Zhunan Shuai Part 4. Utility

Question 1:

convert km/h to min/km

- Rocky terrain: 60 / 2 = 30 minutes/km
- Sandy terrain: 60 / 3 = 20 minutes/km
- Smooth terrain: 60 / 5 =12 minutes/km

Calculate time travel for each route

- route 1
- 20% chance of being sandy, 30% chance of being smooth, and 50% chance of being rocky.
 - expected time per km = $0.2 \times 20 + 0.3 \times 12 + 0.5 \times 30 = 22.6$ min
 - total time = $5\times22.6=113$ min
- route 2
 - 40% chance of being sandy, 20% chance of being smooth, and 40% chance of being rocky.
 - Expected time per km: 0.4×20+0.2×12+0.4×30=22.4 min
 - Total time: $7 \times 22.4 = 156.8 \ 7 \times 22.4 = 156.8 \ min$
- route 3
 - 50% chance of being sandy, 40% chance of being smooth, and 10% chance of being rocky.
 - Expected time per km: 0.5×20+0.4×12+0.1×30=17.8 min
 - total time: $6 \times 17.8 = 106.8 6 \times 17.8 = 106.8 \text{ min}$

So we should pick route 3 since it only took 106.8 min

Question2

Now we can add crater and bridge, we can calculate the time again

- route 1
 - Wall is intact and not damaged: 70% chance of saving 20 minutes.
 - Wall is damaged: 30% chance of adding 15 minutes.
 - (0.3×15)–(0.7×20)=4.5–14=–9.5 minutes, means we expect to save 9.5 minutes on Route 1 due to the crater conditions.
 - 113-9.5=103.5 minutes
- route 3
 - bridge is damaged: 60% chance of adding 40 minutes.
 - 0.6×40=24 minutes, means we expect to add 24 minutes to Route 3 due to the bridge condition.
 - 106.8+24=130.8 minutes

After adding create and bridge, time adjusted, Route 1 becomes the best option with an expected travel time of 103.5 minutes.

Question 3

- Calculate Expected Time for Route 2 if Not Rocky
- Only sandy (40%) and smooth (20%) terrains remain, giving a combined probability of 60%.
 - Sandy = 0.4 / 0.6 = 0.6667
 - Smooth = 0.2 / 0.6 = 0.3333
 - Expected Time per km if not rocky:
 - 0.6667×20+0.3333×12=17.33 min/km
 - Total Expected Time: 7×17.33=121.3 min

If the satellite said that route 2 was not rocky, if will take 7×17.33=121.3 minutes

Question 4

- Probability the Satellite Confirms Route 2 is Not Rocky 1 - 0.4 = 0.6, so **60**% probability that the satellite will confirm Route 2 is not rocky.

Question 5

- if Route 2 is rocky, we can go with original probability distribution, which will take $7 \times 22.4 = 156.87 \times$

Question 6

- Calculate Expected Travel Time for Route 2 if We Wait for Satellite Info
 - If Route 2 is not rocky
 - Expected time per km = $(0.6667 \times 20) + (0.3333 \times 12) = 13.334 + 3.9996 = 17.33$ minutes/km
 - Total expected time = 7×17.33=121.3 minutes
 - Combined Expected Time for Route 2
 - 60% chance that Route 2 is not rocky, with expected time of 121.3 minutes.
 - 40% chance that Route 2 is rocky, with expected time of 156.8 minutes.
 - Total time = $(0.6 \times 121.3) + (0.4 \times 156.8) = 72.78 + 62.72 = 135.5$ minutes

Given all above calculation, it will take 135.5 min if we wait for satellite, Since the adjusted travel time for Route 1 is 103.5 minutes, which is faster than the expected 135.5 minutes for Route 2, we should not wait for the satellite at all.