

4.5 (z) (a)

GIVEN: $X(z) = \frac{z-1}{(z-0.707)^2}$ find $x[n]$ using P.F.E.

$$\frac{X(z)}{z} = \frac{z-1}{z(z-0.707)^2} = \frac{D_1}{z} + \frac{D_2}{z-0.707} + \frac{D_3}{(z-0.707)^2}$$

$$D_1 = \left. \frac{X(z)}{z} \cdot z \right|_{z=0} = \frac{0-1}{(0-0.707)^2} = -2$$

$$D_2 = \left. \frac{d}{dz} \left[\frac{X(z)}{z} (z-0.707)^2 \right] \right|_{z=0.707} = \left. \frac{d}{dz} \left[\frac{(z-0.707)^2(z-1)}{z(z-0.707)^2} \right] \right|_{z=0.707}$$

$$= \left. \frac{d}{dz} \left[\frac{z-1}{z} \right] \right|_{z=0.707} = \left. \frac{d}{dz} (1-z^{-1}) \right|_{z=0.707} = (0.707)^{-2} = 2$$

$$D_3 = \left. \frac{X(z)}{z} (z-0.707)^2 \right|_{z=0.707} = \left. \frac{(z-1)(z-0.707)^2}{z(z-0.707)^2} \right|_{z=0.707}$$

$$= \frac{(0.707)-1}{0.707} = -0.414$$

$$\frac{X(z)}{z} = \frac{-2}{z} + \frac{2}{(z-0.707)} - \frac{0.414}{(z-0.707)^2}$$

$$X(z) = -2 + \frac{2z}{(z-0.707)} - \frac{0.414z}{(z-0.707)^2} \rightarrow \frac{kz}{(z-\alpha)^2}$$

$$k = \frac{0.414}{0.707} = 0.586$$

$$x[n] = [-2\delta[n] + 2(0.707)^n u[n] - 0.586(0.707)^n n u[n]]$$