

# CS 314 Principles of Programming Languages

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## Lecture 3: Syntax Analysis (Scanning)

Prof. Zheng Zhang



*Rutgers University*

September 12, 2018

# Class Information

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## Homework 1

- Due 9/18 11:55pm EST.
- Only accepted in **pdf** format.
- No late submission will be accepted.

## TA office hours announced

- See Sakai course page

# Review: Formalisms for Lexical and Syntactic Analysis

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Two issues in *Formal Languages*:

- Language Specification → formalism to describe what a valid program (word/sentence) looks like.
- Language Recognition → formalism to describe a machine and an algorithm that can verify that a program is valid or not.

We use **regular expression** to specify tokens (words)

# A Formal Definition

## *Regular Expressions (RE) over an Alphabet $\Sigma$*

If  $\underline{x} = a \in \Sigma$ , then  $\underline{x}$  is an **RE** denoting the set  $\{ a \}$   
or, the language  $L = \{ a \}$

Assuming  $\underline{x}$  and  $\underline{y}$  are both **REs** then

1.  $\underline{xy}$  is an **RE** denoting  $L(\underline{x})L(\underline{y}) = \{ pq \mid p \in L(\underline{x}) \text{ and } q \in L(\underline{y}) \}$

2.  $\underline{x} \mid \underline{y}$  is an **RE** denoting  $L(\underline{x}) \cup L(\underline{y})$

3.  $\underline{x}^*$  is an **RE** denoting

$$L(\underline{x})^* = \bigcup_{0 \leq k < \infty} L(\underline{x})^k \quad (\text{Kleene Closure})$$

*Set of all strings that are zero or more concatenations of  $\underline{x}$*

4.  $\underline{x}^+$  is an **RE** denoting

$$L(\underline{x})^+ = \bigcup_{1 \leq k < \infty} L(\underline{x})^k \quad (\text{Positive Closure})$$

*Set of all strings that are one or more concatenations of  $\underline{x}$*

$\varepsilon$ is an <b>RE</b> denoting the empty set
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# Review: Regular Expressions

A syntax (notation) to specify regular languages.

***RE***  $p$

**Language**  $L(p)$

$x$  |  $y$

$L(\underline{x}) \cup L(\underline{y})$

$xy$

$\{RS \mid R \in L(\underline{x}), S \in L(\underline{y})\}$

$x$ <sup>+</sup>

$L(\underline{x}) \cup L(\underline{xx}) \cup L(\underline{xxx}) \cup \dots$

$x$ <sup>\*</sup> ( $\underline{x}^* = \underline{x}^+ \mid \epsilon$ )

$\{\epsilon\} \cup L(\underline{x}) \cup L(\underline{xx}) \cup \dots$

*The symbols underlined denotes a regular expression, i.e.,  $x$*

**$(s)$**

$L(s)$

**$a$**

**$\{a\}$**

**$\epsilon$**

**$\{\epsilon\}$**

*The symbols in bold-face denotes a letter from the alphabet, i.e.,  **$a$***

# Review: Formalisms for Lexical and Syntactic Analysis

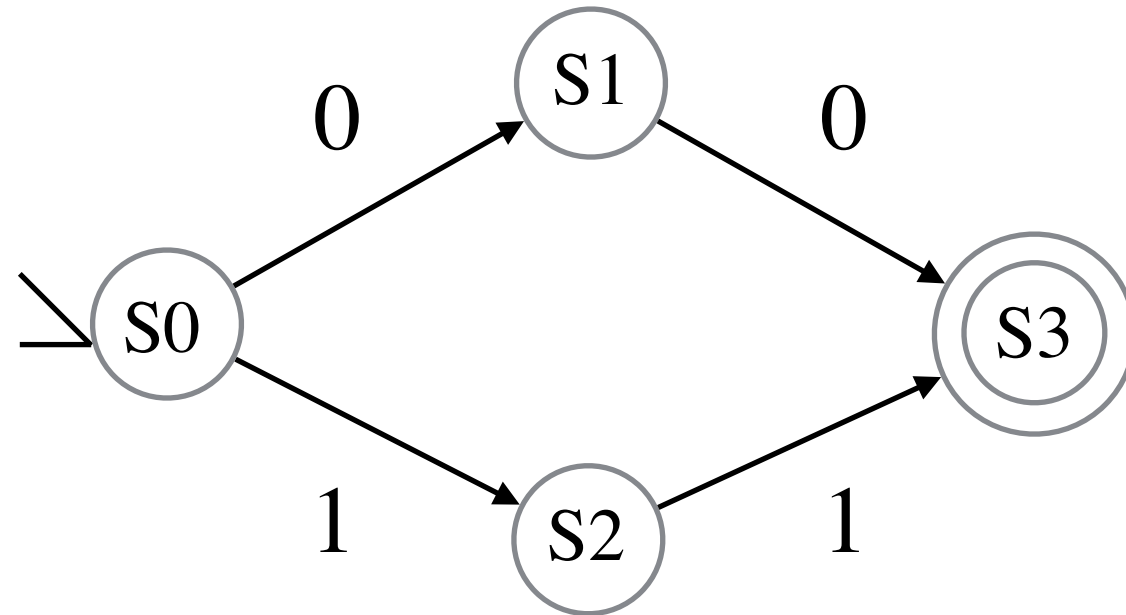
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Two issues in *Formal Languages*:

- Language Specification → formalism to describe what a valid program (word/sentence) looks like.
- Language Recognition → formalism to describe a machine and an algorithm that can verify that a program is valid or not.

**We use finite state automata to recognize regular language**

# Finite State Automata



A Finite-State Automaton is a quadruple:  $\langle S, s, F, T \rangle$

- $S$  is a set of states, e.g.,  $\{S0, S1, S2, S3\}$
- $s$  is the start state, e.g.,  $S0$
- $F$  is a set of final states, e.g.,  $\{S3\}$
- $T$  is a set of labeled transitions, of the form  $(\text{state}, \text{input}) \rightarrow \text{state}$  [i.e.,  $S \times \Sigma \rightarrow S$ ]

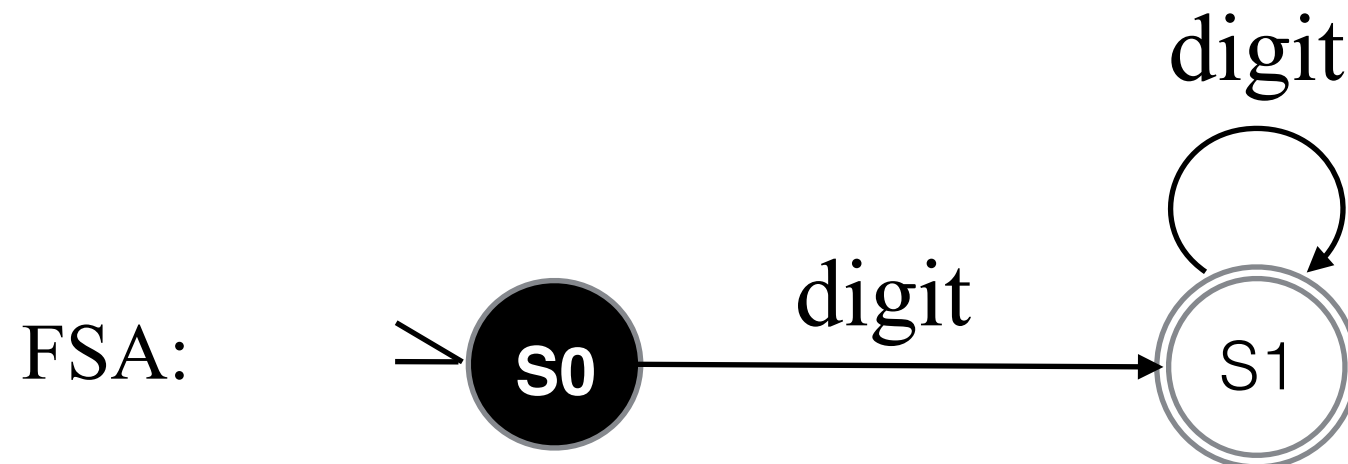
# Recognizers for Regular Expressions

Let *letter* stand for  $A \mid B \mid C \mid \dots \mid Z$

Let *digit* stand for  $0 \mid 1 \mid 2 \mid \dots \mid 9$

**Integer Constant**

Regular Expression:  $\text{digit}^+$





# From RE to Scanner

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**Classic approach is a three-step method:**

1. Build automata for each piece of the **RE** using a **template**.  
Multiple automata can be pasted using  $\epsilon$ -transition.  
This construction is called “**Thompson’s construction**”
2. Convert the newly built automaton into a deterministic automaton.  
This construction is called the “**subset construction**”
3. Given the deterministic automaton, minimize the number of states.  
Minimization is a **space optimization**.

# Non-deterministic Finite Automaton (NFA)

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- NFA might have transitions on  $\epsilon$
- Non-deterministic choice: multiple transition from the same state on the same symbol

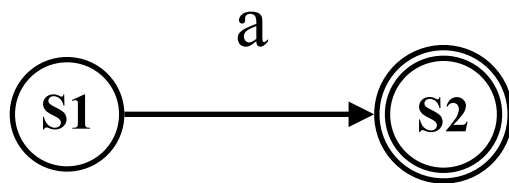
Deterministic finite automaton (DFA) has no  $\epsilon$ -transitions and all choices are single-valued.

# Thompson's Construction

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- From each RE symbol and operator, we have a template
- Build them, in precedence order, and join them with  $\epsilon$ -transition

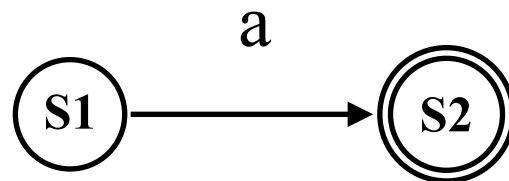
NFA for **a**



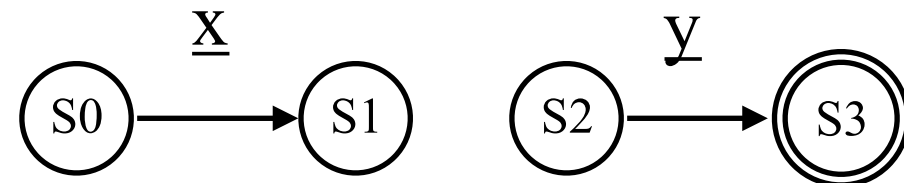
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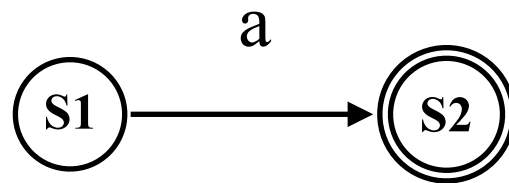
NFA for xy



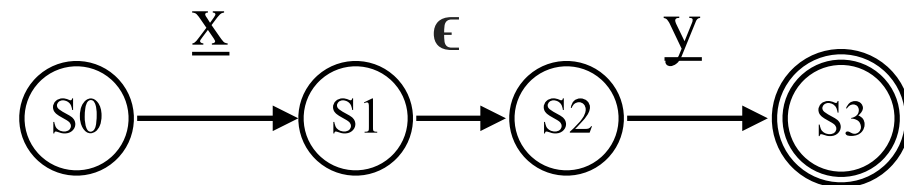
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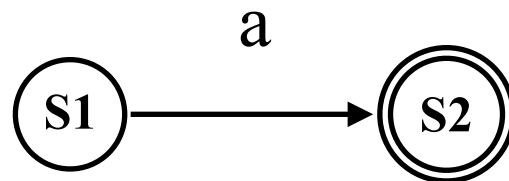
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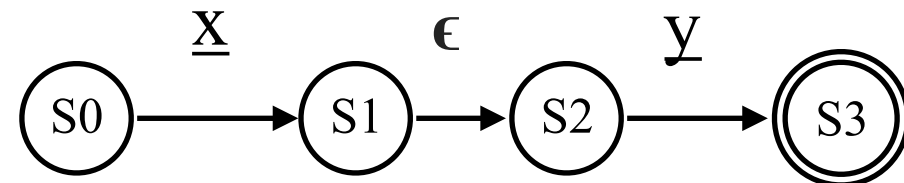
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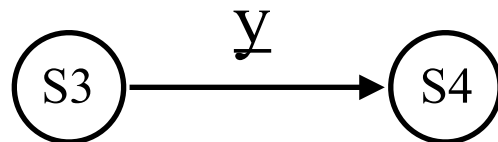
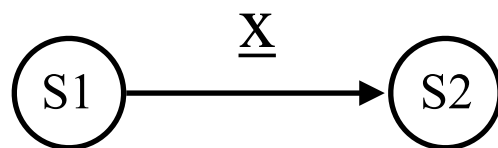
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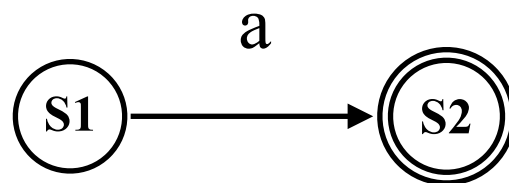
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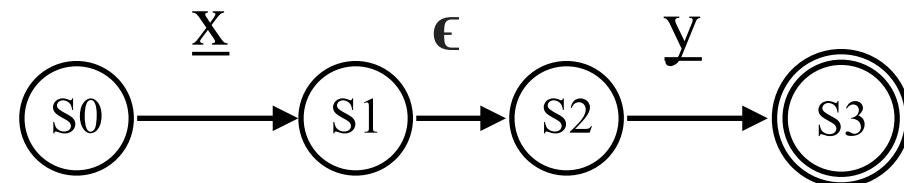
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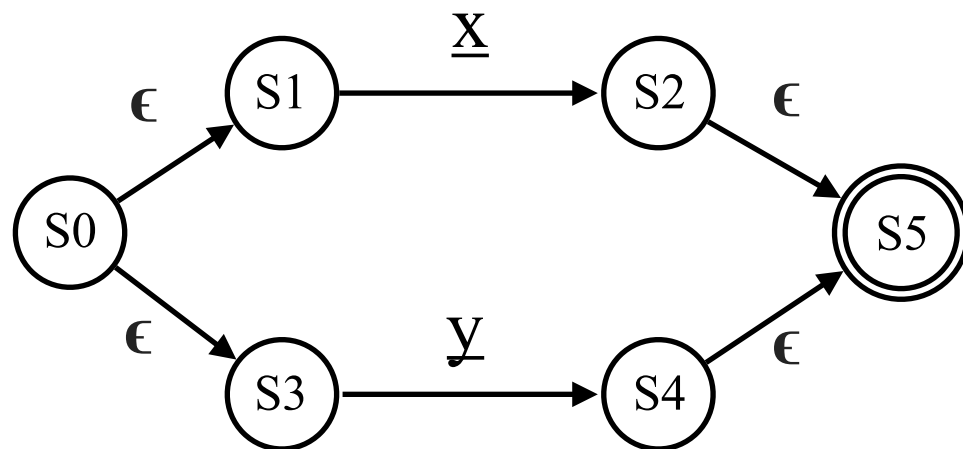
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NFA for **x****y**



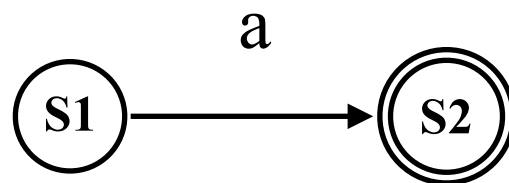
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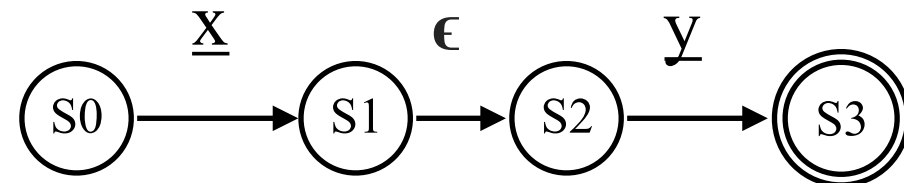
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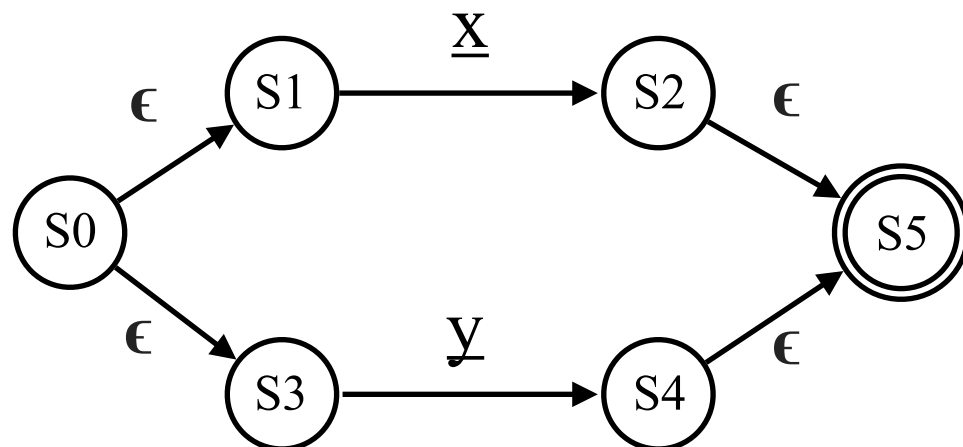
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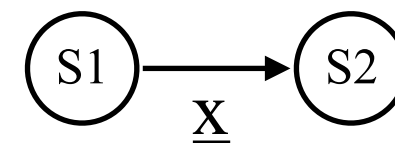
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NFA for x|y



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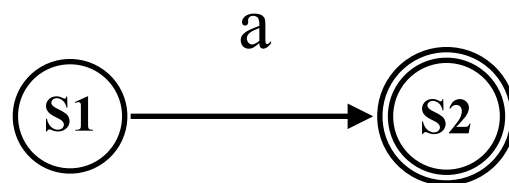




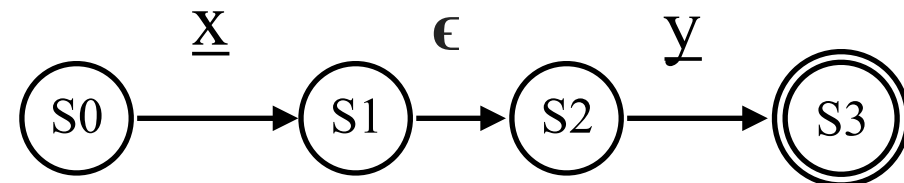
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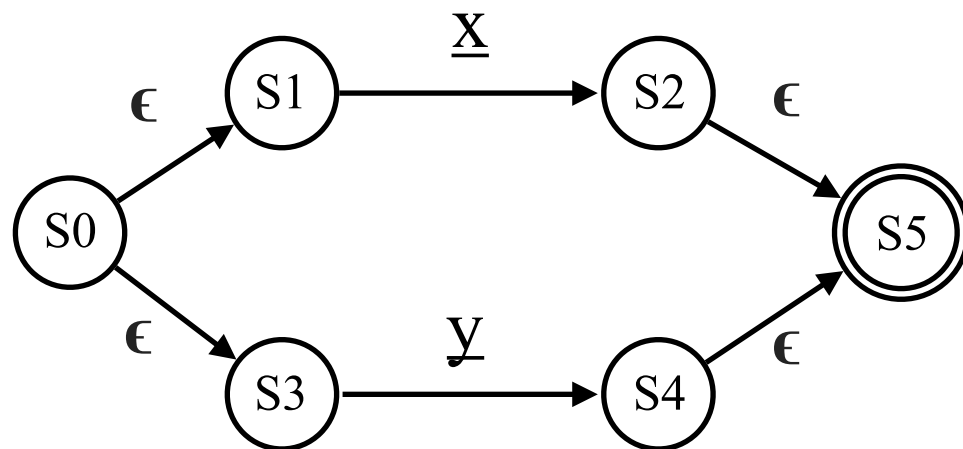
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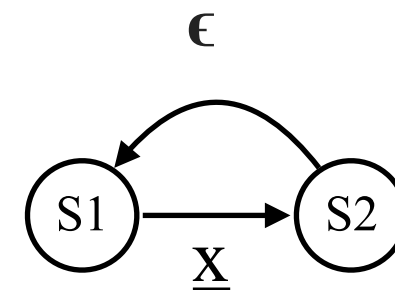
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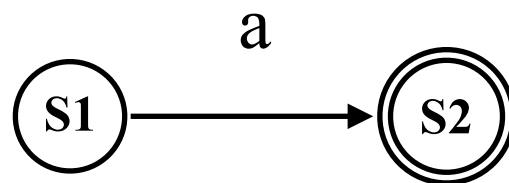
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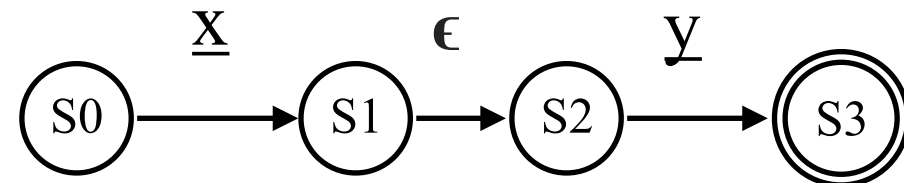
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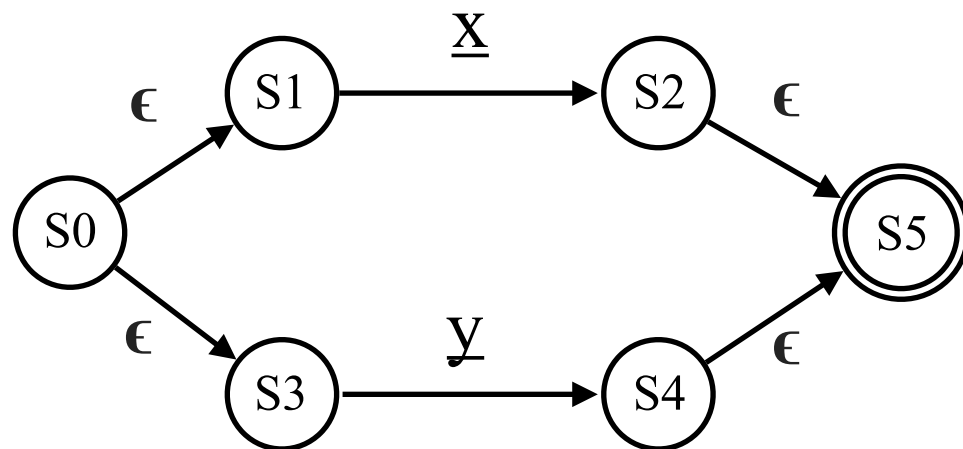
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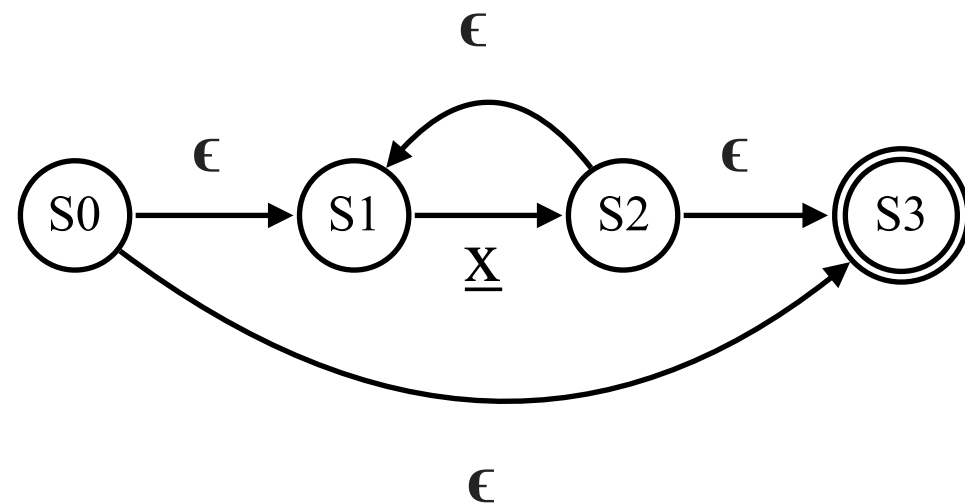
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# Thompson's Construction

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- Let's build an NFA for  $a(b|c)^*$

**1.  $a, b, \& c$**

**2.  $b | c$**

**3.  $(b | c)^*$**

**4.  $a (b | c)^*$**

# Thompson's Construction

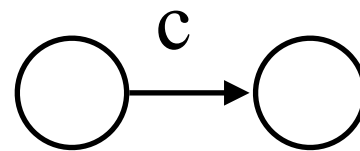
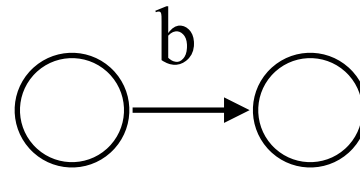
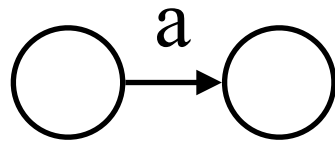
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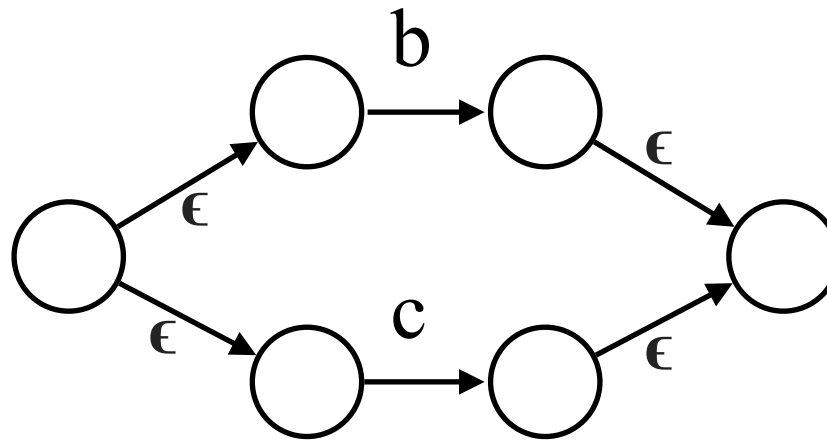
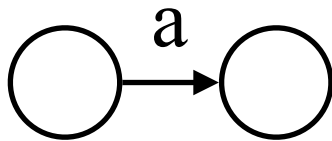
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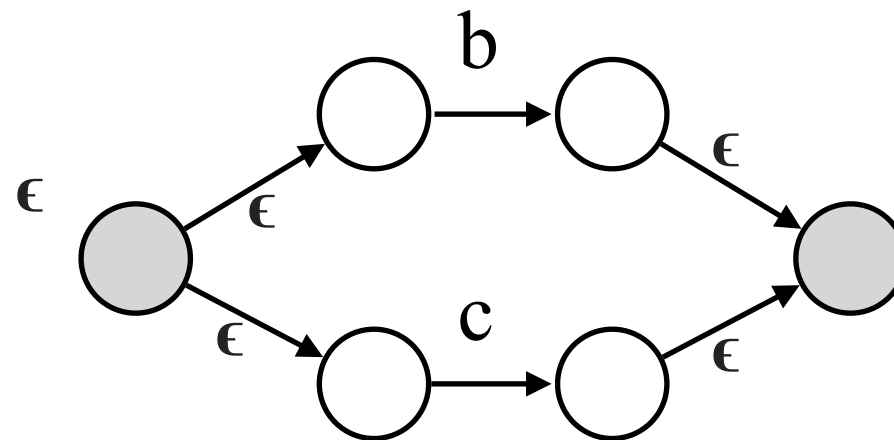
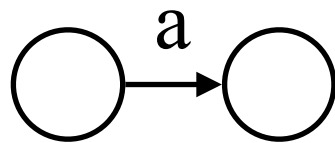
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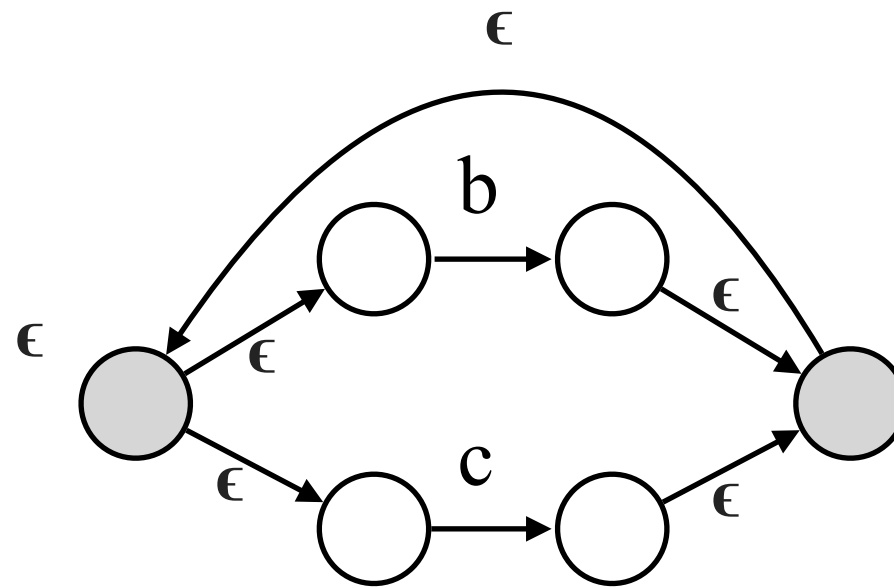
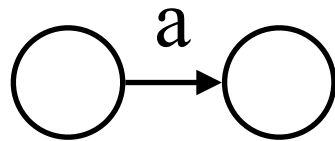
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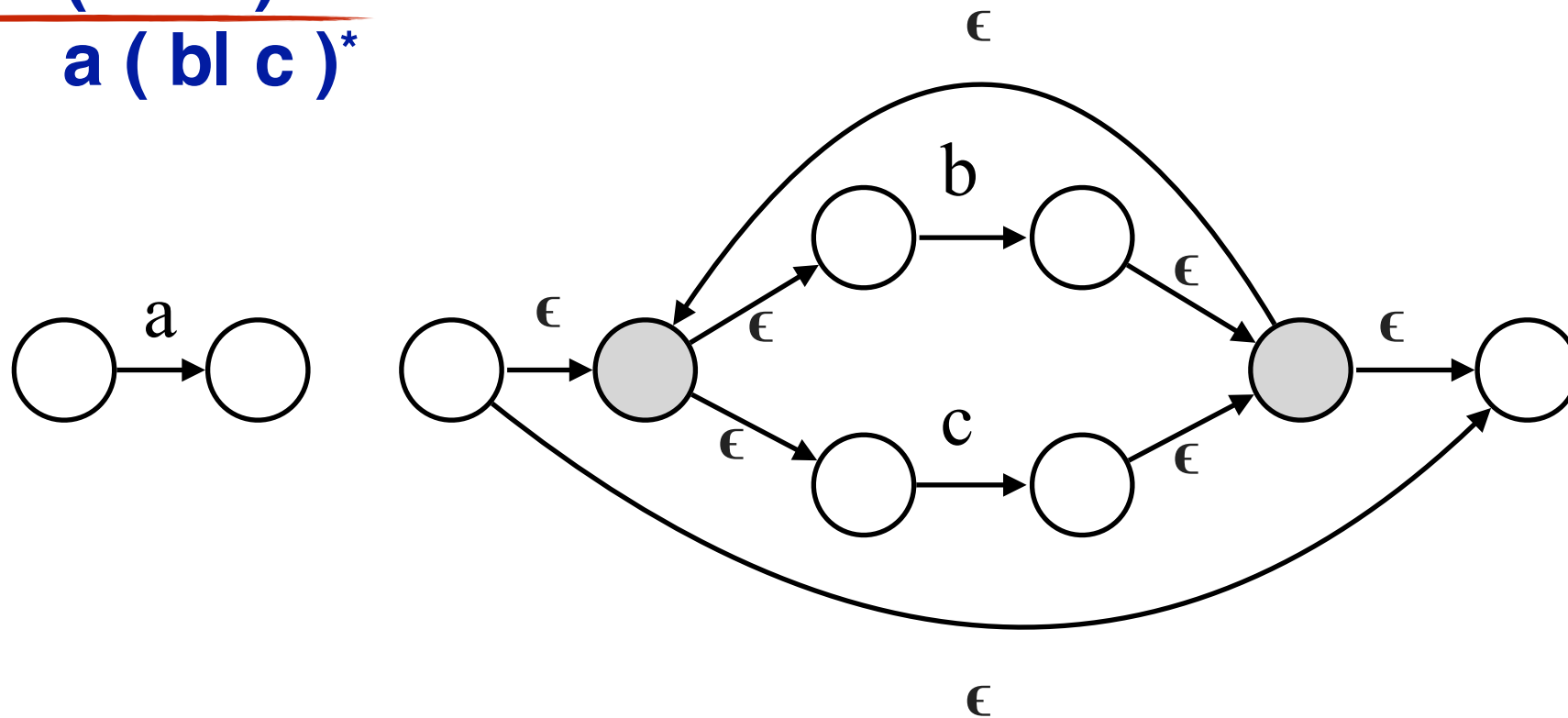
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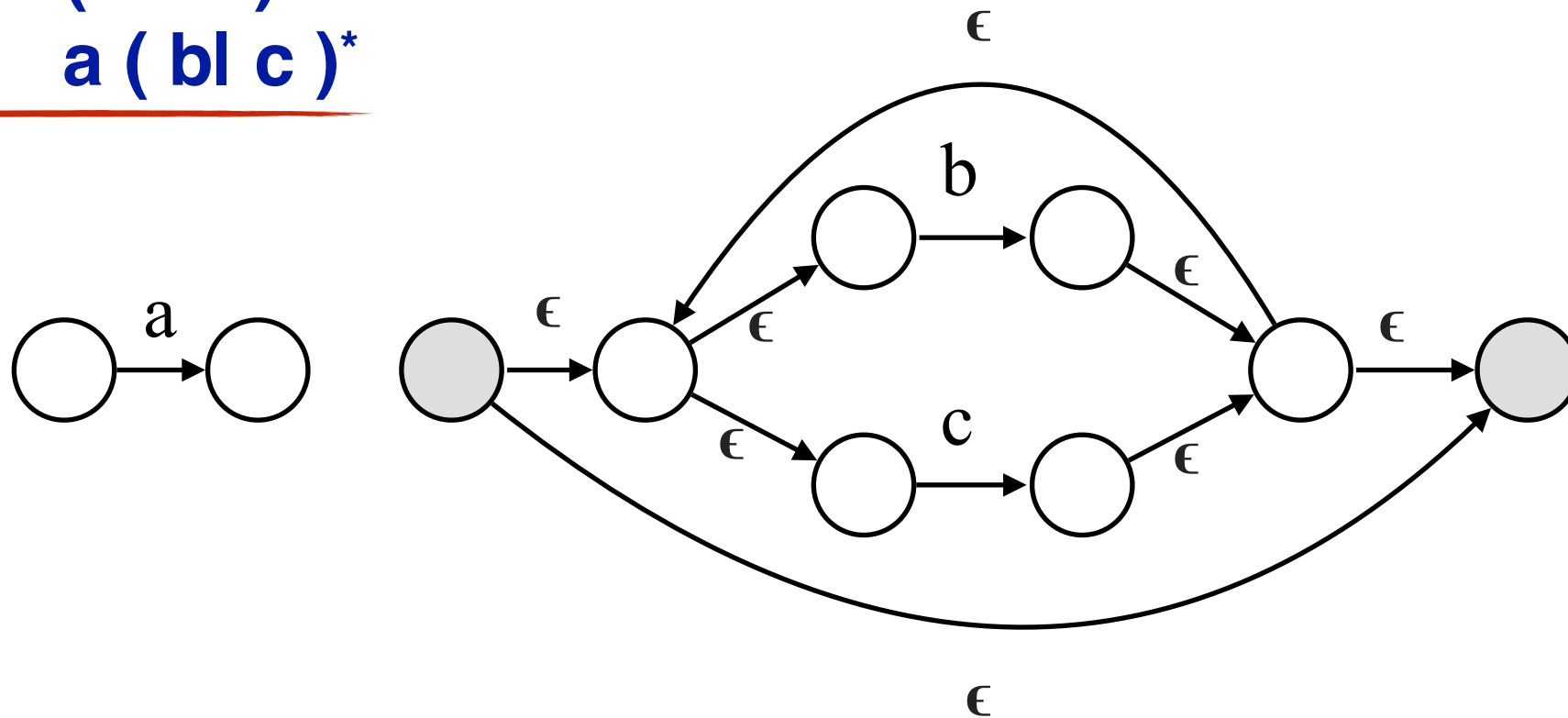
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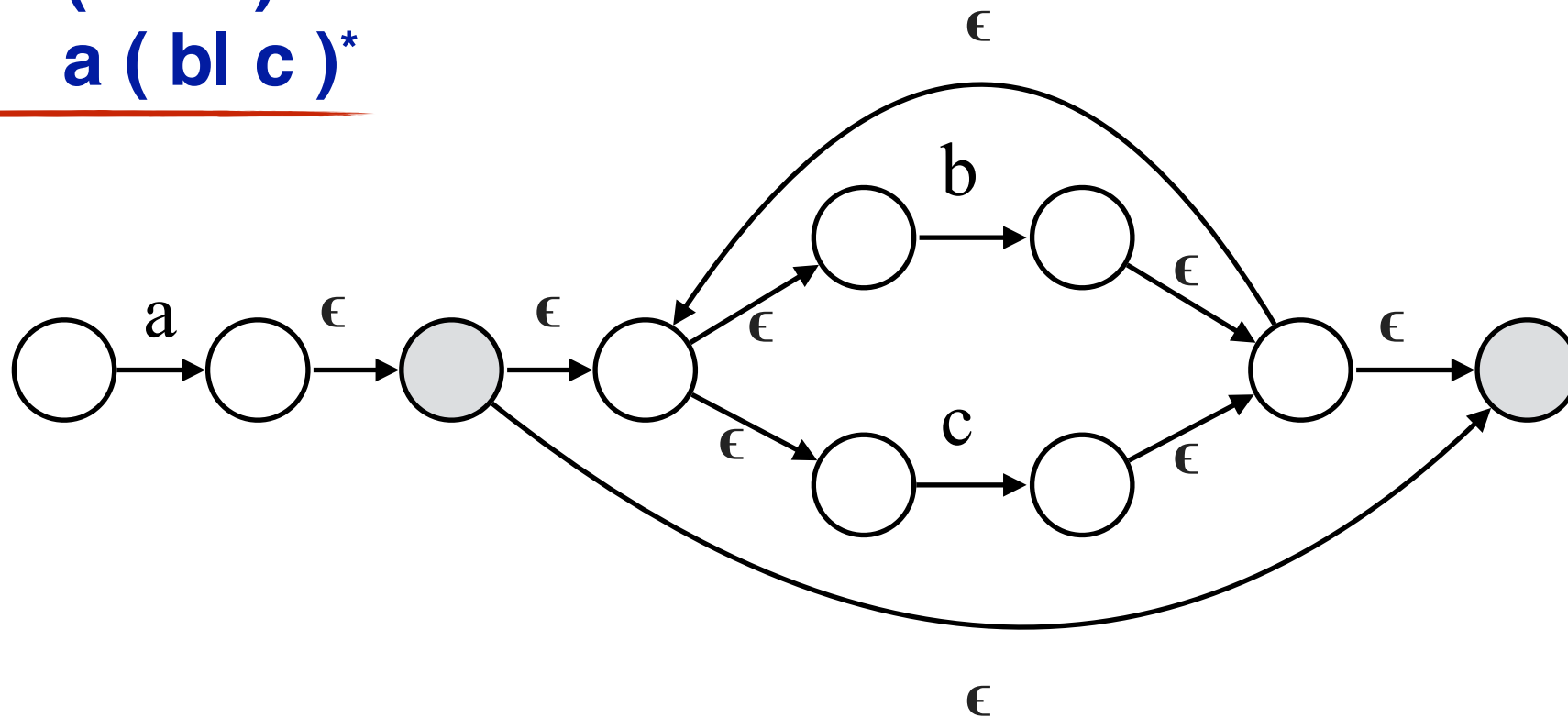
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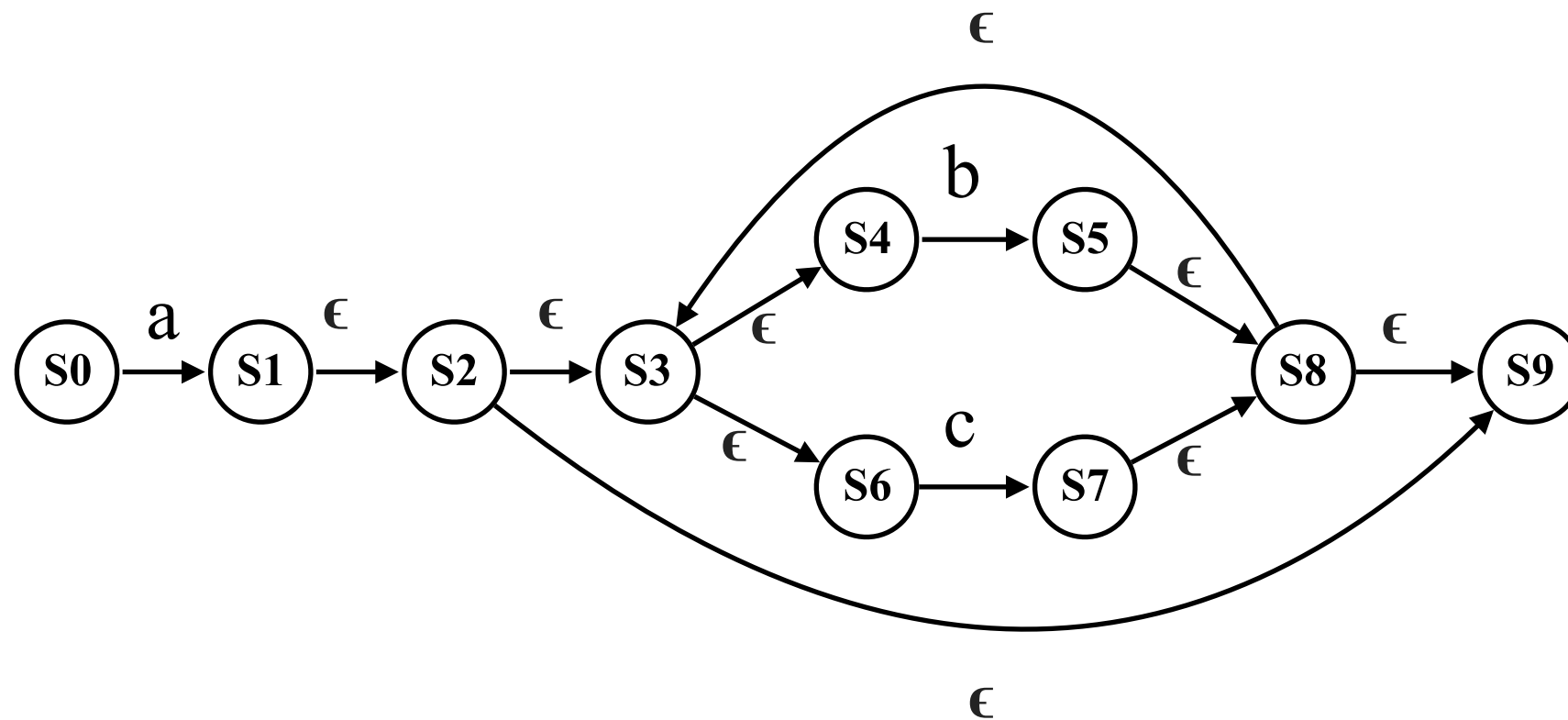
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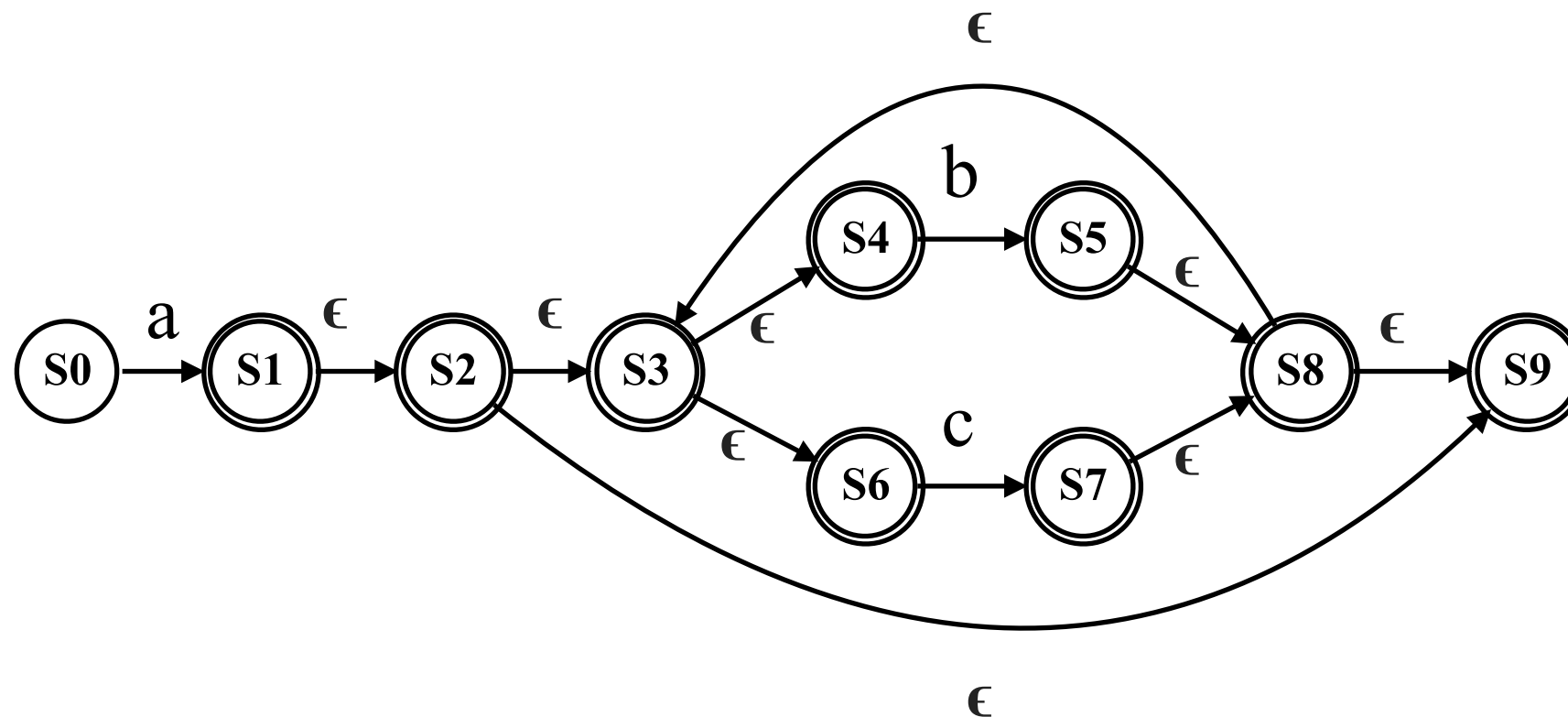
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# Thompson's Construction

- Let's build an NFA for  $a(b|c)^*$



Final states are double circled

# From RE to Scanner

---

**Classic approach is a three-step method:**

1. Build automata for each piece of the **RE** using a **template**.

Multiple automata can be pasted using  $\epsilon$ -transition.

This construction is called “**Thompson’s construction**”

2. Convert the newly built automaton into a deterministic automaton.

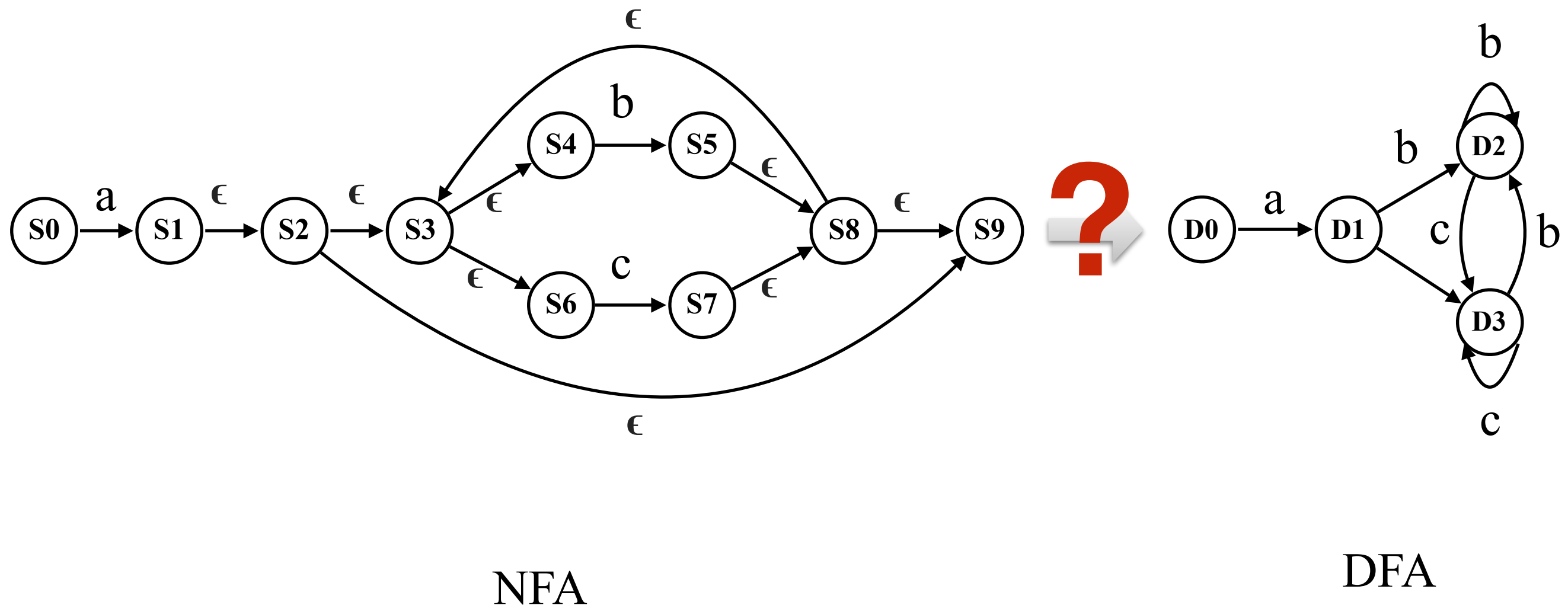
This construction is called the “**subset construction**”

3. Given the deterministic automaton, minimize the number of states.

Minimization is a **space optimization**.

# Subset Construction

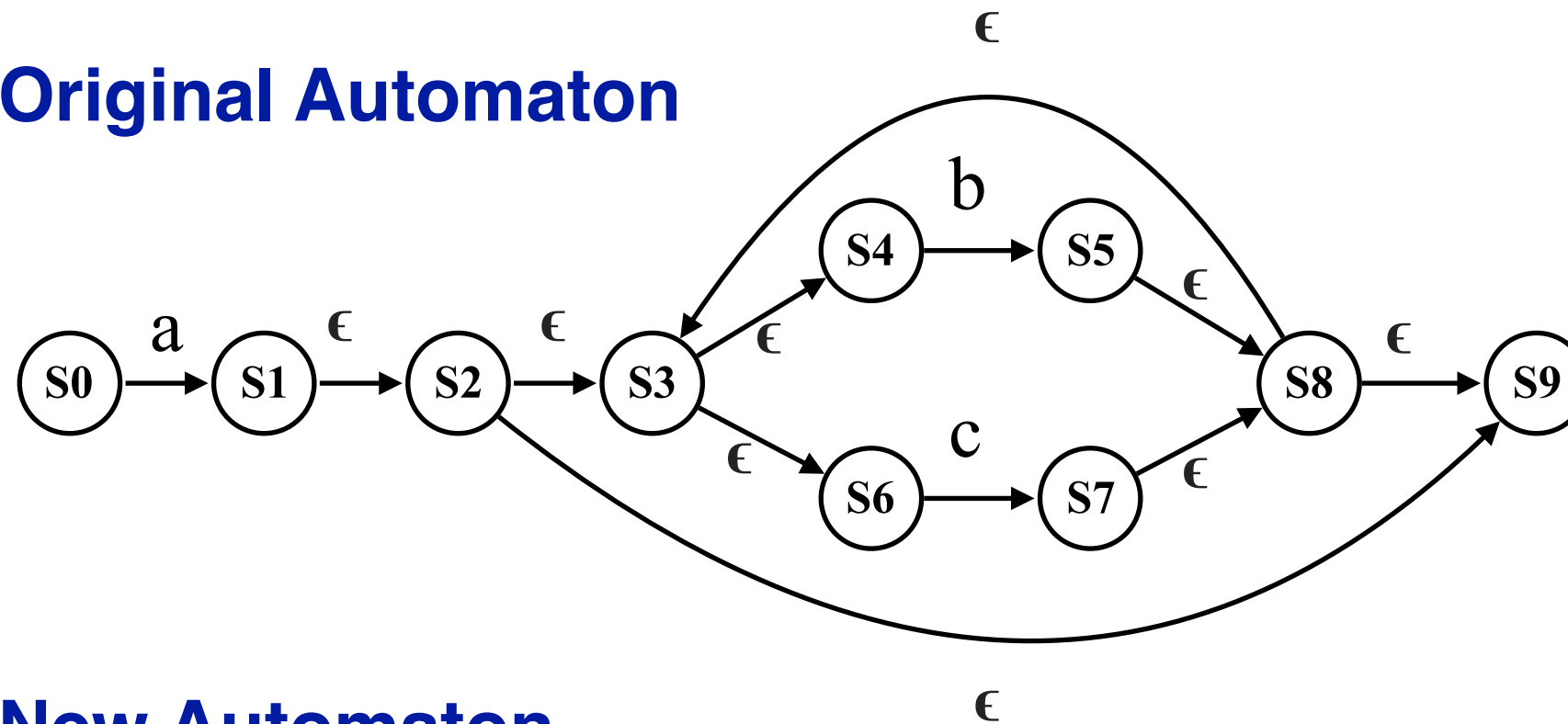
- Build a deterministic automaton that simulates the non-deterministic one
- Each state in the new one represents a set of states in the original one



# Subset Construction

- Each state in the new one represents a set of states in the original one

## Original Automaton



## New Automaton

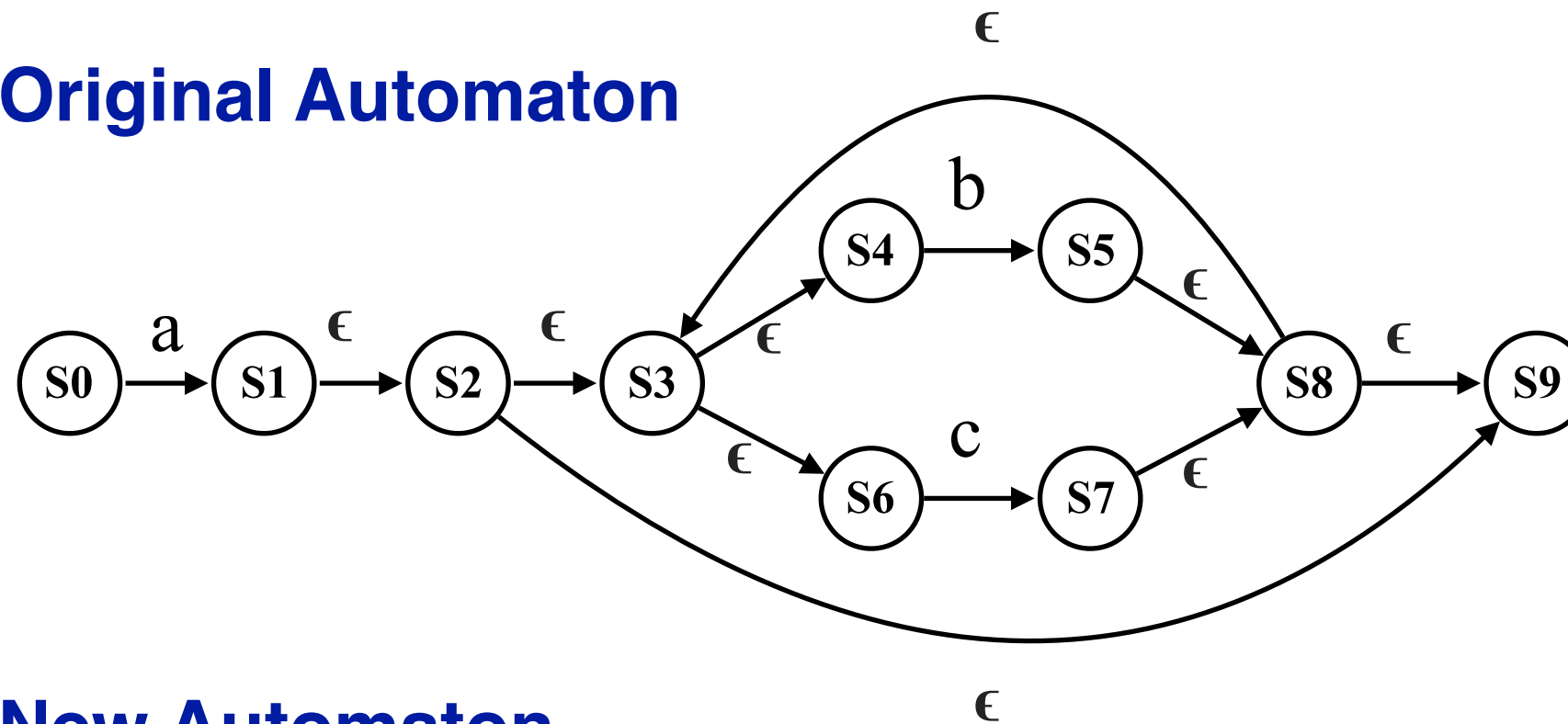


DFA	NFA
<i>D0</i>	<i>S0</i>

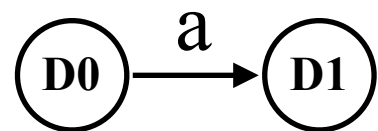
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## Original Automaton



## New Automaton



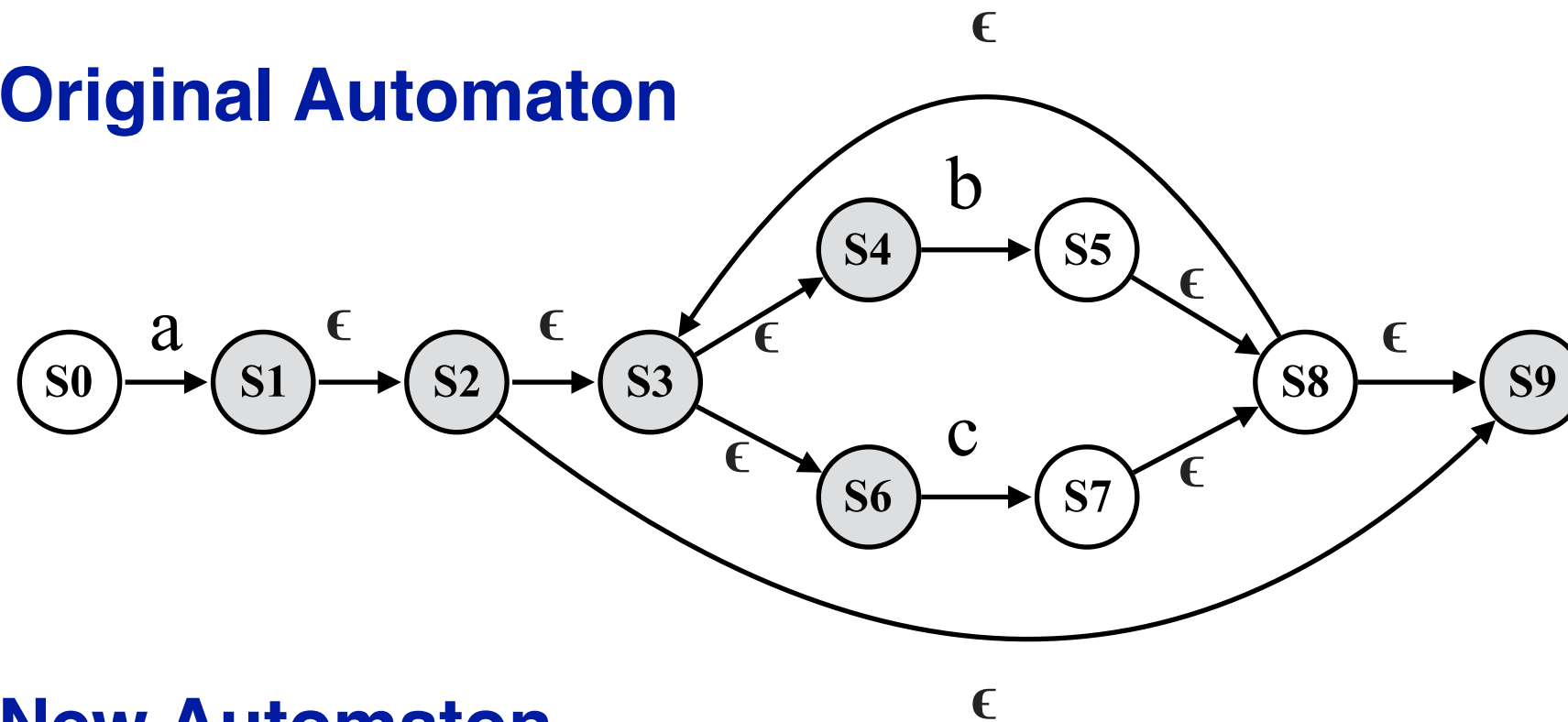
DFA	NFA
$D0$	$S0$
$D1$	$S1, S2, S3, S9, S4, S6$



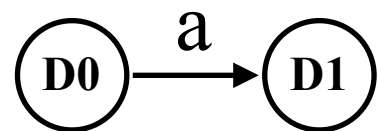
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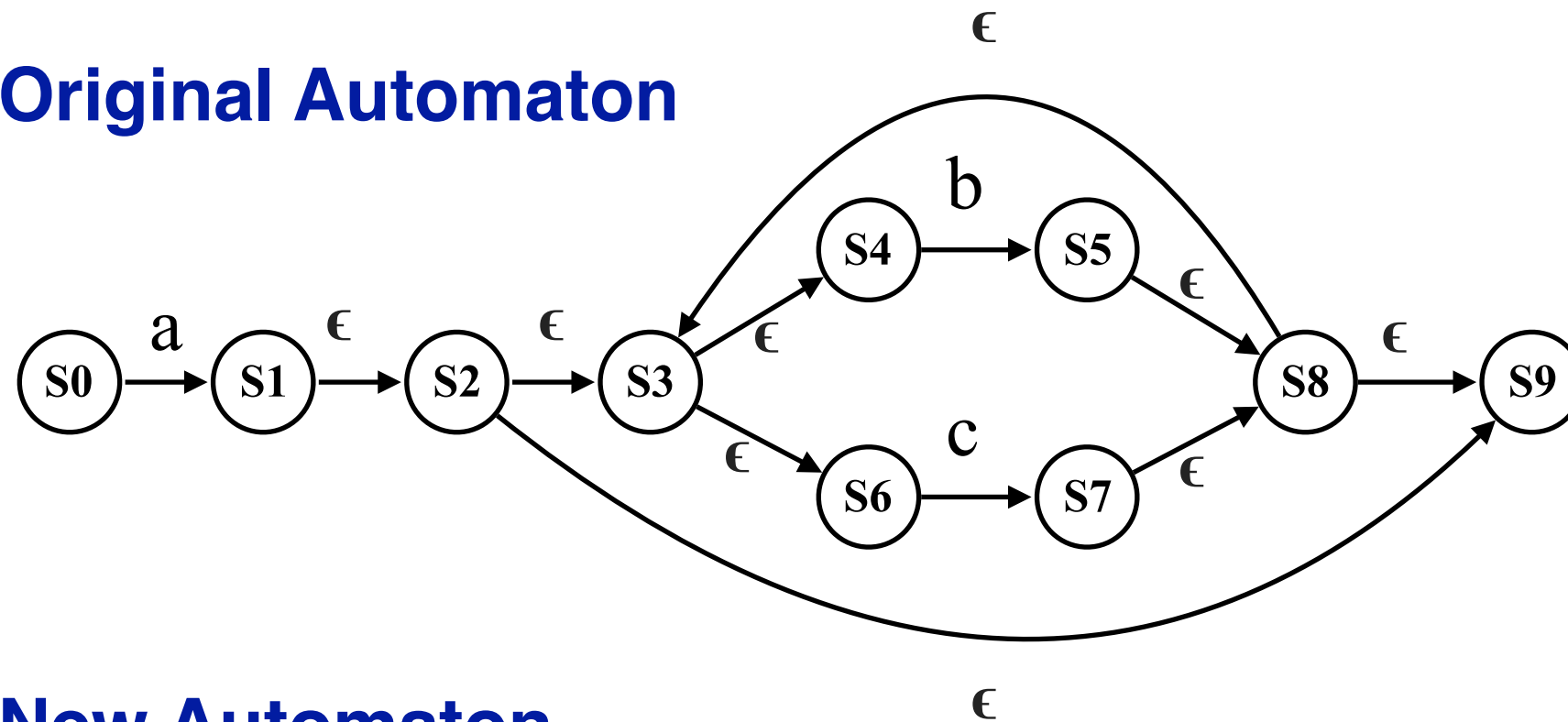


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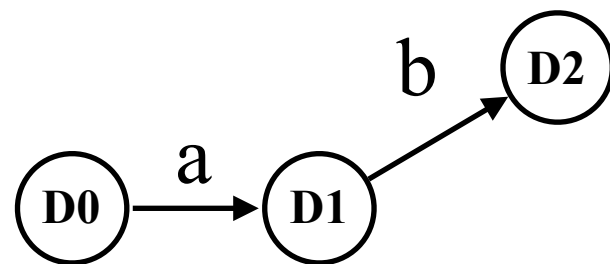
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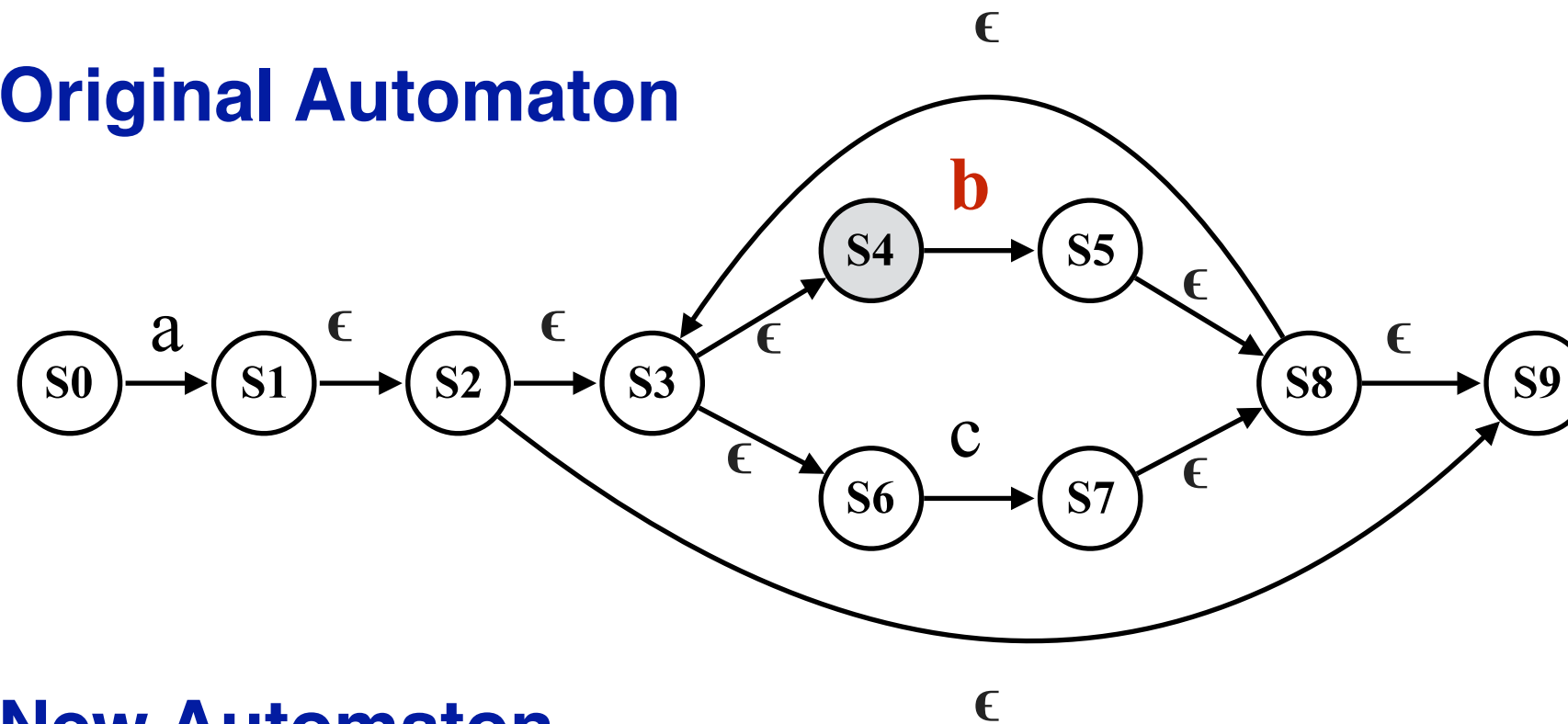


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$D2$	

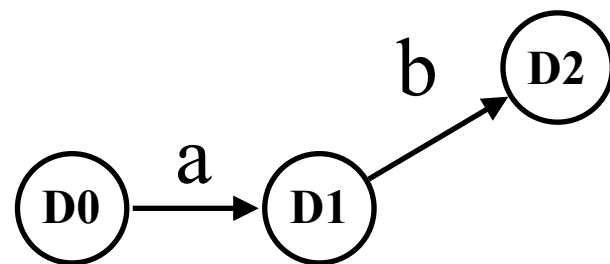
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## New Automaton

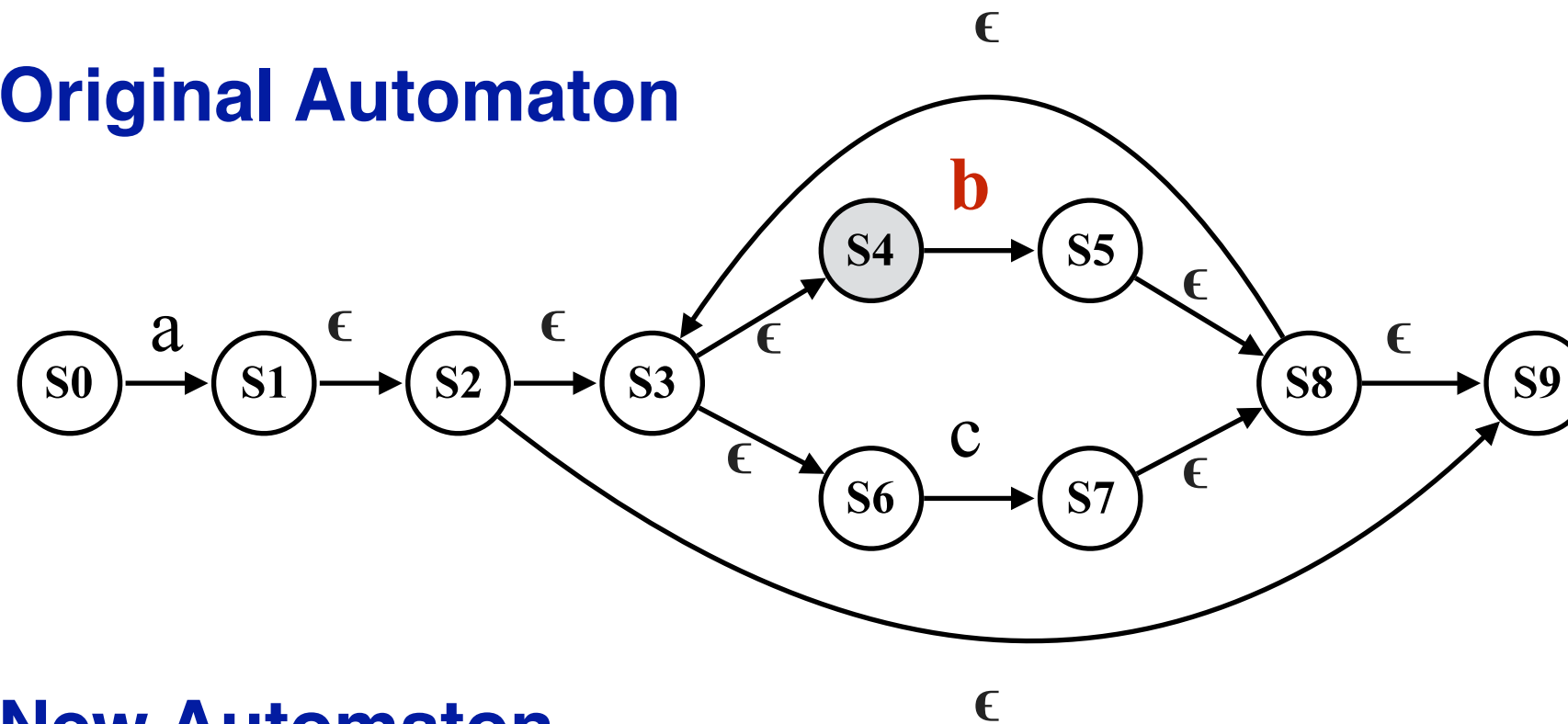


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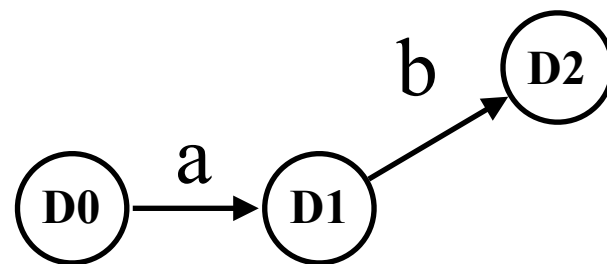
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## New Automaton

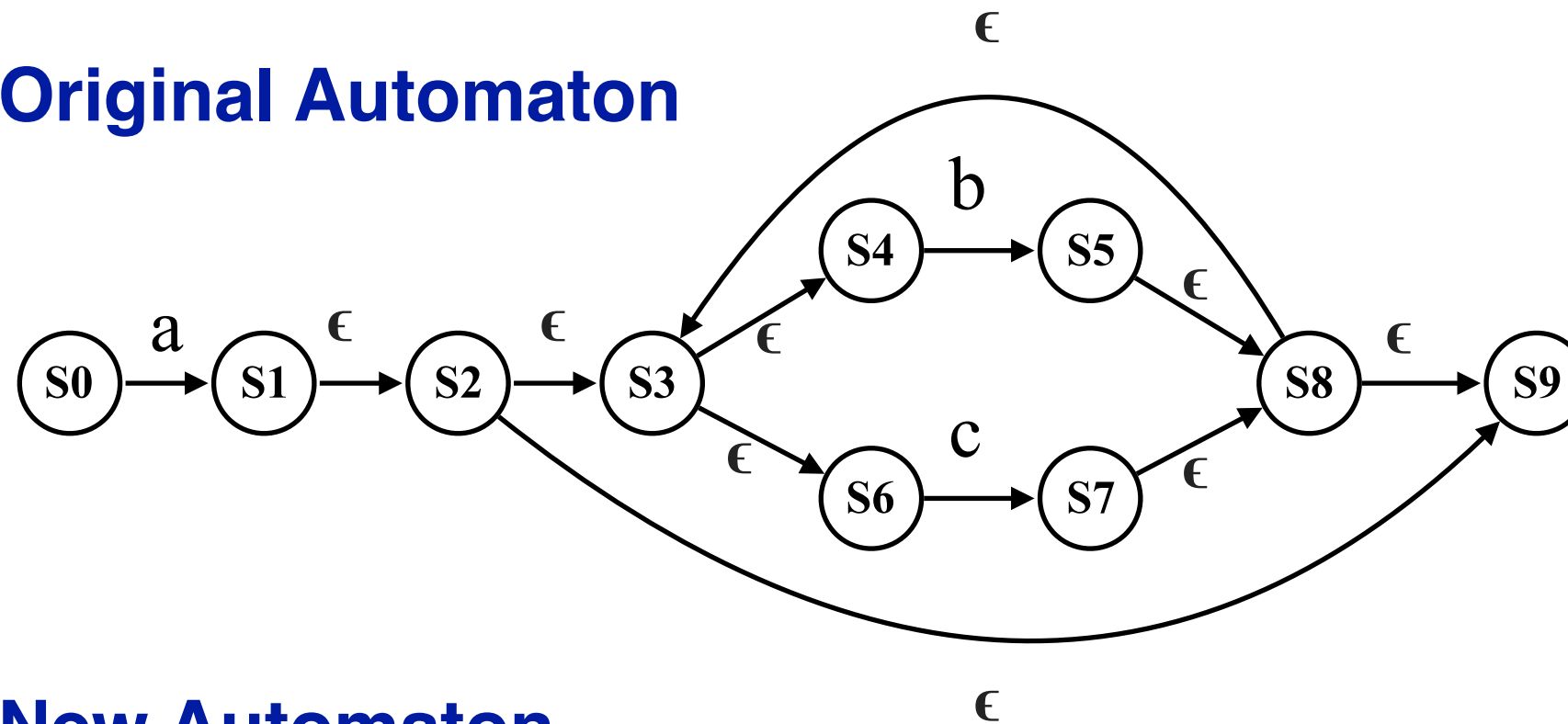


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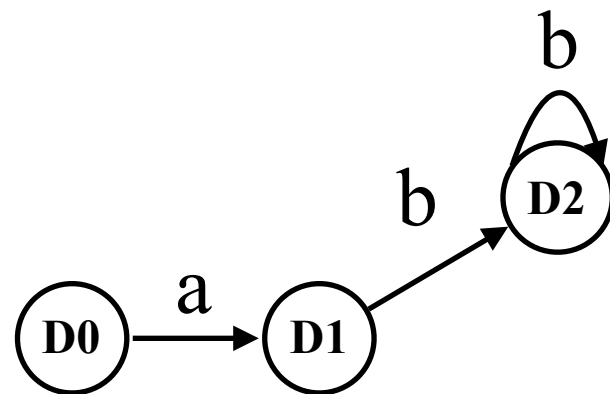
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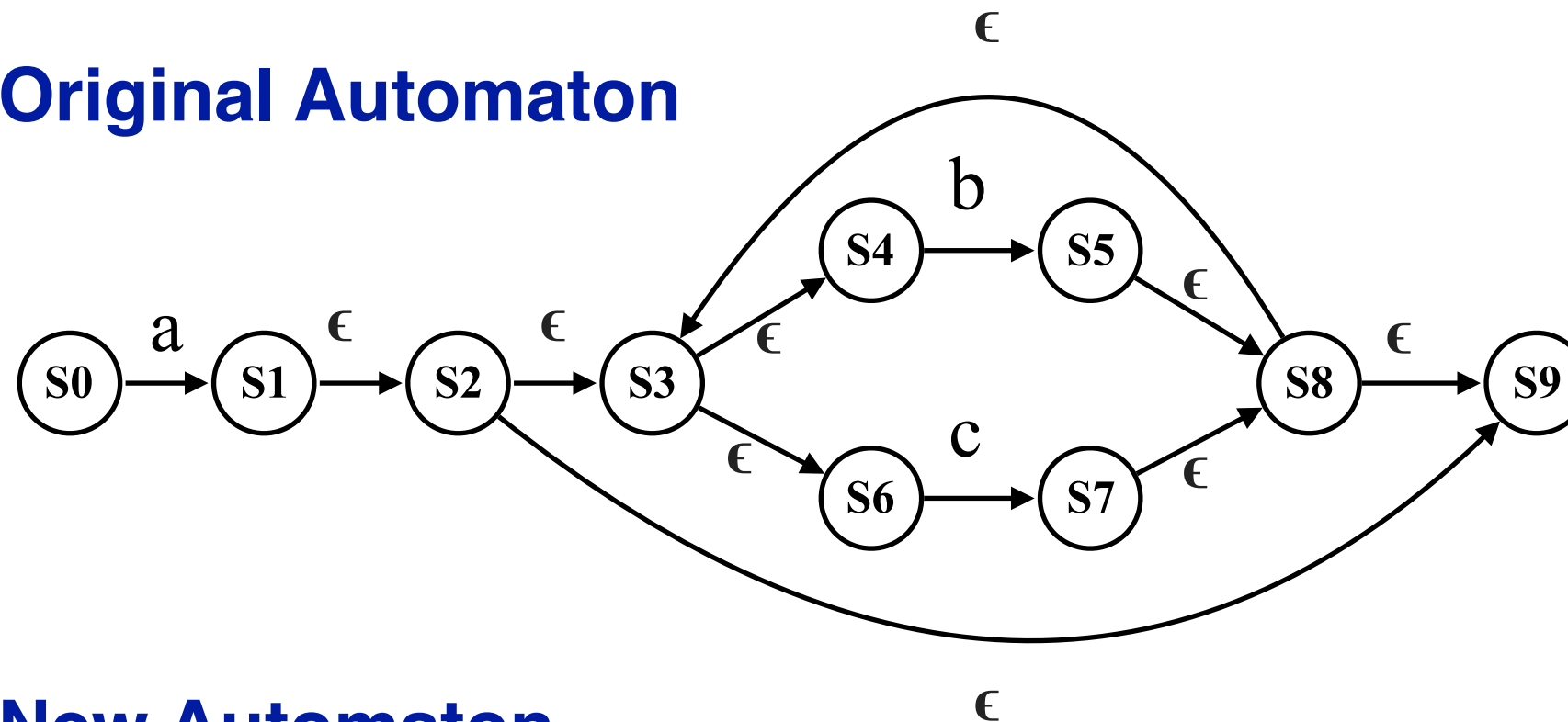


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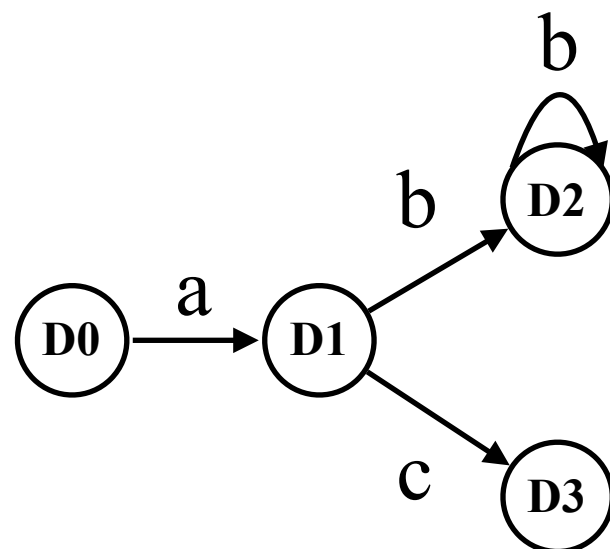
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## Original Automaton



## New Automaton

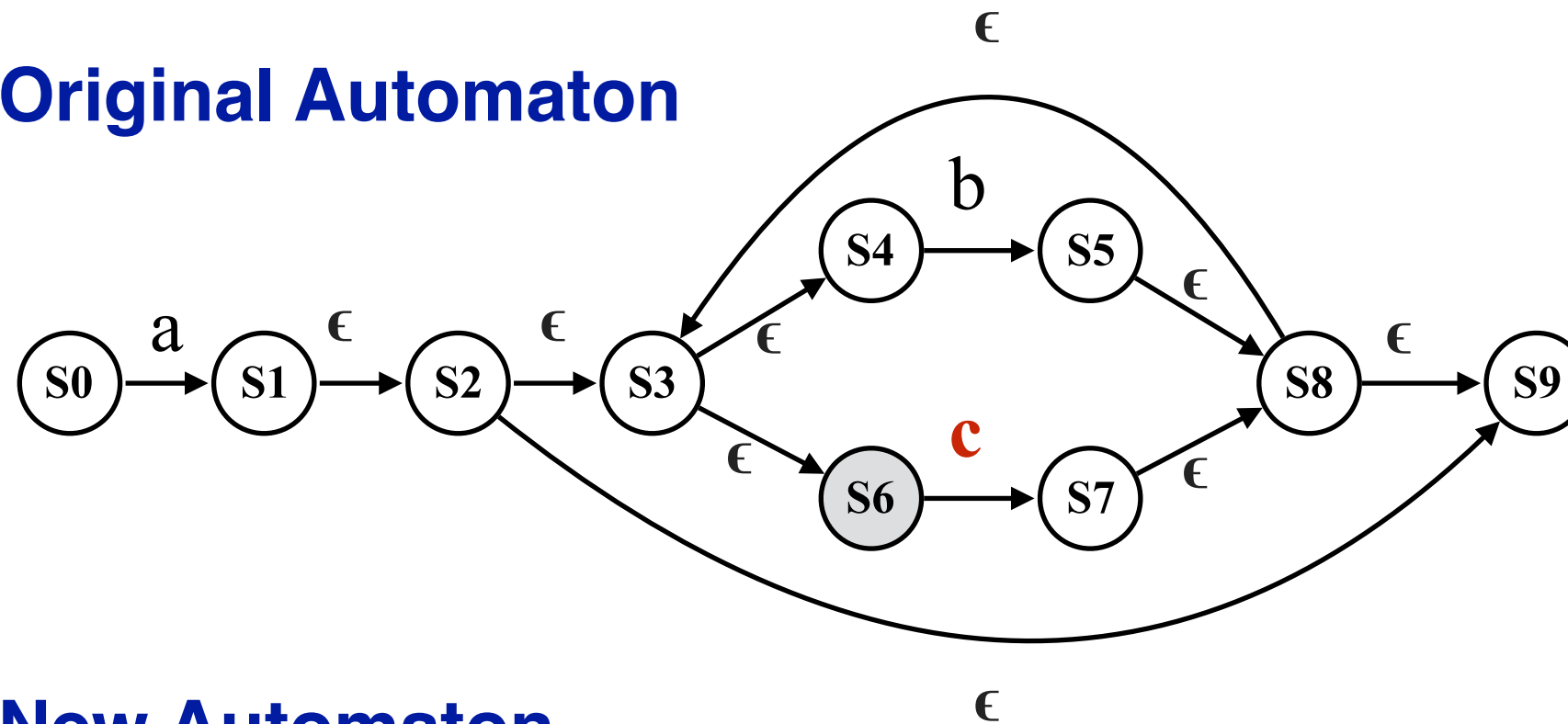


DFA	NFA
$D0$	$S0$
$D1$	$S1, S2, S3, S9, S4, S6$
$D2$	$S5, S8, S3, S9, S4, S6$
$D3$	

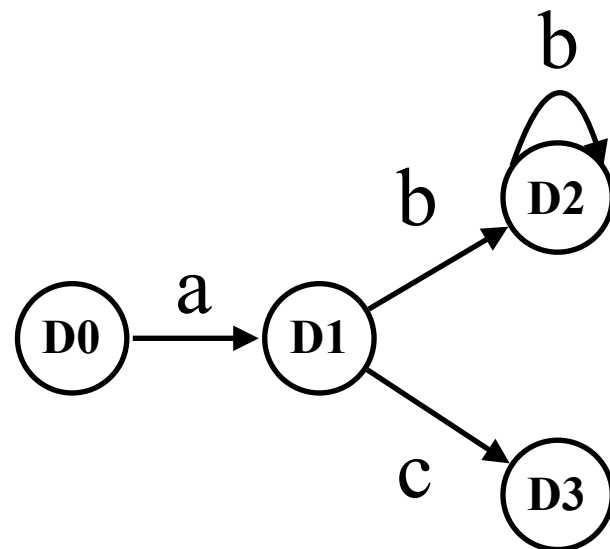
# Subset Construction

- Each state in the new one represents a set of states in the original one

## Original Automaton



## New Automaton

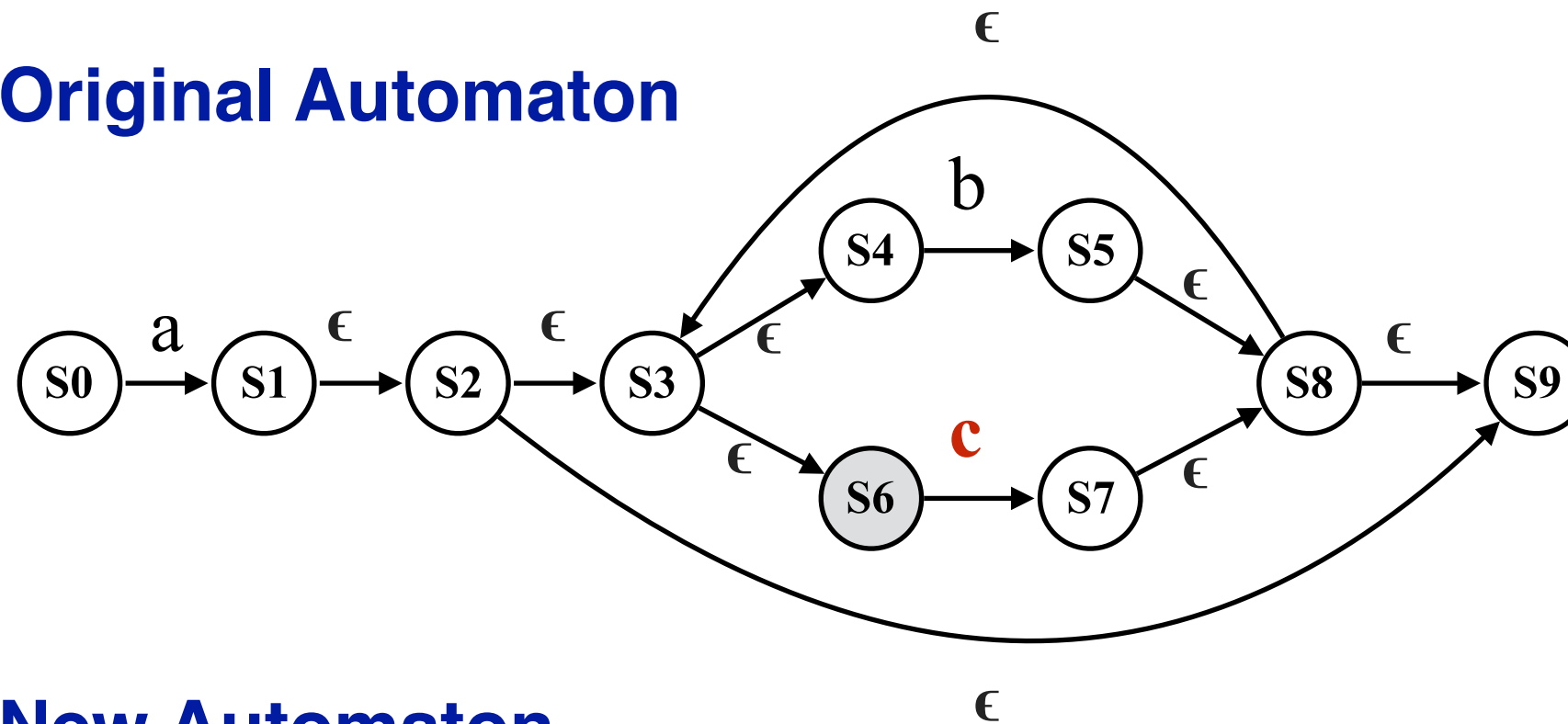


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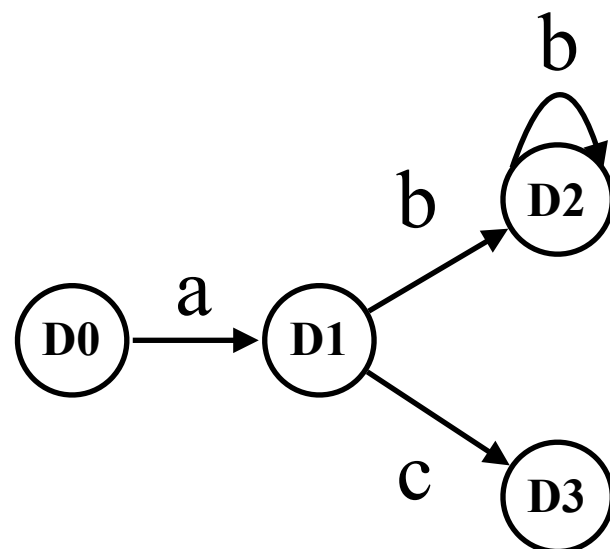
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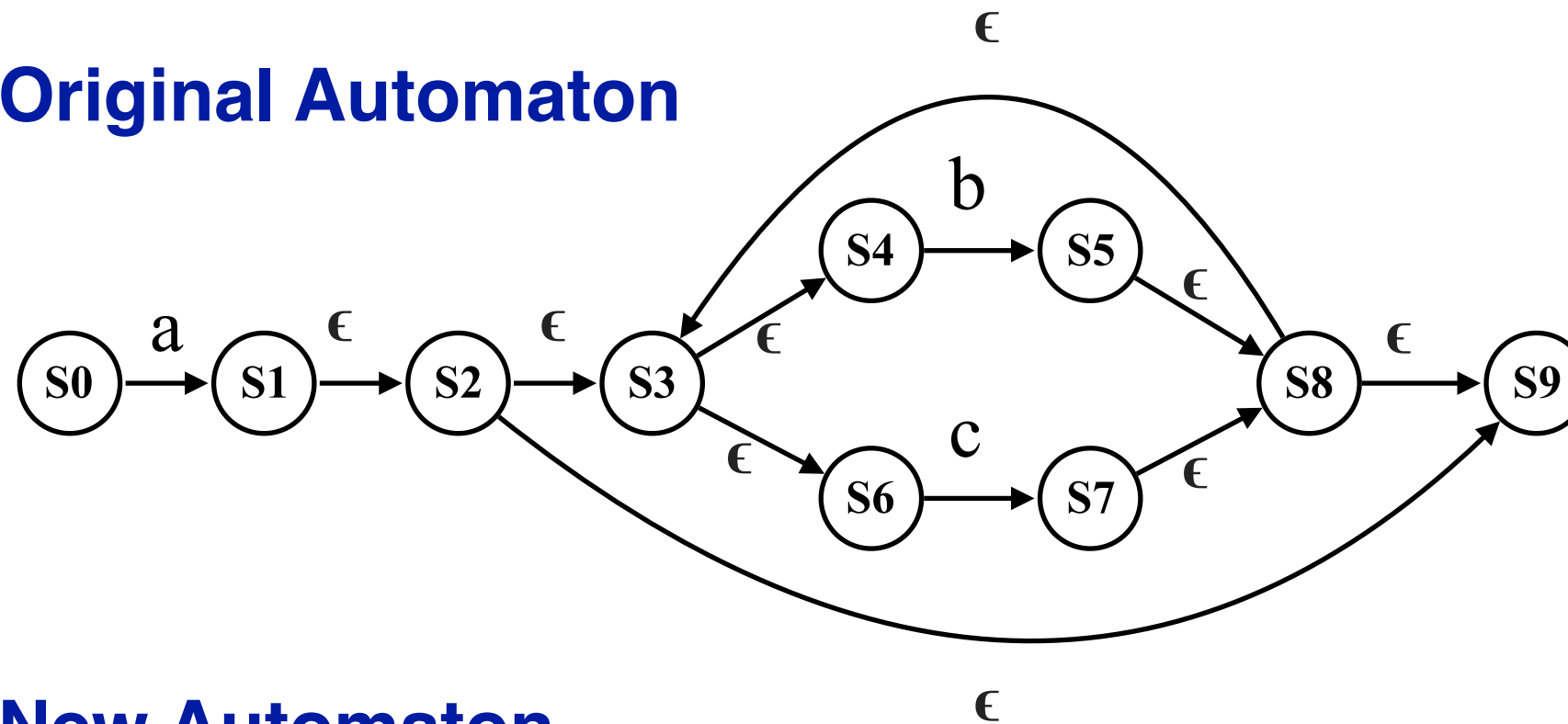
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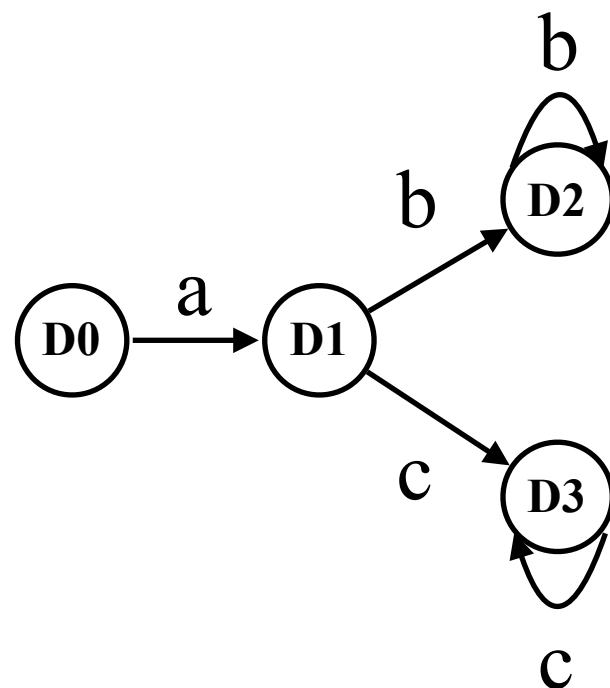
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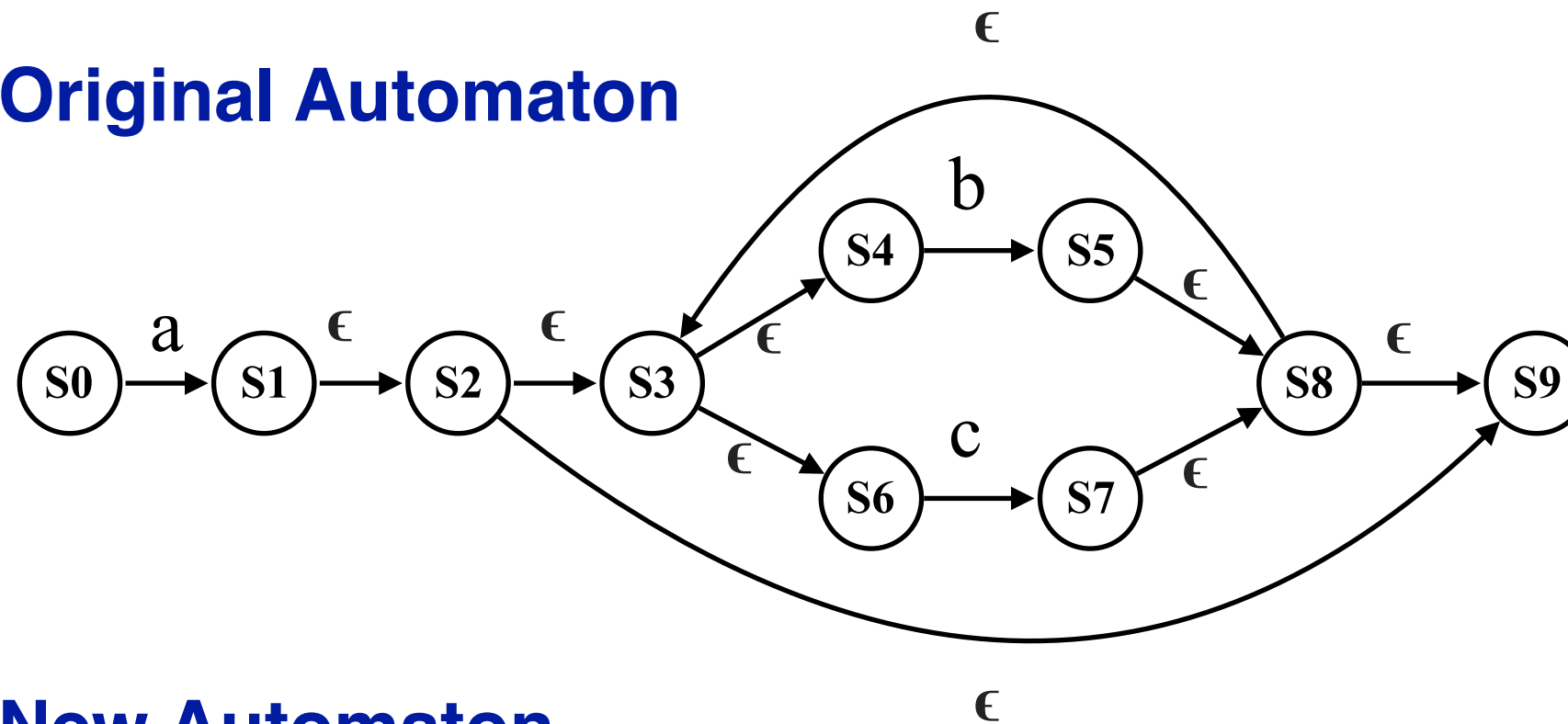


DFA	NFA
$D_0$	$S_0$
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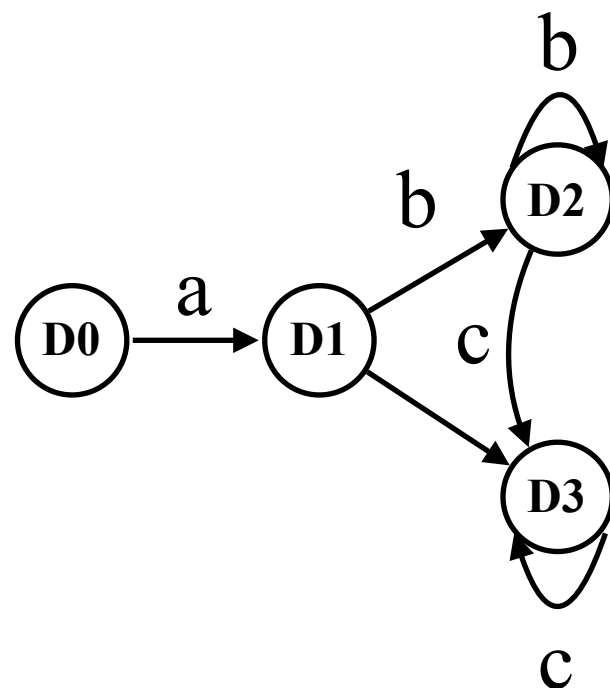
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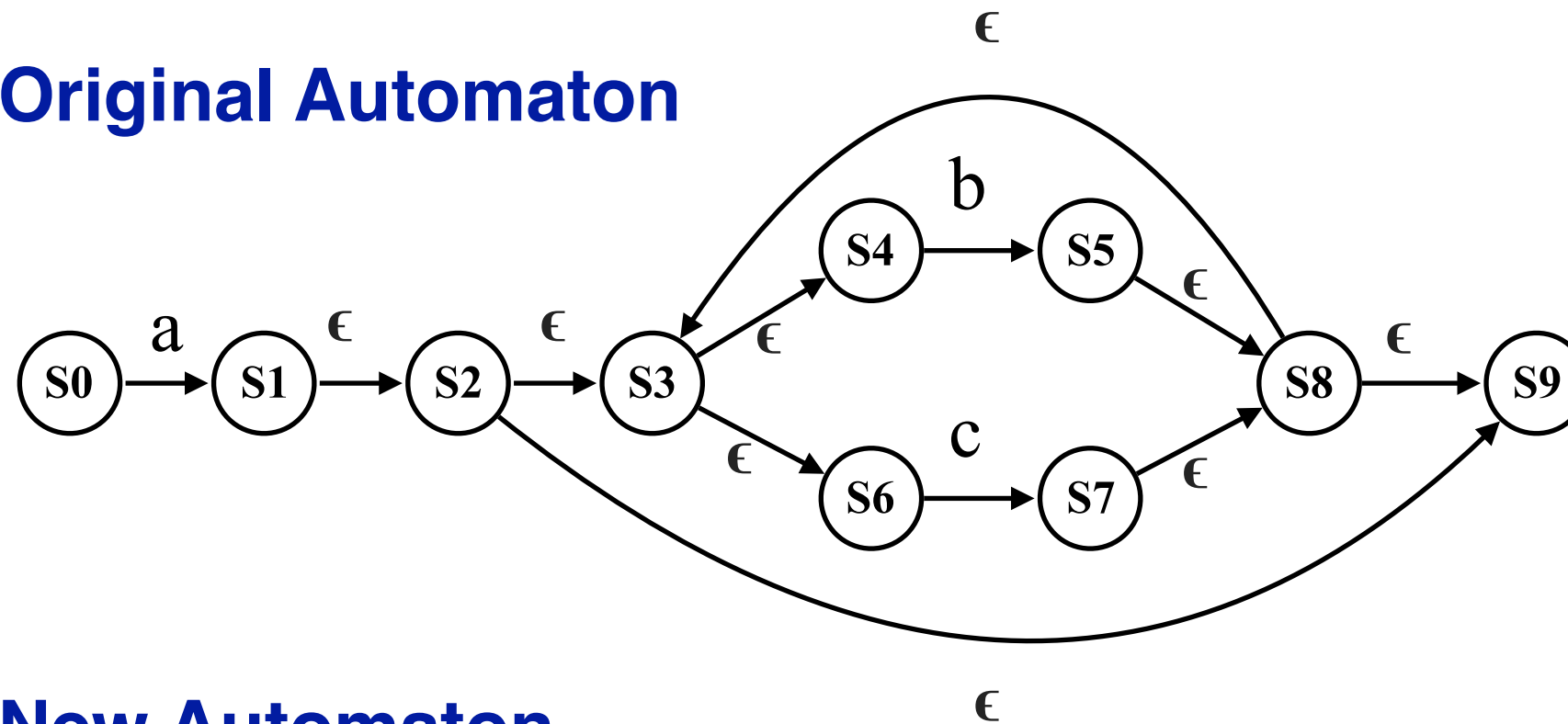


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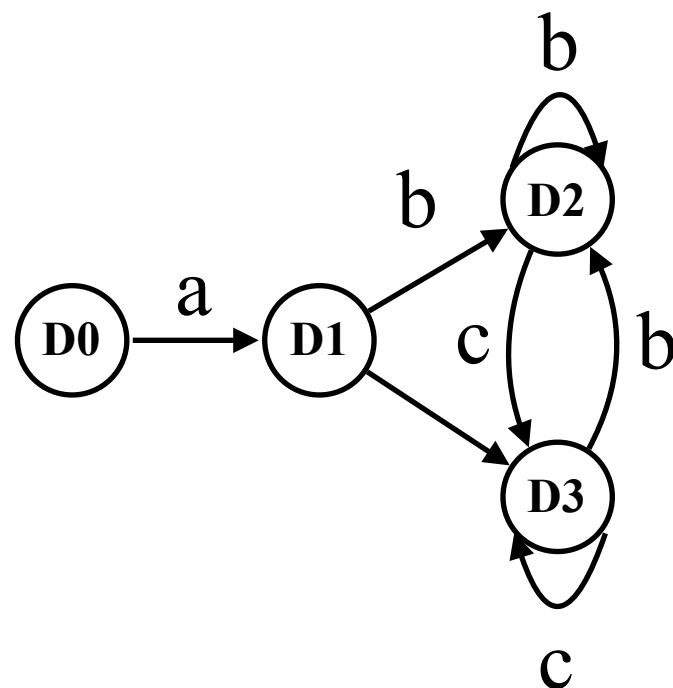
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## Original Automaton



## New Automaton

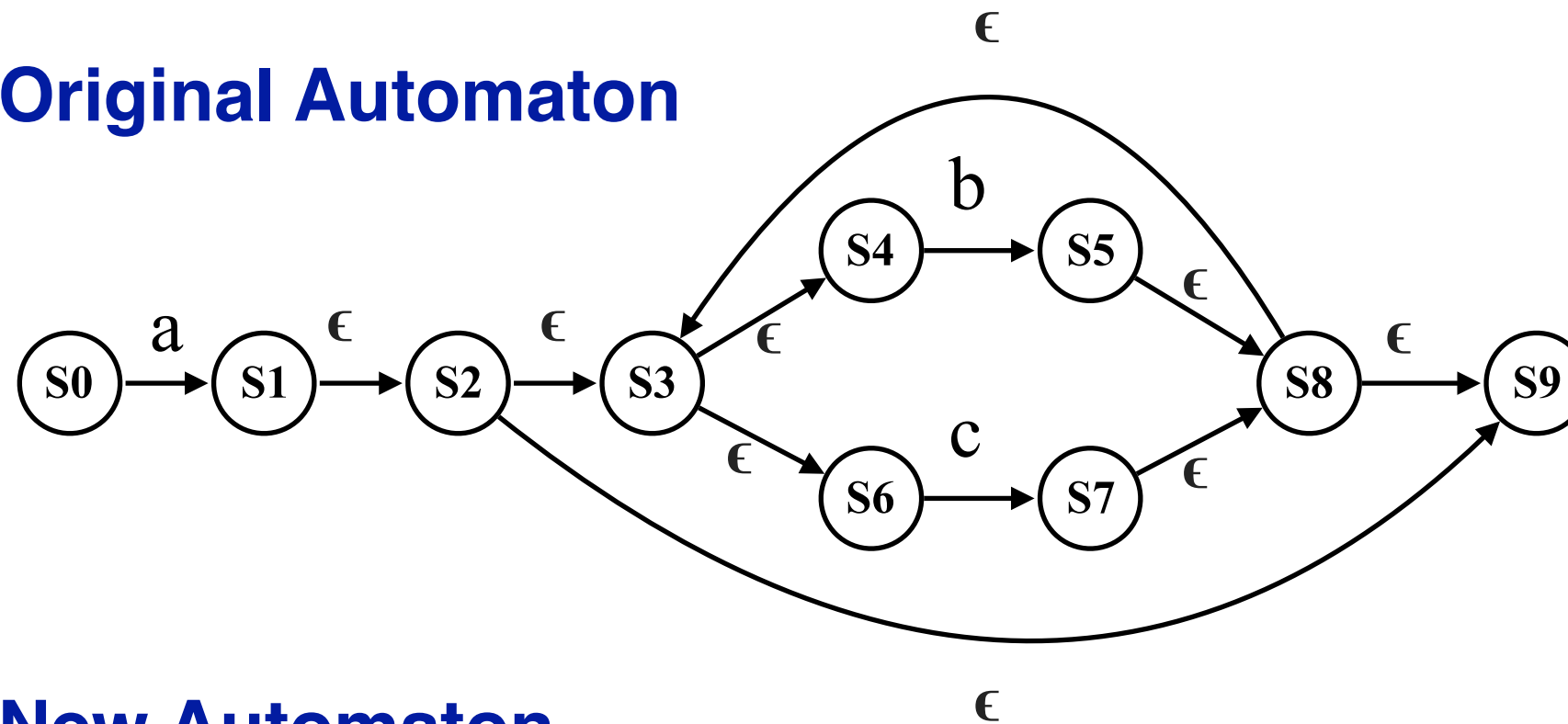


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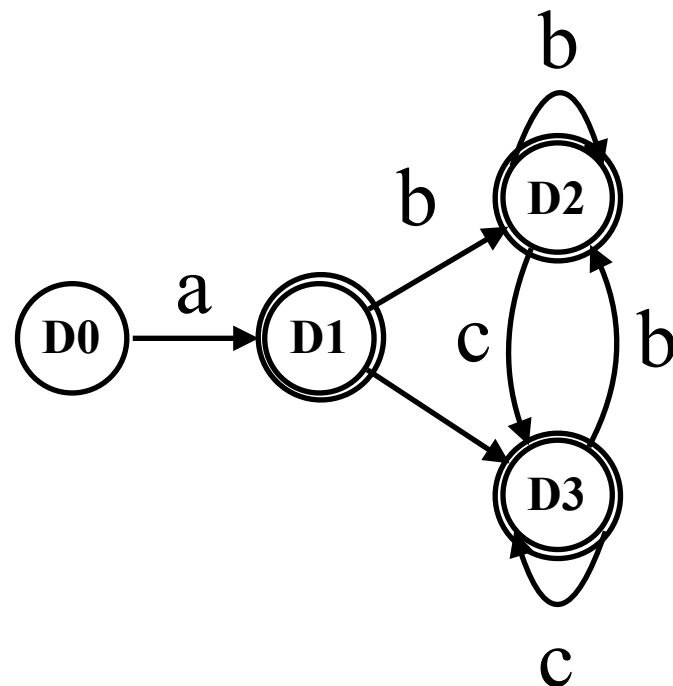
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- Each state in the new one represents a set of states in the original one

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## New Automaton



DFA	NFA
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# From RE to Scanner

---

**Classic approach is a three-step method:**

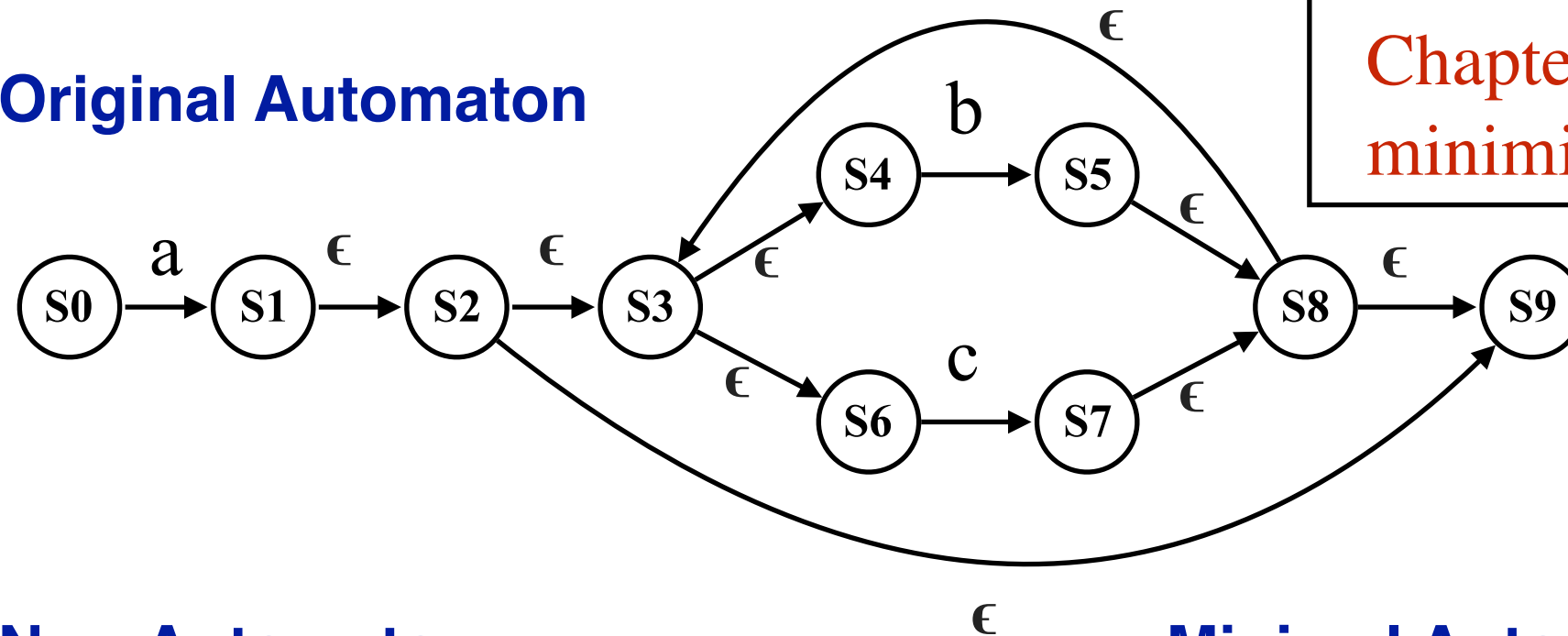
1. Build automata for each piece of the **RE** using a **template**.  
Multiple automata can be pasted using  $\epsilon$ -transition.  
This construction is called “**Thompson’s construction**”
2. Convert the newly built automaton into a deterministic automaton.  
This construction is called the “**subset construction**”
3. Given the deterministic automaton, minimize the number of states.  
Minimization is a **space optimization**.

# DFA Minimization

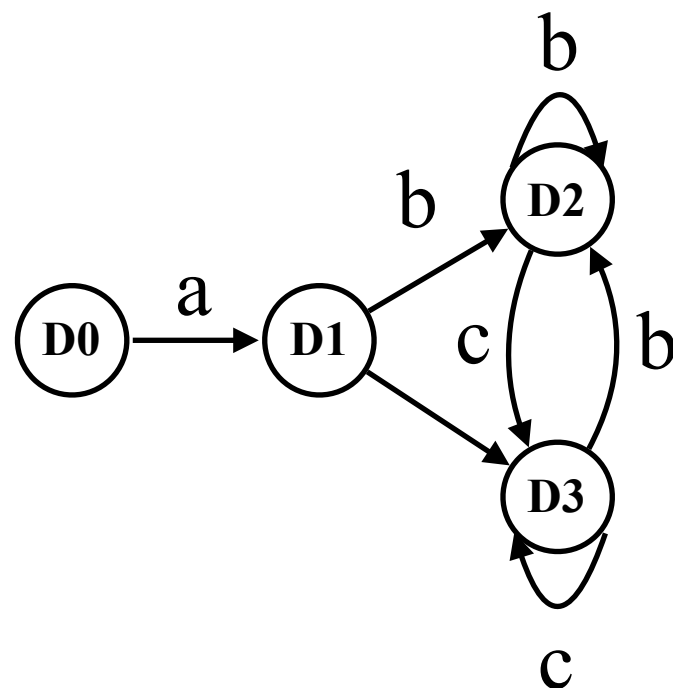
- Discover states that are equivalent in their contexts and replace multiple states with a single one

Read Textbook: Scott,  
Chapter 2.2.1 for the DFA  
minimization algorithm.

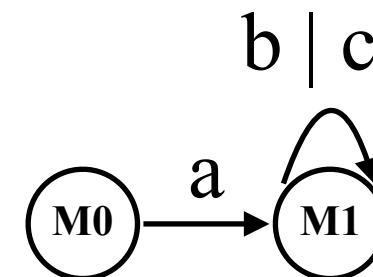
## Original Automaton



## New Automaton



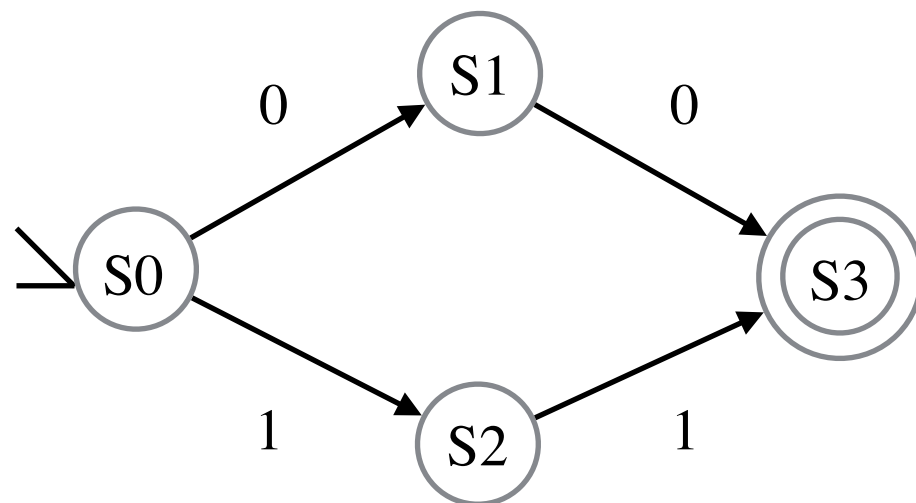
## Minimal Automaton



Minimal DFA	Original DFA
$M_0$	$D_0$
$M_1$	$D_1, D_2, D_3$

# Review: Scanner Implementation

Transitions can be represented using a transition table:

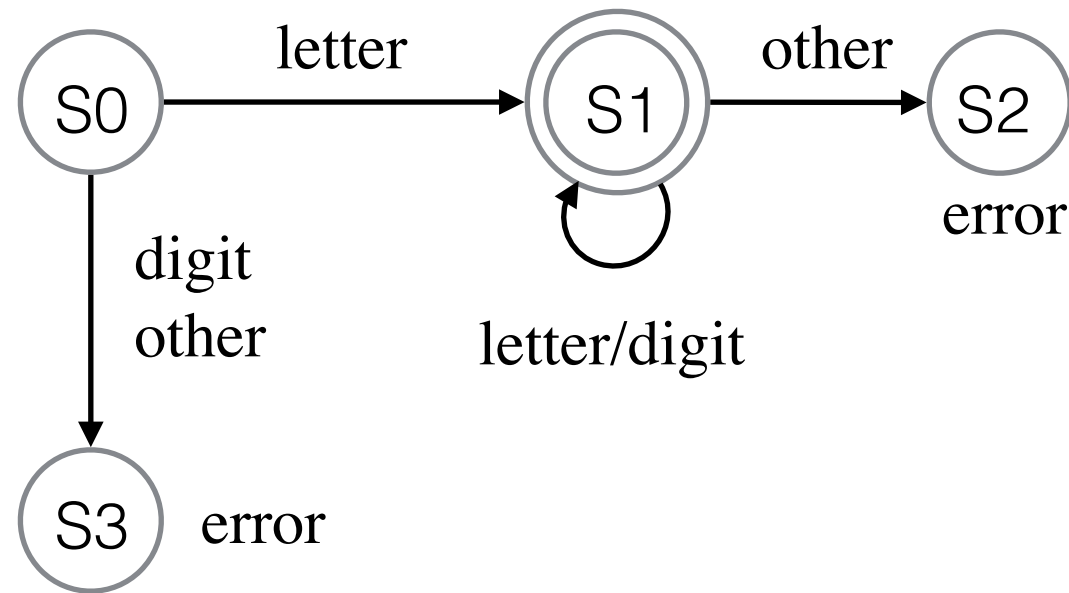


State	Input	
	0	1
<i>S0</i>	<i>S1</i>	<i>S2</i>
<i>S1</i>	<i>S3</i>	-
<i>S2</i>	-	<i>S3</i>

Two red arrows point from the dashed circles around the '-' entries in the table to the word "Error".

An FSA *accepts/recognizes* an input string **iff** there is some path from start state to a final state such that the labels on the path are that string. Lack of entry in the table (or no arc for a given character) indicates an *error—reject*.

# Review: Code for the scanner



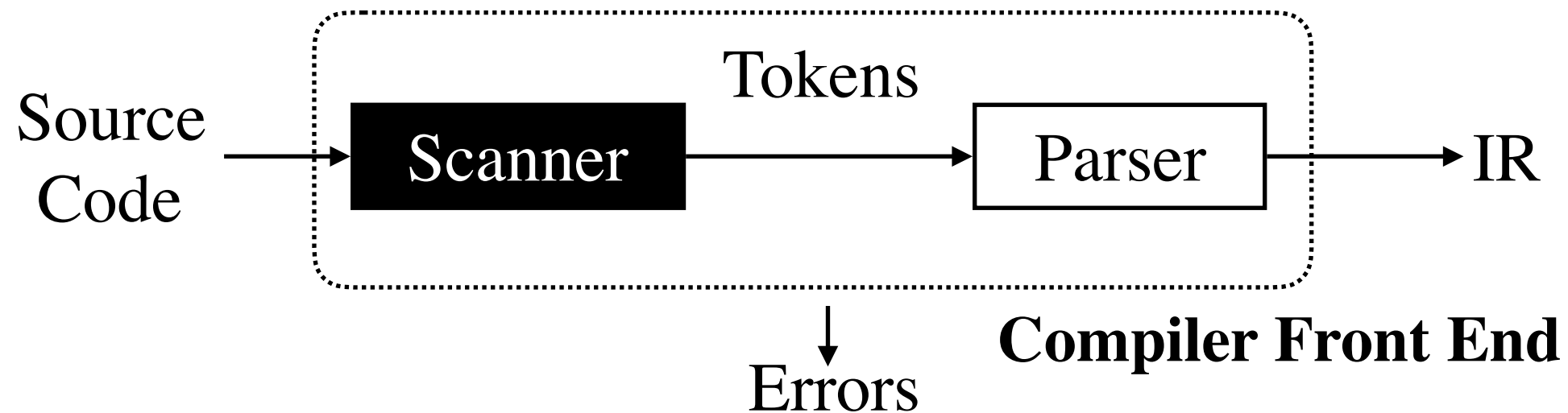
## Implementation:

```
char ← next_char();
state ← S0;
done ← false;
while( not done ) {
    class ← char_class[char];
    state ← next_state[class,state];
    switch(state) {
        case S1:
            /* building an id */
            token_value ← token_value + char;
            char ← next_char();
            if (char == DELIMITER)
                done = true;
            break;
        case S2: /* error state */
        case S3: /* error state */
            token type = error;
            done = true;
            break;
    }
}
return token_type;
```

<i>class</i>	<i>S0</i>	<i>S1</i>	<i>S2</i>	<i>S3</i>
<i>letter</i>	<i>S1</i>	<i>S1</i>	—	—
<i>digit</i>	<i>S3</i>	<i>S1</i>	—	—
<i>other</i>	<i>S3</i>	<i>S2</i>	—	—



# Compiler Front End

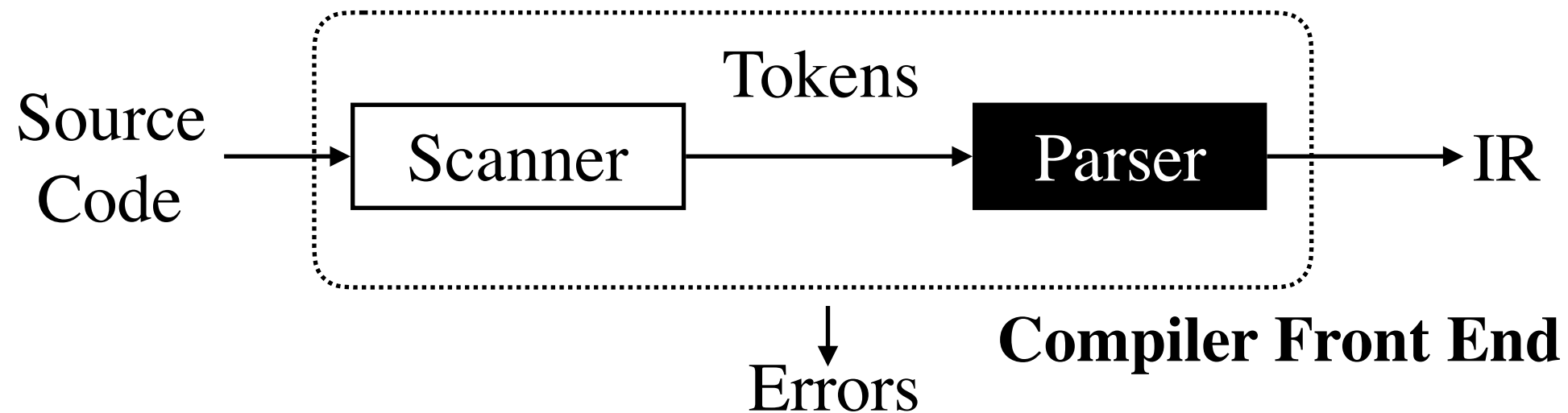


**Syntax & semantic analyzer, IR code generator**

*Front End Responsibilities:*

- Recognize legal programs
- Report errors
- Produce intermediate representation

# Compiler Front End



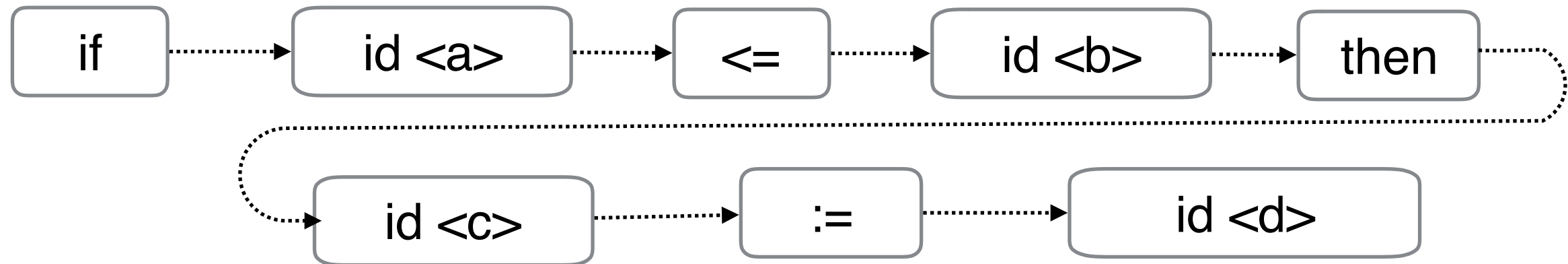
**Syntax & semantic analyzer, IR code generator**

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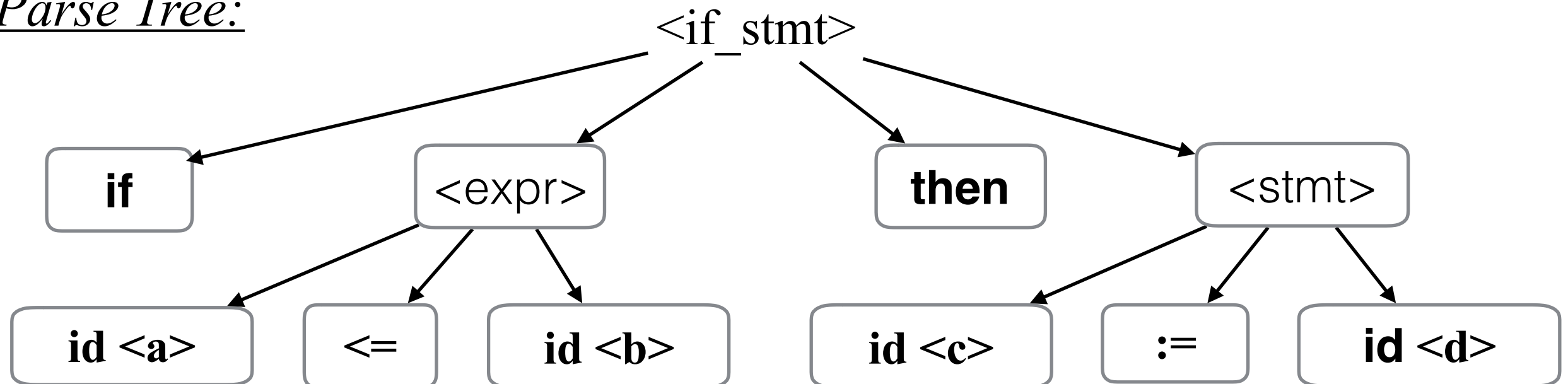
# Syntax Analysis ( Scott, Chapter 2.1, 2.3)

Token Sequence:



**parser**

Parse Tree:



# Context Free Grammars (CFGs)

---

- Another formalism to for describing languages
- A CFG  $G = \langle T, N, P, S \rangle$ :
  1. A set  $T$  of terminal symbols (tokens).
  2. A set  $N$  of nonterminal symbols.
  3. A set  $P$  production (rewrite) rules.
  4. A special start symbol  $S$ .
- The language  $L(G)$  is the set of sentences of terminal symbols in  $T^*$  that can be **derived** from the start symbol  $S$ :

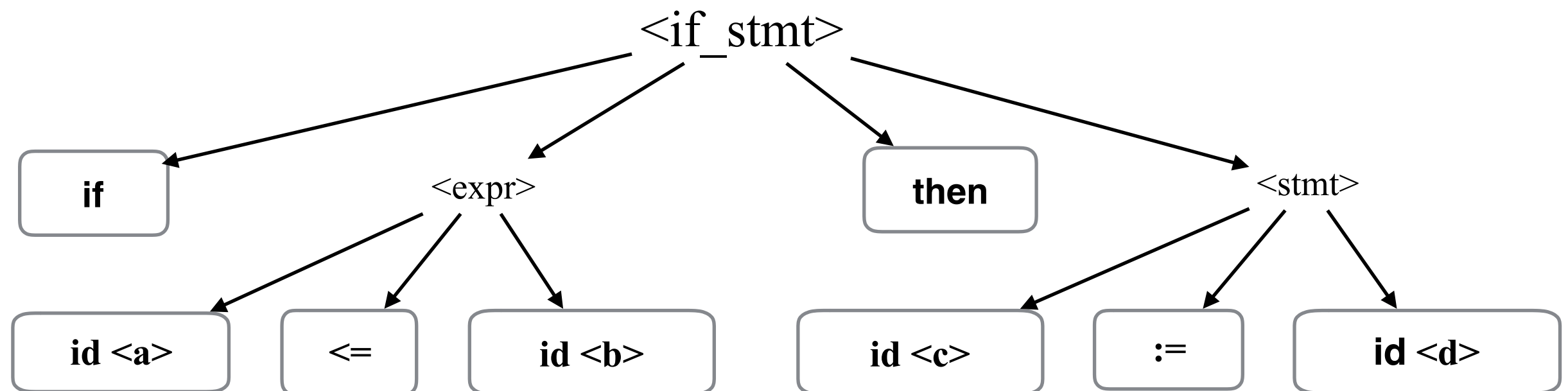
$$L(G) = \{w \in T^* \mid S \Rightarrow^* w\}$$

# An Example of a Partial Context Free Grammar

$\langle \text{if-stmt} \rangle ::= \text{if } \langle \text{expr} \rangle \text{ then } \langle \text{stmt} \rangle$

$\langle \text{expr} \rangle ::= \text{id } \leq \text{id}$

$\langle \text{stmt} \rangle ::= \text{id } := \text{id}$



# Context Free Grammars (CFGs)

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$$L(G) = \{w \in T^* \mid S \Rightarrow^* w\}$$

CFGs are rewrite systems with restrictions on the form of rewrite (production) rules that can be used.

# A Partial Context Free Grammar

...

$\langle \text{if-stmt} \rangle ::= \text{if } \langle \text{expr} \rangle \text{ then } \langle \text{stmt} \rangle$

$\langle \text{expr} \rangle ::= \text{id} \leq \text{id}$

$\langle \text{stmt} \rangle ::= \text{id} := \text{num}$

...

Context free grammar

Rule 1      $\$1 \Rightarrow 1\&$

Rule 2      $\$0 \Rightarrow 0\$$

Rule 3      $\&1 \Rightarrow 1\$$

Rule 4      $\&0 \Rightarrow 0\&$

Rule 5      $\$ \# \Rightarrow \rightarrow A$

Rule 6      $\& \# \Rightarrow \rightarrow B$

Not a context free grammar

CFGs are rewrite systems with restrictions on the form of rewrite (production) rules that can be used. The left hand side of a production rule can only be **one non-terminal symbol**.

# Elements of BNF Syntax

Terminal Symbol:	<b>Symbol-in-Boldface</b>
Non-Terminal Symbol:	<i>Symbol-in-Angle-Brackets</i>
Production Rule:	Non-Terminal ::= Sequence of Symbols or Non-Terminal ::= Sequence   Sequence   ...

## Example:

...

$\langle \text{if-stmt} \rangle ::= \text{if } \langle \text{expr} \rangle \text{ then } \langle \text{stmt} \rangle$

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$\langle \text{stmt} \rangle ::= \text{id } := \text{num}$



# How a BNF Grammar Describes a Language

---

**A sentence is a sequence of terminal symbols (tokens).**

**The language  $L(G)$  of a BNF grammar  $G$  is the set of sentences generated using the grammar:**

- Begin with start symbol.
- Iteratively replace non-terminals with terminals according to the rules (rewrite system).

*This replacing process is called a derivation ( $\Rightarrow$ ).  
Zero or multiple derivation steps are written as  $\Rightarrow^*$ .*

Formally,  $L(G) = \{w \in T^* \mid S \Rightarrow^* w\}$

# Derivation in a Grammar ( $G$ )

Is  $X2 := 0 \in L(G)$ , in another word, can  $X2 := 0$  be derived in  $G$ ?

**Start Symbol :**  $\langle \text{stmt} \rangle$



⋮



$X2 := 0$

Grammar  $G$ :

1.  $\langle \text{letter} \rangle ::= A \mid B \mid C \mid \dots \mid Z$
2.  $\langle \text{digit} \rangle ::= 0 \mid 1 \mid 2 \mid \dots \mid 9$
3.  $\langle \text{identifier} \rangle ::= \langle \text{letter} \rangle \mid$
4.  $\qquad \qquad \qquad \langle \text{identifier} \rangle \langle \text{letter} \rangle \mid$
5.  $\qquad \qquad \qquad \langle \text{identifier} \rangle \langle \text{digit} \rangle$
6.  $\langle \text{stmt} \rangle ::= \langle \text{identifier} \rangle := 0$

In which order to apply the rules?

Typically, there are three options:

leftmost ( $\Rightarrow_L$ ), rightmost ( $\Rightarrow_R$ ), any ( $\Rightarrow$ )

# Derivation in a Grammar ( $G$ )

Is  $X2 := 0 \in L(G)$ , i.e., can  $X2 := 0$  be derived in  $G$ ?

leftmost derivation	rule
$\langle \text{stmt} \rangle \Rightarrow_L$	
$X2 := 0$	

1.  $\langle \text{letter} \rangle ::= A \mid B \mid C \mid \dots \mid Z$
2.  $\langle \text{digit} \rangle ::= 0 \mid 1 \mid 2 \mid \dots \mid 9$
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Is  $X2 := 0 \in L(G)$ , i.e., can  $X2 := 0$  be derived in  $G$ ?

leftmost derivation		rule
$\langle \text{stmt} \rangle$	$\Rightarrow_L$	6
$\langle \text{identifier} \rangle := 0$	$\Rightarrow_L$	
$X2 := 0$		

1.  $\langle \text{letter} \rangle ::= A \mid B \mid C \mid \dots \mid Z$
2.  $\langle \text{digit} \rangle ::= 0 \mid 1 \mid 2 \mid \dots \mid 9$
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leftmost derivation		rule
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$\langle \text{identifier} \rangle := 0$	$\Rightarrow_L$	5
$\langle \text{identifier} \rangle \langle \text{digit} \rangle := 0$	$\Rightarrow_L$	
$X2 := 0$		

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$\langle \text{identifier} \rangle := 0$	$\Rightarrow_L$	5
$\langle \text{identifier} \rangle \langle \text{digit} \rangle := 0$	$\Rightarrow_L$	3
$\langle \text{letter} \rangle \langle \text{digit} \rangle := 0$	$\Rightarrow_L$	
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leftmost derivation		rule
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$\langle \text{identifier} \rangle := 0$	$\Rightarrow_L$	5
$\langle \text{identifier} \rangle \langle \text{digit} \rangle := 0$	$\Rightarrow_L$	3
$\langle \text{letter} \rangle \langle \text{digit} \rangle := 0$	$\Rightarrow_L$	1
$X \langle \text{digit} \rangle := 0$	$\Rightarrow_L$	
$X2 := 0$		

1.  $\langle \text{letter} \rangle ::= A \mid B \mid C \mid \dots \mid Z$
2.  $\langle \text{digit} \rangle ::= 0 \mid 1 \mid 2 \mid \dots \mid 9$
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$X \langle \text{digit} \rangle := 0$	$\Rightarrow_L$	
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$\langle \text{identifier} \rangle \langle \text{digit} \rangle := 0$	$\Rightarrow_L$	3
$\langle \text{letter} \rangle \langle \text{digit} \rangle := 0$	$\Rightarrow_L$	1
$X \langle \text{digit} \rangle := 0$	$\Rightarrow_L$	2
$X2 := 0$		

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3.  $\langle \text{identifier} \rangle ::= \langle \text{letter} \rangle \mid$
4.  $\langle \text{identifier} \rangle \langle \text{letter} \rangle \mid$
5.  $\langle \text{identifier} \rangle \langle \text{digit} \rangle$
6.  $\langle \text{stmt} \rangle ::= \langle \text{identifier} \rangle := 0$

# Derivation in a Grammar ( $G$ )

Is  $X2 := 0 \in L(G)$ , i.e., can  $X2 := 0$  be derived in  $G$ ?

leftmost derivation		rule
$\langle \text{stmt} \rangle$	$\Rightarrow_L$	6
$\langle \text{identifier} \rangle := 0$	$\Rightarrow_L$	5
$\langle \text{identifier} \rangle \langle \text{digit} \rangle := 0$	$\Rightarrow_L$	3
$\langle \text{letter} \rangle \langle \text{digit} \rangle := 0$	$\Rightarrow_L$	1
$X \langle \text{digit} \rangle := 0$	$\Rightarrow_L$	2
$X2 := 0$		

1.  $\langle \text{letter} \rangle ::= A \mid B \mid C \mid \dots \mid Z$
2.  $\langle \text{digit} \rangle ::= 0 \mid 1 \mid 2 \mid \dots \mid 9$
3.  $\langle \text{identifier} \rangle ::= \langle \text{letter} \rangle \mid$
4.  $\langle \text{identifier} \rangle \langle \text{letter} \rangle \mid$
5.  $\langle \text{identifier} \rangle \langle \text{digit} \rangle$
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# Next Lecture

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Things to do:

- Read Scott, Chapter 2.2 - 2.5 (skip 2.3.3 bottom-up Parsing)