc.

The hydraulic shear stress calculated at the bottom of the first table in Figure 16 is for the mean initial pipe diameter. Because some uncertainty is always present regarding the input parameters for this type of analysis, the second table reports the hydraulic shear stress using the same hydraulic conditions as the first table but for pipe diameters ranging from 1 millimeter to 150 millimeters. The second table is used in step 6 to estimate the critical pipe diameter.

Step 4: Estimate the hydraulic shear stress on the surface of a cylindrical pipe

Hydraulic gradient across the core, $i = (H_1 - H_2)/L$

where Hydraulic head at US end of pipe, H_1 (ft) = HW − B

Hydraulic head at DS end of pipe, H_2 (ft) = TW − B

Length of pipe, L (see above figure)

Hydraulic shear stress, $\tau = \rho_w g[(H_1 - H_2)/L](A/P_w) = (\gamma_w)(i)(\pi D^2/4) / (\pi D) = \gamma_w(i)(D/4)$

HW (ft)	204.0	213.5	221.0	228.5	231.0	235.0	239.0
TW (ft)	202.0	202.0	202.0	202.0	202.0	204.0	206.0
H_1 (ft)	10.0	19.5	27.0	34.5	37.0	41.0	45.0
H_2 (ft)	8.0	8.0	8.0	8.0	8.0	10.0	12.0
L (ft)	316.5	316.5	316.5	316.5	316.5	316.5	316.5
i	0.006	0.036	0.060	0.084	0.092	0.098	0.104
τ (psf)	0.002	0.011	0.017	0.024	0.027	0.028	0.030

Note: Calculations for hydraulic shear stress are based on mean initial pipe diameter.

Hydraulic Shear Stress, τ (psf)

D (mm)	Headwater Level (ft-NGVD29)						
	204.0	213.5	221.0	228.5	231.0	235.0	239.0
1	0.000	0.002	0.003	0.004	0.005	0.005	0.005
2	0.001	0.004	0.006	0.009	0.009	0.010	0.011
5	0.002	0.009	0.015	0.021	0.023	0.025	0.027
10	0.003	0.019	0.031	0.043	0.047	0.050	0.053
25	0.008	0.046	0.077	0.107	0.117	0.125	0.133
50	0.016	0.093	0.154	0.214	0.234	0.251	0.267
75	0.024	0.139	0.230	0.321	0.352	0.376	0.400
100	0.032	0.186	0.307	0.429	0.469	0.501	0.534
150	0.049	0.279	0.461	0.643	0.703	0.752	0.800

Figure 16. Step 4 of Cylindrical Pipe worksheet: Hydraulic shear stress.

5.5. Likelihood of Initiation of Concentrated Leak Erosion

Step 5 compares the calculated hydraulic shear stress on the surface of the cylindrical pipe from flow of water in the pipe to the critical shear stress for initiation of concentrated leak erosion. The factor of safety (FS) against initiation of concentrated leak erosion is calculated as shown in Equation 4.

$$FS = \frac{\tau_{cr}}{\tau} \qquad (4)$$

where:

τ_{cr} = critical shear stress for initiation of concentrated leak erosion
τ = hydraulic shear stress on the surface of the cylindrical pipe from flow of water in the pipe

For deterministic analysis, the mean hydraulic shear stress is plotted for each headwater level as a function of pipe diameter and results in a series of straight lines. The mean and mode initial pipe diameter and critical shear stress defined in step 2 are plotted as a point on the chart. When this point plots above the line of the headwater level being considered, the FS is greater than 1, and initiation of concentrated leak erosion is not predicted. When this point plots below the line, the FS is less than 1, and initiation is predicted. Cells that do not apply have a gray background. Figure 17 illustrates the graphical output of hydraulic shear stress for deterministic analysis.

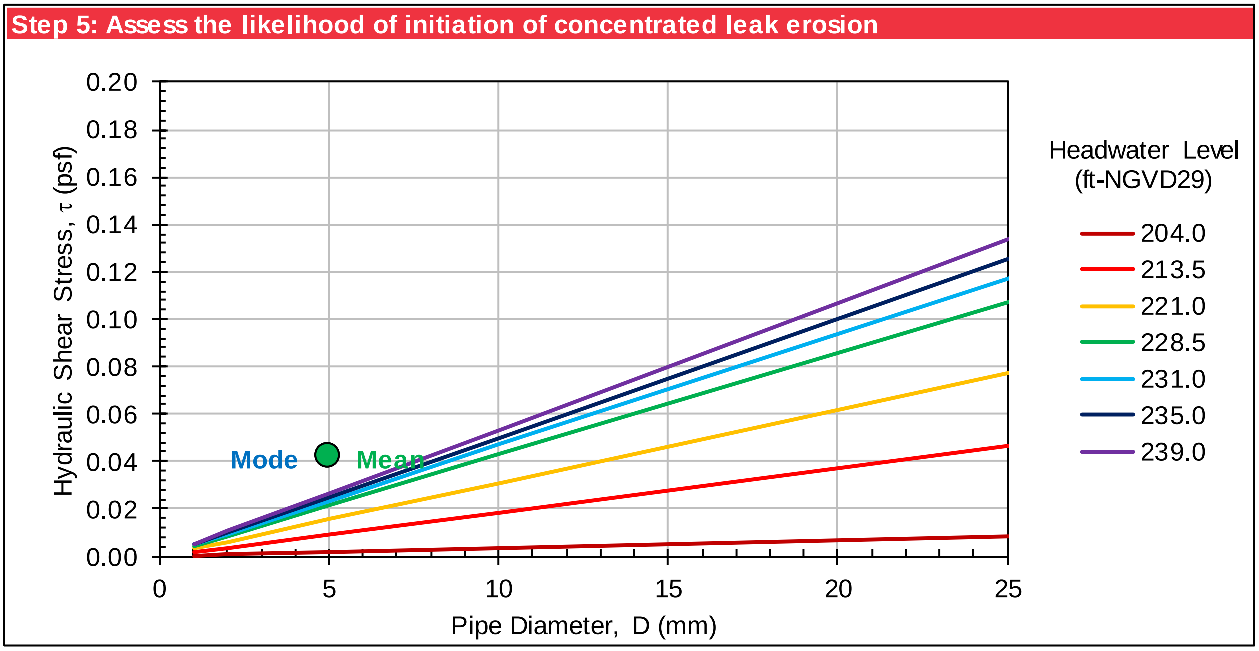


Figure 17. Step 5 of Cylindrical Pipe worksheet:
Graphical output of hydraulic shear stress for deterministic analysis.

For probabilistic analysis, a black box is also plotted showing the distribution limits for initial pipe diameter and critical shear stress. Initiation is predicted for initial pipe diameter and critical shear stress combinations that plot inside the black box and below the line corresponding to the headwater level being evaluated. If cycling through iterations using @RISK, the displayed mean pipe diameter and critical shear stress are no longer mean values; they are the selected iteration’s values. Figure 18 provides an example of the graphical output of hydraulic shear stress for probabilistic analysis.

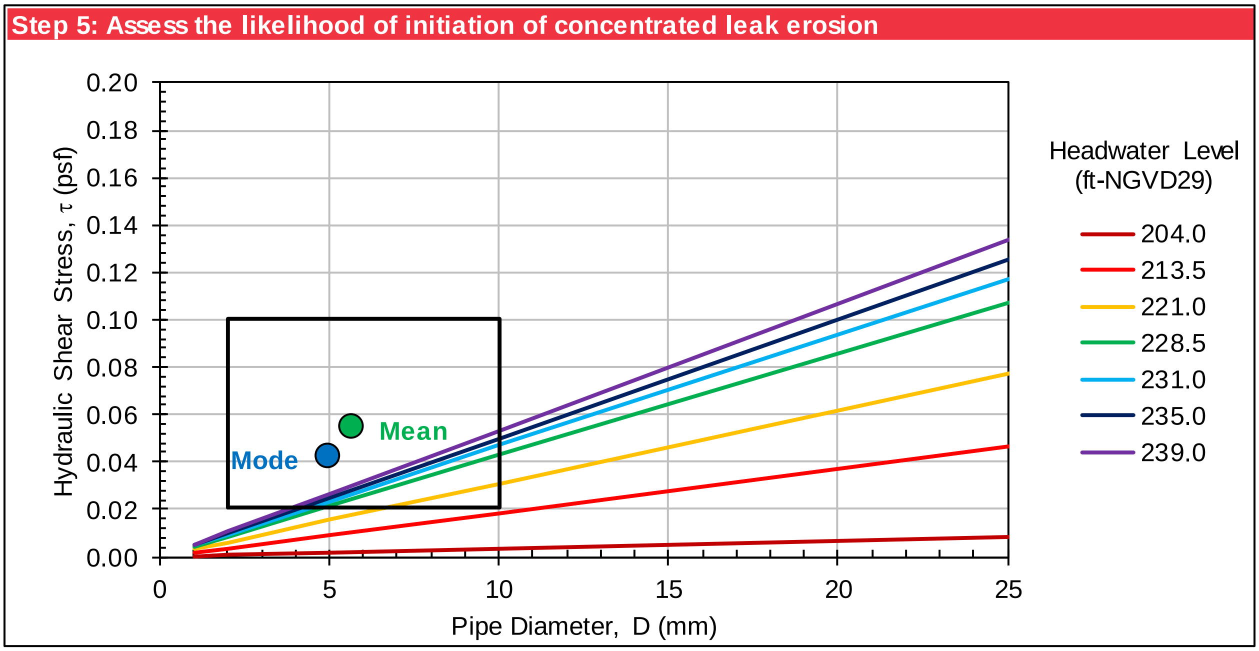


Figure 18. Step 5 of Cylindrical Pipe worksheet:
Graphical output of hydraulic shear stress for probabilistic analysis.

Figure 19 illustrates the plot options for this chart. The maximum value for the y-axis (hydraulic shear stress) and maximum value for the x-axis (pipe diameter) are user-specified.

Worksheet	Cylindrical Pipe		
y-axis bounds			
minimum	0		Value Primary Min: 0
maximum	0.20	◄ Enter maximum hydraulic shear stress.	Value Primary Max: 0.2
x-axis bounds			
minimum	0		Category Primary Min: 0
maximum	25	◄ Enter maximum pipe diameter.	Category Primary Max: 25

Figure 19. Step 5 of Cylindrical Pipe worksheet: Plot options for hydraulic shear stress.

For deterministic analysis, the FS is calculated using the most likely values of the random variables and summarized in separate tables. Cells that do not apply have a gray background. For probabilistic analysis, the FS is calculated as described for the deterministic analysis but for the mean values of the random variables, and multiple iterations are performed by sampling the distributions in step 6. The probability of initiation is equal to the percentage of iterations that resulted in a FS less than 1 [(P(FS < 1)]. For probabilistic analysis performed without using @RISK, 1,000 iterations are used. For probabilistic analysis using @RISK, the number of iterations is user-specified, and "@RISK" displays in parentheses after the number of iterations for this scenario. If cycling through iterations using @RISK, the displayed results are no longer mean values; they are the selected iteration's values. For deterministic and probabilistic analyses, cells with FS less than 1 have an orange background. Figure 20 illustrates the deterministic tabular output, and Figure 21 illustrates the probabilistic tabular output without using @RISK.

Factor of safety against initiation of concentrated leak erosion, FS = τ_c/τ

Probability of a factor of safety against initiation of concentrated leak erosion less than 1, P(FS < 1)

Iterations: #N/A

HW (ft)	204.0	213.5	221.0	228.5	231.0	235.0	239.0
FS	25.97	4.52	2.73	1.96	1.79	1.68	1.57
P(FS < 1)	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A

Note: Probabilities obtained from this method should not be used directly in risk analyses. Rather, the values should be used to help develop a list of more and less likely factors.

Figure 20. Step 5 of Cylindrical Pipe worksheet:
Tabular output of hydraulic shear stress for deterministic analysis.

Factor of safety against initiation of concentrated leak erosion, FS = τ_c/τ

Probability of a factor of safety against initiation of concentrated leak erosion less than 1, P(FS < 1)

Iterations: 1000

HW (ft)	204.0	213.5	221.0	228.5	231.0	235.0	239.0
FS	29.65	5.16	3.12	2.24	2.04	1.91	1.80
P(FS < 1)	0.00E+00	0.00E+00	2.00E-03	2.60E-02	5.10E-02	7.80E-02	1.02E-01

Note: Probabilities obtained from this method should not be used directly in risk analyses. Rather, the values should be used to help develop a list of more and less likely factors.

Figure 21. Step 5 of Cylindrical Pipe worksheet:
Tabular output of hydraulic shear stress for probabilistic analysis without using @RISK.

At the end of step 5, summary plots are generated. The first plot is the mean FS against initiation of concentrated leak erosion as a function of headwater level. A deterministic analysis uses only the mean critical shear stress and the mean initial pipe diameter. The result is plotted as a blue line, and a gray background is applied to the drop-down list above the plot signifying it does not apply. A horizontal reference line is displayed in black for a FS of 1.0. Figure 22 illustrates graphical output for deterministic analysis.

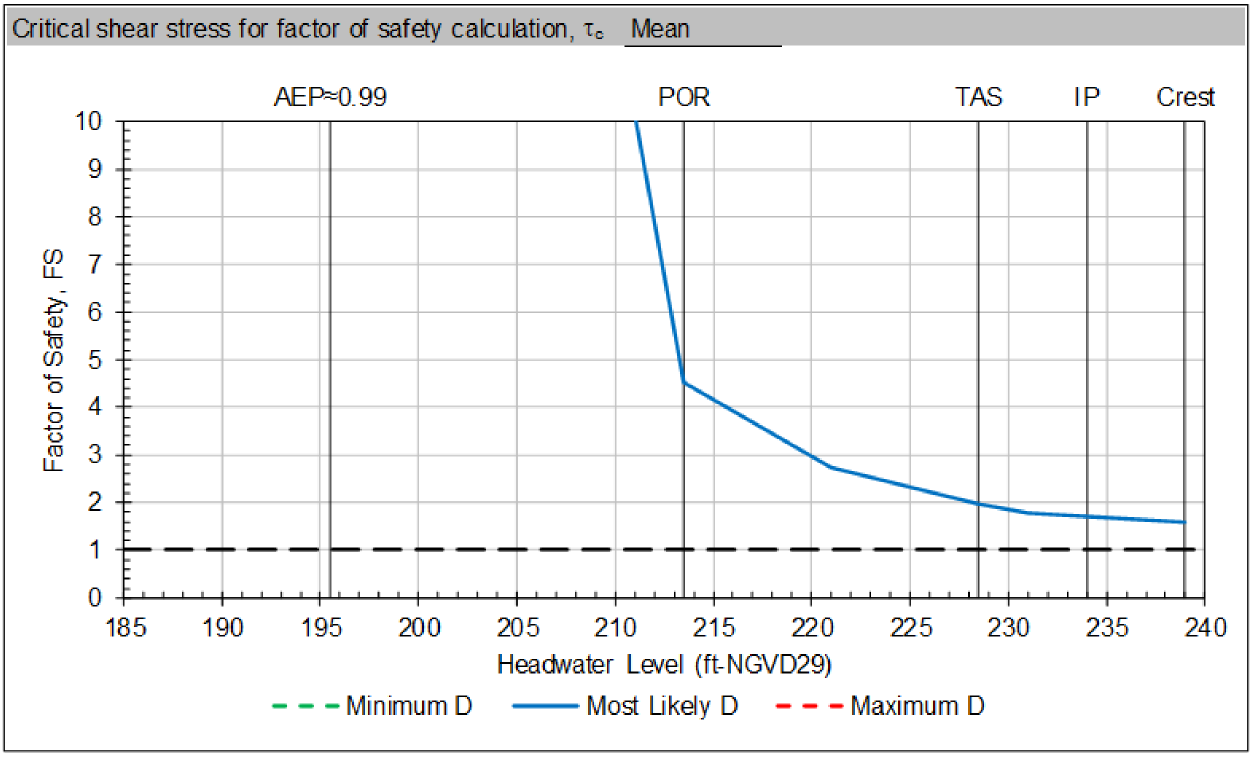


Figure 22. Step 5 of Cylindrical Pipe worksheet:
Graphical output of factor of safety for deterministic analysis.

For probabilistic analysis, use the drop-down list above the plot to select a critical shear stress other than the mean for the FS calculation. Options include the minimum, most likely, and maximum critical shear stress. Additionally, along with the mean initial pipe diameter, the FS against for initiation of concentrated leak erosion is plotted for the minimum (green dashed line) and maximum (red dashed line) initial pipe diameter. If cycling through iterations using @RISK, the plotted FS is calculated using the hydraulic shear stress for the selected iteration and the critical shear stress from the drop-down list. Figure 23 provides an example of the graphical output for probabilistic analysis.

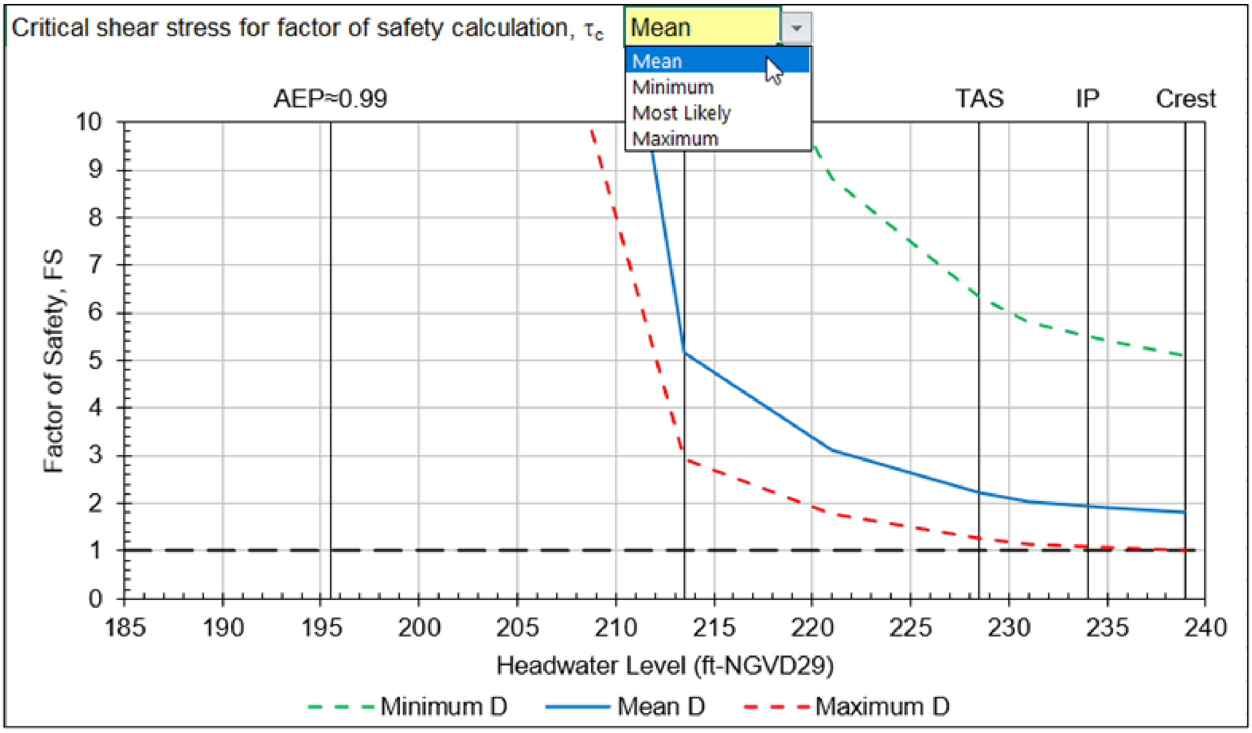


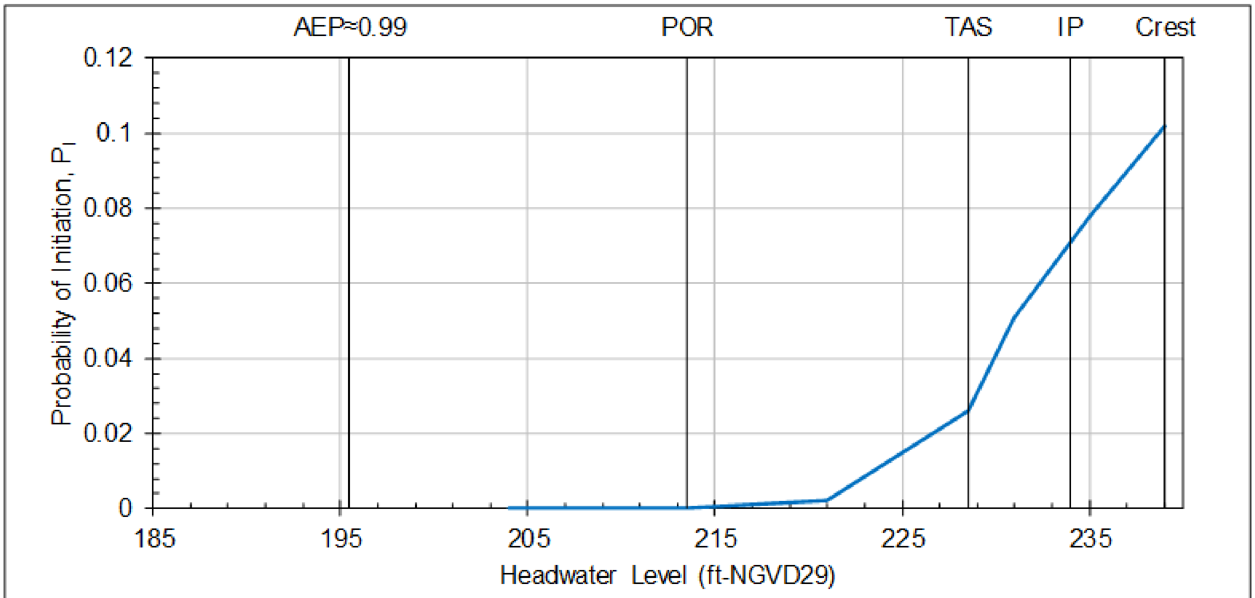
Figure 23. Step 5 of Cylindrical Pipe worksheet:
Graphical output of factor of safety for probabilistic analysis.

Figure 24 illustrates the plot options for Figure 23. The maximum value for the y-axis (FS against initiation of concentrated leak erosion) and minimum and maximum values for the x-axis (headwater level) are user-specified. Up to five vertical reference elevations can be defined with user-specified labels displayed at the top of the chart.

Worksheet	Cylindrical Pipe					
y-axis bounds						
minimum	0					Value Primary Min: 0
maximum	10.00	◄ Enter maximum FS.				Value Primary Max: 10
x-axis bounds						
minimum	185	◄ Enter minimum headwater.				Category Primary Min: 185
maximum	240	◄ Enter maximum headwater.				Category Primary Max: 240
Enter up to 5 vertical reference lines for headwater levels of interest.						
AEP≈0.99	POR	TAS	IP	Crest	◄ Enter headwater description.	
195.5	213.5	228.5	234.0	239.0	◄ Enter headwater level.	

Figure 24. Step 5 of Cylindrical Pipe worksheet: Plot options for factor of safety.

The second plot is the probability of initiation of concentrated leak erosion as a function of headwater level. For deterministic analysis, a probability of initiation of concentrated leak erosion is not calculated, and this plot has a gray background. For probabilistic analysis, the mean probability of initiation of concentrated leak erosion displays. If cycling through iterations using @RISK, this plot has a gray background because the probability of initiation cannot be calculated from a single iteration. Similarly, this plot has a gray background for deterministic analysis. Figure 25 illustrates the graphical output for probabilistic analysis.



Note: Probabilities obtained from this method should not be used directly in risk assessments. Rather, the values should be used to help develop a list of more and less likely factors.

Figure 25. Step 5 of Cylindrical Pipe worksheet:
Graphical output of probability of initiation for probabilistic analysis.

Figure 26 illustrates the plot options for this chart. The vertical reference elevations and minimum and maximum values for the x-axis (headwater level) are the same as the previous chart. Only the maximum value for the y-axis (probability of initiation of concentrated leak erosion) is user-specified.

Worksheet	Cylindrical Pipe		
y-axis bounds			
minimum	0		Value Primary Min: 0
maximum	0.12	◄ Enter maximum probability.	Value Primary Max: 0.12
x-axis bounds			
minimum	185.0	◄ Enter minimum headwater level.	Category Primary Min: 185
maximum	240.0	◄ Enter maximum headwater level.	Category Primary Max: 240

Figure 26. Step 5 of Cylindrical Pipe worksheet: Plot options for probability of initiation.

5.6. Headwater Level for Initiation of Concentrated Leak Erosion

Step 6 calculates the headwater level for initiation of concentrated leak erosion. The results for different combinations of the initial pipe diameter and critical shear stress are linearly interpolated from the tables in step 4. The first table (Distribution) considers the combinations created by the distribution input in step 2 and is available only when probabilistic analysis is selected. For deterministic analysis, this table has a gray background. The second table (Mean) considers the mean initial pipe diameter and the mean critical shear stress and is available for both deterministic and probabilistic analyses. If the critical shear stress is less than the hydraulic shear stress for the minimum specified headwater level, the headwater level for initiation so indicates. If the critical shear stress is greater than the hydraulic shear stress for the maximum specified headwater level or if the hydraulic shear stress does not increase with an increase in headwater level (e.g., because of an increase in the tailwater level), the headwater level for initiation cannot be calculated, and an error displays. For deterministic analysis, the mean value in the second table is equal to the most likely (or mode) value. Figure 27 and Figure 28 provide examples of critical headwater levels for deterministic and probabilistic analyses, respectively.

Step 6: Estimate the headwater level for initiation of concentrated leak erosion

Distribution	Headwater Level (ft) for $\tau > \tau_c$		
D	τ_c (psf)		
(mm)	0.021	0.042	0.100
2.0	-	-	-
5.0	228.0	-	-
10.0	215.0	228.0	-

Mean	Headwater Level (ft) for $\tau > \tau_c$		
D	τ_c (psf)		
(mm)		0.042	
5.0		-	

Notes: Results for selected crack widths are obtained from 2-way interpolation of table in Step 4.
An error will be returned if hydraulic shear stress does not increase with stage.

Figure 27. Step 6 of Cylindrical Pipe worksheet: Critical headwater level for deterministic analysis.

Step 6: Estimate the headwater level for initiation of concentrated leak erosion

Distribution	Headwater Level (ft) for $\tau > \tau_c$		
D	τ_c (psf)		
(mm)	0.021	0.042	0.100
2.0	-	-	-
5.0	228.0	-	-
10.0	215.0	228.0	-

Mean	Headwater Level (ft) for $\tau > \tau_c$		
D	τ_c (psf)		
(mm)		0.054	
5.7		-	

Notes: Results for selected crack widths are obtained from 2-way interpolation of table in Step 4.
An error will be returned if hydraulic shear stress does not increase with stage.

Figure 28. Step 6 of Cylindrical Pipe worksheet: Critical headwater level for probabilistic analysis.

5.7. Critical Pipe Diameter for Initiation of Concentrated Leak Erosion

Step 7 calculates the critical pipe diameter for initiation of concentrated leak erosion (D_{cr}) for each headwater level using Equation 5.

$$D_{cr} = \frac{4\tau_c}{i\gamma_w} \tag{5}$$

This equation was derived by solving Equation 3 for the pipe diameter and setting the hydraulic shear stress equal to the critical shear stress.

For deterministic analysis, the calculations for the minimum and maximum critical pipe diameters have a gray background, and the most likely (mode) and mean critical pipe diameter are equal at each headwater level evaluated because the mode and mean critical shear stress are equal. Figure 29 provides an example for deterministic analysis.

Step 7: Estimate the critical pipe diameter for initiation of concentrated leak erosion

Critical pipe diameter for initiation of concentrated leak erosion, D_{cr}

$D_{cr} = 4\tau_c/(i\gamma_w)$

HW (ft)	204.0	213.5	221.0	228.5	231.0	235.0	239.0
$D_{cr,min}$ (mm)	64.9	11.3	6.8	4.9	4.5	4.2	3.9
$D_{cr,mode}$ (mm)	129.9	22.6	13.7	9.8	9.0	8.4	7.9
$D_{cr,max}$ (mm)	309.2	53.8	32.5	23.3	21.3	19.9	18.7
$D_{cr,mean}$ (mm)	129.9	22.6	13.7	9.8	9.0	8.4	7.9

Figure 29. Step 7 of Cylindrical Pipe worksheet: Deterministic analysis.

For probabilistic analysis, the critical pipe diameter for each headwater level is calculated using the minimum, most likely (mode), maximum, and mean critical shear stress, where $D_{cr,min}$ is based on the minimum τ_c; $D_{cr,mode}$ is based on the most likely (mode) τ_c; $D_{cr,max}$ is based on the maximum τ_c; and $D_{cr,mean}$ is based on the mean τ_c. Figure 30 provides an example for probabilistic analysis.

Step 7: Estimate the critical pipe diameter for initiation of concentrated leak erosion

Critical pipe diameter for initiation of concentrated leak erosion, D_{cr}

$D_{cr} = 4\tau_c/(i\gamma_w)$

HW (ft)	204.0	213.5	221.0	228.5	231.0	235.0	239.0
$D_{cr,min}$ (mm)	64.9	11.3	6.8	4.9	4.5	4.2	3.9
$D_{cr,mode}$ (mm)	129.9	22.6	13.7	9.8	9.0	8.4	7.9
$D_{cr,max}$ (mm)	309.2	53.8	32.5	23.3	21.3	19.9	18.7
$D_{cr,mean}$ (mm)	168.0	29.2	17.7	12.7	11.6	10.8	10.2

Figure 30. Step 7 of Cylindrical Pipe worksheet: Probabilistic analysis.

6. Horizontal Crack

6.1. Method of Analysis

The method of analysis is the same as that described for the Cylindrical Pipe worksheet in section 5.1.

6.2. Critical Shear Stress and Initial Crack Dimensions

The critical shear stress and initial crack dimensions are very similar to that described for the Cylindrical Pipe worksheet in section 5.2. Instead of entering an initial pipe diameter, input the initial crack width. In addition, step 2 includes an input for the crack width parallel to the embankment centerline (X) as Figure 31 illustrates.

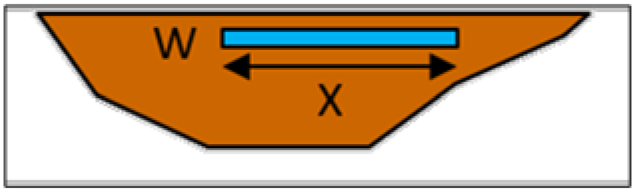


Figure 31. Horizontal crack geometry.

6.3. Core Geometry

The core geometry is the same as that described for the Cylindrical Pipe worksheet in section 5.3.

6.4. Hydraulic Shear Stress

The hydraulic shear stress on the surface of the horizontal crack from flow of water in the crack is the same as that described for the Cylindrical Pipe worksheet in section 5.4, but using a different equation and crack width instead of pipe diameter. The hydraulic shear stress on the rectangular surface of the horizontal crack is calculated using Equation 6.

$$\tau = \gamma_w i \frac{WX}{(2W + 2X)} \tag{6}$$

where:

W = initial crack width
X = crack width parallel to the embankment centerline

6.5. Likelihood of Initiation of Concentrated Leak Erosion

The likelihood of initiation of concentrated leak erosion is the same as that described for the Cylindrical Pipe worksheet in section 5.5, but using crack width instead of pipe diameter.

6.6. Headwater Level for Initiation of Concentrated Leak Erosion

The headwater level for initiation of concentrated leak erosion is the same as that described for the Cylindrical Pipe worksheet in section 5.6, but using crack width instead of pipe diameter.

6.7. Critical Crack Width for Initiation of Concentrated Leak Erosion

The critical pipe diameter for initiation of concentrated leak erosion is the same as that described for the Cylindrical Pipe worksheet in section 5.7, but using a different equation and crack width instead of pipe diameter. Equation 7 calculates the critical crack width for initiation of concentrated leak erosion for each headwater level.

$$W_{cr} = \frac{-2L\tau_c X}{2L\tau_c - \gamma_w(H_1 - H_2)X} \tag{7}$$

7. Vertical Rectangular Crack

7.1. Method of Analysis

The method of analysis is the same as that described for the Cylindrical Pipe worksheet in section 5.1.

7.2. Critical Shear Stress and Initial Crack Dimensions

In step 2, input the critical shear stress for the embankment core and initial crack dimensions. The selections in step 1 affect the input for step 2, and cells that do not apply have a gray background. These cells are not used in subsequent calculations even if data is present.

The first table in step 2 is the correlation between the initial crack width at the top of the core and the crack depth from the top of the core. The default values in the toolbox were adapted from Table 5.25 of Fell et al. (2008) and are based on a review of the literature on observed cracking and the results of numerical modeling by Bui et al. (2005). A user-specified relationship can also be used. For transverse cracks or gaps adjacent to walls, use a constant depth for all crack widths.

For deterministic analysis, input only the most likely values. The mean value used for subsequent calculations is set equal to the most likely (or mode) value. Figure 32 illustrates the deterministic input.

Step 2: Estimate the critical shear stress and initial crack dimensions

Correlation between initial crack width at top of core and crack depth

W (mm)	0	10	25	50	75	100	250
D (ft)	0	5	10	15	22.5	30	75
Use D (ft)	0	5	10	15	22.5	30	75

Parameter	Units	Minimum	Most Likely	Maximum
Critical shear stress, τ_c	psf	0.021	0.042	0.100
Initial crack width at top of core, W	mm	5.0	10.0	15.0
Crack depth from top of core, D	ft	2.5	5.0	6.7

Parameter	Units	Mean	@RISK Formula	Mean
τ_c	psf	#NAME?		0.042
W	mm	#NAME?		10.0
D	ft			5.0

Figure 32. Step 2 of Vertical Rectangular Crack worksheet: Deterministic analysis.

For probabilistic analysis without @RISK, input the minimum and maximum values in addition to the most likely value, and triangular distributions represent the random variables. The mean values used in subsequent calculations are the average of the minimum, most likely, and maximum values. Figure 33 illustrates the probabilistic input without @RISK.

Step 2: Estimate the critical shear stress and initial crack dimensions

Correlation between initial crack width at top of core and crack depth

W (mm)	0	10	25	50	75	100	250
D (ft)	0	5	10	15	22.5	30	75
Use D (ft)	0	5	10	15	22.5	30	75

Parameter	Units	Minimum	Most Likely	Maximum
Critical shear stress, τ_c	psf	0.021	0.042	0.100
Initial crack width at top of core, W	mm	5.0	10.0	15.0
Crack depth from top of core, D	ft	2.5	5.0	6.7

Parameter	Units	Mean	@RISK Formula	Mean
τ_c	psf	#NAME?		0.054
W	mm	#NAME?		10.0
D	ft			4.7

**Figure 33. Step 2 of Vertical Rectangular Crack worksheet:
Probabilistic analysis without using @RISK.**

For probabilistic analysis using @RISK, input the minimum, most likely, and maximum values, and use an @RISK formula for a triangular distribution in the third column as a default. Alternatively, input a valid @RISK distribution in lieu of this default formula, and the user-specified input displays in the fourth column. The mean values used for subsequent calculations are the means for the @RISK distributions entered in the third column. Figure 34 illustrates the probabilistic input using @RISK.

Step 2: Estimate the critical shear stress and initial crack dimensions

Correlation between initial crack width at crest and crack depth

W (mm)	0	10	25	50	75	100	250
D (ft)	0	5	10	15	22.5	30	75
Use D (ft)	0	5	10	15	22.5	30	75

Parameter	Units	Minimum	Most Likely	Maximum
Critical shear stress, τ_c	psf	0.021	0.042	0.100
Initial crack width, W	mm	5.0	10.0	15.0
Crack depth from crest, D	ft	2.5	5.0	6.7

Parameter	Units	Mean	@RISK Formula	Mean
τ_c	psf	0.054	=@RiskTriang(F28,G28,H28)	0.054
W	mm	10.0	=@RiskTriang(F29,G29,H29)	10.0
D	ft			5.0

Figure 34. Step 2 of Vertical Rectangular Crack worksheet: Probabilistic analysis using @RISK.

The RMC Erodibility Parameters Toolbox helps estimate the critical shear stress of the embankment core. The toolbox contains empirical relationships and published values based on field and laboratory testing. The RMC Concentrated Leak Erosion (Cracking) Toolbox helps estimate the initial crack dimensions for a variety of initiating mechanisms. Because it is very difficult to predict the depth and width of cracking, sensitivity analysis is recommended. Numerical modeling can also inform judgment.

The crack depths from the top of the core reported in the second and third tables are linearly interpolated from the correlation table defined at the beginning of step 2 using the initial crack width input.

7.3. Core Geometry

The core geometry is the same as that described for the Cylindrical Pipe worksheet in section 5.3, but the base elevation of the crack is not an input because it is calculated by the toolbox from the step 2 input. Step 4 uses the slopes of the embankment core to calculate the equivalent length over which the hydraulic head difference occurs. Therefore, the lowest core elevation (minimum z_{bot}) must be less than or equal to the lowest crack depth in step 2 to compute the length. A caution displays with an orange background if the minimum user-specified z_{bot} for the upstream or downstream slope of the core is greater than lowest crack depth.

7.4. Hydraulic Shear Stress

Step 4 calculates the hydraulic shear stress on the surface of the vertical rectangular crack from flow of water in the crack using Equation 8.

$$\tau = \frac{\gamma_w W\left(H_1^2 - H_2^2\right)}{2L(H_1 + H_2 + W)} \tag{8}$$

The length used in the calculation of the average hydraulic gradient across the core is the length of crack over which the net hydraulic head occurs and is referred to as the equivalent length. When there is no tailwater above the base of crack, the hydraulic head loss occurs over a length measured from the downstream face of the core at the base of the crack to a projection of the point where the headwater level intersects the upstream face of the core. When tailwater is above the base of crack elevation, the length is measured between the projection point where the headwater level intersects the upstream face of the core and the projection point where the tailwater intersects the downstream face of the core. Figure 35 illustrates both hydraulic conditions for homogeneous embankments and zoned embankments where the upstream and downstream zones are very permeable (e.g., rockfill). Override the formula for equivalent length with a user-specified length for embankments with sloping cores, crack orientations that are not perpendicular to the embankment centerline, etc.

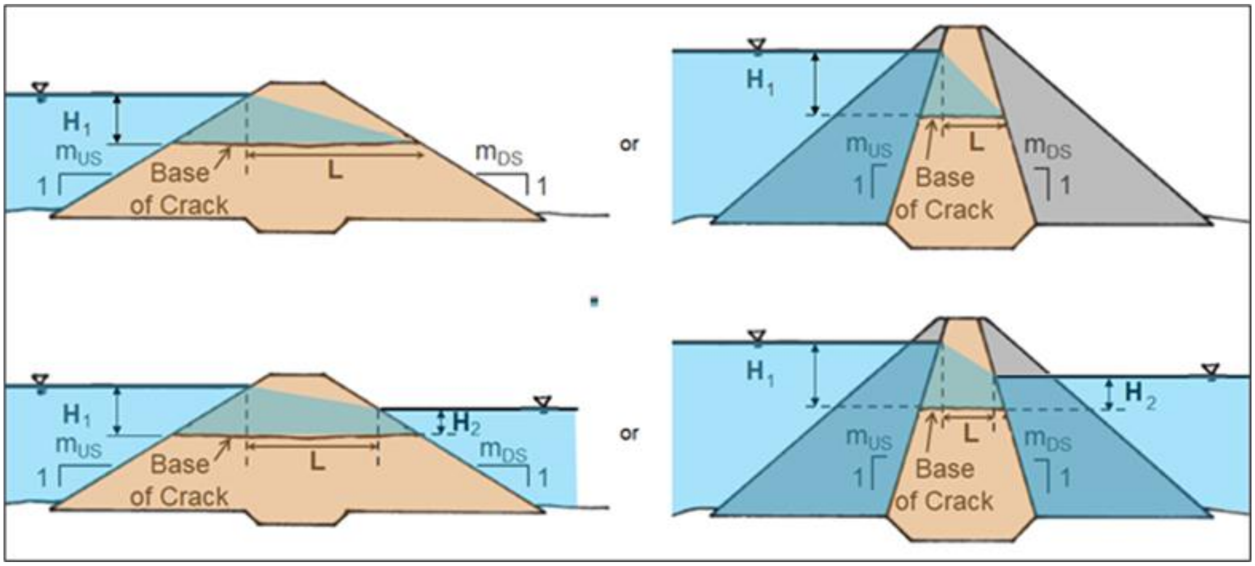


Figure 35. Equivalent crack length without tailwater (top) and with tailwater (bottom) (adapted from Foster et al. 2002).

The hydraulic shear stress calculated at the bottom of the first table is for the mean crack width at the top of the core and the mean crack depth from the top of the core. Because some uncertainty is always present regarding the input parameters for this type of analysis, the second table reports the hydraulic shear stress using the same hydraulic conditions as the first table but for pipe diameters ranging from 1 millimeter to 150 millimeters. Because the crack depth is correlated to the crack width at the top of the core, supporting tables are also provided for hydraulic head at the upstream and downstream ends of the crack, equivalent length for net hydraulic head, and average hydraulic gradient across the core. The table of hydraulic shear stress as a function of crack width at the top of the core is used in step 6 to estimate the critical crack width.

7.5. Likelihood of Initiation of Concentrated Leak Erosion

The likelihood of initiation of concentrated leak erosion is the same as that described for the Cylindrical Pipe worksheet in section 5.5, but using a different equation and crack width instead of pipe diameter.

7.6. Headwater Level for Initiation of Concentrated Leak Erosion

The headwater level for initiation of concentrated leak erosion is the same as that described for the Cylindrical Pipe worksheet in section 5.6, but using a different equation and crack width instead of pipe diameter.

7.7. Critical Crack Width for Initiation of Concentrated Leak Erosion

The critical crack width at the top of the core for initiation of concentrated leak erosion is the same as that described for the Cylindrical Pipe worksheet in section 5.7, but using a different equation and crack width instead of pipe diameter. Input the crack depth from the top of the core since the hydraulic head at the upstream and downstream ends of the core and equivalent length for net hydraulic head are functions of crack depth. Equation 9 calculates the critical crack width at the top of the core for initiation of concentrated leak erosion for each headwater level and the specified crack depth from the top of the core.

$$W_{cr} = \frac{2L\tau_c(H_1 + H_2)}{\gamma_w\left(H_1^2 - H_2^2\right) - 2L\tau_c} \tag{9}$$

8. Vertical Triangular Crack

8.1. Method of Analysis

The method of analysis is the same as that described for the Cylindrical Pipe worksheet in section 5.1.

8.2. Critical Shear Stress and Initial Crack Dimensions

The critical shear stress and initial crack dimensions are the same as that described for the Vertical Rectangular Crack worksheet in section 7.2.

8.3. Core Geometry

The core geometry is the same as that described for the Vertical Rectangular Crack worksheet in section 7.3.

8.4. Hydraulic Shear Stress

The hydraulic shear stress on the surface of the vertical triangular crack from flow of water in the crack is the same as that described for the Vertical Rectangular Crack worksheet in section 7.4, but using a different equation. Equation 10 calculates the hydraulic shear stress on the isosceles triangular surface of the vertical triangular crack.

$$\tau = \frac{\gamma_w W}{6DL} \frac{\left(H_1^3 - H_2^3\right)}{(H_1 + H_2)\sqrt{1 + \frac{W^2}{4D^2}}} \tag{10}$$

8.5. Likelihood of Initiation of Concentrated Leak Erosion

The likelihood of initiation of concentrated leak erosion is the same as that described for the Cylindrical Pipe worksheet in section 5.5, but using a different equation and crack width instead of pipe diameter.

8.6. Headwater Level for Initiation of Concentrated Leak Erosion

The headwater level for initiation of concentrated leak erosion is the same as that described for the Cylindrical Pipe worksheet in section 5.6, but using a different equation and crack width instead of pipe diameter.

8.7. Critical Crack Width for Initiation of Concentrated Leak Erosion

The critical crack width at the top of the core for initiation of concentrated leak erosion is the same as that described for the Vertical Rectangular Crack worksheet in section 7.7, but using a different equation. Equation 11 calculates the critical crack width at the top of the core for initiation of concentrated leak erosion for each headwater level and the specified crack depth from the top of the core.

$$W_{cr} = \frac{\tau_c (H_1 + H_2)}{\sqrt{\frac{\gamma_W^2 \left(H_1^3 - H_2^3\right)^2}{36 D^2 L^2} - \frac{\tau_c{}^2 (H_1 + H_2)^2}{4 D^2}}} \tag{11}$$

9. Probability Tables

This worksheet estimates the probability of initiation of concentrated leak erosion using the probability tables from by Fell et al. (2008). See section S5.4.2.4 of Fell et al. (2008) for the assumptions of the Monte Carlo analyses used to develop the probability tables. For this simplified approach, tailwater is assumed to be below the base of the crack, gap, or pipe, and horizontal cracks cannot be evaluated.

9.1. Flaw and Embankment Core Characterization

In step 1, use the drop-down lists to select the flaw type and soil properties in the embankment core as shown in Figure 36. The flaw types that can be evaluated are cylindrical pipe, vertical rectangular crack, and vertical triangular crack. The soil properties that best describe the embankment core are based on Fell et al. (2008).

Step 1: Characterize the flaw and soil properties of the embankment core	
Flaw (continuous transverse crack, gap, or pipe)	Cylindrical pipe
Soil properties of the embankment core	CL

Figure 36. Step 1 of Probability Tables worksheet: Flaw type and embankment core soil properties input.

9.2. Core Geometry and Initial Pipe or Crack Dimensions

In step 2, input pipe diameters or crack widths to evaluate, width at top of core, top of core elevation, and slope data for the embankment core. Input (in ascending order) up to six different pipe diameters or crack widths to evaluate. When evaluating a vertical rectangular crack or a vertical triangular crack, the crack width refers to the maximum width at the top of the core. Slopes are defined by elevation, starting at the top of the core and ending at the base of the embankment. Step 3 uses the slopes of the embankment core to calculate the pipe or crack length over which the hydraulic head difference occurs. Therefore, the lowest core elevation (minimum z_{bot}) must be less than or equal to the base of pipe or crack elevation to compute the length. A caution displays with an orange background if the minimum user-specified z_{bot} for the upstream or downstream slope of the core is greater than base of pipe elevation.

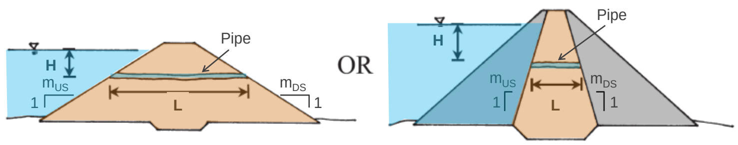
Input the base of pipe elevation if the selected flaw is cylindrical pipe. The length of base of pipe is calculated using the core geometry and the base of pipe elevation. If the selected flaw is a vertical rectangular crack or a vertical triangular crack, this input and calculation have a gray background. Figure 37 illustrates the geometric input for a cylindrical pipe.

Step 2: Assess the core geometry and initial pipe dimensions

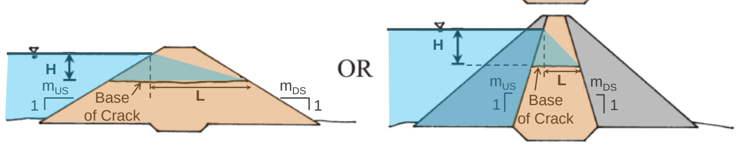
Pipe Diameter, D (mm)	1	2	5	10	25	50

Width at top of core	30.0	ft
Top of core elevation, C	239.0	ft-NGVD29
Base of pipe elevation, B	194.0	ft-NGVD29
Length of base of pipe, L	316.5	ft
Crack depth from top of core, D	18.0	ft
Base of crack elevation, B = C − D	#N/A	ft-NGVD29
Total length at base of crack, L_B	#N/A	ft

Definition of Hydraulic Gradient across the Core at Level of a Pipe



Definition of Hydraulic Gradient across the Core for a Vertical Crack



Upstream Slope of Core		
z_{top} (ft)	z_{bot} (ft)	m_{US} = H/V
239.0	228.5	2.00
228.5	221.0	2.50
221.0	194.0	3.00
-		

Downstream Slope of Core at Exit		
z_{top} (ft)	z_{bot} (ft)	m_{DS} = H/V
239.0	228.5	3.00
228.5	221.0	3.50
221.0	194.0	4.00
-		

Note: z_{bot} must extend below lowest base of pipe/crack elevation of 194.0 ft-NGVD29

Figure 37. Step 2 of Probability Tables worksheet: Cylindrical pipe input.

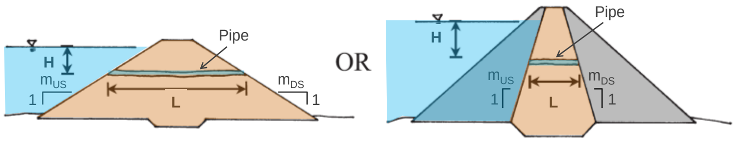
Input the crack depth from the top of the core if the selected flaw is a vertical rectangular crack or a vertical triangular crack. The base of crack elevation is calculated by subtracting the crack depth from the top of core elevation. The total length at the base of the crack is calculated using the core geometry and the base of crack elevation. If the selected flaw is a cylindrical pipe, this input and calculations have a gray background. Figure 38 illustrates the geometric input for a vertical rectangular crack or a vertical triangular crack.

Step 2: Assess the core geometry and initial crack dimensions

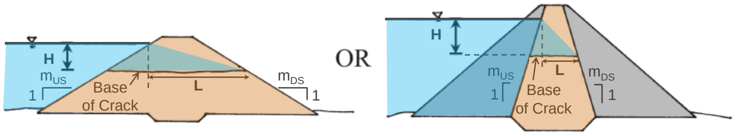
Crack Width, W (mm)	1	2	5	10	25	50

Width at top of core	30.0	ft
Top of core elevation, C	239.0	ft-NGVD29
Base of pipe elevation, B	194.0	ft-NGVD29
Length of base of pipe, L	127.5	ft
Crack depth from top of core, D	18.0	ft
Base of crack elevation, B = C − D	221.0	ft-NGVD29
Total length at base of crack, L_B	127.5	ft

Definition of Hydraulic Gradient across the Core at Level of a Pipe



Definition of Hydraulic Gradient across the Core for a Vertical Crack



Upstream Slope of Core		
z_{top} (ft)	z_{bot} (ft)	m_{US} = H/V
239.0	228.5	2.00
228.5	221.0	2.50
221.0	194.0	3.00
-		

Downstream Slope of Core at Exit		
z_{top} (ft)	z_{bot} (ft)	m_{DS} = H/V
239.0	228.5	3.00
228.5	221.0	3.50
221.0	194.0	4.00
-		

Note: z_{bot} must extend below lowest base of pipe/crack elevation of 221.0 ft-NGVD29

Figure 38. Step 2 of Probability Tables worksheet: Vertical crack input.

9.3. Hydraulic Gradient in the Pipe or Crack

Step 3 calculates the average hydraulic gradient in the pipe or crack by dividing the hydraulic head above the base of the pipe or crack by the equivalent length of the pipe or crack. The length used in the calculation of the average hydraulic gradient across the core is the length of crack over which the net hydraulic head occurs and is referred to as the equivalent length.

For a cylindrical pipe, the equivalent length is the full distance along the base of the pipe calculated in step 2. Since it is not a function of the embankment geometry, the calculated length as a function of headwater level displays a gray background. For a homogenous embankment, the pipe length is measured at the base elevation of the pipe as a straight line from the upstream slope of the embankment to the downstream slope. For a zoned embankment where the upstream and downstream zones are very permeable (e.g., rockfill), the pipe length is measured as a straight line at the base elevation of the pipe from the upstream slope of the impervious core to the downstream slope of the impervious core. Override the formula for pipe length with a user-specified length for embankments with sloping cores, pipe orientations that are not perpendicular to the embankment centerline, etc.

Figure 39 illustrates the average hydraulic gradient calculations for a cylindrical pipe.

Step 2: Estimate the hydraulic gradient in the pipe							
Hydraulic gradient across the core, i = H/L							
HW (ft)	204.0	213.5	221.0	228.5	231.0	235.0	239.0
HW_{mid} (ft)	-	-	-	-	-	-	-
H (ft)	10.0	19.5	27.0	34.5	37.0	41.0	45.0
L (ft)	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A
i	0.032	0.062	0.085	0.109	0.117	0.130	0.142

Figure 39. Step 3 of Probability Tables worksheet: Average hydraulic gradient for cylindrical pipe.

For a vertical rectangular crack or a vertical triangular crack, the equivalent length is measured from the downstream face of the core at the base of the crack to a projection of the point where the headwater level intersects the upstream face of the core. Figure 35 illustrates both hydraulic conditions for homogeneous embankments and zoned embankments where the upstream and downstream zones are very permeable (e.g., rockfill). Override the formula for equivalent length with a user-specified length for embankments with sloping cores, crack orientations that are not perpendicular to the embankment centerline, etc. The headwater level at the midpoint of the flow path under consideration (HW_{mid}), which occurs at H/2, is also calculated for a vertical rectangular crack and a vertical triangular crack. HW_{mid} displays a gray background if the selected flaw is a cylindrical pipe. Figure 40 illustrates the average hydraulic gradient calculations for a vertical rectangular crack or a vertical triangular crack.

Step 2: Estimate the hydraulic gradient in the crack							
Hydraulic gradient across the core, i = H/L							
HW (ft)	204.0	213.5	221.0	228.5	231.0	235.0	239.0
HW_{mid} (ft)	-	-	221.0	224.8	226.0	228.0	230.0
H (ft)	0.0	0.0	0.0	7.5	10.0	14.0	18.0
L (ft)	127.5	127.5	127.5	108.8	103.8	95.8	87.8
i	0.000	0.000	0.000	0.069	0.096	0.146	0.205

Figure 40. Step 3 of Probability Tables worksheet: Average hydraulic gradient for vertical cracks.

9.4. Likelihood of Initiation of Concentrated Leak Erosion

Step 4 calculates the crack width at HW_{mid} (W_{mid}) for a vertical rectangular crack or a vertical triangular crack, as shown in Figure 41. W_{mid} for a vertical rectangular crack is the same as the crack width at the top of the core. This table is not used for cylindrical pipes, and if the selected flaw is a cylindrical pipe, a gray background displays.

Step 4: Assess the likelihood of initiation of concentrated leak erosion							
	Crack Width at HW_{mid}, W_{mid} (mm)						
W (mm)	Headwater Level (ft-NGVD29)						
	204.0	213.5	221.0	228.5	231.0	235.0	239.0
1	-	-	0.0	0.2	0.3	0.4	0.5
2	-	-	0.0	0.4	0.6	0.8	1.0
5	-	-	0.0	1.0	1.4	1.9	2.5
10	-	-	0.0	2.1	2.8	3.9	5.0
25	-	-	0.0	5.2	6.9	9.7	12.5
50	-	-	0.0	10.4	13.9	19.4	25.0

Figure 41. Step 4 of Probability Tables worksheet:
Crack width at HW_{mid} for a vertical triangular crack.

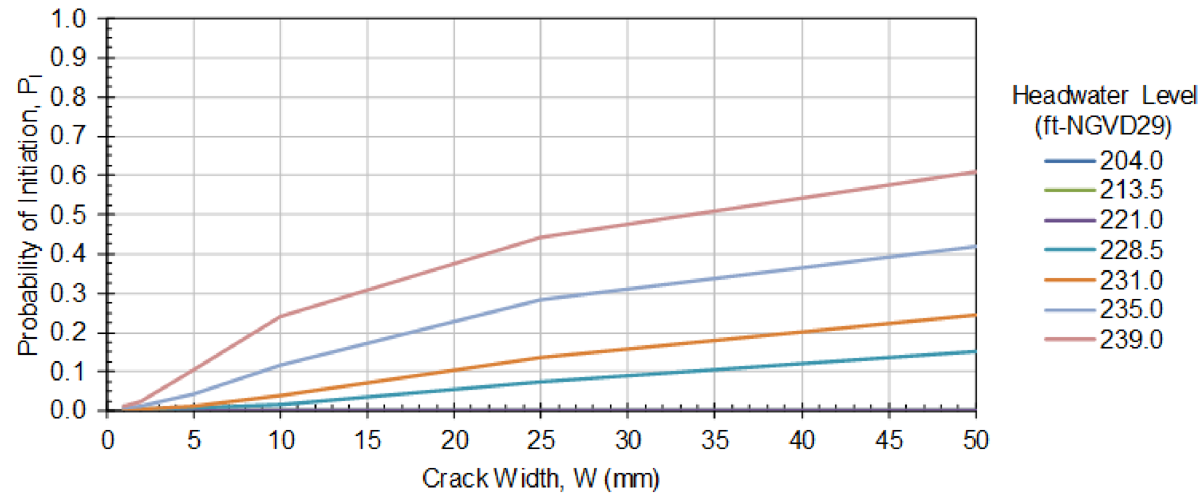
The second table in step 4 is from Fell et al. (2008) and is based on the user-specified soil properties of the embankment core in step 1. It provides the best estimate of the probability of initiation of concentrated leak erosion as a function of hydraulic gradient and pipe diameter or crack width. For vertical triangular cracks, the crack width in this table is W_{mid}. Figure 42 illustrates an example of one of these tables for an embankment core characterized using the Unified Soil Classification (USCS) group symbol of CL (lean clay).

Best Estimate of Probability of Initiation of CLE for CL or MH						
W (mm)	Hydraulic Gradient, i					
	0.1	0.25	0.5	1.0	2.0	5.0
0	0	0	0	0	0	0
1	0.01	0.03	0.1	0.2	0.3	0.7
2	0.02	0.1	0.2	0.5	0.6	0.9
5	0.1	0.3	0.5	0.7	0.9	1.0
10	0.2	0.5	0.7	0.95	1.0	1.0
25	0.4	0.7	0.95	1.0	1.0	1.0
50	0.7	1.0	1.0	1.0	1.0	1.0
75	0.9	1.0	1.0	1.0	1.0	1.0
100	0.95	1.0	1.0	1.0	1.0	1.0

Figure 42. Step 4 of Probability Tables worksheet:
Best estimate probabilities from Fell et al. (2008).

The third table in step 4 calculates the best estimate of the probability of initiation of concentrated leak erosion as a function of the user-specified pipe diameter of crack width at the top of the core and headwater level, by performing two-way linear interpolation using the second table. The results are plotted as a function of pipe diameter or crack width at the top of the core and headwater level, as in Figure 43.

Best Estimate of Probability of Initiation of Scour for CL or MH							
W	Headwater Level (ft-NGVD29)						
(mm)	204.0	213.5	221.0	228.5	231.0	235.0	239.0
1	0.000	0.000	0.000	0.002	0.003	0.006	0.012
2	0.000	0.000	0.000	0.003	0.005	0.013	0.024
5	0.000	0.000	0.000	0.008	0.013	0.043	0.103
10	0.000	0.000	0.000	0.016	0.040	0.118	0.240
25	0.000	0.000	0.000	0.076	0.135	0.285	0.444
50	0.000	0.000	0.000	0.150	0.245	0.418	0.610



Note: Probabilities obtained from this method should not be used directly in risk analyses. Rather, the values should be used to help develop a list of more and less likely factors.

Figure 43. Step 4 of Probability Tables worksheet:
Tabular and graphical output of best estimate probabilities.

Figure 44 illustrates the plot options for Figure 43. The maximum value for the y-axis (probability of initiation of concentrated leak erosion) and maximum value for the x-axis (pipe diameter or maximum crack width at the top of the core) are user-specified.

Worksheet	Probability Tables		
y-axis bounds			
minimum	0		Value Primary Min: 0
maximum	1.00	◄ Enter maximum probability.	Value Primary Max: 1
x-axis bounds			
minimum	0		Category Primary Min: 0
maximum	50	◄ Enter maximum crack width.	Category Primary Max: 50

Figure 44. Step 4 of Probability Tables worksheet: Plot options for best estimate probability.

9.5. Likelihood of Initiation of Concentrated Leak Erosion Considering Uncertainty

Fell et al. (2008) also developed lower bound and upper bound probability estimates to quantify the uncertainty. The first and second tables in step 5 provide probabilities of initiation of concentrated leak erosion as a function of hydraulic gradient and pipe diameter or crack width for the lower bound estimate and upper bound estimate, respectively. These tables are also based on the soil properties of the embankment core in step 1. The third and fourth tables calculate the lower bound and upper bound estimates of the probabilities of initiation of concentrated leak erosion, as a function of the user-specified pipe diameter or crack width at the top of the core and headwater level, by performing two-way linear interpolation using the first and second tables, respectively. These four tables are like those shown in step 4.

Use the drop-down list to select the pipe diameter or crack width at the top of the core for plotting the uncertainty bounds. The best, lower bound, and upper bound estimates of probabilities of initiation of concentrated leak erosion are plotted as a function of headwater level for the user-specified pipe diameter or crack width at the top of the core. Figure 45 illustrates the graphical summary of the probability estimates.

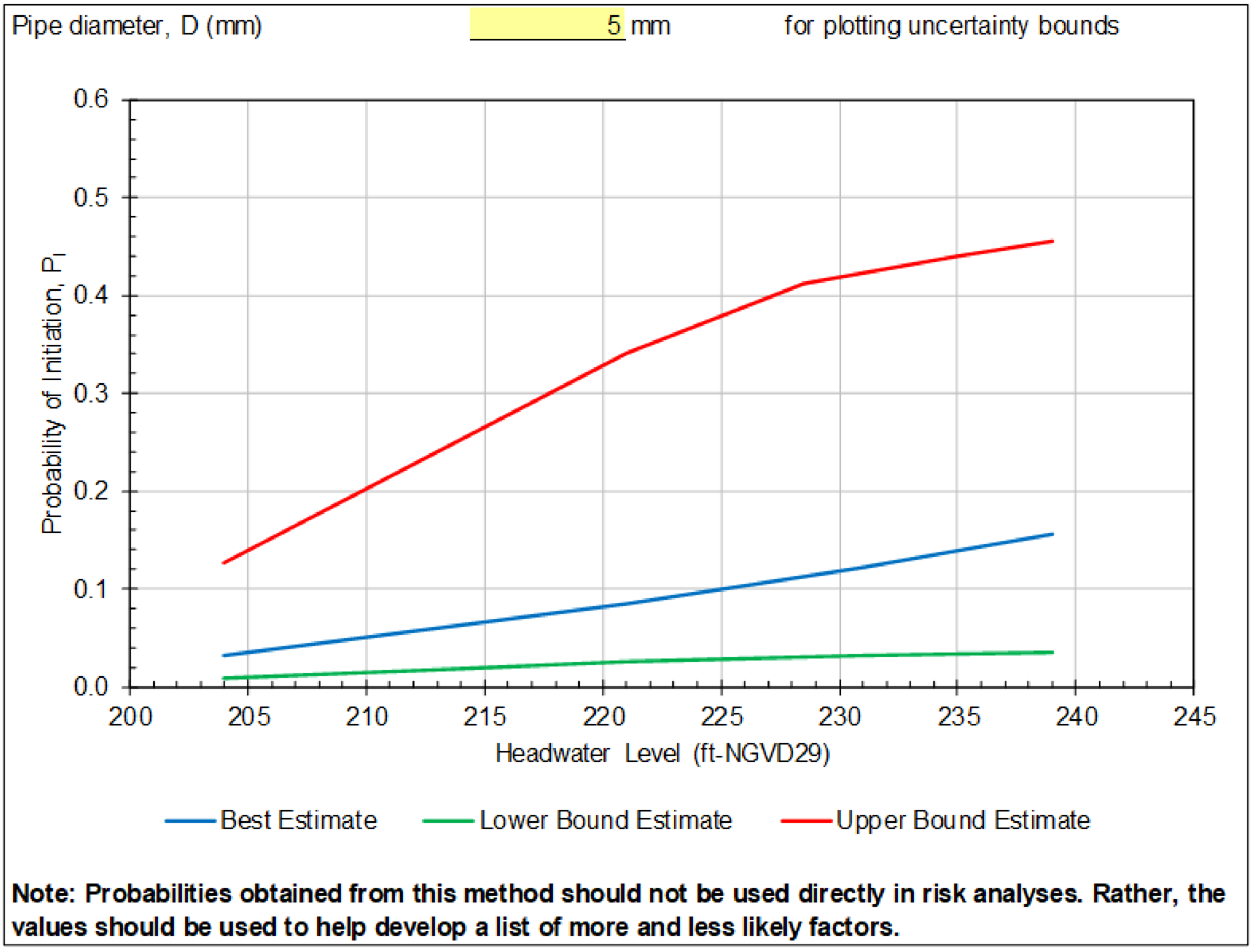


Figure 45. Step 5 of Probability Tables worksheet: Graphical output of probability estimates.

Figure 46 illustrates the plot options for Figure 45. The maximum value for the y-axis (probability of initiation of concentrated leak erosion) and minimum and maximum values for the x-axis (headwater level) are user-specified.

Worksheet	Probability Tables		
y-axis bounds			
minimum	0		Value Primary Min: 0
maximum	0.60	◄ Enter maximum probability.	Value Primary Max: 0.6
x-axis bounds			
minimum	200.0	◄ Enter minimum headwater level.	Category Primary Min: 200
maximum	245.0	◄ Enter maximum headwater level.	Category Primary Max: 245

Figure 46. Step 5 of Probability Tables worksheet: Plot options for probability estimates.

10. References

Bui, H., Tandjiria, V., Fell, R., Song, C., and Khalili, N. (2005). *Two- and three-dimensional numerical analysis of the potential for cracking of embankment dams-supplementary report* (UNICIV Report No. R-438). School of Civil and Environmental Engineering, University of New South Wales. http://vm.civeng.unsw.edu.au/uniciv/R-438.pdf.

Fell, R., MacGregor, P., Stapledon, D., Bell, G., and Foster, M. (2015). *Geotechnical engineering of dams* (2nd ed.). CRC Press/Balkema. https://doi.org/10.1201/b17800.

Fell, R., Foster, M.A., Cyganiewicz, J., Sills, G.L., Vroman, N.D., Davidson, R.R. (2008). *Risk analysis for dam safety: A unified method for estimating probabilities of failure of embankment dams by internal erosion and piping*. (UNICIV Report No. R-446). http://vm.civeng.unsw.edu.au/uniciv/R-446.pdf.

Foster, M., Fell, R., and Spannagle, M. (2002). A method for assessing the relative likelihood of failure of embankment dams by piping: Reply. *Canadian Geotechnical Journal*, *39*(2), 497–500. https://doi.org/10.1139/t01-109.

Wan, C.F. (2006). *Experimental investigations of piping erosion and suffusion of soils in embankment dams and their foundations*. [Doctoral thesis, School of Civil and Environmental Engineering, University of New South Wales]. UNSW Library. https://doi.org/10.26190/unsworks/4828.

Appendix A. Derivation of Hydraulic Shear Stress Equations

Each of the following four derivations of hydraulic shear stress on the surface of a continuous flaw (crack, gap, or pipe) from flow of water in the flaw is based on these simplifying assumptions:

- Cross-section of the flaw is uniform from upstream to downstream (waterside to landside).
- Steady uniform flow occurs through the flaw.
- Head loss is linear from upstream to downstream (waterside to landside).
- Frictional resistance is uniform along the surface of the flaw.
- Frictional resistance is equal to the driving force.

A.1. Hydraulic Shear Stress on the Surface of a Cylindrical Pipe

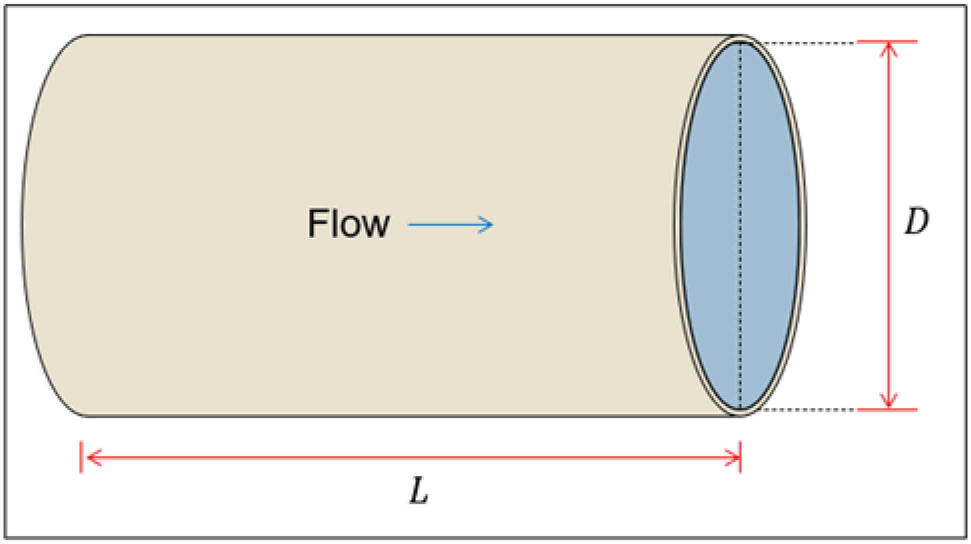


Figure A-1. Flow through a cylindrical pipe.

Equation A-1 approximates the average hydraulic shear stress acting on the pipe by assuming static equilibrium.

$$\sum F = \gamma_w H_1 \frac{\pi D^2}{4} - \tau(\pi D)L - \gamma_w H_2 \frac{\pi D^2}{4} = 0 \tag{A-1}$$

where:

F = force

D = diameter of the pipe
L = length of the pipe
H_1 = hydraulic head at upstream end of pipe (headwater elevation minus base of pipe elevation)
H_2 = hydraulic head at downstream end of pipe (tailwater elevation minus base of pipe elevation)
γ_w = unit weight of water
τ = average hydraulic shear stress

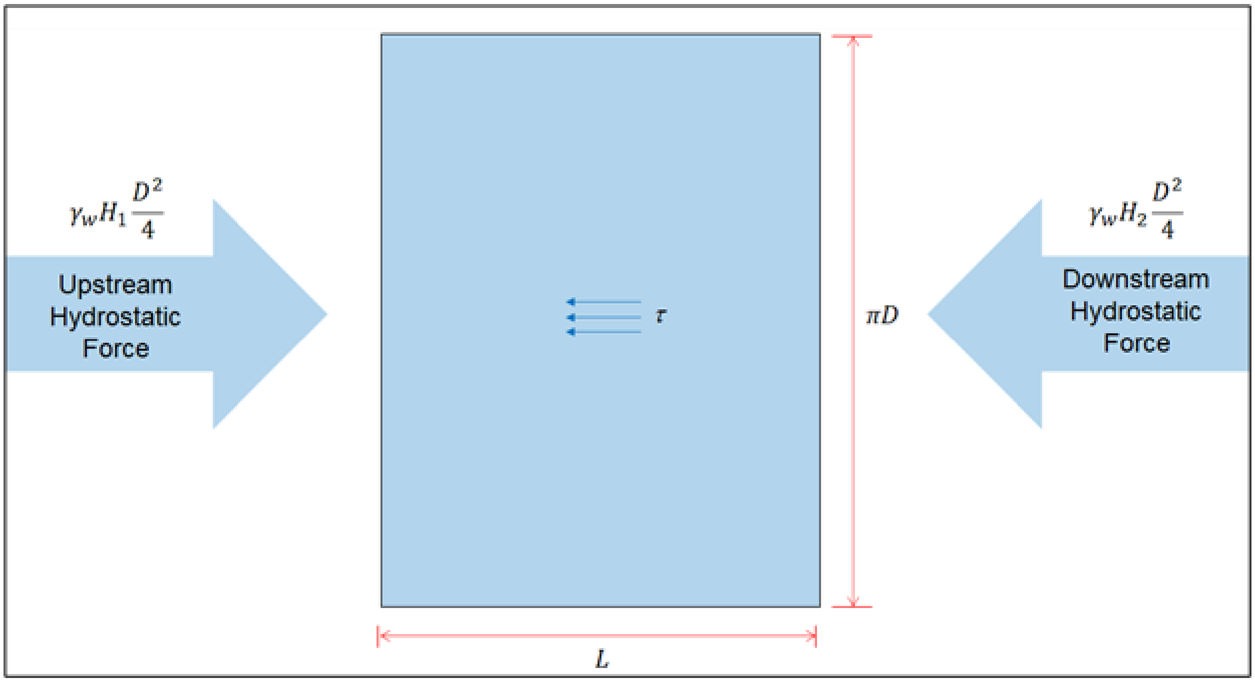


Figure A-2. Free-body diagram for flow through a cylindrical pipe.

Solving for the average hydraulic shear stress results in Equations A-2 and A-3.

$$\tau(\pi D)L = \gamma_w(H_1 - H_2)\frac{\pi D^2}{4} \tag{A-2}$$

$$\tau = \frac{\gamma_w(H_1 - H_2)D}{4L} \tag{A-3}$$

Equation A-4 calculates the average hydraulic gradient across the flow path.

$$i = \frac{H_1 - H_2}{L} \tag{A-4}$$

Therefore, Equation A-3 can be rewritten as Equation A-5.

$$\tau = \frac{\gamma_w i D}{4} \tag{A-5}$$

Alternatively, the average hydraulic shear stress acting on the walls of the pipe can be approximated by a more general formula for the hydraulic shear stress along a pipe as shown in Equation A-6.

$$\tau = \gamma_w i \frac{A}{P_w} \tag{A-6}$$

where:

A = average cross-sectional area of the flow

$$A = \frac{\pi D^2}{4} \tag{A-7}$$

P_w = average wetted perimeter

$$P_w = \pi D \tag{A-8}$$

Substituting Equations A-4, A-7, and A-8 into Equation A-6 results in Equations A-9, A-10, and A-11.

$$\tau = \gamma_w \left(\frac{H_1 - H_2}{L}\right)\left(\frac{\left(\frac{\pi D^2}{4}\right)}{\pi D}\right) \tag{A-9}$$

$$\tau = \frac{\gamma_w (H_1 - H_2) D}{4L} \tag{A-10}$$

$$\tau = \frac{\gamma_w i D}{4} \tag{A-11}$$

Solving Equation A-10 for the pipe diameter and setting $\tau = \tau_c$ (critical shear stress) results in the critical pipe diameter for initiation of concentrated leak erosion (D_{cr}) as shown in Equations A-12, A-13, and A-14.

$$\gamma_w (H_1 - H_2) D = 4L\tau_c \tag{A-12}$$

$$D_{cr} = \frac{4L\tau_c}{\gamma_w (H_1 - H_2)} \tag{A-13}$$

$$D_{cr} = \frac{4L\tau_c}{\gamma_w (H_1 - H_2)} \tag{A-14}$$

A.2. Hydraulic Shear Stress on the Surface of a Horizontal Crack

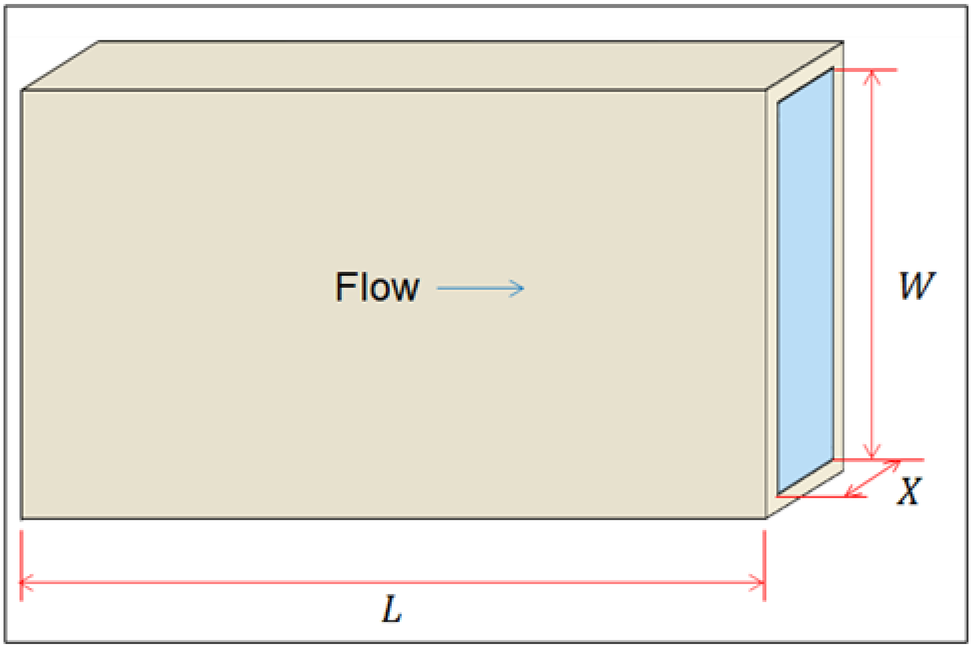


Figure A-3. Flow through a horizontal crack.

Equation A-15 approximates the average hydraulic shear stress acting on the crack by assuming static equilibrium.

$$\sum F = \gamma_w H_1 WX - \tau(2WL + 2XL) - \gamma_w H_2 WX = 0 \tag{A-15}$$

where:

F = force
W = vertical crack dimension (crack width)
X = horizontal crack dimension (crack width parallel to the embankment centerline)
L = length of the crack
H_1 = hydraulic head at upstream end of crack (headwater elevation minus base of crack elevation)
H_2 = hydraulic head at downstream end of crack (tailwater elevation minus base of crack elevation)
γ_w = unit weight of water
τ = average hydraulic shear stress

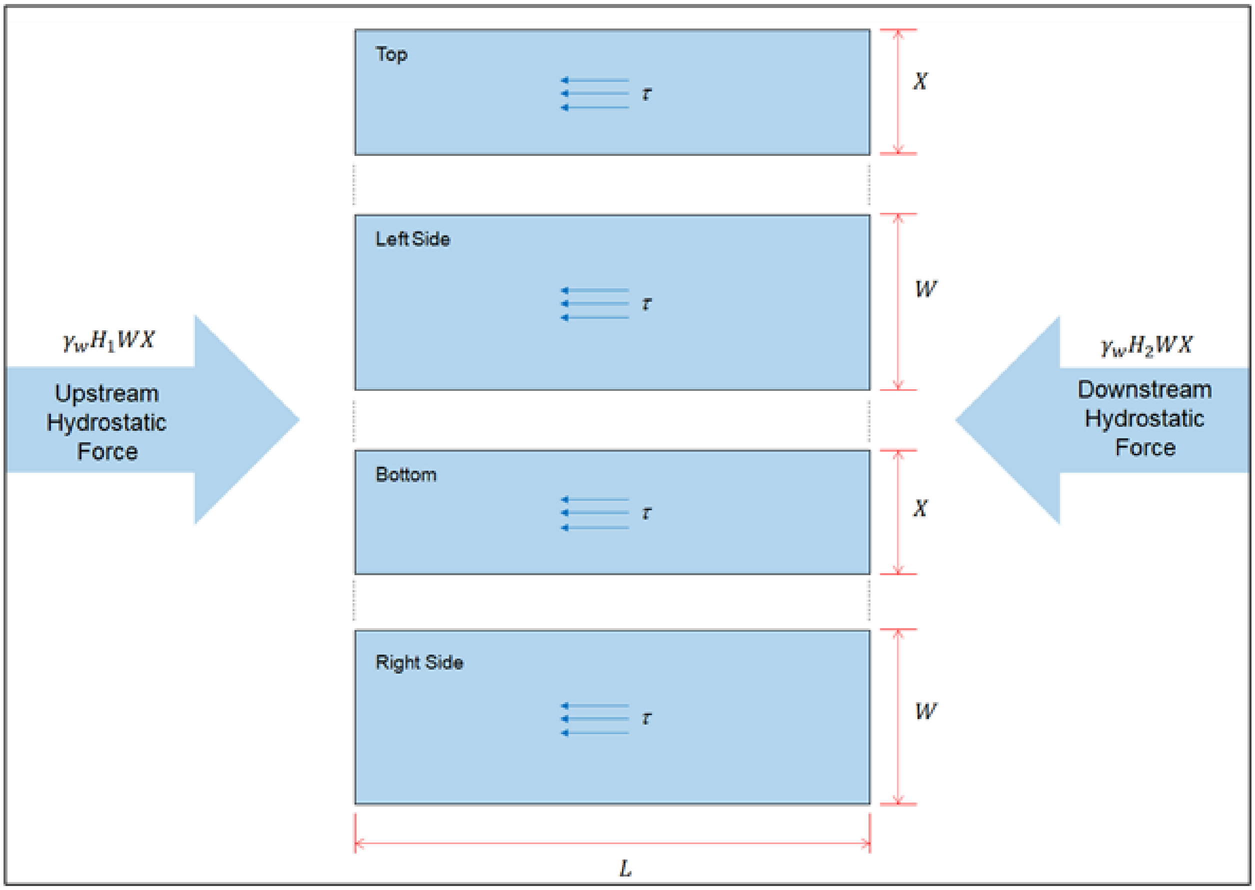


Figure A-4. Free-body diagram for flow through a horizontal crack.

Solving for the average hydraulic shear stress results in Equations A-16 and A-17.

$$\tau(2WL + 2XL) = \gamma_w(H_1 - H_2)WX \quad \text{(A-16)}$$

$$\tau = \frac{\gamma_w(H_1 - H_2)WX}{L(2W + 2X)} \quad \text{(A-17)}$$

Equation A-18 calculates the average hydraulic gradient across the flow path.

$$i = \frac{H_1 - H_2}{L} \quad \text{(A-18)}$$

Therefore, Equation A-17 can be rewritten as Equation A-19.

$$\tau = \frac{\gamma_w iWX}{2W + 2X} \quad \text{(A-19)}$$

Alternatively, the average hydraulic shear stress acting on the walls of the crack can be approximated by a more general formula for the hydraulic shear stress along a crack as shown in Equation A-20.

$$\tau = \gamma_w i \frac{A}{P_w} \quad \text{(A-20)}$$

where:

A = average cross-sectional area of the flow

$$A = WX \tag{A-21}$$

P_w = average wetted perimeter

$$P_w = 2W + 2X \tag{A-22}$$

Substituting Equations A-18, A-21, and A-22 into Equation A-20 results in Equations A-23, A-24, and A-25.

$$\tau = \gamma_w \left(\frac{H_1 - H_2}{L}\right)\left(\frac{WX}{2W + 2X}\right) \tag{A-23}$$

$$\tau = \frac{\gamma_w (H_1 - H_2) WX}{L(2W + 2X)} \tag{A-24}$$

$$\tau = \frac{\gamma_w i WX}{2W + 2X} \tag{A-25}$$

Solving Equation A-24 for the crack width (vertical crack dimension) and setting $\tau = \tau_c$ (critical shear stress) results in the critical crack width for initiation of concentrated leak erosion (W_{cr}) as shown in Equations A-26 to A-31.

$$L\tau_c(2W + 2X) = \gamma_w (H_1 - H_2) WX \tag{A-26}$$

$$L\tau_c(2W + 2X) - \gamma_w (H_1 - H_2) WX = 0 \tag{A-27}$$

$$2L\tau_c W + 2L\tau_c X - \gamma_w (H_1 - H_2) WX = 0 \tag{A-28}$$

$$W[2L\tau_c - \gamma_w (H_1 - H_2) X] = -2L\tau_c X \tag{A-29}$$

$$W_{cr} = \frac{-2L\tau_c X}{2L\tau_c - \gamma_w (H_1 - H_2) X} \tag{A-30}$$

$$W_{cr} = \frac{-2\tau_c X}{2L\tau_c - \gamma_w i X} \tag{A-31}$$

Because $W \ll X$, the average hydraulic shear stress in Equation A-25 can be approximated as shown in Equations A-32 and A-33.

$$\tau = \frac{\gamma_w i WX}{2(0) + 2X} \tag{A-32}$$

$$\tau \approx \frac{\gamma_w i W}{2} \tag{A-33}$$

Solving Equation A-33 for the crack width and setting $\tau = \tau_c$ (critical shear stress) results in the critical crack width for initiation of concentrated leak erosion as shown in Equation A-34.

$$W_{cr} \approx \frac{2\tau_c}{\gamma_w i} \tag{A-34}$$

A.3. Hydraulic Shear Stress on the Surface of a Vertical Rectangular Crack

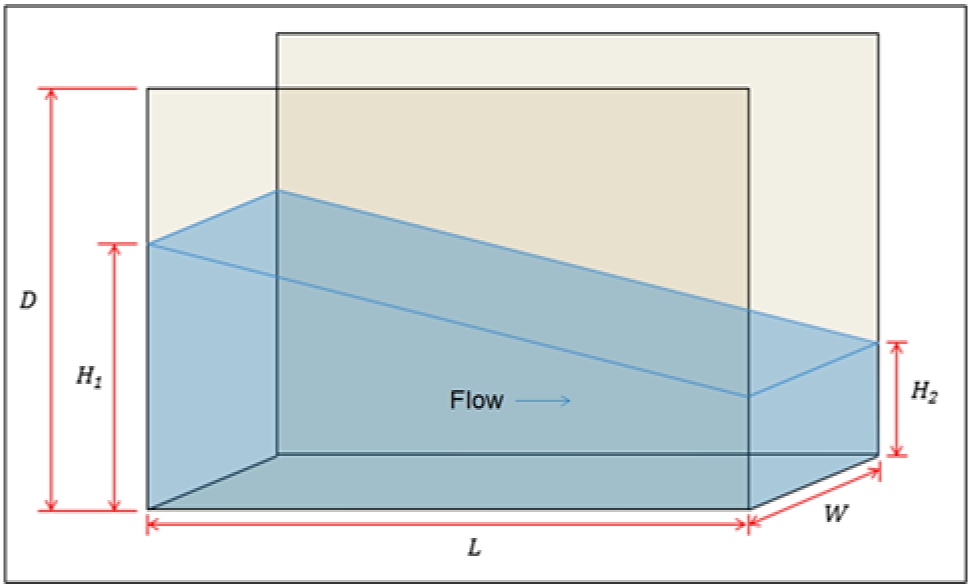


Figure A-5. Flow through a vertical rectangular crack.

Equation A-35 approximates the average hydraulic shear stress acting on the crack by assuming static equilibrium.

$$\sum F = \frac{1}{2}\gamma_w H_1^2 W - \tau\left(2L\left(\frac{H_1 + H_2}{2}\right) + WL\right) - \frac{1}{2}\gamma_w H_2^2 W = 0 \tag{A-35}$$

where:

F = force
D = depth of crack from top of core
H_1 = hydraulic head at upstream end of crack (headwater elevation minus base of crack elevation)
H_2 = hydraulic head at downstream end of crack (tailwater elevation minus base of crack elevation)
L = equivalent length of crack for net hydraulic head
W = width of crack at top of core
γ_w = unit weight of water
τ = average hydraulic shear stress

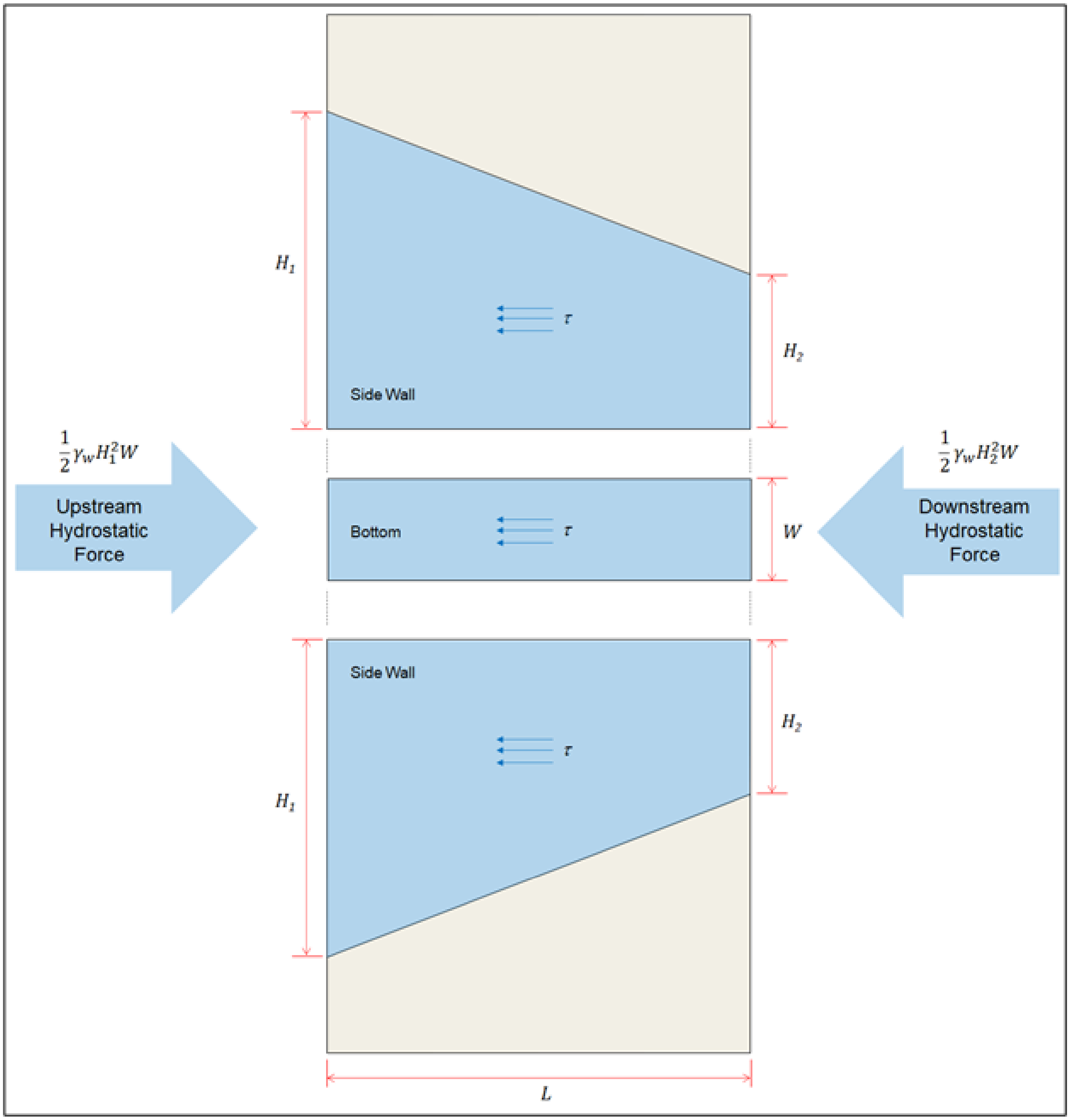


Figure A-6. Free-body diagram for flow through a vertical rectangular crack.

Solving for the average hydraulic shear stress results in Equations A-36 and A-37.

$$\tau L(H_1 + H_2 + W) = \frac{1}{2}\gamma_w W(H_1^2 - H_2^2) \quad \text{(A-36)}$$

$$\tau = \frac{\gamma_w W(H_1^2 - H_2^2)}{2L(H_1 + H_2 + W)} \quad \text{(A-37)}$$

Alternatively, the average hydraulic shear stress acting on the walls of the crack can be approximated by a more general formula for the hydraulic shear stress along a crack as shown in Equation A-38.

$$\tau = \gamma_w i \frac{A}{P_w} \tag{A-38}$$

where:

i = average hydraulic gradient across the flow path

$$i = \frac{H_1 - H_2}{L} \tag{A-39}$$

A = average cross-sectional area of the flow

$$A = \frac{1}{2}(WH_1 + WH_2) \tag{A-40}$$

$$A = \frac{W}{2}(H_1 + H_2) \tag{A-41}$$

P_w = average wetted perimeter

$$P_w = \frac{1}{2}((2H_1 + W) + (2H_2 + W)) \tag{A-42}$$

$$P_w = H_1 + H_2 + W \tag{A-43}$$

Substituting Equations A-39, A-41, and A-43 into Equation A-38 results in Equations A-44 and A-45.

$$\tau = \gamma_w \left(\frac{H_1 - H_2}{L}\right) \frac{\frac{W}{2}(H_1 + H_2)}{H_1 + H_2 + W} \tag{A-44}$$

$$\tau = \frac{\gamma_w W(H_1^2 - H_2^2)}{2L(H_1 + H_2 + W)} \tag{A-45}$$

Solving Equation A-45 for the crack width at the top of the core and setting $\tau = \tau_c$ (critical shear stress) results in the critical crack width for initiation of concentrated leak erosion (W_{cr}) as shown in Equations A-46 to A-50.

$$2L\tau_c(H_1 + H_2 + W) = \gamma_w W(H_1^2 - H_2^2) \tag{A-46}$$

$$2L\tau_c(H_1 + H_2) + 2L\tau_c W = \gamma_w W(H_1^2 - H_2^2) \tag{A-47}$$

$$\gamma_w W(H_1^2 - H_2^2) - 2L\tau_c W = 2L\tau_c(H_1 + H_2) \tag{A-48}$$

$$W(\gamma_w(H_1^2 - H_2^2) - 2L\tau_c) = 2L\tau_c(H_1 + H_2) \tag{A-49}$$

$$W_{cr} = \frac{2L\tau_c(H_1 + H_2)}{\gamma_w(H_1^2 - H_2^2) - 2L\tau_c} \tag{A-50}$$

Because H_2 is often zero (i.e., no tailwater) and $W << H_1$, the average hydraulic shear stress in Equation A-45 can be approximated as shown in Equations A-51, A-52, and A-53.

$$\tau \approx \frac{\gamma_w W(H_1^2 - 0)}{2L(H_1 + 0 + 0)} \tag{A-51}$$

$$\tau \approx \frac{\gamma_w W H_1}{2L} \tag{A-52}$$

$$\tau \approx \frac{\gamma_w i W}{2} \tag{A-53}$$

Solving Equation A-53 for the crack width and setting $\tau = \tau_c$ (critical shear stress) results in the critical crack width for initiation of concentrated leak erosion as shown in Equation A-54.

$$W_{cr} \approx \frac{2\tau_c}{\gamma_w i} \tag{A-54}$$

A.4. Hydraulic Shear Stress on the Surface of a Vertical Triangular Crack

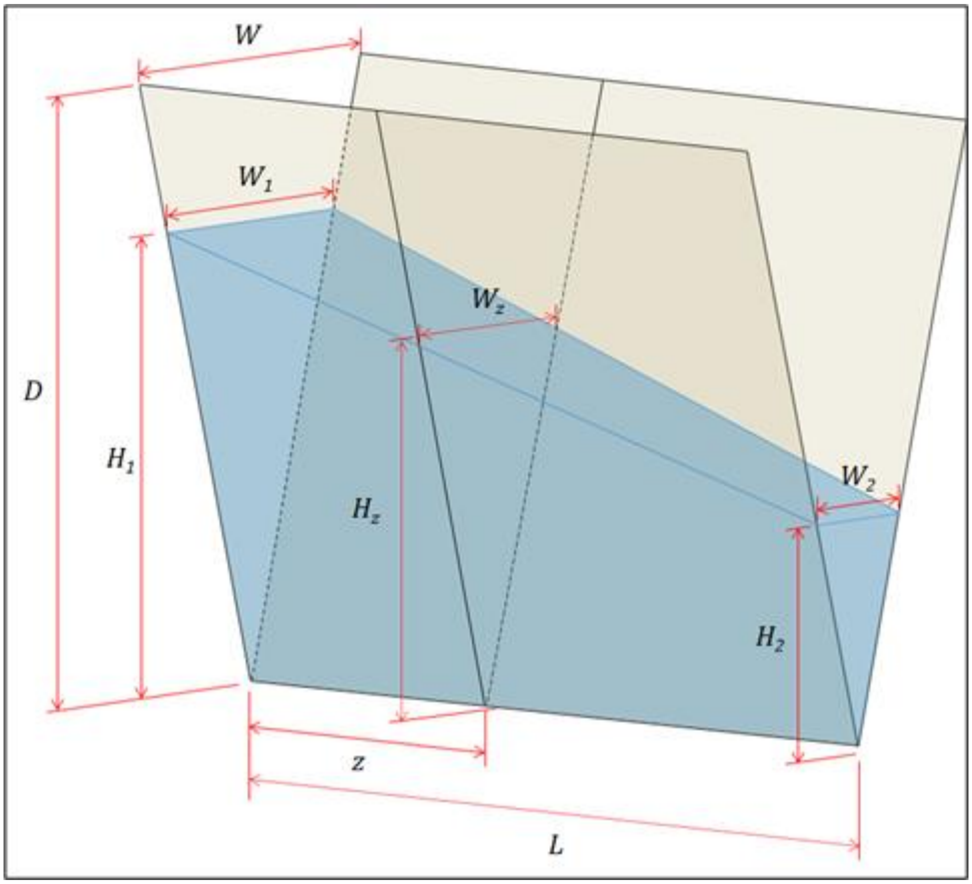


Figure A-7. Flow through a vertical triangular crack.

Equation A-55 approximates the average hydraulic shear stress acting on the crack by assuming static equilibrium.

$$\sum F = \frac{1}{6}\gamma_w H_1^2 W_1 - \tau L\left(\sqrt{H_1^2 + \left(\frac{W_1}{2}\right)^2} + \sqrt{H_2^2 + \left(\frac{W_2}{2}\right)^2}\right) - \frac{1}{6}\gamma_w H_2^2 W_2 = 0 \tag{A-55}$$

where:

F = force
D = depth of crack from top of core
H_1 = hydraulic head at upstream end of crack (headwater elevation minus base of crack elevation)
H_2 = hydraulic head at downstream end of crack (tailwater elevation minus base of crack elevation)
H_z = height of water in crack at a distance z from upstream end of crack
L = equivalent length of crack for net hydraulic head
W = width of crack at top of core
W_1 = width of crack at upstream end of crack
W_2 = width of crack at downstream end of crack
W_z = width of crack at a distance z from upstream end of crack
Z = distance measured along base of crack from upstream end of crack
γ_w = unit weight of water
τ = average hydraulic shear stress

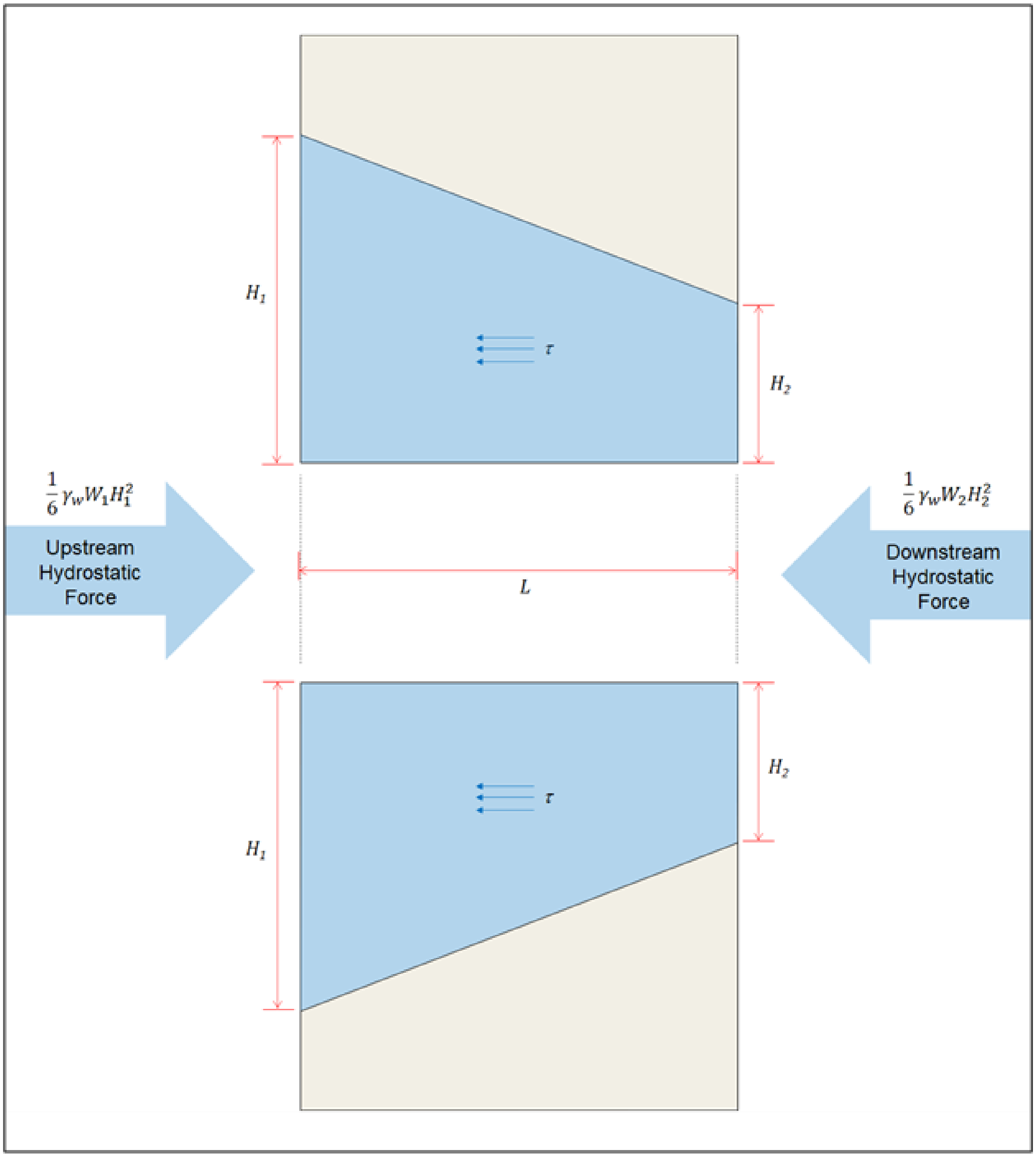


Figure A-8. Free-body diagram for flow through a vertical triangular crack.

Solving for the average hydraulic shear stress results in Equations A-56 and A-57.

$$\tau L\left(\sqrt{H_1^2+\left(\frac{W_1}{2}\right)^2}+\sqrt{H_2^2+\left(\frac{W_2}{2}\right)^2}\right)=\frac{1}{6}\gamma_w\left(H_1^2W_1-H_2^2W_2\right) \tag{A-56}$$

$$\tau=\frac{\gamma_w}{6L}\frac{\left(H_1^2W_1-H_2^2W_2\right)}{\sqrt{H_1^2+\left(\frac{W_1}{2}\right)^2}+\sqrt{H_2^2+\left(\frac{W_2}{2}\right)^2}} \tag{A-57}$$

Solving for crack width at H_1 and H_2 using similar triangles results in Equations A-58, A-59, and A-60.

$$\frac{W}{D} = \frac{W_1}{H_1} = \frac{W_2}{H_2} \tag{A-58}$$

$$W_1 = \frac{WH_1}{D} \tag{A-59}$$

$$W_2 = \frac{WH_2}{D} \tag{A-60}$$

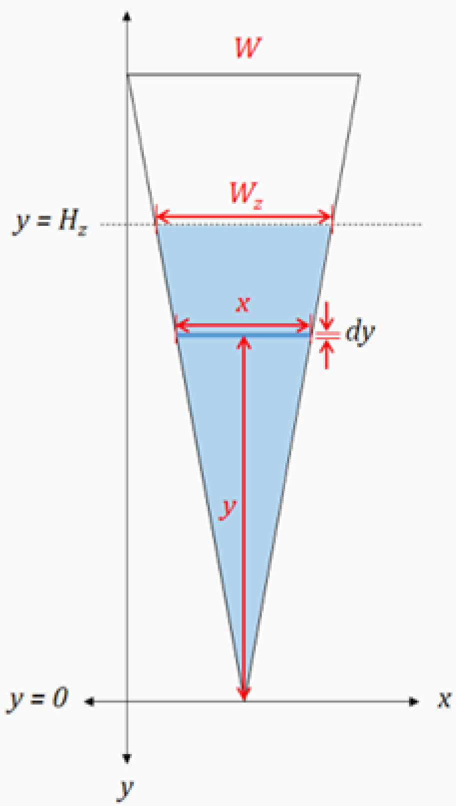
Substituting these crack widths into Equation A-57 results in Equations A-61, A-62, and A-63.

$$\tau = \frac{\gamma_w}{6L} \frac{\left(H_1^2\left(\frac{WH_1}{D}\right) - H_2^2\left(\frac{WH_2}{D}\right)\right)}{\sqrt{H_1^2 + \left(\frac{WH_1}{2D}\right)^2} + \sqrt{H_2^2 + \left(\frac{WH_2}{2D}\right)^2}} \tag{A-61}$$

$$\tau = \frac{\gamma_w W}{6DL} \frac{\left(H_1^3 - H_2^3\right)}{H_1\sqrt{1 + \frac{W^2}{4D^2}} + H_2\sqrt{1 + \frac{W^2}{4D^2}}} \tag{A-62}$$

$$\tau = \frac{\gamma_w W}{6DL} \frac{\left(H_1^3 - H_2^3\right)}{\left(H_1 + H_2\right)\sqrt{1 + \frac{W^2}{4D^2}}} \tag{A-63}$$

The derivation of the equation used to calculate the hydrostatic forces is shown in Figure A-9.



Hydrostatic Force on a Triangular Face

$$F = \int_0^{H_z} \gamma_w (H_z - y) x \, dy$$

$$\frac{x}{y} = \frac{W_z}{H_z} \rightarrow x = \frac{W_z y}{H_z}$$

$$F = \int_0^{H_z} \gamma_w (H_z - y) \left(\frac{W_z y}{H_z}\right) dy$$

$$F = \int_0^{H_z} \gamma_w \left(W_z y - \frac{W_z y^2}{H_z}\right) dy$$

$$F = \gamma_w \left(\frac{W_z y^2}{2} - \frac{W_z y^3}{3H_z}\right)\Bigg|_0^{H_z} = \gamma_w \left(\frac{W_z H_z^2}{2} - \frac{W_z H_z^3}{3H_z}\right)$$

$$F = \frac{1}{6}\gamma_w W_z H_z^2$$

Figure A-9. Hydrostatic force on a triangular face.

Alternatively, the average hydraulic shear stress acting on the walls of the crack can be approximated by a more general formula for the hydraulic shear stress along a crack as shown Equation A-64.

$$\tau = \gamma_w i \frac{A}{P_w} \tag{A-64}$$

where:

i = average hydraulic gradient across the flow path

$$i = \frac{H_1 - H_2}{L} \tag{A-65}$$

A = average cross-sectional area of the flow

$$A = \frac{1}{L}\int_0^L \frac{1}{2} H_z W_z \, dz \tag{A-66}$$

where:

H_z = height of water in crack at a distance z from upstream end of crack
W_z = width of crack at a distance z from upstream end of crack

$$H_z = H_1 - \frac{(H_1 - H_2)z}{L} \tag{A-67}$$

$$W_z = \frac{W H_z}{D} = \left[H_1 - \frac{(H_1 - H_2)z}{L}\right]\left(\frac{W}{D}\right) \tag{A-68}$$

$$A = \frac{1}{L}\int_0^L \frac{1}{2}\left[H_1 - \frac{(H_1 - H_2)z}{L}\right]\left[H_1 - \frac{(H_1 - H_2)z}{L}\right]\left(\frac{W}{D}\right) dz \tag{A-69}$$

$$A = \frac{W}{2DL}\int_0^L \left(H_1 - \frac{H_1 z}{L} + \frac{H_2 z}{L}\right)\left(H_1 - \frac{H_1 z}{L} + \frac{H_2 z}{L}\right) dz \tag{A-70}$$

$$A = \frac{W}{2DL}\int_0^L \left(H_1^2 - \frac{H_1^2 z}{L} + \frac{H_1 H_2 z}{L} - \frac{H_1^2 z}{L} + \frac{H_1^2 z^2}{L^2} - \frac{H_1 H_2 z^2}{L^2} + \frac{H_1 H_2 z}{L} - \frac{H_1 H_2 z^2}{L^2} + \frac{H_2^2 z^2}{L^2}\right) dz \tag{A-71}$$

$$A = \frac{W}{2DL}\left(H_1^2 z - \frac{H_1^2 z^2}{2L} + \frac{H_1 H_2 z^2}{2L} - \frac{H_1^2 z^2}{2L} + \frac{H_1^2 z^3}{3L^2} - \frac{H_1 H_2 z^3}{3L^2} + \frac{H_1 H_2 z^2}{2L} - \frac{H_1 H_2 z^3}{3L^2} + \frac{H_2^2 z^3}{3L^2}\right)\Bigg|_0^L \tag{A-72}$$

$$A = \frac{W}{2DL}\left(H_1^2 L - \frac{H_1^2 L^2}{2L} + \frac{H_1 H_2 L^2}{2L} - \frac{H_1^2 L^2}{2L} + \frac{H_1^2 L^3}{3L^2} - \frac{H_1 H_2 L^3}{3L^2} + \frac{H_1 H_2 L^2}{2L} - \frac{H_1 H_2 L^3}{3L^2} + \frac{H_2^2 L^3}{3L^2}\right) \tag{A-73}$$

$$A = \frac{W}{12DL}\begin{pmatrix} 6H_1^2 L - 3H_1^2 L + 3H_1 H_2 L - 3H_1^2 L \\ + 2H_1^2 L - 2H_1 H_2 L + 3H_1 H_2 L - 2H_1 H_2 L + 2H_2^2 L \end{pmatrix} \tag{A-74}$$

$$A = \frac{W}{12D}\left(2H_1^2 + 2H_1 H_2 + 2H_2^2\right) \tag{A-75}$$

$$A = \frac{W}{6D}\left(H_1^2 + H_1 H_2 + H_2^2\right) \tag{A-76}$$

P_w = average wetted perimeter

$$P_w = \frac{1}{2}\left(2\sqrt{H_1^2 + \left(\frac{W_1}{2}\right)^2} + 2\sqrt{H_2^2 + \left(\frac{W_2}{2}\right)^2}\right) \tag{A-77}$$

$$P_w = \frac{1}{2}\left(2\sqrt{H_1^2 + \left(\frac{WH_1}{2D}\right)^2} + 2\sqrt{H_2^2 + \left(\frac{WH_2}{2D}\right)^2}\right) \tag{A-78}$$

$$P_w = \sqrt{H_1^2 + \left(\frac{WH_1}{2D}\right)^2} + \sqrt{H_2^2 + \left(\frac{WH_2}{2D}\right)^2} \tag{A-79}$$

$$P_w = H_1\sqrt{1 + \frac{W^2}{4D^2}} + H_2\sqrt{1 + \frac{W^2}{4D^2}} \tag{A-80}$$

$$P_w = (H_1 + H_2)\sqrt{1 + \frac{W^2}{4D^2}} \tag{A-81}$$

Substituting Equations A-65, A-76, and A-81 into Equation A-64 results in Equations A-82, A-83, and A-84.

$$\tau = \gamma_w\left(\frac{H_1 - H_2}{L}\right)\frac{\frac{W}{6D}\left(H_1^2 + H_1H_2 + H_2^2\right)}{(H_1 + H_2)\sqrt{1+\frac{W^2}{4D^2}}} \tag{A-82}$$

$$\tau = \frac{\gamma_w W}{6DL}\frac{\left(H_1^3 + H_1^2H_2 + H_1H_2^2 - H_1^2H_2 - H_1H_2^2 - H_2^3\right)}{(H_1 + H_2)\sqrt{1+\frac{W^2}{4D^2}}} \tag{A-83}$$

$$\tau = \frac{\gamma_w W}{6DL}\frac{\left(H_1^3 - H_2^3\right)}{(H_1 + H_2)\sqrt{1 + \frac{W^2}{4D^2}}} \tag{A-84}$$

Solving Equation A-84 for the crack width and setting $\tau = \tau_c$ (critical shear stress) results in the critical crack width for initiation of concentrated leak erosion (W_{cr}) as shown in Equations A-85 to A-91.

$$\tau_c(H_1 + H_2)\sqrt{1 + \frac{W^2}{4D^2}} = \frac{\gamma_w W\left(H_1^3 - H_2^3\right)}{6DL} \tag{A-85}$$

$$\tau_c{}^2(H_1 + H_2)^2\left(1 + \frac{W^2}{4D^2}\right) = \frac{\gamma_w^2 W^2\left(H_1^3 - H_2^3\right)^2}{36D^2L^2} \tag{A-86}$$

$$\tau_c{}^2(H_1 + H_2)^2 + \frac{\tau^2(H_1 + H_2)^2W^2}{4D^2} = \frac{\gamma_w^2 W^2\left(H_1^3 - H_2^3\right)^2}{36D^2L^2} \tag{A-87}$$

$$\frac{\gamma_w^2 W^2\left(H_1^3 - H_2^3\right)^2}{36D^2L^2} - \frac{\tau_c{}^2(H_1 + H_2)^2W^2}{4D^2} = \tau_c{}^2(H_1 + H_2)^2 \tag{A-88}$$

$$W^2\left(\frac{\gamma_w^2\left(H_1^3 - H_2^3\right)^2}{36D^2L^2} - \frac{\tau_c{}^2(H_1 + H_2)^2}{4D^2}\right) = \tau_c{}^2(H_1 + H_2)^2 \tag{A-89}$$

$$W^2 = \frac{{\tau_c}^2(H_1 + H_2)^2}{\left(\frac{\gamma_w^2 (H_1^3 - H_2^3)^2}{36D^2L^2} - \frac{{\tau_c}^2(H_1+H_2)^2}{4D^2}\right)} \tag{A-90}$$

$$W_{cr} = \frac{\tau_c(H_1 + H_2)}{\sqrt{\frac{\gamma_w^2 (H_1^3 - H_2^3)^2}{36D^2L^2} - \frac{{\tau_c}^2(H_1 + H_2)^2}{4D^2}}} \tag{A-91}$$

Because H_2 is often zero (i.e., no tailwater) and $W<<D$, the average hydraulic shear stress in Equation A-84 can be approximated as shown in Equations A-92, A-93, and A-94.

$$\tau \approx \frac{\gamma_w W}{6DL} \frac{\left(H_1^3 - 0\right)}{(H_1 + 0)\sqrt{1 + 0}} \tag{A-92}$$

$$\tau \approx \frac{\gamma_w W H_1^2}{6DL} \tag{A-93}$$

$$\tau \approx \frac{\gamma_w i W H_1}{6D} \tag{A-94}$$

Solving Equation A-94 for the crack width and setting $\tau = \tau_c$ (critical shear stress) results in the critical crack width for initiation of concentrated leak erosion as shown in Equation A-95.

$$W_{cr} \approx \frac{6D\tau_c}{\gamma_w i H_1} \tag{A-95}$$

Appendix B. Summary of Hydraulic Shear Stress Equations

Table B-1
Hydraulic shear stress in pipe or crack.

Transverse Flaw	Tailwater above Base of Pipe/Crack (H_2>0)	Tailwater below Base of Pipe/Crack (H_2=0)	Approximation for No Tailwater and Small Flaw
Cylindrical Pipe	$\tau = \gamma_w \left(\frac{H_1 - H_2}{L}\right)\frac{D}{4} = \frac{\gamma_w iD}{4}$	$\tau = \gamma_w \left(\frac{H_1 - H_2}{L}\right)\frac{D}{4} = \frac{\gamma_w iD}{4}$	$\tau = \gamma_w \left(\frac{H_1 - H_2}{L}\right)\frac{D}{4} = \frac{\gamma_w iD}{4}$
Horizontal Crack	$\tau = \gamma_w \left(\frac{H_1 - H_2}{L}\right)\frac{WX}{2W + 2X} = \frac{\gamma_w iWX}{(2W + 2X)}$	$\tau = \gamma_w \left(\frac{H_1 - H_2}{L}\right)\frac{WX}{2W + 2X} = \frac{\gamma_w iWX}{(2W + 2X)}$	Since W << X $\tau \approx \frac{\gamma_w iW}{2}$
Vertical Rectangular Crack	$\tau = \frac{\gamma_w W(H_1^2 - H_2^2)}{2L(H_1 + H_2 + W)}$	$\tau = \frac{\gamma_w H_1^2 W}{2(H_1 + W)L}$	Since W << H_1 $\tau \approx \frac{\gamma_w WH_1}{2L} = \frac{\gamma_w iW}{2}$
Vertical Triangular Crack	$\tau = \frac{\gamma_w W}{6DL}\frac{(H_1^3 - H_2^3)}{(H_1 + H_2)\sqrt{1 + \frac{W^2}{4D^2}}}$	$\tau = \frac{\gamma_w W}{6DL}\frac{H_1^2}{\sqrt{1 + \frac{W^2}{4D^2}}}$	Since W << D $\tau \approx \frac{\gamma_w WH_1^2}{6DL} = \frac{\gamma_w iWH_1}{6D}$

Table B-2
Critical pipe diameter or crack width.

Transverse Flaw	Tailwater above Base of Pipe/Crack (H_2>0)	Tailwater below Base of Pipe/Crack (H_2=0)	Approximation for No Tailwater and Small W
Cylindrical Pipe	$D_{cr} = \frac{4\tau_c}{\gamma_w i}$	$D_{cr} = \frac{4\tau_c}{\gamma_w i}$	$D_{cr} = \frac{4\tau_c}{\gamma_w i}$
Horizontal Crack	$W_{cr} = \frac{-2L\tau_c X}{2L\tau_c - \gamma_w (H_1 - H_2) X}$	$W_{cr} = \frac{-2L\tau_c X}{2L\tau_c - \gamma_w H_1 X}$	Since W << X $W_{cr} \approx \frac{2\tau_c}{\gamma_w i}$
Vertical Rectangular Crack	$W_{cr} = \frac{2L\tau_c (H_1 + H_2)}{\gamma_w \left(H_1^2 - H_2^2\right) - 2L\tau_c}$	$W_{cr} = \frac{2L\tau_c H_1}{\gamma_w H_1^2 - 2L\tau_c}$	Since W << H_1 $W_{cr} \approx \frac{2\tau_c}{\gamma_w i}$
Vertical Triangular Crack	$W_{cr} = \frac{\tau_c (H_1 + H_2)}{\sqrt{\frac{\gamma_w^2 \left(H_1^3 - H_2^3\right)^2}{36 D^2 L^2} - \frac{\tau_c^2 (H_1 + H_2)^2}{4D^2}}}$	$W_{cr} = \frac{\tau_c H_1}{\sqrt{\frac{\gamma_w^2 H_1^6}{36 D^2 L^2} - \frac{\tau_c^2 H_1^2}{4D^2}}}$	Since W << D $W_{cr} \approx \frac{6D\tau_c}{\gamma_w i H_1}$

Appendix C. Acronym List

CPD	Computer Program Document
FS	Factor of Safety
HEC	Hydrologic Engineering Center
HW	Headwater
IWR	Institute for Water Resources
NAVD88	North American Vertical Datum of 1988
NGVD29	National Geodetic Vertical Datum of 1929
QC	Quality Control
RMC	Risk Management Center
TW	Tailwater
UDF	User-Defined Function
U.S.	United States
USCS	Unified Soil Classification System
USACE	United States Army Corps of Engineers