7.4 *(1) Obtain the coefficients of an FIR lowpass digital filter to meet the following specifications using the window method:*

stopband attenuation	*50 dB*
passband edge frequency	*3.4 kHz*
transition width	*0.6 kHz*
sampling frequency	*8 kHz*

Include in your answer the type of window used and the reason for your choice.

(2) Assuming that the filter coefficients are stored in contiguous memory locations in a microcomputer, list the values of coefficients in the order in which they are stored.

(3) Draw and briefly describe a flowchart of the direct software implementation of the filter in real time, and suggest two ways of improving the efficiency of the software implementation.

Note: you may use the information given inn Table 7.2 in your design.

Solution

(1) From Table 7.2, the ideal impulse response is given by:

$$h_D(n) = 2f_c \frac{\sin(n\omega_c)}{n\omega_c}, \quad n \neq 0$$

$$h_D(n) = 2f_c, \quad n=0$$

Three window functions may be used to design the filter (Hamming, Blackman or Kaiser). Kaiser window will be used here because it is the most flexible and should lead to fewer coefficients. The stop band and band pass edge frequencies are $f_p = 1.5kHz$ and $f_p = 2kHz$, and the transition width $\Delta f = 0.5kHz$; The cut off frequency, $f_c = 1.5 + \Delta f/2 = 1.75$ kHz.

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An estimate of the filter, $N \geq \frac{A-7.95}{14.36\Delta f} = \frac{50-7.95}{14.36(500/8000)} \approx 47$.

The ripple parameter (from Equation 7.10) is given by $\beta = 0.1102(A-8.7) = 4.55126$. The coefficients of the filter are listed in Table s7.4. The coefficients were obtained with the MATLAB m-file given below.

(2) The indices of the filter coefficients run from – 23 to + 23 when the equation for the ideal impulse response, $h_D(n)$, is used. To make filter causal for implementation we add 23 to the indices so that the indices run from 0 to 46. The filter coefficients with the indices adjusted are listed in Table s7.4.

(3) A simplified flowchart for the filter is depicted in Figure s7.4. The basic operation is that at each sampling instant, we update the transversal line with the latest input data point (by shift the last N data points by one place so that the oldest data point drops off at the end and then storing the newest data sample). We then compute the output sample using the FIR equation. Efficiency may be improved by exploiting the symmetry in the filter coefficients to reduce computation time; it may also be improved by using modulo arithmetic/circular buffer to save time spent on data shifts.

```
%  PROB74.m                           File name
%
FS=8000;                              %  Sampling frequency
FN=FS/2;                              %  Nyquist frequency
N=47;                                 %  Filter length
beta=4.55126;                         %  Kaiser ripple parameter
fc=1750/FN;                           %  Normalized cut off frequency
wn=kaiser(N, beta);                   %  Compute Kaiser window values
hd=fir1(N-1, fc, wn, 'noscale');      %  Obtain windowed filter coefficients
[H,f]=freqz(hd, 1, 512, FS);          %  Compute frequency response
mag=20*log10(abs(H));
plot(f, mag), grid on
xlabel('Frequency (Hz)')
ylabel('Magnitude Response (dB)')
```

Table s7.4

Index, n	Filter coeffs, h(n)	Index, n	Filter coeffs, h(n)
0	0.00014762943964	24	0.31101115280726
1	-0.00109011396870	25	0.05998730944155
2	-0.00095116681018	26	-0.08524995456226
3	0.00167501355756	27	-0.05293513047208
4	0.00263371369414	28	0.03213732282821
5	-0.00157950346321	29	0.04266975405307
6	-0.00516903612453	30	-0.00733842331031
7	0.00000000000000	31	-0.03100655134636
8	0.00805239239667	32	-0.00502032213984
9	0.00385483531053	33	0.01979150102735
10	-0.01019036581464	34	0.00990918164395
11	-0.01047730477377	35	-0.01047730477377
12	0.00990918164395	36	-0.01019036581464
13	0.01979150102735	37	0.00385483531053
14	-0.00502032213984	38	0.00805239239667
15	-0.03100655134636	39	0.00000000000000
16	-0.00733842331031	40	-0.00516903612453
17	0.04266975405307	41	-0.00157950346321
18	0.03213732282821	42	0.00263371369414
19	-0.05293513047208	43	0.00167501355756
20	-0.08524995456226	44	-0.00095116681018
21	0.05998730944155	45	-0.00109011396870
22	0.31101115280726	46	0.00014762943964
23	0.43750000000000		

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MATLAB Problems

7.28 Use MATLAB to compute the coefficients, plot the magnitude frequency response in dB, and determine the locations of the zeros of each of the following filters (assume a sampling frequency of 2 kHz and the optimal method):

(i) A 7-point, band pass FIR filter with a pass and stop band edge frequencies of 200 Hz and 500 Hz.
(ii) An 8-point, band pass FIR filter with pass and stop band edge frequencies of 200 Hz and 500 Hz.
(iii) A 7-point, FIR differentiator with a pass and stop band edge frequencies of 200 Hz and 500Hz.
(iv) An 8-point, FIR Hilbert Transformer with band edges of 200 Hz and 500 Hz.

Comment on the differences and/or similarities in the positions of the zeros.

Solution

(i) The coefficients and z-plane zero locations of the band pass FIR filter are (using the optimal method for convenience)

```
b =
  Columns 1 through 4
  -0.30797926917693  -0.16349260648539   0.15270374489248   0.34472447571556
  Columns 5 through 7
   0.15270374489248  -0.16349260648539  -0.30797926917693

z =
 -0.31803520644593 + 0.94807890360502i
 -0.31803520644593 - 0.94807890360502i
 -0.89993451510633 + 0.43602507785714i
 -0.89993451510633 - 0.43602507785714i
  0.95254178070040 + 0.30440787772349i
  0.95254178070040 - 0.30440787772349I
```

The magnitude response and zero plot are depicted in Figure s7.28a and b. The m-file used is listed in Prob7.28a.m

(ii) This is exactly the same as (i) above, except that N=8. The coefficients and z-plane zero locations of the filter are (using the optimal method for convenience)

```
b =
 Columns 1 through 4
 -0.13150199103015 -0.33606584035579 0.01744332375232 0.28146332817016
 Columns 5 through 8
 0.28146332817016 0.01744332375232 -0.33606584035579 -0.13150199103015

-2.35635700178470
-1.00000000000000
-0.33727284884003 + 0.94140693933884i
-0.33727284884003 - 0.94140693933884i
 0.94984576874497 + 0.31271874839428i
 0.94984576874497 - 0.31271874839428i
-0.42438391094499
```

The magnitude response and zero plot are depicted in Figure s7.28c and d.

(iii) The coefficients and z-plane zero locations of the differentiating FIR filter are (using the optimal method for convenience).

```
b =
  Columns 1 through 4
  -0.07698199692482   0.41643371887054   0.24639090125673   0
  Columns 5 through 7
  -0.24639090125673  -0.41643371887054   0.07698199692482

z =
  5.92966979836067
  1.00000000000000
 -0.34440918228340 + 0.93881963931251i
 -0.34440918228340 - 0.93881963931251i
 -1.00000000000000
  0.16864345469565
```

The magnitude response and zero plot are depicted in Figure s7.28e and f. The m-file used is listed as Prob7.28b.m below.

(iv) The coefficients and z-plane zero locations of the Hilbert transforming FIR filter are (using the optimal method for convenience)

```
b =
  Columns 1 through 4
  -0.10981361438978   0.26834199892450   0.31274635055707   0.14899391312229
  Columns 5 through 8
  -0.14899391312229  -0.31274635055707  -0.26834199892450   0.10981361438978

z =
  3.35068883248107
  1.00000000000000
 -0.21247679919600 + 0.97716611167366i
 -0.21247679919600 - 0.97716611167366i
 -0.89028405915489 + 0.45540563678406i
 -0.89028405915489 - 0.45540563678406i
  0.29844609571207
```

The magnitude response and zero plot are depicted in Figure 7.28g and h. The m-file used is listed as Prob7.28c.m below.

The zero positions for linear phase FIR filters are determined by the type of symmetry in the impulse response (positive or negative) whether the filter length, N, is odd or even, the type of filter/band edge frequencies. In general zeros that are located within the unit circle have mirror-image zeros located outside the unit circle. The pass band is located between the first two zeros. The main features that characterise the zeros of the filters are summarised below.

Filter description.	Features of the zeros of filter
7-point, bandpass FIR filter Positive symmetry, N odd	Contains six zeros all on the unit circle. The zeros occur in complex conjugate pairs. The first two zeros mark the pass band edge frequencies
8-point band pass FIR filter Positive symmetry, N even.	Contains seven zeros. These consists of two pairs of complex conjugate zeros on the unit circle and two zeros on the real axis. One of the zeros on the real axis is located inside the unit circle at a radius of –0.4243 and the other, its mirror image, is located at a radius of -2.3563 (i.e. at – 1/0.4243) outside the unit circle. There is a single zero at z = -1.
7-point differentiator Negative symmetry, N odd	Contains six zeros, including a pair of complex conjugate zeros on the unit circle ;a zero at z=1 and at z=-1; a zero inside the unit circle on the real axis at z = 0.1686 and a mirror image zero at z = 1/ 0.1686 = 5.9296.
8-point, Hilbert Transformer negative symmetry, N even.	Contains seven zeros; two pairs of complex conjugate zeros on the unit circle; one at z=1 and two on the real axis – one inside the unit circle at z=0.2984, and a mirror image zero outside the unit circle at z=3.3506.

```
%
%  Program name - prob728a.m
%
Fs=2000;                          %  sampling frequency
FN=Fs/2;
N=7;                                 %  Filter length
f1=150/FN;
f2=200/FN;
f3=500/FN;
f4=550/FN;
Hd=[0 0 1 1 0 0];                 %  Desired magnitude response in the bands
F=[0 f1, f2, f3, f4, 1];          %  Band edge frequencies
b = remez(N-1, F, Hd);            %  Compute the filter coefficients
[H, f] = freqz(b, 1, 512, Fs);    %  Compute the frequency response
mag = 20*log10(abs(H));           %  of filter and plot it
plot(f, mag), grid on
xlabel('Frequency (Hz)')
ylabel('Magnitude (dB)')
a = [1];
z=roots(b)
b
```

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```
%
%  Program name - Prob728c.m  - Linear phase differentiator
%
Fs=2000;                          % sampling frequency
FN=Fs/2;
f1=200/FN;
f2=500/FN;
N=7;                              % Filter length
Hd=[0 1 0 0];                     % Desired magnitude response in the bands
F=[0 f1 f2 1];                    % Band edge frequencies
b = remez(N-1, F, Hd, 'd');       % Compute the filter coefficients
[H, f] = freqz(b, 1, 512, Fs);    % Compute the frequency response
plot(f, abs(H)), grid on
xlabel('Frequency (Hz)')
ylabel('Magnitude')
a = [1];
z=roots(b)
b
```

7.33A linear phase bandpass digital filter is required for feature extaction in a certain signal analyser. The filter is required to meet the following specification:

passband	*12 – 16 kHz*
transition width	*3 kHz*
sampling frequency	*96 kHz*
passband ripple	*0.01 dB*
stop band attenuation	*80 dB*

The coefficients of the filter are to be calculated using the optimal method. Determine, with the aid of MATLAB, the following:

(i) *the number of filter coefficients, N*
(ii) *coefficients of the filter.*

Plot the magnitude frequency response.

Solution

See solution to problem 7.5.