CS 314 Principles of Programming Languages

Lecture 5: Syntax Analysis (Parsing)

Prof. Zheng Zhang

Rutgers University

September 19, 2018

Class Information

- Homework 1 is being graded now.
  The sample solution will be posted soon.
- Homework 2 will be posted tomorrow.

Review: Context Free Grammars (CFGs)

- A formalism to for describing languages
- A CFG $\mathcal{G} = \langle \mathbf{T}, \mathbf{N}, \mathbf{P}, \mathbf{S} \rangle$:
  1. A set T of terminal symbols (tokens).
  2. A set N of nonterminal symbols.
  3. A set P production (rewrite) rules.
  4. A special start symbol S.
- The language $L(\mathcal{G})$ is the set of sentences of terminal symbols in T* that can be derived from the start symbol S:

$$L(\mathcal{G}) = \{w \in T^* \mid S \Rightarrow^* w\}$$

Elements of BNF Syntax

Terminal Symbol:	**Symbol-in-Boldface**
Non-Terminal Symbol:	*Symbol-in-Angle-Brackets*
Production Rule:	Non-Terminal ::= Sequence of Symbols or Non-Terminal ::= Sequence | Sequence | …

Example:

terminal non-terminal

…
<if-stmt> ::= **if** <expr> **then** <stmt>
<expr> ::= **id** **<=** **id**
<stmt> ::= **id** **:=** **num**

terminal

Review: Context Free Grammar

…

<if-stmt> ::= **if** <expr> **then** <stmt>
<expr> ::= **id** **<=** **id**
<stmt> ::= **id** **:=** **num**

…

Rule 1	**$1 ⇒ 1&**
Rule 2	**$0 ⇒ 0$**
Rule 3	**&1 ⇒ 1$**
Rule 4	**&0 ⇒ 0&**
Rule 5	**$# ⇒ →A**
Rule 6	**&# ⇒ →B**

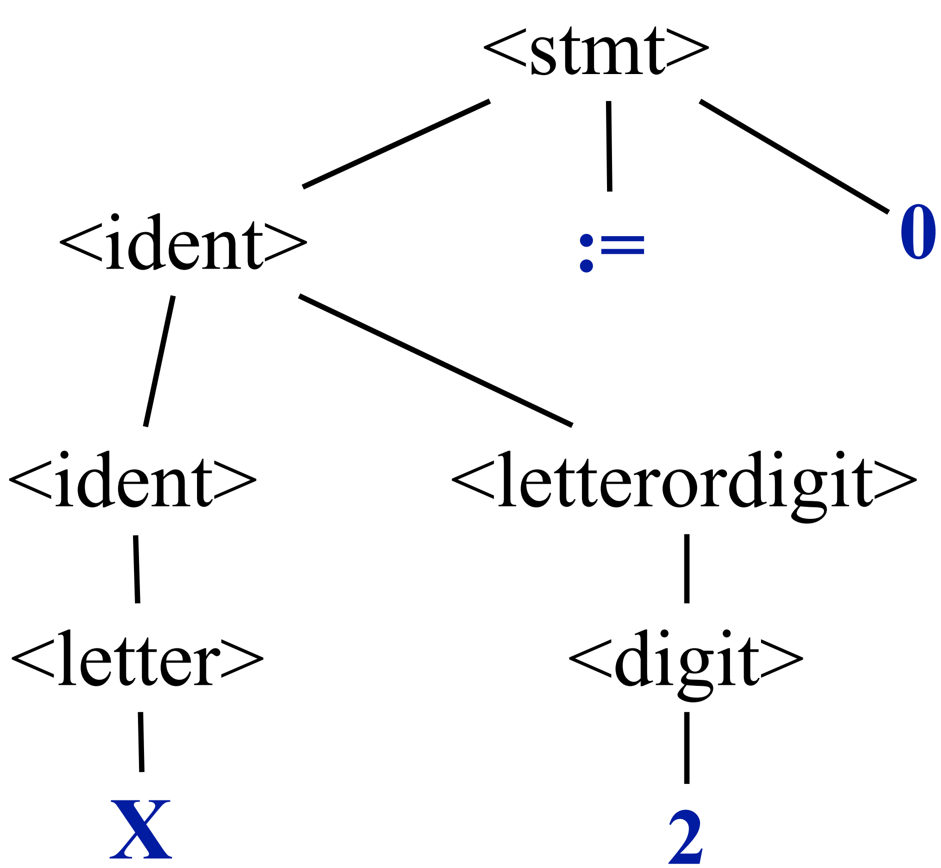
Context free grammar

Not a context free grammar

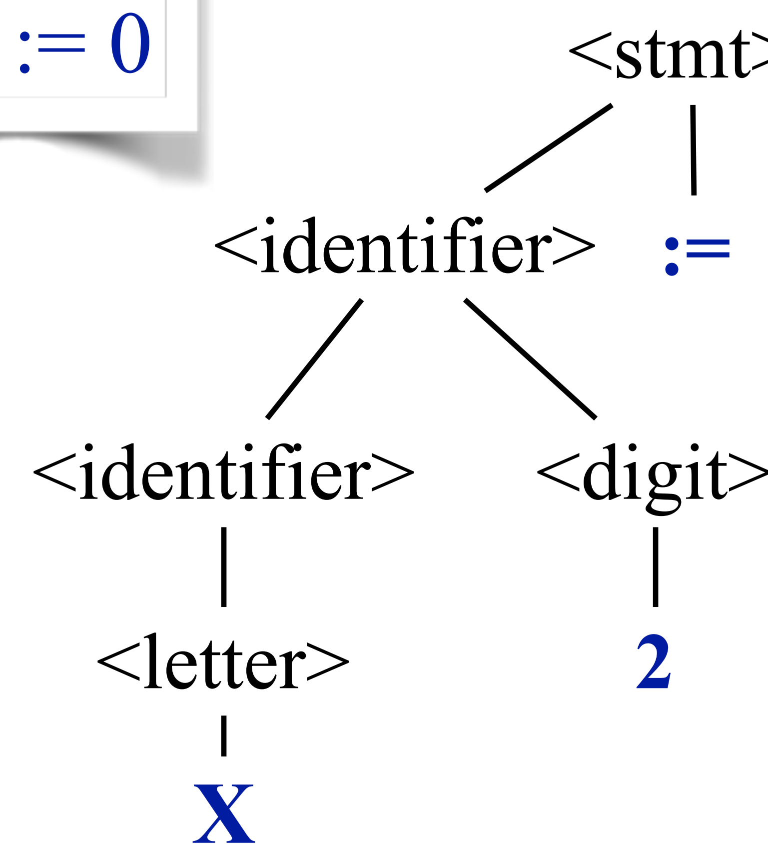
CFGs are rewrite systems with restrictions on the form of rewrite (production) rules that can be used. The left hand side of a production rule can only be **one non-terminal symbol**.

A Language May Have Many Grammars

Consider $\mathcal{G}$':	The Original Grammar $\mathcal{G}$:
1. < letter > ::= **A** | **B** | **C** | … | **Z**	1. < letter > ::= **A** | **B** | **C** | … | **Z**
2. < digit > ::= **0** | **1** | **2** | … | **9**	2. < digit > ::= **0** | **1** | **2** | … | **9**
3. < ident > ::= < letter > |	3. < identifier > ::= < letter > |
4. < ident > < letterordigit >	4. < identifier > < letter > |
5. < stmt > ::= < ident > **:=** 0	5. < identifier > < digit >
6. < letterordigit > ::= < letter > | < digit >	6. < stmt > ::= < identifier > **:=** 0



X2 := 0



Review: Grammars and Programming Languages

Many grammars may correspond to one programming language.

Good grammars:

- Captures the logic structure of the language

  ⇒ structure carries some semantic information
  (example: expression grammar)
- Use meaningful names
- Are easy to read
- Are unambiguous
- …

Review: Ambiguous Grammars

*"Time **flies like** an arrow; fruit **flies like** a banana."*

A grammar $\mathcal{G}$ is ambiguous iff there exists a w $\in$ L($\mathcal{G}$) such that there are:

- two distinct parse trees for w, or
- two distinct leftmost derivations for w, or
- two distinct rightmost derivations for w.

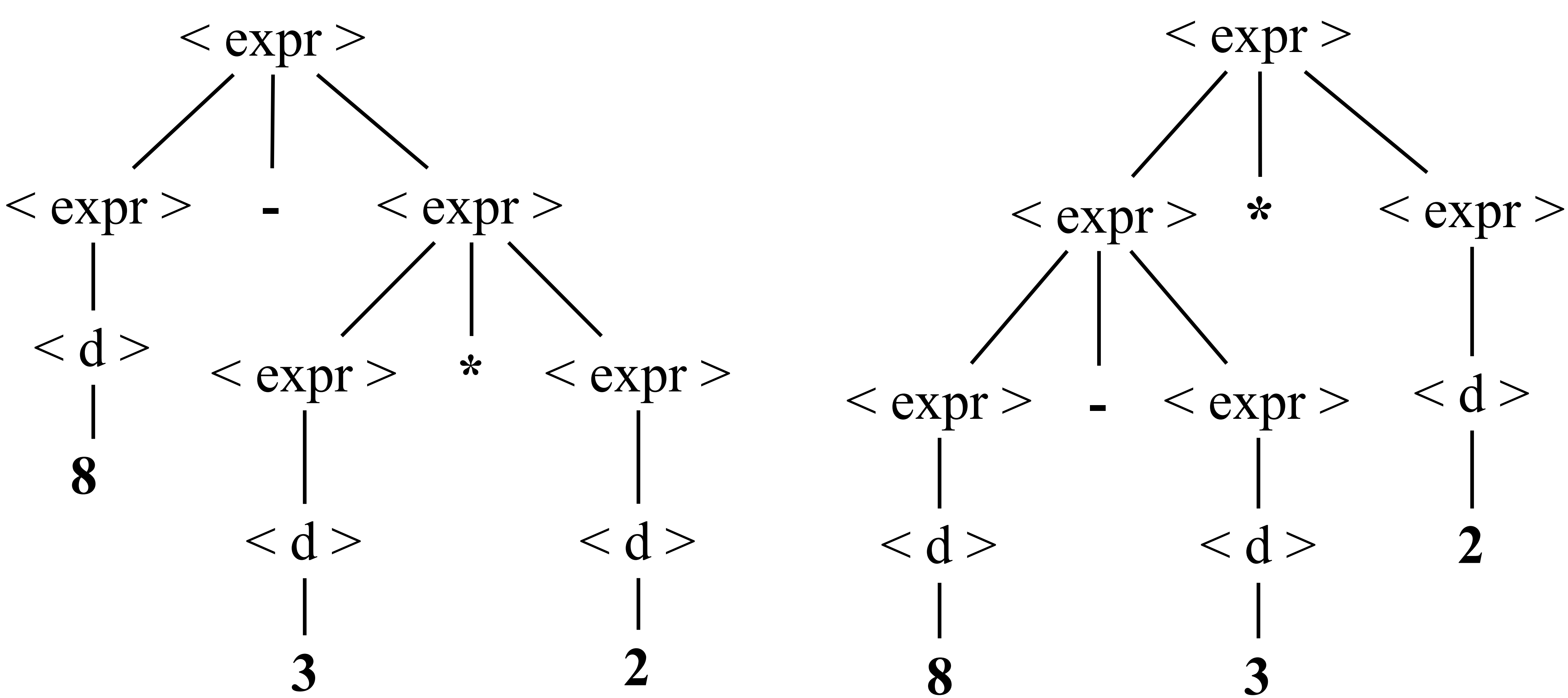
*We want **a unique semantics** of our programs, which typically requires **a unique syntactic structure**.*

Review: Arithmetic Expression Grammar

Parse "8 - 3 * 2":

< start > ::= < expr >
< expr > :: = < expr > **-** < expr > |
< expr > ***** < expr > |
< d > | < l >
< d > :: = **0** | **1** | **2** | **3** | … | **9**
< l > :: = **a** | **b** | **c** | … | **z**

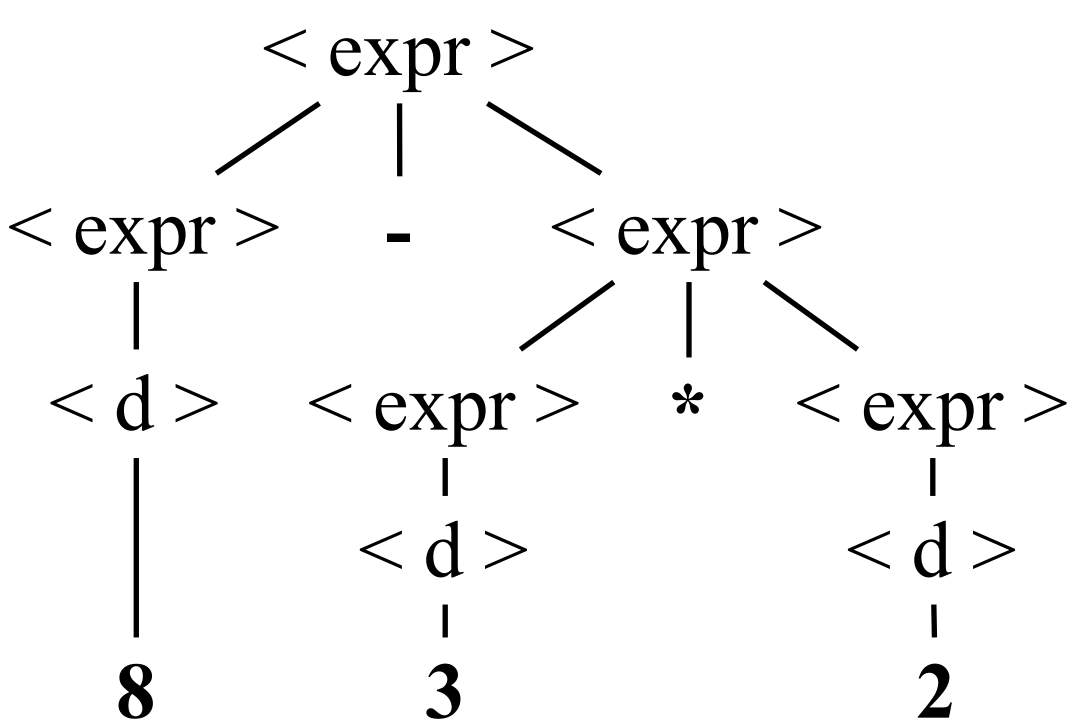
Two parse trees!

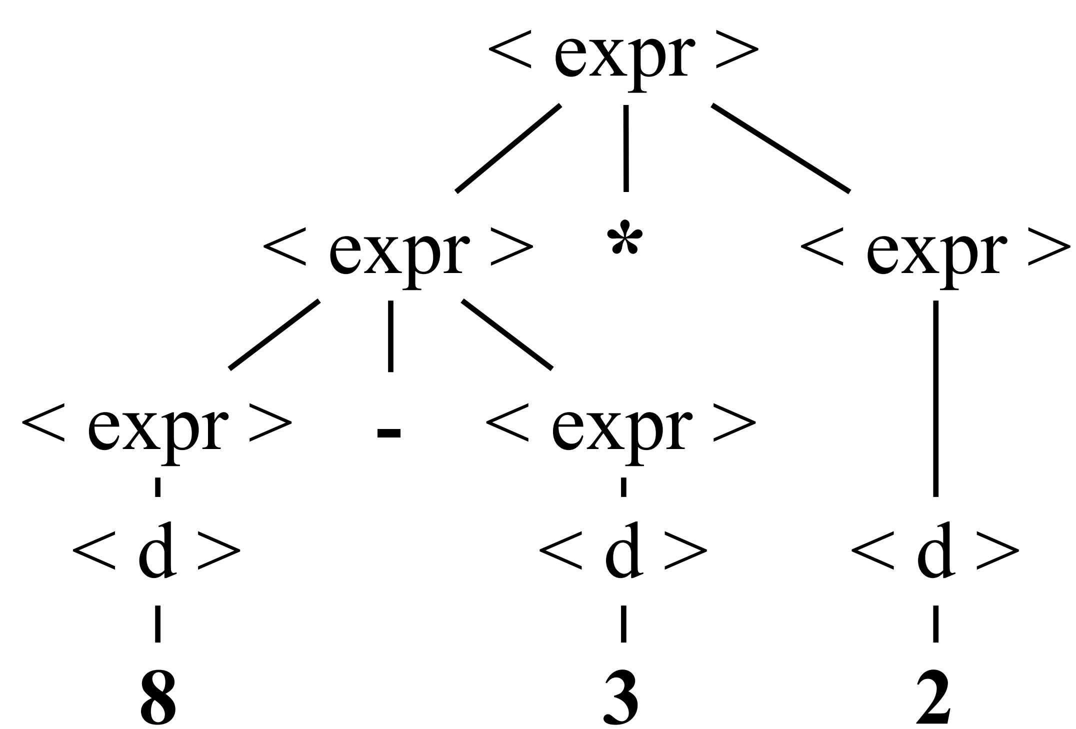


Review: Arithmetic Expression Grammar

Parse “8 - 3 * 2”:

Two Parse Trees —> Two leftmost derivations!





leftmost derivation	
<expr>	$\Rightarrow_L$
<expr> - <expr>	$\Rightarrow_L$
<d> - <expr>	$\Rightarrow_L$
<d> - <expr> * <expr>	$\Rightarrow_L$
<d> - <d> * <expr>	$\Rightarrow_L$
<d> - <d> * <d>	

leftmost derivation	
<expr>	$\Rightarrow_L$
<expr> * <expr>	$\Rightarrow_L$
<expr> - <expr> * <expr>	$\Rightarrow_L$
<d> - <expr> * <expr>	$\Rightarrow_L$
<d> - <d> * <expr>	$\Rightarrow_L$
<d> - <d> * <d>	

Review: Ambiguity

How to deal with ambiguity?

- Change the language
  Example: Adding new terminal symbols as delimiters.
  Fix the *dangling else*, *expression* grammars.
- Change the grammar
  Example: Impose **associativity** and **precedence** in an *arithmetic expression* grammar.

Changing the Grammar to Impose Precedence

< start > ::= < expr >
< expr > ::= < expr > + < expr > |
< expr > - < expr > |
< expr > * < expr > |
< expr > / < expr > |
< d > | < l >
< d > ::= **0** | **1** | **2** | **3** | … | **9**
< l > ::= **a** | **b** | **c** | … | **z**

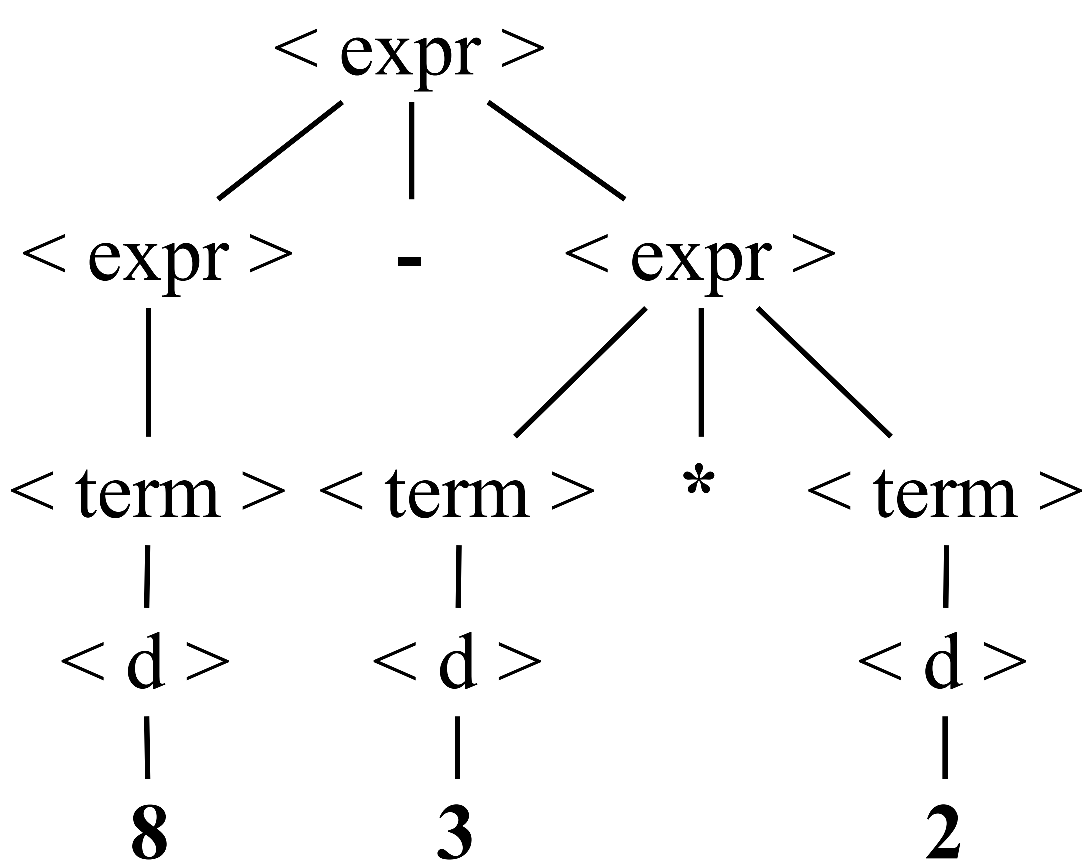
Original Grammar $\mathcal{G}$

< start > ::= < expr >
< expr > ::= < expr > + < expr > |
< expr > - < expr > |
< term >
< term > ::= < term > * < term > |
< term > / < term > |
< d > | < l >
< d > ::= **0** | **1** | **2** | **3** | … | **9**
< l > ::= **a** | **b** | **c** | … | **z**

Modified Grammar $\mathcal{G}$’

Grouping in Parse Tree Now Reflects Precedence

Parse “8 - 3 * 2”:



Only One Possible Parse Tree

< start > ::= < expr >

< expr > :: = < expr > + < expr > |
< expr > - < expr > |
< term >

< term > ::= < term > * < term > |
< term > / < term > |
< d > | < l >

< d > :: = **0** | **1** | **2** | **3** | … | **9**

< l > :: = **a** | **b** | **c** | … | **z**

Modified Grammar $\mathcal{G}$’

Precedence

- *Low Precedence:*
  Addition + and Subtraction -
- *Medium Precedence:*
  Multiplication * and Division /
- *Highest Precedence:*
  Exponentiation ^

$$
\begin{aligned}
&< \text{start} > ::= \; < \text{expr} > \\
&< \text{expr} > ::= \; < \text{expr} > + < \text{expr} > \mid \\
&\qquad\qquad\quad\; < \text{expr} > - < \text{expr} > \mid \\
&\qquad\qquad\quad\; < \text{term} > \\
&< \text{term} > ::= \; < \text{term} > * < \text{term} > \mid \\
&\qquad\qquad\quad\; < \text{term} > / < \text{term} > \mid \\
&\qquad\qquad\quad\; < \text{d} > \mid < \text{l} > \\
&< \text{d} > \quad ::= \mathbf{0} \mid \mathbf{1} \mid \mathbf{2} \mid \mathbf{3} \mid \ldots \mid \mathbf{9} \\
&< \text{l} > \quad ::= \mathbf{a} \mid \mathbf{b} \mid \mathbf{c} \mid \ldots \mid \mathbf{z}
\end{aligned}
$$

Still Have Ambiguity…

How about 3 - 2 - 1 ?

3 - 2 - 1 OR? 3 - 2 - 1

< start > ::= < expr >

< expr > :: = < expr > + < expr > | < expr > - < expr > | < term >

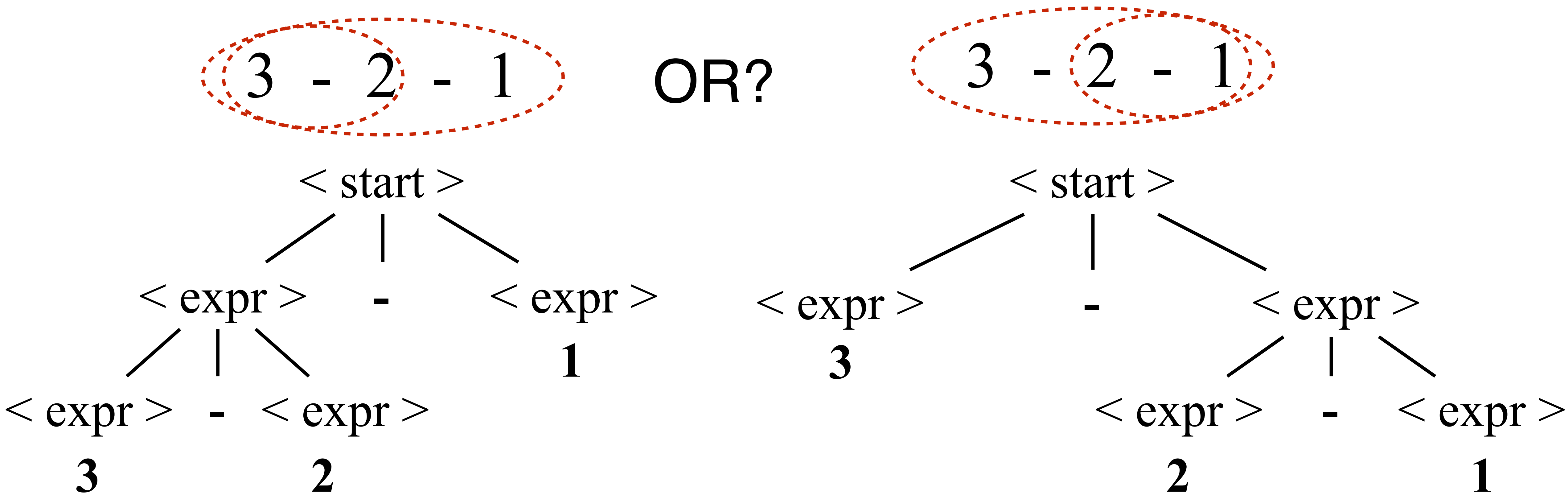
< term > ::= < term > * < term > | < term > / < term > | < d > | < l >

< d > :: = **0** | **1** | **2** | **3** | … | **9**

< l > :: = **a** | **b** | **c** | … | **z**

Still Have Ambiguity…

How about 3 - 2 - 1 ?



< start > ::= < expr >

< expr > :: = < expr > + < expr > | < expr > - < expr > | < term >

< term > ::= < term > * < term > | < term > / < term > | < d > | < l >

< d > :: = **0** | **1** | **2** | **3** | … | **9**

< l > :: = **a** | **b** | **c** | … | **z**

Still Have Ambiguity…

- Grouping of operators of same precedence not disambiguated.
- Non-commutative operators: only one parse tree correct.

< start > ::= < expr >

< expr > :: = < expr > + < expr > | < expr > - < expr > | < term >

< term > ::= < term > * < term > | < term > / < term > | < d > | < l >

< d > :: = **0** | **1** | **2** | **3** | … | **9**

< l > :: = **a** | **b** | **c** | … | **z**

Imposing Associativity

Same grammar with left / right recursion for - :

Our choices:

```
< expr > :: = < d >  -  < expr > |
              < d >
< d >       :: = 0 | 1 | 2 | 3 | … | 9
```

```
<expr>                                ⇒
< d > - <expr>                        ⇒
< d > - < d > - <expr>                ⇒
< d > - < d > - < d > - <expr>        ⇒
…
```

Or:

```
< expr > :: = < expr > - < d > |
              < d >
< d >       :: = 0 | 1 | 2 | 3 | … | 9
```

```
<expr>                                ⇒
<expr> - < d >                        ⇒
<expr> - < d > - < d >                ⇒
<expr> - < d > - < d > - <d>          ⇒
…
```

Which one do we want for - in the calculator language?

Associativity

- Deals with operators of same precedence
- Implicit grouping or parenthesizing
- Left to right: *, /, +, -
- Right to left: ^

Complete, Unambiguous Arithmetic Expression Grammar

$$
\begin{array}{lll}
\langle \text{start} \rangle & ::= & \langle \text{expr} \rangle \\
\langle \text{expr} \rangle & ::= & \langle \text{expr} \rangle + \langle \text{expr} \rangle \mid \\
 & & \langle \text{expr} \rangle - \langle \text{expr} \rangle \mid \\
 & & \langle \text{expr} \rangle * \langle \text{expr} \rangle \mid \\
 & & \langle \text{expr} \rangle \, / \, \langle \text{expr} \rangle \mid \\
 & & \langle \text{expr} \rangle \wedge \langle \text{expr} \rangle \mid \\
 & & \langle \text{d} \rangle \mid \langle \text{l} \rangle \\
\langle \text{d} \rangle & ::= & \mathbf{0} \mid \mathbf{1} \mid \mathbf{2} \mid \mathbf{3} \mid \ldots \mid \mathbf{9} \\
\langle \text{l} \rangle & ::= & \mathbf{a} \mid \mathbf{b} \mid \mathbf{c} \mid \ldots \mid \mathbf{z}
\end{array}
$$

Original Ambiguous Grammar $\mathcal{G}$

$$
\begin{array}{lll}
\langle \text{start} \rangle & ::= & \langle \text{expr} \rangle \\
\langle \text{expr} \rangle & ::= & \langle \text{expr} \rangle + \langle \text{term} \rangle \mid \\
 & & \langle \text{expr} \rangle - \langle \text{term} \rangle \mid \\
 & & \langle \text{term} \rangle \\
\langle \text{term} \rangle & ::= & \langle \text{term} \rangle * \langle \text{factor} \rangle \mid \\
 & & \langle \text{term} \rangle \, / \, \langle \text{factor} \rangle \mid \\
 & & \langle \text{factor} \rangle \\
\langle \text{factor} \rangle & ::= & \langle \text{g} \rangle \wedge \langle \text{factor} \rangle \mid \\
 & & \langle \text{g} \rangle \\
\langle \text{g} \rangle & ::= & \mathbf{(} \langle \text{expr} \rangle \mathbf{)} \mid \langle \text{d} \rangle \mid \langle \text{l} \rangle \\
\langle \text{d} \rangle & ::= & \mathbf{0} \mid \mathbf{1} \mid \mathbf{2} \mid \ldots \mid \mathbf{9} \\
\langle \text{l} \rangle & ::= & \mathbf{a} \mid \mathbf{b} \mid \mathbf{c} \mid \ldots \mid \mathbf{z}
\end{array}
$$

Unambiguous Grammar $\mathcal{G}$

Abstract versus Concrete Syntax

Concrete Syntax:

Representation of a construct in a particular language, including placement of keywords and delimiters.

Abstract Syntax:

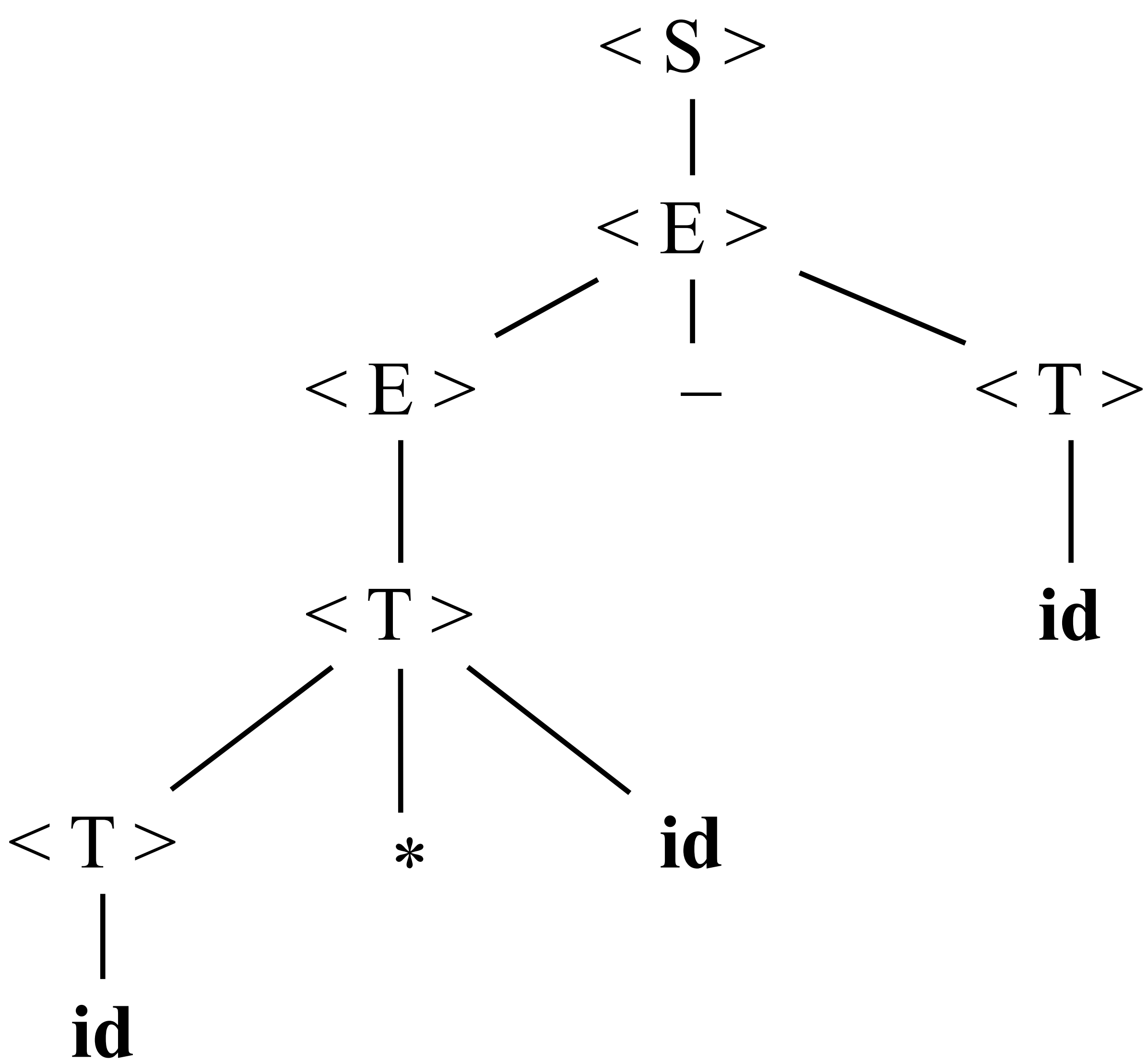
Structure of meaningful components of each language construct.

Example:

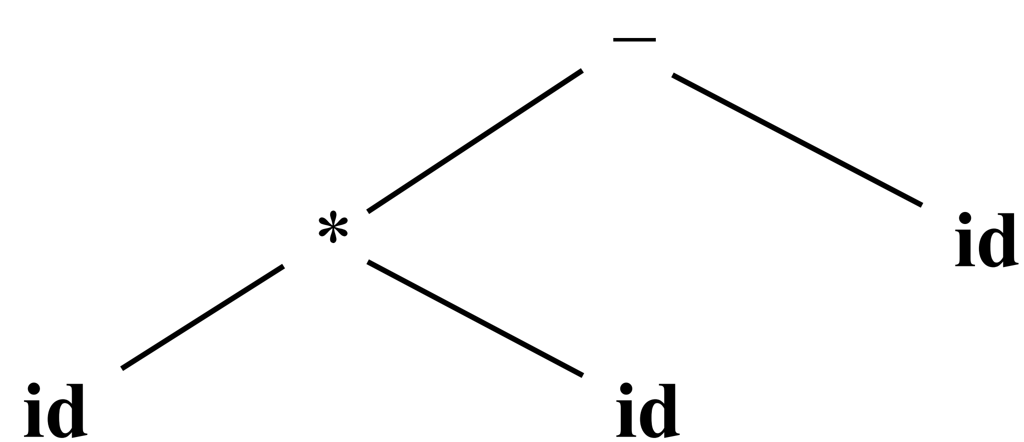
Consider A * B − C:

< S > ::= < E >
< E > ::= < E > − < T > | < T >
< T > ::= < T > * **id** | **id**

Concrete Syntax Tree



Abstract Syntax Tree
(AST)



Abstract versus Concrete Syntax

Same abstract syntax, different concrete syntax:

Pascal

while x <> A[i], **do**
    i := i + 1
end

C

```
while ( x != A[i])
    i = i + 1;
```

Regular vs. Context Free

- All Regular languages are context free languages
- Not all context free languages are regular languages

Example:

<N> ::= <X> | <Y>
<X> ::= **a** | <X> **b**   is equivalent to: a b* | c^+
<Y> ::= **c** | <Y> **c**

Question:

Is $\{a^n b^n \mid n \geq 0\}$ *a context free language?*

Regular vs. Context Free

<Y> ::= **a** <Y> **b** | ϵ

Regular vs. Context Free

- All Regular languages are context free languages
- Not all context free languages are regular languages

Example:

<N> ::= <X> | <Y>
<X> ::= **a** | <X> **b**   is equivalent to: a b* | c^+
<Y> ::= **c** | <Y> **c**

Question:

Is $\{a^n b^n \mid n \geq 0\}$ *a context free language?*

Is $\{a^n b^n \mid n \geq 0\}$ *a regular language?*

Regular Grammars

CFGs with restrictions on the shape of production rules.

Left-linear:

<X> ::= **a** | <X> **b**
<N> ::= <X> **a b**

Right-linear:

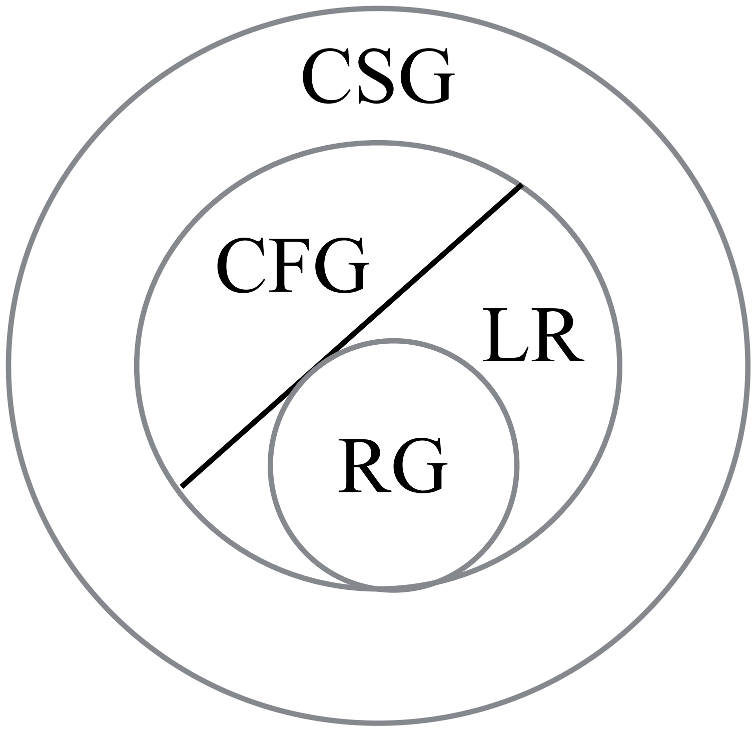
<Y> ::= **a b** | **a b** <Y>
<N> ::= **b** | **b** <Y>

Complexity of Parsing

Classification of languages that can be recognized by specific grammars.

Complexity:

Regular grammars	DFAs	**O**(n)
LR grammars	Kunth's algorithm	**O**(n)
Arbitrary CFGs	Earley's algorithm	**O**(n^3)
Arbitrary CSGs	LBAs	P-SPACE COMPLETE

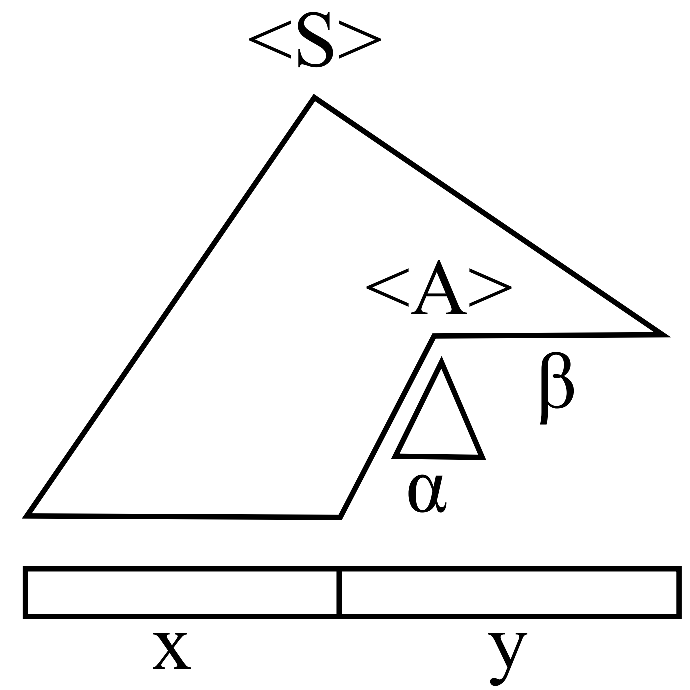


Reading:

Scott Chapter 2.3.4 (for LR parser) and Chapter 2.4.3 for language class.

Earley, Jay (1970), "An efficient context-free parsing algorithm", Communications of the ACM.

Top - Down Parsing - LL(1)



<u>Basic Idea:</u>

- The parse tree is constructed from the root, expanding non-terminal nodes on the tree's frontier following a **leftmost** derivation.
- The input program is read from **left** to right, and input tokens are read (consumed) as the program is parsed.
- The next non-terminal symbol is replaced using one of its rules. The particular choice <u>has to be unique</u> and uses parts of the input (partially parsed program), for instance the first **token** of the remaining input.

Example:

S ::= a S b | ε

How can we parse (automatically construct a leftmost derivation) the input string **a a a b b b** using a PDA (push-down automaton) and only the first symbol of the remaining input?

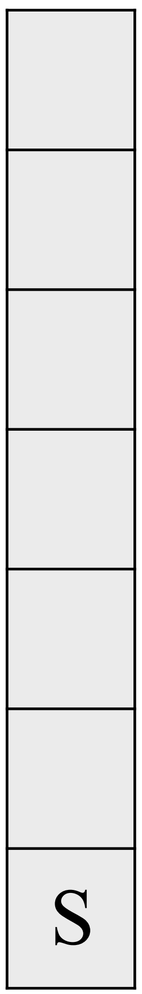
INPUT: $\boxed{\text{a a a b b b eof}}$

LL(1) Parsing Example

S ::= a S b | ε

S

Remaining Input:
a a a b b b

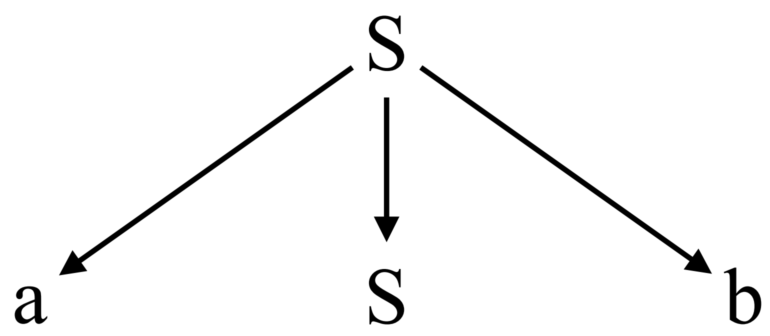


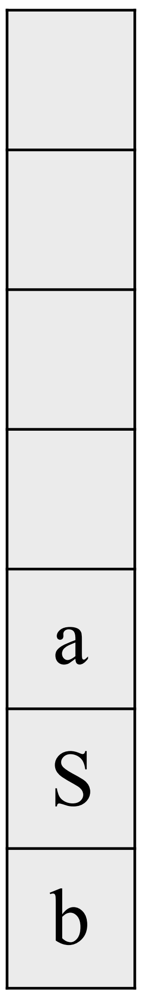
Sentential Form:
S

Applied Production:

LL(1) Parsing Example

S ::= a S b | ε





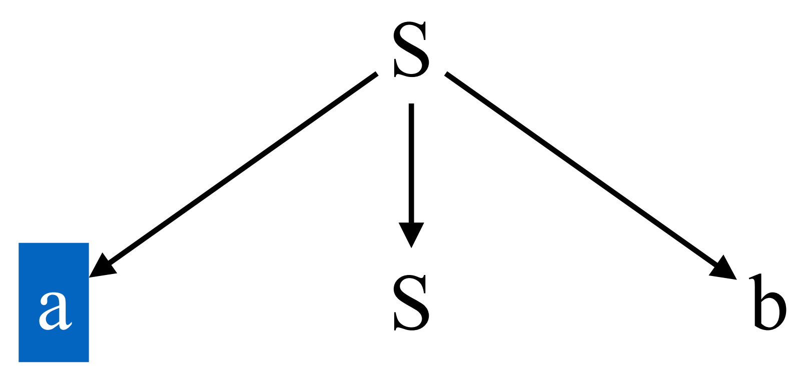
Remaining Input:
a a a b b b

Sentential Form:
a S b

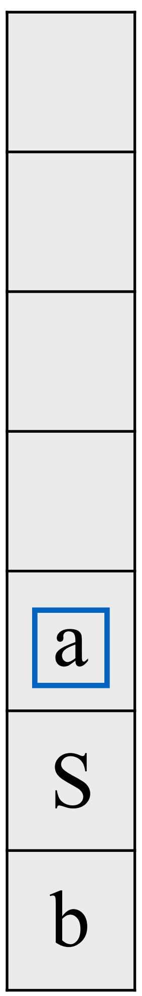
Applied Production:
S ::= a S b

LL(1) Parsing Example

S ::= a S b | ε



Match!



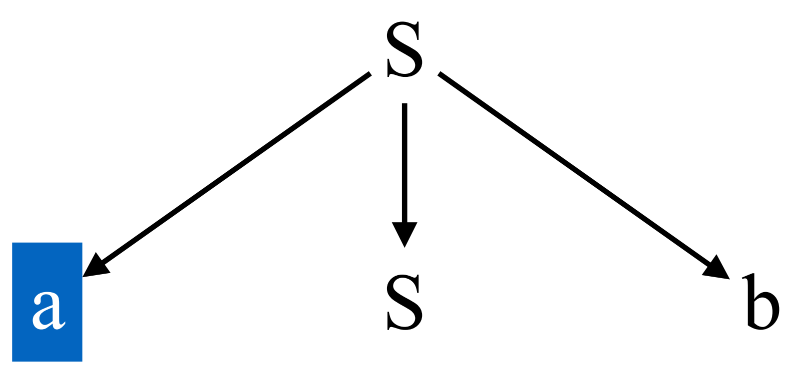
Remaining Input:
a a a b b b

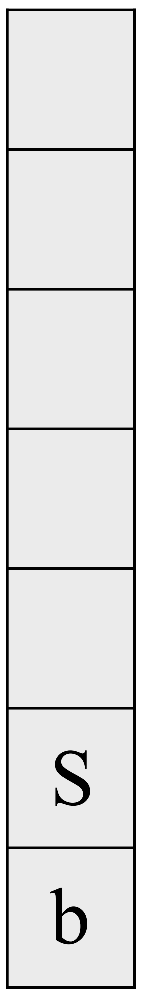
Sentential Form:
a S b

Applied Production:

LL(1) Parsing Example

S ::= a S b | ε





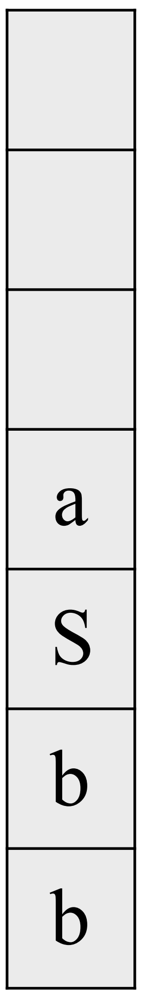
Remaining Input:
a a b b b

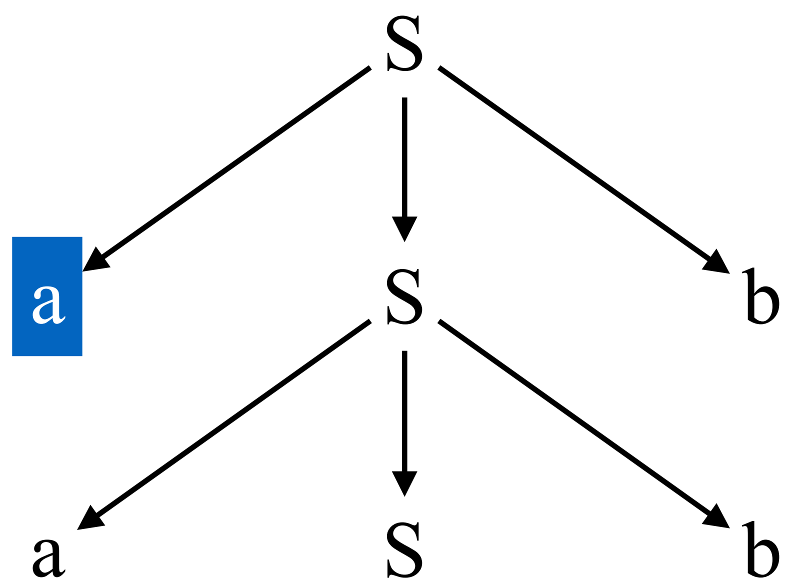
Sentential Form:
a S b

Applied Production:

LL(1) Parsing Example

$S ::= a\ S\ b \mid \varepsilon$





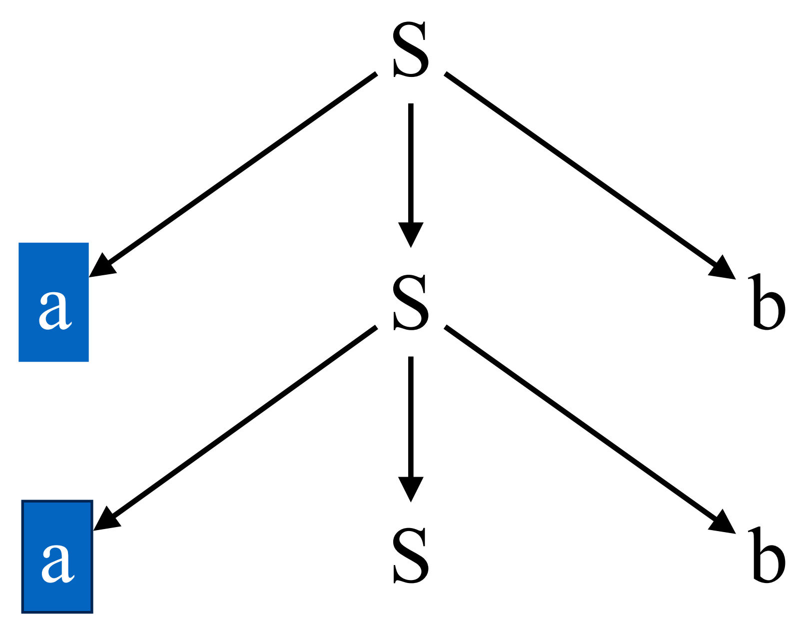
Remaining Input:
a a b b b

Sentential Form:
a a S b b

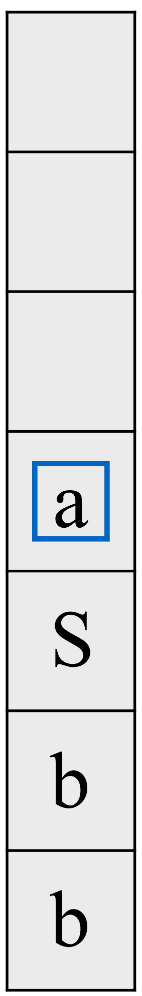
Applied Production:
$S ::= a\ S\ b$

LL(1) Parsing Example

S ::= a S b | ε



Match!



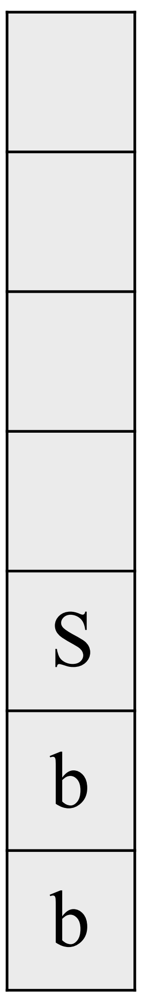
Remaining Input:
a a b b b

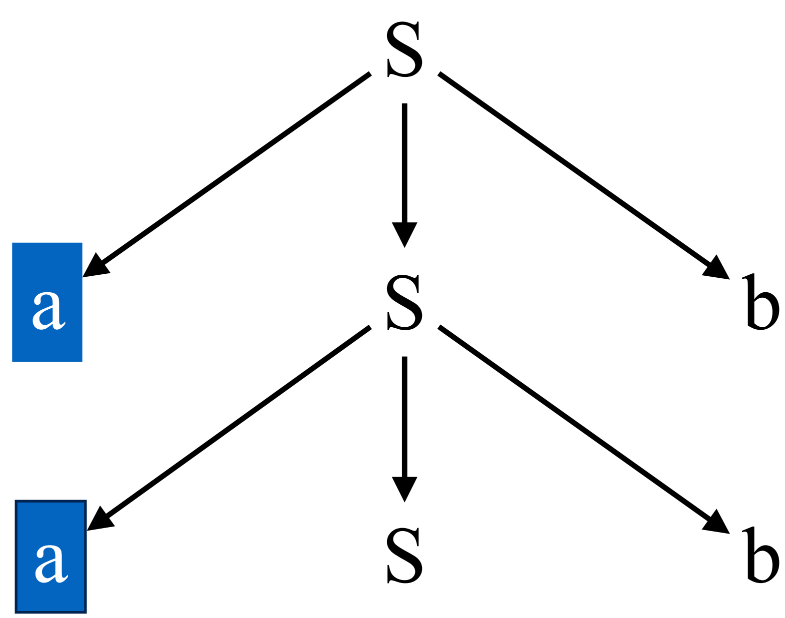
Sentential Form:
a a S b b

Applied Production:

LL(1) Parsing Example

S ::= a S b | ε





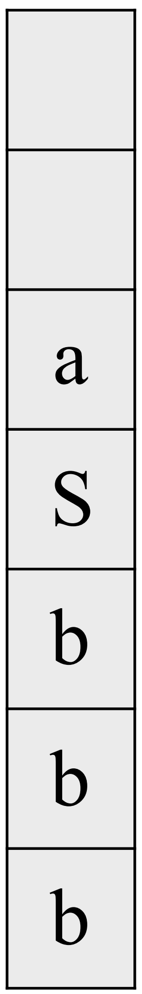
Remaining Input:
a b b b

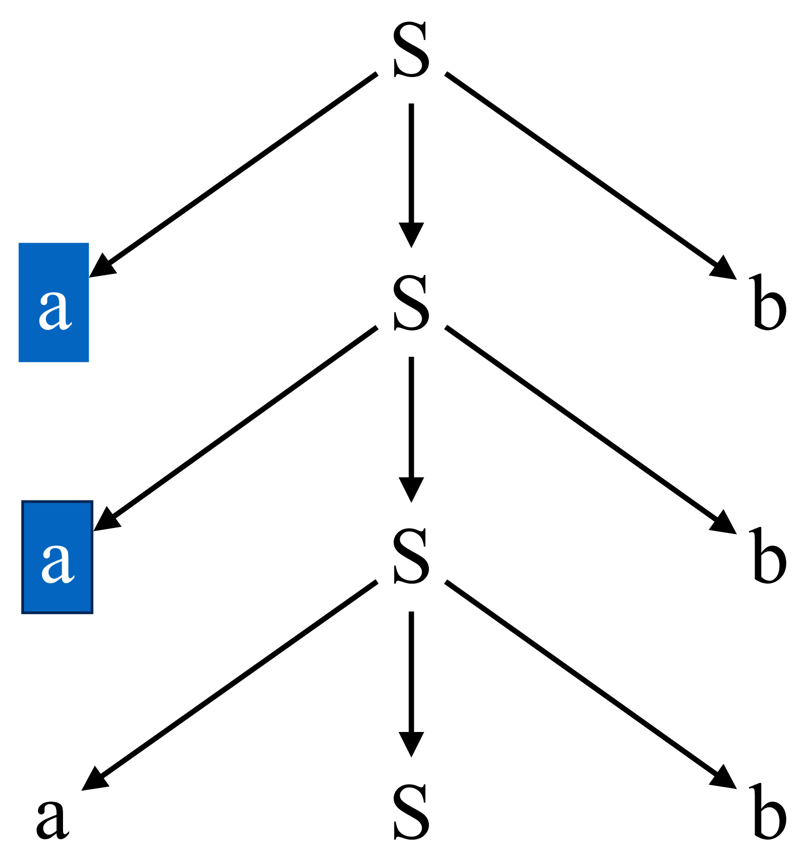
Sentential Form:
a a S b b

Applied Production:

LL(1) Parsing Example

S ::= a S b | ε





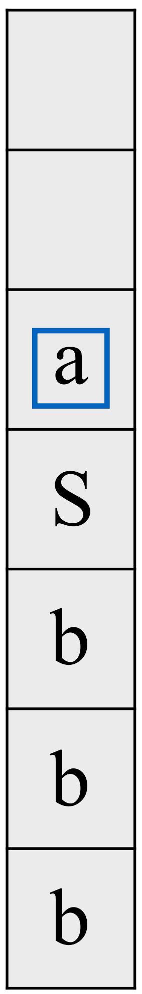
Remaining Input:
a b b b

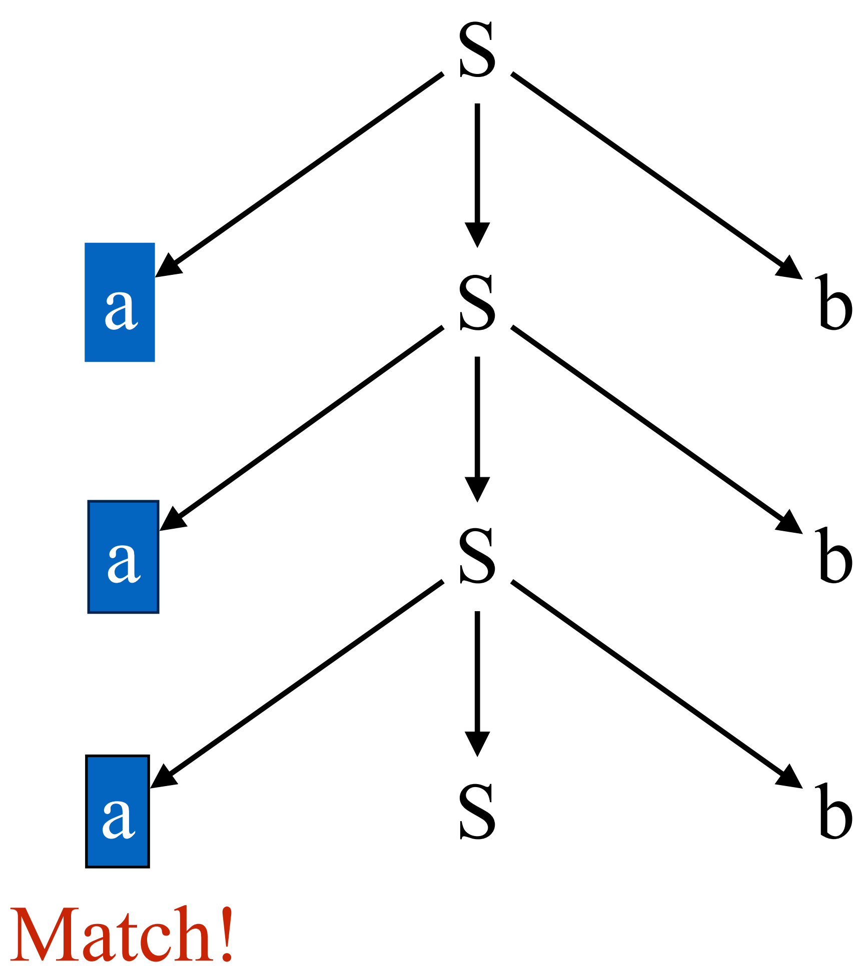
Sentential Form:
a a a S b b b

Applied Production:
S ::= a S b

LL(1) Parsing Example

S ::= a S b | ε





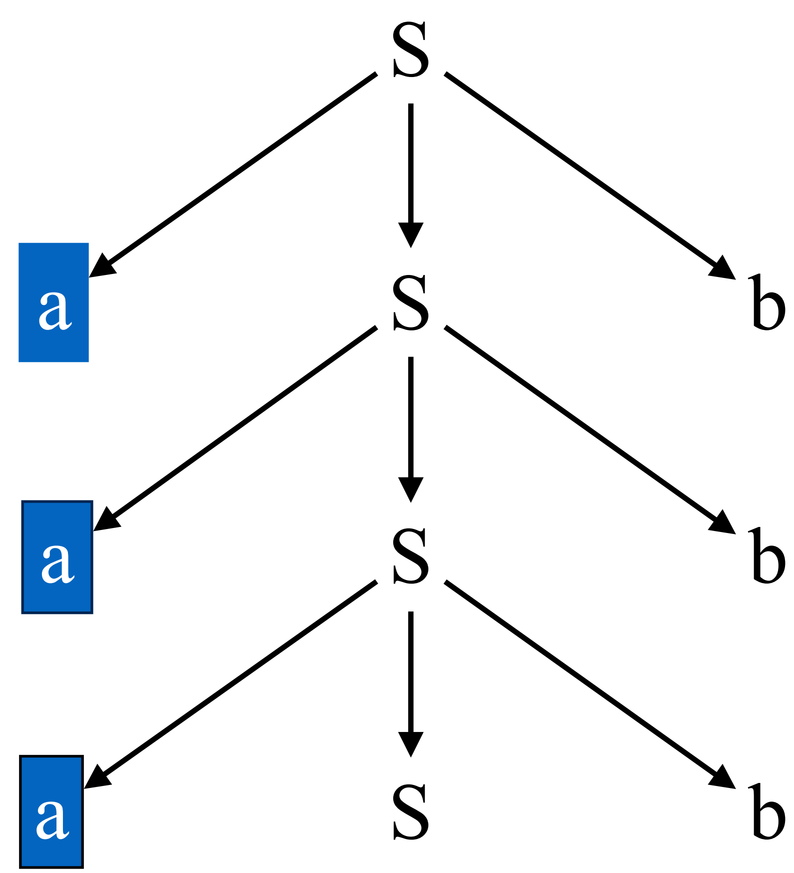
Remaining Input:
a b b b

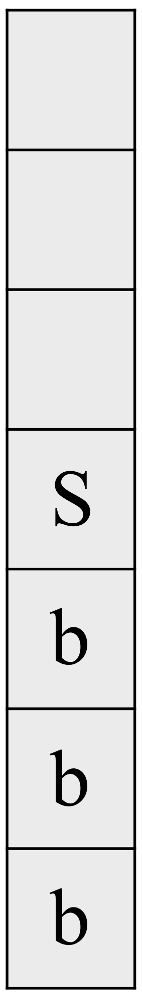
Sentential Form:
a a a S b b b

Applied Production:

LL(1) Parsing Example

S ::= a S b | ε





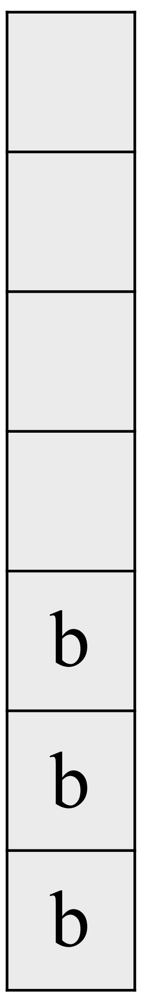
Remaining Input:
b b b

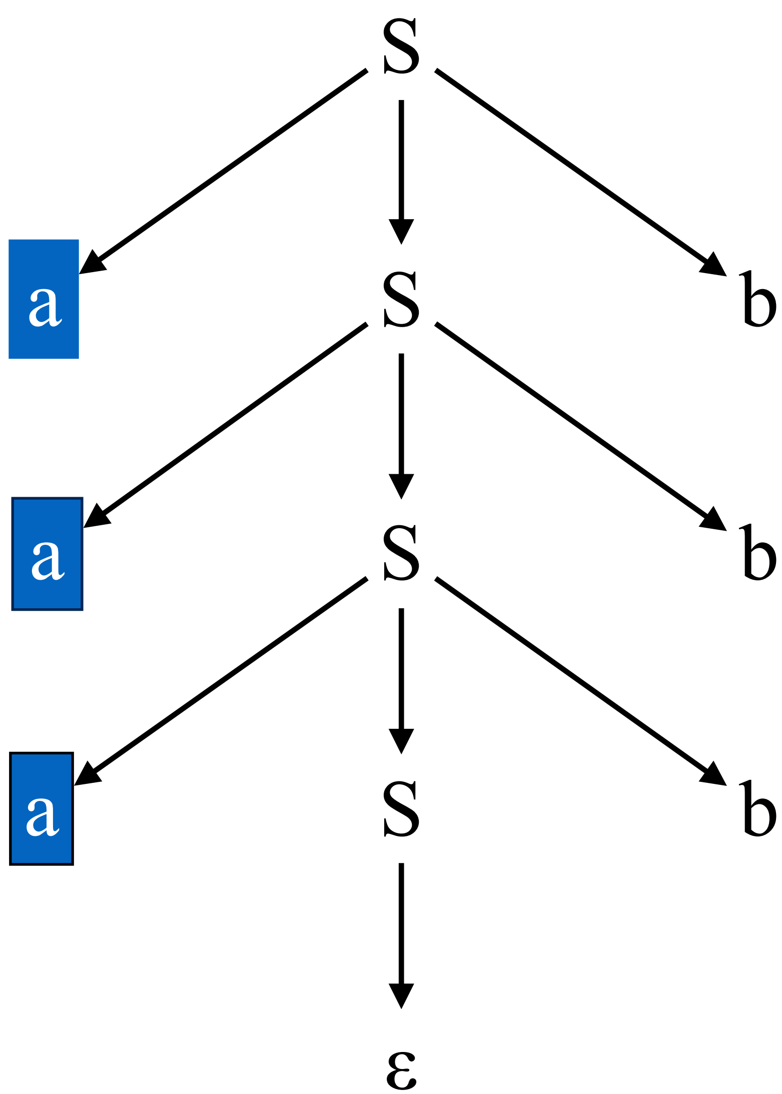
Sentential Form:
a a a S b b b

Applied Production:

LL(1) Parsing Example

S ::= a S b | ε





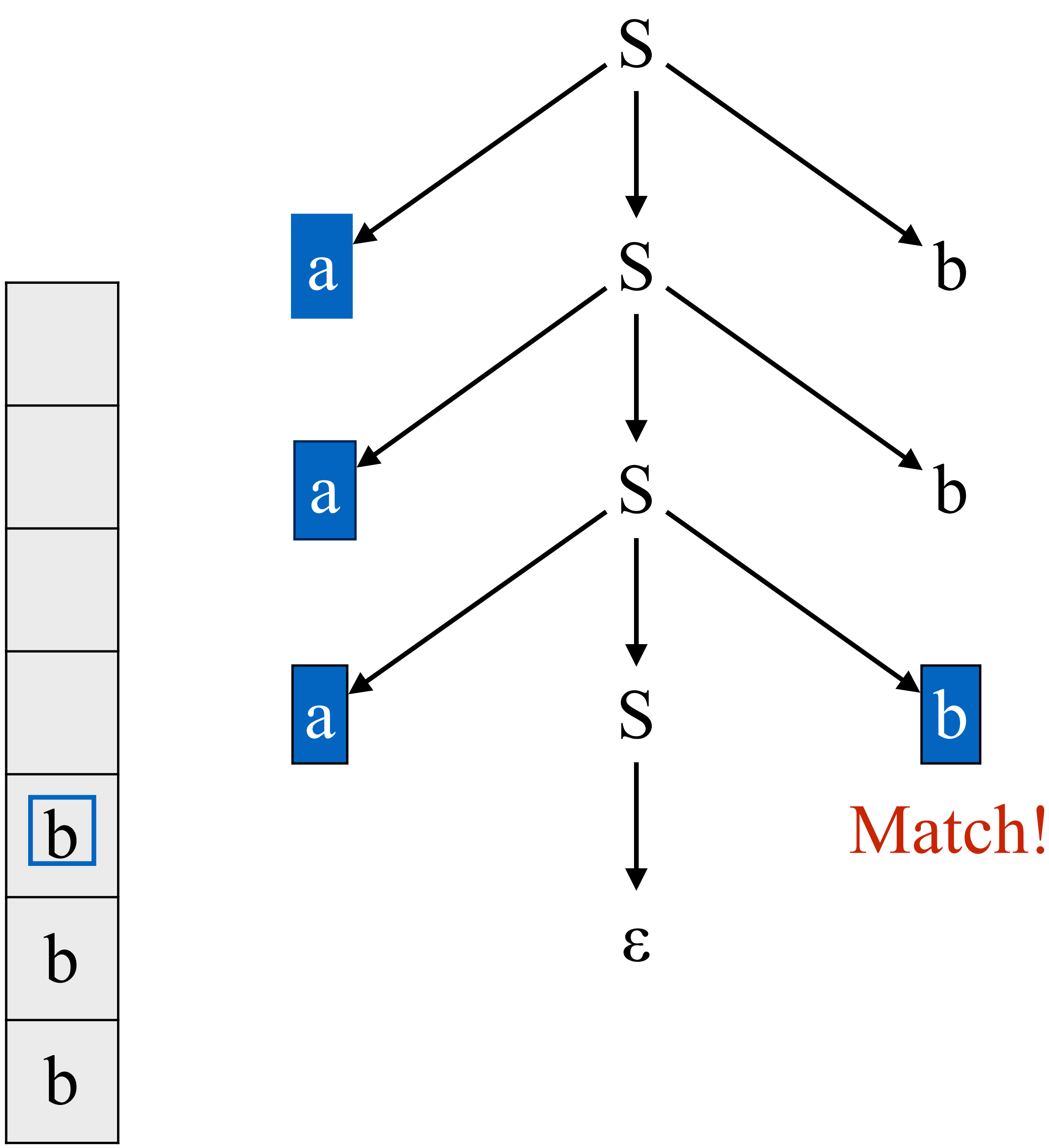
Remaining Input:
b b b

Sentential Form:
a a a b b b

Applied Production:
S ::= ε

LL(1) Parsing Example

S ::= a S b | ε



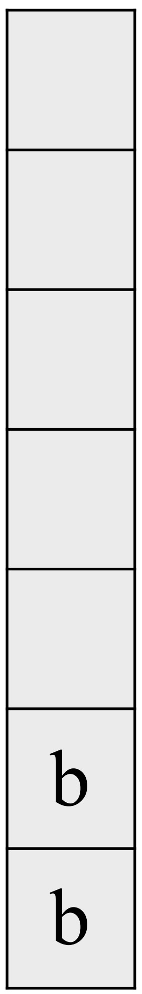
Remaining Input:
b b b

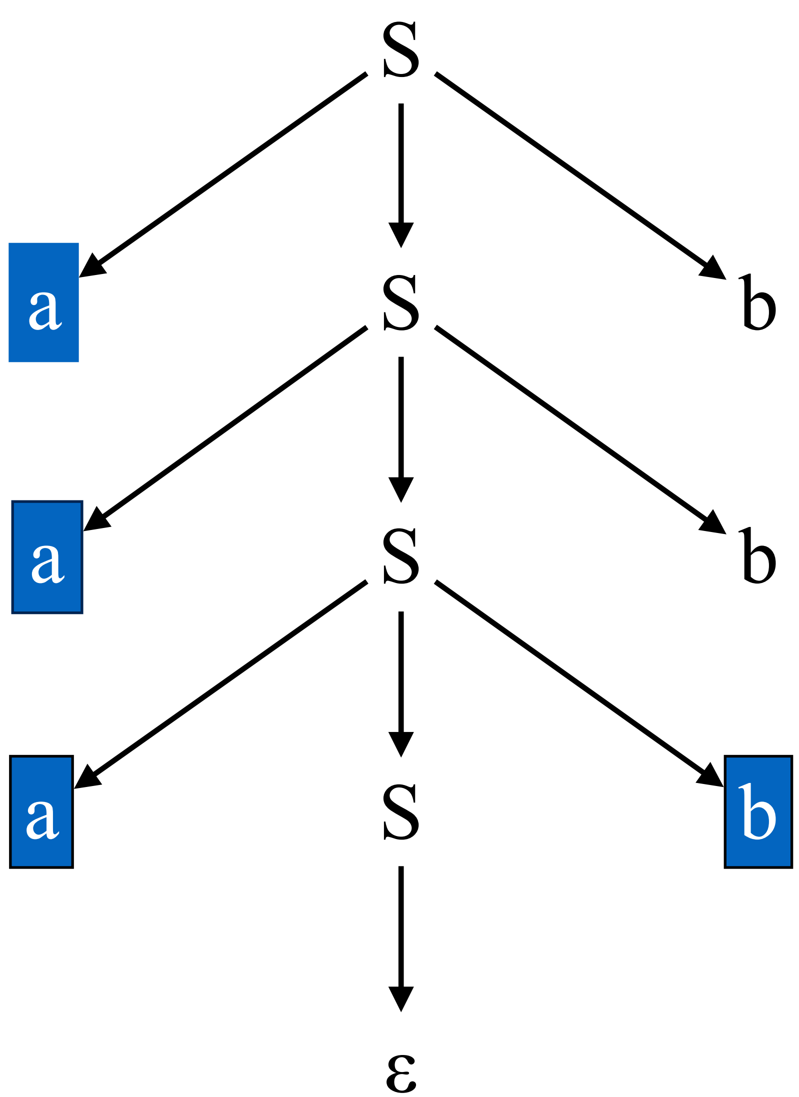
Sentential Form:
a a a b b b

Applied Production:

LL(1) Parsing Example

S ::= a S b | ε





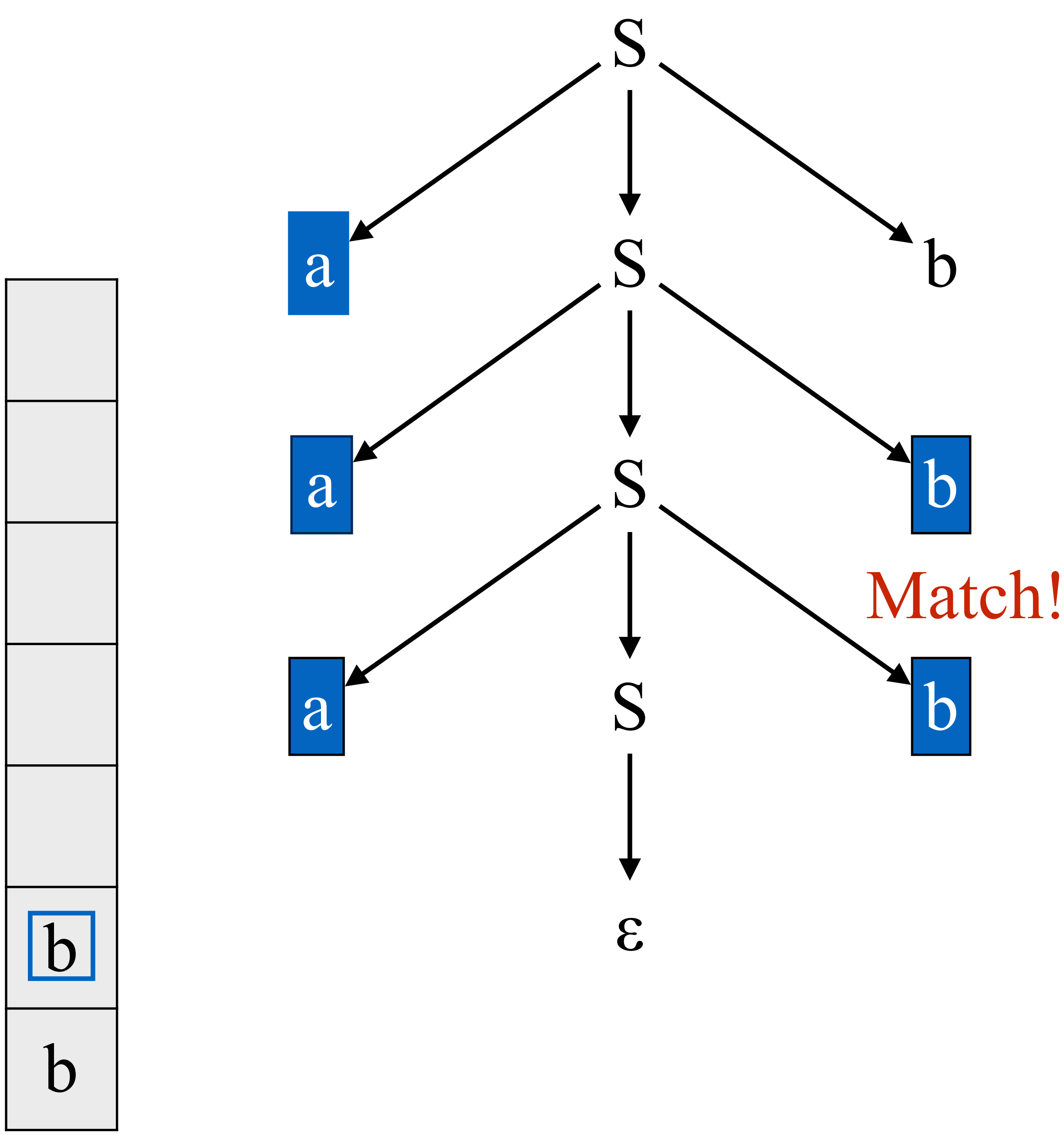
Remaining Input:
b b

Sentential Form:
a a a b b b

Applied Production:

LL(1) Parsing Example

S ::= a S b | ε



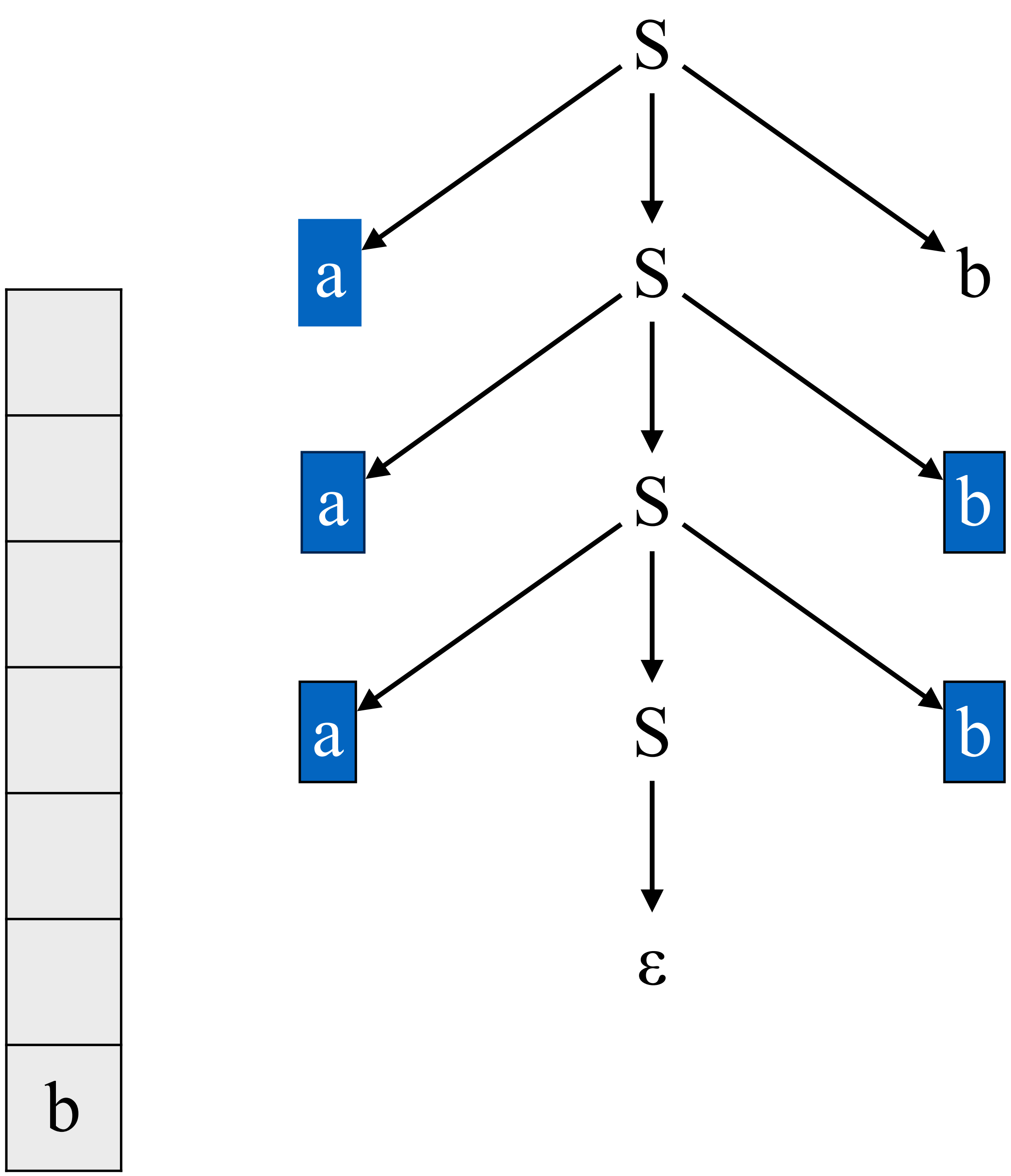
Remaining Input:
b b

Sentential Form:
a a a b b b

Applied Production:

LL(1) Parsing Example

S ::= a S b | ε



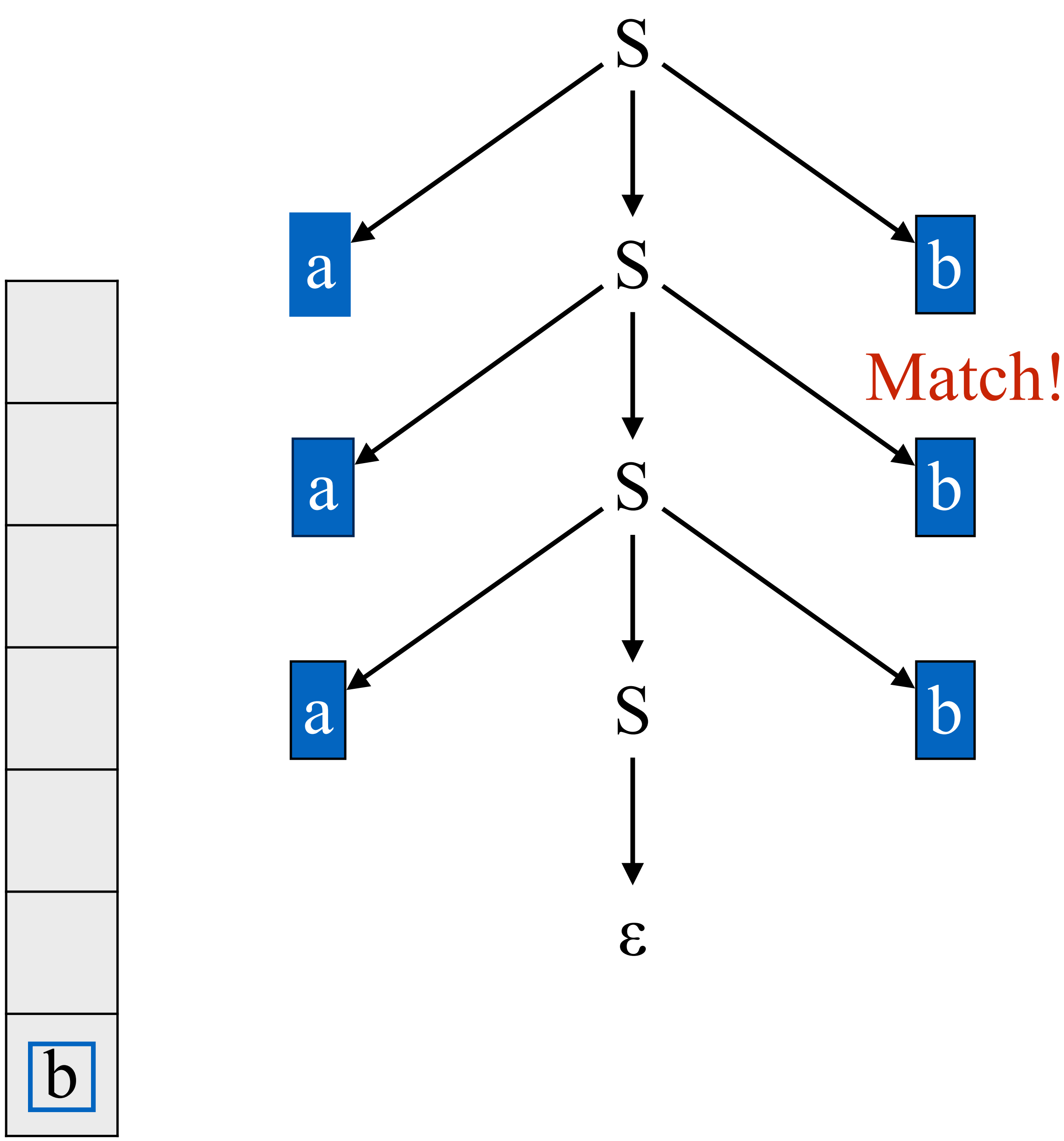
Remaining Input:
b

Sentential Form:
a a a b b b

Applied Production:

LL(1) Parsing Example

S ::= a S b | ε



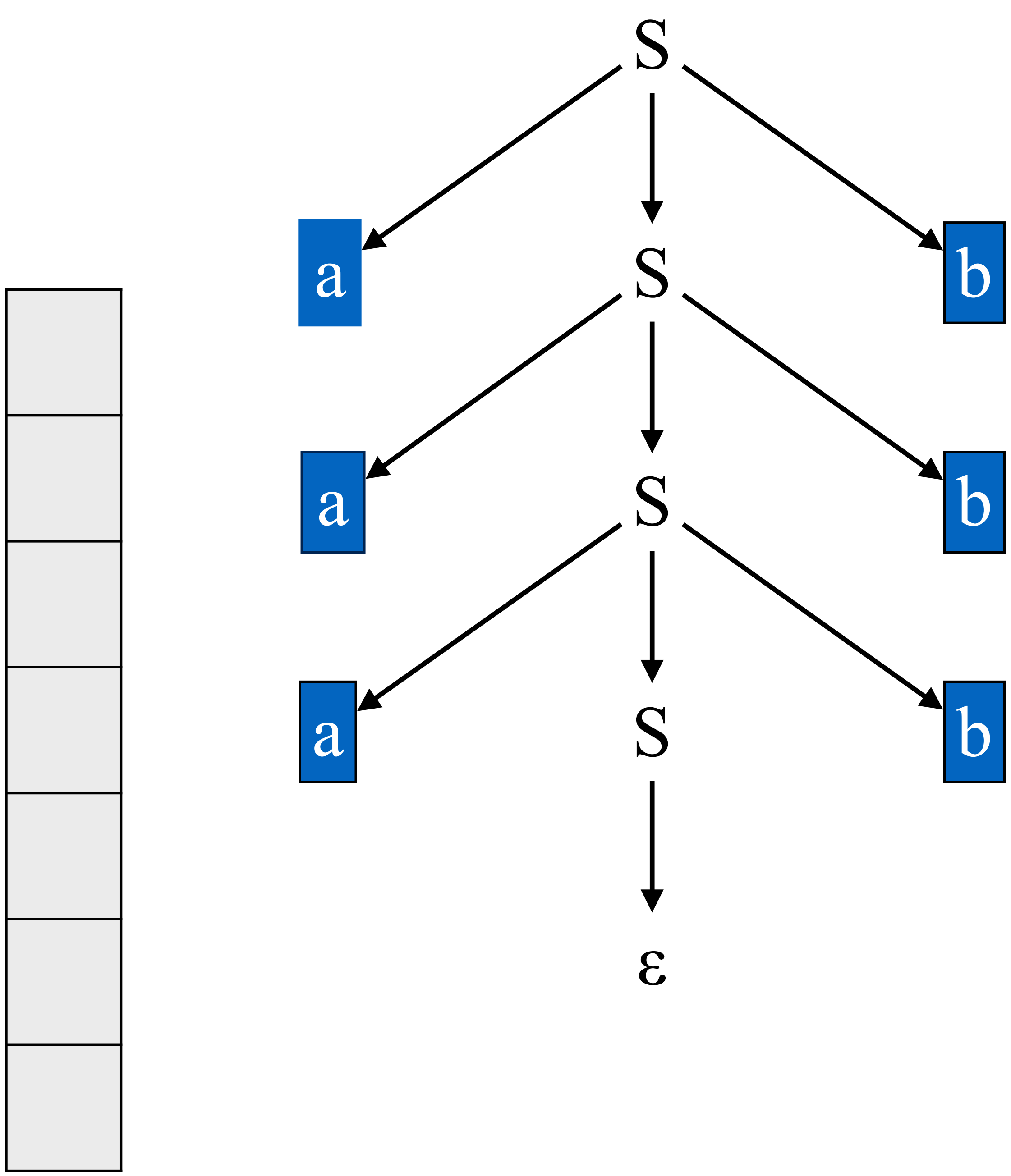
Remaining Input:
b

Sentential Form:
a a a b b b

Applied Production:

LL(1) Parsing Example

S ::= a S b | ε



Remaining Input:

Sentential Form:
a a a b b b

Applied Production:

Next Lecture

Next Time:

- Read Scott, Chapter 2.3.1 - 2.3.2 (and the materials on companion site)