



# Trajectory based routing / Reachability Analysis

CSCI 587: Lecture 17

04/26/2025

# Location-based services are everywhere

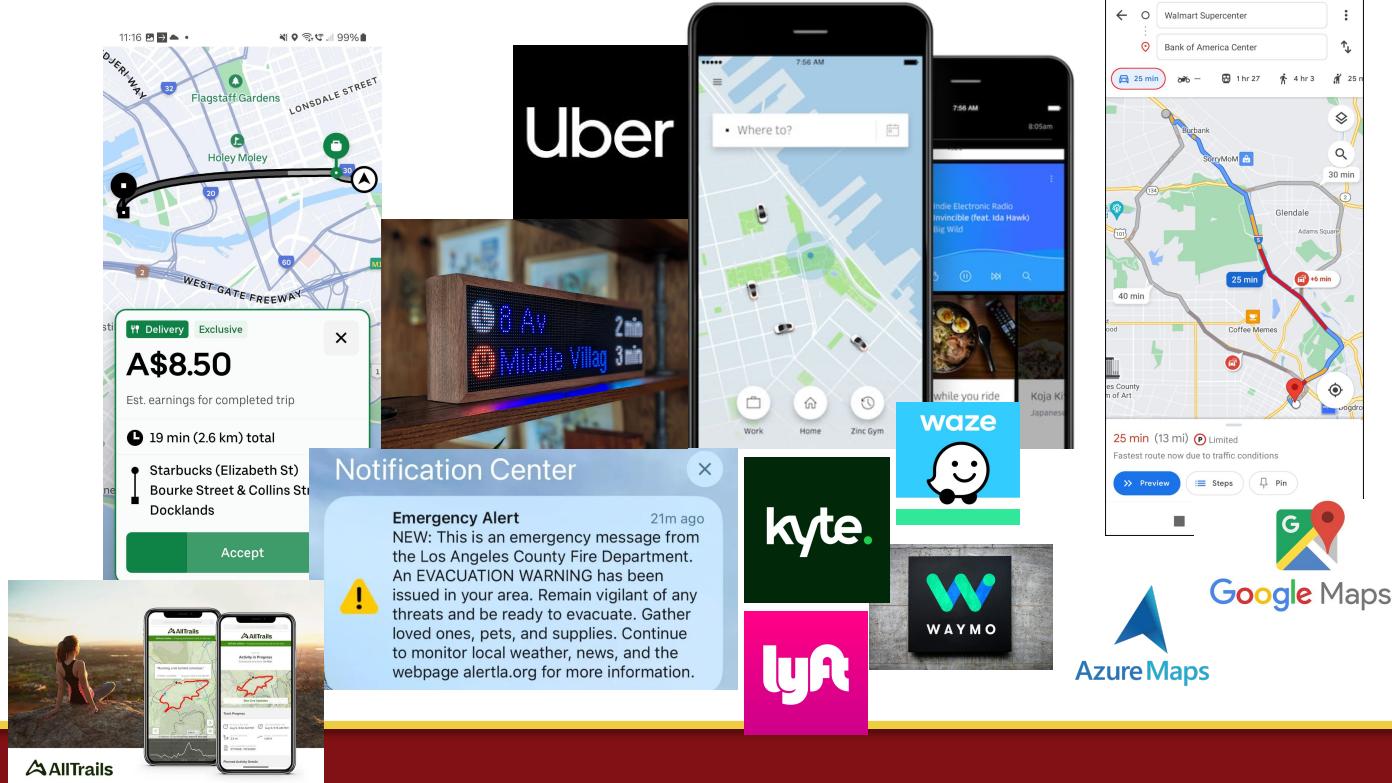


The global location based services market is projected to reach **\$402.4 billion** by 2031



# Location-based services are everywhere

- Delivery services
- Navigation
- Ride hailing / sharing
- Emergency response
- Public Transit
- Fleet Management
- Outdoors & Recreation





# Origin Destination Queries

Origin point

Destination point

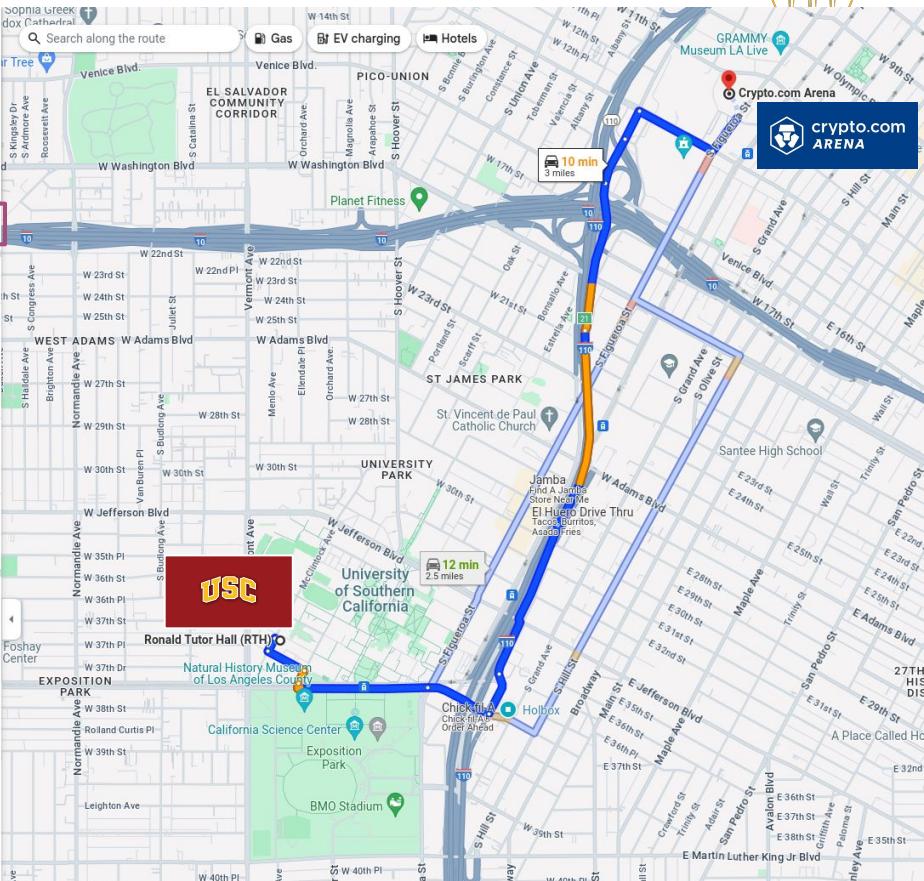
Timestamp

Route &  
Estimated time  
of arrival

The screenshot shows a navigation application interface. At the top, there are icons for Best, 10 min, 18 min, 56 min, 14 min, and a search bar. Below this, the origin point is listed as "Ronald Tutor Hall (RTH), 3710 McClintock" and the destination point is "Crypto.com Arena, 1111 S Figueroa St, Los Angeles, CA". A button to "Add destination" is present. In the middle section, a dropdown menu shows "Leave now" and "Options". Below that, three route options are listed:

- via I-110 N (10 min, 3.0 miles)
- via S Figueroa St (12 min, 2.5 miles)
- via S Hill St (13 min, 3.1 miles)

A callout box highlights the first route. At the bottom, there is a section titled "Explore nearby Crypto.com Arena" with icons for Restaurants, Hotels, Gas stations, Parking Lots, and More.

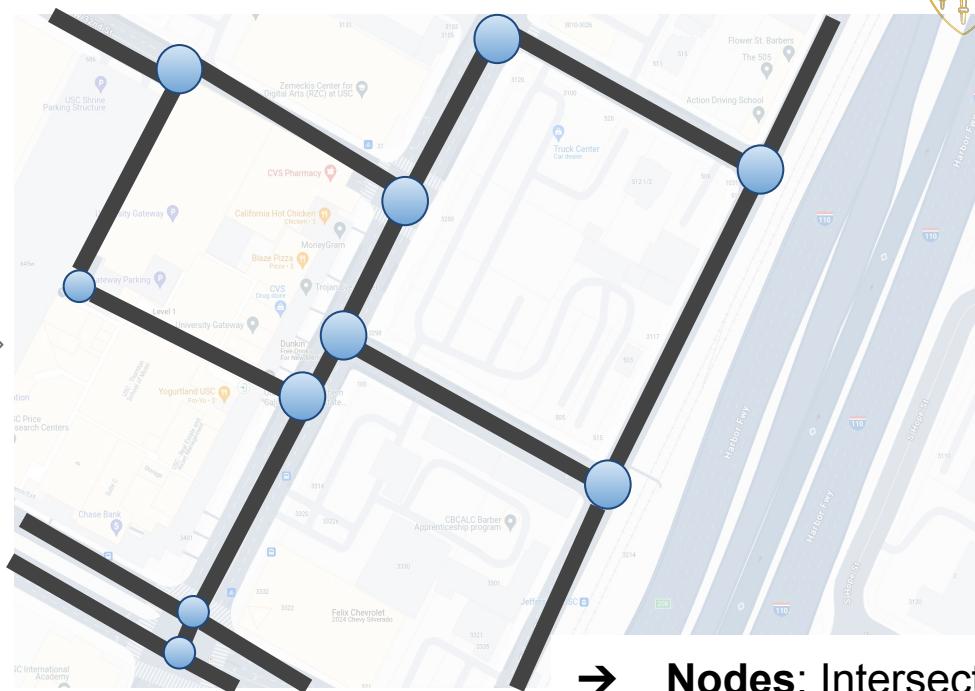




# Road Network as a Graph



city road network extracted from OpenStreetMaps (OSM)

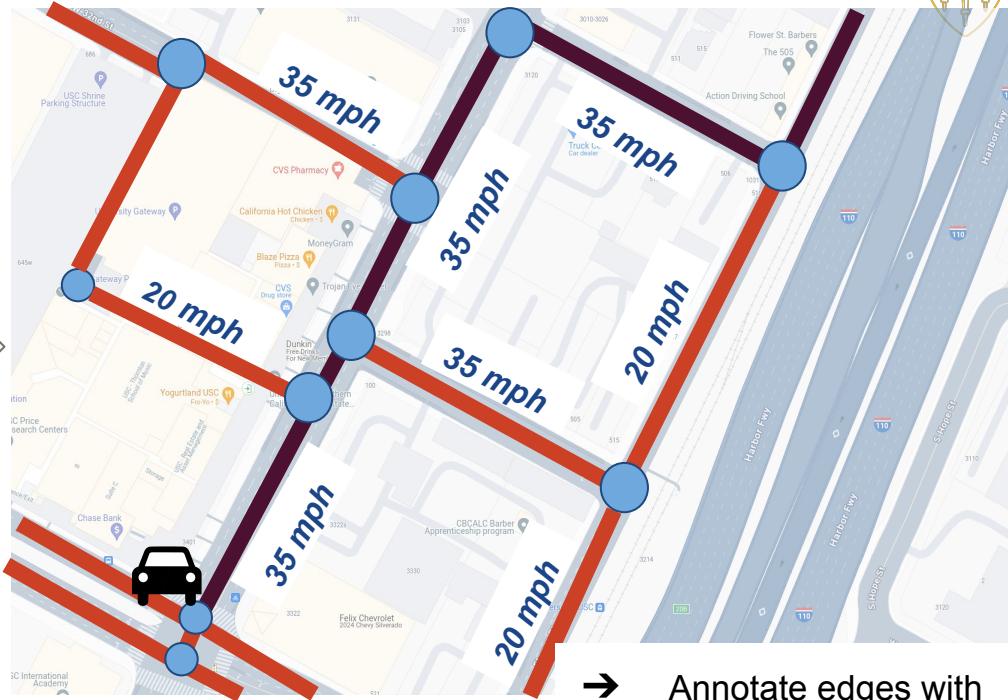


→ **Nodes:** Intersections  
→ **Edges:** Roads

# Road Network as a Graph

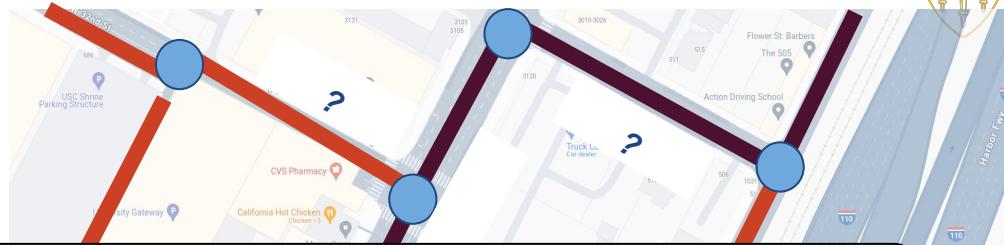


city road network extracted from OpenStreetMaps (OSM)

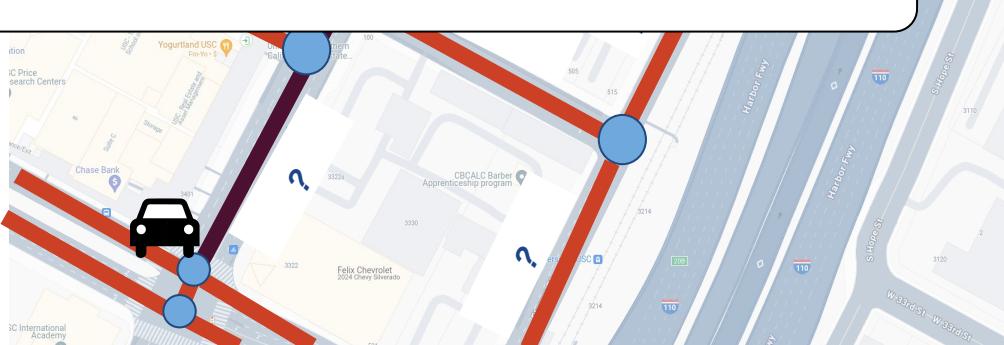


→ Annotate edges with weights e.g. road speed limits

# Road Network as a Graph



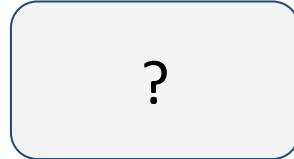
★ How can we get accurate, time dependent weights?



# Origin Destination Queries



**Origin point**  
**Destination point**  
**Timestamp**



**Route**  
**Estimated time of arrival**

# Origin Destination Queries



Origin point  
Destination point  
Timestamp



Route  
Estimated time of arrival

- Spatial / Temporal Information
  - road conditions
  - stop signs / intersection signals
- Traffic conditions

- Personalized Information
  - preferred routes
  - driving style
- Augmented information
  - weather
  - road closures



Third-Party Sources



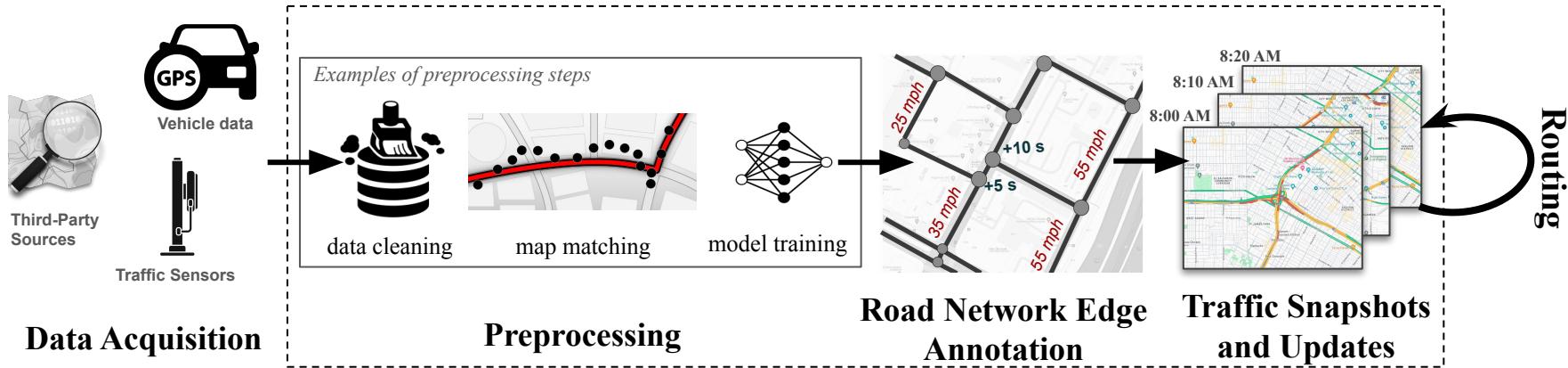
Vehicle data



Traffic Sensors



# Typical Pipeline of Routing Services

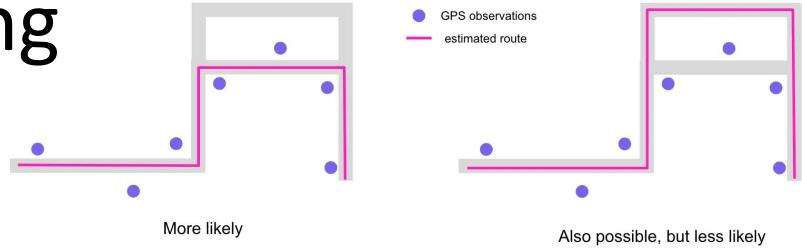


- Large scale, *up-to date GPS data* are continuously collected
- Several cost-intensive preprocessing steps to extract *time dependent “features”*
  - E.g. Map matching: GPS data is aligned with the road network
- Road network edges are dynamically updated (*e.g. every 5 minutes*) and *new traffic snapshots* are created

# Map Matching



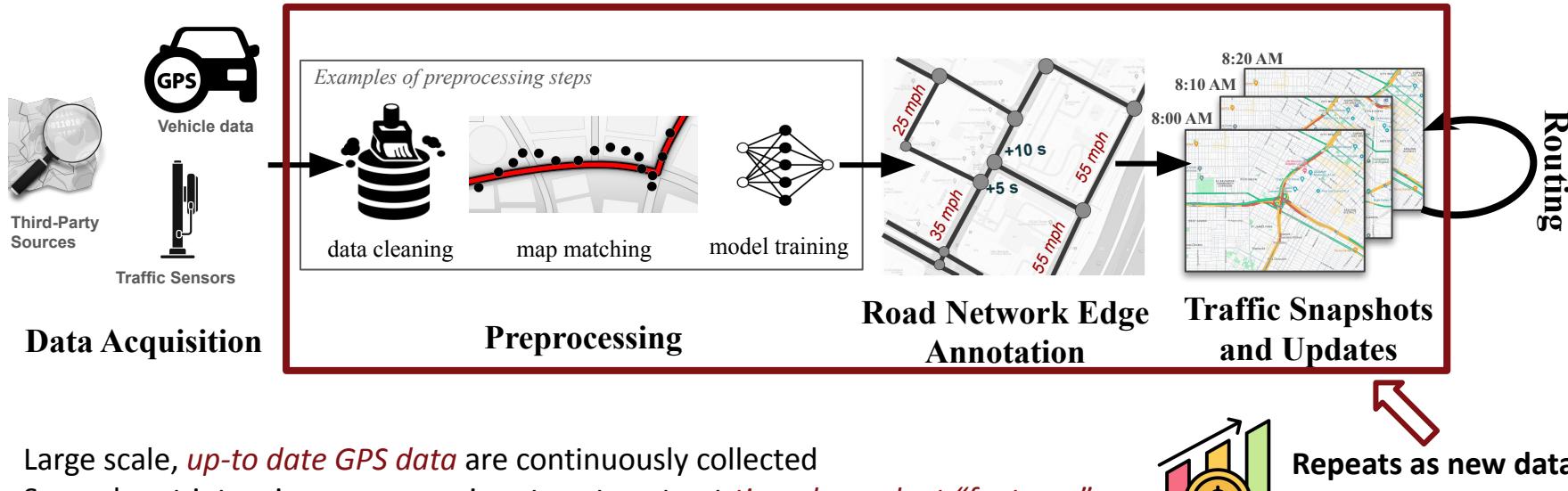
Example of a driver trajectory



- Lyft and Uber use map matching to:
  - To compute the distance travelled by a driver to calculate the fare
  - Dispatch decisions and to display the drivers' cars on the rider app
  - Detect reckless driving
- Why map matching for Origin-Destination Queries?
  - Map past trajectories to road segments
    - Utilize the features of those segments
- Approaches for Map Matching
  - Hidden Markov Model: *Newson & Krumm @ SIGSPATIAL '09* [1]
    - DiDi's IJCAI-19 Tutorial [2]
    - Map Matching @ Uber [3]



# Typical Pipeline of Routing Services



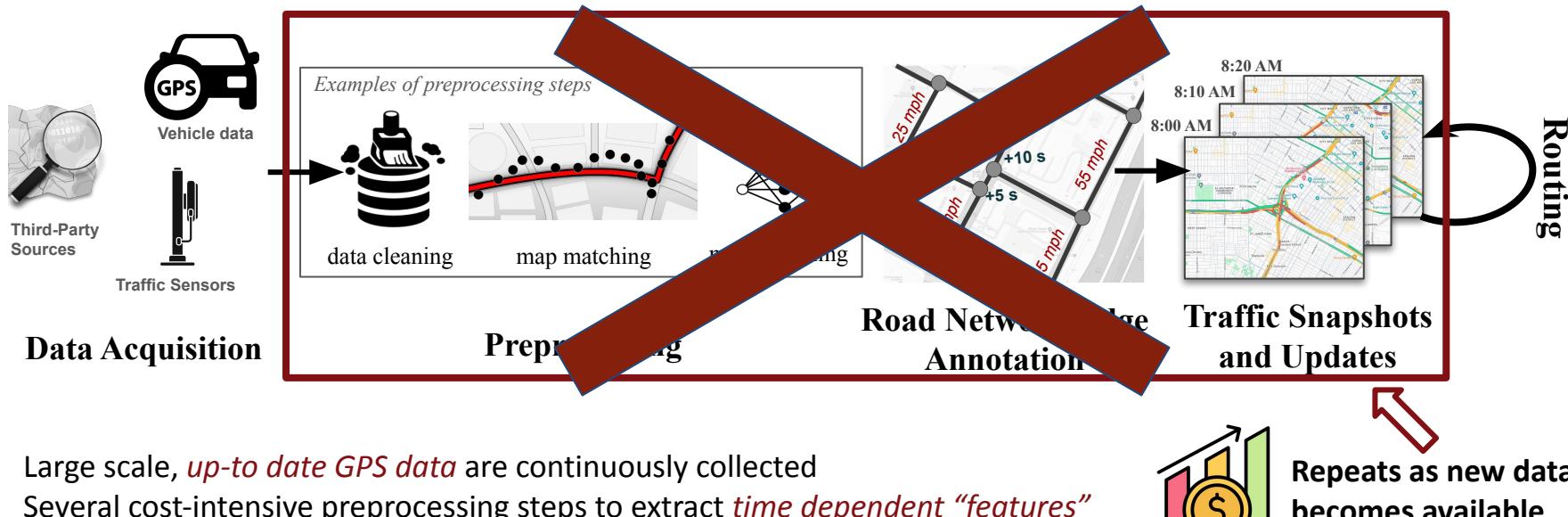
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- Several cost-intensive preprocessing steps to extract *time dependent "features"*
  - E.g. Map matching: GPS data is aligned with the road network
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Repeats as new data becomes available

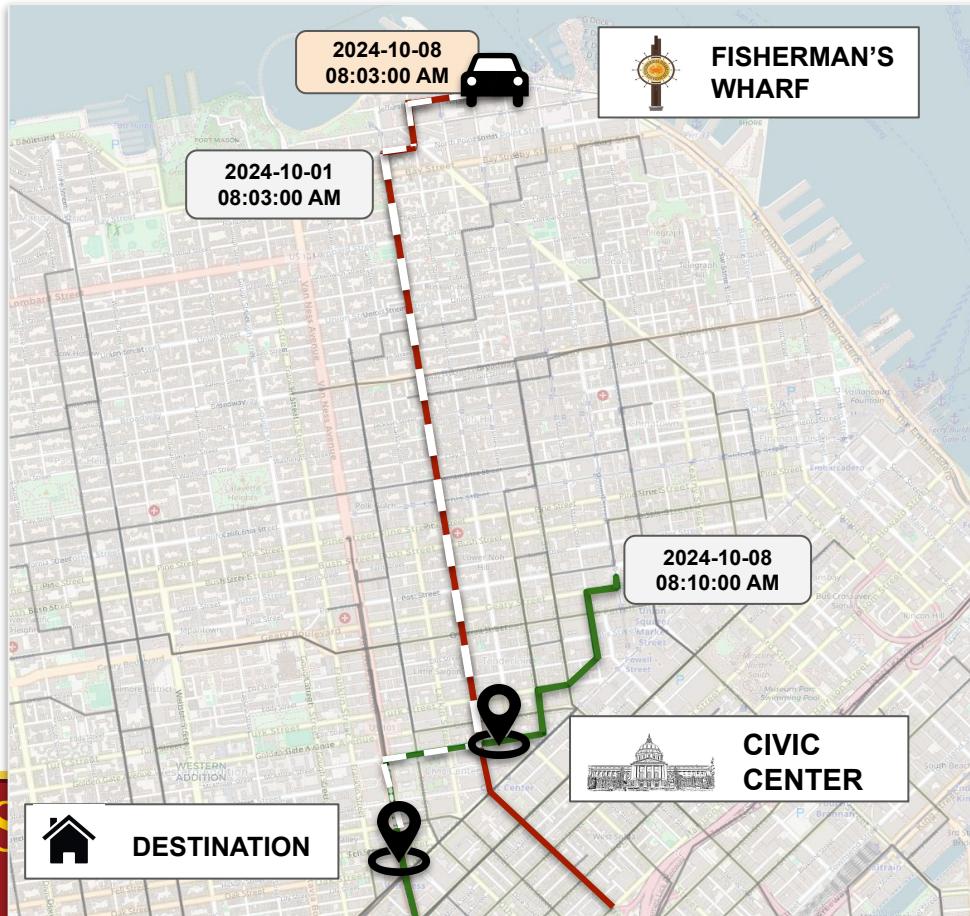


# Typical Pipeline of Routing Services





# TrajRoute (*Motivation*)

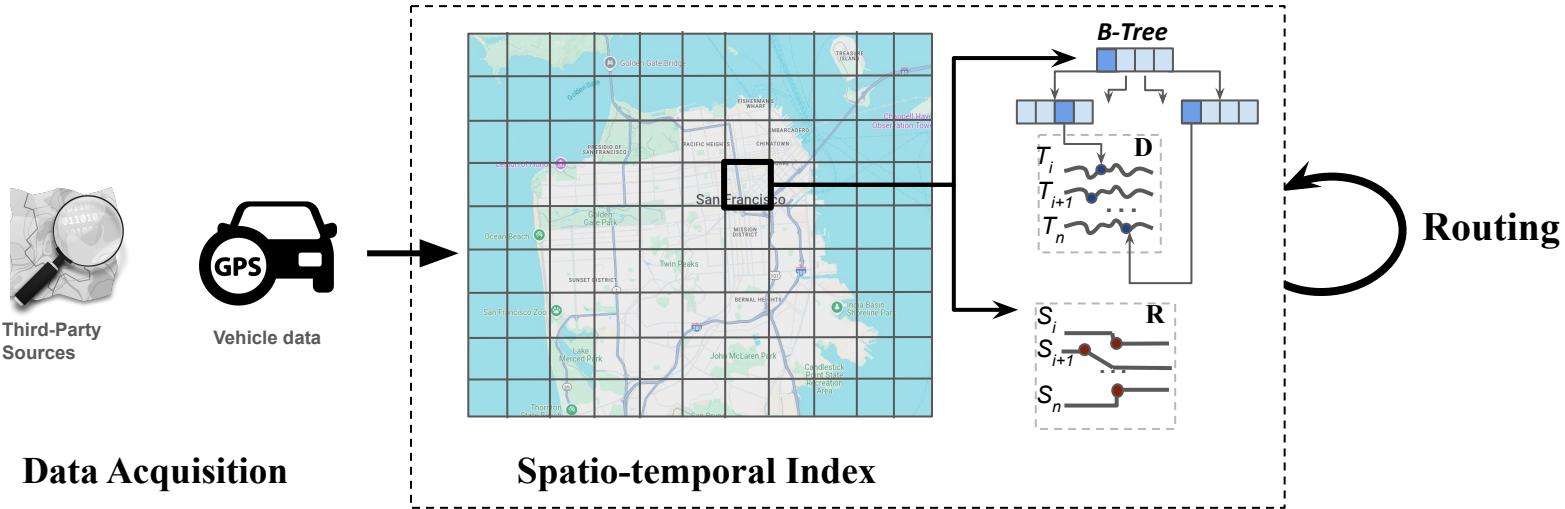


**OR:** Fisherman's Wharf  
**DEST:** Home  
**Time:** 08:03:00 AM





# TrajRoute: Approach



- Routing based on *raw historical trajectories*
  - Ensure that only trajectories that are *spatially* and *temporally* close to current position are considered
- *Fallback* to the road network when trajectories are not available

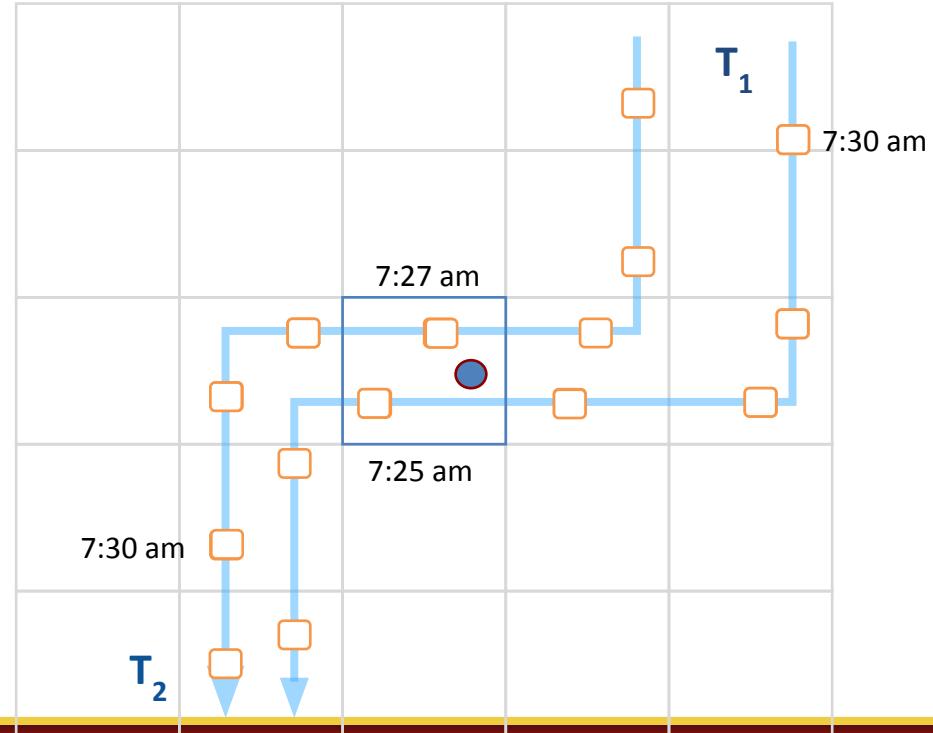
# TrajRoute: Approach



□ GPS Point:  $\langle p_j = (\text{lat}_j, \text{lon}_j), \text{ts}_j \rangle$

Query Point:  $\langle p_{\text{OR}}, p_{\text{DEST}}, \text{dtime} \rangle$

Current Position( ● , 7:30am)



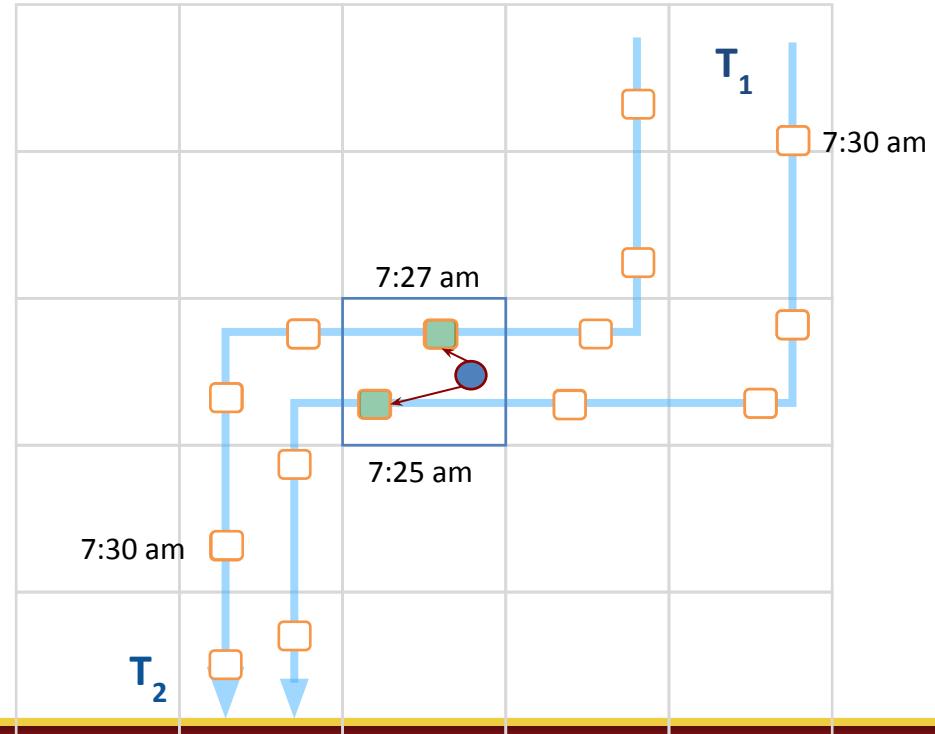
# TrajRoute: Approach



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# TrajRoute: Approach

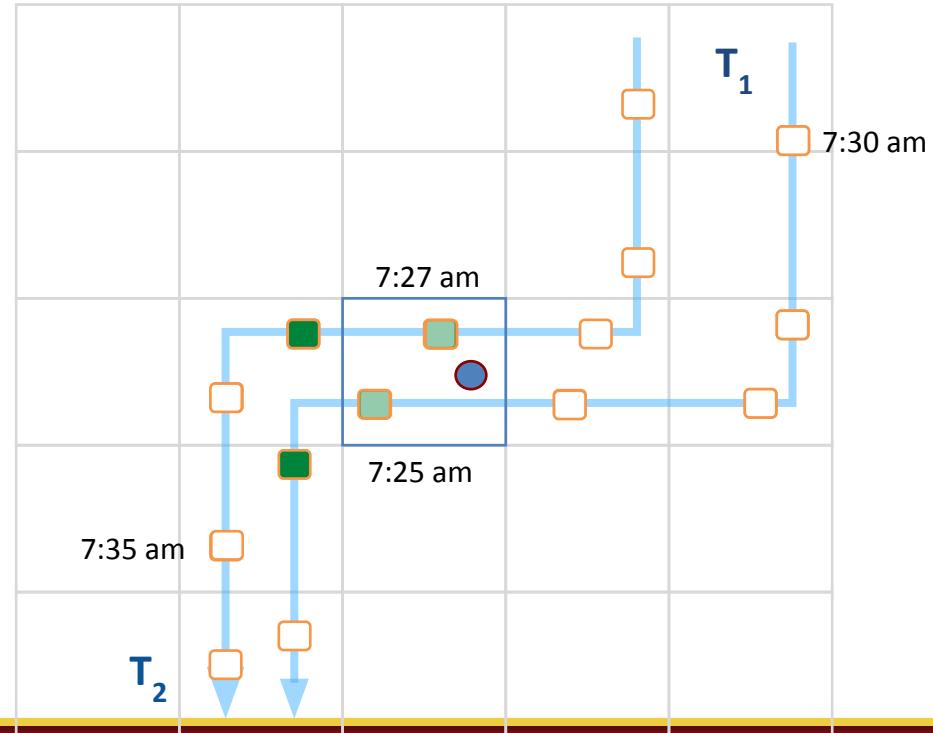


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Query Point:  $\langle p_{\text{OR}}, p_{\text{DEST}}, \text{dtime} \rangle$

■ Trajectory neighbors to  $\bullet$

Current Position(  $\bullet$  , 7:30am)



# TrajRoute: Approach



□ GPS Point:  $\langle p_j = (\text{lat}_j, \text{lon}_j), \text{ts}_j \rangle$

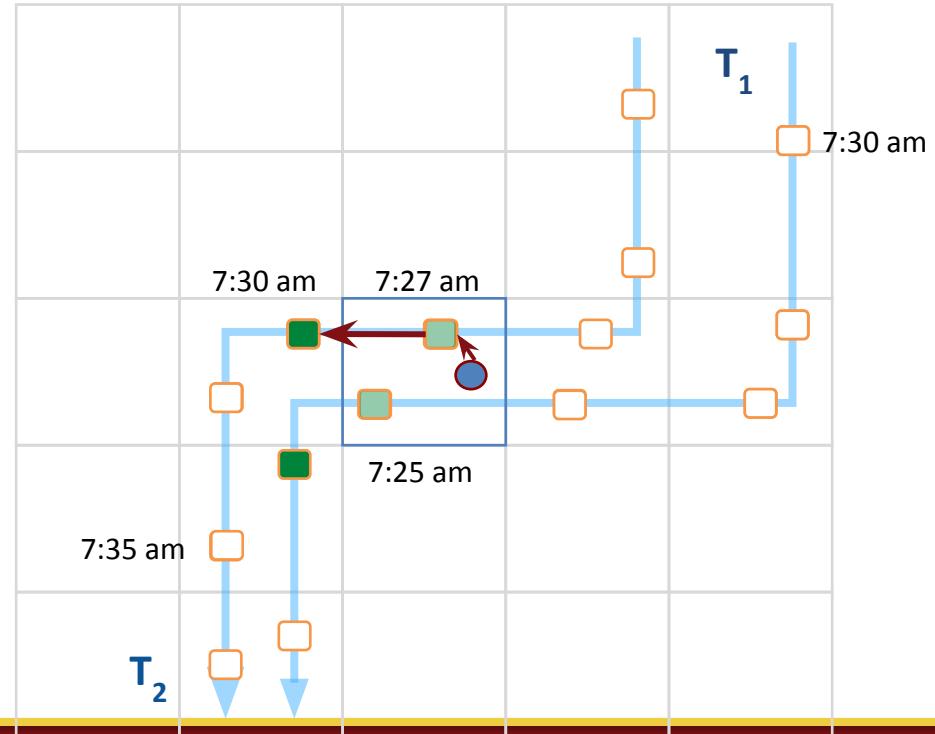
Query Point:  $\langle p_{\text{OR}}, p_{\text{DEST}}, \text{dtime} \rangle$

■ Trajectory neighbors to

$$C_{\text{traj}}(\text{Current Position}, \text{Trajectory neighbors}) = T_C + ts(\text{neighbor}) - ts(\text{Current Position})$$

-  $T_C$ : Cost of transition, constant, depends on the size of the cell

Current Position(, 7:30am)



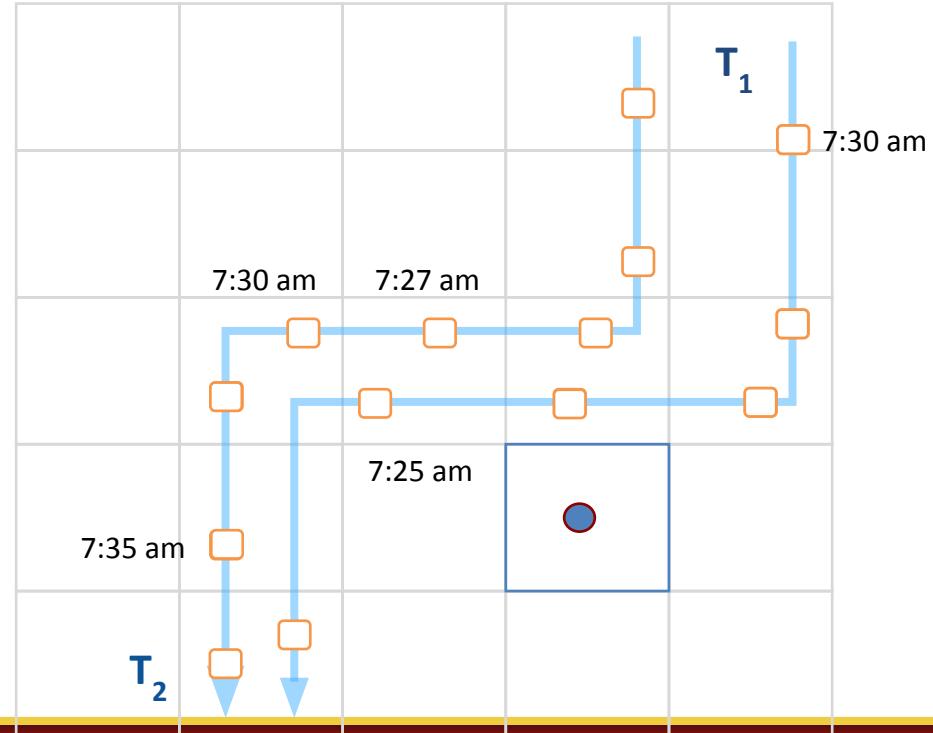
# TrajRoute: Approach



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Current Position( ● , 7:30am)



# TrajRoute: Approach

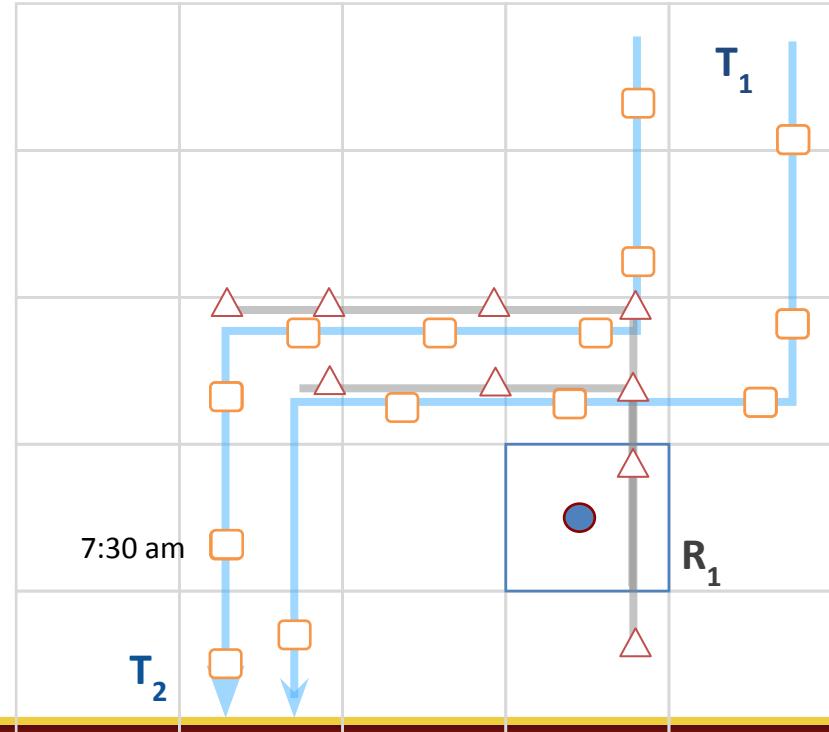


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△ Road Point:  $\langle p_i = (\text{lat}_i, \text{lon}_i) \rangle$

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Current Position( ● , 7:30am)



# TrajRoute: Approach



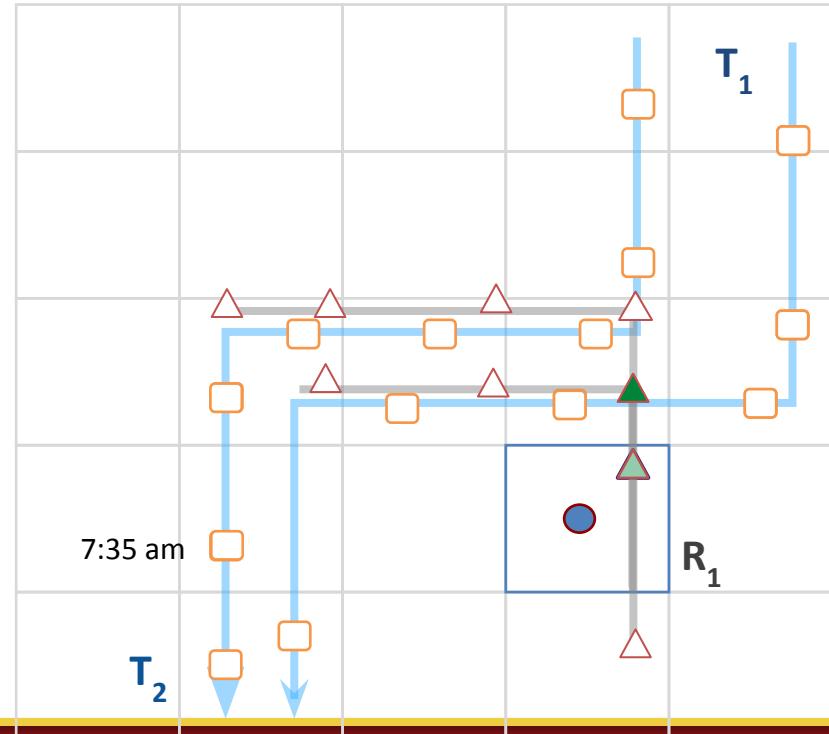
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▲ Road neighbors to

Current Position( , 7:30am)



# TrajRoute: Approach



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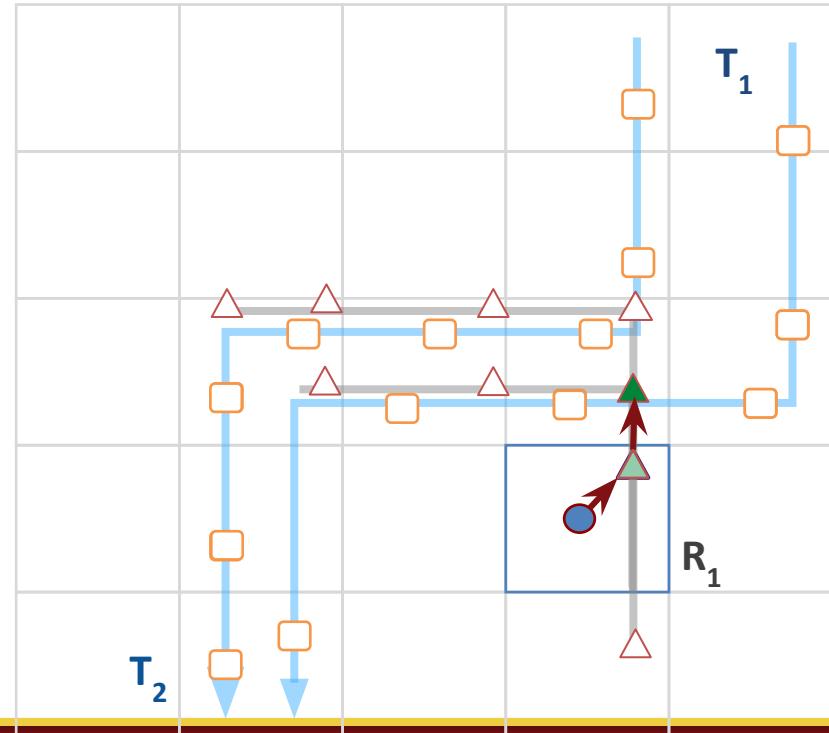
▲ Road neighbors to ●

$$C_{\text{road}}(\bullet, \triangle) = \tau c + [\text{dist}(\triangle, \triangle) / v(\triangle)]$$

- dist: Haversine distance between road points

- v: Speed limit of road segment

Current Position(●, 7:30am)



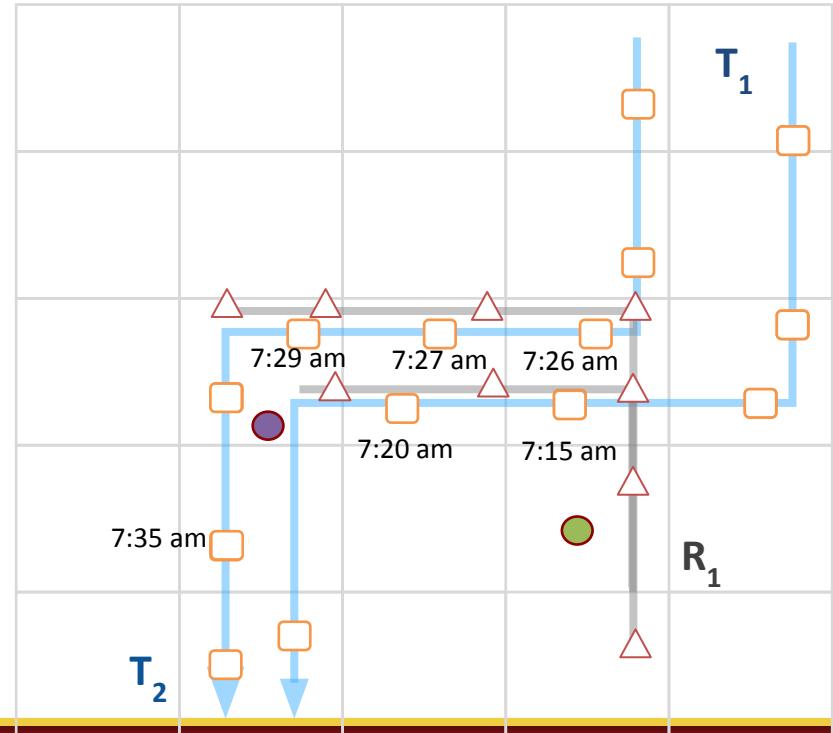
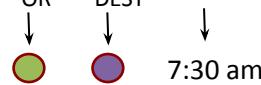
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# TrajRoute: Approach



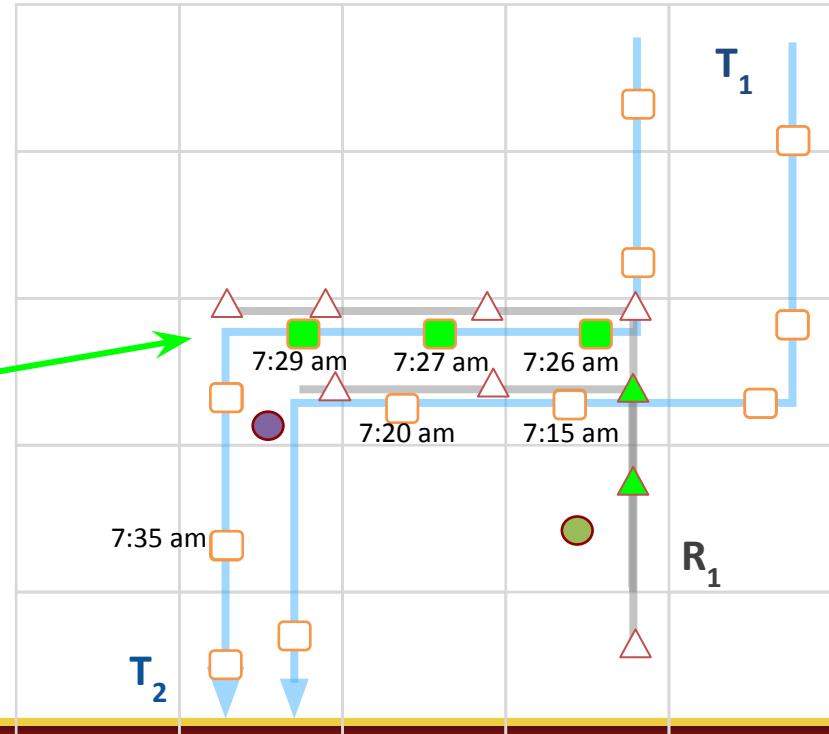
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Query Point:  $\langle p_{\text{OR}}, p_{\text{DEST}}, \text{dtime} \rangle$

↓  
7:30 am  
↓  
7:30 am

Ideal Path



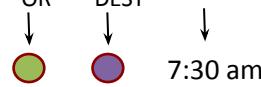
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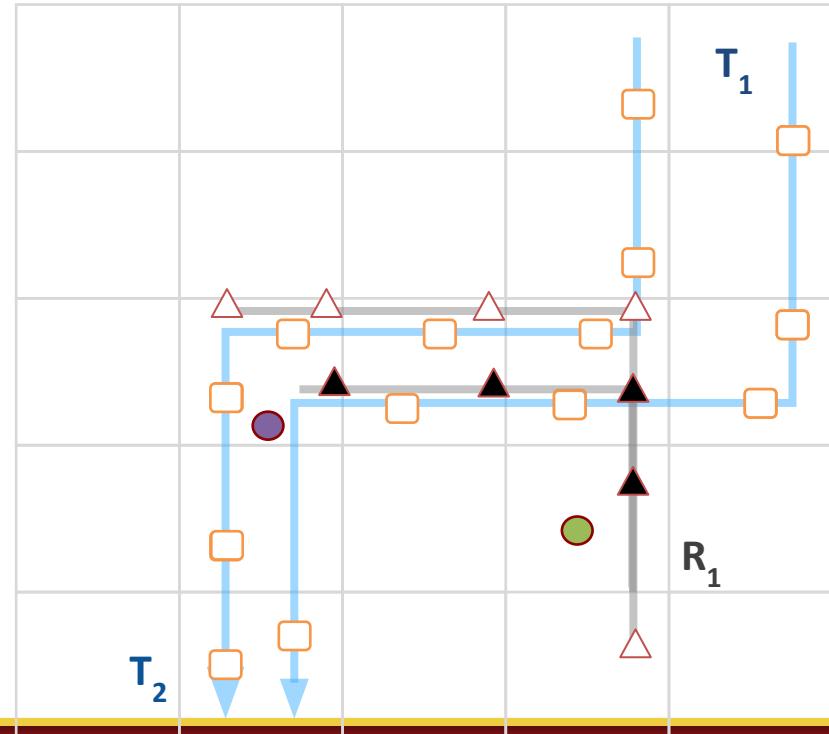
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$C_{\text{road}} < C_{\text{traj}}$ . Why?



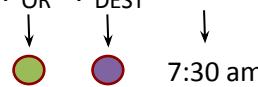
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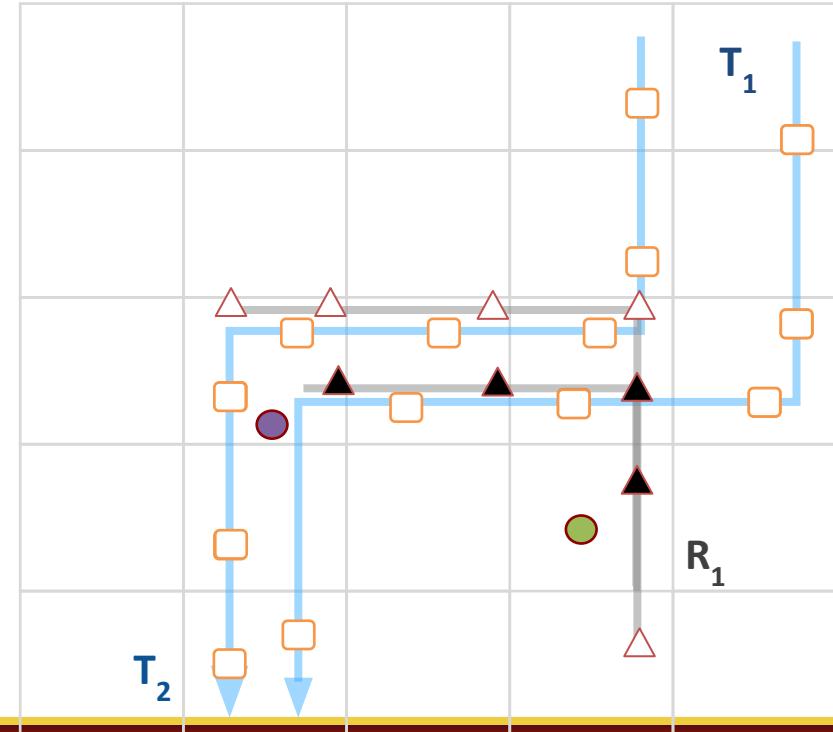
Query Point:  $\langle p_{\text{OR}}, p_{\text{DEST}}, \text{dtime} \rangle$



Using speed limit does not account for:

- Intersection costs
- Acceleration/Deceleration
- Traffic Lights
- Traffic Congestion etc.

Inherently  
encoded in  
trajectory  
timestamps



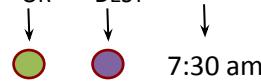
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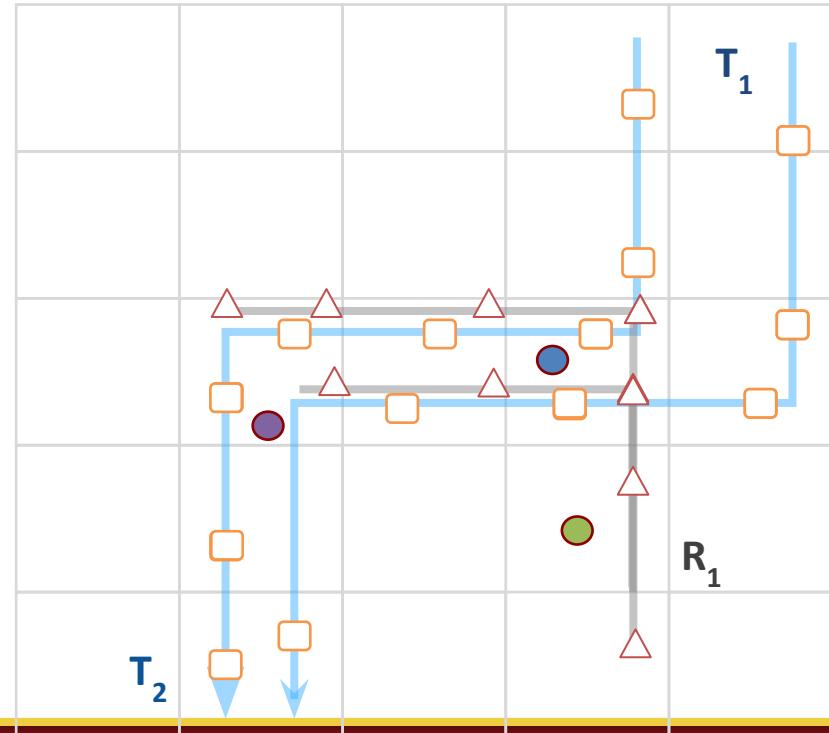
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Current Position( ● , 7:30am)



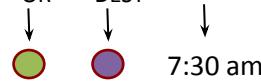
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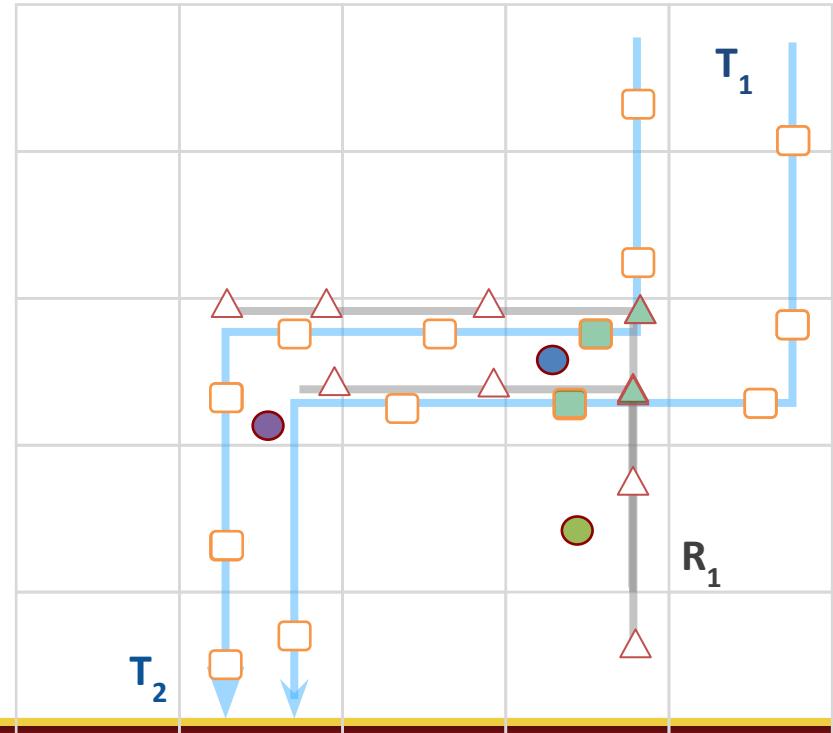
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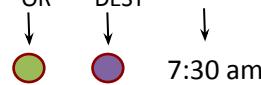
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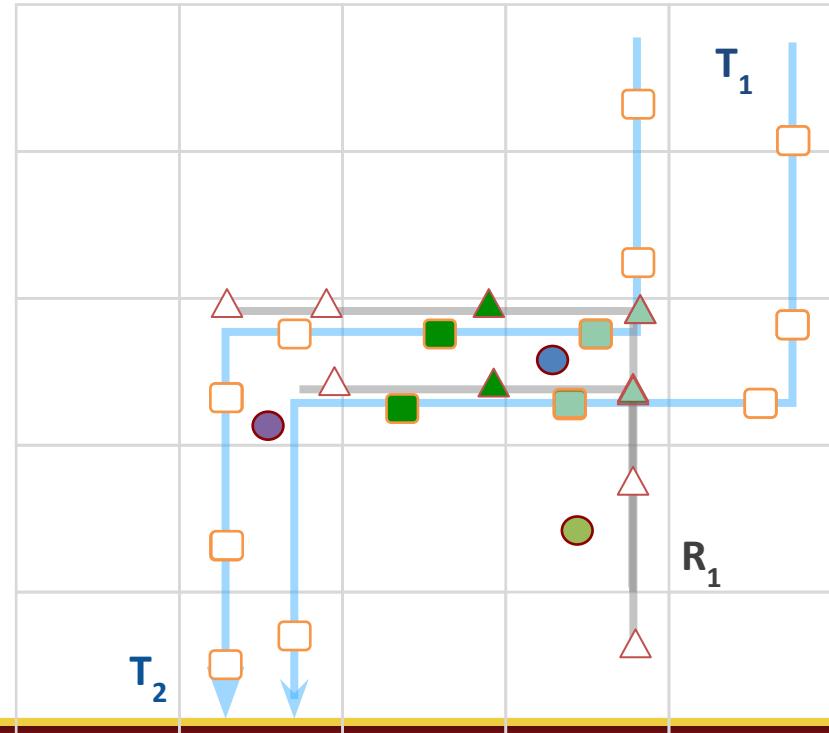
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Query Point:  $\langle p_{\text{OR}}, p_{\text{DEST}}, \text{dtime} \rangle$



$$C_{\text{base}} = \begin{cases} C_{\text{road}} (\text{purple circle}, \text{green triangle}), & \text{green triangle : Road Neighbor} \\ C_{\text{traj}} (\text{purple circle}, \text{green square}), & \text{green square : Traj. Neighbor} \end{cases}$$

Current Position( ● , 7:30am)



# TrajRoute: Approach



□ GPS Point:  $\langle p_j = (\text{lat}_j, \text{lon}_j), \text{ts}_j \rangle$

△ Road Point:  $\langle p_i = (\text{lat}_i, \text{lon}_i) \rangle$

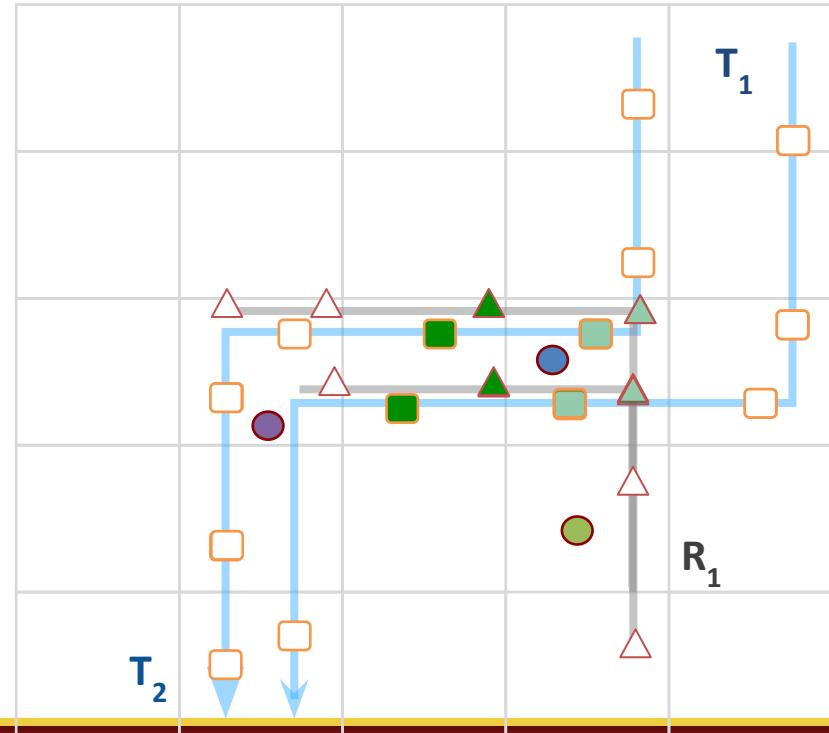
Query Point:  $\langle p_{\text{OR}}, p_{\text{DEST}}, \text{dtime} \rangle$



$$C_{\text{pref}} = \begin{cases} (1 + \alpha) C_{\text{road}}(\text{purple circle}, \text{green triangle}), \text{green triangle} : \text{Road Neighbor} \\ C_{\text{traj}}(\text{purple circle}, \text{green square}), \text{green square} : \text{Traj. Neighbor} \end{cases}$$

-  $\alpha$ : road penalty factor ( $> 0$ )

Current Position( ● , 7:30am)



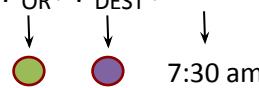


# TrajRoute: Approach

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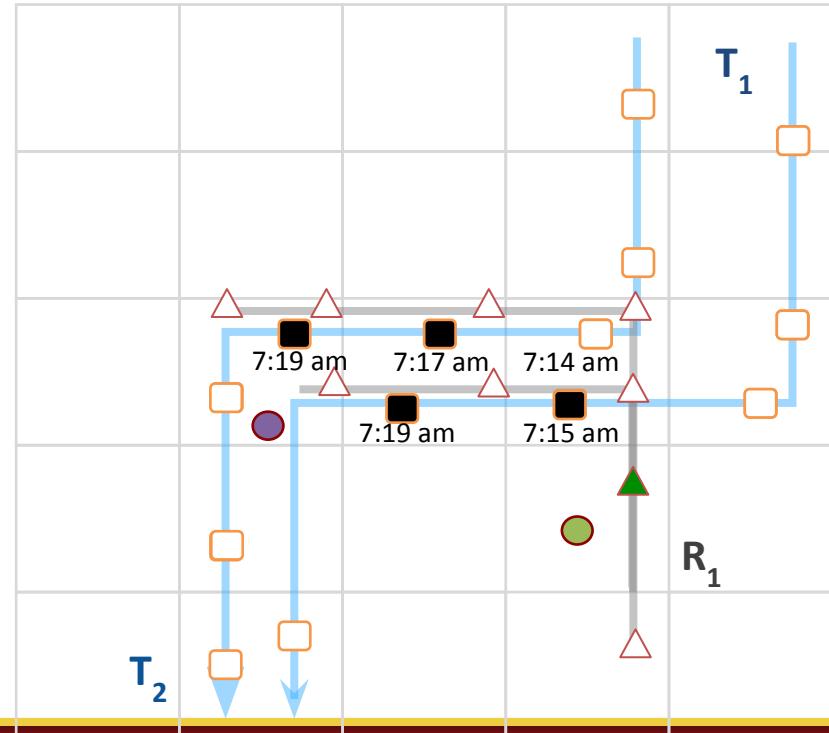
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$$C_{\text{pref}} = \begin{cases} (1 + \alpha) C_{\text{road}}(p_{\text{OR}}, \triangle), \triangle : \text{Road Neighbor} \\ C_{\text{traj}}(p_{\text{OR}}, \square), \square : \text{Traj. Neighbor} \end{cases}$$

-  $\alpha$ : road penalty factor ( $> 0$ )

Current Position (●, 7:30am)



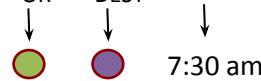
# TrajRoute: Approach



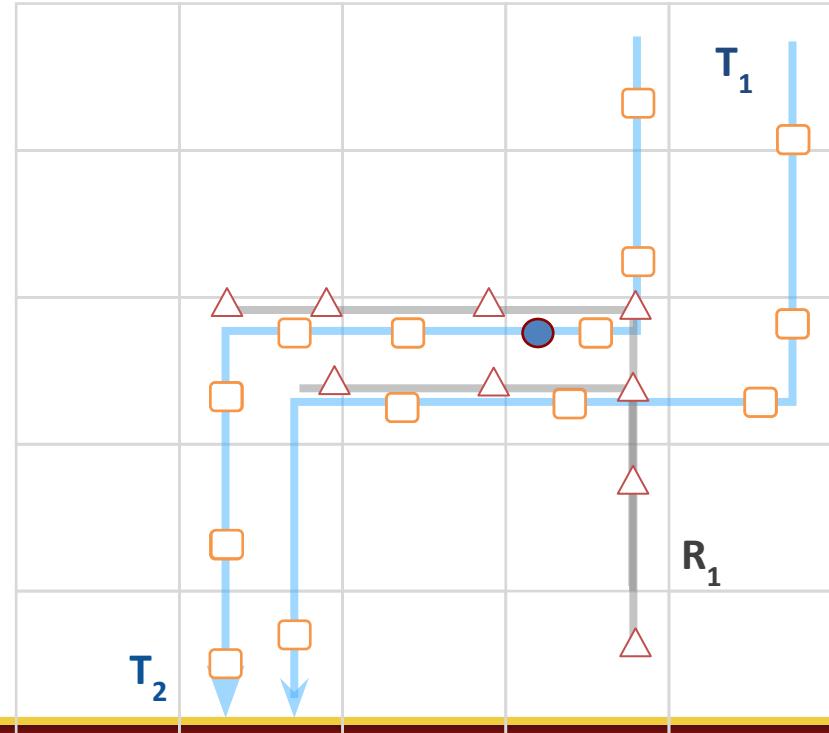
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Query Point:  $\langle p_{\text{OR}}, p_{\text{DEST}}, \text{dtime} \rangle$



Current Position( ● , 7:30am)





# TrajRoute: Approach

□ GPS Point:  $\langle p_j = (\text{lat}_j, \text{lon}_j), ts_j \rangle$

△ Road Point:  $\langle p_i = (\text{lat}_i, \text{lon}_i) \rangle$

Query Point:  $\langle p_{\text{OR}}, p_{\text{DEST}}, \text{dtime} \rangle$

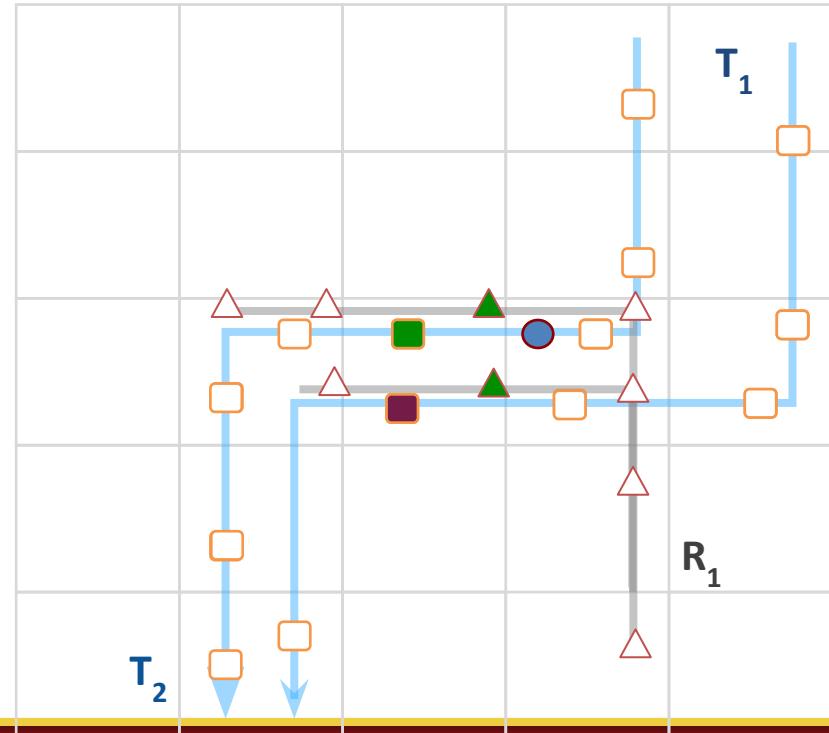


$$C = \begin{cases} (1 + \alpha) C_{\text{road}} (p_{\text{OR}}, \triangle), \triangle : \text{Road Neighbor} \\ e^{-rw} C_{\text{traj}} (p_{\text{OR}}, \square), \square \in T_1, p_{\text{DEST}} \in T_1 \\ C_{\text{traj}} (p_{\text{DEST}}, \blacksquare), \blacksquare \in T_2, p_{\text{OR}} \in T_1 \end{cases}$$

-  $\alpha$ : road penalty factor ( $> 0$ )

-  $rw$ : continuity reward  $\in [0, 1]$

Current Position( ● , 7:30am)



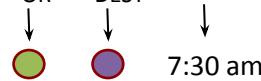
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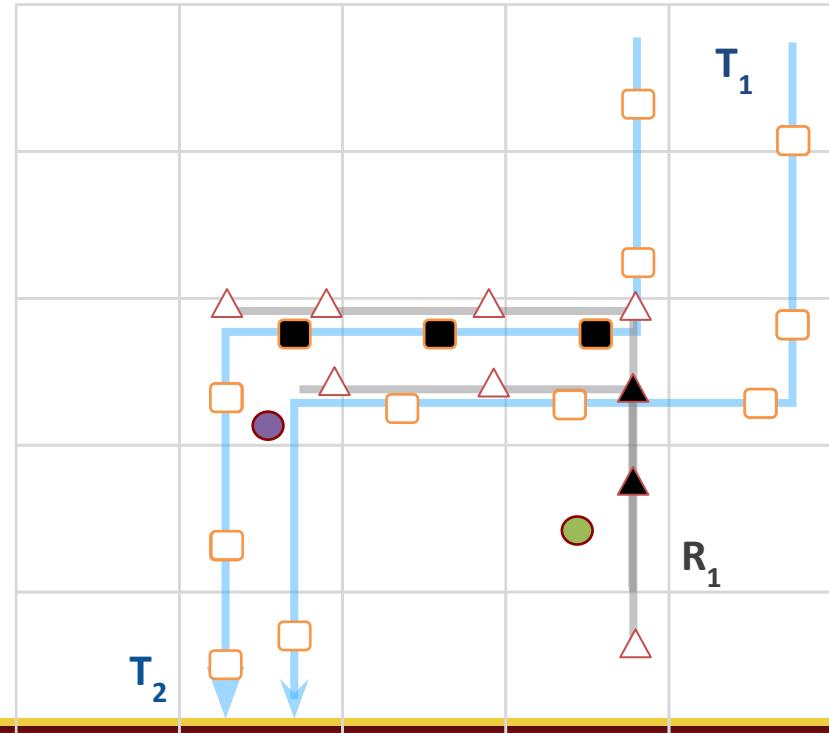
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Current Position( ● , 7:30am)



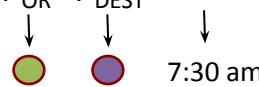
# TrajRoute: Approach



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- Any pathfinding algorithm can be applied.

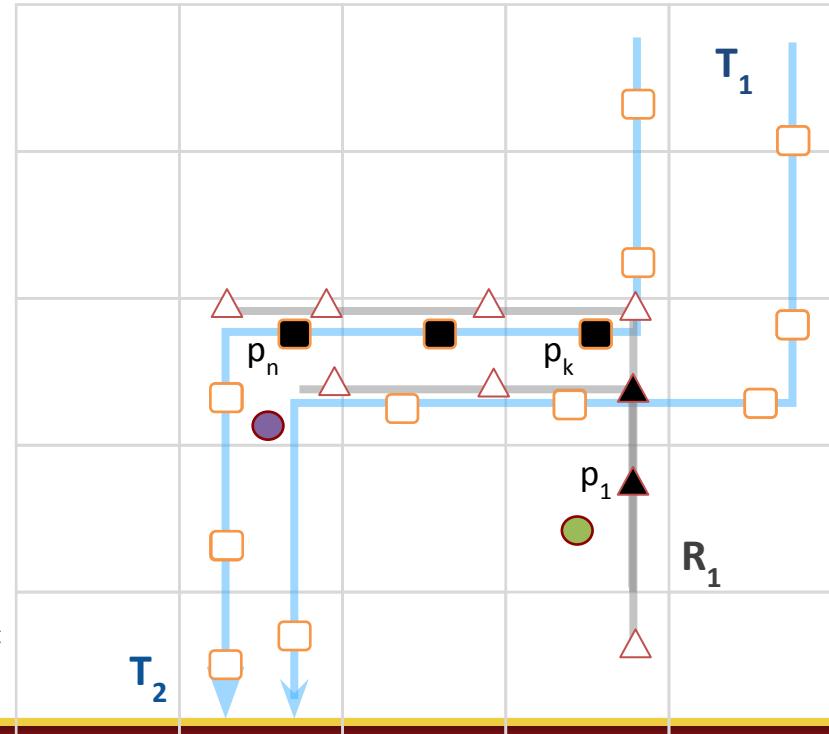
- For Dijkstra:

$$g(p_k) = \sum_{i=1}^{|P|} C(p_{i-1}, p_i), \quad p_i \in P$$

- For A\*:

$$h(p_k) = \frac{\text{dist}(p_k, Q.p_{\text{DEST}})}{v_{\max}} \rightarrow \text{always underestimates the cost}$$

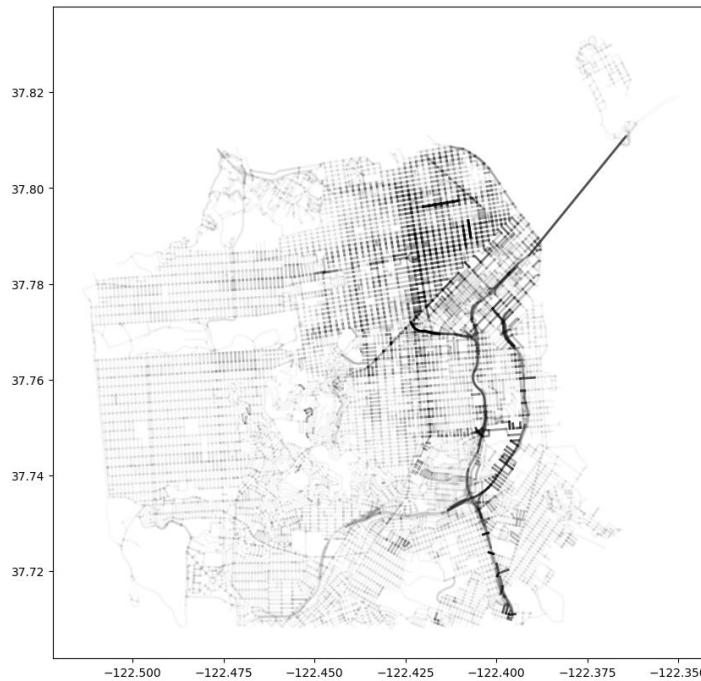
Current Position( ● , 7:30am)





# TrajRoute: Experiments

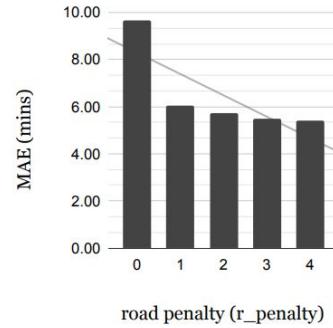
- Data Source
  - San Francisco Taxi Data
  - OSM for road network
- Data Statistics
  - > 1M trajectories, 27.279 roads
  - 99% spatial coverage
  - Peak: ~25%, Off-peak: ~75%
  - Weekend: ~35%, Weekday: ~65%
- Evaluation
  - Random Origin-Destination Queries from trajectories.
  - Comparison of routes with Azure Maps
    - Length of route
    - ETA of route



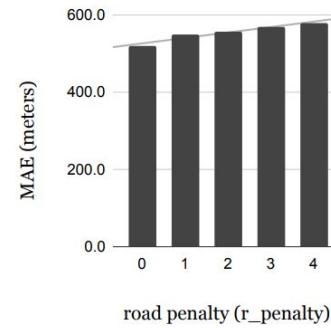


# TrajRoute: Experiments

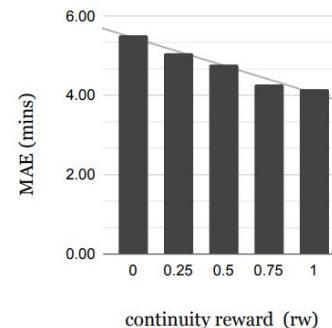
- Data Source
  - San Francisco Taxi Data
  - OSM for road network
- Data Statistics
  - > 1M trajectories, 27.279 roads
  - 99% spatial coverage
  - Peak: ~25%, Off-peak: ~75%
  - Weekend: ~35%, Weekday: ~65%
- Evaluation
  - Random Origin-Destination Queries from trajectories.
  - Comparison of routes with Azure Maps
    - Length of route
    - ETA of route



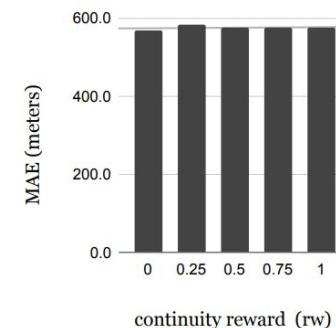
(a) MAE of route travel time



(b) MAE of route distance



(a) MAE of route travel time



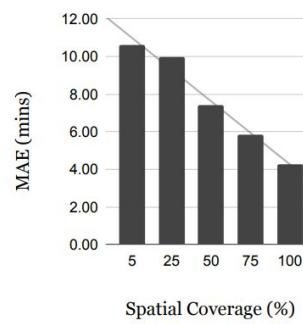
(b) MAE of route distance



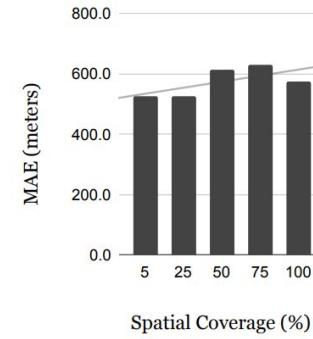
# TrajRoute: Experiments

- Evaluation

- Random Origin-Destination Queries from trajectories.
- Comparison of routes with Azure Maps
  - Length of route
  - ETA of route
- Spatial Coverage
  - Keep trajectories that cover x% of the area
  - Keep  $\alpha=3.0$  and  $rw=0.75$  constant

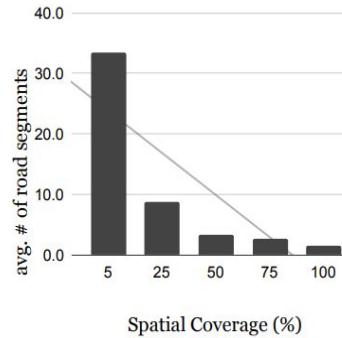


(a) MAE of route travel time



(b) MAE of route distance

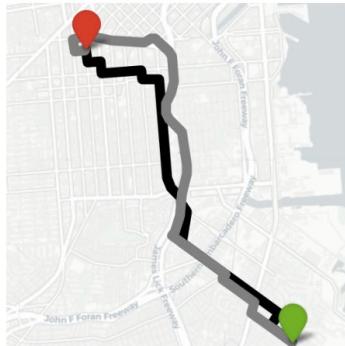
Figure 7: Results for different levels of spatial coverage.



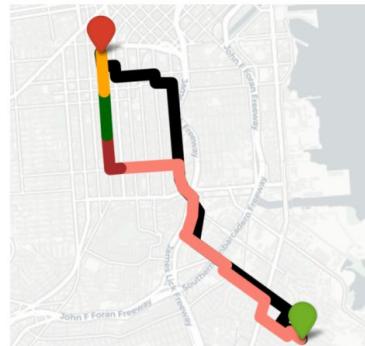


# TrajRoute: Experiments

Query Time: 06:01 PM  
Azure Maps ETA: 21 mins



(a) Route for  $r_{penalty} = 0$  and  $rw = 0$ .  
TrajRoute ETA: 10 mins.



(b) Route for  $r_{penalty} = 3$  and  $rw = 0$ .  
TrajRoute ETA: 16.21 mins.



(c) Route for  $r_{penalty} = 3$  and  $rw = 0.75$ .  
TrajRoute ETA: 19.53 mins.

Query Time: 01:25 AM  
Azure Maps ETA: 12 mins



(a) Route for  $r_{penalty} = 0$  and  $rw = 0$ .  
TrajRoute ETA: 8.3 mins.



(b) Route for  $r_{penalty} = 3$  and  $rw = 0$ .  
TrajRoute ETA: 10.15 mins.

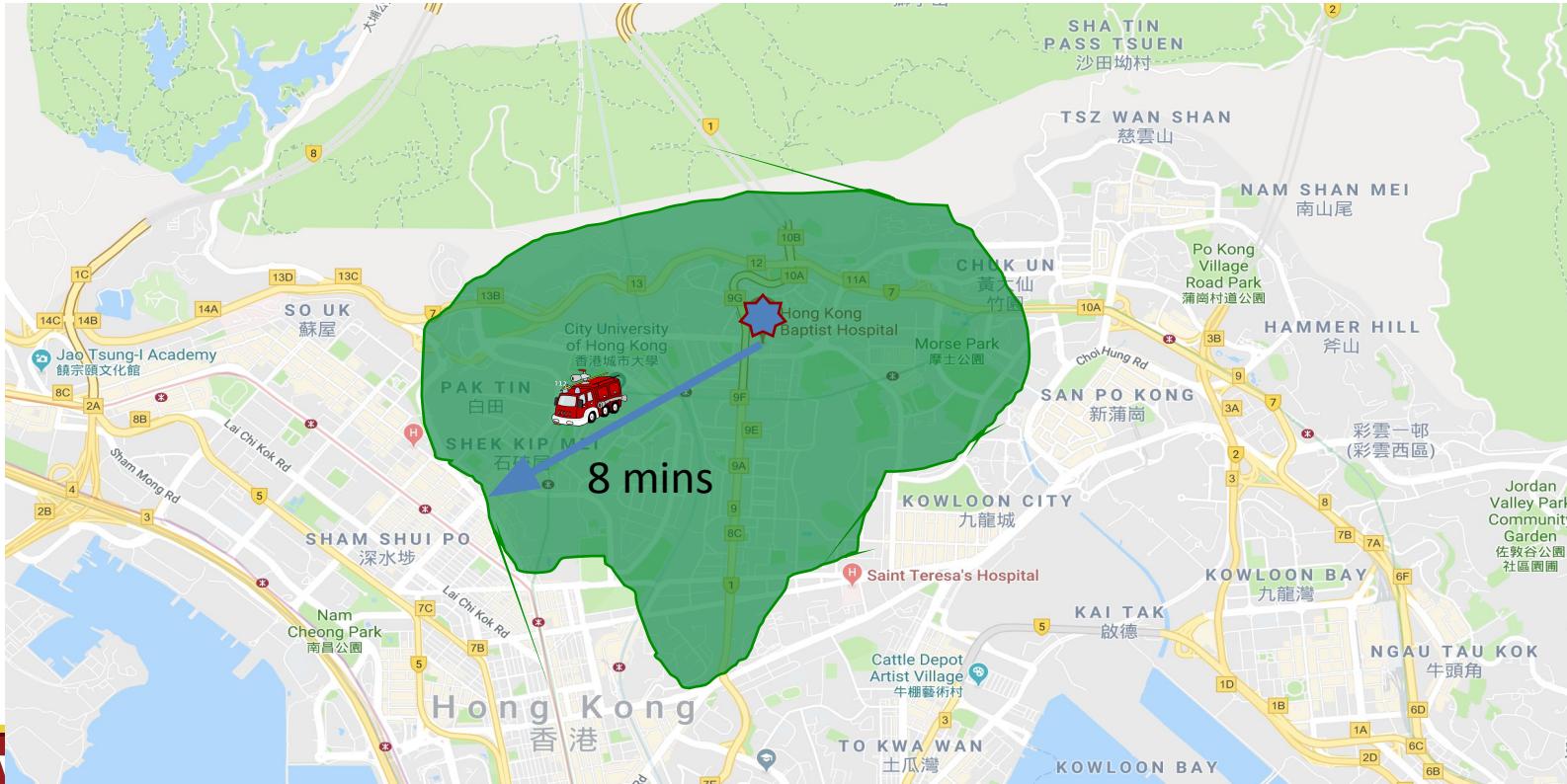


(c) Route for  $r_{penalty} = 3$  and  $rw = 0.75$ .  
TrajRoute ETA: 11.58 mins.

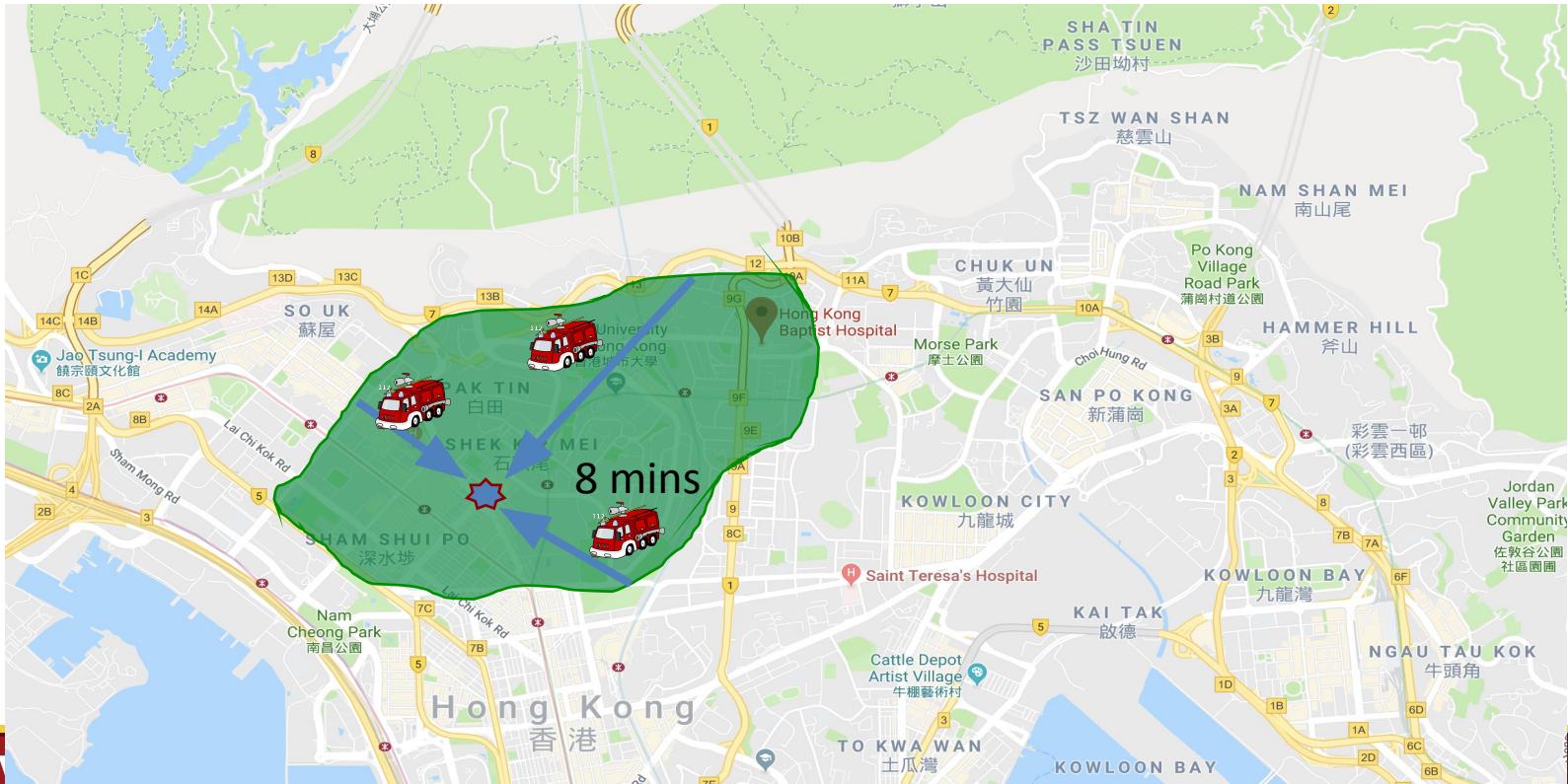


# Isochrone Maps

\*Actual travel times might vary



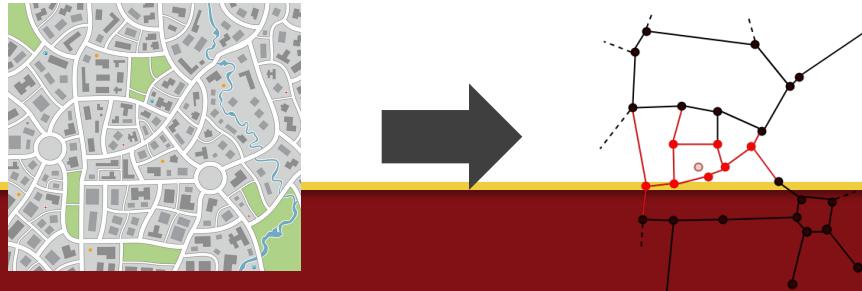
# Reverse Reachability Analysis



# Graph-based Approaches



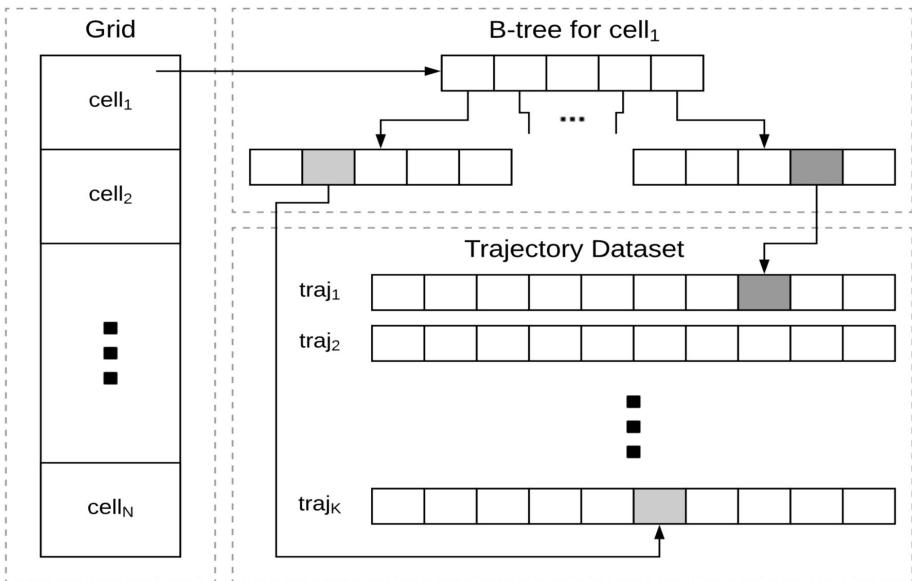
- **Isochrone maps** extensively studied in the databases community
- In **graph theory** defined as the minimal subgraph that can be reached from a query vertex given a limited path cost that is equivalent to travel time.
  - More precisely, it's the set of all reachable vertices, fully traversed edges, and possibly partially traversed edges
- Standard solutions are based on **Dijkstra's** (or **Dreyfus**) shortest path algorithm
  - [ICDE'06] Finding fastest paths on a road network with speed patterns, *Kanoulas et al.*
  - [GIS'08] Computing isochrones in multi-modal, schedule-based transport networks, *Bauer et al.*
  - [EDBT'08] Finding time-dependent shortest paths over large graphs, *Ding et al.*
  - [CIKM'11] Defining isochrones in multi-modal spatial networks, *Gamber et al.*
  - [SEA'16] Fast exact computation of isochrones in road networks, *Baum et al.*



# Data-Driven Reachability



- Remove the *expensive map-matching* step
  - Can take days to compute time-dependent weights for big data
- Remove the traversal step of *complex graphs*
  - The higher the query time limit the more edges need to be explored
- Compute isochrone maps *directly from data*
  - Only process trajectories that satisfy query criteria
- Support multiple *Reachability Queries*
  - Single-Source & Multi-Source (Normal)
  - Single-Target & Multi-Target (Reverse)



# Single-Source & Multi-Source Queries

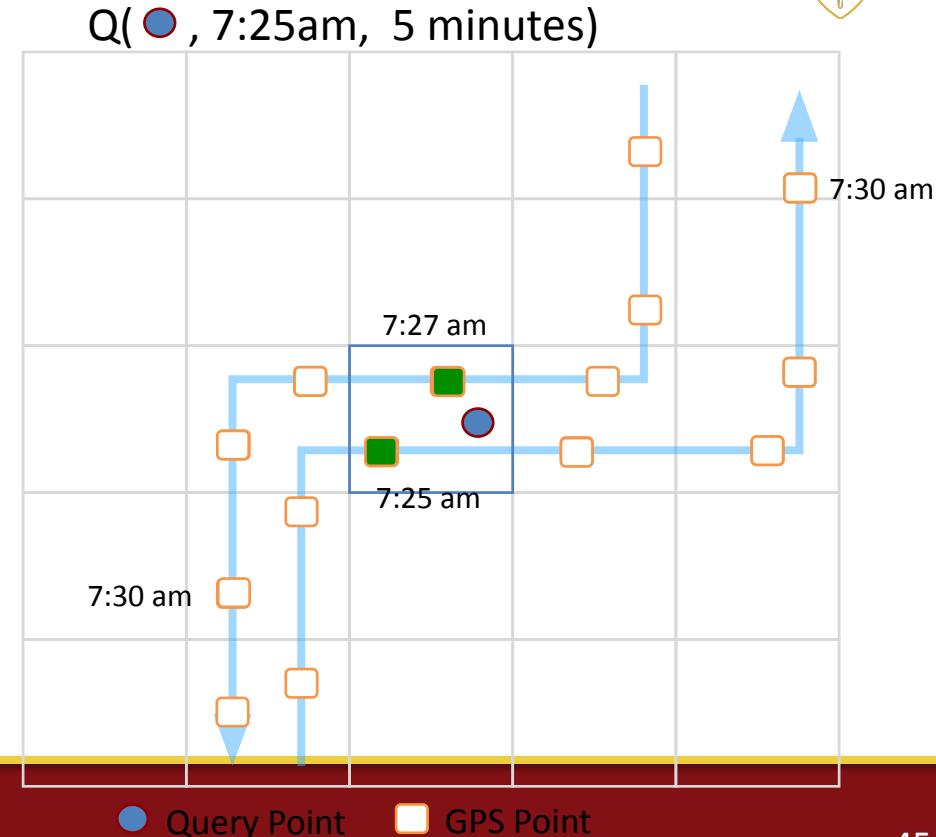


## Reachability Query

-  $Q(s, t, d)$

- s: source location
- t: departure time
- d: time limit in minutes

```
1:  $c \leftarrow findCell(G, Q.s)$ 
2:  $r \leftarrow \{\} // Initialize result to empty set$ 
3: for  $(traj, i) \in c.gpsInWindow(Q.t, Q.t + Q.d)$  do
4:   while  $i < traj.length$  and  $traj[i].ts \leq Q.t + Q.d$  do
5:      $r \leftarrow r \cup \{traj[i].loc\}$ 
6:      $i \leftarrow i + 1$ 
7:   end while
8: end for
9: return  $r //$  Return the set of all reachable points
```



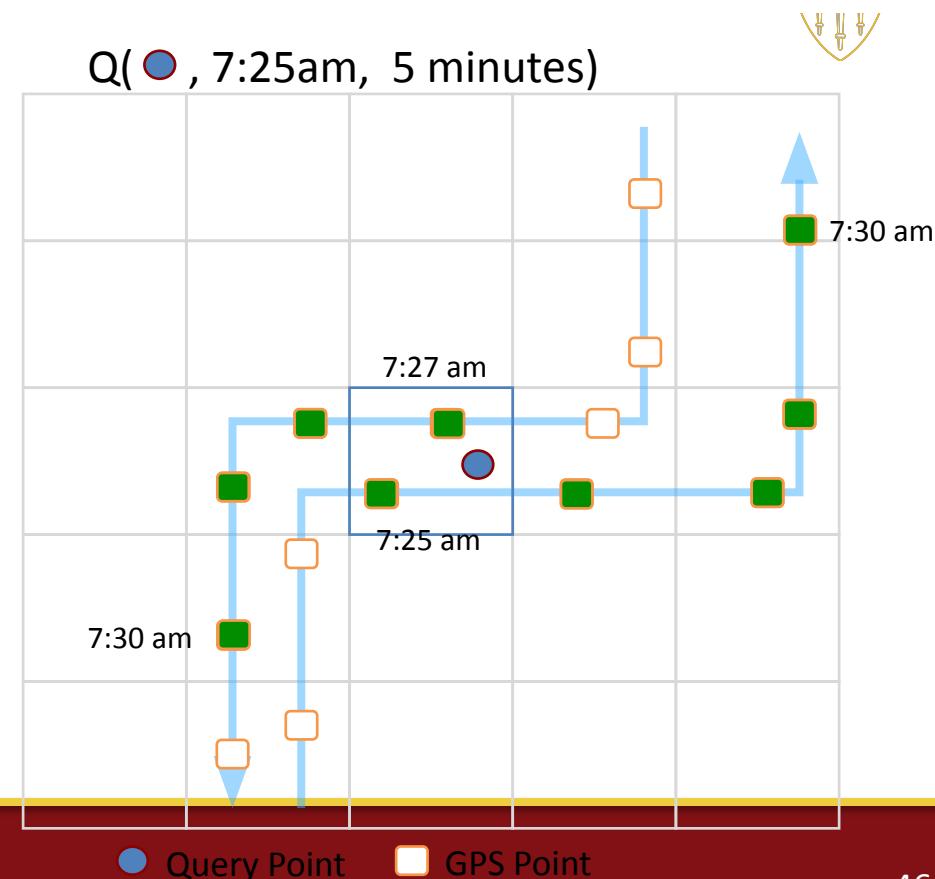
# Single-Source & Multi-Source Queries

## Reachability Query

-  $Q(s, t, d)$

- s: source location
- t: departure time
- d: time limit in minutes

```
1:  $c \leftarrow findCell(G, Q.s)$ 
2:  $r \leftarrow \{\} // Initialize result to empty set$ 
3: for  $(traj, i) \in c.gpsInWindow(Q.t, Q.t + Q.d)$  do
4:   while  $i < traj.length$  and  $traj[i].ts \leq Q.t + Q.d$  do
5:      $r \leftarrow r \cup \{traj[i].loc\}$ 
6:      $i \leftarrow i + 1$ 
7:   end while
8: end for
9: return  $r //$  Return the set of all reachable points
```



# Single-Target & Multi-Target Queries

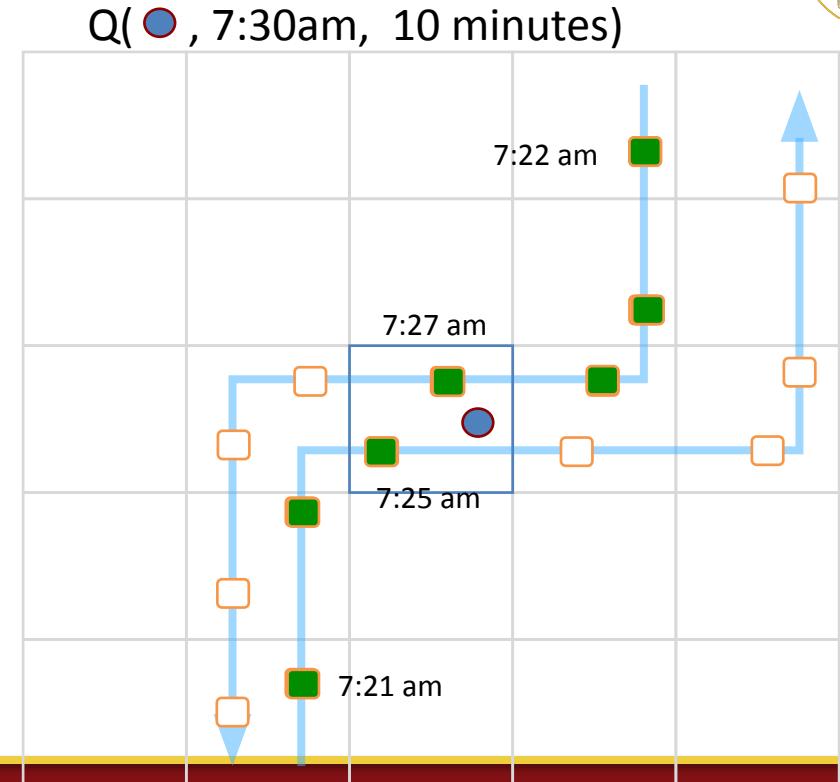


## Reverse Reachability Query

-  $Q(q, t, d)$

- q: target location
- t: arrival time
- d: time limit in minutes

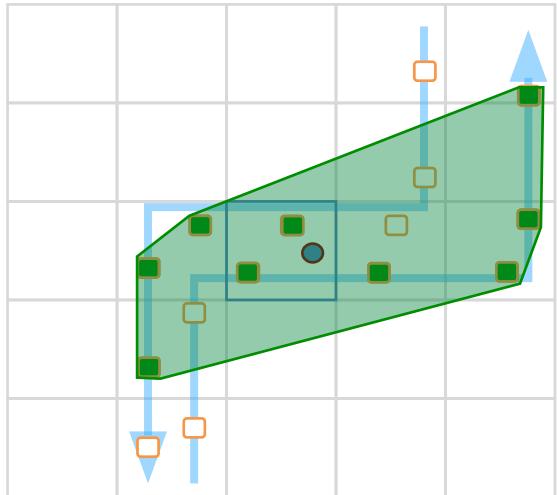
```
1:  $c \leftarrow findCell(G, Q.q)$ 
2:  $r \leftarrow \{\} // Initialize result to empty set$ 
3: for  $(traj, i) \in c.gpsInWindow(Q.t - Q.d, Q.t)$  do
4:   while  $i \geq 0$  and  $Q.t - Q.d \leq traj[i].ts$  do
5:      $r \leftarrow r \cup \{traj[i].loc\}$ 
6:      $i \leftarrow i - 1$ 
7:   end while
8: end for
9: return  $r //$  Return the set of all reachable points
```



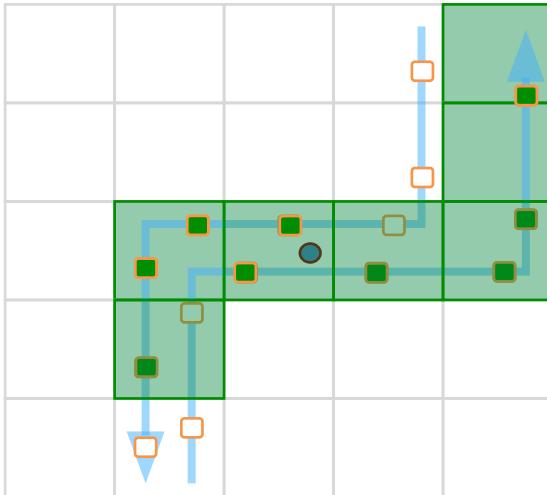
# Visualization Methods



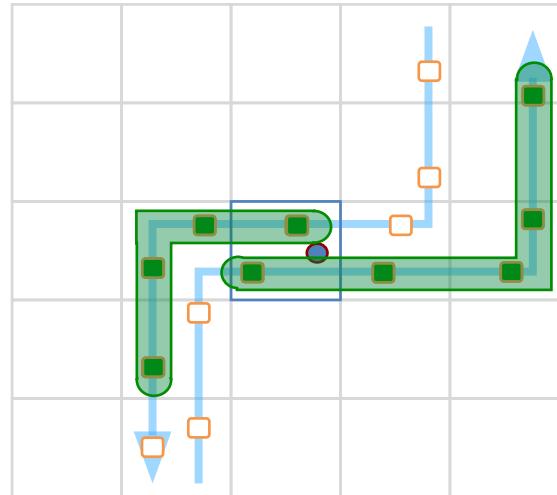
Convex Hull



Cells

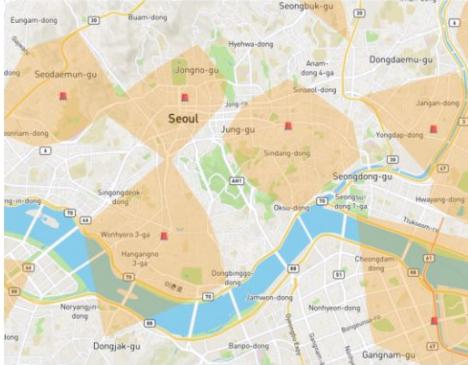


Trajectory Buffer

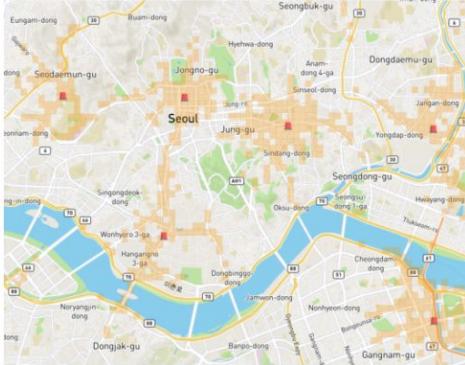


● Query Point    □ GPS Point

# Visualization Methods



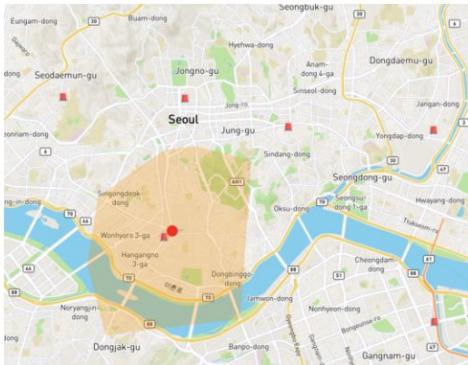
(a) Convex Hull



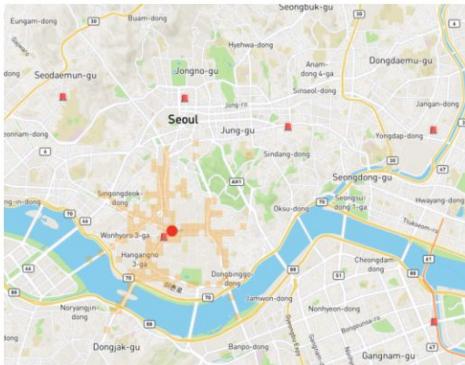
(b) Cells



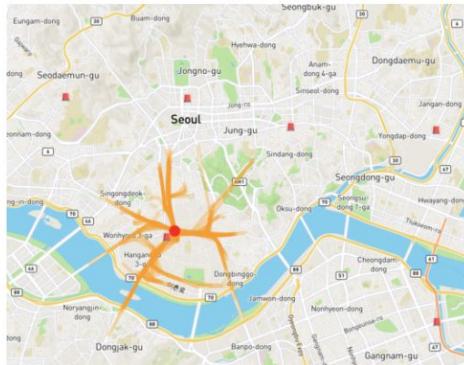
(c) Buffers



(d) Convex Hull Reverse



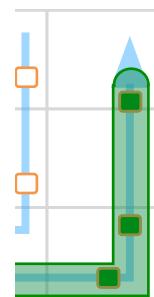
(e) Cells Reverse



(f) Buffers Reverse

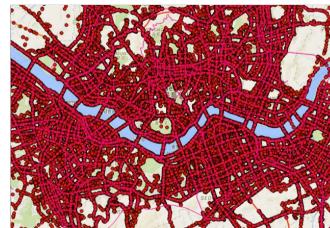


er

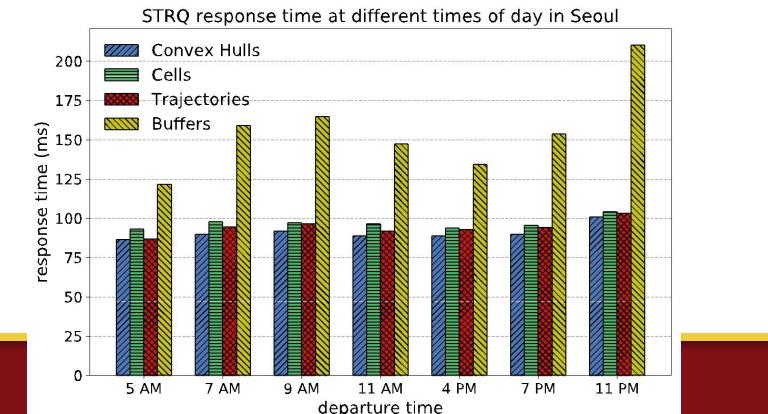
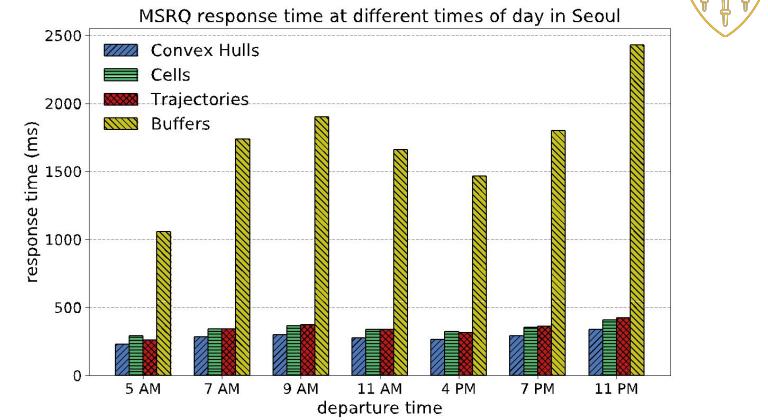
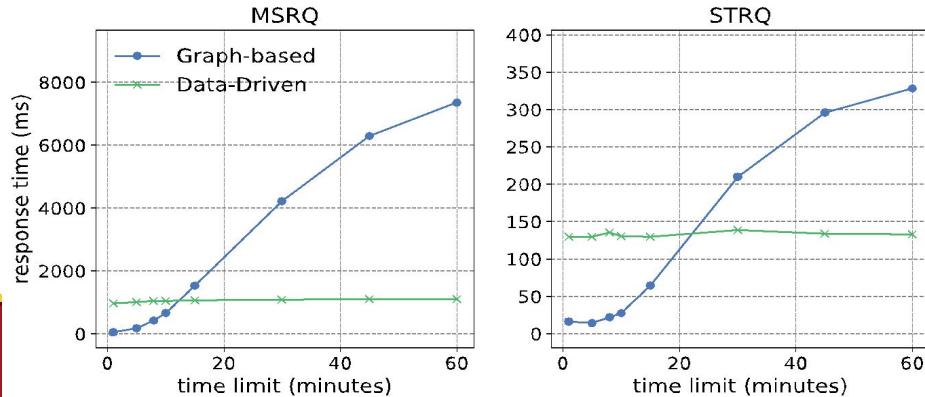


# Experiments

- Data Source
  - Navicall (Seoul Brand Taxi Call Company)
- Data collection period
  - July 2016 – November 2016
- Data Statistics
  - 5,000 taxies
  - 1 min unit sensing data
  - ~600M readings
  - ~50 GB total



Graph-Based vs Data-Driven Query Processing



# References



- [1] Newson P. & Krumm J., "Hidden Markov map matching through noise and sparseness," Proceedings of the 17th ACM SIGSPATIAL International Conference on Advances in Geographic Information Systems GIS 09, 336, 2009
- [2] [DiDi's IJCAI-19 Tutorial: Artificial Intelligence in Transportation](#) (slides 28–40)
- [3] [Map Matching @ Uber](#)
- [4] Anastasiou, C., Huang, C., Kim, S. H., & Shahabi, C. (2019, June). Time-Dependent Reachability Analysis: A Data-Driven Approach. In *2019 20th IEEE International Conference on Mobile Data Management (MDM)* (pp. 138-143). IEEE.
- [5] Siampou, MD., Anastasiou, C., Krumm, J., & Shahabi, C. (2024). TrajRoute: Rethinking Routing with a Simple Trajectory-based Approach: Forget the Maps and Traffic!. To appear @ *IEEE International Conference on Mobile Data Management (MDM)*.