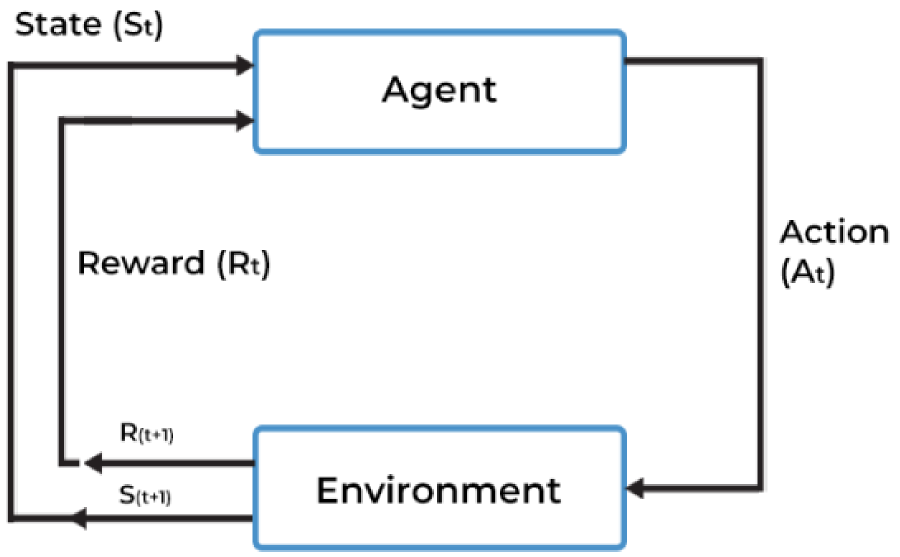
Deep Q Learning

Reinforcement learning

- The challenges encompassing an agent's interaction with an environment, wherein numeric rewards are given.
- Objective of acquiring the ability to make decisions that optimize the reward outcome.



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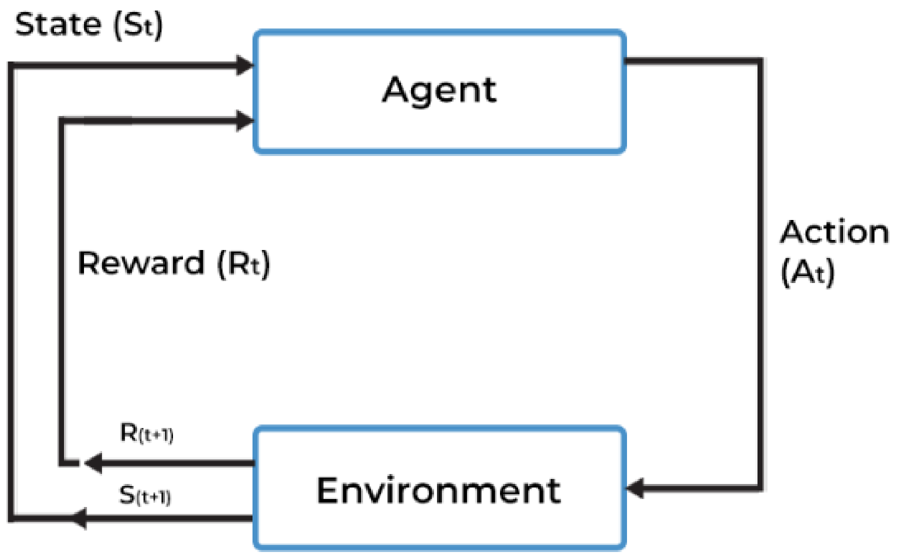
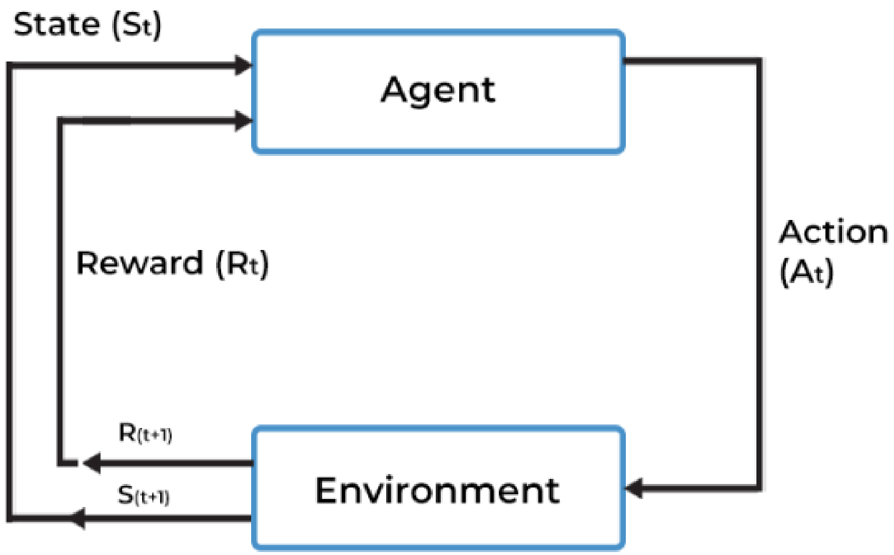
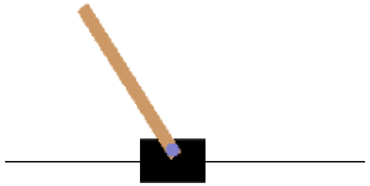


Typically in a RL system one observes the following sequence
state, action, reward, new state…

Reinforcement learning

Examples

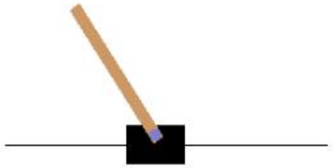


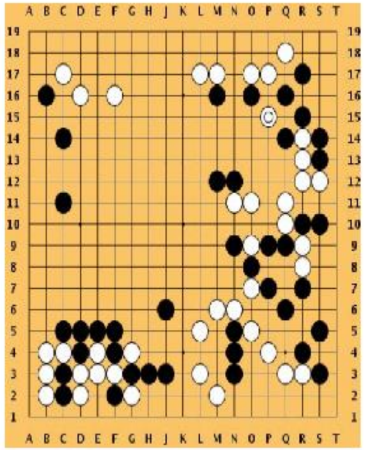


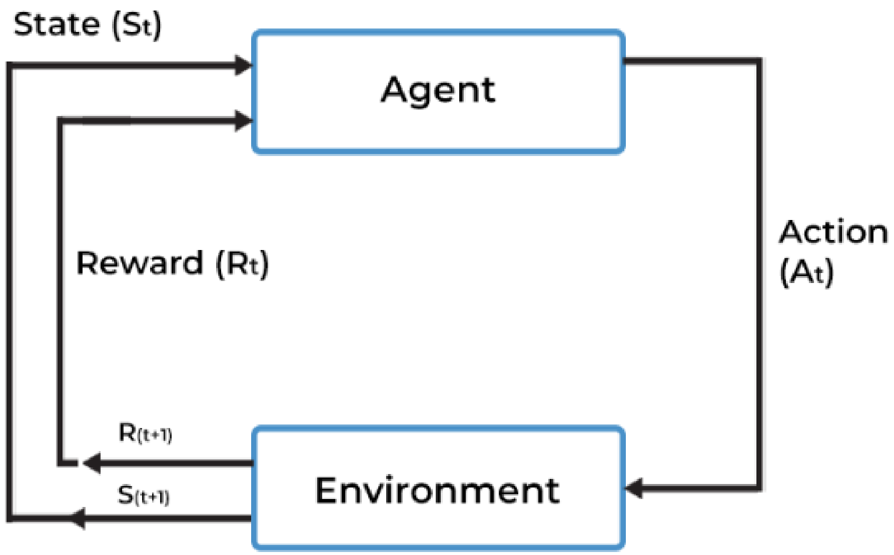
Objective: Balance a pole on top of a movable cart
State: angle, angular speed, position, horizontal velocity
Action: horizontal force applied on the cart
Reward: 1 at each time step if the pole is upright

Reinforcement learning

Examples







Markov Decision Process

Defined by: $(\mathcal{S}, \mathcal{A}, \mathcal{R}, \mathbb{P}, \gamma)$

$\mathcal{S}$: set of possible states
$\mathcal{A}$: set of possible actions
$\mathcal{R}$: distribution of reward given (state, action) pair
$\mathbb{P}$: transition probability i.e. distribution over next state given (state, action) pair
γ : discount factor

Markov Decision Process

At time step t=0, environment samples initial state $s_0 \sim p(s_0)$
Then, for t=0 until done:

Agent selects action a_t
Environment samples reward $r_t \sim R(\,.\,|s_t\,, a_t\,)$
Environment samples next state $s_{t+1} \sim P(\,.\,|s_t\,, a_t\,)$
Agent receives reward rt and moves to next state s_{t+1}

A policy **π** is a function from S to A that specifies what action to take in each state –

Objective: find policy **π*** that maximizes cumulative discounted reward:

$$\sum_{t\geq 0} \gamma^t r_t$$

Markov Decision Process

Typically, we do not have access to most of the information given in an MDP.
Instead, what one typically have in practice is something along the line of the sequence state, action, reward, next state.

Q function

A practical solution to the MDP is given by solving the state-action value function :

$$Q^*(s,a) = \mathbb{E}_{s'}\left[r + \gamma \max_{a'} Q^*(s',a') \mid s,a \right]$$ Bellman Equation

Q(s,a) can be understood as : how “good” a given state, action pair is.

Q function

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$$Q^*(s, a) = \mathbb{E}_{s'}\left[r + \gamma \max_{a'} Q^*(s', a') \mid s, a\right]$$ Bellman Equation

The optimal policy can be found via :

$$\pi^*(s) = \operatorname{argmax} Q^*(s, a)$$

Q function

$$Q^*(s,a) = \mathbb{E}_{s'}\left[r + \gamma \max_{a'} Q^*(s',a') \mid s,a\right]$$ Bellman Equation

How can we learn Q?

Q Learning

The most primitive form of the so-called Q-learning algorithm learns the correct Q-function by iteratively reducing the discrepancy between Q value estimations for two consecutive states. More precisely, the Q learning update rule is given by

$$Q(s_t, a_t) = r_t + \gamma max_a Q(s_{t+1}, a)$$

Q Learning

The most primitive form of the so-called Q-learning algorithm learns the correct Q-function by iteratively reducing the discrepancy between Q value estimations for two consecutive states. More precisely, the Q learning update rule is given by

$$Q(s_t, a_t) = r_t + \gamma max_a Q(s_{t+1}, a)$$

where s_t, a_t, r_t are the state, action, reward received by the agent at time t.

In a deep learning setting, we set the function Q(s, a) as a neural network Q(s, a, θ) and we try to optimize θ by specifying the loss

$$||Q(s_t, a_t) - (r_t + \gamma max_a Q(s_{t+1}, a))||^2$$

Q Learning

Problems:

1. Correlations between samples
2. 2. Non-stationary targets

Q-learning with experience replay

- To remove correlations, build data-set from agent's own experience

s_1, a_1, r_2, s_2
s_2, a_2, r_3, s_3
s_3, a_3, r_4, s_4
$\ldots$
$s_t, a_t, r_{t+1}, s_{t+1}$

$\rightarrow \quad s, a, r, s'$

- Sample experiences from data-set and apply update

$$l = \left(r + \gamma \max_{a'} Q(s', a', \mathbf{w}^-) - Q(s, a, \mathbf{w})\right)^2$$

- To deal with non-stationarity, target parameters w− are held fixed

Q-learning with experience replay

Initialize replay memory D to capacity N
Initialize action-value function Q with random weights θ
Initialize target action-value function $\hat{Q}$ with weights $\theta^- = \theta$

Initialize the replay memory and two identical Q approximators (DNN). $\hat{Q}$ is our target approximator.

For episode $= 1, M$ **do**
  Initialize sequence $s_1 = \{x_1\}$ and preprocessed sequence $\phi_1 = \phi(s_1)$
  For $t = 1$,T **do**
    With probability ε select a random action a_t
    otherwise select $a_t = \text{argmax}_a Q(\phi(s_t), a; \theta)$
    Execute action a_t in emulator and observe reward r_t and image x_{t+1}
    Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$
    Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in D
    Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from D
    Set $y_j = \begin{cases} r_j & \text{if episode terminates at step j+1} \\ r_j + \gamma \max_{a'} \hat{Q}(\phi_{j+1}, a'; \theta^-) & \text{otherwise} \end{cases}$
    Perform a gradient descent step on $\left(y_j - Q(\phi_j, a_j; \theta)\right)^2$ with respect to the network parameters θ
    Every C steps reset $\hat{Q} = Q$
  End For
End For

Q-learning with experience replay

Initialize replay memory D to capacity N
Initialize action-value function Q with random weights θ
Initialize target action-value function $\hat{Q}$ with weights $\theta^{-} = \theta$
For episode $= 1, M$ **do** ← Play *m* episodes (full games)
  Initialize sequence $s_1 = \{x_1\}$ and preprocessed sequence $\phi_1 = \phi(s_1)$
  For $t = 1$,T **do**
    With probability ε select a random action a_t
    otherwise select $a_t = \mathrm{argmax}_a Q(\phi(s_t), a; \theta)$
    Execute action a_t in emulator and observe reward r_t and image x_{t+1}
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    Perform a gradient descent step on $\left(y_j - Q\left(\phi_j, a_j; \theta\right)\right)^2$ with respect to the network parameters θ
    Every C steps reset $\hat{Q} = Q$
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  End For
End For

Start episode from x_1 (pixels at the starting screen).
Preprocess the state (include 4 last frames, RGB to grayscale conversion, downsampling, cropping)

Q-learning with experience replay

Initialize replay memory D to capacity N
Initialize action-value function Q with random weights θ
Initialize target action-value function $\hat{Q}$ with weights $\theta^{-} = \theta$
For episode $= 1, M$ **do**
  Initialize sequence $s_1 = \{x_1\}$ and preprocessed sequence $\phi_1 = \phi(s_1)$
  For $t = 1$,T **do** ← For each time step during the episode
    With probability ε select a random action a_t
    otherwise select $a_t = \operatorname{argmax}_a Q(\phi(s_t), a; \theta)$
    Execute action a_t in emulator and observe reward r_t and image x_{t+1}
    Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$
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Initialize target action-value function $\hat{Q}$ with weights $\theta^- = \theta$
For episode = 1, M **do**
  Initialize sequence $s_1 = \{x_1\}$ and preprocessed sequence $\phi_1 = \phi(s_1)$
  For $t = 1$,T **do**
    $\left.\begin{array}{l}\text{With probability } \varepsilon \text{ select a random action } a_t \\ \text{otherwise select } a_t = \text{argmax}_a Q(\phi(s_t), a; \theta)\end{array}\right\}$
    Execute action a_t in emulator and observe reward r_t and image x_{t+1}
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    Perform a gradient descent step on $\left(y_j - Q(\phi_j, a_j; \theta)\right)^2$ with respect to the network parameters θ
    Every C steps reset $\hat{Q} = Q$
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End For

With small probability select a random action (explore), otherwise select the, currently known, best action (exploit).

Q-learning with experience replay

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    Every C steps reset $\hat{Q} = Q$
  End For
End For

Execute the chosen action and store the (processed) observed transition in the replay memory

Q-learning with experience replay

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For episode $= 1, M$ **do**
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    Perform a gradient descent step on $\left(y_j - Q(\phi_j, a_j; \theta)\right)^2$ with respect to the network parameters θ
    Every C steps reset $\hat{Q} = Q$
  End For
End For

Experience replay:
Sample a random minibatch of transitions from replay memory and perform gradient decent step on Q (not on $\hat{Q}$)

Q-learning with experience replay

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For episode $= 1, M$ **do**
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End For

Once every several steps set the target function, $\hat{Q}$, to equal Q

Q-learning with experience replay

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    Perform a gradient descent step on $\left(y_j - Q(\phi_j, a_j; \theta)\right)^2$ with respect to the network parameters θ
    Every C steps reset $\hat{Q} = Q$
  End For
End For

Such delayed online learning helps in practice:
"This modification makes the algorithm more stable compared to standard online Q-learning, where an update that increases $Q(s_t, a_t)$ often also increases $Q(s_{t+1}, a)$ for all a and hence also increases the target y_j, possibly leading to oscillations or divergence of the policy" [Human-level control through deep reinforcement learning. Nature 518.7540 (2015): 529.]

Refs

lecture_DQL.key (cmu.edu)

12DQN.pptx (live.com)

Lecture 6: CNNs and Deep Q Learning =1[1]With many slides for DQN from David Silver and Ruslan Salakhutdinov and some vision slides from Gianni Di Caro and images from Stanford CS231n, http://cs231n.github.io/convolutional-networks/