Neural Network as a Classifier

MUSTAFA HAJIJ

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Lets recall the feedforward algorithm before first.

Feedforward Neural Network

How do we compute a feedforward neural network on an input x ?

Feedforward Neural Network

Start with an input $x = a^{(0)}$. In the picture, this is represented by the first layer of nodes. We will call this layer 0.

$$x = a^{(0)}$$

Feedforward Neural Network

We apply the weight $W^{(1)}$ coming from the edges between layer 0 and layer 1 and add the biases and then apply the Activation function on the resulting vector coordinate-wise.

$$x = a^{(0)} \longrightarrow \boxed{\sigma(W^{(1)}a^{(0)}+b^{(1)})}$$

$W^{(1)}$: Edges between layer 0 and layer 1
$a^{(0)}$: input
$b^{(1)}$: biases applied to layer 1
σ : activation function

Feedforward Neural Network

We will call the output of this computation $a^{(1)}$. This is now represented by the nodes in layer 1.

$$x = a^{(0)} \longrightarrow \boxed{\sigma(W^{(1)}a^{(0)}+b^{(1)})} \xrightarrow{a^{(1)}}$$

$W^{(1)}$: Edges between layer 0 and layer 1
$a^{(0)}$: input
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Feedforward Neural Network

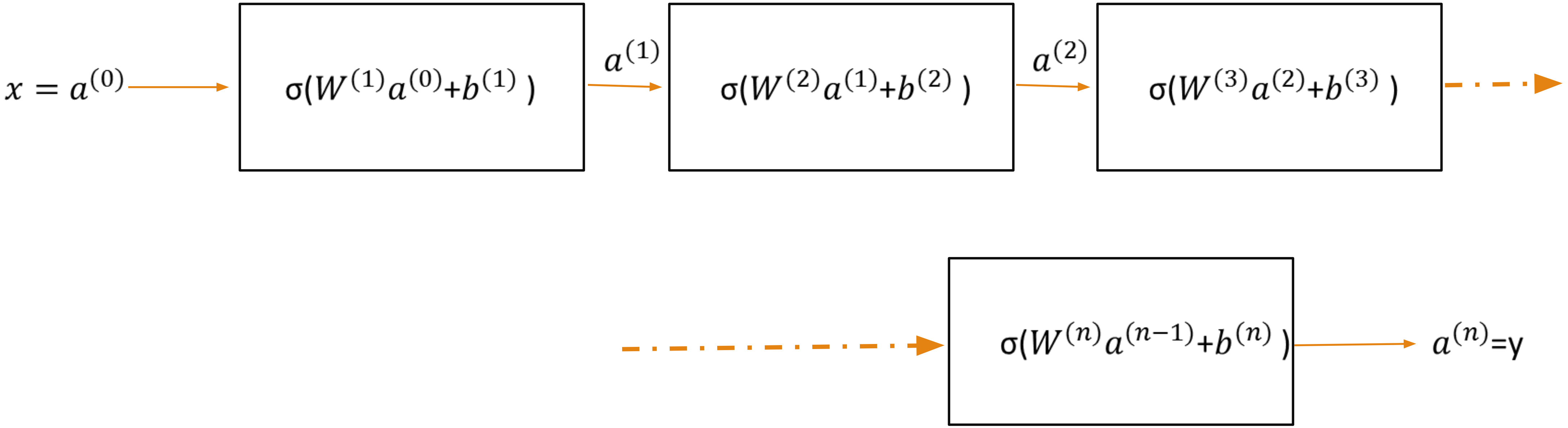
Repeat.

$$x = a^{(0)} \longrightarrow \boxed{\sigma(W^{(1)}a^{(0)}+b^{(1)})} \xrightarrow{a^{(1)}} \boxed{\sigma(W^{(2)}a^{(1)}+b^{(2)})} \xrightarrow{a^{(2)}}$$

$W^{(2)}$: Edges between layer 1 and layer 2
$a^{(1)}$: input from layer 1
$b^{(2)}$: biases applied to layer 2
σ : activation function

Feedforward Neural Network

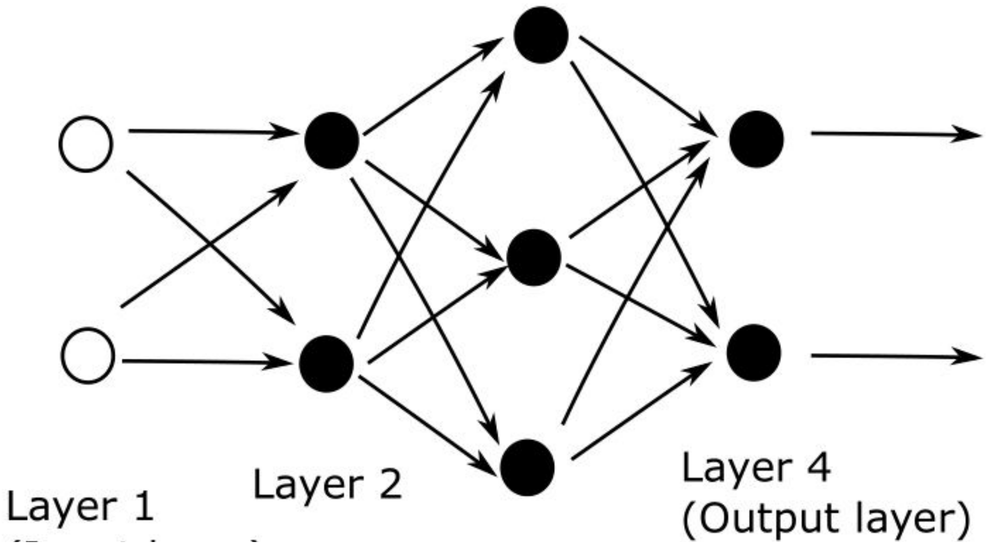
Until you finish the neural network and get the final output.



Example

We will use an example from <u>this</u> paper.

(note that the convention of the index is a little different here)



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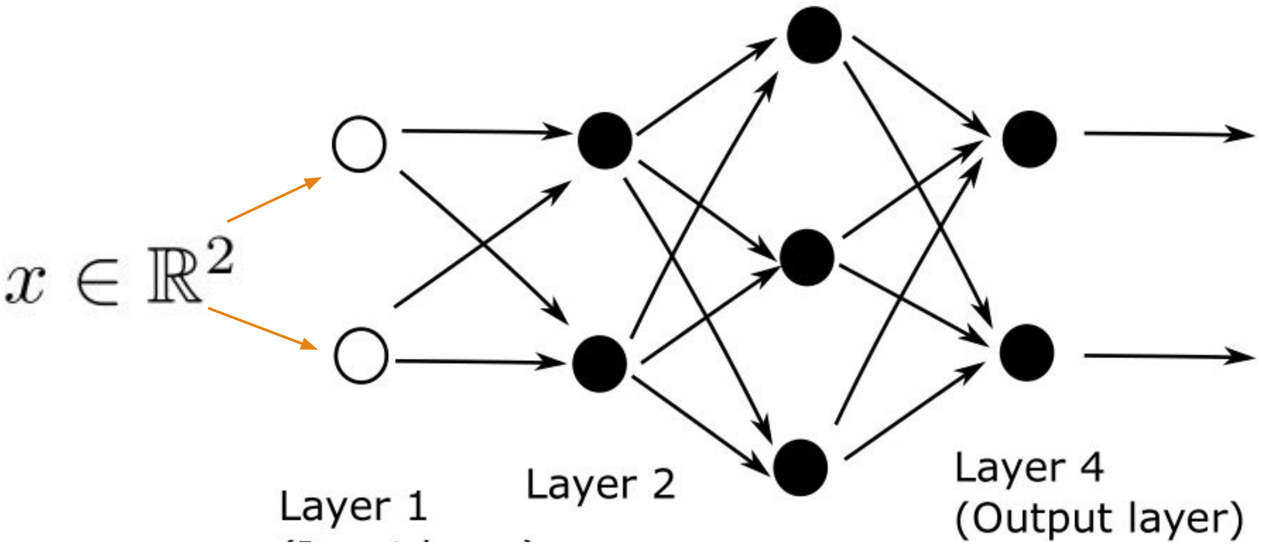

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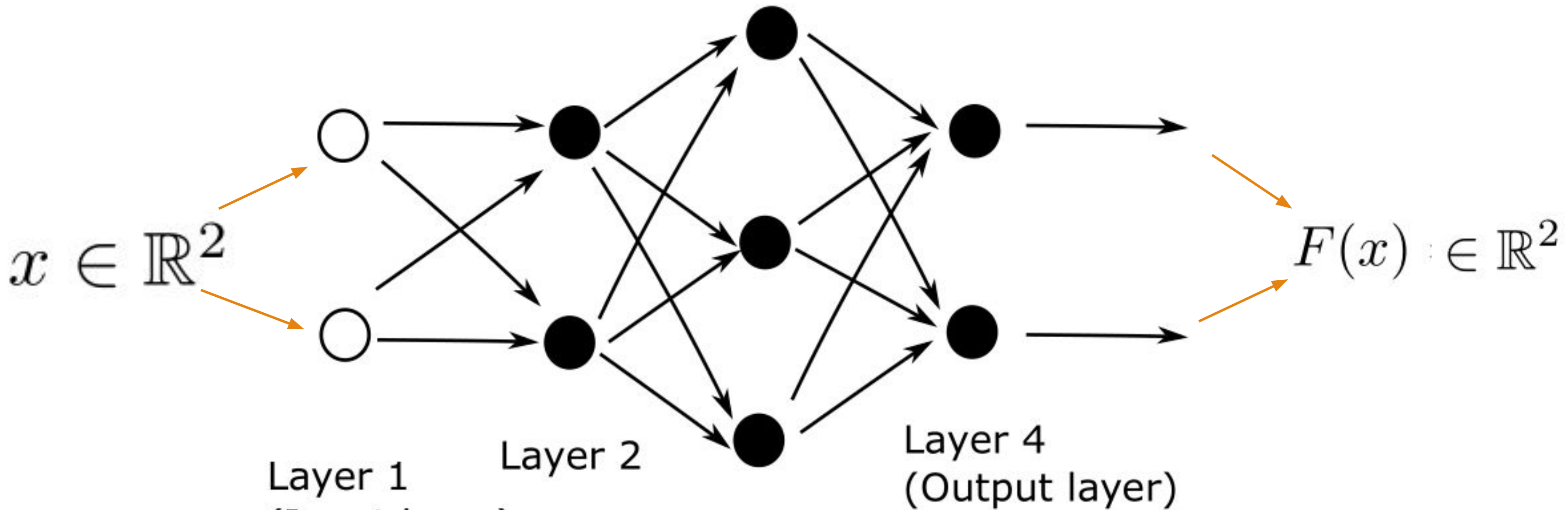

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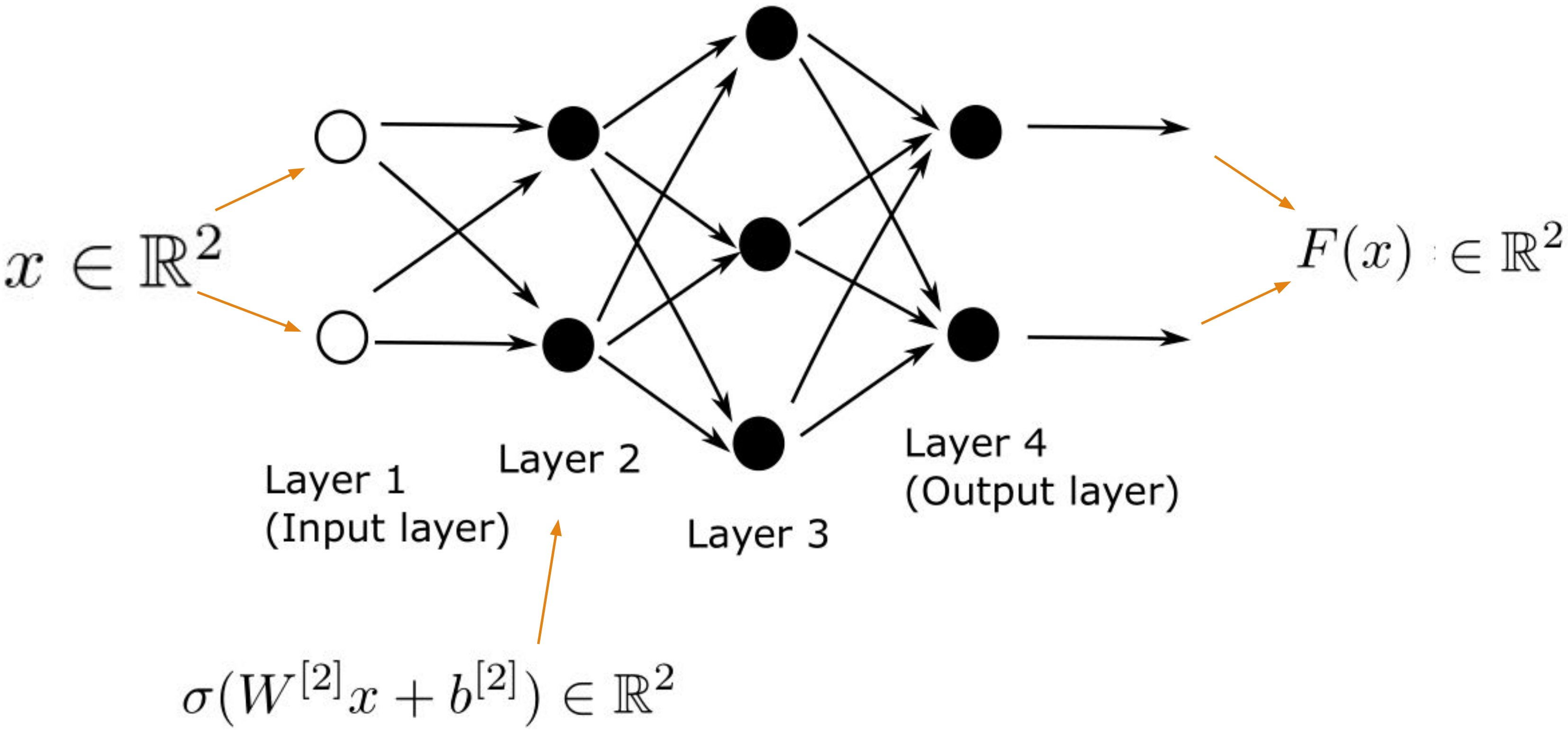

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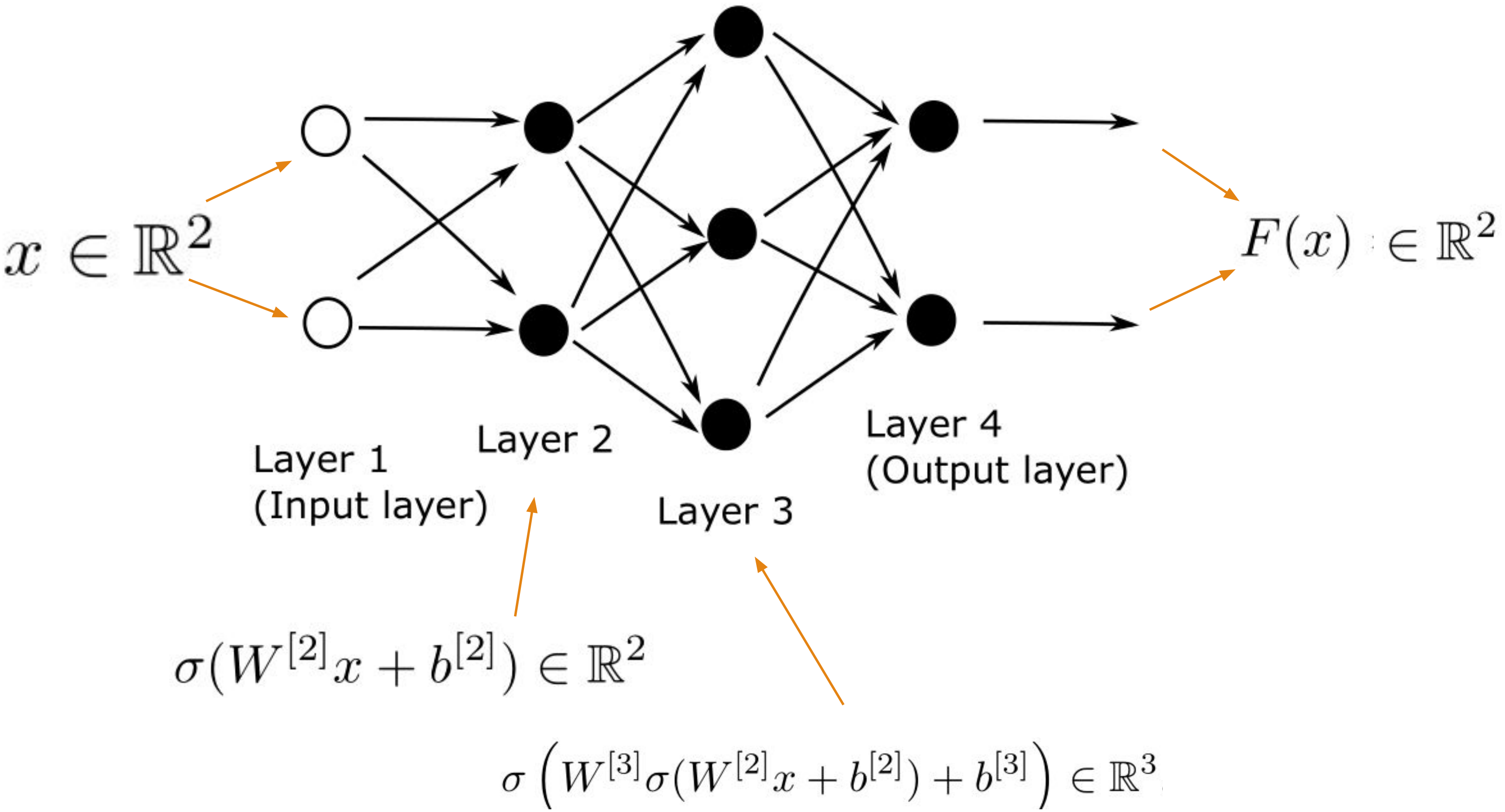

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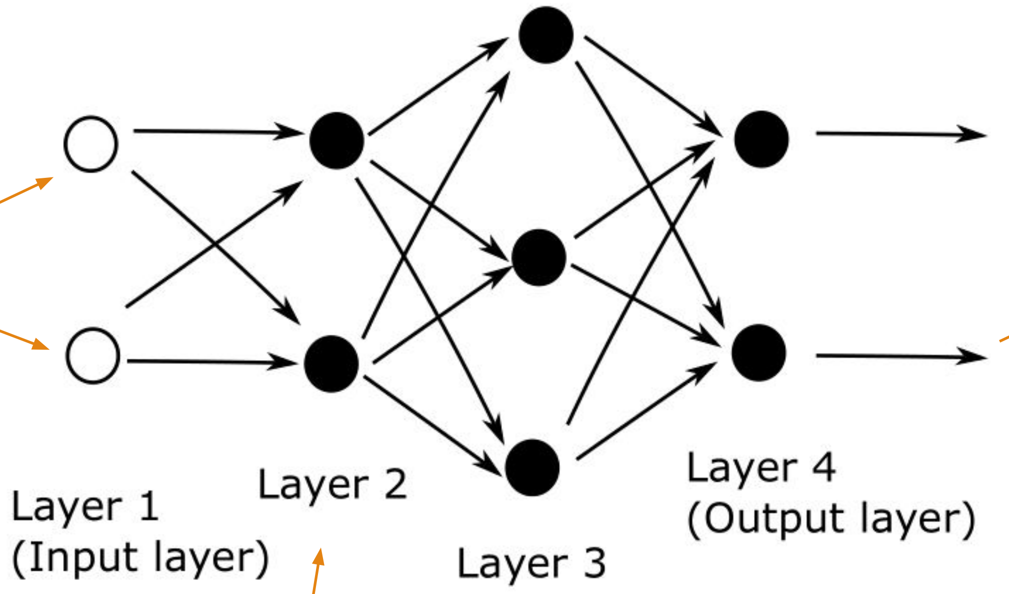
$x \in \mathbb{R}^2$



$F(x) \in \mathbb{R}^2$

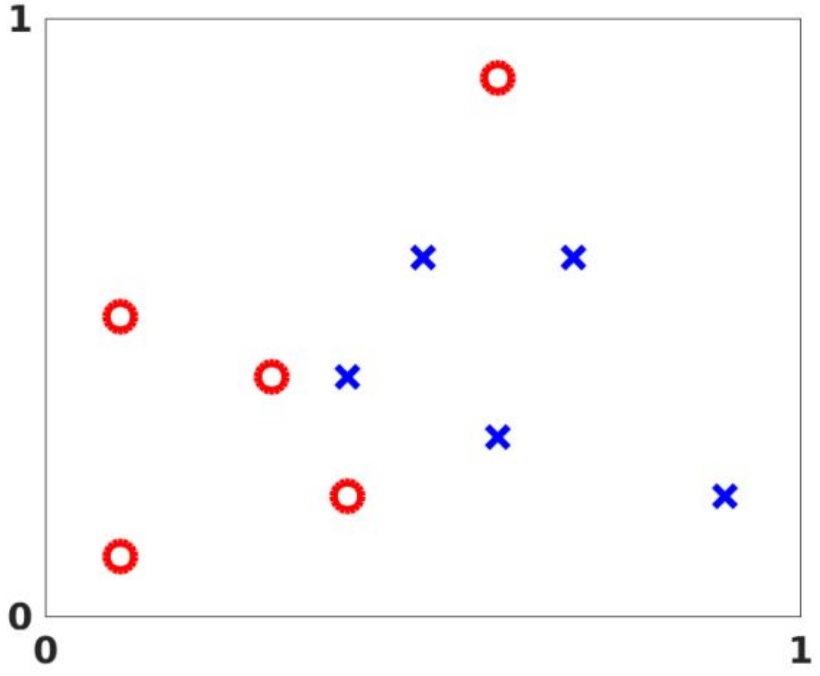
$\sigma(W^{[2]}x + b^{[2]}) \in \mathbb{R}^2$

$\sigma\left(W^{[3]}\sigma(W^{[2]}x + b^{[2]}) + b^{[3]}\right) \in \mathbb{R}^3$

Final function representing the neural network

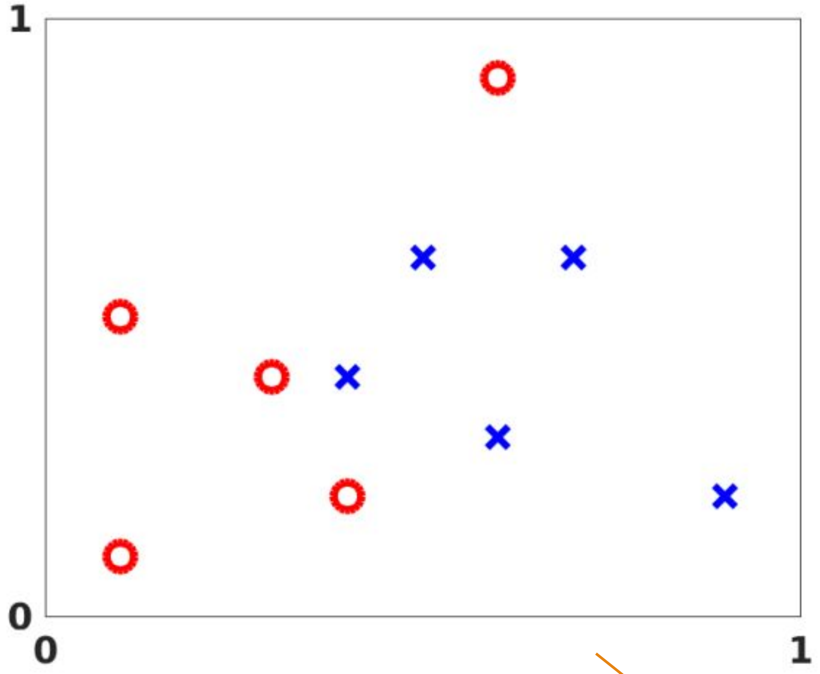
$$F(x) = \sigma\left(W^{[4]}\sigma\left(W^{[3]}\sigma(W^{[2]}x + b^{[2]}) + b^{[3]}\right) + b^{[4]}\right) \in \mathbb{R}^2.$$

Example



Input : labeled data X

Example



Input : labeled data X

$$\mathrm{Cost}\left(W^{[2]}, W^{[3]}, W^{[4]}, b^{[2]}, b^{[3]}, b^{[4]}\right) = \frac{1}{10}\sum_{i=1}^{10} \tfrac{1}{2}\|y(x^{\{i\}}) - F(x^{\{i\}})\|_2^2.$$

the difference between the output given by the network and the actual label

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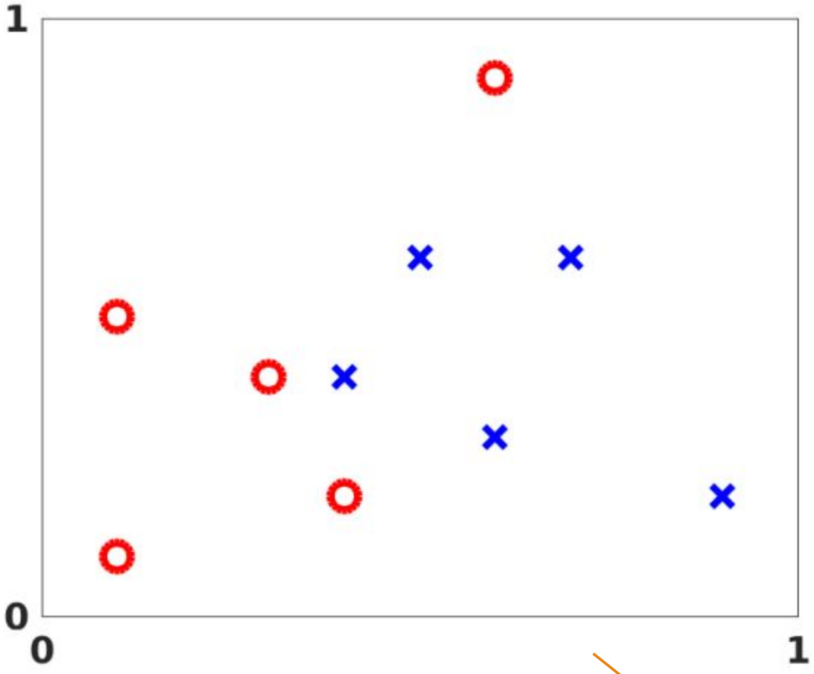

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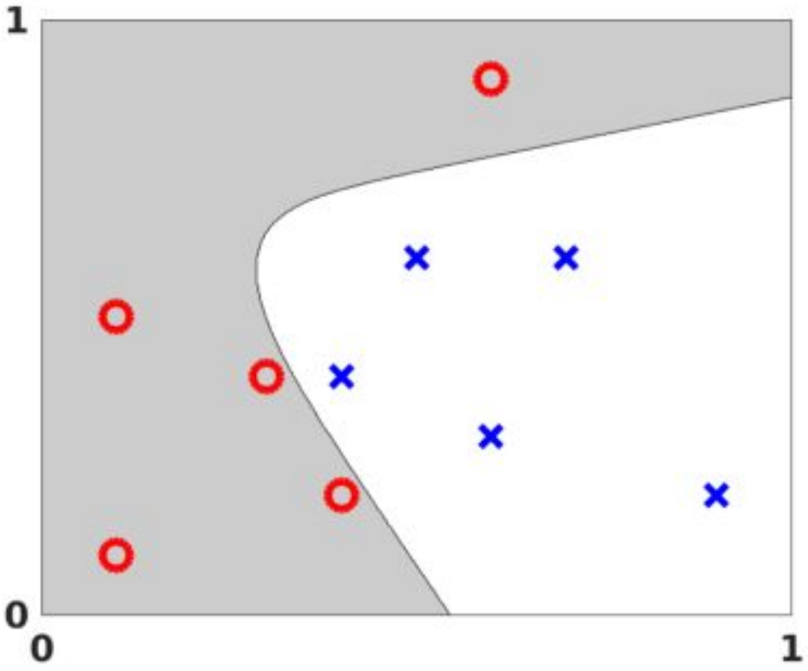

Minimize the cost function

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Binary classification

Now suppose that we have data set that consists of images of cats and dogs and we built a neural network that takes as input an image from this data set and gives out a vector in R^1 (a real number).
How exactly do we use this vector for our classification task ? In general the output f(x) coming from the neural network
Does not match the class $\{\pm 1\}$ of the input point x (it could be any real number).


Binary classification

This function takes a tensor of size input_size and returns a real number.

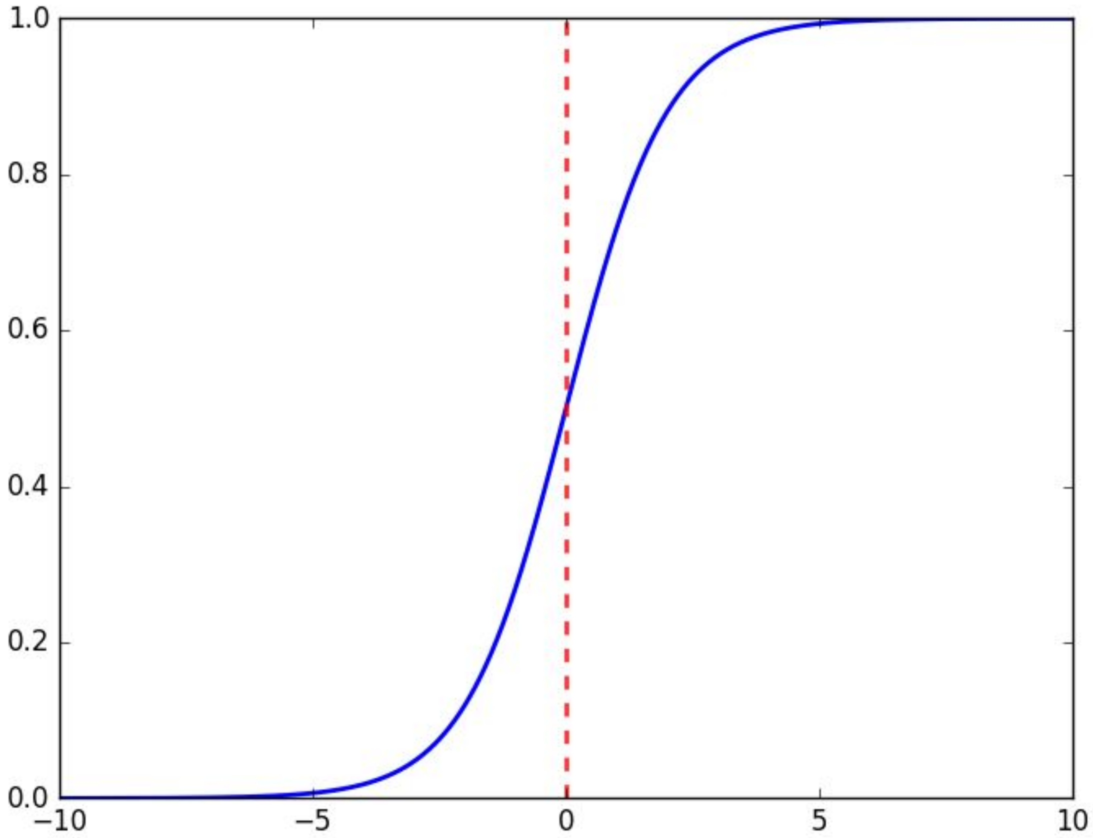
```
import torch
import torch.nn as nn

class Net(nn.Module):
    def __init__(self, input_size, hidden_size):
        super(Net, self).__init__()
        self.fc1 = nn.Linear(input_size, hidden_size)
        self.fc2 = nn.Linear(hidden_size, 1)

    def forward(self, x):
        x = torch.relu(self.fc1(x))
        x = self.fc2(x)
        return x
```

How can we constrain the output to be between -1 and +1?

Binary classification

To obtain the required binary classification, we pass the output f(x) through another function :

$$g(z) = 1/(1 + e^{-z})$$



The graph of the sigmoid function

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But what do we do in the multi-class classification ?

Multi-class classification : the softmax function

In the case of multi-class classification, we use the softmax activation function.
Suppose that we have k classes then the softmax activation function is define by :

$$\mathrm{softmax}(z)_i = \frac{\exp(z_i)}{\sum_{l=1}^{k} \exp(z_l)}$$

Here z_i represents the ith element of the input to softmax, which corresponds to class i.

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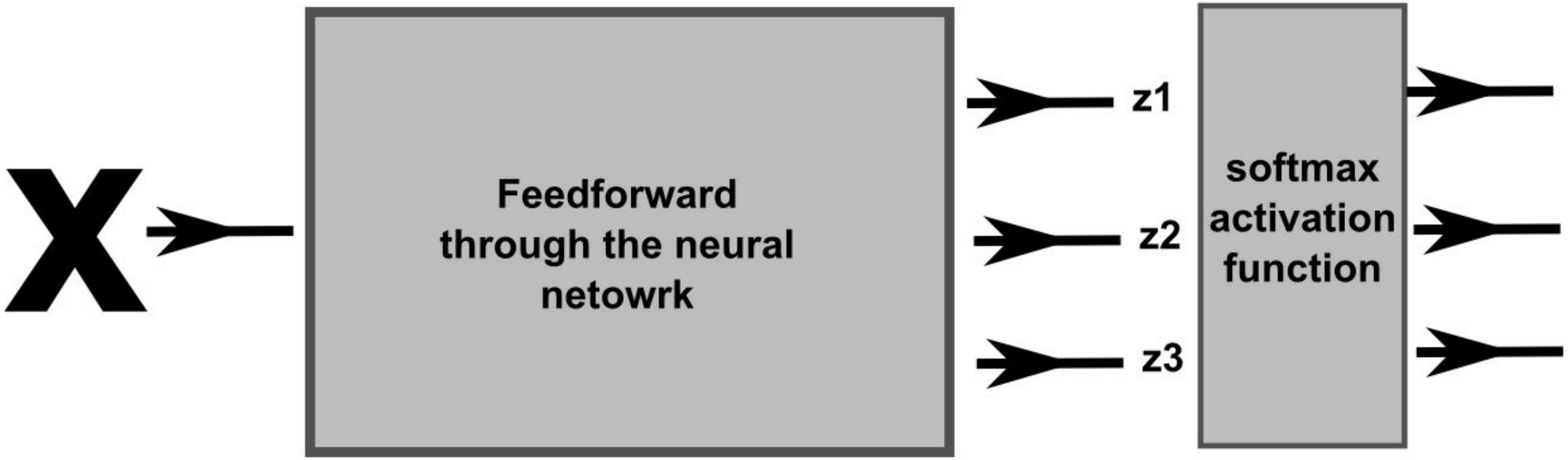

The softmax function in Python

The softmax function is a mathematical function used to convert a vector of real numbers into a probability distribution.

It takes an input vector and returns another vector of the same length, where each element is transformed to a value between 0 and 1, representing the probability of that element being selected. In simple terms, the softmax function normalizes the input vector and makes it easier to interpret as probabilities. Here's a Python example:

```
import numpy as np

def softmax(x):
    exp_values = np.exp(x)
    probabilities = exp_values / np.sum(exp_values)
    return probabilities

input_vector = np.array([2.0, 1.0, 0.5])
output_vector = softmax(input_vector)
print(output_vector)
[0.62842832 0.2312239  0.14034778]
```