# CSCI 570 - HW 5

1.Given Function
10x1+8x2+5x3
Framing wood needed:
1*500+2*300+2*200=1500 ft
1500 ft is the minimum framing wood needed, this month the framing wood has been incresed by 100, so the maximum framing wood which can be used is 1600 ft
So the equation for the framing wood is
x1+2*x2+2*x3<= 1600
x1,x2,x3>=0
Cabinet wood needed:
3*500+2*300+1*200= 2300 ft
Originally 2300 ft is the minimum cabinet wood needed, this month the cabinet wood has been decreased by 600, so the maximum framing wood which can be used is 1700 ft
So the equation for the Cabinet wood is
3*x1+2*x2+x3<= 1700
x1,x2,x3>=0

Min(10x1+8x2+5x3)
x1+2*x2+2*x3<= 1600
3*x1+2*x2+x3<= 1700
X1,x2,x3>=0

2. Dual:
min(4y1+6y2-3y3+3y4+8y5)
y1+2y2+y3-y4+y5>=3
-y1+y2+y5>=2
Y1+3y2-2y3+2y4+y5>=1

3. Min([summation from k,l=1 to m] [summation from i=1,j=2 to 2] Xki + Xlj)
Let Xab be the assignment of ath frequency with the bth station and two adjacent station b and b+1 must have two different frequencies, i.e. Xab!=Xcb+1 where c is the frequency assigned to the station b+1 and c!=a
Objective function:
Min([summation from a=1 to m] [summation from b=1 to n] Xab)
Constraints:
Pick a frequency 1,2,…,m
X1b+X2b+…>=2
Pick a station 1,2,…,n
Xa1+Xc2+….=n
Constraints on Xab
Xab E {0,1}

4.
(i)The given statement is false

Proof:
B is harder than and it is hard to reduce A and find a solution in polynomial time for NP-hard

(ii) The given statement is true
Proof:
B E NP implies that B can be verified in polynomial time and according to non deterministic turing machine A E NP

(iii) The given Statement if False
Proof:
3-SAT has 3 literals per clause
Example: (x1 v x2 v x3) ^ (x1 v ~x2 v x3) ^ (~x1 v x2 v ~x3)
2-SAT, for every clause there will be 2 literals
Example: (x1 v x2) ^ (x1 v ~x2) ^ (~x1 v x2)
The 3-SAT problem is a NP complete and the 2-SAT problem can be solved in polynomial time that is P complete
Therefore the statement is contradiction since 3-SAT cannot be reduced to 2-SAT problem

(iv)The given statement is True
Proof:
NP-complete problems can be solved in O(2^poly(n)) time, where n is the size of the input, if the non deterministic turing machine always finds a solution to the given problem

(v)The given statement is false
Proof:
If A is in P, then it can be solved in polynomial time, and if B is in NP, A<=p B, however B<=p A is not possible because B is NP and any NP problem cannot be solved in polynomial time

5. Suppose A be a polynomial time algorithm that decides whether a SAT instance is satisfiable or not
Let c1,c2,…,cm be the clauses and x1,x2,…,xn be the variables, then the boolean formula of the SAT is
Ø(x1,x2,…,xn)= c1 ^ c2 ^ … ^ cm
If the given instance A(Ø(x1,x2,…,xn)) = 0 or False, then the instance is not satisfiable
If the given instance A(Ø(x1,x2,…,xn)) = 1 or True, then the assignment if satisfying
xi will be either 1 or 0, consider x1 is either 0 or 1
if x1 is 0 then Ø(x1,x2,…,xn) = Ø(0,x2,…,xn) or if x1 is 1 then Ø(x1,x2,…,xn)= Ø(1,x2,…,xn), either of them must be satisfiable
if A(Ø0) is 1, x1=0, but there is xj satisfying to the Ø in which x1=0, now repeat the process to find all the n variables using Ø0
if A(Ø0) is 0, then A(Ø1) must be satisfiable, so now assign x1 as 1 and repeat the process to find all the assignments of n variables using Ø1
To find all the assignments to the variables we iterate n+1 number of times, by this we can possibly find satisfying assignments to the given 3-SAT instance in polynomial time

6. Given:
duv is shortest path between u and v
d(v,P) is the distance between node v to the closest node on a Path P
aggregate remoteness of P
r(P) = (summation vEV)d(v,P)

->Highway E NP
This can be certified by traversing through all the vertices on the Highway and find the min(r(p)) and min(r(p))<=k
-> Hamiltonian path <=p Highway
Given any graph with Hamiltonian path we need to construct with minimum aggregate remoteness min(r(p)) and min(r(p))<=k
Claim: Graph G has a Hamiltonian path if and only if the highway has minimum aggregate remoteness min(r(p)) and min(r(p))<=k
Proof:
Any graph G has Hamiltonian path
Construct a highway using the vertices in the graph G and the highway must have minimum aggregate remoteness min(r(p)) and min(r(p))<=k
Find a Hamiltonian path in the graph G and using the Hamiltonian path construct the highway by adding edges duv
If the constructed highway has minimum aggregate remoteness min(r(p)) and min(r(p))<=k and the highway must be in the Hamiltonian path