Lesson 15

I&J problem 4.1

(2) Find Z-transform of $x[n] = a^n \quad n \geq 0$
$= 0 \quad n < 0$

causal! right-handed!

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

$$= \sum_{n=0}^{\infty} a^n z^{-n} = \sum_{n=0}^{\infty} (a z^{-1})^n$$

Power Series Expansion says

$$\sum_{n=0}^{\infty} \alpha^n = \frac{1}{1-\alpha} \quad \text{for } |\alpha| < 1$$

else it explodes!
- doesn't converge

So let $\alpha = a z^{-1}$

$$\boxed{X(z) = \frac{1}{1 - a z^{-1}}}$$

ROC $|\alpha| < 1$

$$|a z^{-1}| < 1$$

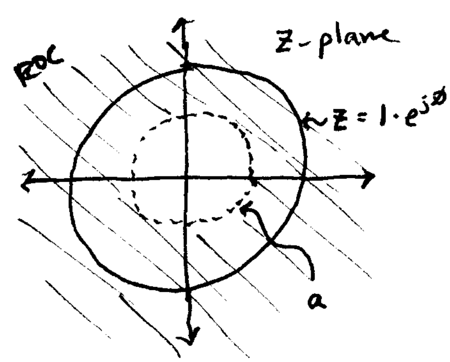
$$\left|\frac{a}{z}\right| < 1$$

$$\boxed{|a| < |z|}$$

If z is evaluated along the unit circle

$$z = e^{j\phi}$$

then a must be < 1
or inside the unit circle
and causal to converge



I&J problem 4.1

(3) Find Z-transform of $x[n] = 1 \quad 0 \le n \le N-1$ causal; right handed

$ = 0 \quad$ elsewhere

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

$$= \sum_{n=0}^{N-1} z^{-n} = \sum_{n=0}^{N-1} (z^{-1})^n$$

Yet another Power Series

$$\sum_{n=0}^{N-1} \alpha^n = \frac{1-\alpha^N}{1-\alpha}$$

for ~~|α| < 1~~ $\alpha \neq \infty$

let $\alpha = z^{-1}$

$$\boxed{X(z) = \frac{1-z^{-N}}{1-z^{-1}}}$$

for $z \neq \infty$