```
In [1]: from numpy import mean, array, cos, sin, arctan, log, sqrt
                          import numpy as np
                          from math import pi
                         from scipy.optimize import curve_fit, OptimizeWarning, fsolve
                         import warnings
                         porosity = 0.60
                         heat_capacity = 4190000
                         heat_capacity_solid = 2000000
                         beta = 0.001
                         shallow_mid_dist = 0.1
                         conductivity = 0.84
                         str_temp_time_to = """7/12/2014 0:00,7/12/2014 1:00,7/12/2014 2:00,7/12/2014 3:00,7/12/2014 4:00,
                                                                                                \frac{7}{12}/2014 \ 5:00, \frac{7}{12}/2014 \ 6:00, \frac{7}{12}/2014 \ 7:00, \frac{7}{12}/2014 \ 8:00, \frac{7}{12}/2014 \ 9:00, \frac{7}{12}/2014 \ 10:00, \frac{7}{12}/2014 \ 11:00, \frac{7}{12}/2014 \ 12:00, \frac{7}{12}/2014 \ 13:00, \frac{7}{12}/2014 \ 1
                                                                                                 7/12/2014 \ 15:00, 7/12/2014 \ 16:00, 7/12/2014 \ 17:00, 7/12/2014 \ 18:00, 7/12/2014 \ 1
                                                                                                 7/12/2014 \ 20:00, 7/12/2014 \ 21:00, 7/12/2014 \ 22:00, 7/12/2014 \ 23:00, 7/13/2014 \ 0
                         \texttt{str\_temp\_to} = \texttt{"""24.51,24.38,24.20,24.07,23.88,23.70,23.51,23.45,23.45,23.63,23.95,24.26,24.51,24.26,24.51,24.26,24.51,24.26,24.51,24.26,24.51,24.26,24.51,24.26,24.51,24.26,24.51,24.26,24.51,24.26,24.51,24.26,24.51,24.26,24.51,24.26,24.51,24.26,24.51,24.26,24.51,24.26,24.51,24.26,24.51,24.26,24.51,24.26,24.51,24.26,24.51,24.26,24.51,24.26,24.51,24.26,24.51,24.26,24.51,24.26,24.51,24.26,24.51,24.26,24.51,24.26,24.51,24.26,24.51,24.26,24.51,24.26,24.51,24.26,24.26,24.51,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,24.26,
                                                                                24.70,24.82,24.88,24.88,24.88,24.76,24.63,24.51,24.45,24.32,24.20,24.13"""
= """7/12/2014 0:00,7/12/2014 1:00,7/12/2014 2:00,7/12/2014 3:00,7/12/2014 4:00,
                          str temp time tz =
                                                                                                 7/12/2014 5:00,7/12/2014 6:00,7/12/2014 7:00,7/12/2014 8:00,7/12/2014 9:00,
                                                                                                 7/12/2014 10:00,7/12/2014 11:00,7/12/2014 12:00,7/12/2014 13:00,7/12/2014 1
                                                                                                 7/12/2014 \ 15:00,7/12/2014 \ 16:00,7/12/2014 \ 17:00,7/12/2014 \ 18:00,7/12/2014 \ 1
                                                                                                 7/12/2014\ 20:00, 7/12/2014\ 21:00, 7/12/2014\ 22:00, 7/12/2014\ 23:00, 7/13/2014\ 0
                          \mathtt{str\_temp\_tz} = \texttt{"""21.07,21.07,21.07,21.07,21.01,21.01,20.94,20.94,20.88,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,20.82,
                                                                                20.82, 20.88, 20.88, 20.94, 20.94, 21.01, 21.01, 21.07, 21.07, 21.07, 21.07, 21.01, ""
                          expected = 10.3008
In [2]: def get_frequency(data):
                                          ""Derive the frequency to be used in the transient functions."""
                                       # Frequency is derived from the frequency of the data. Hourly is 1/24, every two hours might
                                       # One of the examples given works well with frequency = 1/24
                                       # All the tests seem to utilize data from a 24 hour period, so the frequency might always be
                                      return 2 / (len(data) - 1)
                         def parse_csv_str(csv_str, data_type):
                                       Receiving a CSV string.
                                      This function ignores all newlines and tabs before parsing
                                       the data into arrays of type 'datetime' or 'float'.
                                       # Strip all special (newline, tab, etc.) characters.
                                      csv_str = csv_str.replace('\n', '')
csv_str = csv_str.replace('\r', '')
                                      csv_str = csv_str.replace('\t', '')
                                      csv_str = csv_str.rstrip(',')
                                       # Split on commas
                                       if data_type == 'datetime':
                                                   return csv_str.split(',')
                                       elif data_type == 'float':
                                                   # Data type is numerical (float), so we need to parse each value
                                                   csv_array = [float(x) for x in csv_str.split(',')]
                                                   return csv_array
                         def transient_ydata_func(parms, a, b, c, d, e, f, g, h):
                                        return (a * cos((parms[2] * 1) * pi * parms[0]) + b * sin((parms[2] * 1) * pi * parms[0]) + c * cos((parms[2] * 2) * pi * parms[0]) + d * sin((parms[2] * 2) * pi * parms[0]) + e * cos((parms[2] * 3) * pi * parms[0]) + f * sin((parms[2] * 3) * pi * parms[0]) + g * cos((parms[2] * 4) * pi * parms[0]) + b * sin((parms[2] * 3) * pi * parms[0]) + g * cos((parms[2] * 4) * pi * parms[0]) + parms[0]) + parms[0] * parms[0] 
                                       return (a
In [3]: # Parse the time series strings into arrays of strings (dates) and floats (temperatures)
                         temp_time_to = parse_csv_str(str_temp_time_to, 'datetime')
temp_time_tz = parse_csv_str(str_temp_time_tz, 'datetime')
                                      temp_to = parse_csv_str(str_temp_to, 'float')
                                       temp_tz = parse_csv_str(str_temp_tz, 'float')
                          except ValueError:
                                      print("Could not parse provided time series data. Please check your input.")
                         if len(temp_time_to) + len(temp_time_tz) + len(temp_to) + len(temp_tz) < 12 * 4:
```

```
print("Not enough time series data provided. Please check your input. There must be at least
        elif len(temp_time_to) != len(temp_time_tz) or len(temp_time_tz) != len(temp_to) or len(temp_to)
            print("Please check your input. The four time series inputs should have the same number of po
In [4]: RN1 = len(temp_to)
        RN2 = len(temp_tz)
        RN3 = 15
        RN4 = 15
        for i in range(16, RN1, 1):
    if temp_to[i] - temp_to[0] <= 1:
        RN3 = RN3 + 1</pre>
        for i in range(16, RN2, 1):
            if temp_tz[i] - temp_tz[0] <= 1:
    RN4 = RN4 + 1</pre>
        RN5 = len(temp_to)
        RN6 = len(temp_tz)
        L15 = mean(temp_to)
        L20 = mean(temp_tz)
        frequency = get_frequency(temp_to)
        # If scipy==1.2.1 AND python > 3.6 (not working):
        # L15_init = [L15] * 8
# L20_init = [L20] * 8
        # sigma = [frequency] * RN1
        xdata = []
        i = 0.0
        while i < RN5:
            xdata.append(i)
            i = i + 1.0
        xdata = np.array(xdata, dtype=int)
In [6]: alb1_array = [xdata, L15, frequency]
        a2b2_array = [xdata, L20, frequency]
        with warnings.catch_warnings():
            warnings.simplefilter("error", OptimizeWarning)
                 # If scipy==1.2.1 AND python > 3.6 (not working):
                 # alb1_array = xdata
                 \# a2b2_array = xdata
                 # AlB1_calculated, pcov = curve_fit(f=transient_ydata_func, xdata=alb1_array, ydata=temp_
                 # A2B2_calculated, pcov = curve_fit(f=transient_ydata_func, xdata=a2b2_array, ydata=temp_
                 # If python == 3.6 (working):
                 AlB1_calculated, pcov = curve_fit(f=transient_ydata_func, xdata=alb1_array, ydata=temp_to
                A2B2_calculated, pcov = curve_fit(f=transient_ydata_func, xdata=a2b2_array, ydata=temp_tz
            except OptimizeWarning as err:
                print(err)
            except RuntimeError:
                print("Least-Squares minimization has failed.")
In [7]: # calculate amplitude & phase angle of the shallow depth
        A1 = A1B1_calculated[0]
        B1 = A1B1_calculated[1]
        Po = 0
        Pz = 0
        Ao = (A1 ** 2 + B1 ** 2) ** 0.5
        if A1 < 0:
            Po = arctan(B1 / A1) + pi
        else:
            Po = arctan(B1 / A1)
        # calculate amplitude & phase angle of the deeper depth
        A2 = A2B2_calculated[0]
        B2 = A2B2_calculated[1]
        Az = (A2 ** 2 + B2 ** 2) ** 0.5
        if A2 < 0:
```

```
Pz = arctan(B2 / A2) + pi
             Pz = arctan(B2 / A2)
         # Calculate Amplitude ratio & phase shift
         AR = Az / Ao
         PS = (Pz - Po) / (2 * pi) # phase shift unit is "day"
         if PS <= 0:
              PS = (2 * pi + Pz - Po) / (2 * pi)
In [8]: # ***start Hatch (2006) calculation***
         dz = shallow_mid_dist # S23 depth
         por = porosity # 02 porosity
PfCf = heat_capacity # 04 volumetric heat capacity of fluid
PsCs = heat_capacity_solid # 06 volumetric heat capacity of solid
         PC = por * PfCf + (1 - por) * PsCs # heat capacity of saturated media
         r = PC / PfCf
         P = 1 # Period 1 day
         O38 = PC # ok
O39 = AR # WRONG
X38 = 1 # initialization of v
         S2 = conductivity # = K conductivity = J/(m.s.C)
         S4 = beta # Beta
         S23 = shallow_mid_dist # S23 depth
         # NOTE: 038 is OK, but 039 is wrong here:
         func = lambda X38: (X38 + ((2 * ((86400 * S2 / O38) + S4 * abs(X38)) / S23) * log(O39)) + sqrt( ((sqrt(X38 ** 4 + (8 * pi * ((86400 * S2 / O38) + S4 * abs(X38)) / 1) ** 2) + X38 ** 2) / 2))
         v_array = fsolve(func, X38)
         if len(v_array) > 1:
              if v_array.ier != 1:
                  print(v_array.mesg)
         v = v_array[0]
         # Darcy velocity (seepage flux)
         qz = v * r # in m/day
         X12 = qz * 100 # in cm/day
         print(f"Expected: {expected}")
         print(f"Actual: {X12} cm/day")
Expected: 10.3008
Actual: 10.300831575612353 cm/day
In [ ]:
```