COT 4521: Introduction to Computational Geometry

Polygons (part 2)

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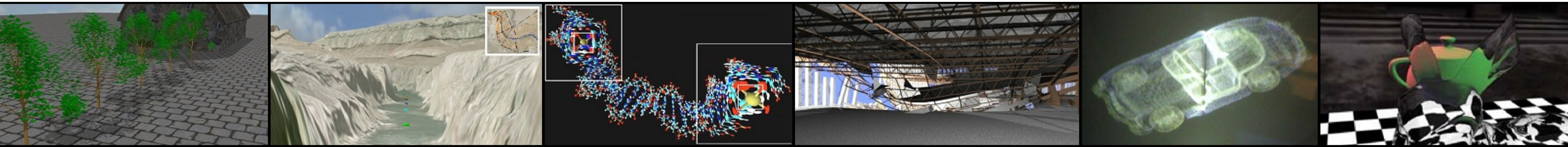
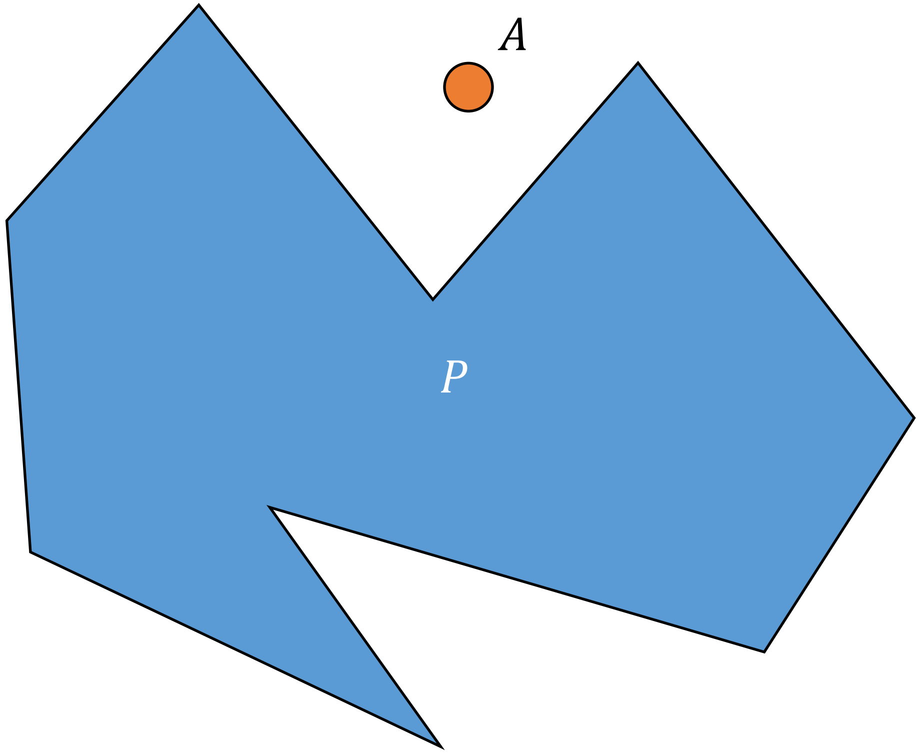
Some slides from Valentina Korzhova



Point inside a polygon test

- How can we tell if point A is inside of polygon P?



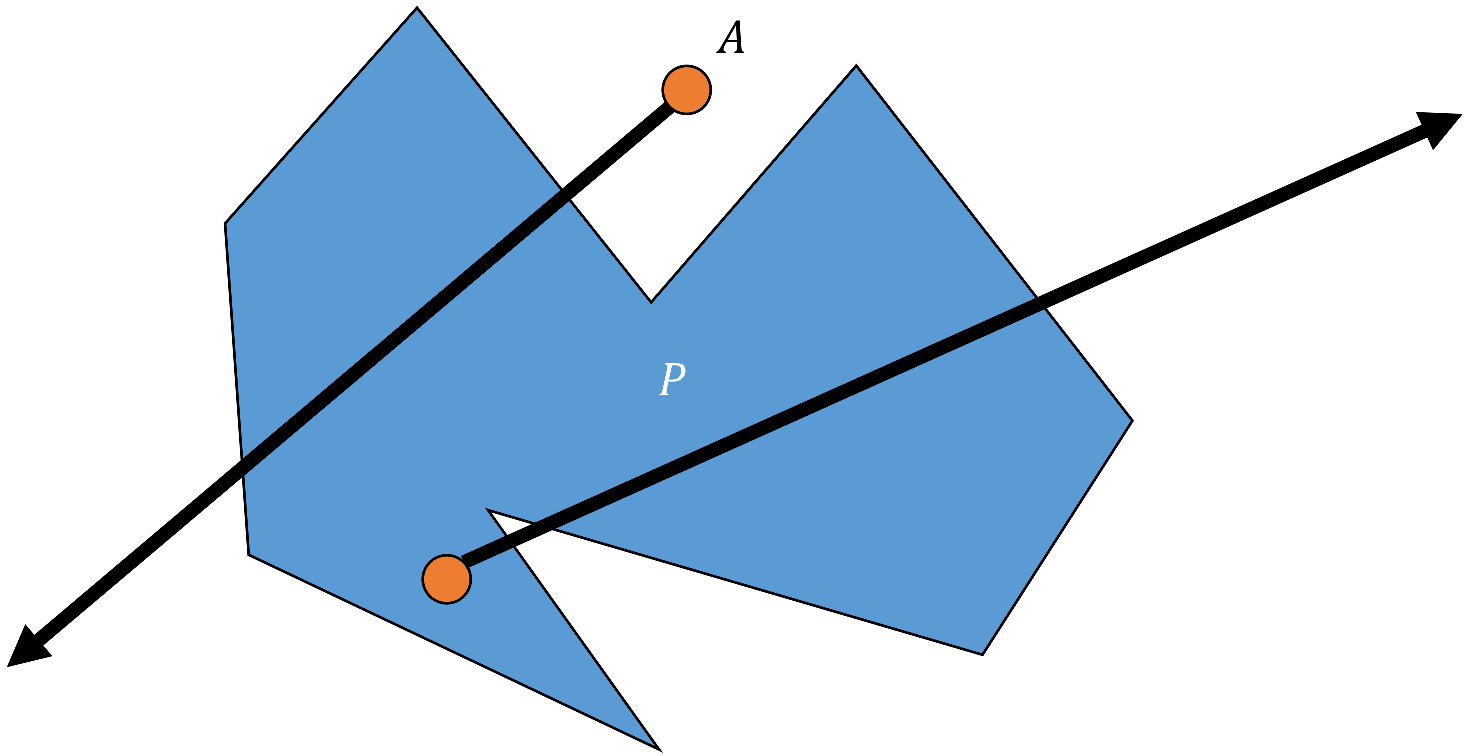
POINT INSIDE A POLYGON TEST

- EVEN–ODD RULE ALGORITHM
  - If a point A moves along a ray from infinity to A
  - If it crosses the boundary of a polygon, possibly several times, then it alternately goes from the outside to inside to outside
  - After every two "border crossings" the moving point goes outside.

  - This observation may be mathematically proved using the Jordan curve theorem.

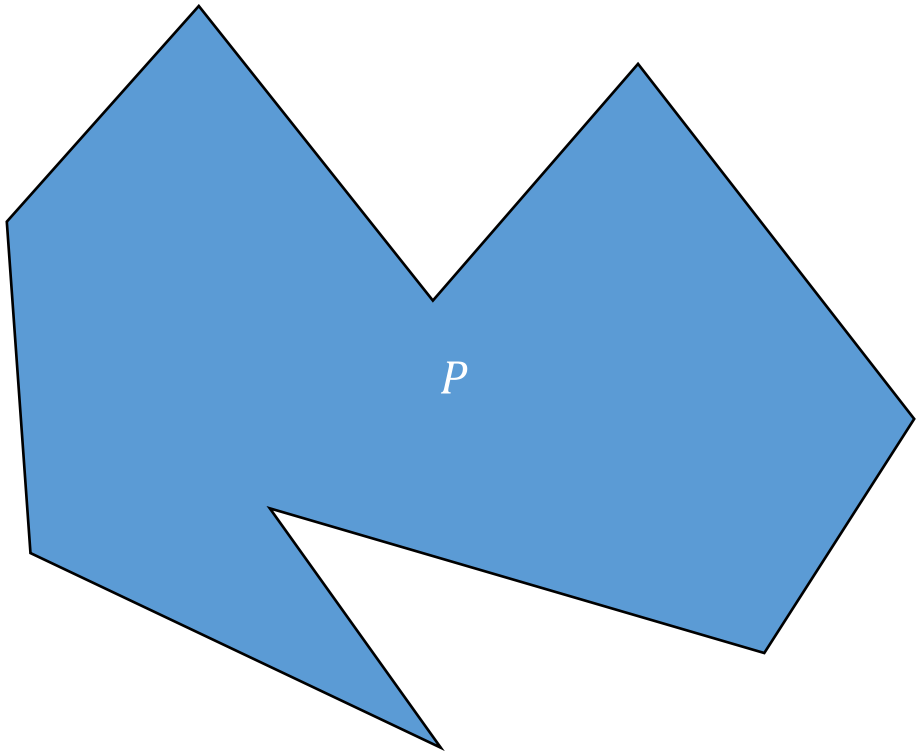
Point inside a polygon test

- How can we tell if point A is inside of polygon P?



SUM OF ANGLES

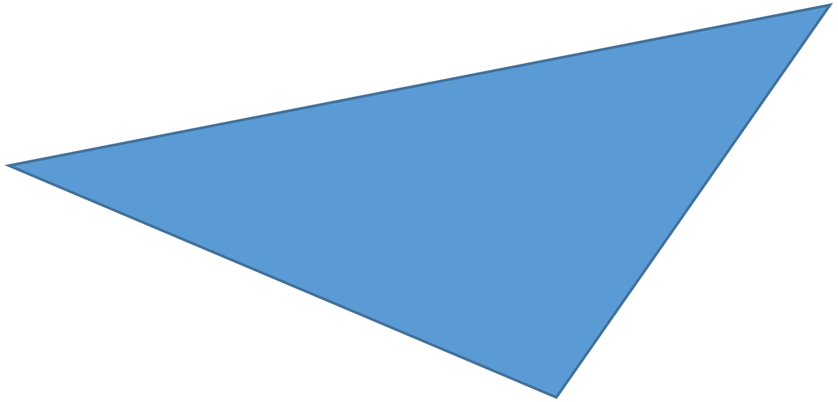
- THE SUM OF THE INTERNAL ANGLES OF A POLYGON OF N VERTICES IS?



Sum of Angles

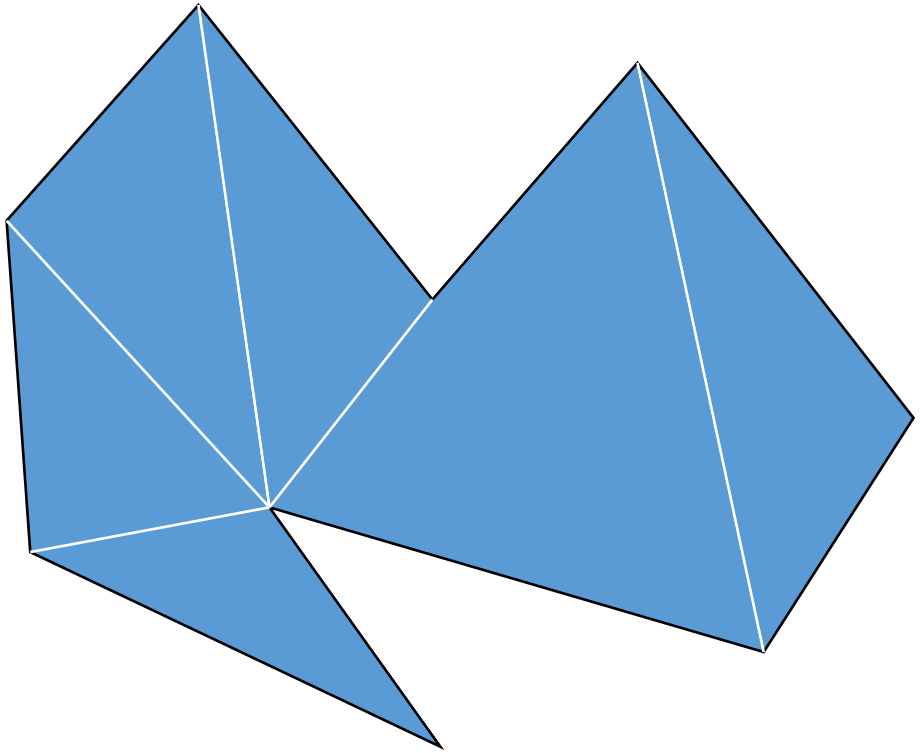
- Use the Triangle Angle-Sum Theorem to find the sum of the measures of the angles of a polygon.

- Triangle Angle-Sum Theorem
  - The sum of the measures of the angles of a triangle measure 180°



Sum of Angles

- Theorem: The sum of the measures of the internal angles of an n-gon is (n - 2) *180.



- Proof by induction

TRIANGULATION THEORY

- **THEOREM**: EVERY TRIANGULATION OF AN N-VERTEX POLYGON P USES $n - 3$ DIAGONALS AND CONSISTS OF $n - 2$ TRIANGLES.
  - Proof by induction:
    - Base case $N = 3$
    - Assume true for any polygon $< N$ sides
    - Given a N sided polygon partition it into two ($N1$ and $N2$) by adding a diagonal

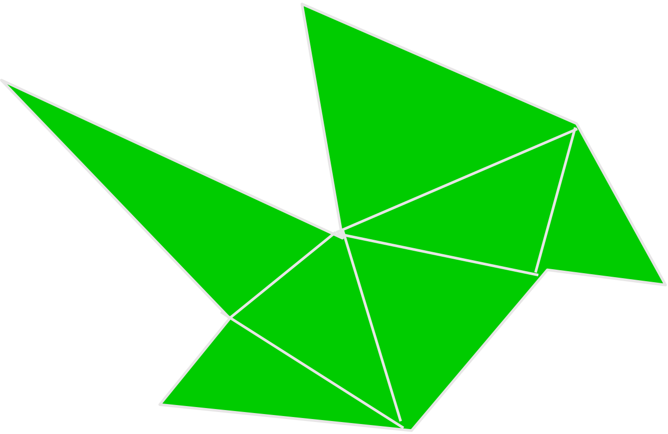
      Total number of diagonals:

      $(N1 - 3) + (N2 - 3) + 1 = (N1 + N2 - 2) - 3 = N - 3$

      Total number of triangles:

      $(N1 - 2) + (N2 - 2) = (N1 + N2) - 4 = N + 2 - 4 = N - 2$

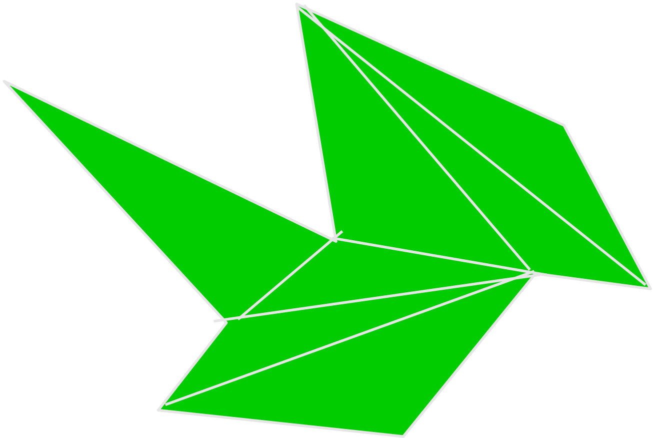
Triangulation Example



9 vertices

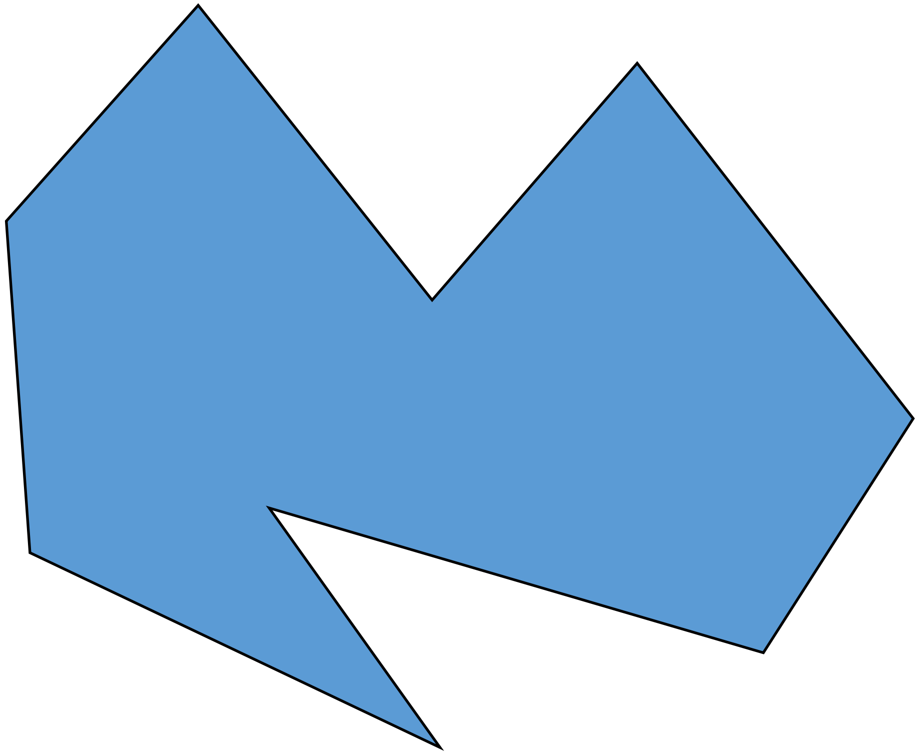
7 triangles

6 diagonals

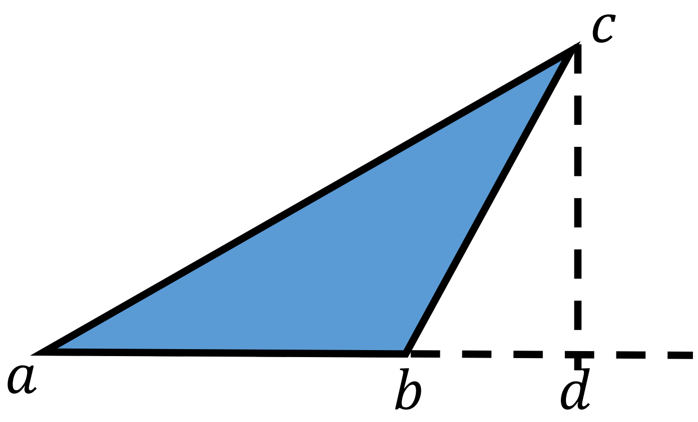


Area of a Polygon

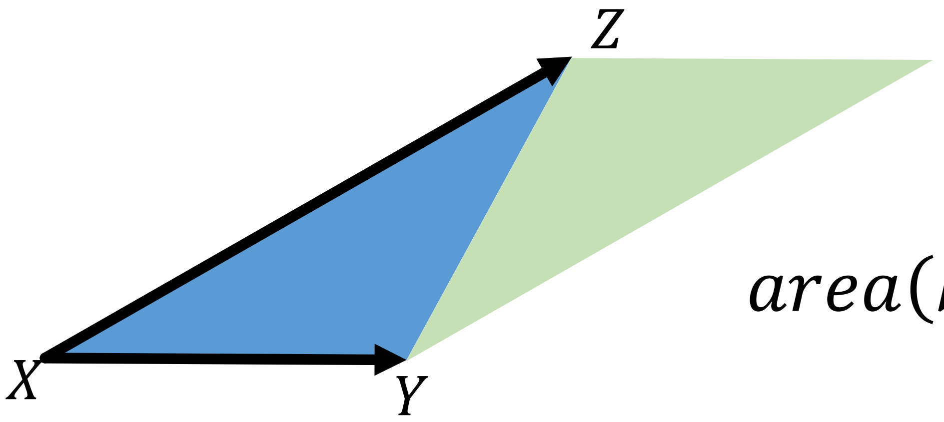
- What is the area of the polygon



AREA OF A TRIANGLE



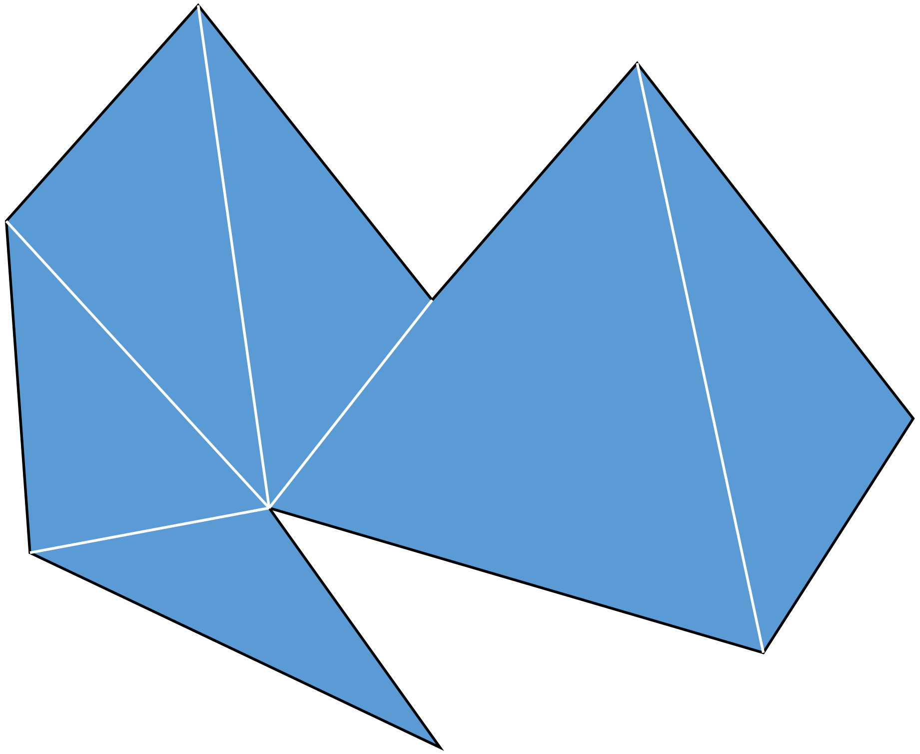
$$area = 0.5|a - b||c - d|$$



$$area(blue + green) = \left|\left(\overrightarrow{b - a}\right)\times\left(\overrightarrow{c - a}\right)\right|$$

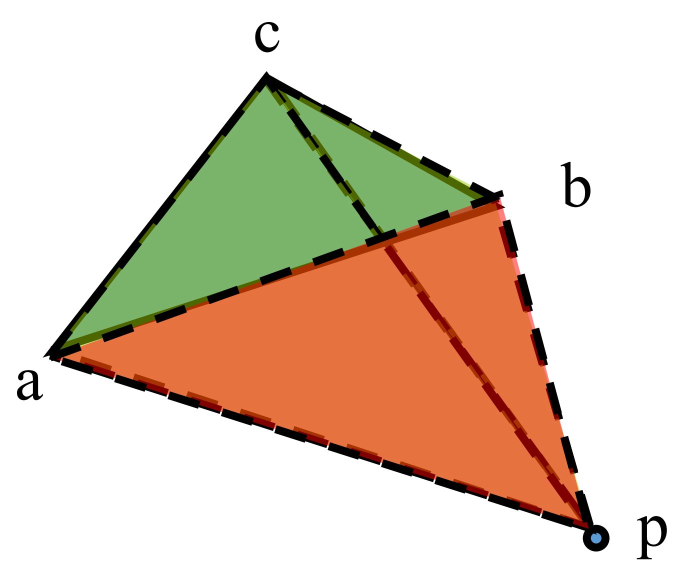
Sum of Areas

- Area of the polygon can be found by adding the areas of a triangulation



ANOTHER WAY OF COMPUTING AREA

$$Area\ (T) = Area(p, b, c) + Area(p, c, a) + Area(p, a, b)$$



Area of Polygon

- Theorem: Let a polygon (convex or non-convex) P have vertices $v_0, v_1, \ldots, v_{n-1}$

$$Area(P) = A(p, v_0, v_1) + A(p, v_1, v_2) + A(p, v_2, v_3) + \cdots + A(p, v_{n-2}, v_{n-1}) + A(p, v_{n-1}, v_0)$$

- Proof is by induction