COT 4521: Introduction to Computational Geometry

Segment Intersection

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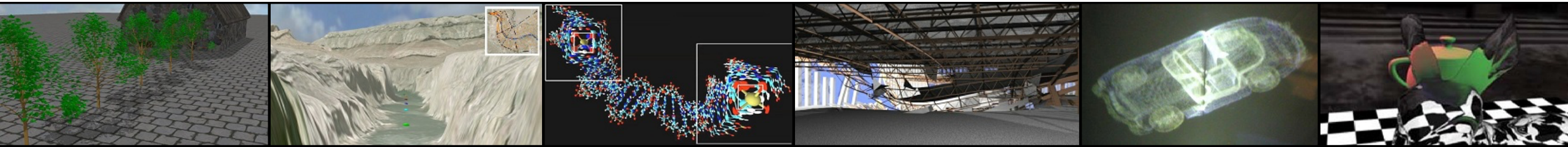
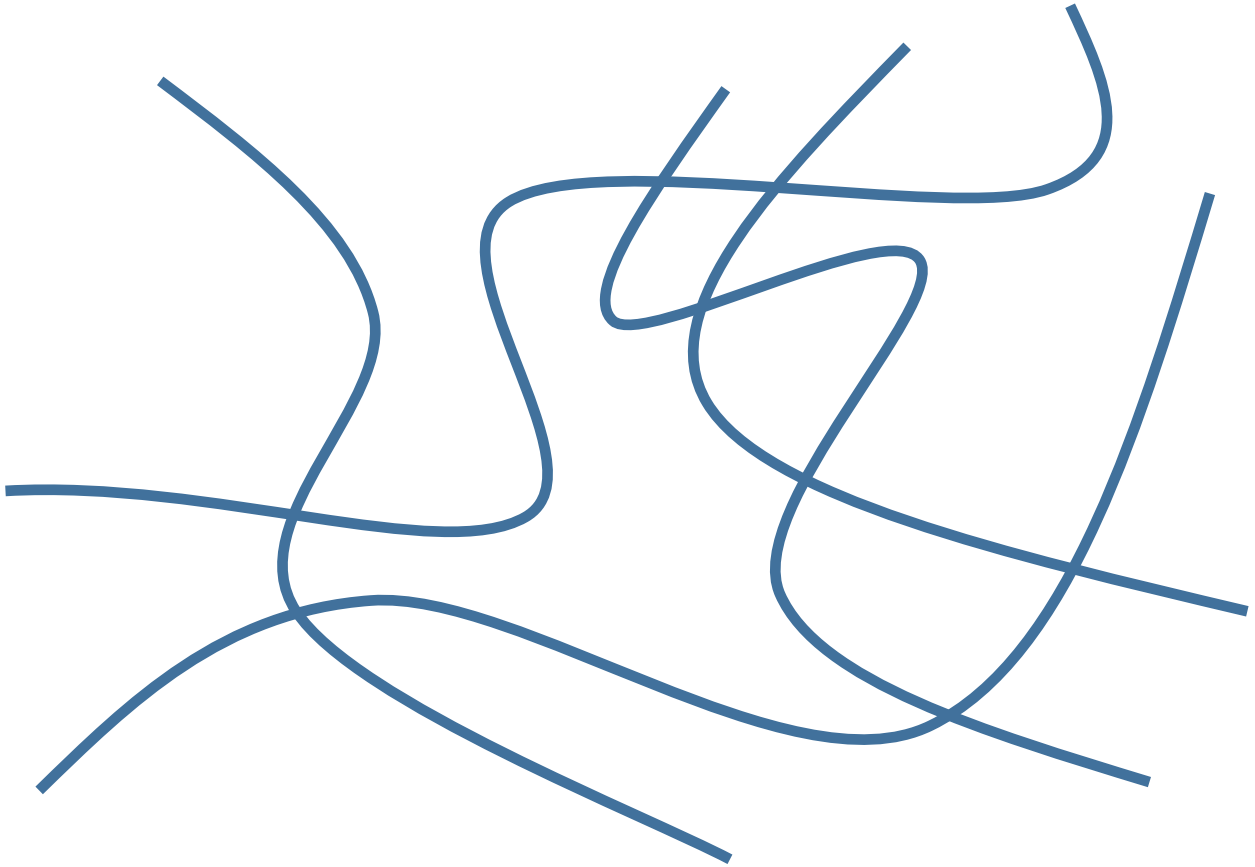


Problem Statement



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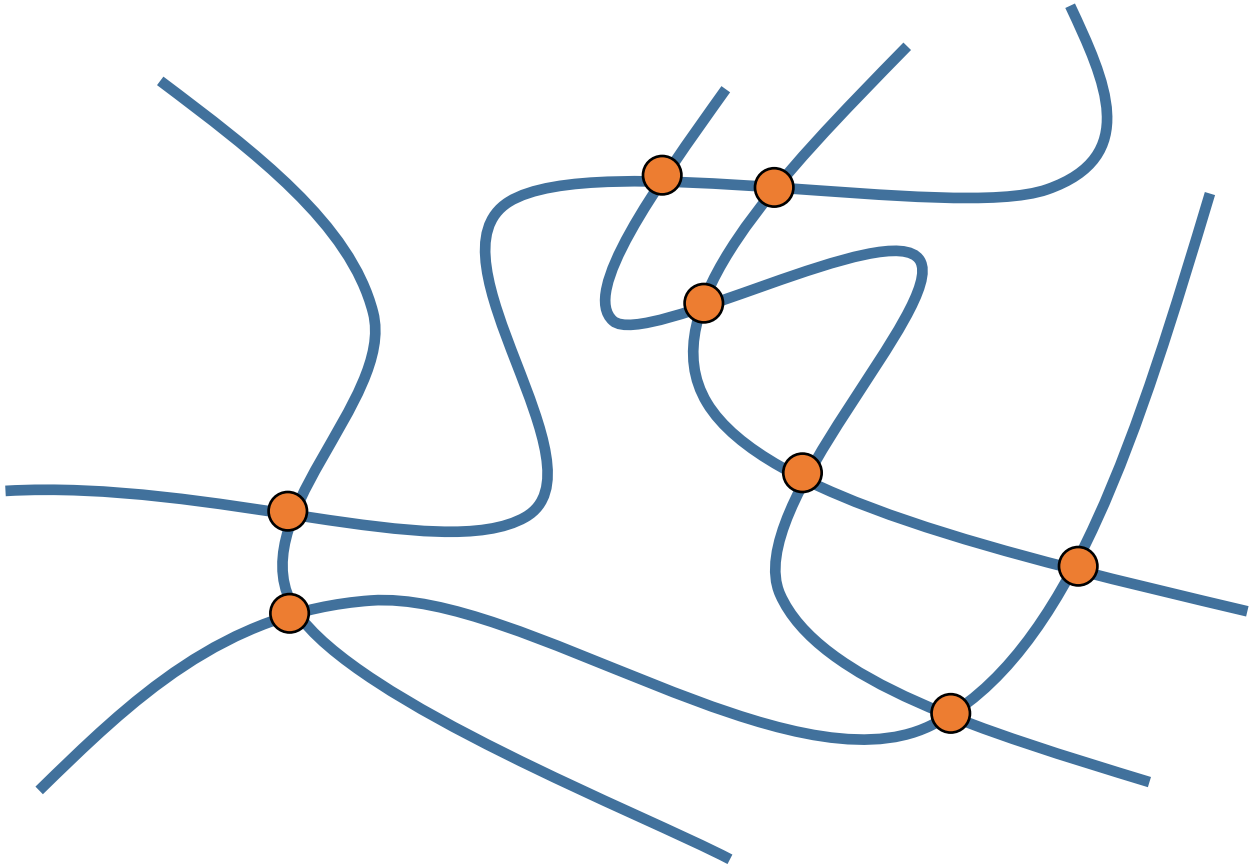
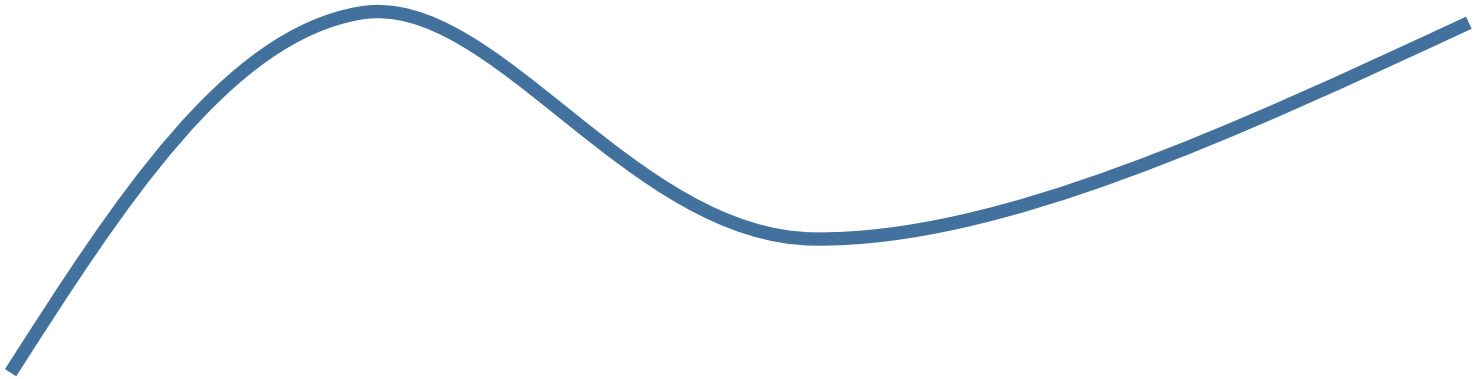


Representing Curves

- Curves often represented by a polynomial or polynomial spline
  - Bezier, NURBS, etc.
- Tesselate Curve into many small line segments



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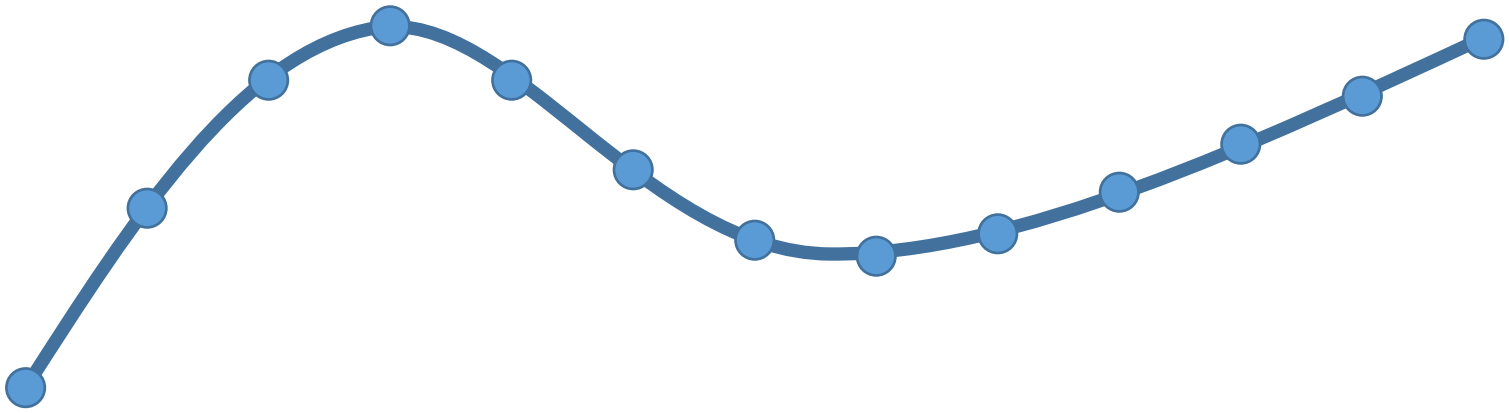

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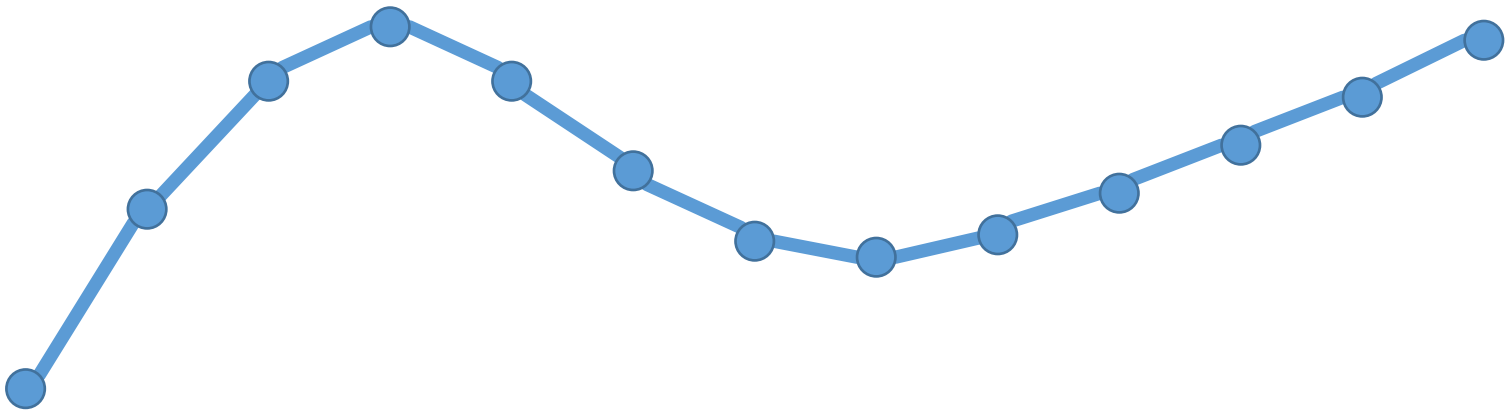
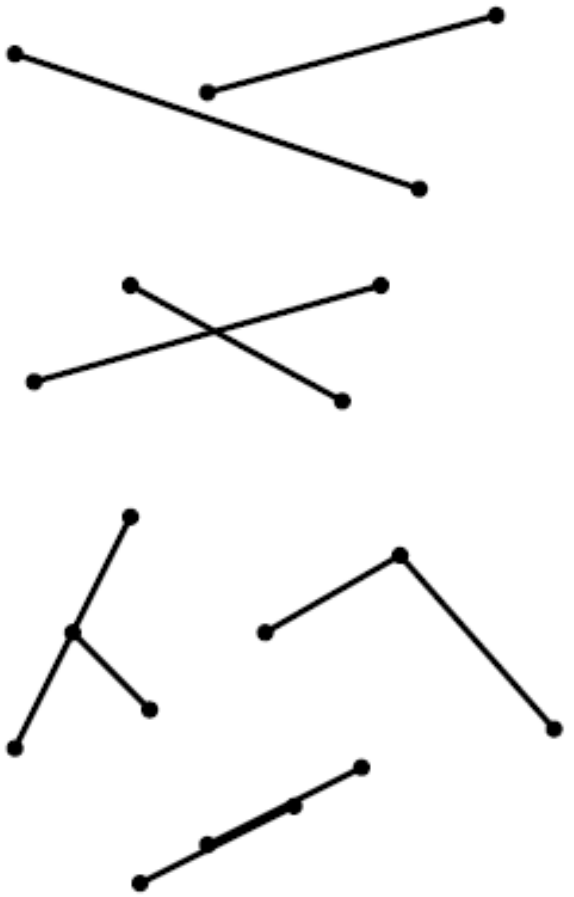


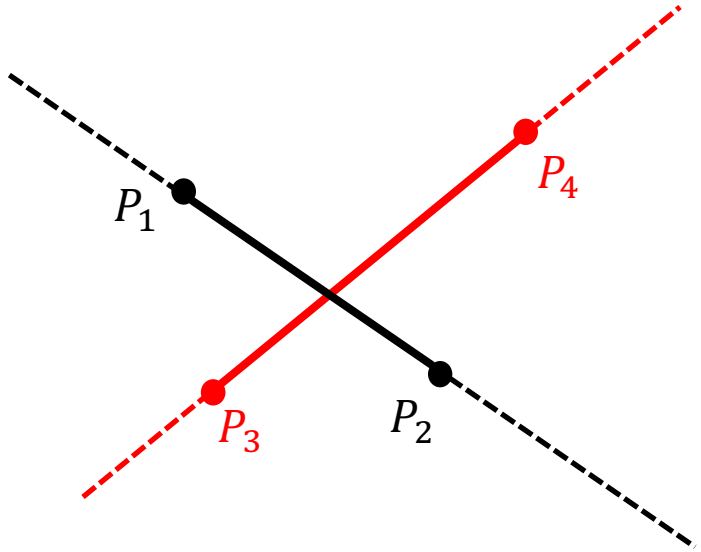
Segment-Segment Intersection

- A line segment $\overline{pq}$ is denoted by its two endpoints p and q:
  - $\alpha\, p_x + (1-\alpha) q_x$
  - $\alpha p_y + (1-\alpha)\ q_y)$
  - where $0 \leq \alpha \leq 1$
- Line segments are assumed to be closed with endpoints, not open
- Two line segments *intersect if they have some point in common*.
- It is a *proper intersection* if it is *exactly one interior point of each line segment*



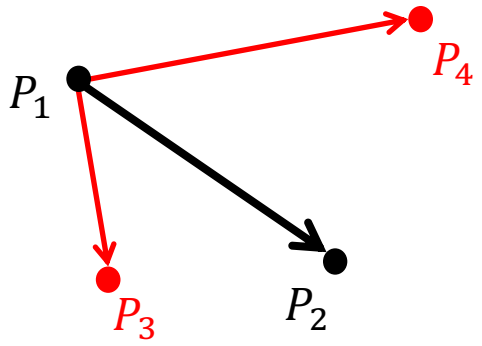
Do they intersect?

- Observation: If the two segments intersect, the two red points must lie on different sides of the black line (or lie exactly on it)
- The same holds with black/red switched



Do they intersect?

- What does "different sides" mean?
- Use the cross product to determine sidedness



Representing a Line

- Slope-Intercept Form:

$$y = mx + b$$

- Given 2 points, P_1 and P_2, how do you compute m and b?
- Given 2 lines, m_1, b_1 and m_2, b_2, how do you compute the intersection point, P_I, between them?
- How do you know if P_I is on the segment defined by P_1 and P_2?

Representing a Line

- Standard Form:

$$Ax + By + C = 0$$

- Given 2 points, P_1 and P_2, how do you compute A, B and C?
- Given 2 lines, A_1, B_1, C_1 and A_2, B_2, C_2, how do you compute the intersection point, P_I, between them?
- How do you know if P_I is on the segment defined by P_1 and P_2?

Representing a Line

- Parametric Form:

$$P = P_0 + Dt$$

- Given 2 points, P_1 and P_2, how do you represent the parametric line?
- Given 2 lines, how do you compute the intersection point, P_I, between them?
- How do you know if P_I is on the segment defined by P_1 and P_2?