Taming the Duck: Can Stochastic Programming Help?

1. Formulations

1.1. Decision Variables and Parameters

Let $\mathcal{G}$ represent the set of generators, and $\mathcal{T}$ denote the set of discretized time periods. The set of buses is denoted with $\mathcal{B}$ and the set of lines is denoted with $\mathcal{L}$. The generators that are located at a particular bus j are represented with the set $\mathcal{G}_j$. The following variables are defined for each generator $g \in \mathcal{G}$, and period $t \in \mathcal{T}$:

x_{gt}: 1 if generator g is operational at t, 0 otherwise,
s_{gt}: 1 if generator g becomes online at t, 0 otherwise,
z_{gt}: 1 if generator g becomes offline at t, 0 otherwise,
G_{gt}: Production level of generator g at t,

For each bus $j \in \mathcal{B}$ and period $t \in \mathcal{T}$, we define the following variables:

θ_{jt}: Voltage angle at bus j at t,
L_{jt}: Amount of load shed at bus j at t.
O_{jt}: Amount of over-generation at bus j at t.

The latter two variables are penalized in the objective with the coefficients $L^{penalty}$ and $O^{penalty}$, respectively.

Each generator $g \in \mathcal{G}$ is characterized by the following parameters:

$G_g^{\max}$: Maximum generation capacity,
$G_g^{\min}$: Minimum generation requirement when the generator is online,
$\Delta G_g^{\max}$: Ramp up limit,
$\Delta G_g^{\min}$: Ramp down limit,
UT_g: Minimum required uptime before the generator can become offline,
DT_g: Minimum required downtime before the generator can become online,
c_g^{gen}: Generation cost,
c_g^{start}: Start up cost,
c_g^{noload}: No load cost.

The problem is defined over a topology that is defined by the following characteristics:

$F_{ij,t}^{\min}$ / $F_{ij,t}^{\max}$: Flow lower / upper limits over line $(i,j) \in \mathcal{L}$ at t,
B_{ij}: Susceptance of line $(i,j) \in \mathcal{L}$.

Finally, each bus $j \in \mathcal{B}$ faces a demand D_{jt} at period $t \in \mathcal{T}$.

1.2. Unit Commitment

$$
\min \quad \sum_{g \in \mathcal{G}} \sum_{t \in \mathcal{T}} \left(c_g^{gen} G_{gt} + c_g^{start} s_{gt} + c_g^{noload} x_{gt} \right) + \sum_{j \in \mathcal{B}} \sum_{t \in \mathcal{T}} \left(L^{penalty} L_{jt} + O^{penalty} O_{jt} \right) \tag{1a}
$$

s.t.

State Equations
$$
\left\{ x_{gt} - x_{gt-1} = s_{gt} - z_{gt}, \quad \forall g \in \mathcal{G},\, t \in \mathcal{T}, \right. \tag{1b}
$$

Minimum Up/Downtime Restrictions
$$
\begin{cases} \displaystyle\sum_{j=t-UT_g+1}^{t-1} s_{gt} \le x_{gt}, & \forall g \in \mathcal{G},\, t \in \mathcal{T}, \\ \displaystyle\sum_{j=t-DT_g}^{t} s_{gt} \le 1 - x_{gt}, & \forall g \in \mathcal{G},\, t \in \mathcal{T}, \end{cases} \tag{1c}
$$

Generation Limits
$$
\left\{ G_g^{\min} x_{gt} \le G_{gt} \le G_g^{\max} x_{gt}, \quad \forall g \in \mathcal{G},\, t \in \mathcal{T}, \right. \tag{1d}
$$

Ramping Limits
$$
\left\{ \Delta G_g^{\min} \le G_{gt} - G_{gt-1} \le \Delta G_g^{\max}, \quad \forall g \in \mathcal{G},\, t \in \mathcal{T}, \right. \tag{1e}
$$

Flow Limits
$$
\left\{ F_{ij,t}^{\min} \le F_{ij,t} \le F_{ij,t}^{\max}, \quad \forall (i,j) \in \mathcal{L}, t \in \mathcal{T}, \right. \tag{1f}
$$

Flow Balance
$$
\left\{ \sum_{i \in \mathcal{B}:(i,j) \in \mathcal{L}} F_{ij,t} - \sum_{i \in \mathcal{B}:(j,i) \in \mathcal{L}} F_{ji,t} + \sum_{g \in \mathcal{G}_j} G_{gt} + L_{jt} - O_{jt} = D_{jt}, \quad j \in \mathcal{B}, t \in \mathcal{T}, \right. \tag{1g}
$$

Power Flow Approximation
$$
\left\{ F_{ij,t} = B_{ij}(\theta_{it} - \theta_{jt}), \quad \forall (i,j) \in \mathcal{L}, t \in \mathcal{T}, \right. \tag{1h}
$$

Integrality Restrictions & Variable Bounds
$$
\begin{cases} x_{gt},\, s_{gt},\, z_{gt} \in \{0,1\}, & \forall g \in \mathcal{G},\, t \in \mathcal{T}, \\ G_{gt} \ge 0, & \forall g \in \mathcal{G},\, t \in \mathcal{T}, \\ -\pi \le \theta_{jt} \le \pi,\; L_{jt} \ge 0,\, O_{jt} \ge 0 & \forall j \in \mathcal{B},\, t \in \mathcal{T}, \end{cases} \tag{1i}
$$

In deterministic UC formulations, alternative representations of the above constraints were used to gain computational performance. For brevity, such representations are omitted from this paper.

1.3. Economic Dispatch