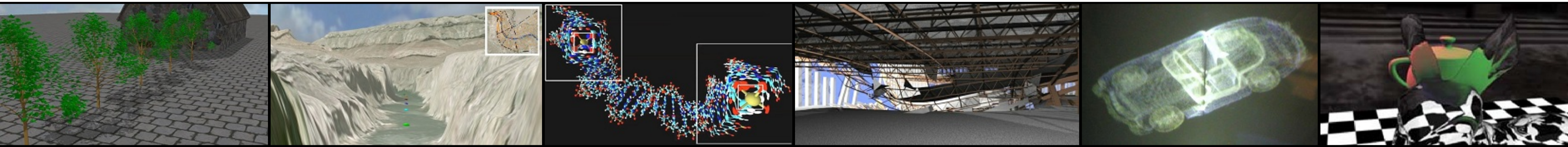
COT 4521: Introduction to Computational Geometry

Polygons

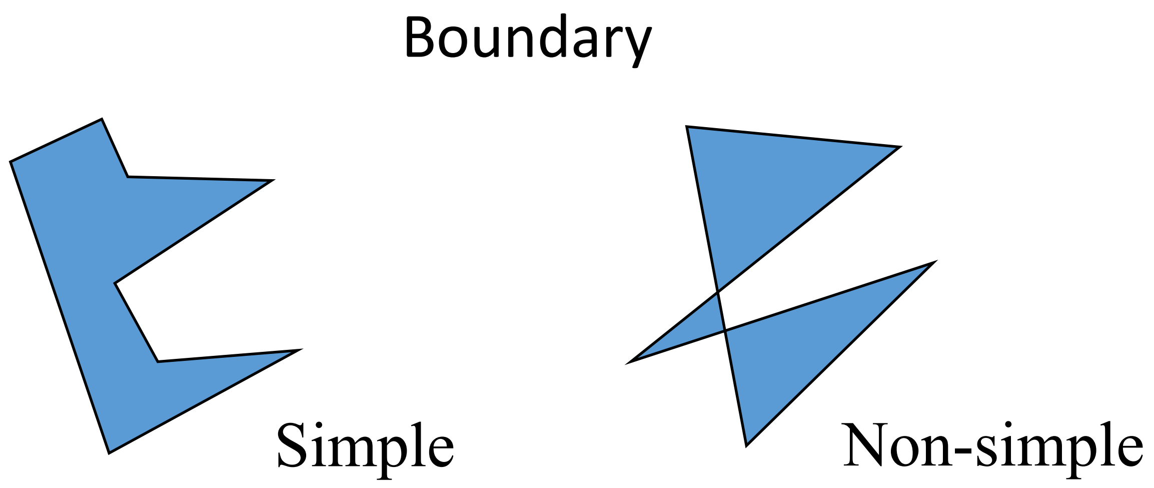
Paul Rosen
Assistant Professor
University of South Florida

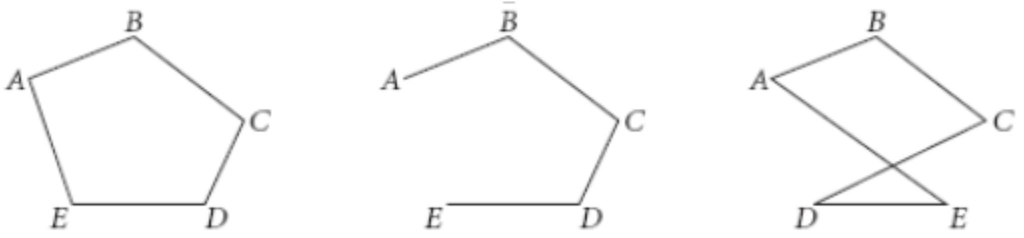
Some slides from Valentina Korzhova



POLYGON

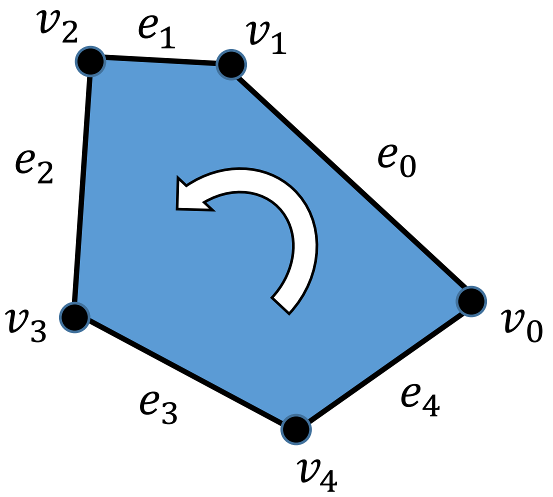
- **POLYGON** IS A REGION OF A PLANE BOUNDED BY A FINITE COLLECTION OF LINE SEGMENTS FORMING A SIMPLE CLOSED CURVE.





POLYGON

- EDGES – THE LINE SEGMENTS $(e_0, e_1, \cdots, e_{n-1})$
- VERTICES – THE POINTS WHERE ADJACENT EDGES MEET
  - Start at any vertex and list the vertices consecutively in a counterclockwise direction $(v_0, v_1, \dots, v_{n-1})$
- ANGLES
  - Name by angle naming convention
  - $\measuredangle v_0, \measuredangle v_1, \dots, \measuredangle v_{n-1}$

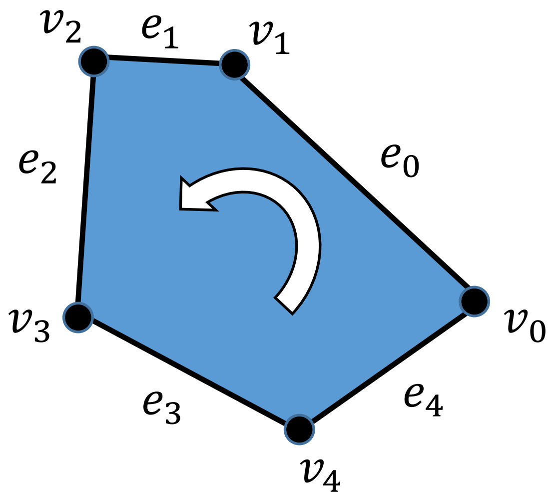


Properties of polygon

- The line segments that make-up a polygon (called sides or edges) meet only at their endpoints, called vertices (singular: vertex) or less formally "corners"
- Exactly two edges meet at every vertex
- The number of edges always equals the number of vertices.
- Two edges meeting at a corner are required to form an angle that is not straight (180°); otherwise, the line segments will be considered parts of a single edge

Formal Definition of a Simple Polygon

- Formally, we are given n vertices (i.e., points) $v0, v1, \ldots, v_{n-1}$, the chain formed by $v0v1 \cdots v_{n-1}$ is a simple polygon iff
  - The segments $e0 = v0v1, \ldots, e_{n-2} = v_{n-2}v_{n-1}$, and $e_{n-1} = v_{n-1}v0$ are disjoint in their interior
  - Consecutive segments intersect only in their endpoints. Namely $ei \cap e_{i+1} = v_{i+1}$ for $i = 0, \ldots, n-2$ and $e_{n-1} \cap e0 = v0$
  - Non adjacent segments do not intersect $ei \cap ej = \emptyset$, for $\forall j \neq i + 1$
- We work $mod\ n$.
  - Namely $v_i = v_i\ mod\ n$



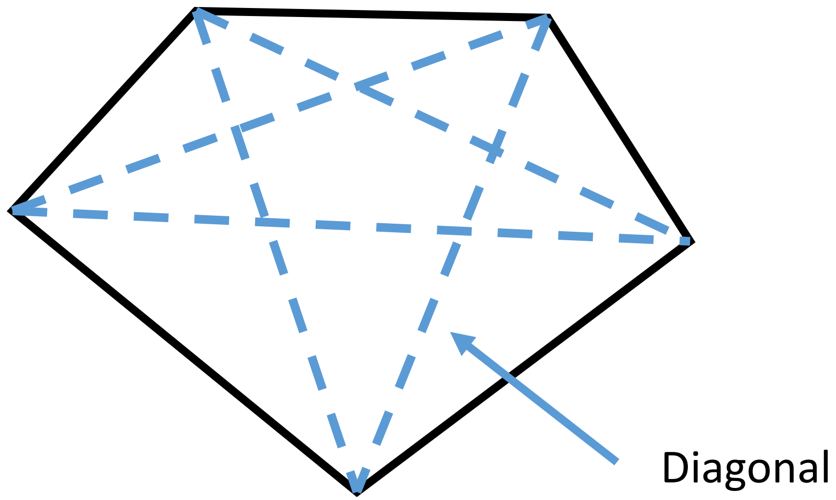
Diagonals

- Definition: The diagonal of a polygon is a line segment linking two non-adjacent vertices, interior to the polygon, and not blocked by portion of polygon's boundary

- Lemma: The segment $s = v_i v_j$ is a diagonal of P iff
  1. For all edges e of P that are not incident to either $v_i or\ v_j$, s and e do not intersect: $s \cap e = \emptyset$;
  2. s is internal to P in neighborhood of v_i and v_j

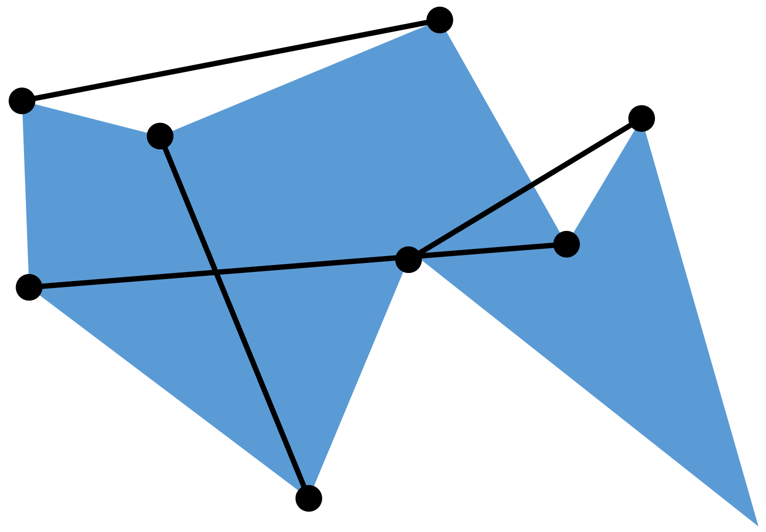
DIAGONALS

- DEFINITION: THE DIAGONAL OF A POLYGON IS A LINE SEGMENT LINKING TWO NON-ADJACENT VERTICES, INTERIOR TO THE POLYGON, AND NOT BLOCKED BY PORTION OF POLYGON'S BOUNDARY



DIAGONALS

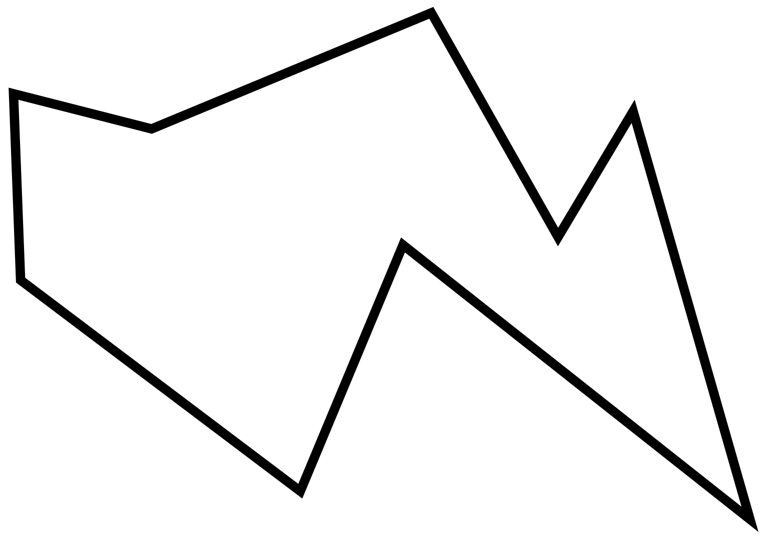
- DEFINITION: THE DIAGONAL OF A POLYGON IS A LINE SEGMENT LINKING TWO NON-ADJACENT VERTICES, INTERIOR TO THE POLYGON, AND NOT BLOCKED BY PORTION OF POLYGON'S BOUNDARY



which are legal diagonals?

Efficient Diagonal Finding

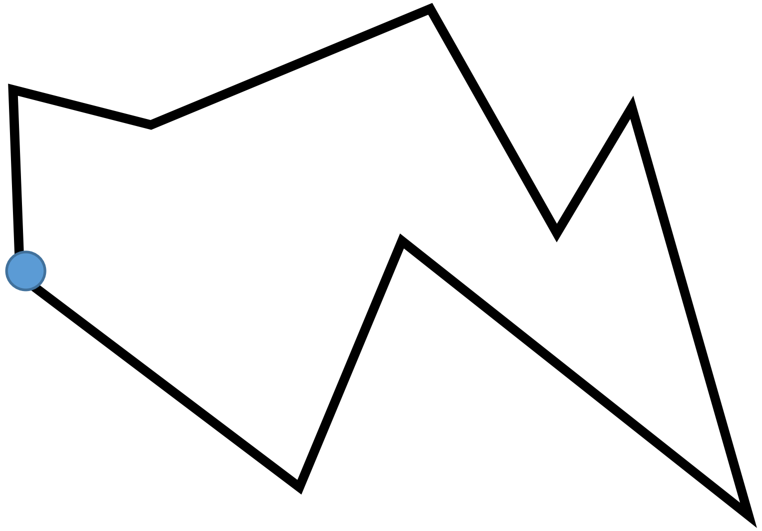
- Brute force
  - Generate each potential diagonal—$O(n^2)$ possible diagonals
  - Check each diagonal against the boundary to determine
    - If it is inside or outside
    - If it intersects the boundary
  - Total performance $O(n^3)$



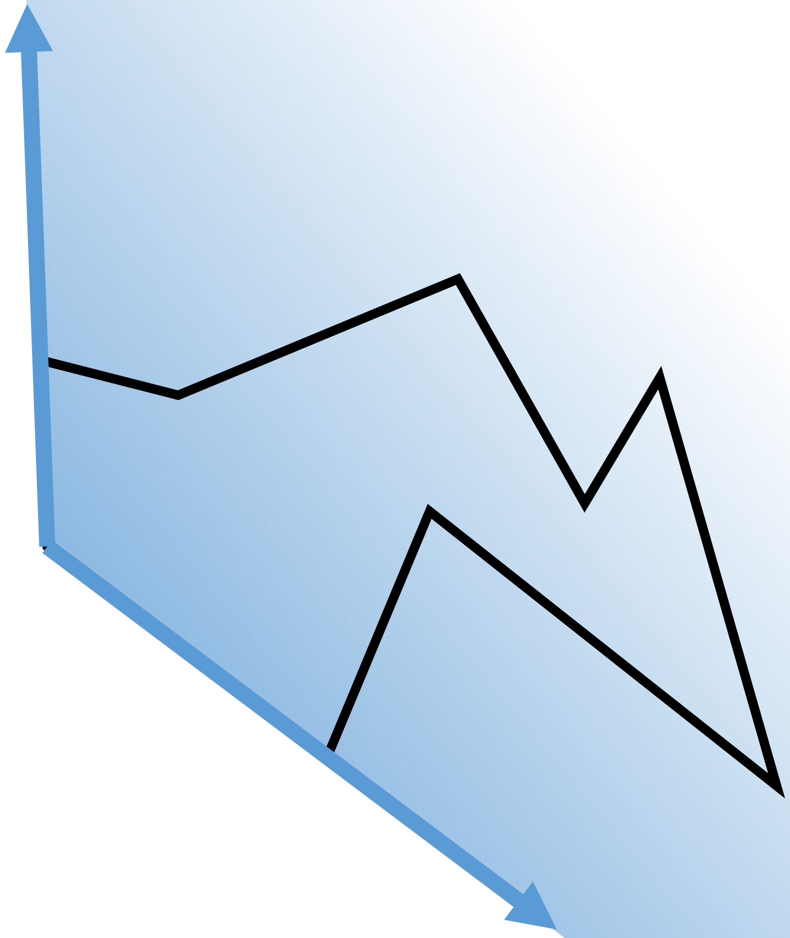
- Can we do Better?

Efficient Diagonal Finding

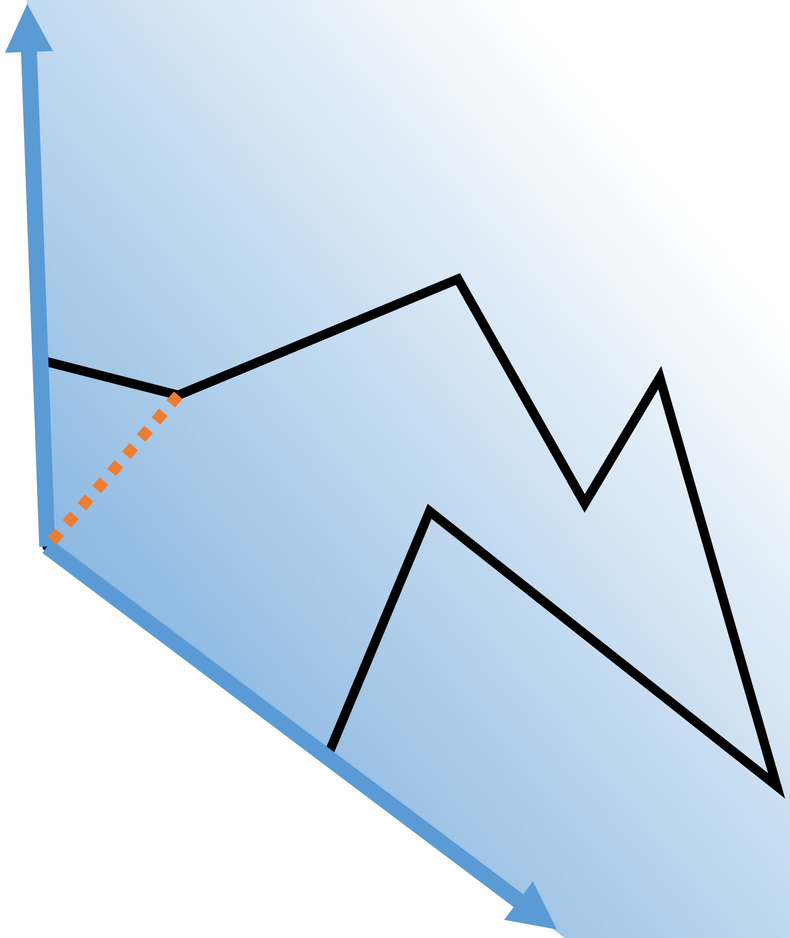
- Variation on a sweep algorithm
  - From a given vertex, sweep both clockwise and counterclockwise, ordered by the distance of points
  - Retain a notion of “valid space” for diagonals



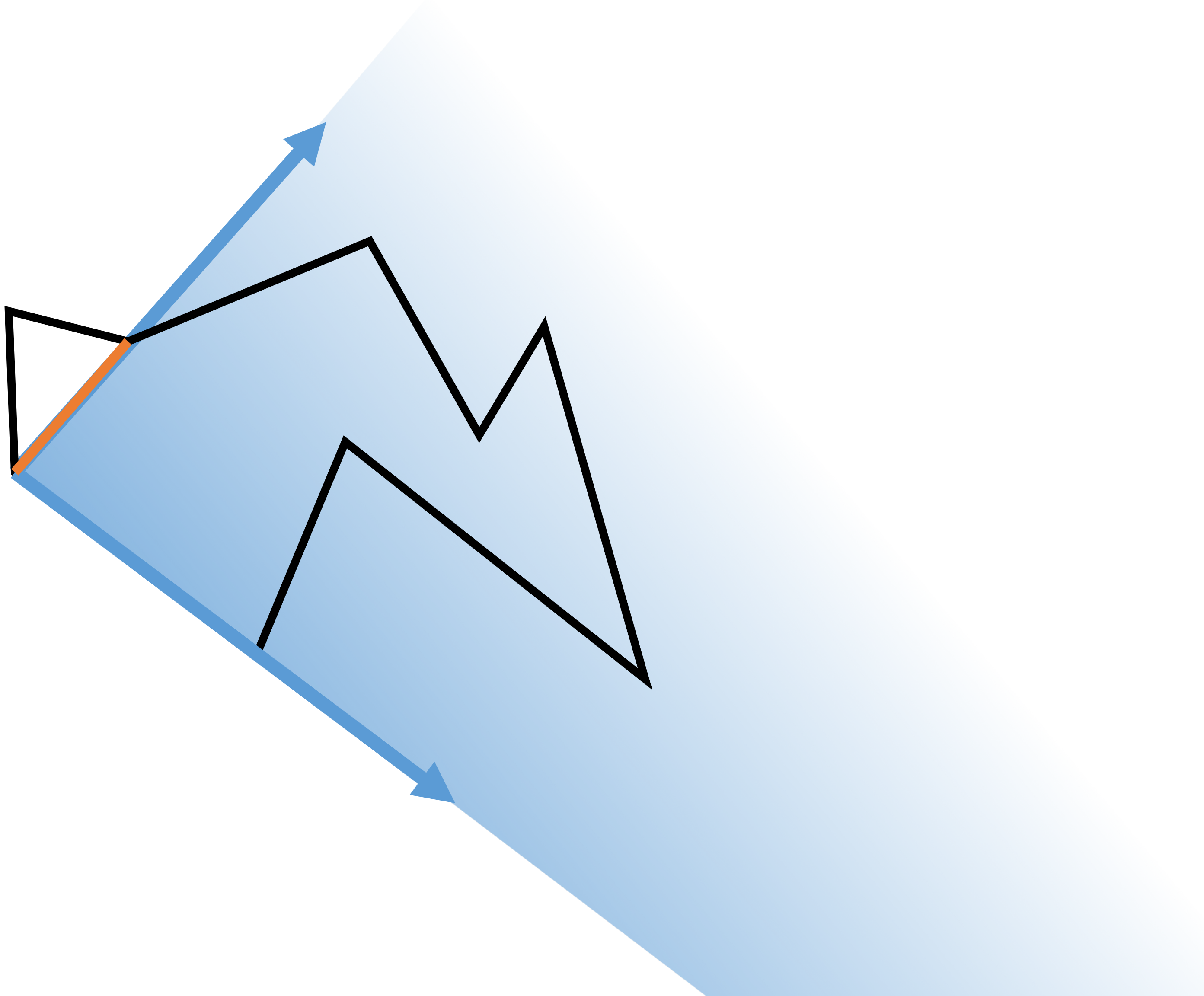
Efficient Diagonal Finding



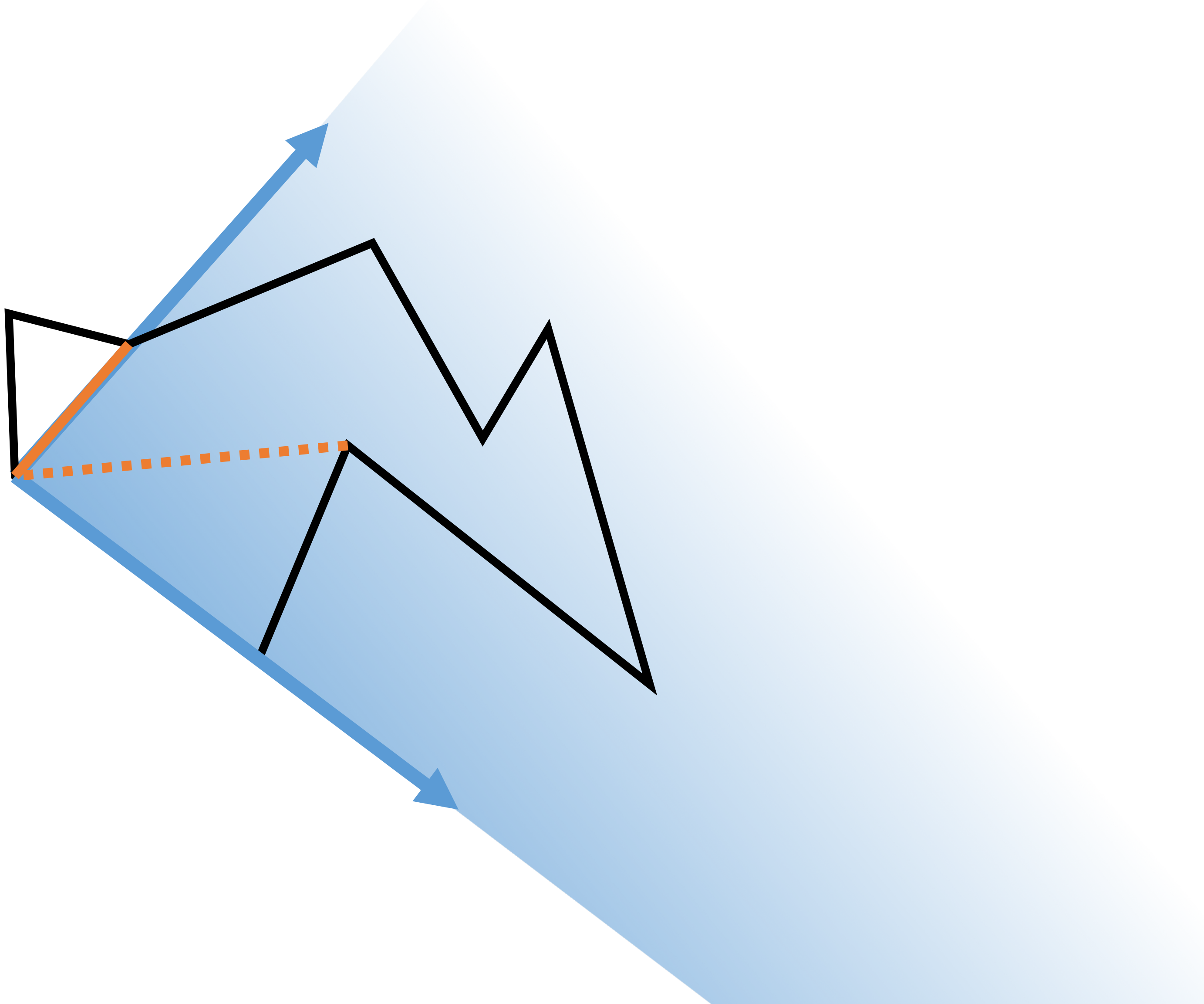
Efficient Diagonal Finding



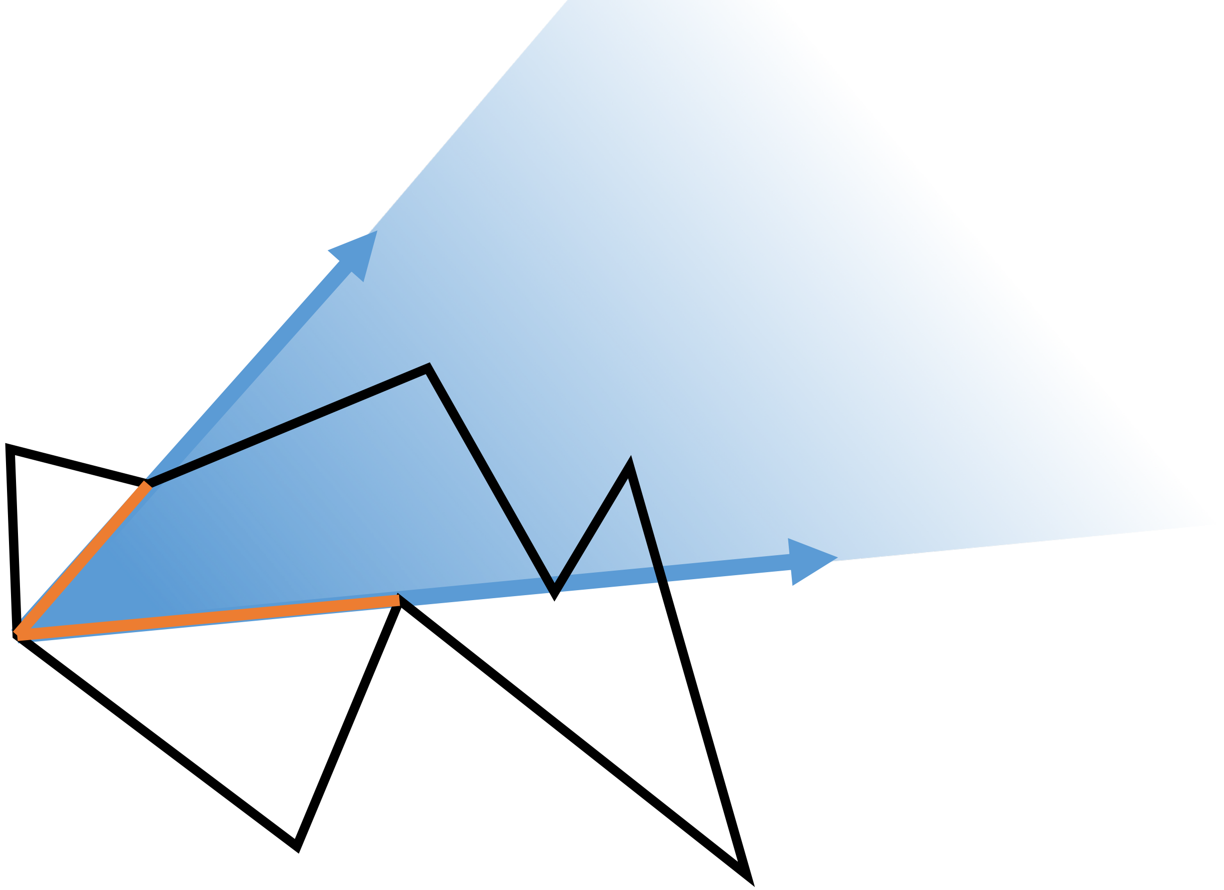
Efficient Diagonal Finding



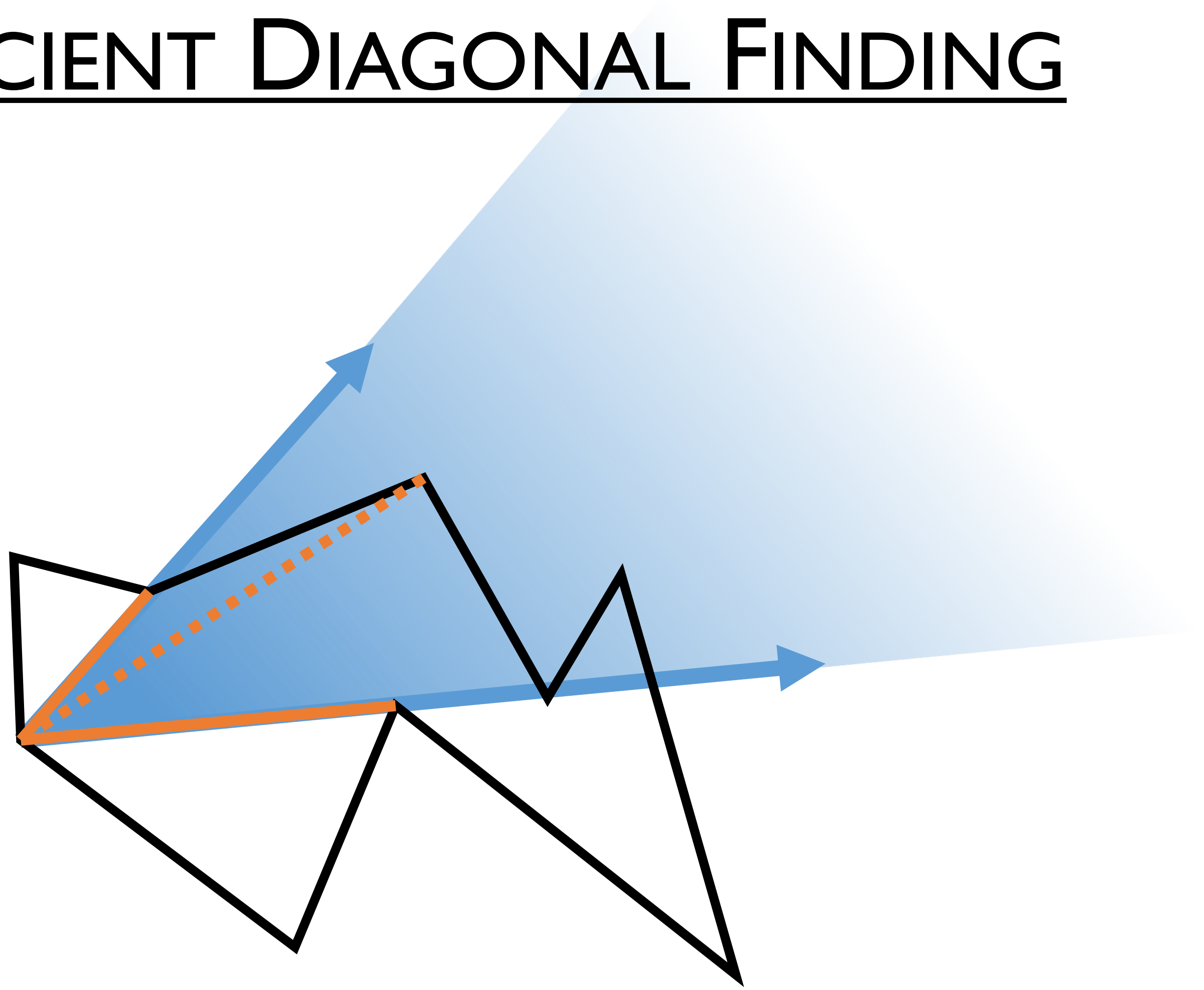
Efficient Diagonal Finding



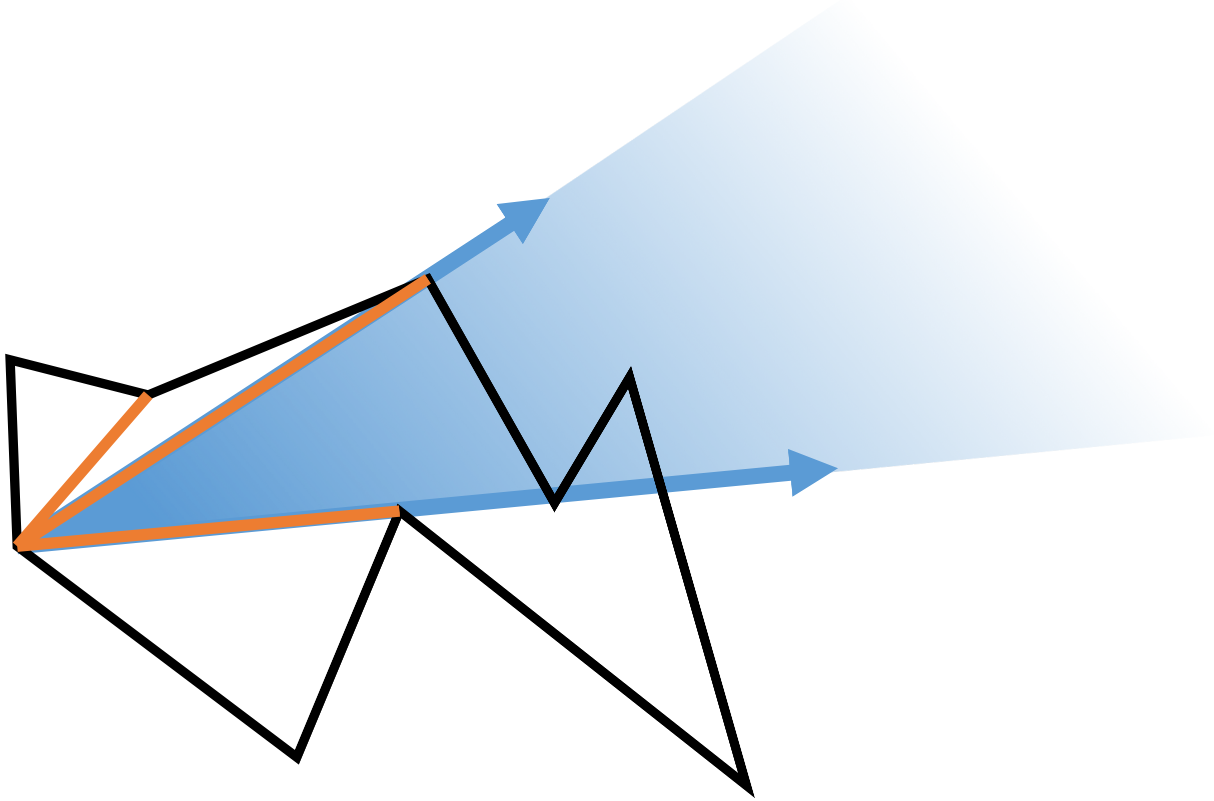
Efficient Diagonal Finding



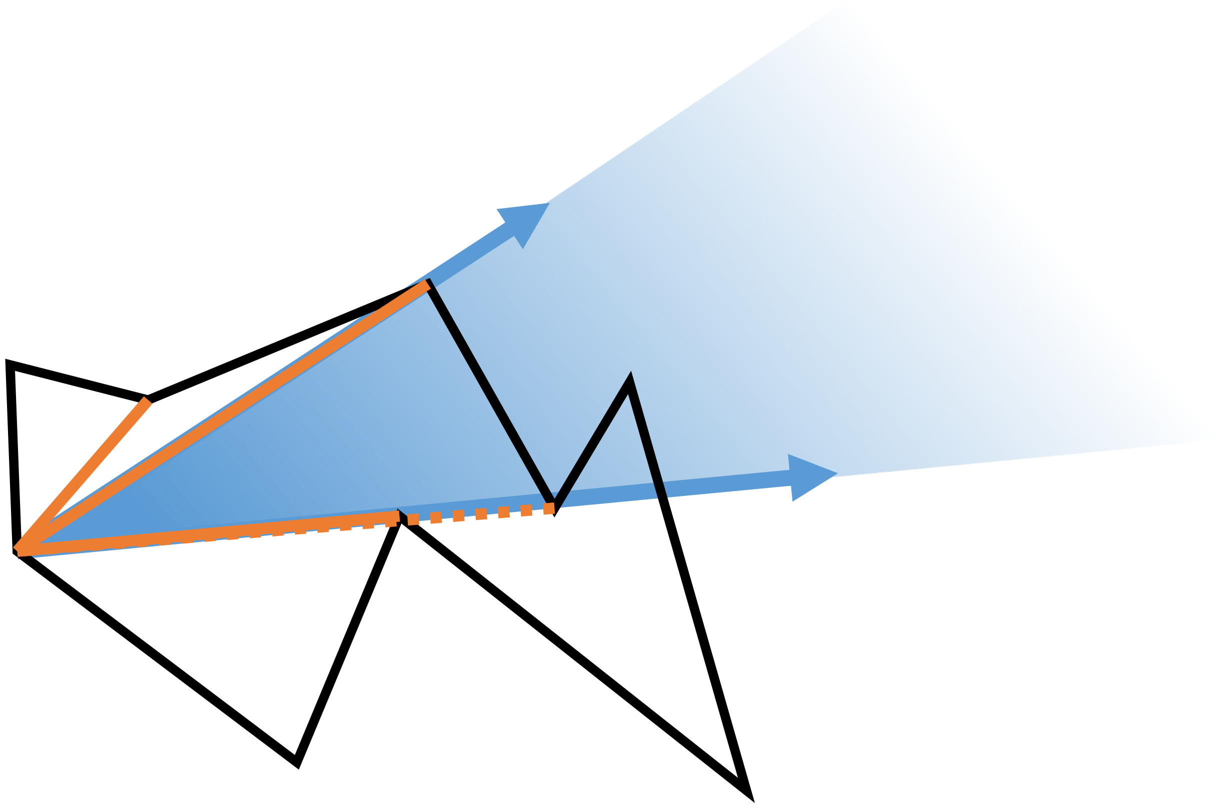
Efficient Diagonal Finding



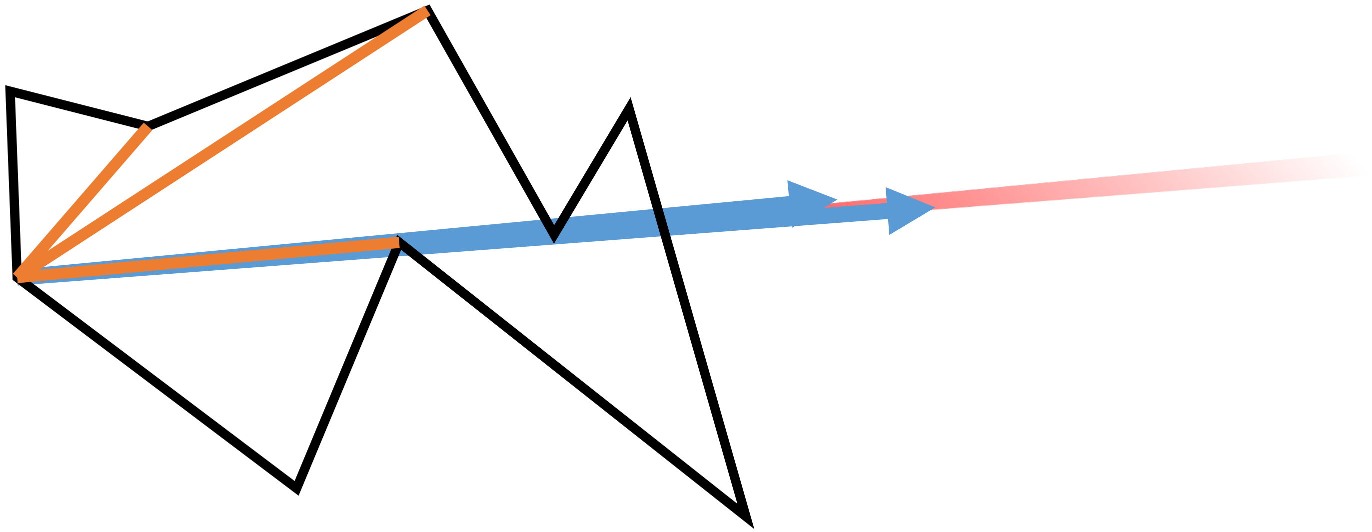
Efficient Diagonal Finding



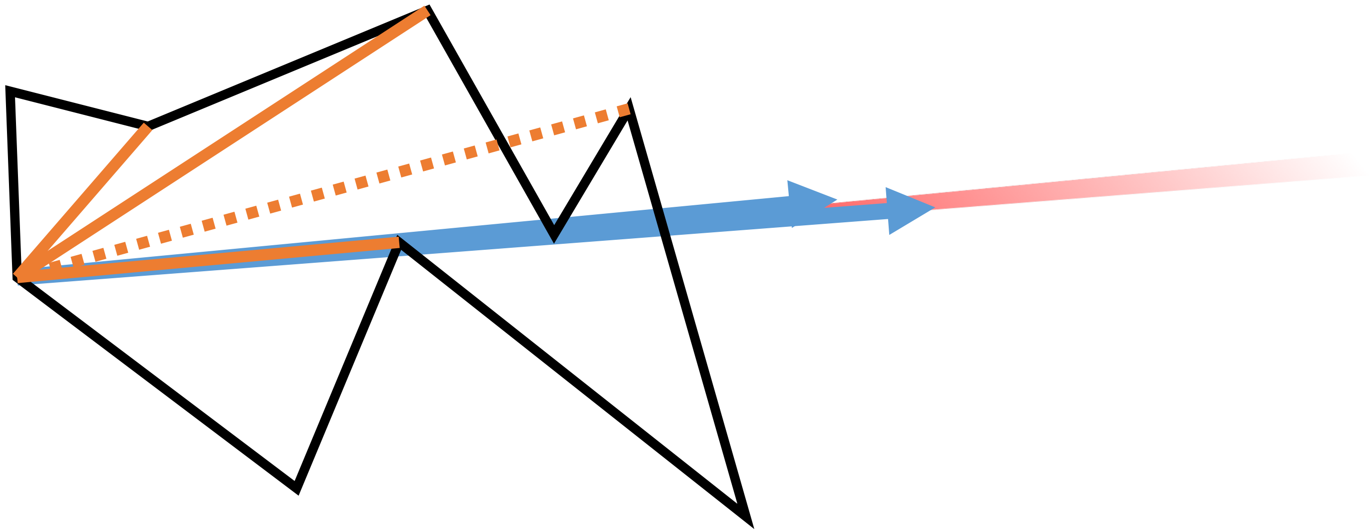
Efficient Diagonal Finding



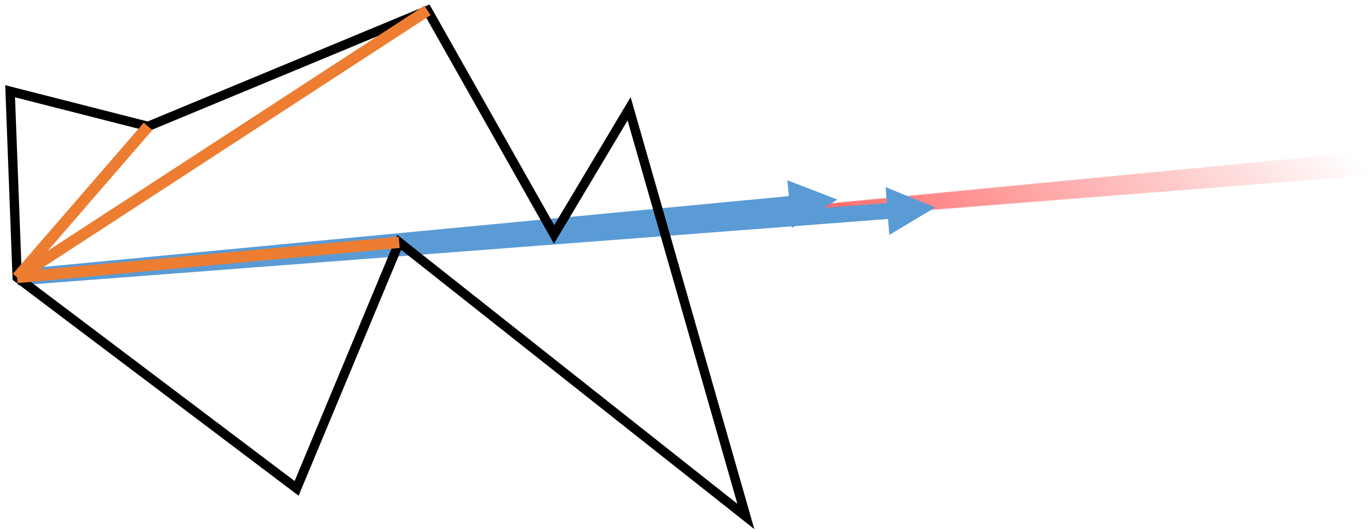
Efficient Diagonal Finding



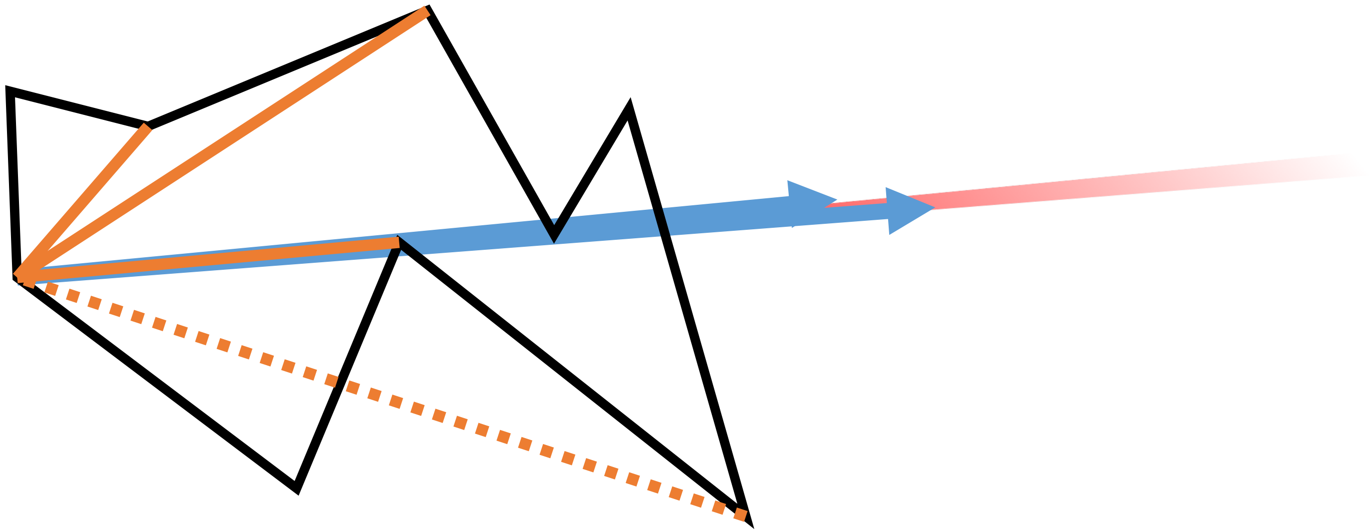
Efficient Diagonal Finding



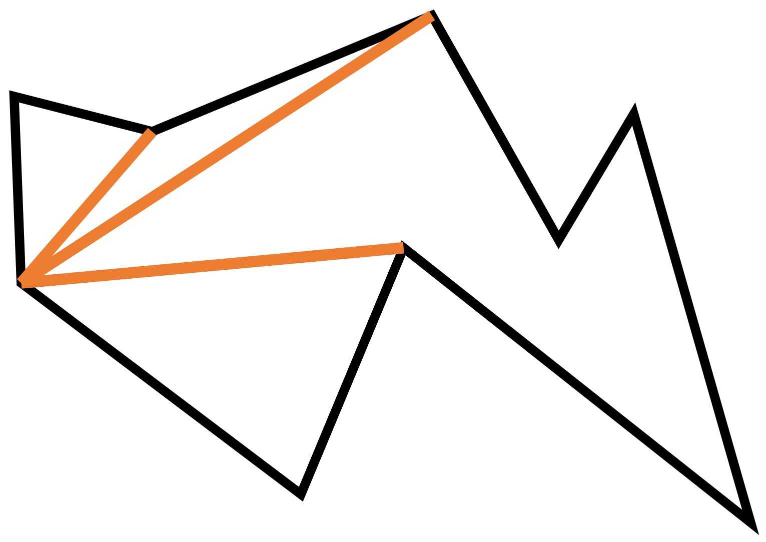
Efficient Diagonal Finding



Efficient Diagonal Finding

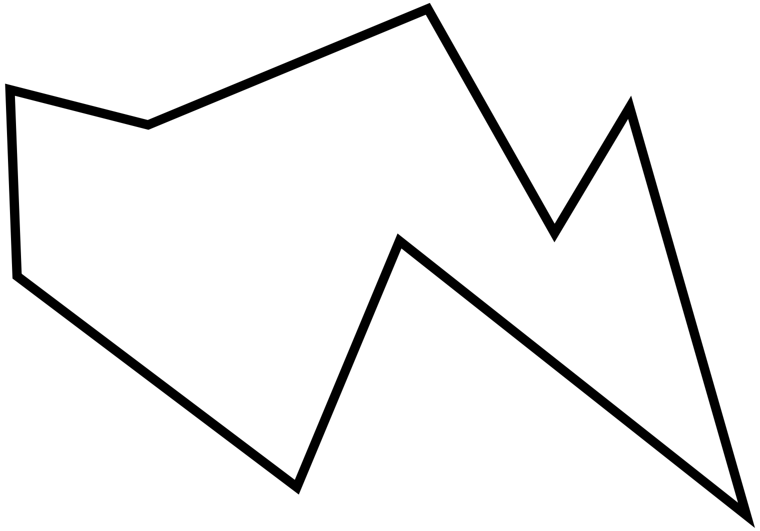


Efficient Diagonal Finding



Efficient Diagonal Finding

- Analysis
  - Preprocessing: None
  - Query: Worst case $O(n^2)$; $O(n)$ per vertex
  - Storage: $O(n)$



Triangulation Theory: Properties

- Lemma: An internal diagonal exists between any two nonadjacent vertices of a polygon P if and only if P is convex polygon.
- Proof: The proof consists of two parts, both established by contradiction.

Classification of Polygons by the Number of Edges
Most Common Polygons

Number of Sides	Name
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
8	Octagon
9	Nonagon
10	Decagon
12	Dodecagon
n	n-gon