

# CS 314 Principles of Programming Languages

---

## Lecture 22: Type Systems, Concurrent Data Structure

Prof. Zheng Zhang

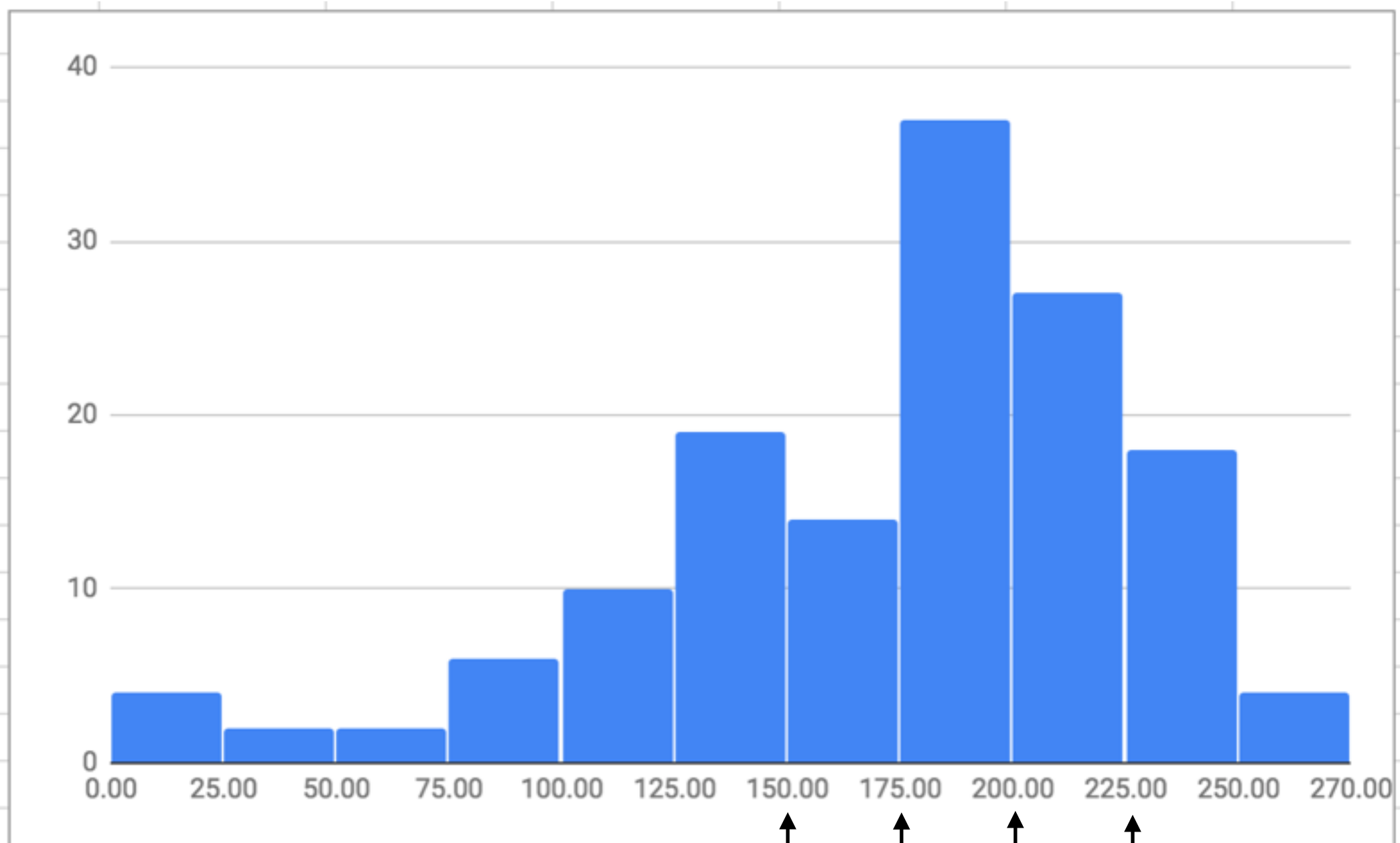


*Rutgers University*

November 28, 2018

# Class Information

- Project 3 and homework 8 will be released this week.
- Midterm grades are released.



100  
D

86  
C

49  
B

22  
A

143 in total.

2

# Type System

---

- **Basic types:**  
integer, real, character, ...
- **Constructed types:**  
arrays, records (union), sets, pointers, functions
- **A type system** is a collection of rules for assigning type expressions to operators, expressions in the program.  
*Type systems are language dependent.*
- **A type checker** implements the type system, i.e., deduces type expressions for program constructs based on the type inference rules of the type system.

**The type checker “computes” or “reconstructs” type expressions.**

# Type Expression

---

1. A basic type is a type expression. A special basic type, `TypeError` will signal an error. A basic type *void* denotes an untyped statement.
2. Since type expressions may be named, a type name is a type expression. (e.g.: `typedef struct foo bar`;) )
3. Type expressions may contain variables whose values are type expressions (e.g.: useful for languages without type declarations, or polymorphism).
4. A type constructor applied to type expressions is a type expression.

Examples:

- (a) arrays
- (b) cartesian products
- (c) records
- (d) pointers
- (e) functions

# Example Type Rules

---

- If both operands of the arithmetic operators of addition, subtraction, and multiplication are of type integer, then the result is of type integer.

Rule for  $+$  (analogue rules for  $-$  and  $*$ ):

$$\frac{E \vdash e1 : \textit{integer} \quad E \vdash e2 : \textit{integer}}{E \vdash e1 + e2 : \textit{integer}}$$

- Where  $E$  is a type environment that maps constants and variables to their types. In combination with the following axiom in the type system for constants, we can now infer that  $(2 + 3)$  is of type integer,  $E = \{2 : \textit{integer}, 3 : \textit{integer}\}$ .

**In general, type deduction proofs work bottom up.**

# Example Type Rules

---

**$\alpha$  is a type variable, a placeholder for other type expressions.**

- The result of the unary  $\&$  operator is a pointer to the object referred to by the operand. If the operand is of type “foo”, then the type of the result is a pointer to “foo”. (C and C++ definition)

$$\frac{E \vdash e : \alpha}{E \vdash \&e : \textit{pointer}(\alpha)}$$

# Example Type Rules

---

**$\alpha$  is a type variable, a placeholder for other type expressions.**

- Two expressions can only be compared if they have the same types. The result is of type boolean.

$$\frac{E \vdash e1 : \alpha \quad E \vdash e2 : \alpha}{E \vdash (e1 == e2) : \text{boolean}}$$

# Type Variables

---

**Type expressions** may contain variables (type variables) whose values are type expressions. **Type variables** are used for implicitly typed languages or languages with polymorphic types.

Programming languages can be

- explicitly typed — every object is declared with its type (**type checking**)
- implicitly typed — type of object is derived from its use (**type reconstruction**)
- monomorphic — every function or data type has a unique, single type
- polymorphic — allows functions or data types to have more than one type (e.g.: *list* in Scheme and *&* in C)



# Type Variables — Polymorphic

---

- **Polymorphic cons:**

$$\frac{E \vdash e1 : \alpha \quad E \vdash e2 : \text{list}(\alpha)}{E \vdash \text{cons}(e1, e2) : \text{list}(\alpha)}$$

- **Polymorphic ‘():**

$$E \vdash '(): \text{list}(\alpha) \quad \forall \alpha \text{ in } E$$

# Type Variables: Implicitly Typed

- Recall:

$$\frac{E \vdash e1 : integer \quad E \vdash e2 : integer}{E \vdash e1 + e2 : integer}$$

where E is a type environment. In other words, "+" has the type expression  $(integer \times integer) \rightarrow integer$ . What are the types of the variables a and b in the following program:

```
read(a);  
read(b);  
... a + b ...;
```

```
{a: α}  
{a: α, b: β}  
unify(α, integer)  
unify(β, integer)  
apply type rule; result integer
```

# Unification

---

**unify** generates a mapping  $U$  from type variables to type expressions such that two type expressions become identical.

Example:

- Two type expressions:

$$\text{type\_expr1} = \alpha \rightarrow \beta$$

$$\text{type\_expr2} = (\beta \times \beta) \rightarrow \text{integer}$$

- Mapping  $U = \text{unify}(\text{type\_expr1}, \text{type\_expr2})$  implies:

$$\{[\alpha, (\text{integer} \times \text{integer})], [\beta, \text{integer}] \}.$$

- $U(\text{type\_expr1}) = U(\text{type\_expr2})$

$$(\text{integer} \times \text{integer}) \rightarrow \text{integer}$$

# Type Inference

---

Here's an untyped program:

$\lambda a. \lambda b. \lambda c. \text{ if } a (b + 1) \text{ then } b \text{ else } c$

Informal inference:

- $b$  must be `int`
- $a$  must be some kind of function
- the argument type of  $a$  must be the same as  $b + 1$
- the result type of  $a$  must be `bool`
- the type of  $c$  must be the same as  $b$

Putting all these pieces together:

$a : \text{int} \rightarrow \text{bool}, b : \text{int}, c : \text{int}$

# Unification

---

## Find unifier for $t_1$ and $t_2$

If  $t_1$  and  $t_2$

- are both type variables  $v_1$  and  $v_2$ 
  - if  $v_1 = v_2$ , return empty substitution
  - otherwise return  $\{v_2 := v_1\}$
- are both primitive types
  - if they are the same, return the empty substitution
  - otherwise, there is no unifier
- both are product types with  $t_1 = (t_{11} * t_{12})$  and  $t_2 = (t_{21} * t_{22})$ 
  - compute the most general unifier  $S$  of  $t_{11}$  and  $t_{21}$
  - compute the most general unifier  $S'$  of  $S t_{12}$  and  $S t_{22}$
  - return  $S \cup S'$
- only one is type variable  $v$ , the other an arbitrary term  $t$ 
  - if  $v$  occurs in  $t$ , there is no unifier (occurs check)
  - otherwise, return  $\{v := t\}$
- otherwise, there is no unifier

# Concurrent Programming Fundamentals

---

- A THREAD is a potentially-active execution context
- One thread can run concurrently with other threads
- A thread can be thought of as an abstraction of a physical processor
- Classic von Neumann model of computing has single thread of control, while parallel programs have more than one

# Concurrent Programming Fundamentals

---

- Threads can run asynchronously
- The steps of different threads can be interleaved arbitrarily
- Synchronization is a way to ensure that events in different threads happen in a desired order

## Thread 1

```
r1 := shared_counter  
  r1 := r1 + 1  
shared_counter := r1
```

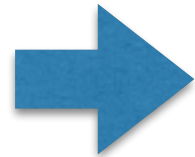
## Thread 2

```
r1 := shared_counter  
  r1 := r1 + 1  
shared_counter := r1
```

# Concurrent Programming Fundamentals

- Make a sequence of operations **atomic** for shared memory objects
- One possible way to do this is to use locks

```
r1 := shared_counter  
r1 := r1 + 1  
shared_counter := r1
```



```
acquire(Lock);  
r1 := shared_counter  
r1 := r1 + 1  
shared_counter := r1  
release(Lock);
```

A **lock** is a construct that, at any point in time, is unowned or owned by a single thread. If a thread  $t1$  wishes to acquire ownership of a lock that is already owned by another thread  $t2$ , then  $t1$  must wait until  $t2$  releases ownership of the lock.



# Compare and Swap (CAS)

- (Intrinsic) atomic instruction available on most processors
- Most common building block for non-blocking algorithms

```
int compare_and_swap(int* reg, int oldval, int newval)
{
    ATOMIC();
    int old_reg_val = *reg;
    if (old_reg_val == oldval)
        *reg = newval;
    END_ATOMIC();
    return old_reg_val;
}
```

- Other types of atomic operations exist:  
increment, decrement, exchange, fetch-and-add, ...

# Blocking V.S. Non-blocking

- **Blocking algorithms**

If the thread that currently holds the lock is delayed, then all other threads attempting to access the shared data object are also delayed.

- **Non-blocking algorithms**

The delay of a thread does not cause the delay of others. By definition, these algorithms cannot use locks.

```
acquire(Lock)
```

```
r1 := shared_counter
```

```
r1 := r1 + 1
```

```
shared_counter := r1
```

```
release(Lock)
```

blocking implementation

```
do
```

```
  r1 := shared_counter
```

```
while CAS(shared_counter, r1, r1+1)
```

non-blocking implementation

# Blocking V.S. Non-blocking

---

## Non-blocking algorithm

The key

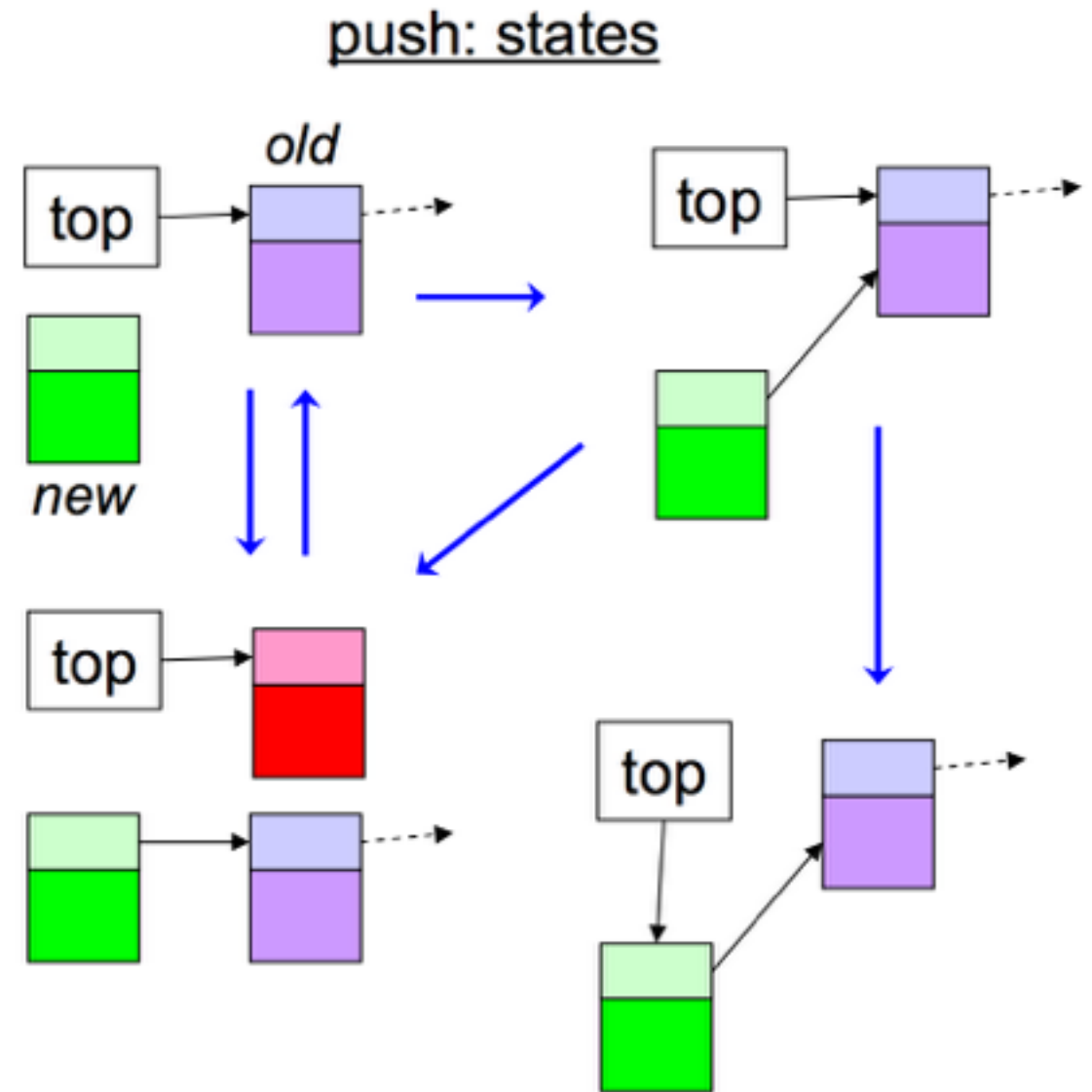
- Try to compute speculatively.
- CAS before committing the result.
- Retry if CAS fails.

Good practice:

- Work with a state-machine.
- Every state must be consistent.
- States = committed (intermediate) results.

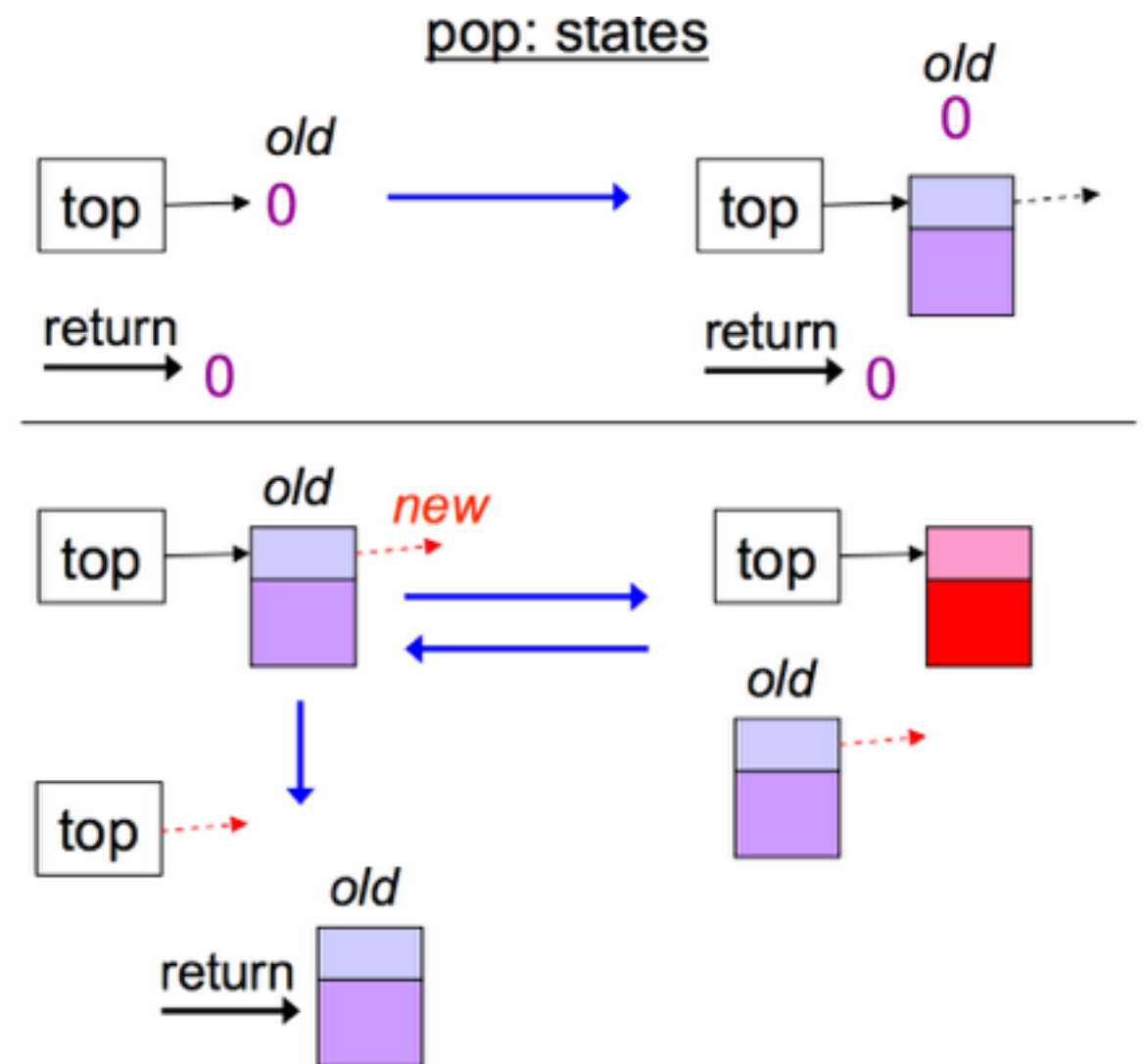
# Non-blocking Stack (Treiber's algorithm)

```
proc push(new)
  do
    old = top
    new.next = old
    while not CAS(top, old, new)
  end
proc pop
  do
    old = top
    return null if old == null
    new = old.next
    while not CAS(top, old, new)
  return old
end
```



# Non-blocking Stack (Treiber's algorithm)

```
proc push(new)
  do
    old = top
    new.next = old
    while not CAS(top, old, new)
  end
proc pop
  do
    old = top
    return null if old == null
    new = old.next
    while not CAS(top, old, new)
  return old
end
```



# Concurrent Specification

---

- Given a concurrent data object, each of its methods takes time.
- For example, the **enqueue** and **dequeue** operations for a FIFO queue.
- For a concurrent data object, if we let its methods
  - “take effect”
  - as if instantaneously
  - between invocation and response events

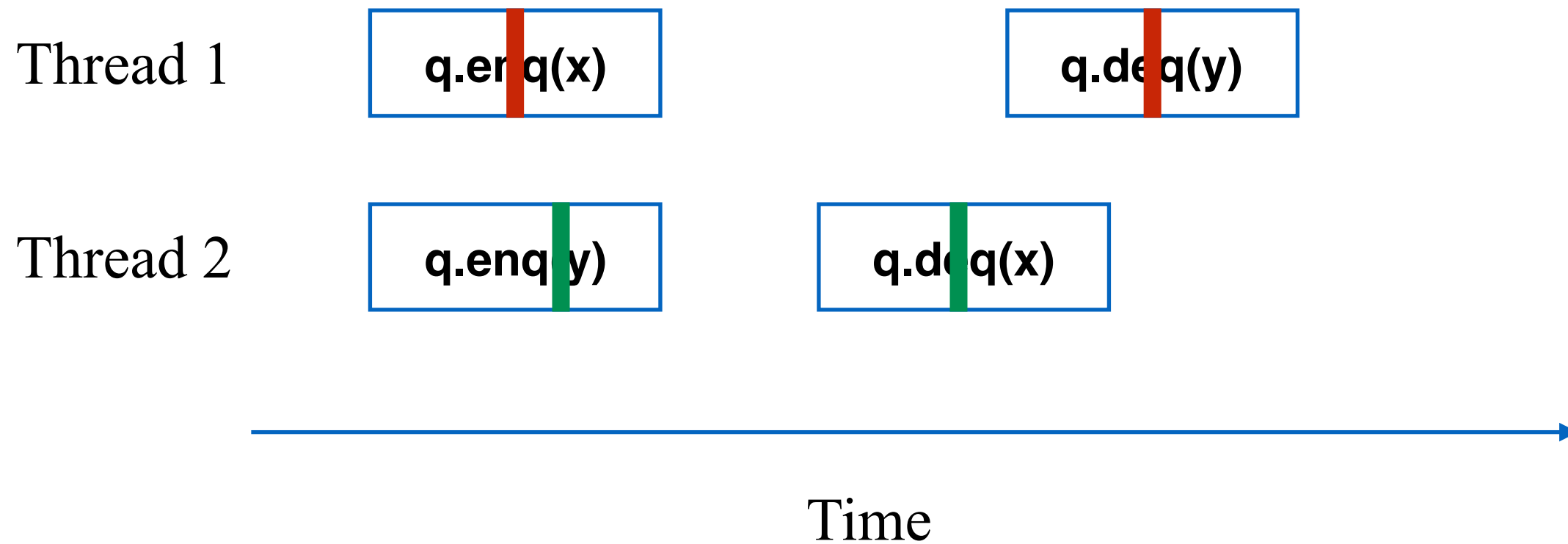
If the corresponding “sequential” behavior can be proved correct, then we say this concurrent data object is linearizable!

# Linearizability

---

- Concurrent FIFO queue examples:

**linearizable!**

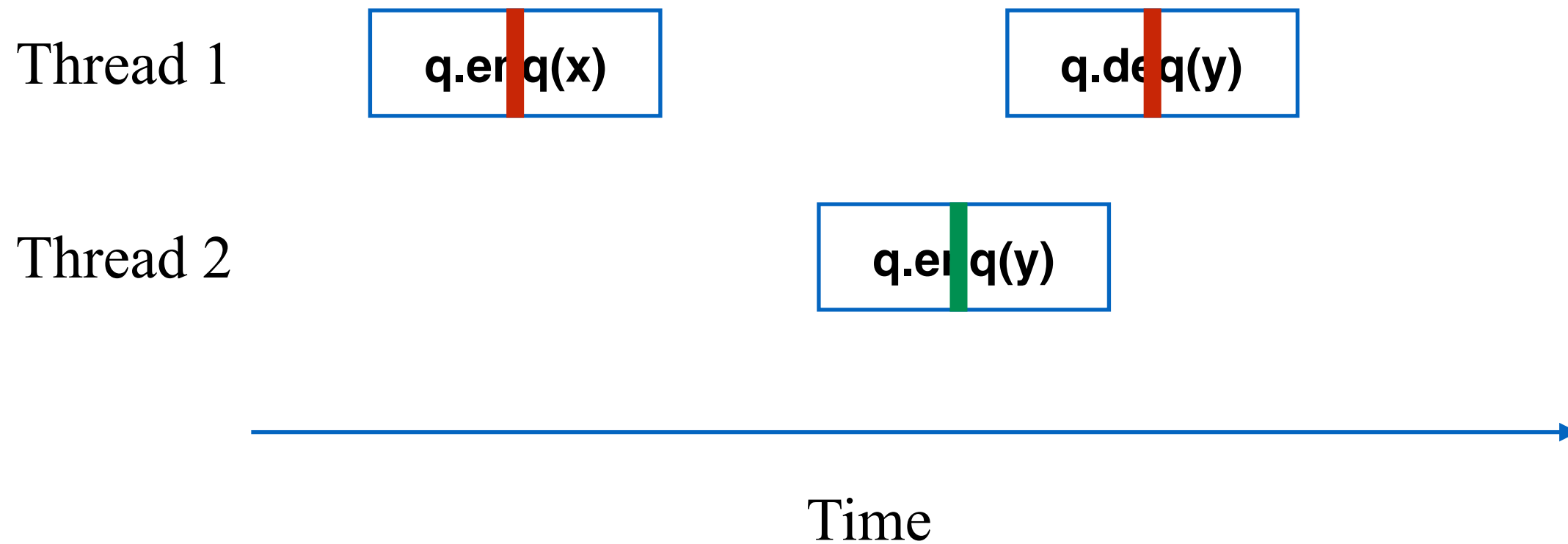


# Linearizability

---

- Concurrent FIFO queue examples:

**not linearizable!**

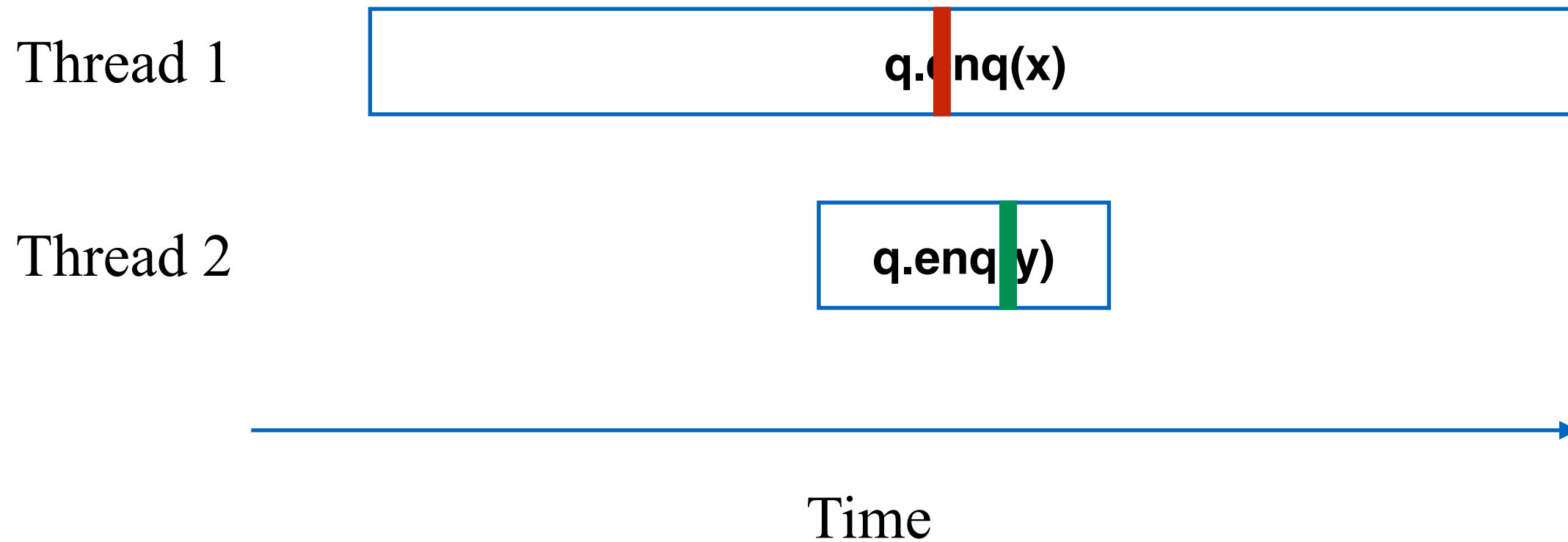




# Linearizability

- Concurrent FIFO queue examples:

**linearizable!**

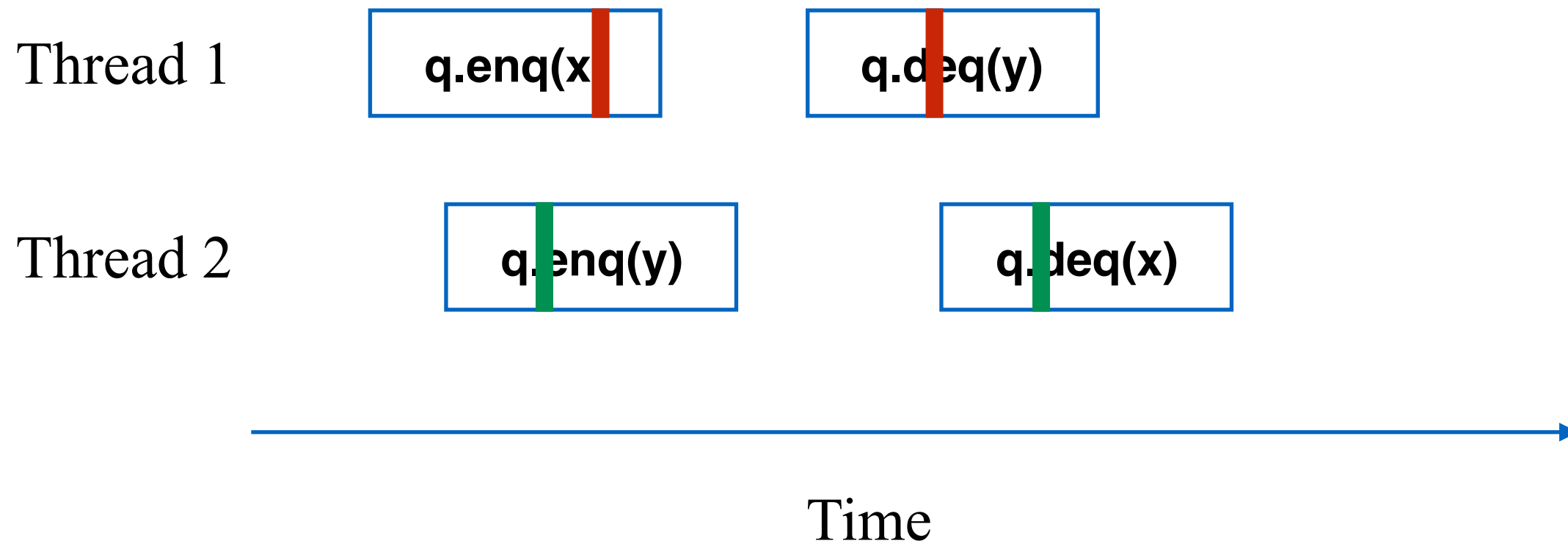


# Linearizability

---

- Concurrent FIFO queue examples:

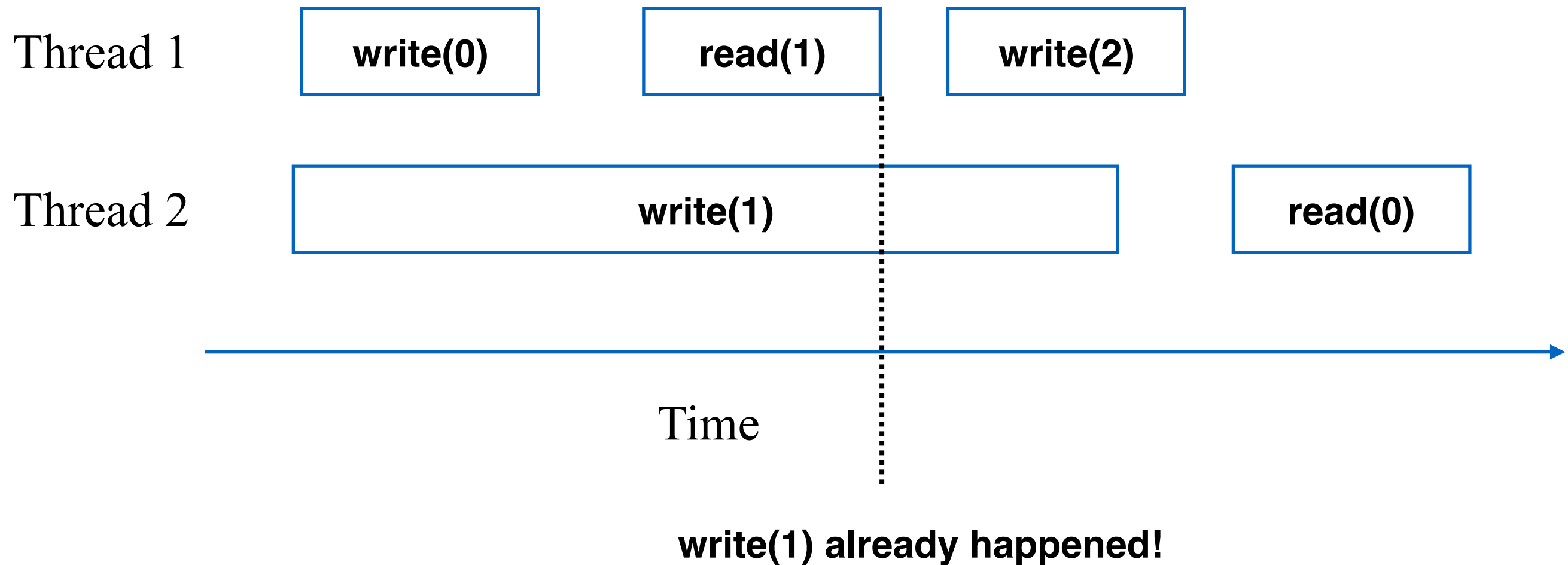
**linearizable!**



# Linearizability

- Read/Write register examples:

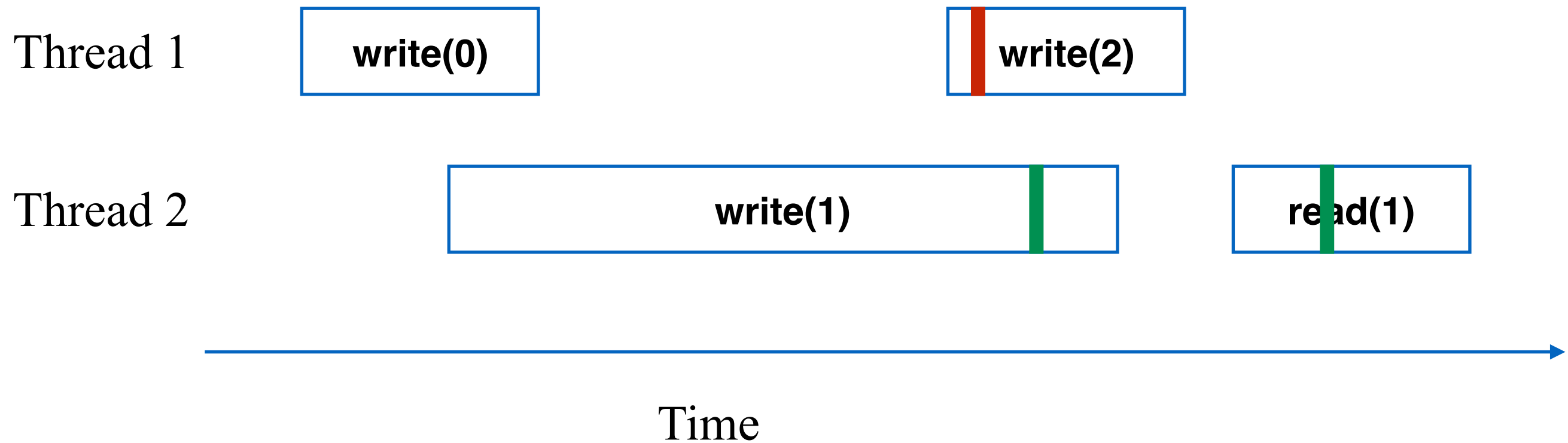
**not linearizable!**



# Linearizability

- Read/Write register examples:

**linearizable!**



# Find Linearization Point

---

- Easy for data structures that use locks for synchronization: let the linearization point be at where the lock is released
- May not be so easy for those that does not use locks, linearization point might depend on the execution of program
- Nonetheless, linearization analysis is a powerful specification tool for concurrent data objects. It allows us to capture the notion of objects being “atomic” and reason about the correctness.

```
acquire(Lock)
```

```
r1 := shared_counter
```

```
r1 := r1 + 1
```

```
shared_counter := r1
```

```
release(Lock)
```

← linearization point

# Next Class

---

## Reading

- Scott, Chapter 7.2, 13.1 - 13.3;