

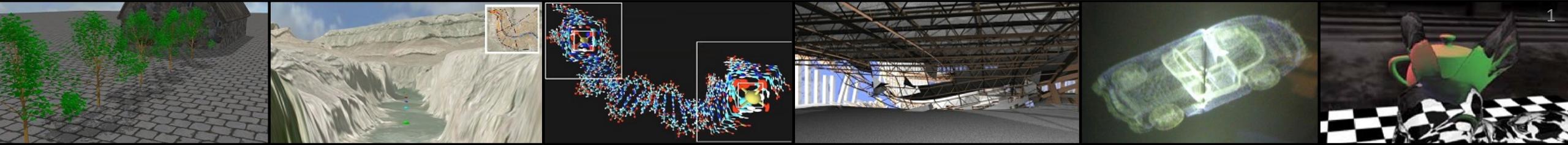
# COT 4521: INTRODUCTION TO COMPUTATIONAL GEOMETRY

---



## Introduction

Paul Rosen  
Assistant Professor  
University of South Florida





2004

## Problem Domain

Computer  
Graphics

## Problem Solving Approach

Geometric  
Approaches



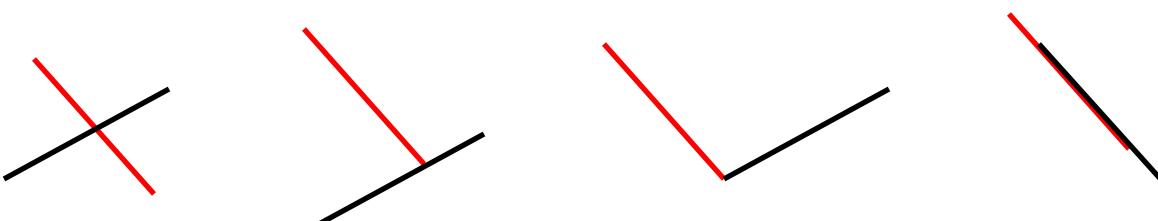
# INTRODUCTION

- **WHAT IS COMPUTATIONAL GEOMETRY?**
  - A subfield of the Analysis of Algorithms
  - Design of efficient algorithms and data structures for geometrical problems
- **STARTED IN MID 70's**
  - started out by developing solid theoretical foundations, but became more and more applied over the last years
- **DEALS WITH GEOMETRICAL STRUCTURES**
  - Points, lines, line segments, vectors, planes, etc.
- **WE'LL FOCUS MOSTLY ON 2-DIMENSIONAL GEOMETRY, BUT 3D WILL BE COVERED WHEN POSSIBLE**



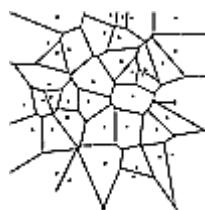
# WHAT MAKES COMPUTATIONAL GEOMETRY PROBLEMS SO INTERESTING/ANNOYING?

- COMPLEXITY
  - Curse of dimensionality
- PRECISION ERROR
  - Avoid floating-point computations whenever possible
- DEGENERACY
  - Boundary cases
  - For example, imagine how 2 line segments can intersect

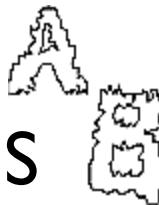


# TYPICAL COMPUTATIONAL GEOMETRY PROBLEMS

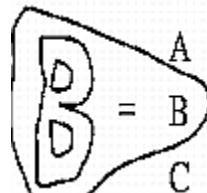
- VORONOI DIAGRAM



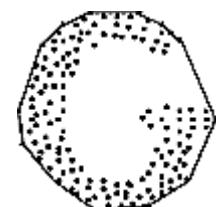
- SIMPLIFYING POLYGONS



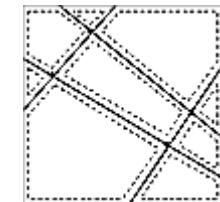
- SHAPE SIMILARITY



- CONVEX HULL



- LINE ARRANGEMENTS

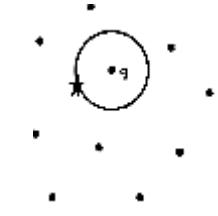


- POLYGON PARTITIONING

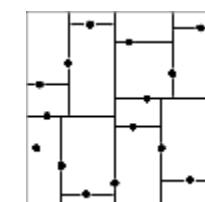


- NEAREST NEIGHBOR

SEARCH

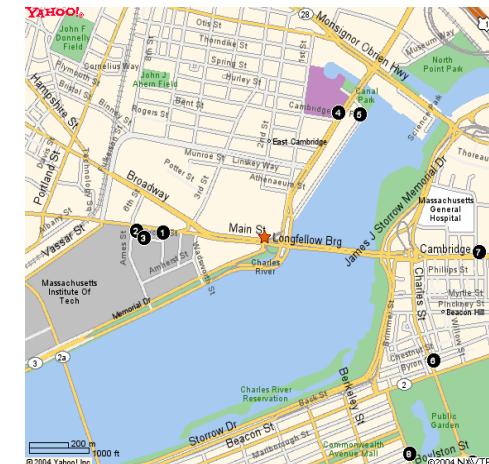


- SPATIAL ORGANIZATIONS



# APPLICATIONS

- Computer Graphics
- Robotics motion planning
- Geographic Information
- CAD/CAM
- Collision detection
- Computer vision
- Molecular modeling
- VLSI design
- Data Mining, Machine Learning, and Visualization



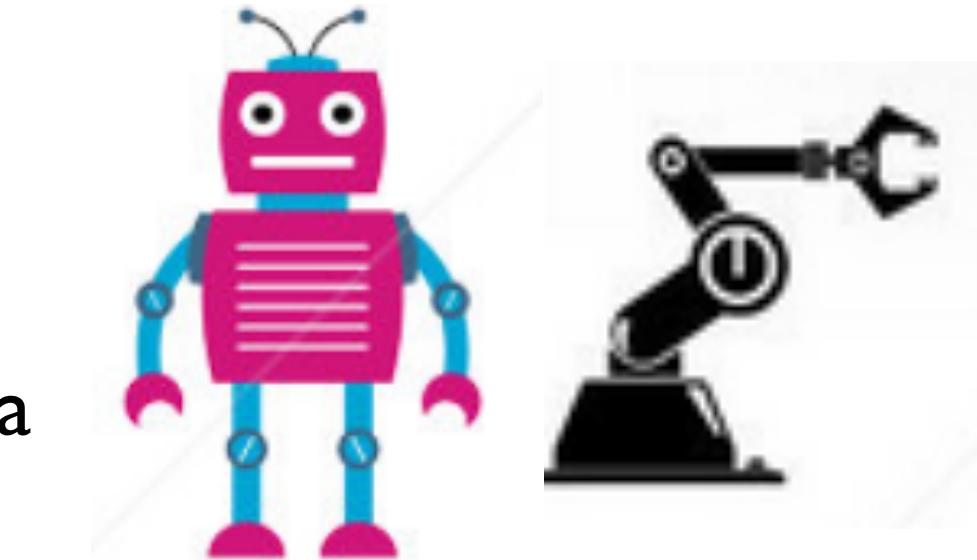
# APPLICATIONS OF COMPUTATIONAL GEOMETRY

- COMPUTER GRAPHICS
  - For 2D graphics, typical questions involve the intersection of certain primitives, determining the subset of primitives that lie within a particular region, etc.
  - For 3D graphics, hidden surface removal



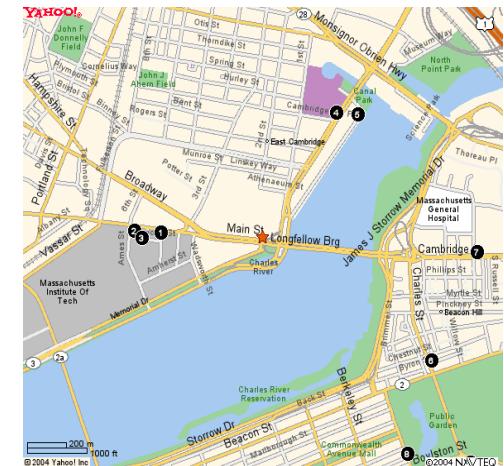
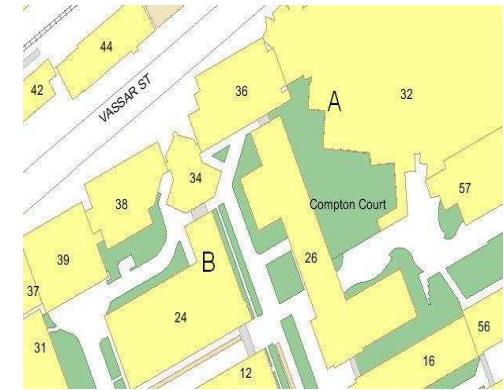
# APPLICATIONS OF COMPUTATIONAL GEOMETRY

- ROBOTIC MOTION PLANNING
  - The geometric problem arise at many places because robots are geometric objects that operate in a 3D space



# APPLICATIONS OF COMPUTATIONAL GEOMETRY

- **GEOGRAPHIC INFORMATION SYSTEM (GIS)**
  - Stores geographical data
  - Can be used to extract information about certain regions and to obtain information between different types of data



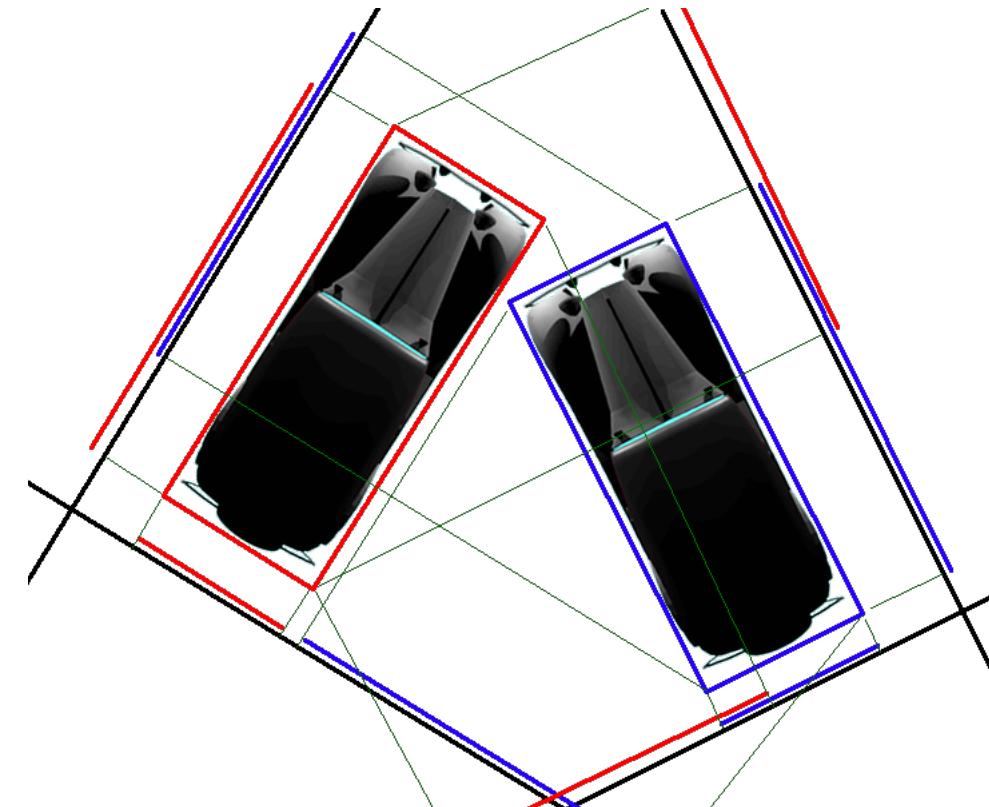
# APPLICATIONS OF COMPUTATIONAL GEOMETRY

- CAD/CAM
  - Computer-Aided Design
    - Deal with intersections and unions of objects, with decompositions objects and object boundaries into simpler shapes, and with visualizing the designed products
  - Computer-Aided Manufacturing
    - Computer controls the actual manufacturing process



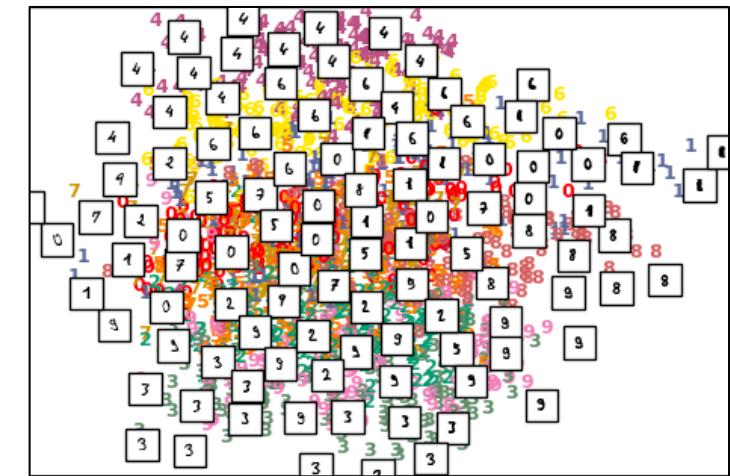
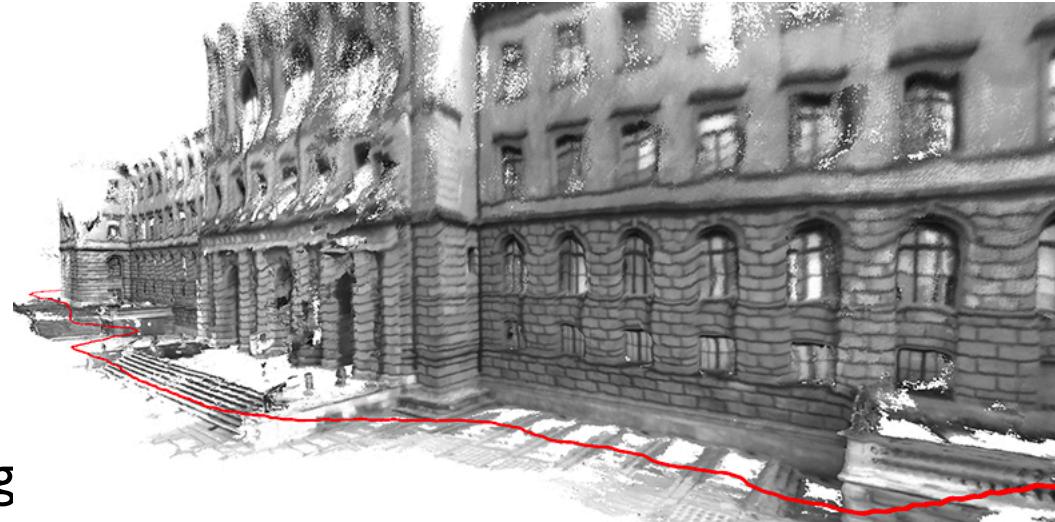
# APPLICATIONS OF COMPUTATIONAL GEOMETRY

- COLLISION DETECTION
  - Check whether two (possibly complicated) 3D objects intersect!
    - Approximate the objects by simple ones that enclose them (bounding volumes)
    - Popular bounding volumes: boxes, spheres, ellipsoids,...
    - If bounding volumes don't intersect, the objects don't intersect, either
    - Only if bounding volumes intersect, apply more expensive intersection test(s)



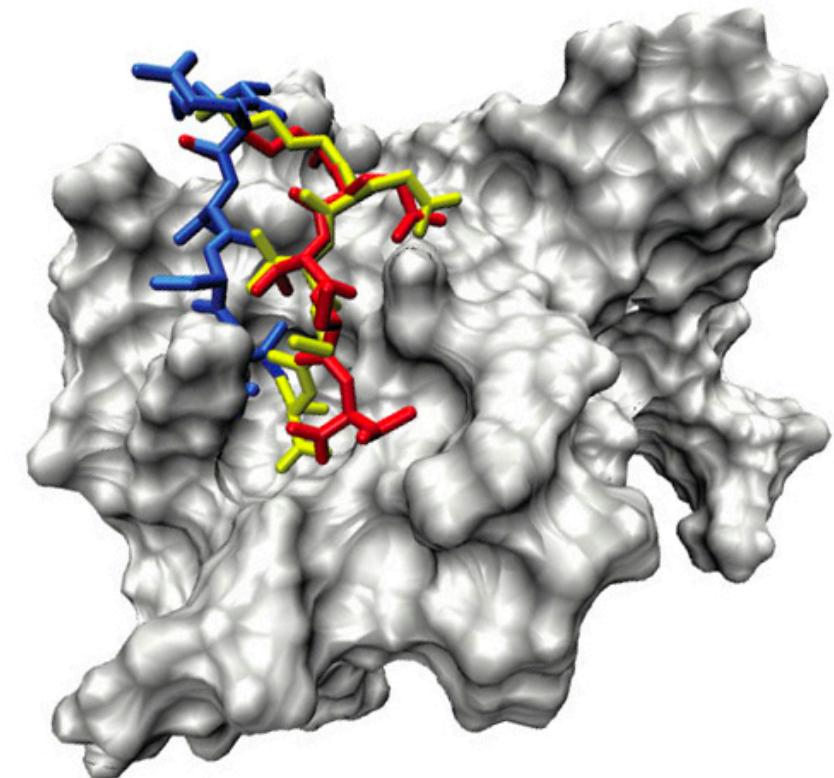
# APPLICATIONS OF COMPUTATIONAL GEOMETRY

- COMPUTER VISION
  - Surface Reconstruction
    - Step 1: Scan the object (3D laser scanner)
    - Step 2: Create a triangulation
    - Step 3: process the triangulation (rendering smooth surface in  $\mathbb{R}^3$ )
  - Major Computational Geometry task:  
Create a “good” triangulation
- Pattern recognition
  - Example: an optical character recognition system



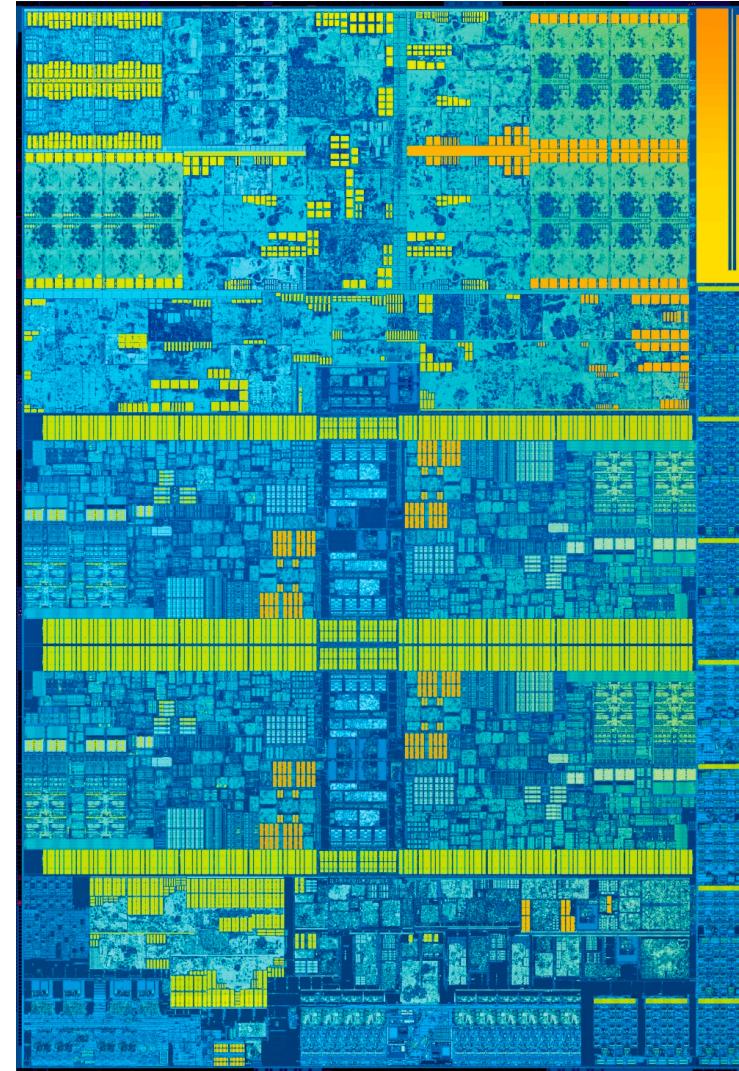
# APPLICATIONS OF COMPUTATIONAL GEOMETRY

- MOLECULAR MODELING
  - Typical questions are to compute the union of the atom balls to obtain the molecule surface or to compute where 2 molecules can touch each other



# APPLICATIONS OF COMPUTATIONAL GEOMETRY

- **VLSI DESIGN**
  - The process of creating an integrated circuit (IC) by combining thousands of transistors into a single chip



# APPLICATIONS OF COMPUTATIONAL GEOMETRY

- DATA MINING, MACHINE LEARNING, AND VISUALIZATION
  - Many algorithms for operations such as clustering, dimension reduction, classification, and segmentation are based upon geometric approaches.

