Primary units in this guide are metric (SI – the International System of units):

Length - m - (meter)
Mass - g - (gram)
Force - mN - (millinewton)
Torque - mN•m - (millinewton meter)
Inertia - g•m^2 - (gram $meter^2$)

In this system, mass is always in kilograms or grams. Force, or weight, is always in newtons or millinewtons.

Force (or weight) = Mass x Acceleration

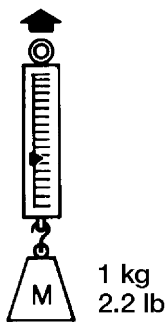
F = ma

when a = 9.81 m/sec^2 (acceleration due to gravity), then F would be the weight in newtons.

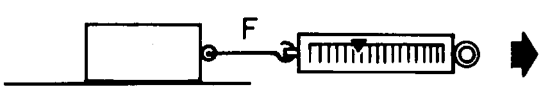
How to measure Mass or Force.

A spring scale reading of 1 kg means that you are measuring a mass of 1 kg.

A spring scale reading of 2.2 lb also is measuring a mass of 1 kg.



If you use that same spring scale to measure a force, the 1 kg reading must be multiplied by 9.8 to give a force of 9.8 newtons.



The reading of 2.2 lb is a force and is equal to 9.8 newtons.

If the same scale is used to measure torque (T = FR) at a one meter radius, the reading of

1 kilogram x 1 meter = 1 kgm

must be multiplied by 9.8 to give a torque of 9.8 newton meters (N•m).

	Given Unit				Units Used in this Manual (Metric SI)
Length	1 inch	=	2.54 cm	=	2.54 X 10^{-2}m
Force	1 oz	=		=	278 mN
	1 lb	=	4.45 N	=	4,450 mN
	1 g•cm	=		=	9.8 mN
Mass	1 lb	=		=	454g
	1oz	=		=	28.4g
	1kg	=		=	1,000g
	1 slug	=	14.6 kg	=	14,600g
Inertia	1 g•cm^2	=		=	10^{-4} g•m^2
	1 oz-in-sec^2	=		=	7.06 g•m^2
	1 slug ft^2	=		=	.29 g•m^2
Torque	1 oz-in	=	72.01 g•cm	=	7.06 mN•m
	1 lb-ft	=		=	1.356 x N•m
	1 g•cm	=		=	9.8 x 10^{-2} mN•m
			10.2 g•cm	=	1 mN•m
			141.6 oz-in	=	1 N•m

1. Torque (mN•m) = Force (mN) x Radius (m)

Torque = FR

2. Torque required to accelerate inertial load

$$\mathbf{T\ (mN \cdot m) = J\ \alpha}$$

J = Inertia in g•m^2

α = Acceleration in radians/sec^2

EXAMPLE:
If a rotor inertia plus load inertia = J = 2 x 10^{-3} g•m^2, and the motor is to be accelerated at 6,000 radians per sec, what torque is required?

T = Jα = 2 x 10^{-3} x 6000
T = 12 mN•m

For stepper motors, α can be converted to radians/sec^2 from steps/sec^2.

$$\alpha \text{ (radians/sec)} = \frac{\Delta v \text{ (steps/sec)}}{\Delta t \text{ (accel. time)}} \times \frac{2\pi}{\text{steps/rev}}$$

$$\textbf{TORQUE} = J\ \frac{\Delta v}{\Delta t} \times \frac{2\pi}{\text{steps/rev}}$$

EXAMPLE:
For a 48-step per revolution motor accelerating from zero to steps/sec running rate v in Δt seconds.

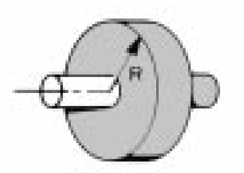
$$\text{TORQUE} = J\ \frac{v}{\Delta t} \times \frac{\pi}{24}$$

If no *acceleration time* is provided, then a maximum 2-step lag can occur.

Δt (sec) $= \dfrac{2 \text{ (steps)}}{v \text{ (steps/sec)}}$ giving the following equation.

$$\textbf{TORQUE} = J \frac{V^2}{2} \times \frac{2\pi}{\text{steps/rev}}$$

3. Moment of Inertia

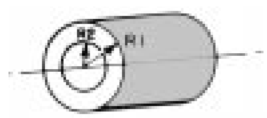


Disc or shaft

M = Mass in grams

R = Radius in meters

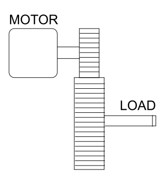
$$J \ (g \bullet m^2) = \frac{MR^2}{2}$$



Cylinder

$$J \ (g \bullet m^2) = \frac{M^2}{2} (R_1^2 + R_2^2)$$

4. Reflected loads when using gears or pulleys

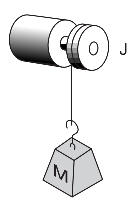


$$\textbf{Torque required of motor} = \frac{\textbf{Load Torque}}{\textbf{GR}}$$

$$\text{gear or pulley ratio GR} = \frac{\text{motor shaft revolutions}}{\text{load shaft revolutions}}$$

$$\textbf{Inertia reflected to motor} = \frac{\textbf{Load intertia}}{\textbf{(GR)}^2}$$

5. Equivalent Inertial Load
For a pulley and weight or a rack and pinion



J eqv. $(g \bullet m^2) = MR^2$

M = Mass of load in grams
R = Radius of pulley in meters

6. Total Load
Note: Be sure to include all load components.

J_T = Rotor Inertia + all J Loads

T_F = Frictional and Forces

Note: In the pulley example above, the total load would be:

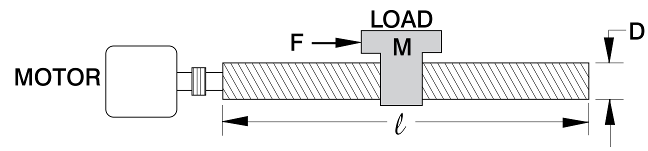
J_T = J rotor + J pulley + J eqv.

T_F = T frictional + Load Weight x Radius

Total T = $J_T \alpha + T_F$

The load weight = mass x 9.8 millinewtons.

7. Axial Force of Lead Screw



$$F = \frac{2\pi \times T}{L} \times \text{eff.}$$

F (mN) when T = Torque in mN•m

L = Lead of screw in meters

F (oz) when T = Torque in oz-in

L = Lead of screw in inches

efficiency = from .9 for ballnut to .3 for Acme

Inertia of lead screw load

J = J rotor + J steel screw + J reflected

$$\text{J steel screw} = D^4 \times \backslash \times \frac{\pi}{32} \times \text{Density}$$

Density for steel = 7.83 x 106 g/m^3

then:

$$\mathbf{J \ (g \bullet m^2) = D^4 \ \backslash \ \times 7.7 \times 10^5}$$

The reflected inertia of the load is:

$$\textbf{J reflected } (g \bullet m^2) = \textbf{M (load) } L^2 \times .025$$

Total Torque Load from lead screw (T) in mN•m

T = (J rotor + J screw + J reflected) α + T friction

8. Motor watts output

Watts out = Torque output x speed in radians/sec

1 watt = 1 Nm/sec

For a given output Torque (mN•m) and converting v (steps/sec) to radians/sec

$$\textbf{Watts out} = \textbf{Torque (mN•m)} \times v \frac{\textbf{(motor step angle)}}{\textbf{57.3}} \times 10^{-3}$$

If the speed is in RPM then:

Watts out = 1.05 x 10^{-4} x torque (mN•m) x RPM

9. Steps/sec to RPM

$$\textbf{RPM} = \frac{\textbf{v (steps/sec) x 60}}{\textbf{motor steps/rev}}$$