8.2 *Digitize, using the impulse invariant method, the analog filter with the transfer function:*

$$H(s) = \frac{\alpha}{s(s+\alpha)}, \quad \alpha = 0.5$$

Assume a sampling frequency of 1 (normalized).

Solution

Now, $H(s) = \frac{\alpha}{s(s+\alpha)} = \frac{\alpha}{s^2 + \alpha s}$. Expressed as a partial fraction, this becomes:

$$H(s) = \frac{\alpha}{s^2 + \alpha s} = \frac{c_1}{s - p_1} + \frac{c_2}{s - p_2}$$

where p_1 and p_2 are the poles of H(s). If we multiply both sides by $(s-p_1)(s-p_2)$ we have $\alpha = c_1(s-p_2) + c_2(s-p_1) = s(c_1 + c_2) - (c_1 p_2 + c_2 p_1)$. Equating terms, we obtain the values of c_1 and c_2:

$\alpha = -c_1 p_2$ (since $p_1 = 0$) and $c_1 = -\alpha / p_2$; $c_1 + c_2 = 0$ and $c_2 = -c_1$

With $\alpha = 0.5$ and $p_2 = -\alpha$, then $c_1 = 1 \quad c_2 = -1$.

Thus, $H(s) = \frac{1}{s} - \frac{1}{s+0.5}$

Applying the impulse invariant transform, we obtain H(z) as:

$$H(z) = \frac{1}{1 - z^{-1}} - \frac{1}{1 + e^{-0.5} z^{-1}} = \frac{(1 - e^{-0.5} z^{-1}) - (1 - z^{-1})}{(1 - z^{-1})(1 - e^{-0.5} z^{-1})}$$

$$= H(z) = \frac{-0.3935 z^{-1}}{1 - 1.6065 z^{-1} + 0.6065 z^{-2}}$$