

CS314-Fall-2018-Assign7-Solution

1 Scheme Programming

1.1

`((lambda(x)(lambda(y)((lambda(z)e)v3)v2))v1)`

1.2

```
(define maxAbsoluteVal
  (lambda(l)
    (let
      ((mal
        (map
          (lambda(x)(abs x))
          l)
        ))
      (reduce
        (lambda(x y)
          (if (> x y) x y)
        )
        mal -inf.0
      )
    )
  )
)
```

2 Lambda Calculus

2.1

$((\lambda x.x)(\lambda x.28))(\lambda z.z) = ((\lambda x.28)(\lambda z.z)) = 28$
No other order

2.2

$((\lambda x.((\lambda z.((\lambda x.(z\ x))\ 2))(\lambda y.(*\ x\ y))))\ 6)$
 $= ((\lambda x.((\lambda z.(z\ 2))(\lambda y.(*\ x\ y))))\ 6)$

$$= ((\lambda x. ((\lambda y. (* x y)) 2)) 6) = ((\lambda x. (* x 2)) 6)$$

$$= (* 2 6) = 12$$
 Other order:

$$((\lambda x. ((\lambda z. ((\lambda x. (z x)) 2)) (\lambda y. (* x y)))) 6)$$

$$= ((\lambda x. ((\lambda z. ((\lambda m. (z m)) 2)) (\lambda y. (* x y)))) 6)$$

$$= (\lambda z. ((\lambda m. (z m)) 2)) (\lambda y. (* 6 y)) = (\lambda m. (\lambda y. (* 6 y) m)) 2$$

$$= (\lambda m. (* 6 m) 2) = 12$$
 result should be the same

2.3

$$((\lambda z. ((\lambda y. z) ((\lambda x. (x x)) (\lambda x. (x x))))) 11)$$

$$= ((\lambda y. 11) ((\lambda x. (x x)) (\lambda x. (x x))))$$

$$= 11$$
 Other order:

$$((\lambda z. ((\lambda y. z) ((\lambda x. (x x)) (\lambda x. (x x))))) 11)$$

$$= ((\lambda z. ((\lambda y. z) ((\lambda x. (x x)) (\lambda m. (m m))))) 11)$$

$$= ((\lambda z. ((\lambda y. z) ((\lambda m. (m m)) (\lambda m. (m m))))) 11)$$

$$= ((\lambda y. 11) ((\lambda m. (m m)) (\lambda m. (m m)))) = 11$$
 result should be the same

3 Programming in Lambda Calculus

3.1

$$((\text{and true}) \text{true})$$

$$= (((\lambda u. (\lambda v. ((u v) \text{false}))) (\lambda x. (\lambda y. x))) \text{true})$$

$$= ((\lambda v. (((\lambda x. (\lambda y. x)) v) \text{false})) \text{true})$$

$$= (((\lambda x. (\lambda y. x)) \text{true}) \text{false})$$

$$= ((\lambda y. \text{true}) \text{false}) = \text{true}$$

3.2

or: $\lambda x. \lambda y. ((x \text{true}) y)$
 prove:

$$((\text{or true}) \text{false})$$

$$= (((\lambda x. \lambda y. ((x \text{true}) y)) \text{true}) \text{false})$$

$$= ((\lambda y. ((\text{true true}) y)) \text{false})$$

$$= ((\text{true true}) \text{false})$$

$$= (((\lambda a. \lambda b. a) \text{true}) \text{false})$$

$$= ((\lambda b. \text{true}) \text{false})$$

$$= \text{true}$$

$$((\text{or true}) \text{true})$$

$$= (((\lambda x. \lambda y. ((x \text{true}) y)) \text{true}) \text{true})$$

$$= ((\lambda y. ((\text{true true}) y)) \text{true})$$

$$= \text{true}$$

```

((or false) true)
=(((λx.λy.((x true) y)) false) true)
=((λy.((false true) y)) true)
=(((λa.λb.b) true) true)
=((λb.b)true)
=true
((or false) false)
=(((λx.λy.((x true) y)) false) false)
=((λy.((false true) y)) false)
=((false true) false)
=(((λa.λb.b) true) false)
=((λb.b) false)
=false

```

3.3

```

NOT:λx.((x false) true)
XOR:λx.λy.((x(NOT y)) y)
prove:
((xor true) false)
=((true (NOT false)) false)
=((true true) false)
=((λb.true) false)
=true
((xor true) true)
=((true (NOT true)) true)
=((true false) true)
=((λb.false) true)
=false
((xor false) true)
=((false (NOT false)) true)
=((false false) true)
=((λb.b) true)
=true
((xor false) false)
=((false (NOT false)) false)
=((false true) false)
=((λb.b) false)
=false

```

4 Lambda Calculus and Combinators S & K

```

((SK)K)
=(((λxyz.((x z)(y z)))K)K)
=((λyz.((K z)(y z)))K)

```

$$\begin{aligned}
&= (\lambda z. ((K\ z)(K\ z))) \\
&= (\lambda z. (((\lambda xy.x)\ z)(K\ z))) \\
&= (\lambda z. ((\lambda y.z)(K\ z))) \\
&= (\lambda z.z) = (\lambda x.x) = I
\end{aligned}$$