ECE 447 Fall 2025

Lesson 04

Signals and Signal Space

U.S. AIR FORCE ACADEMY

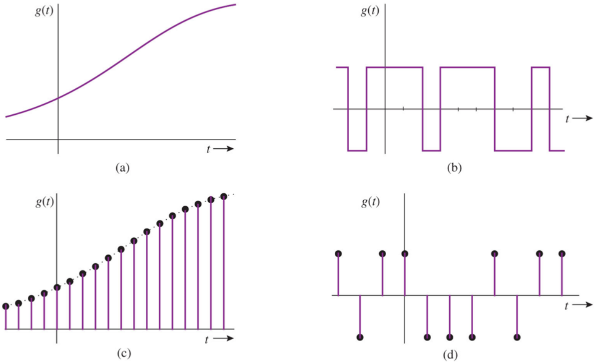
Life/Leadership Lesson of the Day

- Lead!
  - When you believe you can do it better than anyone else – step up!
- Example
  - Qatar built several large, nice dorms at AUAB
  - Internet to each room responsibility of 379th CS
  - No internet = low morale!
  - Capt Josh Thomas (in the CC's Support Staff) had the expertise; got permission from Wg/CC, organized cable patching teams
  - Completed within a few days
  - Fast internet = high morale!

Schedule and Admin

- Schedule
- Admin
    - **SDR Setup**. If unable to get FM radio station playing through GQRX last lesson - get help!
    - **Lab 1 Assignment**. The assignment associated with Lab 1 is due Lesson 6 - specifically 21 Aug by 2359 via Gradescope upload. Make sure you submit a **narrowband FM** signal - not a wideband stereo FM radio signal.
    - **Future HW problems**...

Classification of Signals



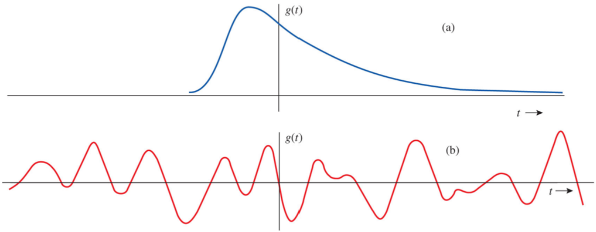
- Periodic vs. Aperiodic
- Deterministic vs. Random

SIGNAL SIZE

- Signal Energy: $E_g = \int_{-\infty}^{+\infty} |x(t)|^2 \, dt = \int_{-\infty}^{+\infty} x(t)x^*(t)dt$

- Signal Power:

$$E_g = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)x^*(t)dt$$



Normalized Resistance

We typically assume that $R = 1\,\Omega$, i.e. *normalized*, so that

$$p(t) = |v(t)|^2 = |i(t)|^2 = x(t)^2 \text{ (Watts)}$$

Energy simply becomes

$$E = \int_{t_1}^{t_2} |x(t)|^2 dt \text{ (Joules)}$$

Normalized Resistance (cont.)

Average power in the interval t_1 to t_2 is given by:

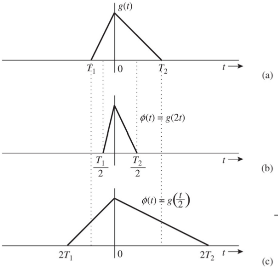
$$P_{avg} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} |x(t)|^2 \, dt$$

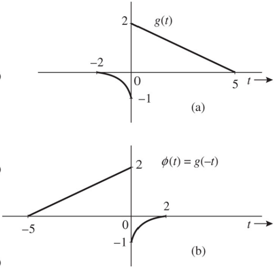
If the signal is periodic, $x(t+T) = x(t) \ \forall t$, then

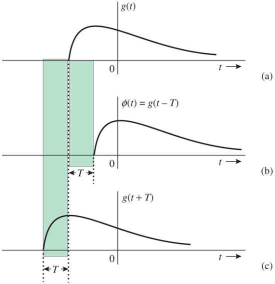
$$P_{avg} = \frac{1}{T} \int_{T} |x(t)|^2 \, dt$$

Useful Signal Operations

Name that operation!







Important Signals

- Unit Impulse:

$$\delta(t) = 0, t \neq 0 \qquad \int_{-\infty}^{\infty} \delta(t)dt = 1 \qquad \phi(t)\delta(t-T) = \phi(T)\delta(t-T)$$

- Unit Step:

$$\int_{-\infty}^{t} \delta(\tau)d\tau = u(t) =$$

BASIS FUNCTIONS

- Vector Analogy: two vectors are perpendicular or "orthogonal" when their dot product is zero

$$A_x\hat{x} \cdot A_y\hat{y} = 0$$

- They have nothing in common
- Cannot write an estimate of one in terms of the other
- Two signals are orthogonal on interval (t_1, t_2) if

$$\int_{t_1}^{t_2} x(t)y^*(t)dt = 0$$

Basis Function Example

- Consider functions $\sin(n\pi t)$ and $\sin(m\pi t)$ on interval $(0,2)$
- Trig identity: $\sin(A)\sin(B) = \frac{1}{2}\cos(A-B) - \frac{1}{2}\cos(A+B)$

$$\int_0^2 \sin(n\pi t)\sin(m\pi t) = \frac{1}{2}\int_0^2 \cos[(n-m)\pi t] - \cos[(n+m)\pi t]\,dt$$

$$= \frac{1}{2\pi(n-m)}\sin[(2\pi(n-m)t] - \frac{1}{2\pi(n+m)}\sin[2\pi(n+m)t]\Big|_0^2$$

$$= 0 \text{ for } n \neq m$$

Basis Function Example (cont.)

- Consider functions $\sin(n\pi t)$ and $\sin(m\pi t)$ on interval $(0, 2)$
- Trig identity: $\sin(A)\sin(A) = \frac{1}{2}[1 - \cos(2A)]$

$$\int_0^2 \sin(n\pi t)\sin(n\pi t)\,dt = \frac{1}{2}\int_0^2 1 - \cos(2n\pi t)\,dt$$

$$= 1 - \frac{1}{4\pi n}\sin(4\pi n t)\Big|_0^2$$

$$= 1 \text{ for } n = m$$

Basis Function Example (cont.)

- Therefore, $\sin(n\pi t)$ and $\sin(m\pi t)$ are orthogonal from $(0, 2)$
- Because inner product $= 1$, they are orthonormal
- Mathematically, $\sin(n\pi t)$ and $\sin(m\pi t)$ have nothing in common
- Because there are an infinite number of n's and m's, we have an infinite number of basis functions
- *Do these functions span the space of all functions on* $(0, 2)$?

Complex Signal Models

- Rotating Phasor Representation:

$$\tilde{x}(t) = Ae^{j\theta}e^{j\omega_o t} = Ae^{j(\omega_o t+\theta)} = A\cos(\omega_o t + \theta) + jA\sin(\omega_o t + \theta)$$

- For orthogonality,

$$\int_{t_1}^{t_2} x_m(t)x_n^*(t)dt = 0 \quad m \neq n$$

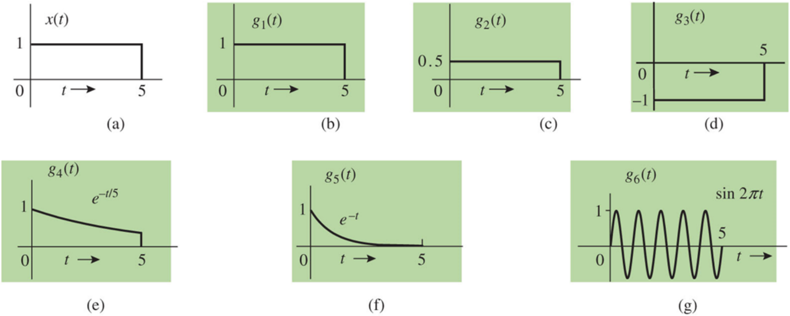
$$\int_{t_1}^{t_2} x_m(t)x_n^*(t)dt = E_n \quad m = n$$

- If $E_n = 1$, then signal set is *orthonormal*

Correlation of Signals

- Generalized correlation definition:

$$\rho = \frac{1}{\sqrt{E_g E_x}} \int_{-\infty}^{\infty} g(t)x^*(t)dt$$



Fourier Series Review

- Synthesis

$$x(t) = \sum_{n=-N}^{N} X_n e^{jn\omega_0 t}$$

- Analysis

$$X_n = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} x(t) e^{-jn\omega_0 t} dt, \quad \omega_0 = \frac{2\pi}{T} = 2\pi f_0$$

- Average Power

$$P_{avg} = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |x(t)|^2 \, dt = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x(t) x^*(t) dt$$

$$P_{avg} = \sum_{n=-\infty}^{+\infty} X_n X_n^* = \sum_{n=-\infty}^{+\infty} |X_n|^2$$