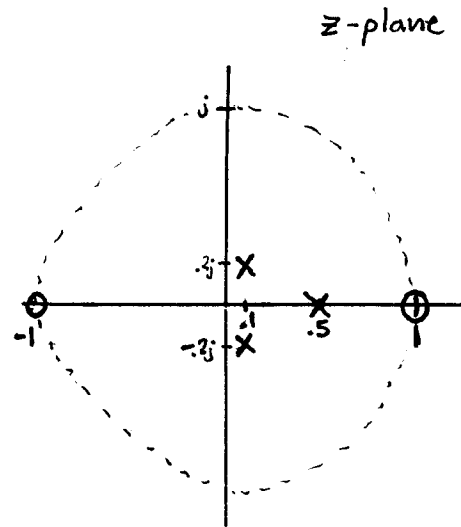4.12 The transfer function of a discrete-time system has poles at $z = 0.5$, $z = 0.1 \pm j\,0.2$ and zeros at $z = -1$ and at $z = 1$.

(1) Sketch the pole–zero diagram for the system.

(2) Derive the system transfer function, $H(z)$, from the pole–zero diagram.

(3) Develop the difference equation.

(4) Draw the realization diagram in signal flowgraph form.

① z-plane

②
$$H(z) = \frac{(z-1)(z+1)}{(z-0.5)\left(z-(0.1+j0.2)\right)\left(z-(0.1-j0.2)\right)}$$

$$H(z) = \frac{z^2 - 1}{z^3 - .250z^2 + .060z - 0.025}$$

$$H(z) = \frac{z^{-1} - z^{-3}}{1 - 0.250z^{-1} + 0.060z^{-2} - 0.025z^{-3}}$$

③ $H(z) = \dfrac{Y(z)}{X(z)} = \dfrac{z^{-1} - z^{-3}}{1 - 0.250z^{-1} + 0.060z^{-2} - 0.025z^{-3}}$
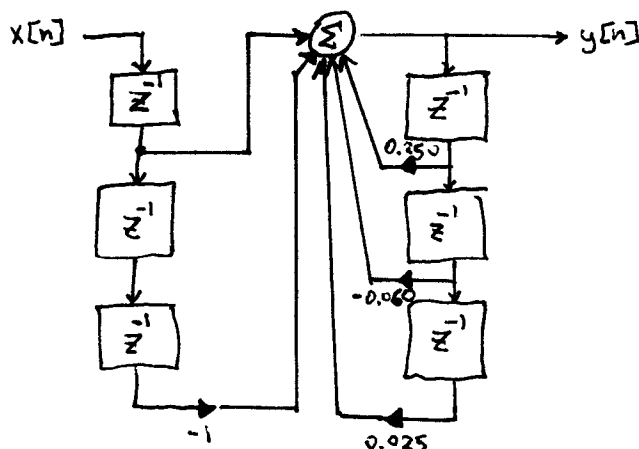
$$Y(z) - 0.250z^{-1}Y(z) + 0.060z^{-2}Y(z) - 0.025z^{-3}Y(z) = z^{-1}X(z) - z^{-3}X(z)$$

$$Y(z) = z^{-1}X(z) - z^{-3}X(z) + 0.250z^{-1}Y(z) - 0.060z^{-2}Y(z) + 0.025z^{-3}Y(z)$$

Diff Equation

$$y[n] = x[n-1] - x[n-3] + 0.250\,y[n-1] - 0.060\,y[n-2] + 0.025\,y[n-3]$$

④

← Actually "Block Diagram" form not "flow graph" form