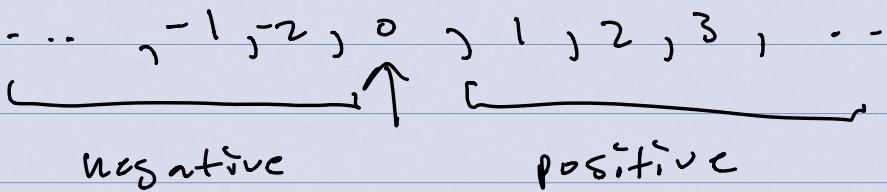


Negative Numbers

int whole numbers



3.781 -1.5 3.1×10^{-2}

Integers

n-bit numbers

3-bit numbers unsigned

| Bin | Dec | n-bit number |
|-----|-----|-------------------------------------|
| 000 | 0 | 0 |
| 001 | 1 | 2^n different values |
| 010 | 2 | |
| 011 | 3 | $0 \rightarrow 2^n - 1$ |
| 100 | 4 | $\rightarrow 0 \rightarrow 2^3 - 1$ |
| 101 | 5 | $0 \rightarrow 8 - 1$ |
| 110 | 6 | $0 \rightarrow 7$ |
| 111 | 7 | |

Negatives

1/2 positive 1/2 negative

Sign magnitude

| Bin | Dec | pluses |
|-----|-----|----------------------|
| 000 | 0 * | 1) simple |
| 001 | 1 | 2) easy to |
| 010 | 2 | determine if a value |
| 011 | 3 | is negative |

$$\rightarrow \begin{array}{|c|c|} \hline & 00 \\ \hline & 01 \\ \hline & 10 \\ \hline & 11 \\ \hline \end{array} \quad \begin{array}{c} -0 * \\ -1 \\ -2 \\ -3 \\ \end{array}$$

$$\text{if } (x == 0) \leq \quad x == 0 \quad | \quad x == +0 \\ \Rightarrow$$

2

Arithmetic with sign magnitude

$$-1 + 3 = 2$$

$$\begin{array}{r}
 \boxed{1} \begin{smallmatrix} 1 \\ 0 \\ 1 \end{smallmatrix} \\
 + \begin{smallmatrix} 0 \\ 1 \\ 1 \end{smallmatrix} \\
 \hline \boxed{1} \begin{smallmatrix} 0 \\ 0 \\ 0 \end{smallmatrix}
 \end{array}
 \leftarrow -1 + 3 \cancel{\neq} 0$$

Two's Complement

Bin

000 0

001 1

010 2

011 3

100 -4 *

101 -3 *

110 -2

111 -1

Converting pos to neg and
neg to pos

2 (pos) what is the 2's comp
neg representation

Invert(v) + 1 = 2's comp

3 → 0111

~0111 = 01100

$$\begin{array}{r} +1 \\ \hline 01101 (-3) \end{array}$$

$$-3 = 0b101$$

$$\sim 0b101 = 0b010$$

$$\begin{array}{r} +1 \\ \hline \boxed{0b011} \end{array} \quad (3)$$

Two's Complement Arithmetic

$$-1 + 3 = \boxed{0b111 + 0b011}$$

$$\begin{array}{r} 1\ 1\ 1 \\ 1\ 1\ 1 \quad (-1) \\ + 0\ 1\ 1 \quad (3) \quad 1\ 1 \\ \hline \boxed{0\ 1\ 0} \quad 0\ 1\ 0 = \boxed{-2} \end{array}$$

$$2 - 4$$

$$2 + (-4)$$

$$2 \quad (-4)$$

$$0b010 + 0b100$$

$$0\ 1\ 0 \quad (2)$$

$$\begin{array}{r} + 1\ 0\ 0 \\ \hline 1\ 1\ 0 \quad (-2) \end{array}$$

$$\begin{array}{r} 0\ 0\ 1 + 1 = \boxed{1\ 0\ 1} \\ \hline 2 \end{array}$$

Range for 2's comp signed ints

$$\underline{-\underline{2^{n-1}}}) \text{ to } \underline{\underline{(2^{n-1}-1)}}$$

... 0 ...

Sign Extension and ASR

3-bit 4-bit 8-bit

001

011

0001

0011

0000 0001

0000 0011

3-bit -2

$010 \text{ I}(01) + 1 = 110$

110

4-bit -2

0010 1101 + 1 1110

1110

8-bit -2

1111 1110

sign extension

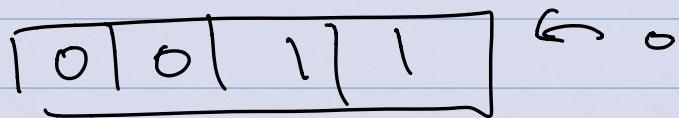
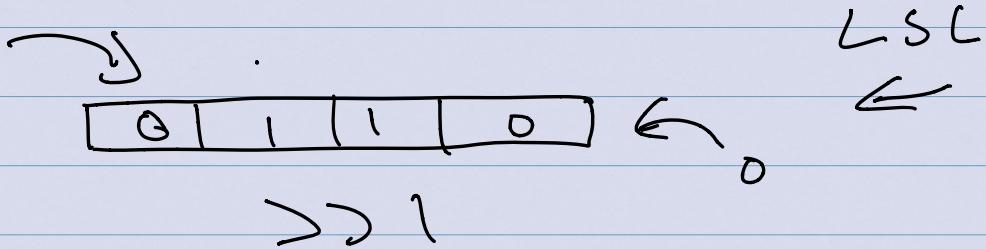
sign

36-bit \rightarrow 8-bit ↓

111111111101110

ASR Arithmetic Shift Right

LSR Logical Shift Right



in C

$$\text{int } x, y - (2^{31}) \dots 0 \dots (2^{31} - 1)$$

unsigned int ux, uy $0 \dots 2^{32} - 1$

$$x + y \quad x = -4 \quad y = -8$$

$$ux + uy \quad ux = (\text{unsigned int}) x : \\ uy = (\text{unsigned int}) y$$

$$\text{int } r : \underline{r = (\text{int})(ux + uy)}$$

$$r = -12$$

ASR

in

$x \gg 1$ (ASR) >-

$ux \gg 1$ (LSR) >>

ASR

eval

$vint32-t$

↓

$((int) v1) \gg v2$

↑
int

↑↑

ASR

$1100 \rightarrow 0011 + \underbrace{[0000]}_4$

↑

neg