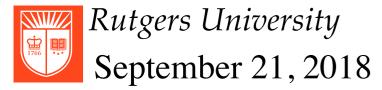
CS 314 Principles of Programming Languages

Lecture 6: LL(1) Parsing

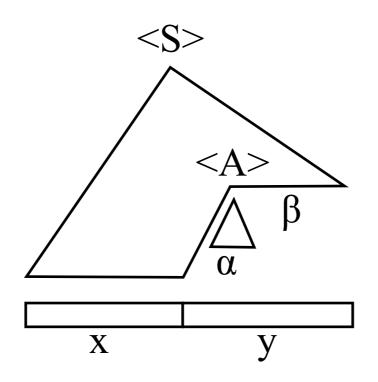
Prof. Zheng Zhang



Class Information

• Homework 2 posted, due Tuesday 9/25/2018 11:55pm.

Review: Top-Down Parsing - LL(1)



Basic Idea:

- The parse tree is constructed from the root, expanding non-terminal nodes on the tree's frontier following a **leftmost** derivation.
- The input program is read from **left** to right, and input tokens are read (consumed) as the program is parsed.
- The next non-terminal symbol is replaced using one of its rules. The particular choice <u>has to be unique</u> and uses parts of the input (partially parsed program), for instance the first token of the remaining input.

Consider this example grammar:

```
<id_list> ::= id <id_list_tail> ::= , id <id_list_tail> ::= , id <id_list_tail> ::= ;
```

Consider this example grammar:

```
id_list ::= id id_list_tail
id_list_tail ::= , id id_list_tail
id_list_tail ::= ;
```

How to parse the following input string?

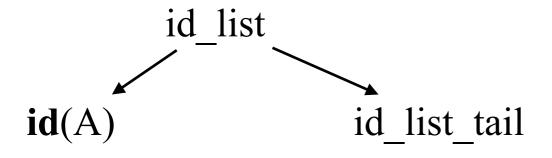
```
id_list ::= id id_list_tail
id_list_tail ::= , id id_list_tail
id_list_tail ::= ;

id_list_
```

Remaining Input: A, B, C;

Sentential Form: id list

```
id_list ::= id id_list_tail
id_list_tail ::= , id id_list_tail
id_list_tail ::= ;
```

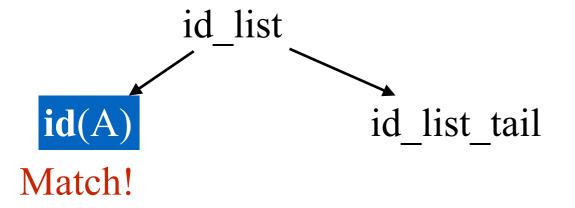


Remaining Input: A, B, C;

Sentential Form: **id**(A) id list tail

Applied Production: id_list ::= id id_list_tail

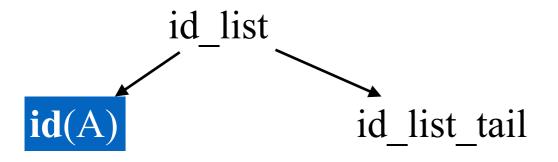
```
id_list ::= id id_list_tail
id_list_tail ::= , id id_list_tail
id_list_tail ::= ;
```



Remaining Input: A, B, C;

Sentential Form: **id**(A) id_list_tail

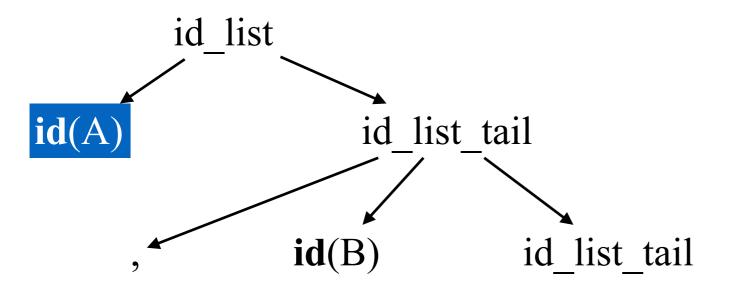
```
id_list ::= id id_list_tail
id_list_tail ::= , id id_list_tail
id_list_tail ::= ;
```



Remaining Input: , B , C ;

Sentential Form: **id**(A) id list tail

```
id list ::= id id list tail
id list tail ::= , id id list tail
id_list_tail ::= ;
```



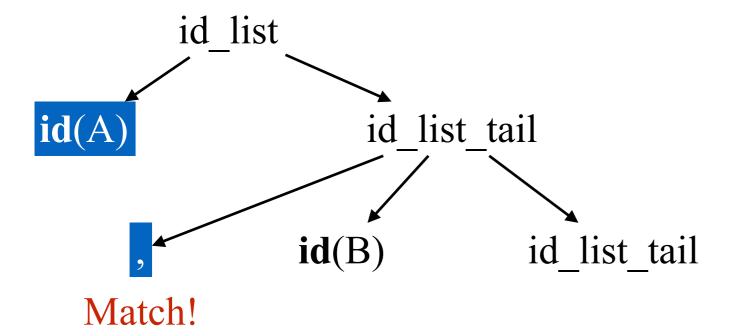
Remaining Input: , B , C ;

Sentential Form: id(A), id(B) id_list_tail

Applied Production: id_list_tail ::= , id id_list_tail

```
id list ::= id id list tail
id list tail ::= , id id list tail)
id_list_tail ::= ;
```

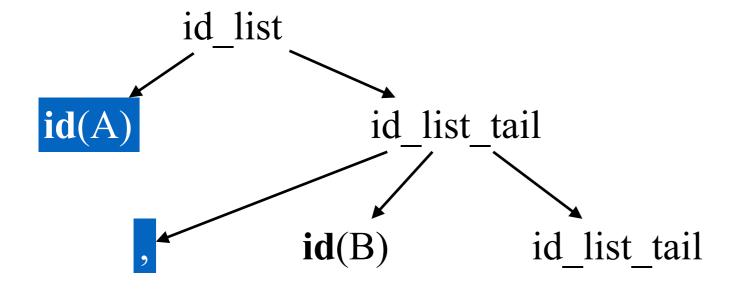
Remaining Input: ,B,C;



Sentential Form: id(A), id(B) id_list_tail

```
id list ::= id id list tail
id list tail ::= , id id list tail
id_list_tail ::= ;
```

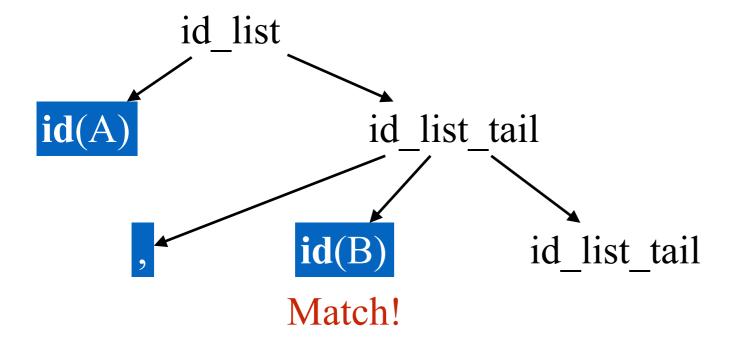
Remaining Input: B, C;



Sentential Form: id(A), id(B) id_list_tail

```
id list ::= id id list tail
id list tail ::= , id id list tail)
id_list_tail ::= ;
```

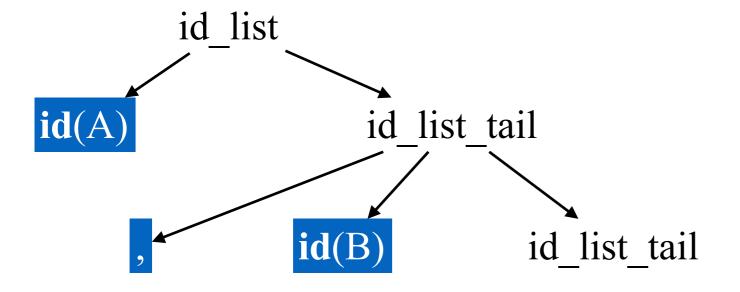
Remaining Input: B, C;



Sentential Form: id(A), id(B) id_list_tail

```
id list ::= id id list tail
id list tail ::= , id id list tail)
id_list_tail ::= ;
```

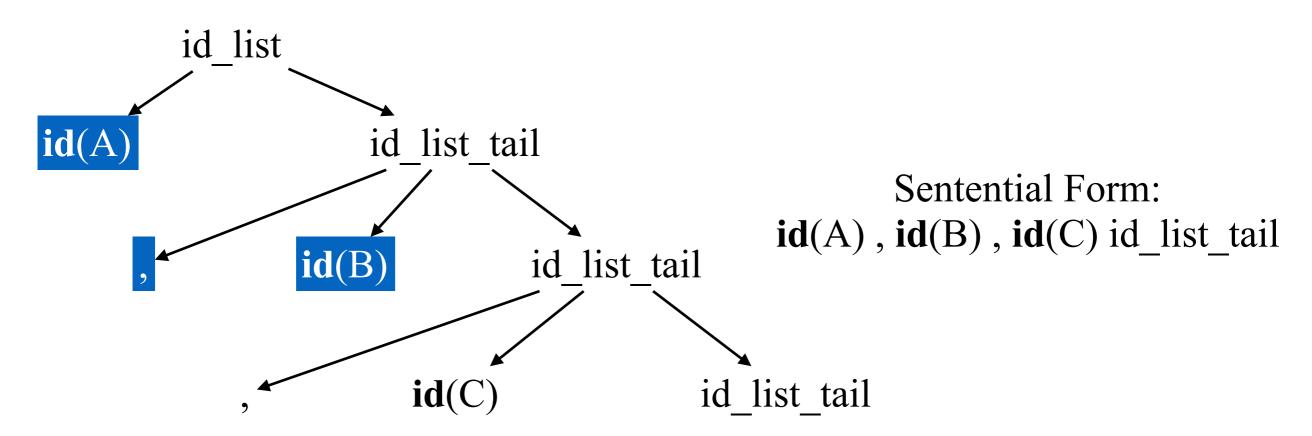
Remaining Input: , C;



Sentential Form: id(A) , id(B) id_list_tail

```
id list ::= id id list tail
id list tail ::= , id id list tail
id_list_tail ::= ;
```

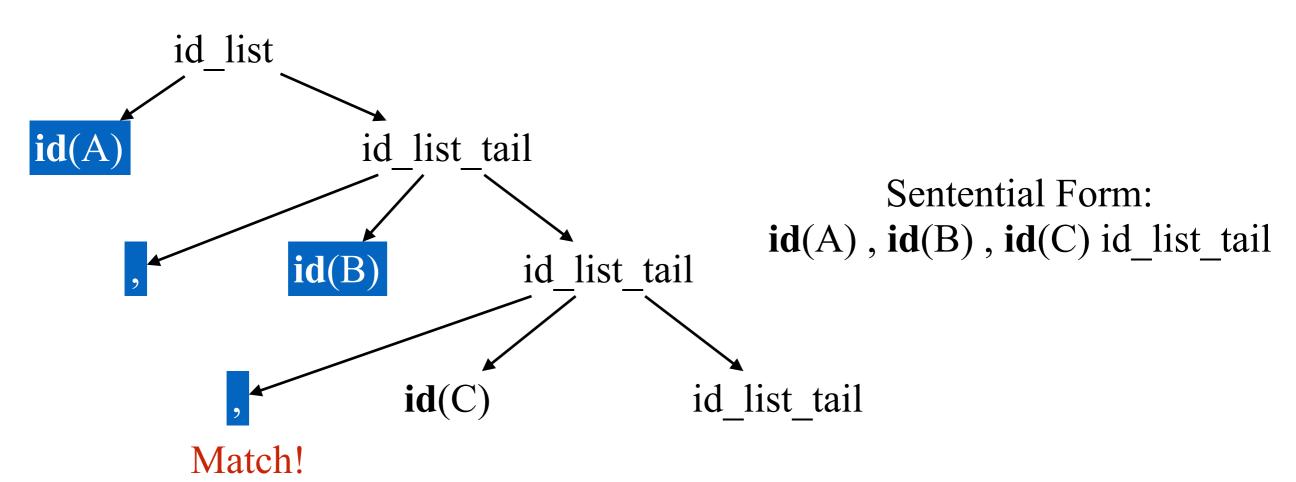
Remaining Input: , C;



Applied Production: id list tail ::=, id id list tail

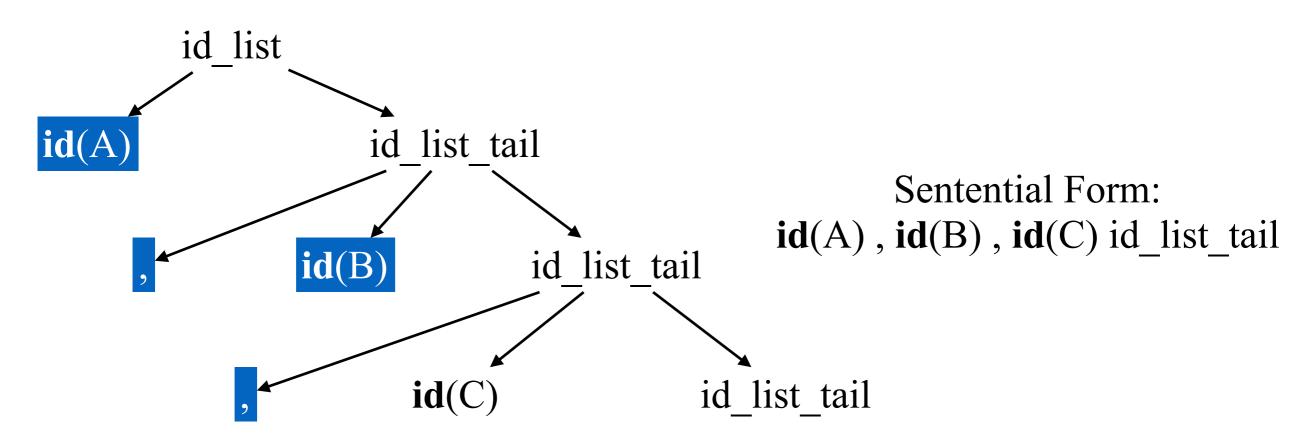
```
id list ::= id id list tail
id list tail ::= , id id list tail
id_list_tail ::= ;
```

Remaining Input: , C;



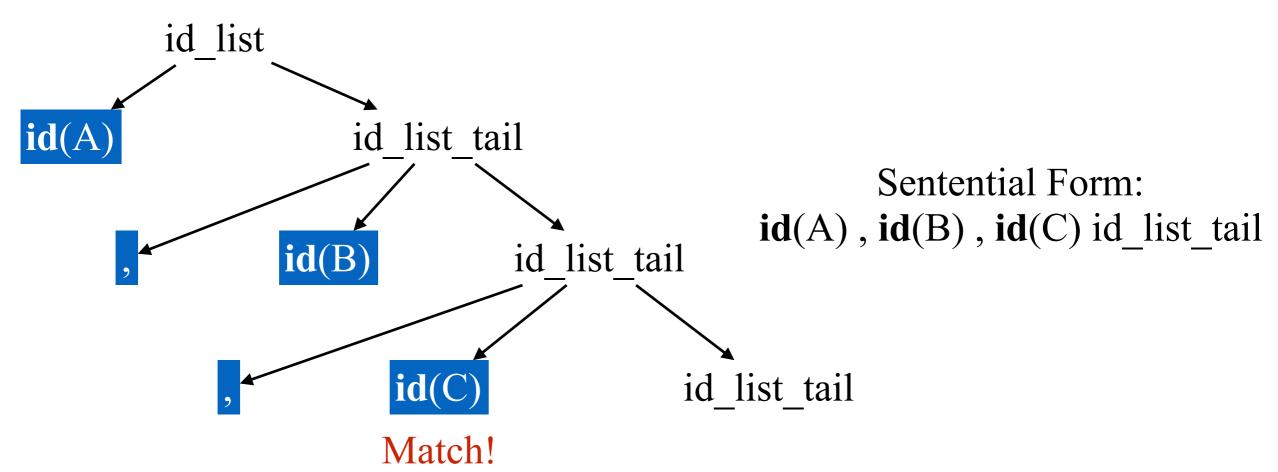
```
id list ::= id id list tail
id list tail ::= , id id list tail
id_list_tail ::= ;
```

Remaining Input: C;



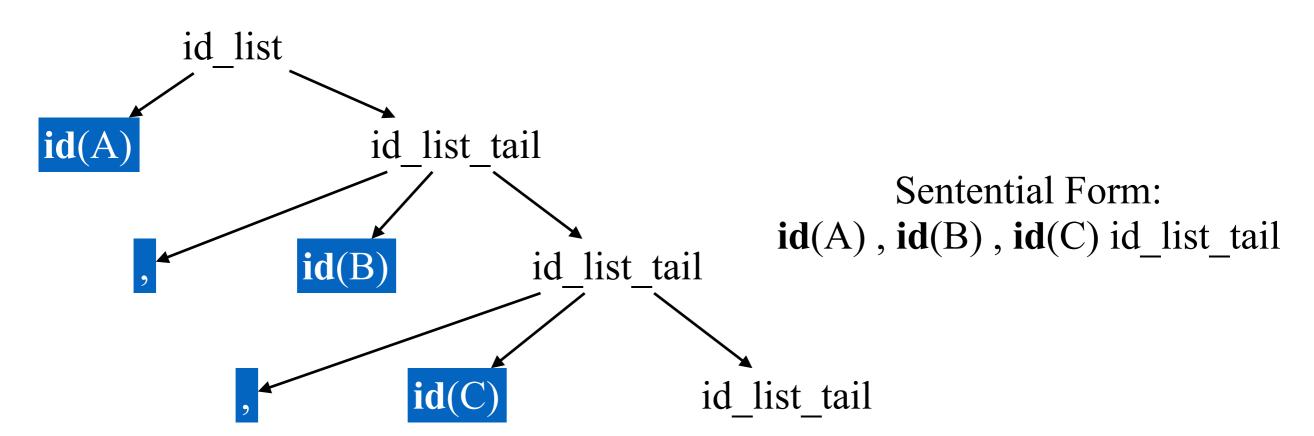
```
id list ::= id id list tail
id list tail ::= , id id list tail
id_list_tail ::= ;
```

Remaining Input: C;



```
id_list ::= id id_list_tail
id_list_tail ::= , id id_list_tail
id_list_tail ::= ;
```

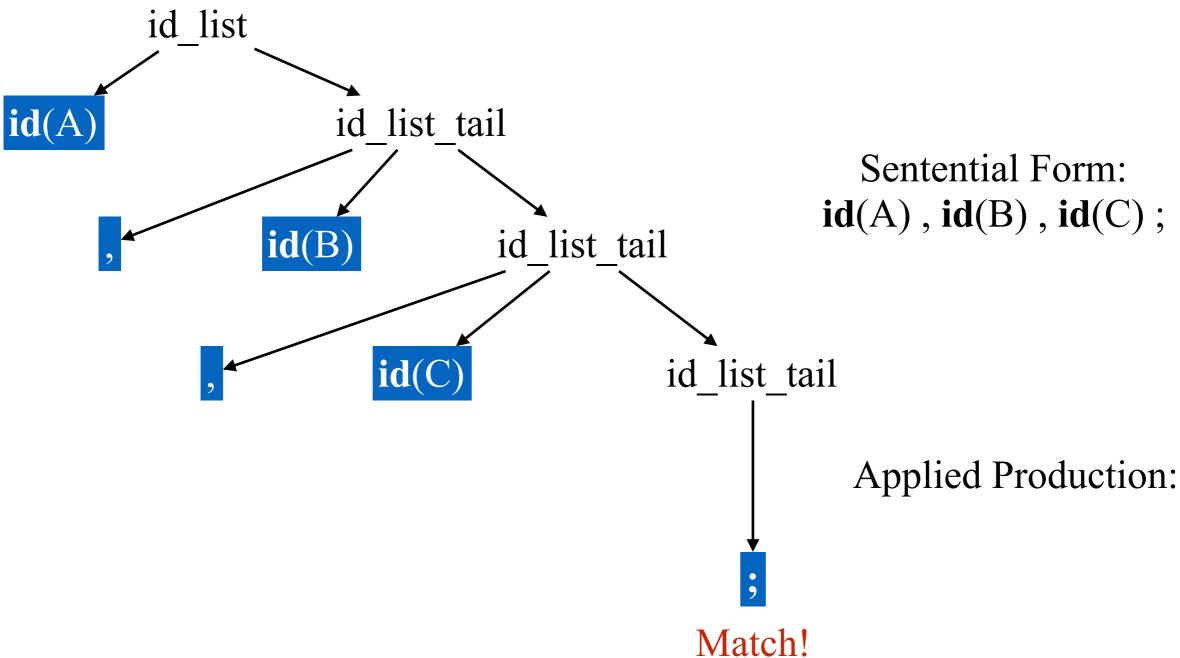
Remaining Input:



```
id list ::= id id list tail
                                                        Remaining Input:
  id_list_tail ::= , id id_list_tail
  id list tail ::=;
         id list
id(A
                      id list tail
                                                         Sentential Form:
                                                      id(A), id(B), id(C);
                 id(B)
                                id list tail
                        id(C
                                            id list tail
                                                        Applied Production:
                                                           id list tail ::=;
```

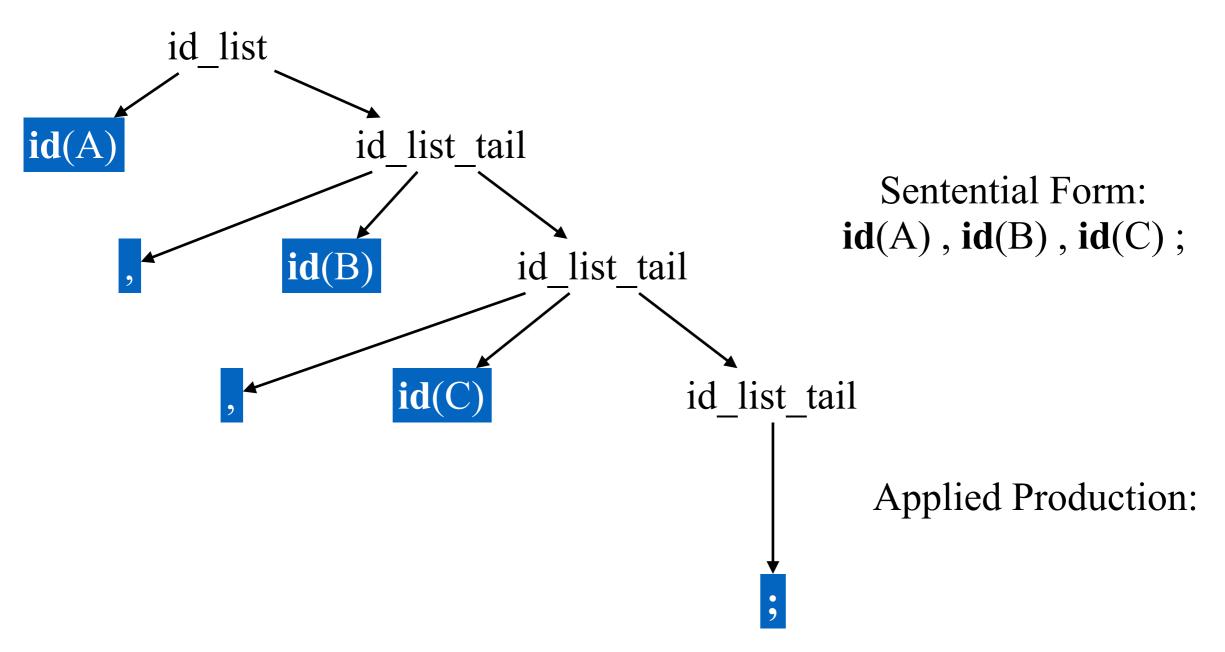
```
id_list ::= id id_list_tail
id_list_tail ::= , id id_list_tail
id_list_tail ::= ;
```

Remaining Input:



```
id_list ::= id id_list_tail
id_list_tail ::= , id id_list_tail
id_list_tail ::= ;
```

Remaining Input:



Predictive Parsing

Basic idea:

a string of symbols

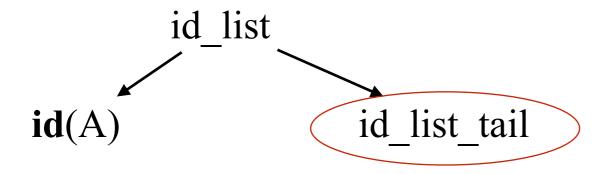
For any two productions $A := \alpha \mid \beta$, we would like a distinct way of choosing the correct production to expand.

For some rhs $\alpha \in G$, define **FIRST**(α) as the set of tokens that appear as the first symbol in some string derived from α .

That is

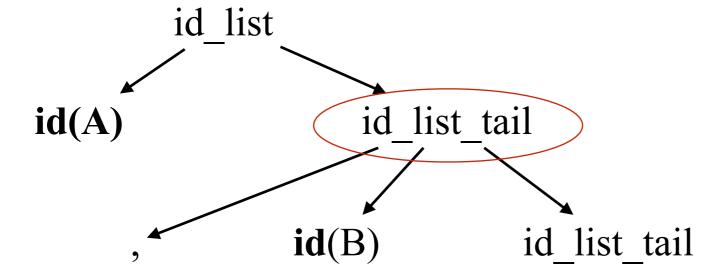
 $x \in FIRST(\alpha)$ iff $\alpha \Rightarrow^* x \gamma$ for some γ

```
id_list ::= id id_list_tail
id_list_tail ::= , id id_list_tail
id_list_tail ::= ;
```



Remaining Input: , B , C ;

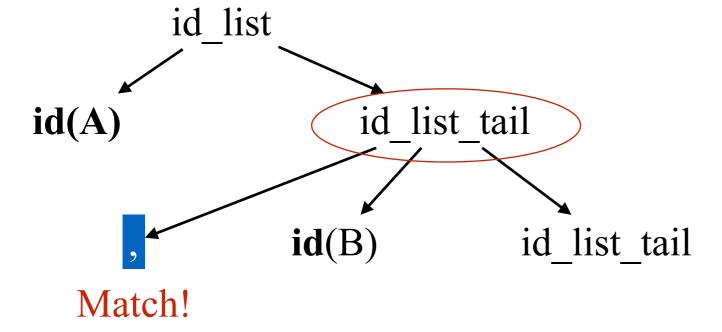
```
id_list ::= id id_list_tail
id_list_tail ::= , id id_list_tail
id_list_tail ::= ;
```



Remaining Input: , B , C ;

Applied Production: id_list_tail ::= , id id_list_tail

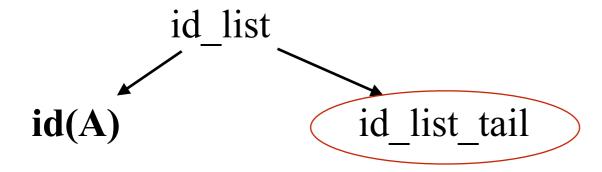
```
id_list ::= id id_list_tail
id_list_tail ::= , id id_list_tail
id_list_tail ::= ;
```



Remaining Input: ,B,C;

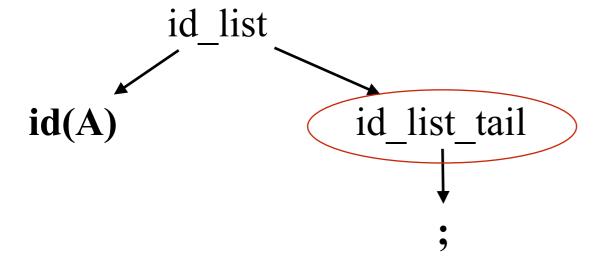
Applied Production: id_list_tail ::= , id id_list_tail

```
id_list ::= id id_list_tail
id_list_tail ::= , id id_list_tail
id_list_tail ::= ;
```



Remaining Input: ,B,C;

```
id_list ::= id id_list_tail
id_list_tail ::= , id id_list_tail
id_list_tail ::= ;
```



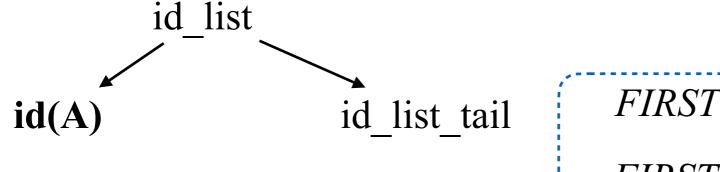
Mismatch!

Remaining Input: ,B,C;

Applied Production: id_list_tail::=;

```
id_list ::= id id_list_tail
id_list_tail ::= , id id_list_tail
id_list_tail ::= ;
```

Remaining Input: , B , C ;



Given id_list_tail as the first non-terminal to expand in the tree:

If the first token of remaining input is "," we choose the rule id_list_tail ::= , id id_list_tail

If the first token of remaining input is ";" we choose the rule id list tail ::=;

Predictive Parsing

Key Property:

Whenever two productions $A := \alpha$ and $A := \beta$ both appear in the grammar, we would like

• $FIRST(\alpha) \cap FIRST(\beta) = \emptyset$

```
id_list ::= id id_list_tail
id_list_tail ::= , id id_list_tail
id_list_tail ::= ;
```

Remaining Input: , B , C ;



$$FIRST($$
, id id_list_tail $\cap FIRST($; $) = \emptyset$

Given id_list_tail as the first **non-terminal** to expand in the tree:

If the first token of remaining input is, we choose the rue id_list_tail ::=, id id_list_tail

If the first token of remaining input is; we choose the rule id list tail ::=;

Predictive Parsing

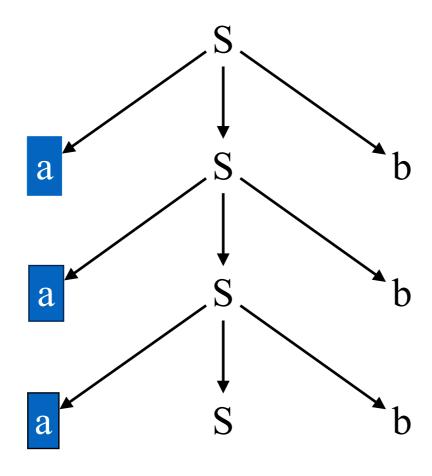
Key Property:

Whenever two productions $A := \alpha$ and $A := \beta$ both appear in the grammar, we would like

•
$$FIRST(\alpha) \cap FIRST(\beta) = \emptyset$$

This rule is intuitive. However, it is **not enough**, because it doesn't handle ε rules. How to handle ε rules?

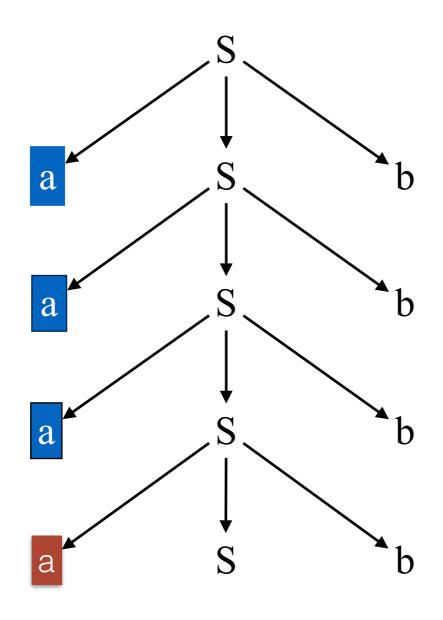
$$S := a S b | \varepsilon$$



Remaining Input:

(b) b

$$S := a S b | \varepsilon$$



Remaining Input: b b b

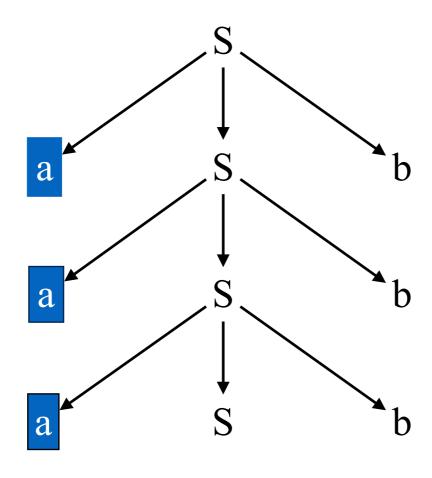
Applied Production:

S := a S b

Mismatch!

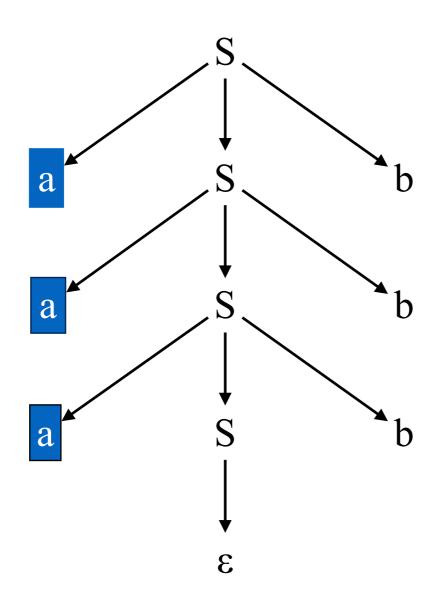
It only means S := aSb is not the right production rule to use!

$$S := a S b | \varepsilon$$



Remaining Input: b b b

$$S := a S b | \varepsilon$$



Remaining Input: b b b

Applied Production:

 $S := \varepsilon$

 $S := \varepsilon$ turns out to be the right rule later.

However, at this point, ε does not match "b" either!

For a non-terminal A, define **FOLLOW**(A) as the set of terminals that can appear immediately to the right of A in some sentential form.

Thus, a non-terminal's **FOLLOW** set specifies the tokens that can legally appear after it. A terminal symbol has no **FOLLOW** set.

FIRST and FOLLOW sets can be constructed automatically

Key Property:

Whenever two productions $A := \alpha$ and $A := \beta$ both appear in the grammar, we would like

• $FIRST(\alpha) \cap FIRST(\beta) = \emptyset$

This would allow the parser to make a correct choice with a lookahead of only one symbol!

Key Property:

Whenever two productions $A := \alpha$ and $A := \beta$ both appear in the grammar, we would like

- $FIRST(\alpha) \cap FIRST(\beta) = \emptyset$, and
- if $\alpha \Rightarrow * \epsilon$, then $FIRST(\beta) \cap FOLLOW(A) = \emptyset$

This would allow the parser to make a correct choice with a lookahead of only one symbol!

Key Property:

Whenever two productions $A := \alpha$ and $A := \beta$ both appear in the grammar, we would like

- $FIRST(\alpha) \cap FIRST(\beta) = \emptyset$, and
- if $\alpha \Rightarrow * \epsilon$, then $FIRST(\beta) \cap FOLLOW(A) = \emptyset$
- Analogue case for $\beta \Rightarrow^* \epsilon$. Note: due to first condition, at most one of α and β can derive ϵ .

This would allow the parser to make a correct choice with a lookahead of only one symbol!

LL(1) Grammar

Define $PREDICT(A := \delta)$ for rule $A := \delta$

- $FIRST(\delta)$ { ε } U Follow (A), if $\varepsilon \in FIRST(\delta)$
- $FIRST(\delta)$ otherwise

A Grammar is LL(1) iff

 $(A := \alpha \text{ and } A := \beta) \text{ implies}$

PREDICT(A ::= α) \cap PREDICT(A ::= β) = \emptyset

Start ::=
$$S$$
 eof
 S ::= a S b $\mid \epsilon$

$$FIRST(aSb) = FIRST(\epsilon) = FOLLOW(S) = FOLLOW(S) = FOLLOW(S)$$

$$PREDICT(S := aSb) =$$

$$PREDICT(S := \epsilon) =$$

- $FIRST(\delta)$ { ε } U Follow (A), if $\varepsilon \in FIRST(\delta)$
- $FIRST(\delta)$ otherwise

Start ::= S eof
S ::= a S b |
$$\varepsilon$$

 $FIRST(aSb) = \{a\}$
 $FIRST(\varepsilon) =$
 $FOLLOW(S) =$

$$PREDICT(S := aSb) =$$

$$PREDICT(S := \epsilon) =$$

- $FIRST(\delta)$ { ε } U Follow (A), if $\varepsilon \in FIRST(\delta)$
- $FIRST(\delta)$ otherwise

Start ::= S eof
S ::= a S b |
$$\varepsilon$$

 $FIRST(aSb) = \{a\}$
 $FIRST(\varepsilon) = \{\varepsilon\}$
 $FOLLOW(S) = \{eof, b\}$

$$PREDICT(S := \varepsilon) =$$

- $FIRST(\delta)$ { ε } U Follow (A), if $\varepsilon \in FIRST(\delta)$
- $FIRST(\delta)$ otherwise

```
Start ::= S eof

S ::= a S b | \varepsilon

FIRST(aSb) = \{a\}
FIRST(\varepsilon) = \{\varepsilon\}
FOLLOW(S) = \{eof, b\}
PREDICT(S ::= aSb) = \{a\}
PREDICT(S ::= \varepsilon) = \varepsilon
```

- $FIRST(\delta)$ { ε } U Follow (A), if $\varepsilon \in FIRST(\delta)$
- $FIRST(\delta)$ otherwise

```
Start ::= S eof

S ::= a S b | \varepsilon

FIRST(aSb) = \{a\}
FIRST(\varepsilon) = \{\varepsilon\}
FOLLOW(S) = \{eof, b\}
PREDICT(S ::= aSb) = \{a\}
PREDICT(S ::= \varepsilon) = (FIRST(\varepsilon) - \{\varepsilon\}) \cup FOLLOW(S)
```

- $FIRST(\delta)$ { ε } U Follow (A), if $\varepsilon \in FIRST(\delta)$
- $FIRST(\delta)$ otherwise

```
Start ::= S eof

S ::= a S b | \varepsilon

FIRST(aSb) = \{a\}
FIRST(\varepsilon) = \{\varepsilon\}
FOLLOW(S) = \{eof, b\}
PREDICT(S ::= aSb) = \{a\}
PREDICT(S ::= \varepsilon) = (FIRST(\varepsilon) - \{\varepsilon\}) \cup FOLLOW(S) = \{eof, b\}
```

- *FIRST* (δ) { ϵ } U Follow (A), if $\epsilon \in FIRST(\delta)$
- $FIRST(\delta)$ otherwise

Start ::= S eof
S ::= a S b |
$$\varepsilon$$

 $FIRST(aSb) = \{a\}$
 $FIRST(\varepsilon) = \{\varepsilon\}$
 $FOLLOW(S) = \{eof, b\}$

Is the grammar LL(1)?

$$PREDICT(S := aSb) = \{a\}$$

 $PREDICT(S := \epsilon) = (FIRST(\epsilon) - \{\epsilon\}) \cup FOLLOW(S) = \{eof, b\}$

- $FIRST(\delta)$ { ε } U Follow (A), if $\varepsilon \in FIRST(\delta)$
- $FIRST(\delta)$ otherwise

Example:

$$S ::= \mathbf{a} S \mathbf{b} \mid \varepsilon$$

LL(1) parse table

How to parse input a a a b b b?

Example:

$$S := \mathbf{a} S \mathbf{b} \mid \varepsilon$$

LL(1) parse table

	a	ь	eof	other
S	aSb	3	3	error

How to parse input a a a b b b?

```
Input: a string w and a parsing table M for G
            push eof
              push Start Symbol
             token \leftarrow next \ token()
            X \leftarrow \text{top-of-stack}
             repeat
                 if X is a terminal then
                   if X == token then
                      pop X
                      token \leftarrow next \ token()
                    else error()
                  else /* X is a non-terminal */
                       if M[X, token] == X \rightarrow Y_1Y_2 \dots Y_k then
                           pop X
                           push Y_k, Y_{k-1}, \ldots, Y_1
                        else error()
                   X \leftarrow top-of-stack
              until X = eof
              if token != eof then error()
```

```
Input: a string w and a parsing table M for G
             push eof
             push Start Symbol
             token \leftarrow next \ token()
             X \leftarrow top-of-stack
             repeat
                if X is a terminal then
                   if X == token then
                      pop X
                      token \leftarrow next \ token()
             else error()
                 else /* X is a non-terminal */
                      if M[X, token] == X \rightarrow Y_1Y_2 \dots Y_k then
                          pop X
                          push Y_k, Y_{k-1}, \ldots, Y_1
                       else error()
                   X \leftarrow top-of-stack
             until X = eof
             if token != eof then error()
```

```
Input: a string w and a parsing table M for G
              push eof
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              token \leftarrow next \ token()
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              repeat
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                       if M[X, token] == X \rightarrow Y_1Y_2 \dots Y_k then
                           pop X
                           push Y_k, Y_{k-1}, \ldots, Y_1
                        else error()
                    X \leftarrow \text{top-of-stack}
              until X = eof
              if token != eof then error()
```

```
Input: a string w and a parsing table M for G
            push eof
            push Start Symbol
            token \leftarrow next \ token()
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            repeat
               if X is a terminal then
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                    token \leftarrow next \ token()
                 else error()
                else /* X is a non-terminal */
                    if M[X, token] == X \rightarrow Y_1Y_2 \dots Y_k then
                        pop X
                        push Y_k, Y_{k-1}, \ldots, Y_1
                     else error()
                                  -----,
              X \leftarrow \text{top-of-stack}
            until X = eof
            if token != eof then error()
```

```
Input: a string w and a parsing table M for G
              push eof
              push Start Symbol
              token \leftarrow next \ token()
              X \leftarrow \text{top-of-stack}
              repeat
                  if X is a terminal then
                    if X == token then
                       pop X
                       token \leftarrow next \ token()
                    else error()
                  else /* X is a non-terminal */
                        if M[X, token] == X \rightarrow Y_1Y_2 \dots Y_k then
                           pop X
                           push Y_k, Y_{k-1}, \ldots, Y_1
                        else error()
                    X \leftarrow \text{top-of-stack}
             \int until X = eof
              if token != eof then error()
```

Top - Down Parsing - LL(1) (cont.)

Example:

$$S := a S b | \varepsilon$$

How can we parse (automatically construct a leftmost derivation) the input string **a a a b b b** using a PDA (push-down automaton) and only the first symbol of the remaining input?

INPUT: | a a a b b b eof

 $S := a S b | \varepsilon$

S

Remaining Input: a a a b b b

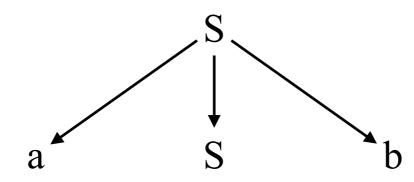
S

Sentential Form: S

Applied Production:

a b eof other S aSb ε ε error

$$S := a S b \mid \varepsilon$$



Remaining Input: a a a b b b

Sentential Form: a S b

Applied Production: S := a S b

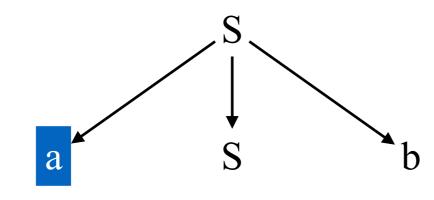
abeofotherSaSbεεerror

a

S

b

$$S := a S b | \varepsilon$$



Remaining Input:

a a b b b

Match! Sentential Form: a S b

Applied Production:

a b eof other
S aSb ε ε error

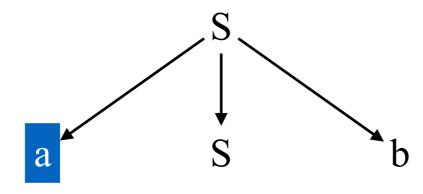
a

S

b

$$S := a S b | \varepsilon$$

b



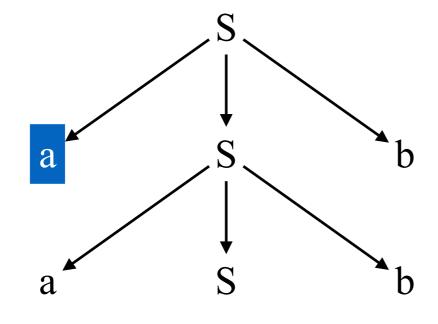
Remaining Input: a a b b b

Sentential Form: a S b

Applied Production:

a b eof other
S aSb ε ε error

$$S := a S b | \varepsilon$$



Remaining Input: a a b b b

Sentential Form: a a S b b

Applied Production: S := a S b

	a	b	eof	other
S	aSb	3	3	error

b

a

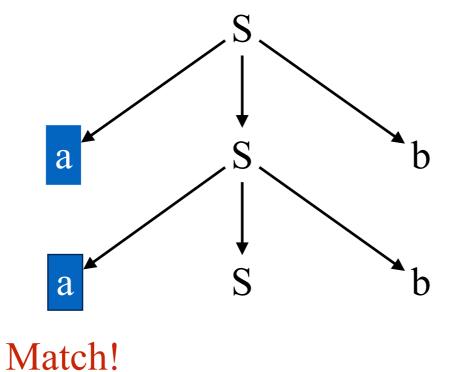
$$S := a S b | \varepsilon$$

a

S

b

b



Remaining Input: a a b b b

Sentential Form: a a S b b

Applied Production:

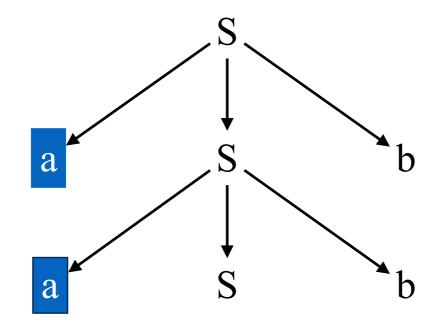
abeofotherSaSbεεerror

$$S := a S b | \varepsilon$$

S

b

b



Remaining Input: a b b b

Sentential Form: a a S b b

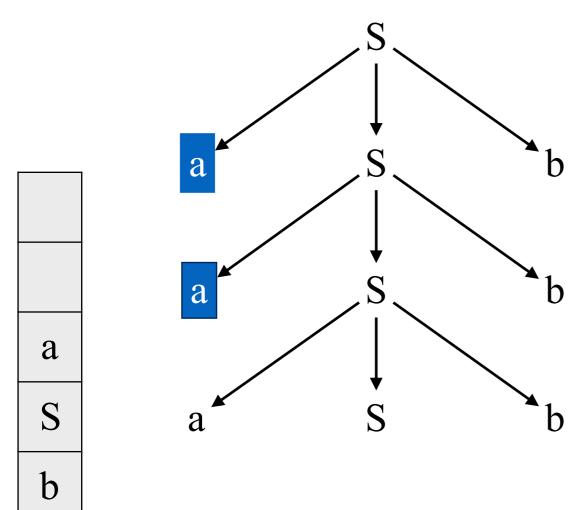
Applied Production:

a b eof other
S aSb ε ε error

 $S := a S b | \varepsilon$

b

b



Remaining Input: a b b b

Sentential Form: a a a S b b b

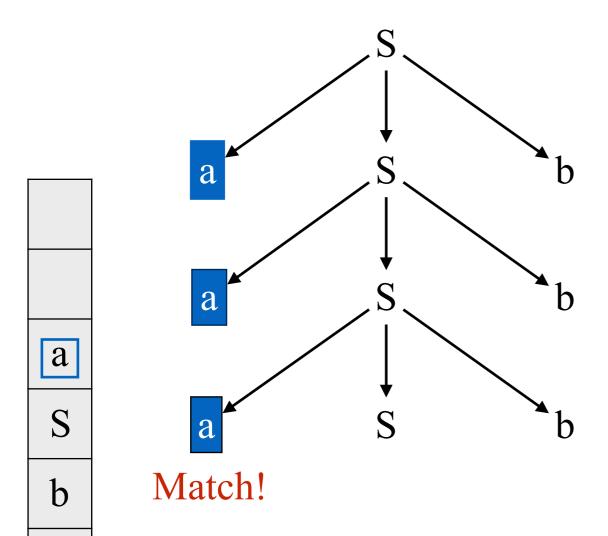
Applied Production: S := a S b

	a	b	eof	other
S	aSb	3	3	error

 $S := a S b | \varepsilon$

b

b



Remaining Input:

a b b b

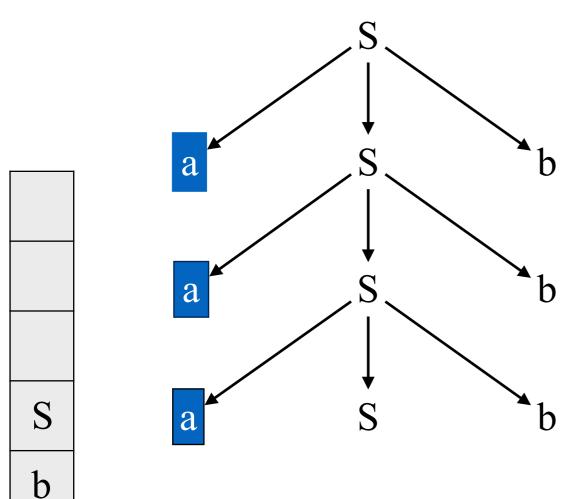
Sentential Form: a a a S b b b

	a	b	eof	other
S	aSb	3	3	error

 $S := a S b | \varepsilon$

b

b



Remaining Input: b b b

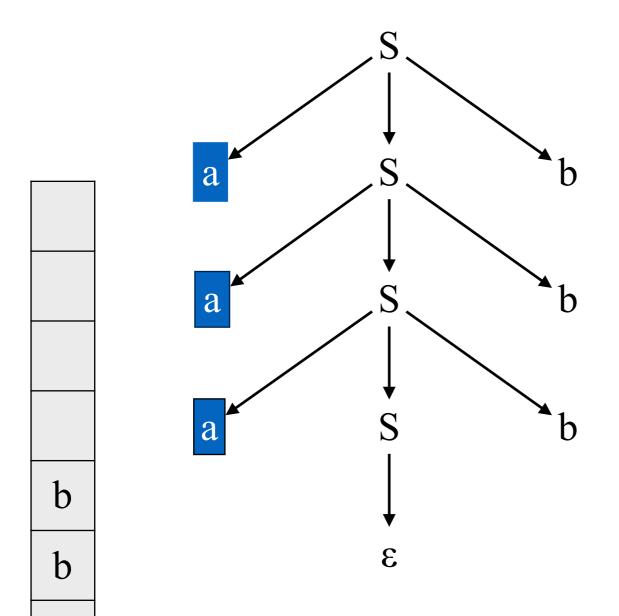
Sentential Form: a a a S b b b

Applied Production:

a b eof other
S aSb ε ε error

$$S := a S b | \varepsilon$$

b



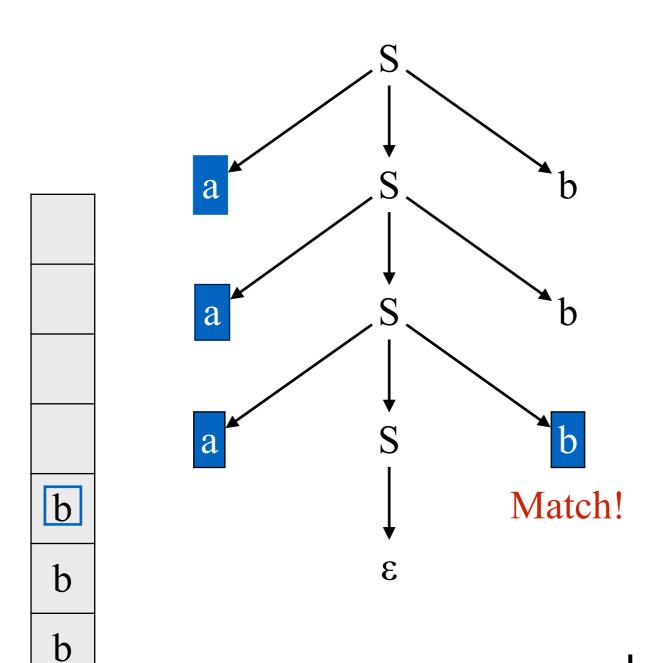
Remaining Input: b b b

Sentential Form: a a a b b b

$$S := \varepsilon$$

	a	b	eof	other
S	aSb	3	3	error

 $S := a S b | \varepsilon$

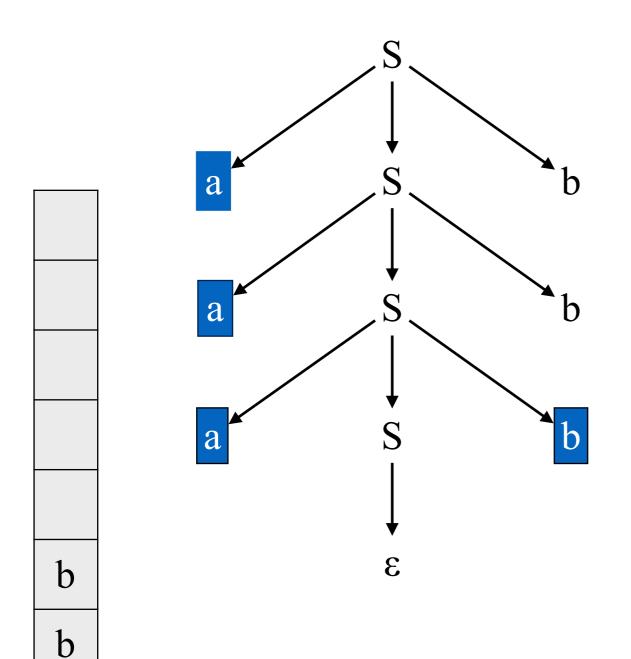


Remaining Input: b b b

Sentential Form: a a a b b b

	a	b	eof	other
S	aSb	3	3	error

$$S := a S b | \varepsilon$$

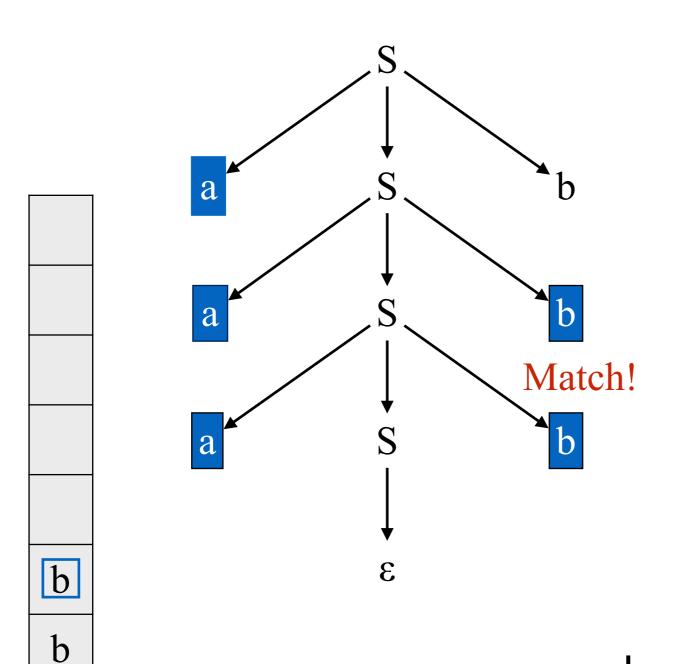


Remaining Input: b b

Sentential Form: a a a b b b

	a	b	eof	other
S	aSb	3	3	error

 $S := a S b | \varepsilon$

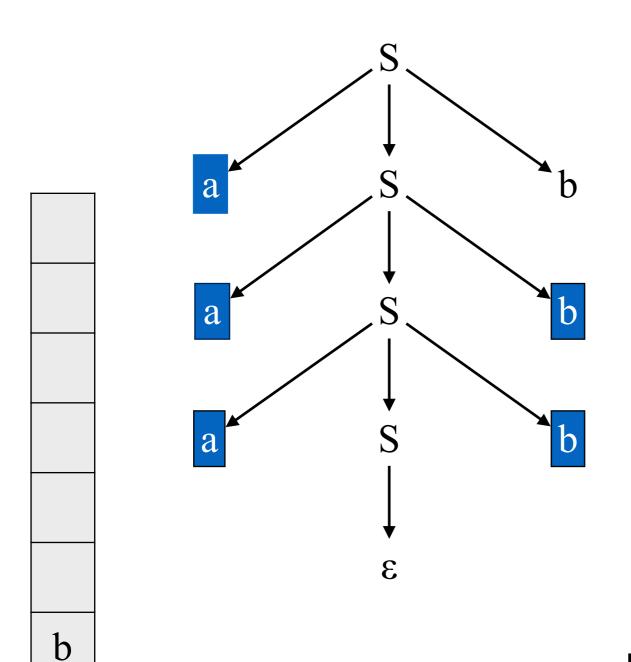


Remaining Input: bb

Sentential Form: a a a b b b

	a	b	eof	other
S	aSb	3	3	error

 $S := a S b | \varepsilon$

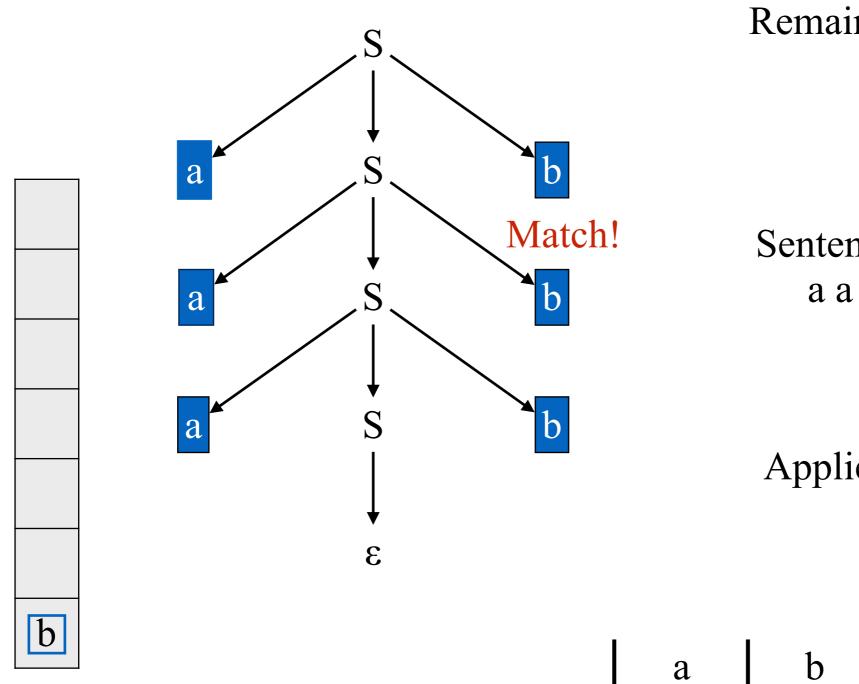


Remaining Input: b

Sentential Form: a a a b b b

	a	b	eof	other
S	aSb	3	3	error

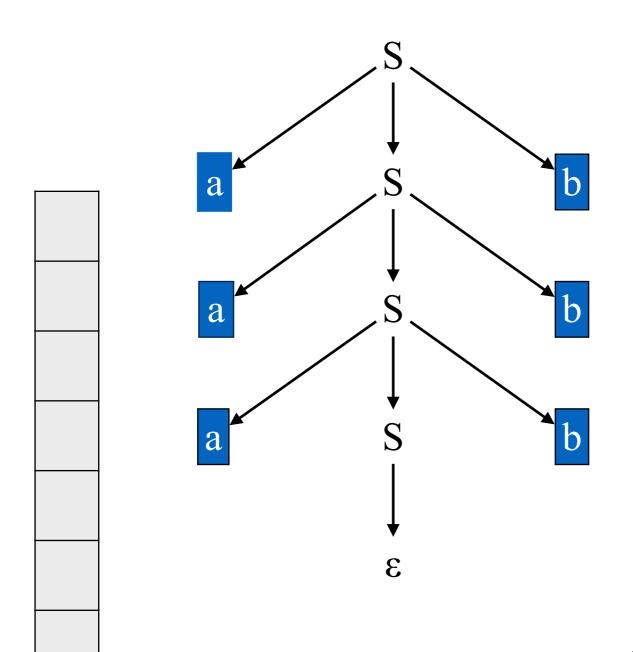
 $S := a S b | \varepsilon$



Remaining Input:

Sentential Form: a a a b b b

$$S := a S b | \varepsilon$$

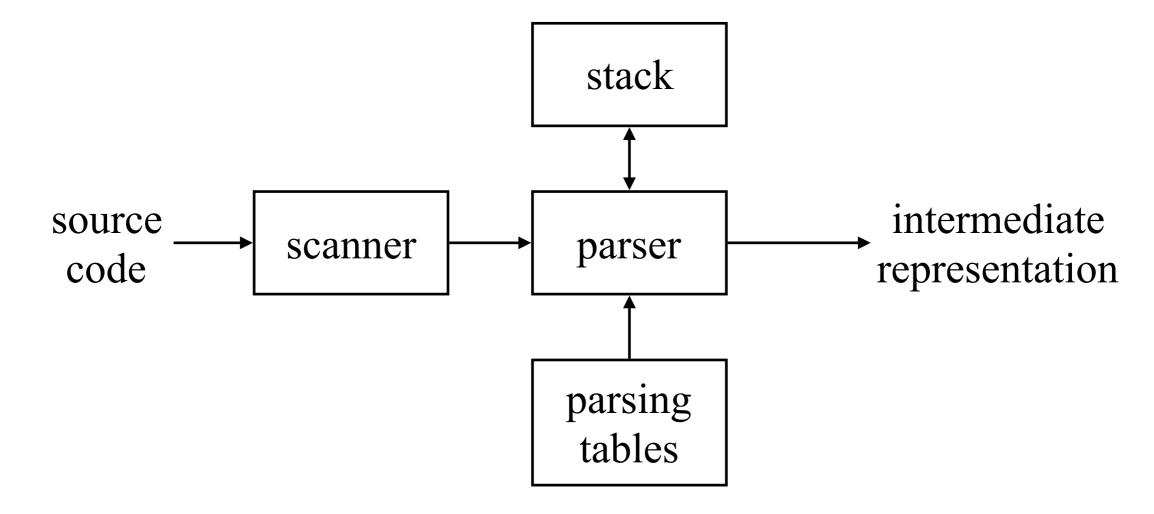


Remaining Input:

Sentential Form: a a a b b b

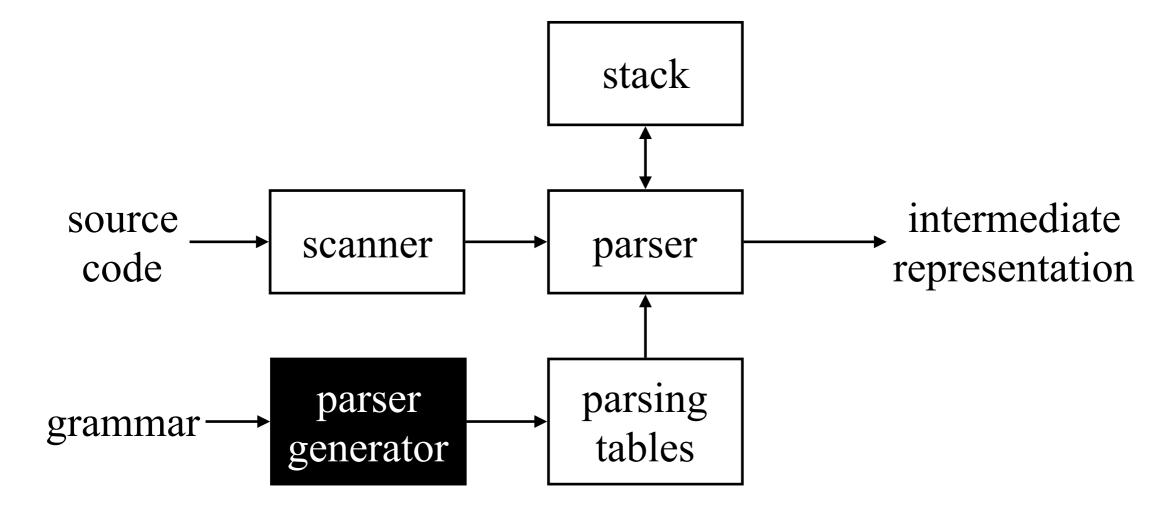
	a	b	eof	other
S	aSb	3	3	error

Now, a predictive parser looks like:



Rather than writing code, we build tables. Building tables can be automated!

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Recursive Descent Parsing

Now, we can produce a recursive descent parser for our LL(1) grammar. Recursive descent is one of the simplest parsing techniques used in practical compilers:

- Each **non-terminal** has an associated parsing procedure that can recognize any sequence of tokens generated by that **non-terminal**
- There is a main routine to initialize all globals (e.g: *tokens*) and call the start symbol. On return, check whether token==eof, and whether errors occurred
- Within a parsing procedure, both **non-terminals** and terminals can be matched:
 - non-terminal A: call procedure for A
 - ▶ token t: compare t with current input token; if matched, consume input, otherwise, ERROR
- Parsing procedure may contain code that performs some useful "computations" (*syntax directed translation*)

Recursive Descent Parsing (pseudo code)

	a	b	eof	other
S	aSb	3	3	error

```
main: {
    token := next_token();
    if (S() and token == eof) print "accept" else print "error";
}
```

Recursive Descent Parsing (pseudo code)

```
eof
                                      other
             a
   S
           aSb
                      3
                               3
                                      error
bool S: {
        switch token {
              case a: token := next token();
                       call S();
                       if (token == b) {
                          token := next_token( );
                           return true;
                      else
                          return false;
                      break;
             case b:
             case eof: return true;
                        break;
             default: return false;
```

Next Lecture

Next Time:

- LL(1) parsing and syntax directed translation
- Read Scott, Chapter 2.3.1 2.3.3