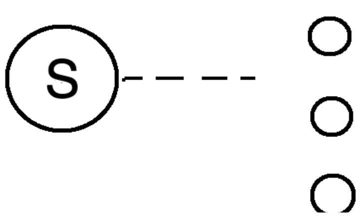
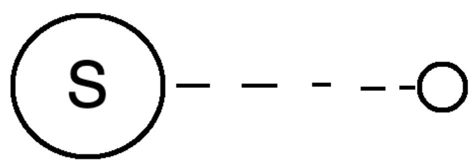
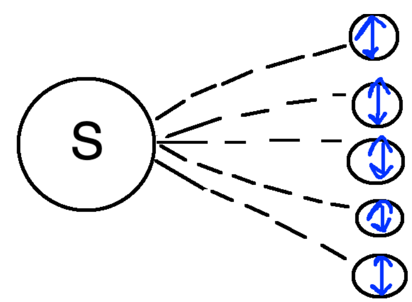
Weak measurement, quantum tunneling spectroscopy (QTS) and full simulation of 1/f noise



Weak measurement



QTS



1/f noise

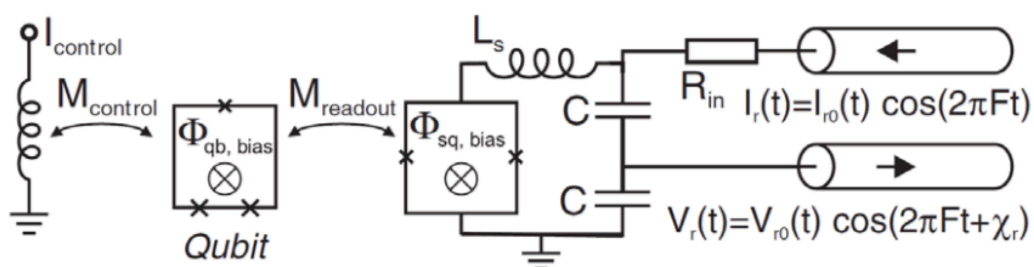
Multi-qubit case

Assume the time-dependent system Hamiltonian is chosen to be the Ising Hamiltonian of N qubits.

$$H_S(s) = -A(s)\sum_{i=1}^{N}\sigma_i^x + B(s)\left(\sum_{i}^{N} h_i\sigma_i^z + \sum_{i<j}^{N} J_{ij}\sigma_i^z\sigma_j^z\right)$$

Weak measurement

The purpose of weak measurements is to obtain the measurement results of a particular measurement operator during quantum annealing, perturbing the system states as little as possible.



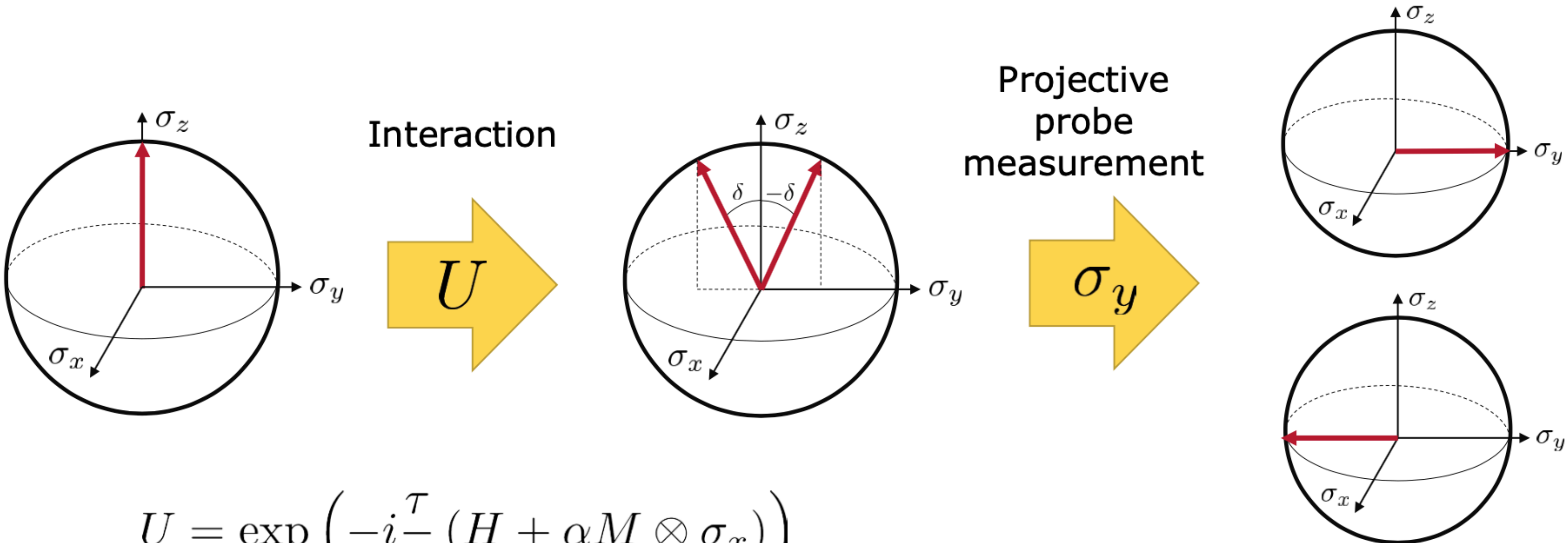
Denote τ as the measurement rate, α as the measurement strength, and M is the measurement operator. The weak measurement of M on a system with Hamiltonian H_S can be simulated by repeated interactions of time τ with a probe qubit.
The total Hamiltonian describing the system and probe can be written as:

$$-A(s)\sum_{i=1}^{N}\sigma_i^x + B(s)(H_{\mathrm{IS}}) + \alpha M \otimes \sigma_{\mathrm{P}}^{\mathrm{x}}$$

where:

$$H_{\mathrm{IS}} = \sum_i h_i \sigma_i^z + \sum_{i<j} J_{ij} \sigma_i^z \sigma_j^z .$$

Can simulate by repeated interactions of time τ with a probe qubit



$$U = \exp\left(-i\frac{\tau}{\hbar}\left(H + \alpha M \otimes \sigma_x\right)\right)$$

$$H(t) = \frac{1}{2}(1 - \frac{t}{t_f})\sigma_x + \frac{1}{2}\frac{t}{t_f}\sigma_z,$$

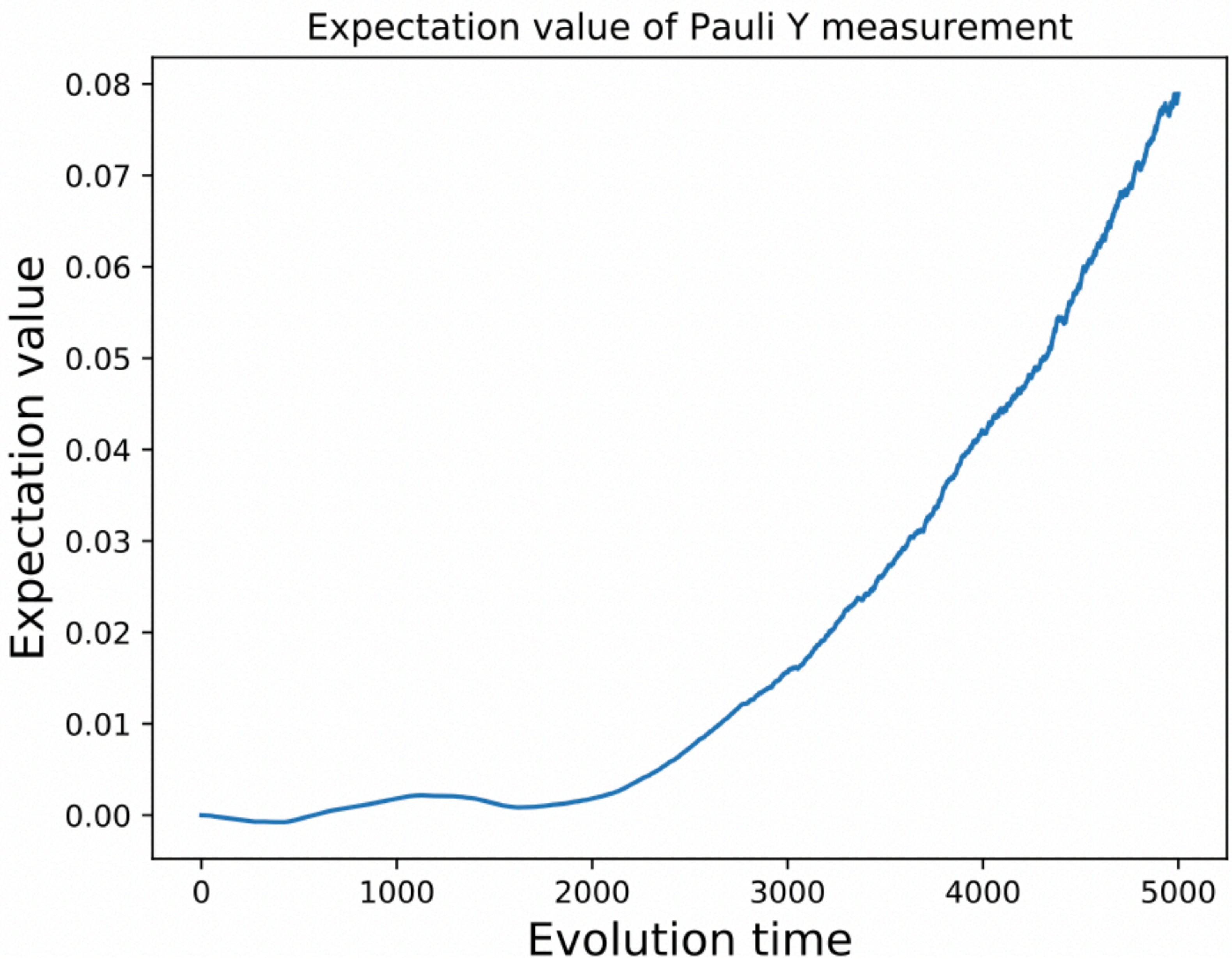
$$M = \frac{1}{2}\sigma_z,$$

The initial state is $|\psi >= \frac{1}{\sqrt{2}}(|0 > +|1 >)$,

and the parameters are

$\tau = 0.00005$

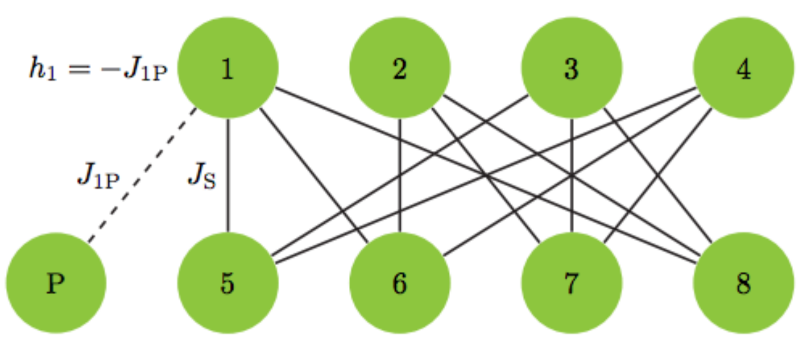
$\alpha = 100$

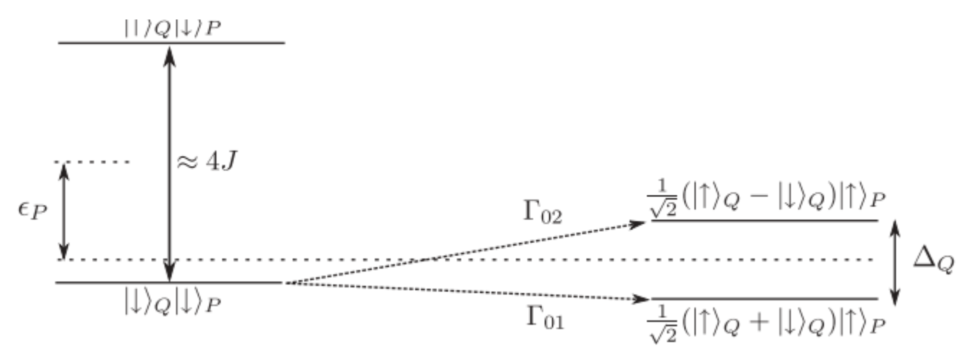


Expectation value of Pauli Y measurement
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.00
Expectation value
0
1000
2000
3000
4000
5000
Evolution time

Quantum tunneling spectroscopy (QTS)

The purpose of the quantum tunneling spectroscopy (QTS) is to detect entanglement by measuring the population transfer to/from a probe qubit due to incoherent tunneling.





Denote the coupling between the system and probe qubit as J_{1P}, and the local transverse field on the probe qubit as $-h_p$ (whose strength is proportional to $-J_{1P}$).
The total Hamiltonian describing the system and probe can be written as:

$$-A(s)\left(\sum_{i=1}^{N} \sigma_i^x + \sigma_{\mathrm{P}}^x\right) + B(s)\left(H_{\mathrm{IS}} + H_{1\mathrm{P}}\right)$$

where $$H_{\mathrm{IS}} = \sum_i h_i \sigma_i^z + \sum_{i<j} J_{ij} \sigma_i^z \sigma_j^z$$

and $$H_{1\mathrm{P}} = J_{1\mathrm{P}} \sigma_1^z \sigma_{\mathrm{P}}^z - J_{1\mathrm{P}} \sigma_1^z - h_{\mathrm{P}} \sigma_{\mathrm{P}}^z$$

Both the weak measurement and quantum tunneling spectroscopy (QTS) use the probe qubit. However, it is used in a different way.

For example:

In weak measurement, a stream of probe qubits is sent to couple with the system, each with a time interval of measurement rate τ, and a projective measurement is done on the probe qubit.

In quantum tunneling spectroscopy (QTS), the probe qubit is coupled with the system for all time, and a projective measurement is done once on both system qubits and probe qubit, at s=1.

There are also other differences, such as:

No bath is assumed in weak measurement, while bath is an essential component in QTS.

Forward annealing schedule vs reverse annealing schedule

	Weak measurement	Quantum tunneling spectroscopy (QTS)
System Hamiltonian	$-A(\mathrm{s})\sum_{i=1}^{N}\sigma_i^x + B(\mathrm{s})(\sum_i h_i\sigma_i^z + \sum_{i<j} J_{ij}\sigma_i^z\sigma_j^z)$	$-A(\mathrm{s})\sum_{i=1}^{N}\sigma_i^x + B(\mathrm{s})(\sum_i h_i\sigma_i^z + \sum_{i<j} J_{ij}\sigma_i^z\sigma_j^z)$
Total Hamiltonian	$-A(\mathrm{s})\sum_{i=1}^{N}\sigma_i^x + B(\mathrm{s})(H_{\mathrm{IS}}) + \alpha M \otimes \sigma_{\mathrm{P}}^{\mathrm{x}}$, $H_{\mathrm{IS}} = \sum_i h_i\sigma_i^z + \sum_{i<j} J_{ij}\sigma_i^z\sigma_j^z$	$-A(\mathrm{s})(\sum_{i=1}^{N}\sigma_i^x + \sigma_{\mathrm{P}}^{\mathrm{x}}) + B(\mathrm{s})(H_{\mathrm{IS}} + H_{1\mathrm{P}})$, $H_{\mathrm{IS}} = \sum_i h_i\sigma_i^z + \sum_{i<j} J_{ij}\sigma_i^z\sigma_j^z$ $H_{1\mathrm{P}} = J_{1\mathrm{P}}\sigma_1^z\sigma_{\mathrm{P}}^z - J_{1\mathrm{P}}\sigma_1^z - h_{\mathrm{P}}\sigma_{\mathrm{P}}^z$
Bath	No	Yes
Evolution time	Successive with rate τ such that at each interval: $U = exp\left(-i\frac{\tau}{h}(H_{\mathrm{s}}(\mathrm{t}) + \alpha M \otimes \sigma_x)\right)$	Continuous

Similarities and differences

Number of Probe qubit	A stream of probe qubits	One
System-probe Coupling	α: measurement strength	$J_{1\mathrm{P}}$
Transverse field on probe qubit	None	$-h_{\mathrm{P}} \propto -J_{1\mathrm{P}}$
Annealing Schedule $A(s)$ and $B(s)$	Forward	Backward to an inversion point s*, then forward to $s = 1$.
Measurement	Projective measurement is done successively (with measurement rate τ) on probe qubits, during the annealing	Projective measurement is done once on both system qubits and probe qubit, at $s = 1$.

Similarities and differences

$1/f$ noise (with Tameem)

Lorentzian power spectrum

Before talking about $1/f$ spectrum, let me talk about the Lorentzian power spectrum:

$$\mathcal{L}_{\gamma_i}(\omega) \equiv \frac{1}{\pi} \frac{\gamma_i}{\omega^2 + \gamma_i^2}.$$

Qubit

The only qubit is subjected to time-independent magnetic field pointing in the z-direction. The initial state of the qubit is $(|0\rangle + |1\rangle)/\sqrt{2}$, with initial Bloch vector pointing in the x-direction: $M(0) = (1, 0, 0)$.
The dynamics of the qubit is described by the precession equation (Schrodinger equation)

$$\dot{\mathbf{M}} = \mathbf{B} \times \mathbf{M} \,.$$

However, a stochastic magnetic field is $b(t)$ exists and is pointing parallel to B such that the total magnetic field is $B + b(t)$.

Fluctuator

The stochastic $b(t)$ originates from the fluctuator, which can be in one of two (meta)stable states, 1 and 2, and **once a while switches** between them.

Symmetric RT process: $\gamma_{1\to2} = \gamma_{2\to1} = \gamma/2$. When the fluctuator is in state 1, the stochastic magnetic field is $b(t) = +b$; similarly when it is in state 2, $b(t) = -b$.

Define $m_+(t) = M_x(t) + iM_y(t)$, the decay of the ensemble average $\langle m_+(t)\rangle$ gives the $T2^*$. The differential equation of $\langle m_+(t)\rangle$ is given as:

$$\langle \ddot{m}_+\rangle + \gamma \langle \dot{m}_+\rangle = -b^2 \langle m_+\rangle$$

with initial conditions $\langle m_+(0)\rangle = 1$, $\langle \dot{m}_+(0)\rangle = 0$.

The differential equation of $\langle m_+(t) \rangle$ is given as:

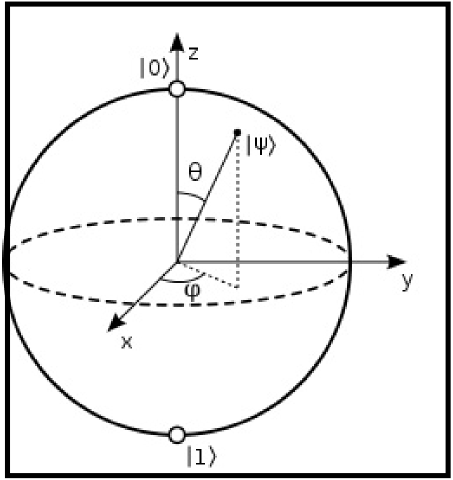
$$\langle \ddot{m}_+ \rangle + \gamma \langle \dot{m}_+ \rangle = -b^2 \langle m_+ \rangle$$

with initial conditions $\langle m_+(0) \rangle = 1$, $\langle \dot{m}_+(0) \rangle = 0$. The solution of differentiation equation with these initial conditions is

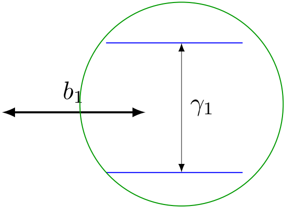
$$\begin{aligned} \langle m_+ \rangle &= (2\mu)^{-1} e^{-\gamma t/2} \left[(\mu + 1) e^{\gamma \mu t/2} + (\mu - 1) e^{-\gamma \mu t/2} \right], \\ \mu &\equiv \sqrt{1 - (2b/\gamma)^2}. \end{aligned}$$

Single fluctuator

Qubit



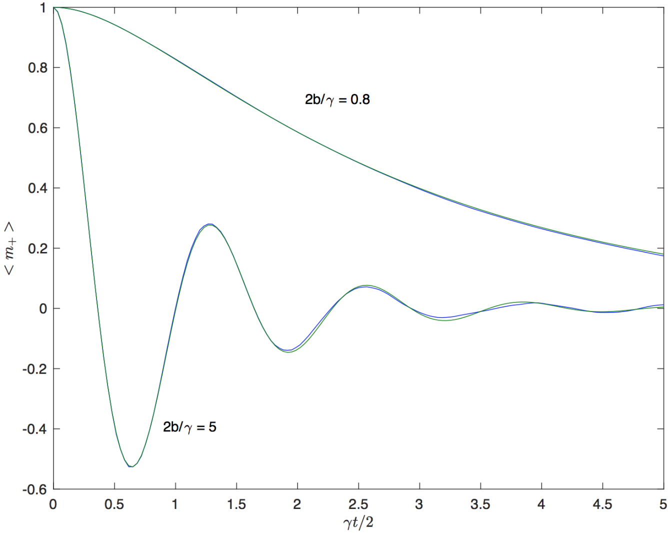
Fluctuator



Strength of a single fluctuator

Define the ratio of the **strength** of fluctuator over the rate of fluctuator as $g = b/(\gamma/2) = \frac{2b}{\gamma}$.
By using Gillispie algorithm to determine the flipping times of fluctuator (thus the sign of the stochastic magnetic field), and averaging over many realizations, the time evolution of ensemble average $\langle m_+(t) \rangle$ with $g = 0.8$ and $g = 5$ is plotted (next slide).
(If the fluctuator starts at either state 1 or 2 with equal probability, the initial condition $\langle \dot{m}_+(0) \rangle = 0$ above is fuifilled.)

Single fluctuator



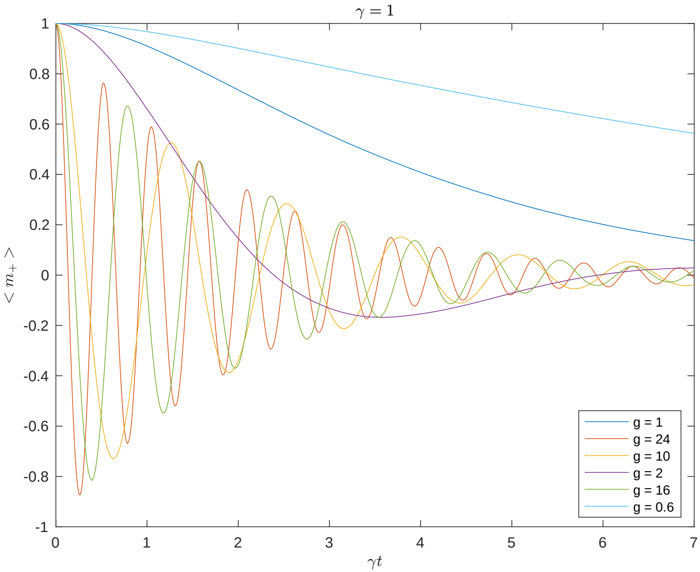
Blue lines: results reproduced from averaging 8000 realizations.
Green lines: analytical solutions.

T_2^*

$$\frac{1}{T_2^*} = \frac{\gamma}{2}\left(1 - \mathrm{Re}\sqrt{1 - \frac{4b^2}{\gamma^2}}\right)$$

Observations

- The stronger the fluctuator is or the slower the fluctuator switches, the resulting ensemble average $\langle m_+ \rangle$ of the qubit decays faster.
- However, its decay rate saturates as the ratio g reaches 1, where the behavior of $\langle m_+(t) \rangle$ enters from strictly decay to damped oscillation.
- $\langle m_+(t) \rangle$s with different g are plotted (next slide).
- γ is 1 switch/(time unit).



$\gamma = 1$. The decay rate is saturated after $g = 2b/\gamma$ reaches 1. Each curves are produced from averaging 10000 realizations.

$1/f$ noise

$1/f$ power spectrum:

$$S(\omega) \approx \frac{A}{\omega}\,.$$

$1/f$ noise

A single two-state fluctuator is described by a stochastic variable $\chi(t) = \pm 1$ with rates $\gamma/2$.
The power spectrum of the noise generated by a **single** i-th fluctuator is:

$$S_i(\omega) = b_i^2 \mathcal{L}_{\gamma_i}(\omega)$$

where $\mathcal{L}_{\gamma_i}(\omega)$ is a Lorentzian function of frequency,

$$\mathcal{L}_{\gamma_i}(\omega) \equiv \frac{1}{\pi} \frac{\gamma_i}{\omega^2 + \gamma_i^2}.$$

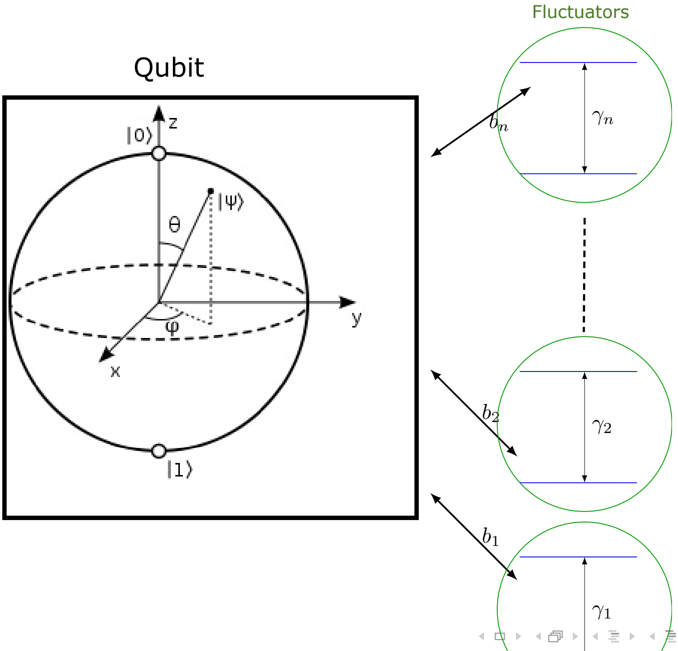
The couplings b_i are distributed around an average value $\overline{b}$. Under these conditions the **total** power spectrum reads

$$S(\omega) = \overline{b^2} \int_{\gamma_m}^{\gamma_M} d\gamma \, \frac{P_0}{2\gamma} \, \mathcal{L}_\gamma(\omega) \approx \frac{\mathcal{A}}{\omega} \, .$$

$1/f$ noise in words:

Let $\xi_k(t)$ be an asymmetric RTN signal switching between values $\pm v_k/2$ with rates $\gamma_k^{(\pm)}$, $\gamma_k = \gamma_k^{(+)} + \gamma_k^{(-)}$. If a distribution $P(\gamma) \propto 1/\gamma$ is assumed for the switching rates $\gamma_k \in [\gamma_{min}, \gamma_{max}]$, the total fluctuation $\Xi(t) = \sum_k \xi_k(t)$ exhibits a $1/f$ power spectrum of the form $S(|\omega|) = A/|\omega|$, $A > 0$, in a frequency range defined by effective cutoffs $\gamma^e_{min} \gg \gamma_{min}$, $\gamma^e_{max} \ll \gamma_{max}$.

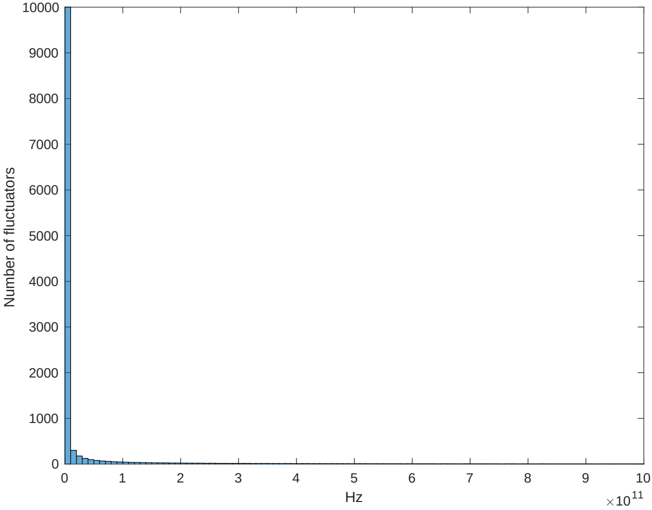
$1/f$ noise: ensemble



Distribution

The $(1/\gamma)$ distribution of the frequencies of the fluctuators corresponds to a uniform distribution of $\ln \gamma$.

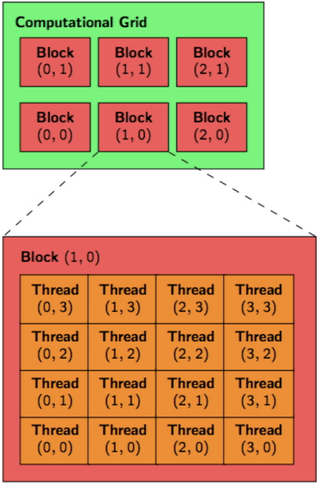
12 decades of noise and the number of fluctuators per decade is $n_d = 100$.



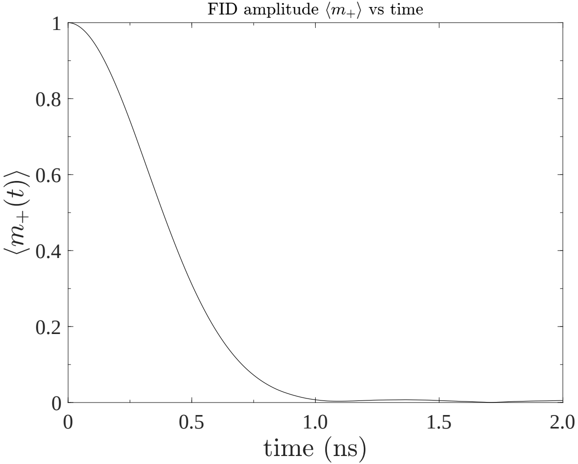
Distribution of frequencies of 1200 fluctuators.

GPU computing

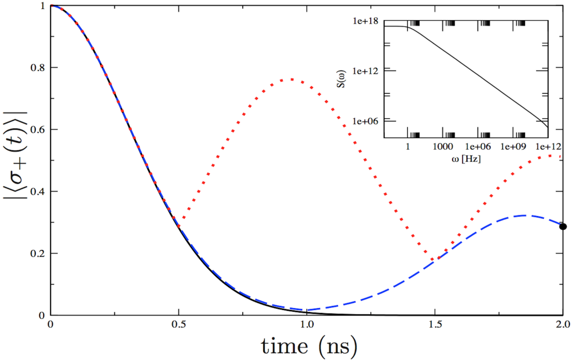
Each GPU thread can compute one fluctuator.



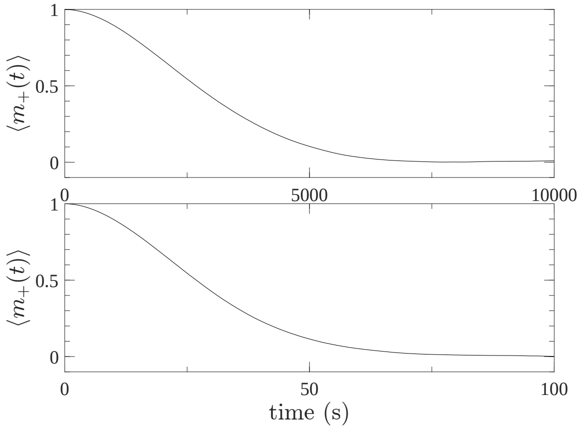
$1/f$ noise: simulation



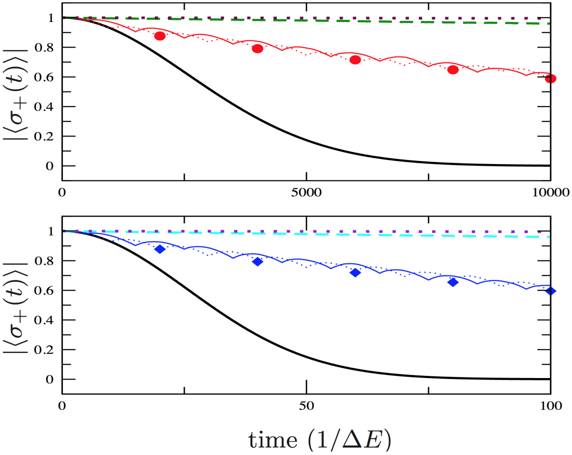
Pure dephasing: γ_{min}=$1/(2\pi)$ Hz, $\gamma_{max} = 10^{12}/2\pi$ Hz. Number of fluctuators per decade is $n_d = 1000$. $\langle b \rangle = (9.2/\pi) \cdot 10^7$ Hz, $\Delta b/\langle b \rangle \sim 0.2$. Results are averaged over $8k$ trajectories. Initial $\delta p_{eq} = 0.08$.



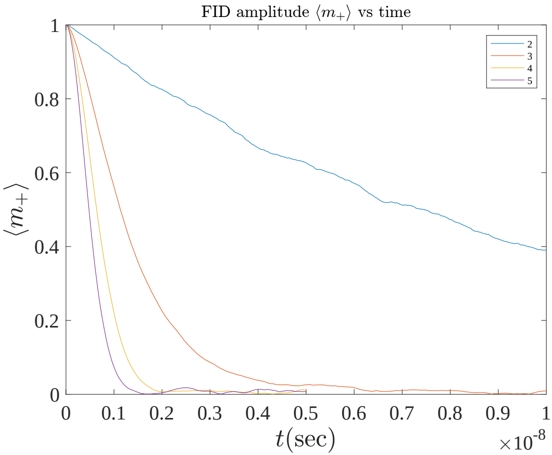
1
0.8
0.6
0.4
0.2
0
0
0.5
1.0
1.5
2.0
$|\langle\sigma_+(t)\rangle|$
time (ns)
1e+18
1e+12
1e+06
S(ω)
1
1000
1e+06
1e+09
1e+12
ω [Hz]



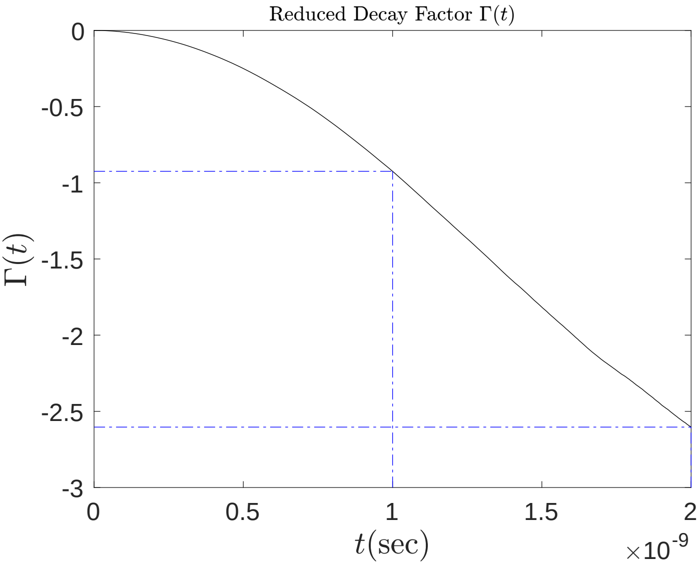
Top: $\gamma_{min} = 10^{-6}$, $\gamma_{max} = 100$, $n_d = 100$, $\langle b \rangle = 10^{-4}/4$; Bottom: $\gamma_{min} = 10^{-4}$, $\gamma_{max} = 100$, $n_d = 100$, $\langle b \rangle = 0.01/4$ Results are averages over $8k$ realizations. Each BC is initially in a pure state randomly sampled according to $\langle \delta p_0 \rangle = \delta p_{eq} = 0.08$. $\Delta b / \langle b \rangle \sim 0.2$.



$|\langle\sigma_+(t)\rangle|$
1
0.8
0.6
0.4
0.2
0
0
5000
10000
$|\langle\sigma_+(t)\rangle|$
1
0.8
0.6
0.4
0.2
0
0
50
100
time $(1/\Delta E)$



Reproduced figures of 2 to 5 decades. Couplings distributed with $\langle \Delta b \rangle / \langle b \rangle = 0.2$ and around $\overline{b} = 4.6 \times 10^7$ Hz. $\gamma_M = 10^{12}$ Hz with different γ_m. $n_d = 1000$. $dp_{0j} = \pm 1$ are distributed according to $\langle dp_{0j} \rangle = dp_{\text{eq}}$.



Reproduced figures of 12 decades. Couplings distributed with $\langle \Delta b \rangle / \langle b \rangle = 0.2$ and around $\overline{b} = (4.6/\pi) \times 10^7$ Hz. $\gamma_M = 10^{12}$ Hz with $\gamma_m = 10^0$ Hz. $n_d = 1000$. $dp_{0j} = \pm 1$ are distributed according to $\langle dp_{0j} \rangle = dp_{\mathrm{eq}}$.

Hamiltonian formalism

$$H_S = -\left[\omega\sigma_z + \delta\sigma_x\right]/2$$
$$H_B = \sum_{k=1}^{\infty} \omega_k b_k^\dagger b_k + \sum_k \left(\epsilon_k b_k^\dagger b_k + T_k \sum_l [c_{kl}^\dagger b_k + \text{h.c.}] + \sum_l \varepsilon_{kl} c_{kl}^\dagger c_{kl} \right)$$
$$H_{SB} = \sum_{i=1}^{N} \sigma_i^z \otimes B_i^{\text{flu}} = \sigma^z \otimes \sum_k \mu_k b_k^\dagger b_k$$

where $b_k(b_k^\dagger)$, $c_{kl}(c_{kl}^\dagger)$ operators for fluctutator and fermionic band respectively. T_k is the coupling to fermonic band. μ_k is the coupling strength of system qubit to fluctuator. γ_k is the fluctuation rate, and μ_k/γ_k corresponds to g_k.

System inherent adiabatic timescale is τ_S. Fluctuator timescale is $\tau_B = \frac{1}{\gamma_{\text{slow}}}$. One important condition to have Lindblad form $\tau_S >> \tau_B$.

Circuit Hamiltonian

$$H_S(s) = E_J\left(\frac{E_C}{2E_J}\hat{n}^2 - (\hat{d}_1 + \hat{d}_1^\dagger) + \alpha\cos\left(\frac{\phi_x(s)}{2}\right)\sqrt{1 + d^2\tan^2\frac{\phi_x(s)}{2}}\left(e^{i(\phi_z(s)-\phi_0(s))}\hat{d}_2 + e^{-i(\phi_z(s)-\phi_0(s))}\hat{d}_2^\dagger\right)\right).$$

where:
E_J is the Josephson junction energy.
E_C is the Capacitive energy.
$\hat{n}$ is the charge operator (with the same dimension as H_S).
d_1 is the charge displacement operator by 1 cooper pair (with the same dimension as H_S).
$\alpha < 1$ is ratio of the current in one of the small x-loop junctions to the current in the large z-loop junction.
$d = \frac{E_{J1}-E_{J2}}{E_{J1}+E_{J2}}$ is the x-loop junction asymmetry.
$\phi_x(s)$ is the time-dependent x (barrier) bias phase
$\phi_z(s)$ is the time-dependent z (tilt) bias phase.
$\phi_0 = \tan^{-1}\left(d\tan\left(\frac{\phi_x(s)}{2}\right)\right).$
$\widehat{d_2}$ is the charge displacement operator by 2 cooper pairs (with the same dimension as H_S).