Big Problems for Small Networks: Small Network Statistics[1]

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[1]Contact: vegayon@usc.edu. We thank members of our MURI research team, USC's Center for Applied Network Analysis, and Andrew Slaughter for their comments.

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Network Science of Teams

a Multidisciplinary University Research Initiative

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Context: A tale about social abilities and team performance

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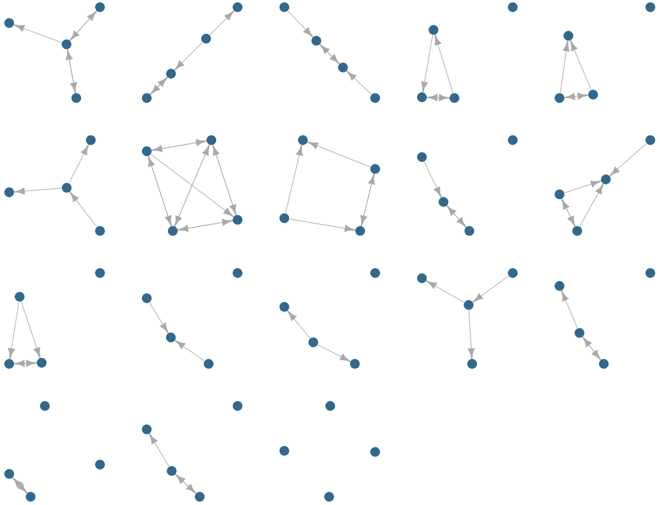
Study motivation

- Overall, a very limited set of SI domains have been tested as predictors of social networks
- Very little research on the emergence of networks in teams.

Context (cont'd)



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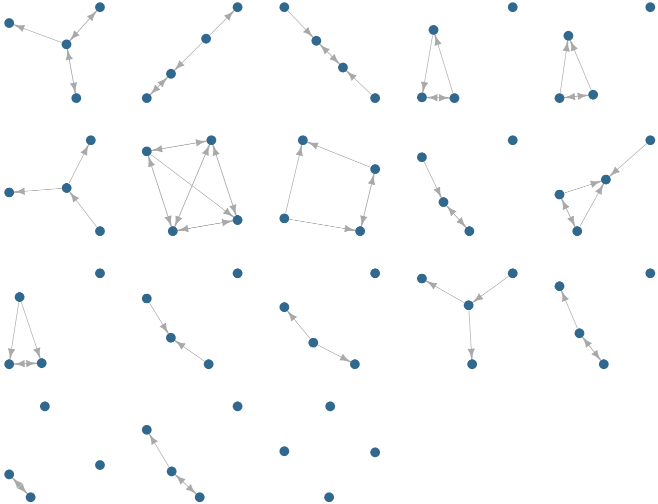

How can we go beyond descriptive statistics?

Small networks and Exponential Random Graph Models

When trying to estimate ERGMs in little networks

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When trying to estimate ERGMs in little networks

- MCMC fails to converge when trying to estimate a block diagonal (structural zeros) model,
- Same happens when trying to estimate an ERGM for a single (little) graph,
- Even if it converges, the Asymptotic properties of MLEs are no longer valid since the sample size is not large enough.

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  - High performing (up to some point): Some components written in C++
  - Very early stage of development...we'll see if it is worth keep working on it!

Example 1

Let's start by trying to estimate an ERGM for a single graph of size 4

```r
library(lergm)
set.seed(12)
x <- sna::rgraph(4)
lergm(x ~ edges + balance + mutual)
```

```
##
## Little ERGM estimates
##
##  Coefficients:
##   edges  balance   mutual
## -1.9443  -0.2417   3.4961
```

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- We propose to solve this by using a pooled version of the ERGM

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- By estimating a pooled version of the ERGM we can recover the asymptotics of MLEs.
- We implemented this in the `lergm` package

Example 2

Suppose that we have 3 little graphs of sizes 4, 5, and 5:

```r
library(lergm)
set.seed(12)
x1 <- sna::rgraph(4)
x2 <- sna::rgraph(5)
x3 <- sna::rgraph(5)

lergm(list(x1, x2, x3) ~ edges + balance + mutual)
```

```
##
## Little ERGM estimates
##
##  Coefficients:
##   edges  balance   mutual
## -0.3941  -0.2085   1.4156
```

Simulation study

Scenario A

1. Draw parameters for edges and mutual from a uniform(-3, 3).
2. Using those parameters, sampled $n \sim \text{Poisson}(30)$ networks of size 4
3. Estimated the pooled ERGMs using both the MLE and the boostrap version.

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Scenario B

1. Idem.
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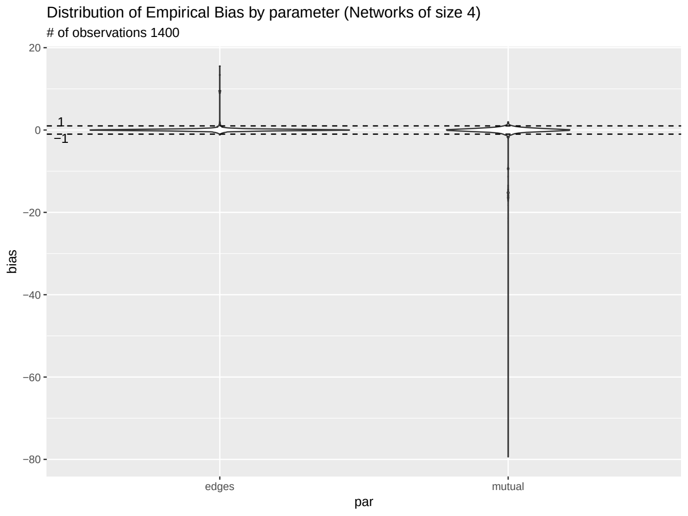
(If anyone asks, I just ran about 3 billion ERGMs... :))

Simulation study: Scenario A

Empirical Bias



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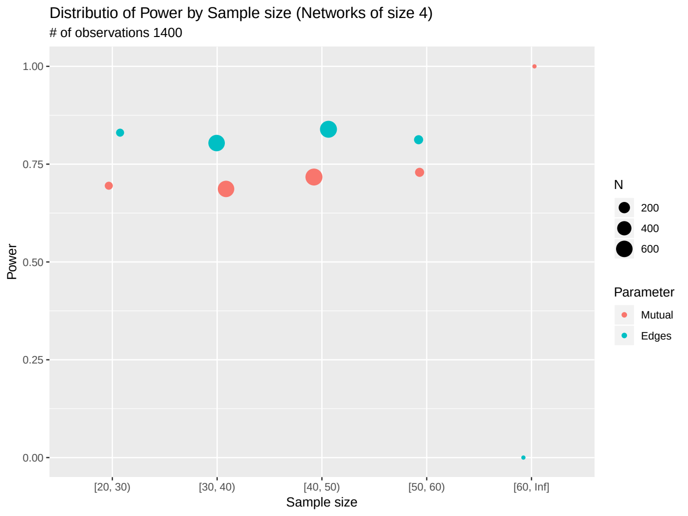
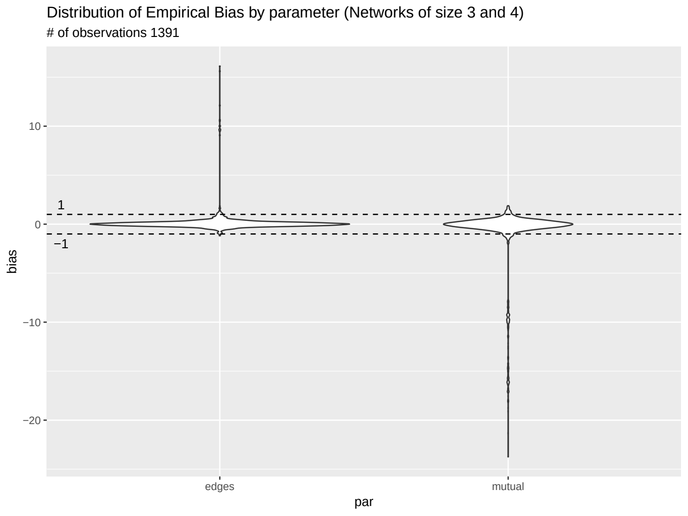


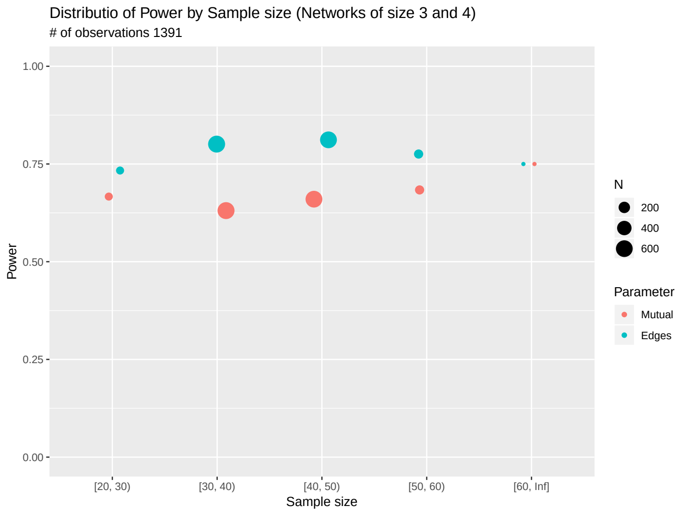
Simulation study: Scenario B

Empirical Bias

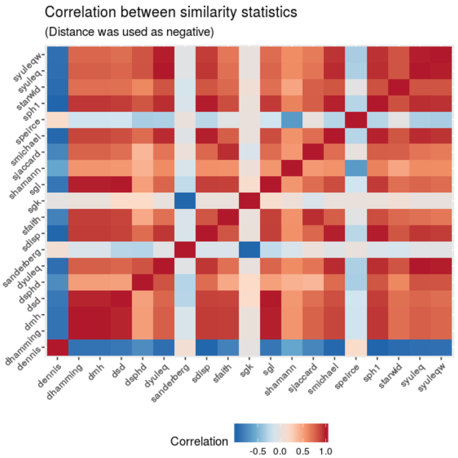


Simulation study: Scenario B

Power



Other approaches



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- Small structures imply a smaller pool of parameters (which is OK), but can be more useful when including nodal attributes.
- When estimating the pooled version, we are essentially hand-waving the fact that parameter estimates implicitly encode size of the graph, i.e.

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- Finally, this work can be extended to other types of small networks, including: families, egonets, etc. And other methods, such as TERGMs.

Thank you!

Big Problems for Small Networks: Small Network Statistics[2]

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What have we got so far?

```
lergm(networks ~ mutual + edges + triangle + nodematch("male") +
    diff("Empathy") + nodematch("nonwhite"))
```

Table 1: Preliminary results with our small teams data. The table shows 95% confidence intervals for the parameter estimates using the pooled ERGM model.

	All (42)		All but size 3 (35)	
	2.5 %	97.5 %	2.5 %	97.5 %
mutual	-0.40	0.55	-0.45	0.55
edges	-0.91	-0.16	-1.04	-0.29
triangle	0.06	0.24	0.09	0.27
nodematch("male")	-0.36	0.31	-0.34	0.36
diff("Empathy")	0.12	0.59	0.09	0.58
nodematch("nonwhite")	-0.26	0.37	-0.29	0.35