Big Problems for Small Networks: Small Network Statistics[1]

George G. Vega Yon, MS Kayla de la Haye, PhD

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[1]Contact: vegayon@usc.edu. We thank members of our MURI research team, USC's Center for Applied Network Analysis, and Andrew Slaughter for their comments.

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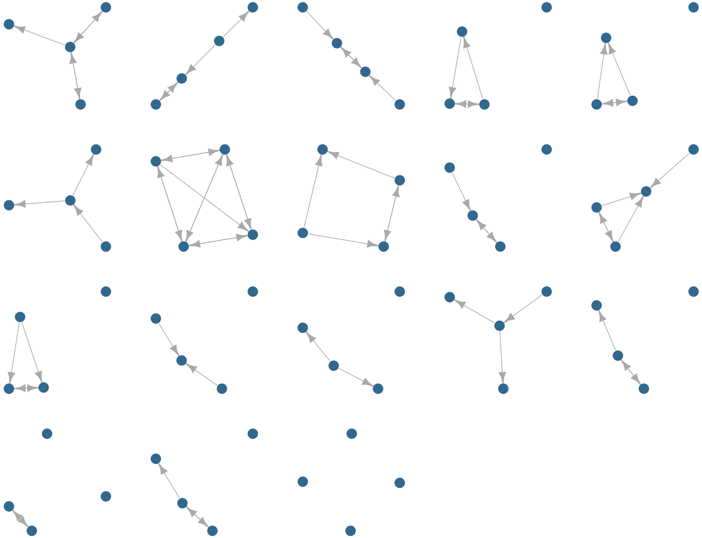
Study motivation

- Overall, a very limited set of social has been tested as predictors of social networks
- Very little research on the emergence of networks in teams

Context (cont'd)



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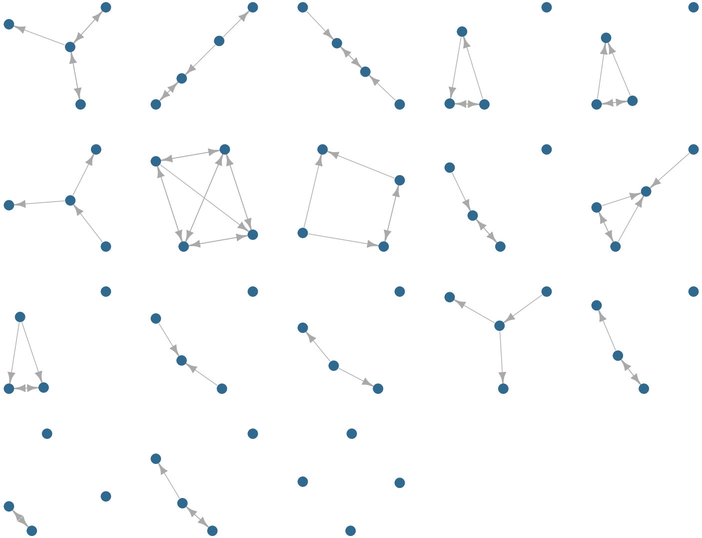

How can we go futher from descriptive statistics?

Small networks and Exponential Random Graph Models

When trying to estimate ERGMs in little networks

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- MCMC fails to converge when trying to estimate a block diagonal (structural zeros) model,
- Same happens when trying to estimate an ERGM for a single (little) graph,
- Even if it converges, the Asymptotic properties of MLEs are no longer valid since the sample size is not large enough.

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- This solves the problem of been able to estimate a small ergm.
- For this we started working on the `lergm` R package (available at https://github.com/USCCANA/lergm).

Example 1

Let's start by trying to estimate an ERGM for a single graph of size 4

```r
library(lergm)
set.seed(12)
x <- sna::rgraph(4)
lergm(x ~ edges + balance + mutual)
```

```
##
## Little ERGM estimates
##
##  Coefficients:
##    edges  balance   mutual
## -1.9443  -0.2417   3.4961
```

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- We still have issues regarding asymptotics.
- We propose to solve this by using a pooled version of the ERGM

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- By estimating a pooled version of the ERGM we can recover the asymptotics of MLEs.
- We implemented this in the `lergm` package

Example 2

Suppose that we have 3 little graphs of sizes 4, 5, and 5:

```r
library(lergm)
set.seed(12)
x1 <- sna::rgraph(4)
x2 <- sna::rgraph(5)
x3 <- sna::rgraph(5)

lergm(list(x1, x2, x3) ~ edges + balance + mutual)
```

```
##
## Little ERGM estimates
##
##  Coefficients:
##   edges  balance   mutual
## -0.3941  -0.2085   1.4156
```

Simulation study

Scenario A

1. Draw parameters for edges and mutual from a uniform(-3, 3).
2. Using those parameters, sampled $n \sim \text{Poisson}(30)$ networks of size 4
3. Estimated the pooled ERGMs using both the MLE and the boostrap version.

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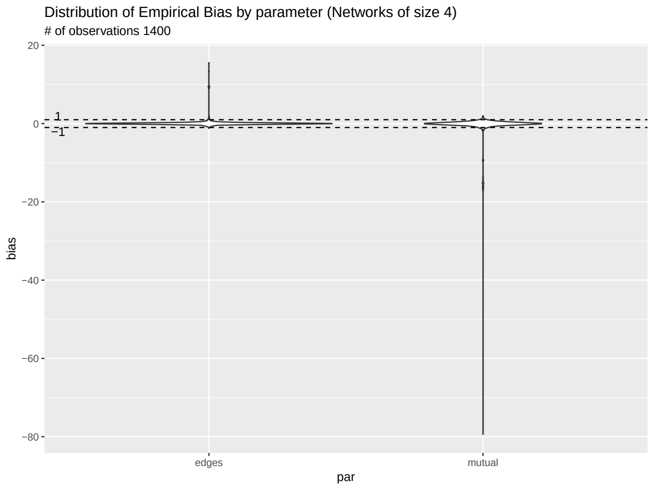
Scenario B

1. Idem.
2. Using those parameters, sampled $n_1 \sim \text{Poisson}(15), n_2 \sim \text{Poisson}(15)$ networks of size 3 and 4 respectively.
3. Idem.

Simulation study: Scenario A

Empirical Bias



Simulation study: Scenario A

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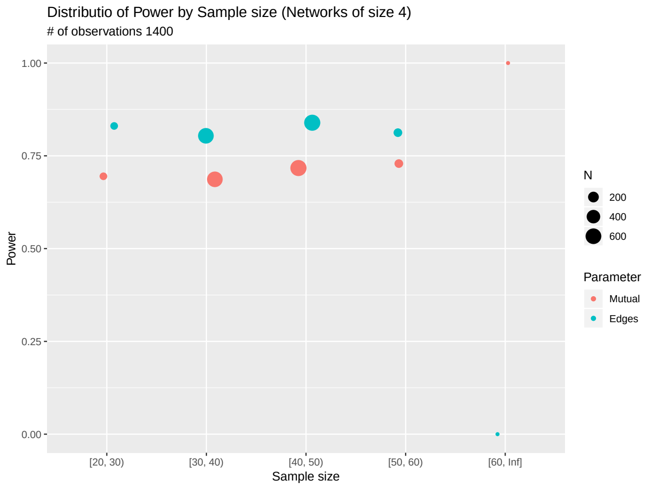
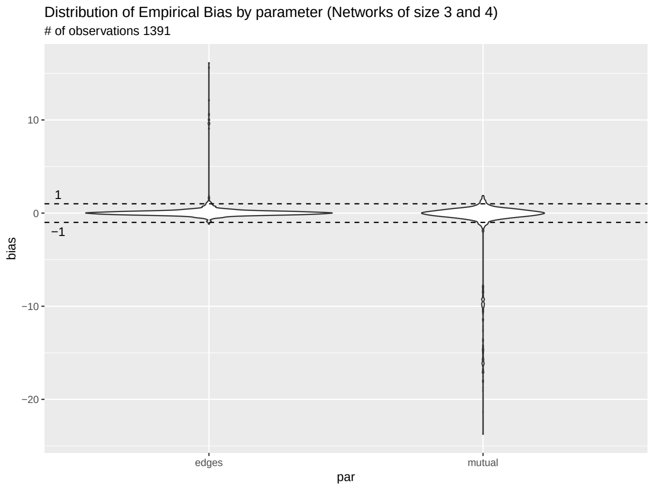


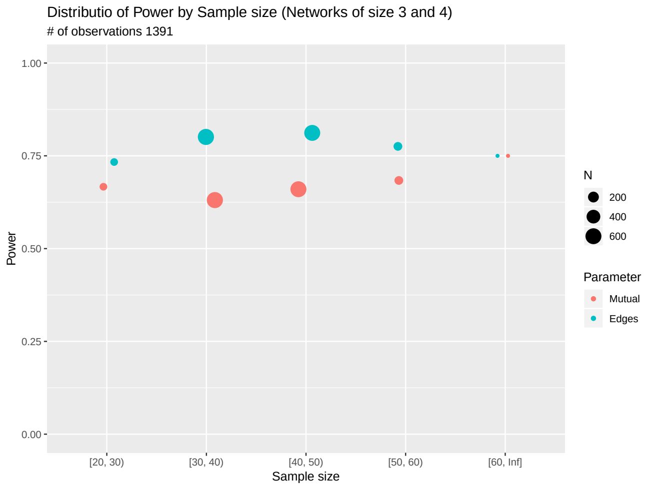
Simulation study: Scenario B

Empirical Bias

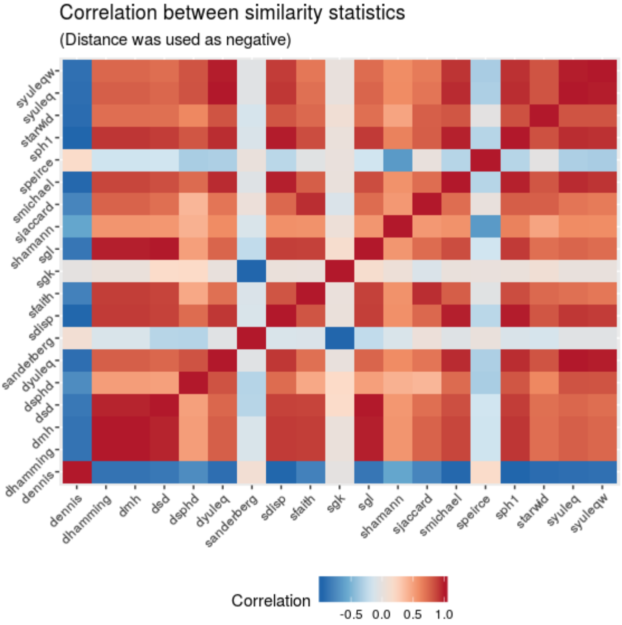


Simulation study: Scenario B

Power



Other approaches



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- Need to conduct more simulations using nodal attributes and networks of size 5 (right now having problems when building the DGP).
- Small structures imply a smaller pool of parameters (which is OK), but can be more useful when including nodal attributes.
- We have to work on parameter interpretation. Right now model statistics are not centered as these are in regular ERGMs (perhaps using the means?).
- When estimating the pooled version, we are essentially hand-waving the fact that parameter estimates implicitly encode size of the graph, i.e.

  Does a the estimate of `edge` = 0.1 has the same meaning for a network of size 3 to a size 5?

Thank you!

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