

First, we convert each speed from km/h to mins/km:

- Rocky terrain: 2 km/h = 30 mins/km
- Sandy terrain: 3 km/h = 20 mins/km
- Smooth terrain: 5 km/h = 12 mins/km

Each route has a different length and different probabilities for each terrain type. We'll calculate the expected travel time per kilometer for each route. Then, we'll multiply this expected time by the total distance of each route to get the expected total travel time.

Route 1: 5 km

- Probability for rocky: 50% or 0.5
- Probability for sandy: 20% or 0.2
- Probability for smooth: 30% or 0.3

Expected time (mins/km) = $(0.5 \times 30) + (0.2 \times 20) + (0.3 \times 12) = 15 + 4 + 3.6 = 22.6$ mins/km

Total expected time for Route 1:

$$22.6 \text{ mins/km} \times 5 \text{ km} = \mathbf{113 \text{ mins}}$$

Route 2: 7 km

- Probability for rocky: 40% or 0.4
- Probability for sandy: 40% or 0.4
- Probability for smooth: 20% or 0.2

Expected time (mins/km) = $(0.4 \times 30) + (0.4 \times 20) + (0.2 \times 12) = 12 + 8 + 2.4 = 22.4$ mins/km

Total expected time for Route 2:

$$22.4 \text{ mins/km} \times 7 \text{ km} = \mathbf{156.8 \text{ mins}}$$

Route 3: 6 km

- Probability for rocky: 10% or 0.1
- Probability for sandy: 50% or 0.5
- Probability for smooth: 40% or 0.4

Expected time (mins/km) = $(0.1 \times 30) + (0.5 \times 20) + (0.4 \times 12) = 3 + 10 + 4.8 = 17.8$ mins/km

Total expected time for Route 3:

$$17.8 \text{ mins/km} \times 6 \text{ km} = \mathbf{106.8 \text{ mins}}$$

The expected travel times for each route are:

- Route 1: 113 mins
- Route 2: 156.8 mins
- Route 3: 106.81 mins

Since Route 3 has the lowest expected travel time, the rover should choose **Route 3** to return to the charging station.

To update our estimates, we need to factor in the potential delays or shortcuts for Routes 1 and 3, given the probabilities of the crater wall or bridge being damaged.

Adjust Route 1 for the Crater Shortcut

For Route 1, we already calculated the baseline expected travel time without considering the crater shortcut or extra detour:

Expected time (without crater adjustment) = 113 mins

Now, with a **30% chance the wall is damaged**, which would add 15 minutes, and a **70% chance the wall is intact**, which would save 20 minutes, the adjusted expected time for Route 1 can be calculated as follows:

Adjusted Expected Time for Route 1 = $133 - (0.7 \times 20) + (0.3 \times 15)$

= $133 - 14 + 4.5$

= 103.5 mins/km

Adjust Route 3 for the Bridge Repair

For Route 3, we previously calculated the expected travel time without considering the bridge condition:

Expected time (without bridge adjustment) = 106.8 mins

There's a **60% chance the bridge is damaged**, which would add 40 minutes to the travel time.

Adjusted Expected Time for Route 3 = $106.8 + (0.6 \times 40)$

= $106.8 + 24$

= 130.8mins/km

With the adjustments, the expected travel times for each route are now:

- Route 1 (with crater shortcut or detour): 103.5 mins
- Route 2: 156.8 mins (unchanged)
- Route 3 (with potential bridge repair): 130.8 mins

With these updated estimates, Route 1 now has the lowest expected travel time at 103.5 mins. Therefore, **Route 1** is the best choice.

To answer this question, let's examine each part carefully and assess how the satellite's information about Route 2 being rocky or not affects the expected travel time and decision-making process.

First: Expected Utility if Route 2 is Not Rocky

If Route 2 is confirmed **not rocky**, it would either be **sandy** or **smooth**. We can calculate the expected travel time under this condition by adjusting our probabilities and recalculating based on the remaining terrains.

Probability adjustments:

Given that Route 2 is not rocky, we update the probabilities as follows:

- **Sandy:** $40/60 = 2/3$
- **Smooth:** $20/60 = 1/3$

Calculate the expected time with adjusted probabilities:

Using the speeds for sandy (20 mins/km) and smooth (12 mins/km) terrains and the updated probabilities:

$$\text{Expected time (mins/km)} = (2/3 \times 20) + (1/3 \times 12) = 13.33 + 4 = 17.33 \text{ mins/km}$$

Total expected time for Route 2 if not rocky:

$$17.33 \times 7 = 121.3 \text{ mins/km}$$

In this case:

- Route 1 (adjusted): 103.5 mins
- Route 2 (not rocky): 121.3 mins

- Route 3 (adjusted): 130.8 mins

Thus, if the satellite confirms that Route 2 is not rocky, **Route 1** is the best choice with a total expected time of 103.5 mins.

Second: Probability That the Satellite Will Tell Route 2 is Not Rocky

The probability that Route 2 is **rocky** is given directly by the problem as **40%**, so The probability that Route 2 is **not rocky** is **60%**

Probability that the satellite will tell route 2 is not rocky = 60% or 0.6

Third: Expected Utility If the Satellite Tells Route 2 is Rocky

If the satellite confirms that Route 2 is **rocky**, we can calculate the expected travel time by adjusting our probabilities and recalculating based on the rocky terrain.

Probability adjustments:

Given that Route 2 is rocky, we update the probabilities as follows:

- **Rocky:** $100/100 = 1$

Calculate the expected time with adjusted probabilities:

Using the speed for rocky (30 mins/km) terrain and the updated probability:

$$\text{Expected time (mins/km)} = (1 \times 30) = 30 \text{ mins/km}$$

Total expected time for Route 2 if rocky:

$$30 \times 7 = \mathbf{210 \text{ mins/km}}$$

In this case:

- Route 1 (adjusted): 103.5 mins
- Route 2 (100% rocky): 210 mins
- Route 3 (adjusted): 130.8 mins

With the new information that Route 2 is rocky, **Route 1** still remains the best choice with a total expected time of 103.5 mins.

Last: How Long to Wait for the Satellite?

1. **Expected Time with Satellite Information:**

- If Route 2 is rocky (40% chance), we choose Route 1: 103.5 mins
- If Route 2 is not rocky (60% chance), we also choose Route 1: 103.5 mins

Expected time with satellite info = $(0.4 \times 103.5) + (0.6 \times 103.5) = \mathbf{103.5 \text{ mins}}$

2. **Expected Time without Satellite Information:**

- Based on initial calculations, the best expected time without satellite info is also Route 1: **103.5 mins.**

Since the expected travel time with satellite information is **103.5 mins**, the same as without it, waiting for satellite data does not change our expected travel time. Therefore, there is no benefit in waiting for the satellite information. We should proceed immediately with **Route 1**, which has the shortest expected travel time of **103.5 mins**.
