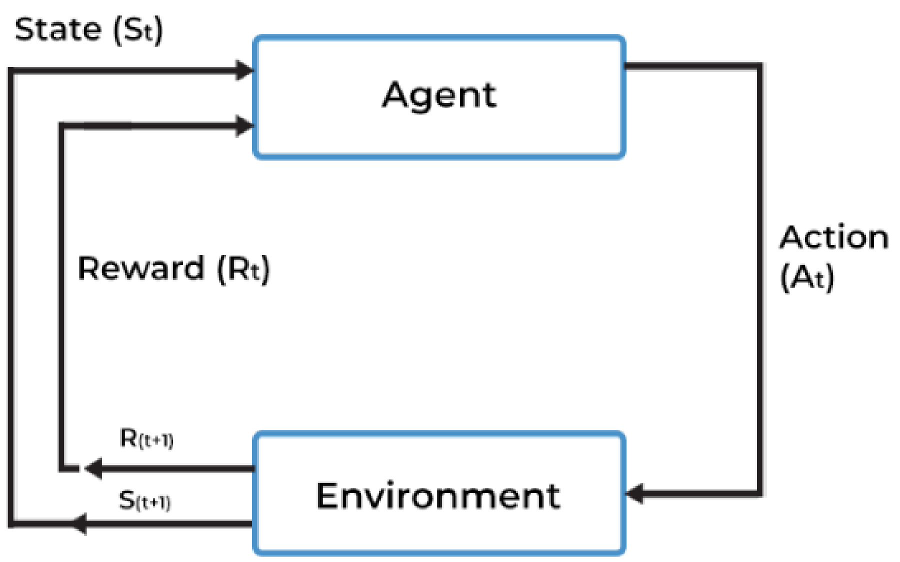
Introduction to RL

Reinforcement learning

- The challenges encompassing an agent's interaction with an environment, wherein numeric rewards are given.
- Objective of acquiring the ability to make decisions that optimize the reward outcome.



Reinforcement learning

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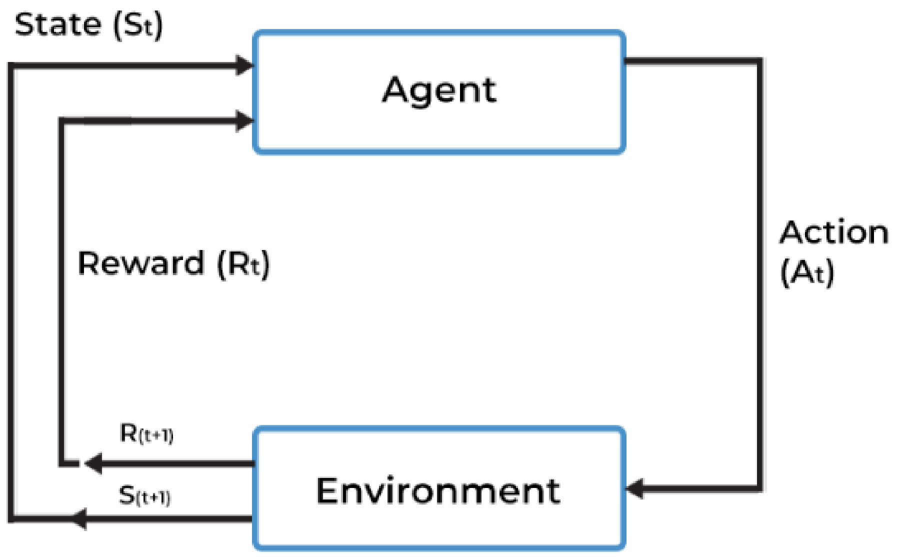
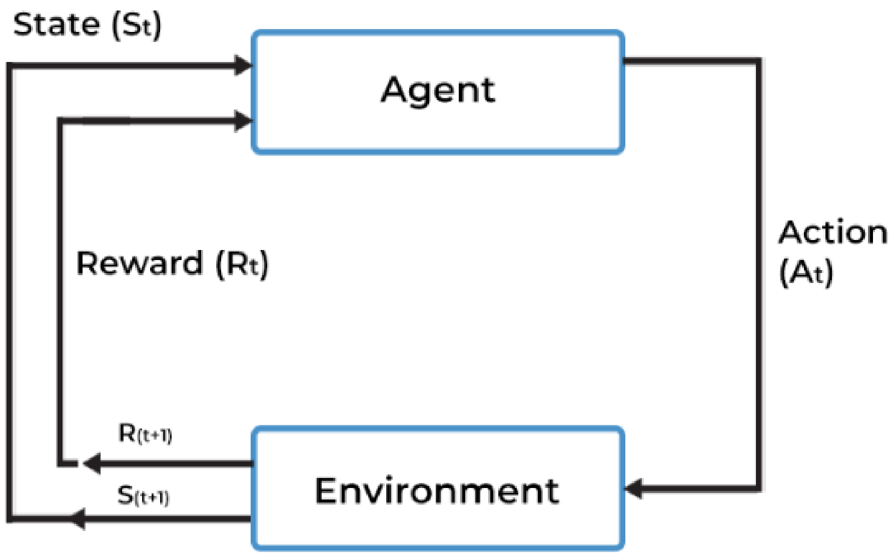
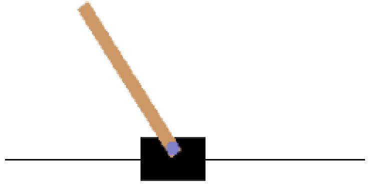


Typically in a RL system one observes the following sequence
state, action, reward, new state…

Reinforcement learning

Examples

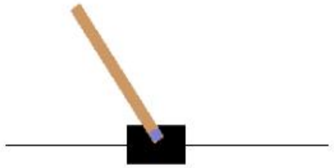


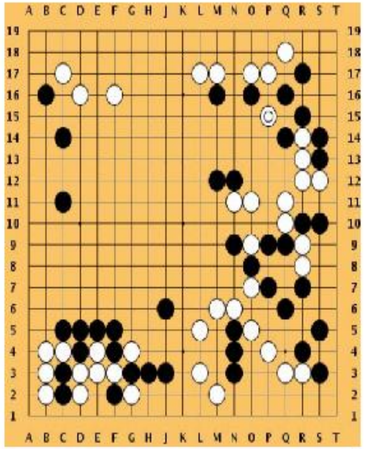


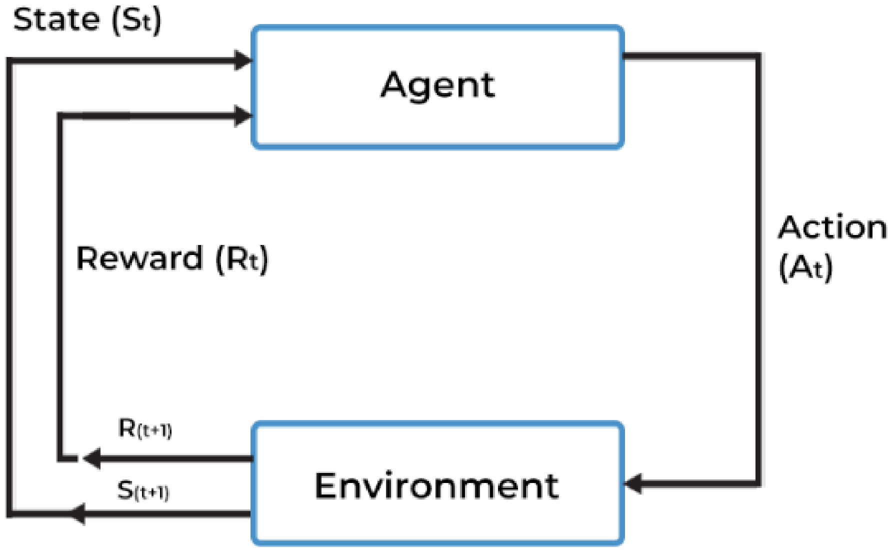
Objective: Balance a pole on top of a movable cart
State: angle, angular speed, position, horizontal velocity
Action: horizontal force applied on the cart
Reward: 1 at each time step if the pole is upright

Reinforcement learning

Examples







Markov Decision Process

Defined by: $(\mathcal{S}, \mathcal{A}, \mathcal{R}, \mathbb{P}, \gamma)$

$\mathcal{S}$: set of possible states
$\mathcal{A}$: set of possible actions
$\mathcal{R}$: distribution of reward given (state, action) pair
$\mathbb{P}$: transition probability i.e. distribution over next state given (state, action) pair
γ : discount factor

Markov Decision Process

At time step t=0, environment samples initial state $s_0 \sim p(s_0)$

Then, for t=0 until done:

Agent selects action a_t

Environment samples reward $r_t \sim R(\,.\,|s_t\,, a_t\,)$

Environment samples next state $s_{t+1} \sim P(\,.\,|s_t\,, a_t\,)$

Agent receives reward rt and moves to next state s_{t+1}

A policy **π** is a function from S to A that specifies what action to take in each state –

Objective: find policy **π*** that maximizes cumulative discounted reward:

$$\sum_{t \geq 0} \gamma^t r_t$$

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Q-function

The total reward is the discounted sum of all rewards obtained from time t:

$$R_t = r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \cdots$$

Where:

- γ is the discount factor.
- R_t is the total reward.

The Q-function is defined as:

$$Q(s_t, a_t) = \mathbb{E}[R_t | s_t, a_t]$$

Where:

- $Q(s_t, a_t)$ captures the expected total future reward an agent in state s_t can receive by executing a certain action a_t.
- s_t is the state.
- a_t is the action.

Q-function

Given the Q function, how would the agent use it to navigate the environment:

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Answer: the agent needs a policy $\pi(s)$ to infer the best action at its state, s

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Strategy : choose an action that maximizes the future reward

$$\pi^* (s) = argmax_a \, Q(s, a)$$

Bellman Equation

$$Q^*(s,a) = \mathbb{E}_{s'}\left[r + \gamma \max_{a'} Q^*(s',a') \mid s,a \right] \quad \text{Bellman Equation}$$

The optimal policy can be found via :

$$\pi^*(s) = \underset{a}{\operatorname{argmax}}\ Q^*(s,a)$$

Q(s,a) can be understood as : how “good” a given state, action pair is.

Deep Q learning

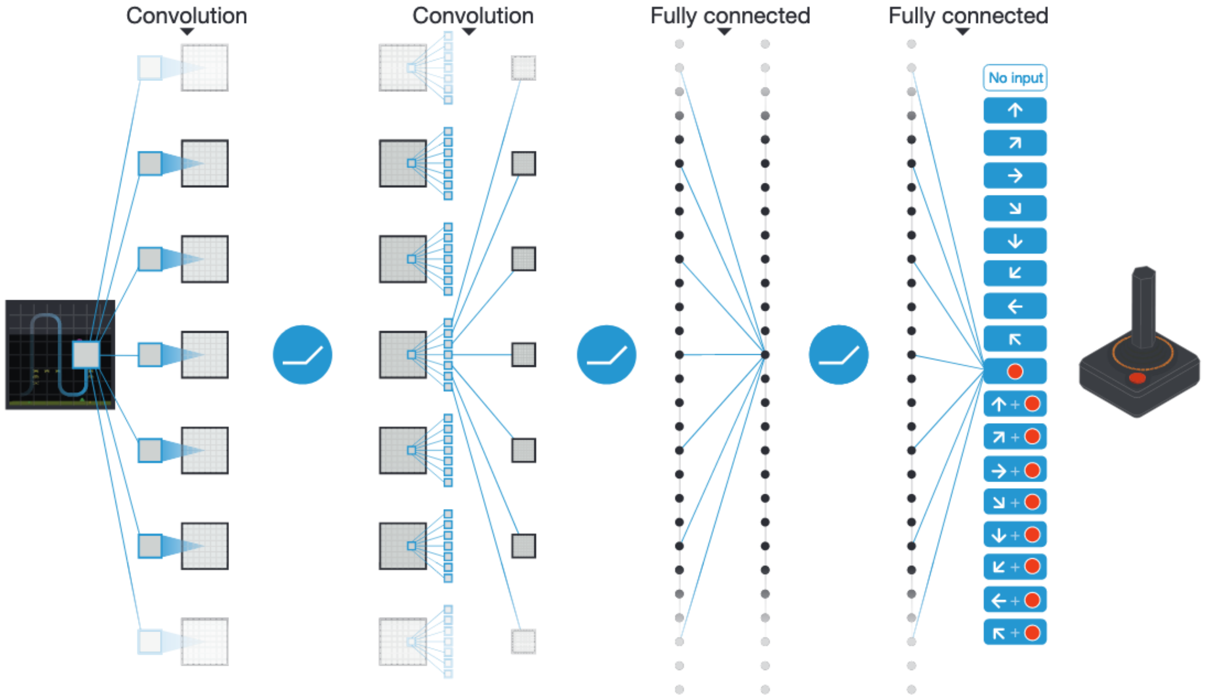
Deep Q learning [Mnih et al. 2015]

- Learn a function approximator (Q network), $\hat{Q}(s, a; \theta) \approx Q^*(s, a)$
- Value propagation: $Q^*(s, a) = \mathbb{E}_{s' \sim \mathcal{E}}\left[r + \gamma \max_{a'} Q^*(s', a')\right]$
  - $\mathcal{E}$ represents the environment (underlying MDP)
- Update $\hat{Q}(s, a; \theta)$ at each step i using SGD with squared loss:
- $L_i(\theta_i) = \mathbb{E}_{s,a \sim \rho(\cdot)}\left[\left(y_i - \hat{Q}(s, a; \theta_i)\right)^2\right]$
  - $y_i = \mathbb{E}_{s' \sim \mathcal{E}}\left[r + \gamma \max_{a'} \hat{Q}(s', a'; \theta_{i-1})\right]$
  - $\rho(s, a)$ is the behavior distribution, e.g., epsilon greedy
  - θ_{i-1} is considered fix when optimizing the loss function (helps when taking the derivative with respect to θ and with stability)

Deep Q learning [Mnih et al. 2015]

- $L_i(\theta_i) = \mathbb{E}_{s,a\sim\rho(\cdot)}\left[\left(\boxed{y_i} - \hat{Q}(s,a;\theta_i)\right)^2\right]$
- $\nabla_{\theta_i} L_i(\theta_i) = \mathbb{E}_{s,a\sim\rho(\cdot),s'\sim\mathcal{E}}\left[\left(r + \gamma \max_{a'} \hat{Q}(s',a';\theta_{i-1}) - \hat{Q}(s,a;\theta_i)\right)\nabla_{\theta_i}\hat{Q}(s,a;\theta_i)\right]$

Independent of θ_i (because θ_{i-1} is considered fix)



Q-learning with experience replay

Initialize replay memory D to capacity N
Initialize action-value function Q with random weights θ
Initialize target action-value function $\hat{Q}$ with weights $\theta^{-} = \theta$

Initialize the replay memory and two identical Q approximators (DNN). $\hat{Q}$ is our target approximator.

For episode $= 1, M$ **do**
  Initialize sequence $s_1 = \{x_1\}$ and preprocessed sequence $\phi_1 = \phi(s_1)$
  For $t = 1$,T **do**
    With probability ε select a random action a_t
    otherwise select $a_t = \mathrm{argmax}_a Q(\phi(s_t), a; \theta)$
    Execute action a_t in emulator and observe reward r_t and image x_{t+1}
    Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$
    Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in D
    Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from D
    Set $y_j = \begin{cases} r_j & \text{if episode terminates at step } j+1 \\ r_j + \gamma \max_{a'} \hat{Q}\left(\phi_{j+1}, a'; \theta^{-}\right) & \text{otherwise} \end{cases}$
    Perform a gradient descent step on $\left(y_j - Q\left(\phi_j, a_j; \theta\right)\right)^2$ with respect to the network parameters θ
    Every C steps reset $\hat{Q} = Q$
  End For
End For

Q-learning with experience replay

Initialize replay memory D to capacity N
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Initialize target action-value function $\hat{Q}$ with weights $\theta^{-} = \theta$
For episode = 1, M **do** Play *m* episodes (full games)
  Initialize sequence $s_1 = \{x_1\}$ and preprocessed sequence $\phi_1 = \phi(s_1)$
  For $t = 1$,T **do**
    With probability ε select a random action a_t
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Start episode from x_1 (pixels at the starting screen).
Preprocess the state (include 4 last frames, RGB to grayscale conversion, downsampling, cropping)

Q-learning with experience replay

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For episode $= 1, M$ **do**
  Initialize sequence $s_1 = \{x_1\}$ and preprocessed sequence $\phi_1 = \phi(s_1)$
  For $t = 1$,T **do** ← For each time step during the episode
    With probability ε select a random action a_t
    otherwise select $a_t = \mathrm{argmax}_a Q(\phi(s_t), a; \theta)$
    Execute action a_t in emulator and observe reward r_t and image x_{t+1}
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  For $t = 1$,T **do**
    $\left.\begin{array}{l}\text{With probability } \varepsilon \text{ select a random action } a_t \\ \text{otherwise select } a_t = \text{argmax}_a Q(\phi(s_t), a; \theta)\end{array}\right\}$
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    Perform a gradient descent step on $\left(y_j - Q\left(\phi_j, a_j; \theta\right)\right)^2$ with respect to the network parameters θ
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With small probability select a random action (explore), otherwise select the, currently known, best action (exploit).

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End For

Execute the chosen action and store the (processed) observed transition in the replay memory

Q-learning with experience replay

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End For

Experience replay:
Sample a random minibatch of transitions from replay memory and perform gradient decent step on Q (not on $\hat{Q}$)

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    Perform a gradient descent step on $\left(y_j - Q(\phi_j, a_j; \theta)\right)^2$ with respect to the network parameters θ
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End For

Once every several steps set the target function, $\hat{Q}$, to equal Q

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$$\text{Set } y_j = \begin{cases} r_j & \text{if episode terminates at step j+1} \\ r_j + \gamma \max_{a'} \hat{Q}(\phi_{j+1}, a'; \theta^-) & \text{otherwise} \end{cases}$$

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  End For
End For

Such delayed online learning helps in practice:
"This modification makes the algorithm more stable compared to standard online Q-learning, where an update that increases $Q(s_t, a_t)$ often also increases $Q(s_{t+1}, a)$ for all a and hence also increases the target y_j, possibly leading to oscillations or divergence of the policy" [Human-level control through deep reinforcement learning. Nature 518.7540 (2015): 529.]