1 Problem Definition

Humans are planning to build their second home on Mars. Suppose that the Mars rovers have detected an area of m*n cells for us, and the air quality index of each cell is M [i; j] for i = 1; : : : ;m and j = 1; : : : ; n. You are tasked to find a rectangle area to build a new base where the total air quality index is maximized. For this you shall solve the following two problems. The solution of the first problem should help design an efficient solution for the second problem. For the rest of this document assume that m is O(n).

Problem1: Given an array A of n integers (positive or negative), find a contiguous subarray whose sum is maximum.

Problem2: Given a two-dimensional array M of size m*n consisting of integers (positive or negative), find a rectangle (two-dimensional sub-array) whose sum is maximum.

2 Algorithm Design Tasks

Alg1 Design a $\Theta(n3)$ time brute force algorithm for solving Problem1.
Alg2 Design a $\Theta(n2)$ time dynamic programming algorithm for solving Problem1.
Alg3 Design a $\Theta(n)$ time dynamic programming algorithm for solving Problem1.
Alg4 Design a $\Theta(n6)$ time brute force algorithm for solving Problem2.
Alg5 Design a $\Theta(n4)$ time algorithm for solving Problem2 using dynamic programming Alg3.
Alg6 Design a $\Theta(n3)$ time algorithm for solving Problem2 using dynamic programming Alg3.

3 Programming Tasks
Implement each of the following programming procedures:

Task1 Give an implementation of Alg1.
Task2 Give an implementation of Alg2.
Task3a Give a recursive implementation of Alg3 using Memoization.
Task3b Give an iterative BottomUp implementation of Alg3.
Task4 Give an implementation of Alg4 using O (1) extra space.
Task5 Give an implementation of Alg5 using O(mn) extra space.
Task6 Give an implementation of Alg6 using O(mn) extra space.