CSCI 570 Homework 2
Spring 2023

Due Date: Feb. 09, 2023 at 11:59 P.M.

1. Consider the following functions:

```
bool function0(int x){
    if (x == 1) return true;
    if (x == 0 || (x % 2 != 0)) return false;
    if (x > 1) return function0(x/2);
}
void function1(int x){
    if (function0(x)){
        for (int i = 0; i < x ; i++)
            print i;
    } else
        print i;
}
void function2(int n){
    for (int i = 1; i <= n; i++)
        function1(i);
}
```

   Compute the amortized time complexity of `function2` in terms of n. Explain your answer. Provide the tightest bound using the big-O notation.

2. We have argued in the lecture that if the table size is doubled when it's full, then the amortized cost per insert is acceptable. Fred Hacker claims that this consumes too much space. He wants to try to increase the size with every insert by just two over the previous size. What is the amortized cost per insertion in Fred's table?

3. Prove by induction that if we remove the root of a k-th order binomial tree, it results in k binomial trees of the smaller orders. You can only use the definition of B_k. Per the definition, B_k is formed by joining two B_{k-1} trees.

4. There are n ropes of lengths $L_1, L_2, \ldots, L_n$. Your task is to connect them into one rope. The cost to connect two ropes is equal to sum of their lengths. Design a greedy algorithm such that it minimizes the cost of connecting all the ropes. No proof is required.

5. Given M sorted lists of different length, each of them contains positive integers. Let N be the total number of integers in M lists. Design an algorithm to create a list of the smallest range that includes at least one element from each list. Explain its worst-case runtime complexity. We define the list range as a difference between its maximum and minimum elements. You are not required to prove the correctness of your algorithm.

6. Design a data structure that has the following properties:

   - Find median takes $O(1)$ time
   - Extract-Median takes $O(\log n)$ time
   - Insert takes $O(\log n)$ time
   - Delete takes $O(\log n)$ time

   where n is the number of elements in your data structure. Describe how your data structure will work and provide algorithms for all aforementioned operations. You are not required to prove the correctness of your algorithm.

7. In a greenhouse, several plants are planted in a row. You can imagine this row as a long line segment. Your task is to install lamps at several places in this row so that each plant receives 'sufficient' light from at least one lamp. A plant receives 'sufficient' if it is within 4 meters of one of the lamps. Design an algorithm that achieve this goal and uses as few numbers of lamps as possible. Prove the correctness of your algorithm.

8. We are back again at the same greenhouse from problem-1, but this time our task is to water the plants. Each plant in the greenhouse has some minimum amount of water (in L) per day to stay alive. Suppose there are n plants in the greenhouse and let the minimum amount of water (in L) they need per day be l1, l2, l3,. . . ln. You generally order n water bottles (1 bottle for each plant) of max(l1, l2, l3,. . . .ln) capacity per day to water the plants, but due to some logistics issue on a bad day, you received n bottles of different capacities (in L). Suppose c1, c2,c3. . . cn be the capacities of the water bottles, and you are required to use one bottle completely to water one plant. In other words, you will allocate one bottle per plant, and use the entire water present (even if it is more than the minimum amount of water required for that plant) in that bottle to water a particular plant. You cannot use more than one bottle (or partial amount of water) to water a single plant (You need to use exactly one bottle per plant). Suggest an algorithm to determine whether it is possible to come up with an arrangement such that every plant receives more than or equal to its minimum water requirement. Prove the correctness of your algorithm.

9. Let's assume that you are worker at a bottled water company named Trojan Waters which supplied water bottles to the greenhouse in problem-2. At the facility, there is a water filter (completely filled with water) which has a capacity of W (in L), and there are n different empty bottles of capacities p1, p2,. . . .pn. Your job as a loyal worker is to design a greedy algorithm, which, given W and p1,p2,. . . ,pn, determine the fewest number of bottles needed to store the entire water W . Prove that your algorithm is correct.

10. **Online Questions.** Please go to DEN (https://courses.uscden.net/) and take the online portion of your assignment.