

# CS 314 Principles of Programming Languages

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## Lecture 9: LL(1) Parsing Review

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# Class Information

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- Homework 4 will be posted after lecture 10.
- Project 1 will be posted after homework 4 is due.

# Review: FIRST and FOLLOW Sets

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## **FIRST( $\alpha$ ):**

For some  $\alpha \in (T \cup NT \cup EOF \cup \varepsilon)^*$ , define **FIRST** ( $\alpha$ ) as the set of tokens that appear as the first symbol in some string that derives from  $\alpha$ .

That is,  $\mathbf{x} \in \text{FIRST}(\alpha)$  iff  $\alpha \Rightarrow^* \mathbf{x}\gamma$  for some  $\gamma$

T: terminals    NT: non-terminals

# First Set Example

Start ::= S eof

S ::= a S b |  $\epsilon$

$$FIRST(\epsilon) = \{\epsilon\}$$

S can be rewritten as the following:

ab  
aaabbb  
aabb  
 $\epsilon$   
...

$$FIRST(S) = \{a, \epsilon\}$$

aSb can be rewritten as the following:

ab  
aabb  
...

$$FIRST(aSb) = \{a\}$$

# Computing *FIRST* Sets

For a production  $A \rightarrow B_1 B_2 \dots B_k$  :

- $\text{FIRST}(A)$  includes  $\text{FIRST}(B_1) - \varepsilon$
- $\text{FIRST}(A)$  includes  $\text{FIRST}(B_2) - \varepsilon$  if  $B_1$  can be rewritten as  $\varepsilon$
- $\text{FIRST}(A)$  includes  $\text{FIRST}(B_3) - \varepsilon$  if both  $B_1$  and  $B_2$  can derive  $\varepsilon$
- ...
- $\text{FIRST}(A)$  includes  $\text{FIRST}(B_m) - \varepsilon$  if  $B_1 B_2 \dots B_{m-1}$  can derive  $\varepsilon$

$\text{FIRST}(A)$  includes  $\text{FIRST}(B_1) \dots \text{FIRST}(B_m)$  excluding  $\varepsilon$  iff  
 $\varepsilon \in \text{FIRST}(B_1), \text{FIRST}(B_2), \text{FIRST}(B_3), \dots, \text{FIRST}(B_{m-1})$

$\text{FIRST}(A)$  includes  $\varepsilon$  iff  
 $\varepsilon \in \text{FIRST}(B_1), \text{FIRST}(B_2), \text{FIRST}(B_3), \dots, \text{FIRST}(B_k)$

# First Set Construction

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Build  $\text{FIRST}(X)$  for all grammar symbols  $X$ :

- For each  $X$  as a terminal, then  $\text{FIRST}(X)$  is  $\{X\}$
- If  $X ::= \varepsilon$ , then  $\varepsilon \in \text{FIRST}(X)$
- For each  $X$  as a non-terminal, initialize  $\text{FIRST}(X)$  to  $\emptyset$
- ***Iterate until*** no more terminals or  $\varepsilon$  can be added to any  $\text{FIRST}(X)$ :

For each rule in the grammar of the form  $X ::= Y_1 Y_2 \dots Y_k$

add  $a$  to  $\text{FIRST}(X)$  if  $a \in \text{FIRST}(Y_1)$

add  $a$  to  $\text{FIRST}(X)$  if  $a \in \text{FIRST}(Y_i)$  and  $\varepsilon \in \text{FIRST}(Y_j)$

for all  $1 \leq j \leq i-1$  and  $i \geq 2$

add  $\varepsilon$  to  $\text{FIRST}(X)$  if  $\varepsilon \in \text{FIRST}(Y_i)$  for all  $1 \leq i \leq k$

EndFor

***End iterate***

# An Example

parentheses grammar

1 Goal ::= List

2 List ::= Pair List

3       | ε

4 Pair ::= LP List RP

Iter. 1 means iteration 1

Where LP is ( and RP is )

## Initialization

- For each X as a terminal, then FIRST(X) is {X}
  - If  $X ::= \epsilon$ , then  $\epsilon \in \text{FIRST}(X)$
  - For each X as a non-terminal, initialize FIRST(X) to  $\emptyset$
  - **Iterate until** no more terminals or  $\epsilon$  can be added to any FIRST(X):

For each rule in the grammar of the form  $X ::= Y_1Y_2...Y_k$ 

add a to FIRST(X) if  $a \in \text{FIRST}(Y_1)$

add a to FIRST(X) if  $a \in \text{FIRST}(Y_i)$  and  $\epsilon \in \text{FIRST}(Y_j)$ 

for all  $1 \leq j \leq i-1$  and  $i \geq 2$

add  $\epsilon$  to FIRST(X) if  $\epsilon \in \text{FIRST}(Y_i)$  for all  $1 \leq i \leq k$

EndFor
- End iterate**

Symbol	Initial	Iter. 1	Iter. 2
Goal	∅		
List	∅		
Pair	∅		
LP	<u>LP</u>	<u>LP</u>	<u>LP</u>
RP	<u>RP</u>	<u>RP</u>	<u>RP</u>
EOF	EOF	EOF	EOF

# An Example

parentheses grammar

1 Goal ::= List

2 List ::= Pair List

3       | ε

4 Pair ::= LP List RP

Iter. 1 means iteration 1

Where LP is ( and RP is )

## Iteration 1 (of the outer loop below):

- For each X as a terminal, then FIRST(X) is {X}
- If  $X ::= \epsilon$ , then  $\epsilon \in \text{FIRST}(X)$
- For each X as a non-terminal, initialize FIRST(X) to  $\emptyset$
- *Iterate until* no more terminals or  $\epsilon$  can be added to any FIRST(X):

For each rule in the grammar of the form  $X ::= Y_1 Y_2 \dots Y_k$ 

add a to FIRST(X) if  $a \in \text{FIRST}(Y_1)$

add a to FIRST(X) if  $a \in \text{FIRST}(Y_i)$  and  $\epsilon \in \text{FIRST}(Y_j)$ 

for all  $1 \leq j \leq i-1$  and  $i \geq 2$

add  $\epsilon$  to FIRST(X) if  $\epsilon \in \text{FIRST}(Y_i)$  for all  $1 \leq i \leq k$

EndFor

Symbol	<i>Initial</i>	Iter. 1	Iter. 2
Goal	$\emptyset$		
List	$\emptyset$		
Pair	$\emptyset$		
LP	<u>LP</u>	<u>LP</u>	<u>LP</u>
RP	<u>RP</u>	<u>RP</u>	<u>RP</u>
EOF	EOF	EOF	EOF



# An Example

parentheses grammar

1

Goal ::= List

2

List ::= Pair List

3

|  $\epsilon$

4

Pair ::= LP List RP

Iter. 1 means iteration 1

Where LP is ( and RP is )

*FIRST sets in progress*

Iteration 1:

The order of the rules do not  
affect the final FIRST set results:

If we visit the rules  
in order 4, 3, 2, 1  $\Rightarrow$

Symbol	<i>Initial</i>	Iter. 1	Iter. 2
Goal	$\emptyset$		
List	$\emptyset$		
Pair	$\emptyset$		
LP	<u>LP</u>	<u>LP</u>	<u>LP</u>
RP	<u>RP</u>	<u>RP</u>	<u>RP</u>
EOF	EOF	EOF	EOF

# An Example

parentheses grammar

```
1 Goal ::= List
2 List ::= Pair List
3       | ε
4 Pair ::= LP List RP
```

Iter. 1 means iteration 1

Where LP is ( and RP is )

*FIRST sets in progress*

Iteration 1:

Symbol	<i>Initial</i>	Iter. 1	Iter. 2
Goal	∅		
List	∅		
Pair	∅	?	
LP	<u>LP</u>	<u>LP</u>	<u>LP</u>
RP	<u>RP</u>	<u>RP</u>	<u>RP</u>
EOF	EOF	EOF	EOF

Applying Rule 4

**Pair ::= LP List RP**

add first(LP list RP) to first(Pair)

# An Example

parentheses grammar

```
1 Goal ::= List
2 List ::= Pair List
3       | ε
4 Pair ::= LP List RP
```

Iter. 1 means iteration 1

Where LP is ( and RP is )

*FIRST sets in progress*

Iteration 1:

Applying Rule 4

Pair ::= LP List RP

add first(LP list RP) to first(Pair)

Symbol	<i>Initial</i>	Iter. 1	Iter. 2
Goal	∅		
List	∅		
Pair	∅	<u>LP</u>	
LP	<u>LP</u>	<u>LP</u>	<u>LP</u>
RP	<u>RP</u>	<u>RP</u>	<u>RP</u>
EOF	EOF	EOF	EOF

# An Example

parentheses grammar

1 Goal ::= List

2 List ::= Pair List

3       | ε

4 Pair ::= LP List RP

Iter. 1 means iteration 1

Where LP is ( and RP is )

*FIRST sets in progress*

Iteration 1:

Symbol	<i>Initial</i>	Iter. 1	Iter. 2
Goal	∅		
List	∅		
Pair	∅	<u>LP</u>	
LP	<u>LP</u>	<u>LP</u>	<u>LP</u>
RP	<u>RP</u>	<u>RP</u>	<u>RP</u>
EOF	EOF	EOF	EOF

Applying Rule 4

Pair ::= LP List RP

add first(LP list RP) to first(Pair)

# An Example

parentheses grammar

```
1 Goal ::= List
2 List ::= Pair List
3       | ε
4 Pair ::= LP List RP
```

Iter. 1 means iteration 1

Where LP is ( and RP is )

*FIRST sets in progress*

Iteration 1:

Applying Rule 2 and Rule 3

```
List ::= Pair List
       | ε
```

add first(Pair List) to first(List)

add first( $\epsilon$ ) to first(List)

Symbol	<i>Initial</i>	Iter. 1	Iter. 2
Goal	$\emptyset$		
List	$\emptyset$	<u>?</u>	
Pair	$\emptyset$	<u>LP</u>	
LP	<u>LP</u>	<u>LP</u>	<u>LP</u>
RP	<u>RP</u>	<u>RP</u>	<u>RP</u>
EOF	EOF	EOF	EOF

# An Example

parentheses grammar

```
1 Goal ::= List
2 List ::= Pair List
3       | ε
4 Pair ::= LP List RP
```

Iter. 1 means iteration 1

Where LP is ( and RP is )

*FIRST sets in progress*

Iteration 1:

Applying Rule 2 and Rule 3

```
List ::= Pair List
       | ε
```

add first(Pair List) to first(List)

add first( $\epsilon$ ) to first(List)

Symbol	<i>Initial</i>	Iter. 1	Iter. 2
Goal	$\emptyset$		
List	$\emptyset$	<u>LP</u> , $\epsilon$	
Pair	$\emptyset$	<u>LP</u>	
LP	<u>LP</u>	<u>LP</u>	<u>LP</u>
RP	<u>RP</u>	<u>RP</u>	<u>RP</u>
EOF	EOF	EOF	EOF

# An Example

parentheses grammar

```
1 Goal ::= List
2 List ::= Pair List
3       | ε
4 Pair ::= LP List RP
```

Iter. 1 means iteration 1

Where LP is ( and RP is )

*FIRST sets in progress*

Iteration 1:

Applying Rule 2 and Rule 3

```
List ::= Pair List
       | ε
```

add first(Pair List) to first(List)

add first( $\epsilon$ ) to first(List)

Symbol	<i>Initial</i>	Iter. 1	Iter. 2
Goal	$\emptyset$		
List	$\emptyset$	<u>LP</u> , $\epsilon$	
Pair	$\emptyset$	<u>LP</u>	
LP	<u>LP</u>	<u>LP</u>	<u>LP</u>
RP	<u>RP</u>	<u>RP</u>	<u>RP</u>
EOF	EOF	EOF	EOF

# An Example

parentheses grammar

1 **Goal ::= List**

2 List ::= Pair List

3       | ε

4 Pair ::= LP List RP

Iter. 1 means iteration 1

Where LP is ( and RP is )

*FIRST sets in progress*

Iteration 1:

Applying Rule 1  
**Goal ::= List**

add first(List) to first(Goal)

Symbol	<i>Initial</i>	Iter. 1	Iter. 2
Goal	∅	?	
List	∅	<u>LP</u> , ε	
Pair	∅	<u>LP</u>	
LP	<u>LP</u>	<u>LP</u>	<u>LP</u>
RP	<u>RP</u>	<u>RP</u>	<u>RP</u>
EOF	EOF	EOF	EOF



# An Example

parentheses grammar

1 Goal ::= List

2 List ::= Pair List

3       | ε

4 Pair ::= LP List RP

Iter. 1 means iteration 1

Where LP is ( and RP is )

*FIRST sets in progress*

Iteration 1:

Applying Rule 1  
Goal ::= List

add first(List) to first(Goal)

Symbol	Initial	Iter. 1	Iter. 2
Goal	∅	<u>LP</u> , ε	
List	∅	<u>LP</u> , ε	
Pair	∅	<u>LP</u>	
LP	<u>LP</u>	<u>LP</u>	<u>LP</u>
RP	<u>RP</u>	<u>RP</u>	<u>RP</u>
EOF	EOF	EOF	EOF

# An Example

parentheses grammar

```
1 Goal ::= List
2 List ::= Pair List
3       | ε
4 Pair ::= LP List RP
```

Iter. 1 means iteration 1

Where LP is ( and RP is )

*FIRST sets in progress*

Symbol	<i>Initial</i>	Iter. 1	Iter. 2
Goal	$\emptyset$	<u>LP</u> , $\epsilon$	
List	$\emptyset$	<u>LP</u> , $\epsilon$	
Pair	$\emptyset$	<u>LP</u>	
LP	<u>LP</u>	<u>LP</u>	<u>LP</u>
RP	<u>RP</u>	<u>RP</u>	<u>RP</u>
EOF	EOF	EOF	EOF

We just finished the first iteration!  
Recall that one iteration reviews all the rules!

# An Example

parentheses grammar

```
1 Goal ::= List
2 List ::= Pair List
3       | ε
4 Pair ::= LP List RP
```

Iter. 1 means iteration 1

Where LP is ( and RP is )

*FIRST sets in progress*

Symbol	<i>Initial</i>	Iter. 1	Iter. 2
Goal	$\emptyset$	<u>LP</u> , $\epsilon$	
List	$\emptyset$	<u>LP</u> , $\epsilon$	
Pair	$\emptyset$	<u>LP</u>	
LP	<u>LP</u>	<u>LP</u>	<u>LP</u>
RP	<u>RP</u>	<u>RP</u>	<u>RP</u>
EOF	EOF	EOF	EOF

Before the second iteration starts...

# An Example

parentheses grammar

```
1 Goal ::= List
2 List ::= Pair List
3       | ε
4 Pair ::= LP List RP
```

Iter. 1 means iteration 1

Where LP is ( and RP is )

*FIRST sets in progress*

Symbol	<i>Initial</i>	Iter. 1	Iter. 2
Goal	$\emptyset$	<u>LP</u> , $\epsilon$	<u>LP</u> , $\epsilon$
List	$\emptyset$	<u>LP</u> , $\epsilon$	<u>LP</u> , $\epsilon$
Pair	$\emptyset$	<u>LP</u>	<u>LP</u>
LP	<u>LP</u>	<u>LP</u>	<u>LP</u>
RP	<u>RP</u>	<u>RP</u>	<u>RP</u>
EOF	EOF	EOF	EOF

Before the second iteration starts...

# An Example

parentheses grammar

1

Goal ::= List

2

List ::= Pair List

3

|  $\epsilon$

4

Pair ::= LP List RP

Where LP is ( and RP is )

*FIRST sets in progress*

Iteration 2:

Symbol	<i>Initial</i>	Iter. 1	Iter. 2
Goal	$\emptyset$	<u>LP</u> , $\epsilon$	<u>LP</u> , $\epsilon$
List	$\emptyset$	<u>LP</u> , $\epsilon$	<u>LP</u> , $\epsilon$
Pair	$\emptyset$	<u>LP</u>	<u>LP</u>
LP	<u>LP</u>	<u>LP</u>	<u>LP</u>
RP	<u>RP</u>	<u>RP</u>	<u>RP</u>
EOF	EOF	EOF	EOF

Applying Rule 4

Pair ::= LP List RP

add first(LP list RP) to first(Pair)

*LP is already in first(Pair)*

# An Example

parentheses grammar

```
1 Goal ::= List
2 List ::= Pair List
3       | ε
4 Pair ::= LP List RP
```

Where LP is ( and RP is )

*FIRST sets in progress*

Iteration 2:

Applying Rule 2 and Rule 3

```
List ::= Pair List
       | ε
```

add first(Pair List) to first(List)

add first( $\epsilon$ ) to first(List)

Symbol	<i>Initial</i>	Iter. 1	Iter. 2
Goal	$\emptyset$	<u>LP</u> , $\epsilon$	<u>LP</u> , $\epsilon$
List	$\emptyset$	<u>LP</u> , $\epsilon$	<u>LP</u> , $\epsilon$
Pair	$\emptyset$	<u>LP</u>	<u>LP</u>
LP	<u>LP</u>	<u>LP</u>	<u>LP</u>
RP	<u>RP</u>	<u>RP</u>	<u>RP</u>
EOF	EOF	EOF	EOF

LP and  $\epsilon$  are already in FIRST(List)

# An Example

parentheses grammar

1 **Goal ::= List**

2 List ::= Pair List

3       | ε

4 Pair ::= LP List RP

Where LP is ( and RP is )

*FIRST sets in progress*

Iteration 2:

Symbol	<i>Initial</i>	Iter. 1	Iter. 2
Goal	∅	<u>LP</u> , ε	<b><u>LP</u>, ε</b>
List	∅	<u>LP</u> , ε	<u>LP</u> , ε
Pair	∅	<u>LP</u>	<u>LP</u>
LP	<u>LP</u>	<u>LP</u>	<u>LP</u>
RP	<u>RP</u>	<u>RP</u>	<u>RP</u>
EOF	EOF	EOF	EOF

Applying Rule 1

**Goal ::= List**

add first(List) to first(Goal)

**LP and ε are already in FIRST(Goal)**

# An Example

parentheses grammar

```
1 Goal ::= List
2 List ::= Pair List
3       | ε
4 Pair ::= LP List RP
```

*FIRST* Sets

Symbol	<i>Initial</i>	Iter. 1	Iter. 2
Goal	$\emptyset$	<u>LP</u> , $\epsilon$	<u>LP</u> , $\epsilon$
List	$\emptyset$	<u>LP</u> , $\epsilon$	<u>LP</u> , $\epsilon$
Pair	$\emptyset$	<u>LP</u>	<u>LP</u>
LP	<u>LP</u>	<u>LP</u>	<u>LP</u>
RP	<u>RP</u>	<u>RP</u>	<u>RP</u>
EOF	EOF	EOF	EOF

Comparing the FIRST sets at the end of iteration 1 and the end of iteration 2, nothing new is added.



Reached fixed point! We have constructed complete FIRST sets!



# FOLLOW Sets

---

## **FOLLOW(A):**

For  $A \in \mathbf{NT}$  , define **FOLLOW(A)** as the set of *tokens* that can occur immediately after A in a valid sentential form.

**FOLLOW** set is defined over the set of non-terminal symbols, **NT**.

## Back to Our Example

---

Start ::= S eof

S ::= a S b |  
           $\epsilon$

*One possible derivation process from the start symbol:*

Start  $\Rightarrow$  S eof  $\Rightarrow$  a S b eof  $\Rightarrow$  a b eof

$FOLLOW(S) = \{ \text{eof}, b \}$

# FIRST and FOLLOW Sets

---

## **FOLLOW(A):**

For  $A \in \mathbf{NT}$  , define **FOLLOW(A)** as the set of tokens that can occur immediately after  $A$  in a valid sentential form.

**FOLLOW** set is defined over the set of non-terminal symbols (**NT**)

# Follow Set Construction

Given a rule  $p$  in the grammar:

$$A \rightarrow B_1 B_2 \dots \underbrace{B_i}_{\text{circled}} \underline{B_{i+1} \dots B_k}$$

If  $B_i$  is a non-terminal, FOLLOW( $B_i$ ) includes

- FIRST( $B_{i+1} \dots B_k$ ) -  $\{\epsilon\}$  U FOLLOW( $A$ ), if  $\epsilon \in \text{FIRST}(B_{i+1} \dots B_k)$
- FIRST( $B_{i+1} \dots B_k$ ) otherwise

Relationship between FOLLOW sets and FIRST sets of different symbols
--

# Follow Set Construction

To Build FOLLOW(X) for non-terminal X:

- Place EOF in FOLLOW(<start>)
- For each X as a non-terminal, initialize FOLLOW(X) to  $\emptyset$

Iterate until no more terminals can be added to any FOLLOW(X):

For each rule  $p$  in the grammar

If  $p$  is of the form  $A ::= \alpha B \beta$ , then

if  $\epsilon \in FIRST(\beta)$

Place  $\{FIRST(\beta) - \epsilon, FOLLOW(A)\}$  in FOLLOW(B)

else

Place  $\{FIRST(\beta)\}$  in FOLLOW(B)

If  $p$  is of the form  $A ::= \alpha B$ , then

Place FOLLOW(A) in FOLLOW(B)

End iterate

# An Example for FOLLOW Set Construction

parentheses grammar

*FOLLOW sets in progress*

- 1

Goal ::= List
- 2

List ::= Pair List
- 3

|  $\epsilon$
- 4

Pair ::= LP List RP

Symbol	<i>Initial</i>	1 <sup>st</sup>
Goal	EOF	
List	$\emptyset$	
Pair	$\emptyset$	

## Initialization

- Place EOF in FOLLOW(<start>)
- For each X as a non-terminal, initialize FOLLOW(X) to  $\emptyset$

*Iterate until* no more terminals can be added to any FOLLOW(X):

For each rule  $p$  in the grammar  
If  $p$  is of the form  $A ::= \alpha B \beta$ , then  
if  $\epsilon \in FIRST(\beta)$   
Place  $\{FIRST(\beta) - \epsilon, FOLLOW(A)\}$  in FOLLOW(B)  
else  
Place  $\{FIRST(\beta)\}$  in FOLLOW(B)  
If  $p$  is of the form  $A ::= \alpha B$ , then  
Place FOLLOW(A) in FOLLOW(B)

*End iterate*

Symbol	<i>FIRST Set</i>
Goal	<u>LP</u> , $\epsilon$
List	<u>LP</u> , $\epsilon$
Pair	<u>LP</u>
LP	<u>LP</u>
RP	<u>RP</u>
EOF	EOF

# An Example for FOLLOW Set Construction

parentheses grammar

*FOLLOW sets in progress*

1	Goal ::= List
2	List ::= Pair List
3	$\epsilon$
4	Pair ::= <u>LP</u> List <u>RP</u>

Symbol	<i>Initial</i>	1 <sup>st</sup>
Goal	EOF	EOF
List	$\emptyset$	
Pair	$\emptyset$	

Iteration 1 (of the outer loop below):

- Place EOF in FOLLOW(<start>)
  - For each X as a non-terminal, initialize FOLLOW(X) to  $\emptyset$   
*Iterate until* no more terminals can be added to any FOLLOW(X):
    - For each rule  $p$  in the grammar
    - If  $p$  is of the form  $A ::= \alpha B \beta$ , then
      - if  $\epsilon \in FIRST(\beta)$   
Place  $\{FIRST(\beta) - \epsilon, FOLLOW(A)\}$  in FOLLOW(B)
      - else  
Place  $\{FIRST(\beta)\}$  in FOLLOW(B)
    - If  $p$  is of the form  $A ::= \alpha B$ , then  
Place FOLLOW(A) in FOLLOW(B)
- End iterate*

Symbol	<i>FIRST Set</i>
Goal	<u>LP</u> , $\epsilon$
List	<u>LP</u> , $\epsilon$
Pair	<u>LP</u>
LP	<u>LP</u>
RP	<u>RP</u>
EOF	EOF

# An Example for FOLLOW Set Construction

parentheses grammar

*FOLLOW sets in progress*

- 1 **Goal ::= List**
- 2 List ::= Pair List
- 3       |  $\epsilon$
- 4 Pair ::= LP List RP

Symbol	<i>Initial</i>	1 <sup>st</sup>
Goal	<b>EOF</b>	<b>EOF</b>
List	$\emptyset$	
Pair	$\emptyset$	

Iteration 1:

*The order of the rules do not affect  
the final FOLLOW set results:*

If we visit the rules  
in order 1, 2, 3, 4

Symbol	<i>FIRST</i> Set
Goal	<u>LP</u> , $\epsilon$
List	<u>LP</u> , $\epsilon$
Pair	<u>LP</u>
LP	<u>LP</u>
RP	<u>RP</u>
EOF	EOF



# An Example for FOLLOW Set Construction

parentheses grammar

```
1 Goal ::= List
2 List ::= Pair List
3       | ε
4 Pair ::= LP List RP
```

Iteration 1:

*FOLLOW sets in progress*

Symbol	<i>Initial</i>	1 <sup>st</sup>
Goal	EOF	EOF
List	∅	?
Pair	∅	

## Rule 1 Goal ::= List

- Add FOLLOW(Goal) to FOLLOW(List)

Symbol	<i>FIRST</i> Set
Goal	<u>LP</u> , ε
List	<u>LP</u> , ε
Pair	<u>LP</u>
LP	<u>LP</u>
RP	<u>RP</u>
EOF	EOF

# An Example for FOLLOW Set Construction

parentheses grammar

*FOLLOW sets in progress*

```
1 Goal ::= List
2 List ::= Pair List
3       | ε
4 Pair ::= LP List RP
```

Symbol	<i>Initial</i>	1 <sup>st</sup>
Goal	EOF	EOF
List	∅	EOF
Pair	∅	

Iteration 1:

Symbol	<i>FIRST</i> Set
Goal	<u>LP</u> , ε
List	<u>LP</u> , ε
Pair	<u>LP</u>
LP	<u>LP</u>
RP	<u>RP</u>
EOF	EOF

## Rule 1 Goal ::= List

- Add FOLLOW(Goal) to FOLLOW(List)

# An Example for FOLLOW Set Construction

parentheses grammar

*FOLLOW sets in progress*

- 1

Goal ::= List
- 2

List ::= Pair List
- 3

|  $\epsilon$
- 4

Pair ::= LP List RP

Iteration 1:

Symbol	<i>Initial</i>	1 <sup>st</sup>
Goal	EOF	EOF
List	$\emptyset$	EOF
Pair	$\emptyset$	

Symbol	<i>FIRST</i> Set
Goal	<u>LP</u> , $\epsilon$
List	<u>LP</u> , $\epsilon$
Pair	<u>LP</u>
LP	<u>LP</u>
RP	<u>RP</u>
EOF	EOF

Rule 2 List ::= Pair List

# An Example for FOLLOW Set Construction

parentheses grammar

```
1 Goal ::= List
2 List ::= Pair List
3       | ε
4 Pair ::= LP List RP
```

Iteration 1:

*FOLLOW sets in progress*

Symbol	<i>Initial</i>	1 <sup>st</sup>
Goal	EOF	EOF
List	∅	EOF
Pair	∅	?

Symbol	<i>FIRST</i> Set
Goal	<u>LP</u> , ε
List	<u>LP</u> , ε
Pair	<u>LP</u>
LP	<u>LP</u>
RP	<u>RP</u>
EOF	EOF

**Rule 2** List ::= Pair List

- Add FIRST(List) to FOLLOW(Pair)
- Add FOLLOW(List) to FOLLOW(Pair)

# An Example for FOLLOW Set Construction

parentheses grammar

```
1 Goal ::= List
2 List ::= Pair List
3       | ε
4 Pair ::= LP List RP
```

Iteration 1:

*FOLLOW sets in progress*

Symbol	<i>Initial</i>	1 <sup>st</sup>
Goal	EOF	EOF
List	∅	EOF
Pair	∅	EOF, LP

Symbol	<i>FIRST</i> Set
Goal	<u>LP</u> , ε
List	<u>LP</u> , ε
Pair	<u>LP</u>
LP	<u>LP</u>
RP	<u>RP</u>
EOF	EOF

**Rule 2** List ::= Pair List

- Add FIRST(List) to FOLLOW(Pair)
- Add FOLLOW(List) to FOLLOW(Pair)

# An Example for FOLLOW Set Construction

parentheses grammar

*FOLLOW sets in progress*

- 1

Goal ::= List
- 2

List ::= Pair List
- 3

|  $\epsilon$
- 4

**Pair ::= LP List RP**

Iteration 1:

Symbol	<i>Initial</i>	1 <sup>st</sup>
Goal	EOF	EOF
List	$\emptyset$	EOF
Pair	$\emptyset$	EOF, LP

Symbol	<i>FIRST</i> Set
Goal	<u>LP</u> , $\epsilon$
List	<u>LP</u> , $\epsilon$
Pair	<u>LP</u>
LP	<u>LP</u>
RP	<u>RP</u>
EOF	EOF

**Rule 4** Pair ::= LP List RP

# An Example for FOLLOW Set Construction

parentheses grammar

*FOLLOW sets in progress*

- 1

Goal ::= List
- 2

List ::= Pair List
- 3

|  $\epsilon$
- 4

Pair ::= LP List RP

Iteration 1:

Symbol	<i>Initial</i>	1 <sup>st</sup>
Goal	EOF	EOF
List	$\emptyset$	EOF, ?
Pair	$\emptyset$	EOF, LP

Symbol	<i>FIRST</i> Set
Goal	<u>LP</u> , $\epsilon$
List	<u>LP</u> , $\epsilon$
Pair	<u>LP</u>
LP	<u>LP</u>
RP	<u>RP</u>
EOF	EOF

**Rule 4** Pair ::= LP List RP

- Add FIRST(RP) to FOLLOW(List)

# An Example for FOLLOW Set Construction

parentheses grammar

*FOLLOW sets in progress*

- 1

Goal ::= List
- 2

List ::= Pair List
- 3

|  $\epsilon$
- 4

Pair ::= LP List RP

Iteration 1:

Symbol	<i>Initial</i>	1 <sup>st</sup>
Goal	EOF	EOF
List	$\emptyset$	EOF, RP
Pair	$\emptyset$	EOF, LP

Symbol	<i>FIRST</i> Set
Goal	<u>LP</u> , $\epsilon$
List	<u>LP</u> , $\epsilon$
Pair	<u>LP</u>
LP	<u>LP</u>
RP	<u>RP</u>
EOF	EOF

**Rule 4** Pair ::= LP List RP

- Add FIRST(RP) to FOLLOW(List)



# An Example for FOLLOW Set Construction

parentheses grammar

```
1 Goal ::= List
2 List ::= Pair List
3       | ε
4 Pair ::= LP List RP
```

*FOLLOW sets in progress*

Symbol	<i>Initial</i>	1 <sup>st</sup>
Goal	<b>EOF</b>	<b>EOF</b>
List	∅	<b>EOF, RP</b>
Pair	∅	<b>EOF, LP</b>

Symbol	<i>FIRST</i> Set
Goal	<u>LP</u> , ε
List	<u>LP</u> , ε
Pair	<u>LP</u>
LP	<u>LP</u>
RP	<u>RP</u>
EOF	EOF

End of First Iteration  
and Before the Second  
Iteration starts

# An Example for FOLLOW Set Construction

parentheses grammar

1	Goal ::= List
2	List ::= Pair List
3	$\epsilon$
4	Pair ::= <u>LP</u> List <u>RP</u>

*FOLLOW sets in progress*

Symbol	<i>Initial</i>	1 <sup>st</sup>	2 <sup>nd</sup>
Goal	EOF	EOF	EOF
List	$\emptyset$	EOF, RP	EOF, RP
Pair	$\emptyset$	EOF, LP	EOF, LP

Symbol	<i>FIRST</i> Set
Goal	<u>LP</u> , $\epsilon$
List	<u>LP</u> , $\epsilon$
Pair	<u>LP</u>
LP	<u>LP</u>
RP	<u>RP</u>
EOF	EOF

End of First Iteration  
and Before the Second  
Iteration starts

# An Example for FOLLOW Set Construction

parentheses grammar

1 **Goal ::= List**  
 2 List ::= Pair List  
 3       |  $\epsilon$   
 4 Pair ::= LP List RP

*FOLLOW sets in progress*

Symbol	<i>Initial</i>	1 <sup>st</sup>	2 <sup>nd</sup>
Goal	<b>EOF</b>	<b>EOF</b>	<b>EOF</b>
List	$\emptyset$	<b>EOF, RP</b>	<b>EOF, RP</b>
Pair	$\emptyset$	<b>EOF, LP</b>	<b>EOF, LP</b>

Iteration 2:

Symbol	<i>FIRST</i> Set
Goal	<u>LP</u> , $\epsilon$
List	<u>LP</u> , $\epsilon$
Pair	<u>LP</u>
LP	<u>LP</u>
RP	<u>RP</u>
EOF	EOF

**Rule 1** Goal ::= List

- Add FOLLOW(Goal) to FOLLOW(List)

EOF already in FOLLOW(list)

# An Example for FOLLOW Set Construction

parentheses grammar

```

1 Goal ::= List
2 List ::= Pair List
3       | ε
4 Pair ::= LP List RP
    
```

Iteration 2:

*FOLLOW sets in progress*

Symbol	<i>Initial</i>	1 <sup>st</sup>	2 <sup>nd</sup>
Goal	EOF	EOF	EOF
List	∅	EOF, RP	EOF, RP
Pair	∅	EOF, LP	EOF, LP, <b>RP</b>

Symbol	<i>FIRST</i> Set
Goal	<u>LP</u> , ε
List	<u>LP</u> , ε
Pair	<u>LP</u>
LP	<u>LP</u>
RP	<u>RP</u>
EOF	EOF

**Rule 2** List ::= Pair List

- Add FIRST(List)-ε to FOLLOW(Pair)
- Add FOLLOW(List) to FOLLOW(Pair)

Added RP

# An Example for FOLLOW Set Construction

parentheses grammar

```

1 Goal ::= List
2 List ::= Pair List
3       | ε
4 Pair ::= LP List RP
    
```

*FOLLOW sets in progress*

Symbol	<i>Initial</i>	1 <sup>st</sup>	2 <sup>nd</sup>
Goal	EOF	EOF	EOF
List	∅	EOF, RP	EOF, RP
Pair	∅	EOF, LP	EOF, RP, LP

Iteration 2:

Symbol	<i>FIRST</i> Set
Goal	<u>LP</u> , ε
List	<u>LP</u> , ε
Pair	<u>LP</u>
LP	<u>LP</u>
RP	<u>RP</u>
EOF	EOF

**Rule 4** Pair ::= LP List RP

- Add FIRST(RP) to FOLLOW(List)

RP already in FOLLOW(list)

# An Example for FOLLOW Set Construction

parentheses grammar

*FOLLOW sets in progress*

- 1

Goal ::= List
- 2

List ::= Pair List
- 3

|  $\epsilon$
- 4

Pair ::= LP List RP

Iteration 2:

Symbol	<i>Initial</i>	1 <sup>st</sup>	2 <sup>nd</sup>
Goal	EOF	EOF	EOF
List	$\emptyset$	EOF, RP	EOF, RP
Pair	$\emptyset$	EOF, LP	EOF, RP, LP

Iteration 3 produces the same result  
⇒ reached a fixed point

We omit the results of Iteration 3.

Symbol	<i>FIRST</i> Set
Goal	<u>LP</u> , $\epsilon$
List	<u>LP</u> , $\epsilon$
Pair	<u>LP</u>
LP	<u>LP</u>
RP	<u>RP</u>
EOF	EOF

# Building Top-down Parsers

## Building the PREDICT set

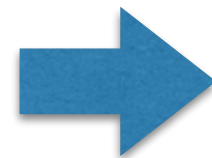
- Need a **PREDICT set** for every rule

Define  $PREDICT(A ::= \delta)$  for rule  $A ::= \delta$

- $FIRST(\delta) - \{ \epsilon \} \cup Follow(A)$ , if  $\epsilon \in FIRST(\delta)$
- $FIRST(\delta)$  otherwise

Symbol	<i>FIRST</i>	<i>FOLLOW</i>
Goal	<u>LP</u> , $\epsilon$	EOF
List	<u>LP</u> , $\epsilon$	EOF, RP
Pair	<u>LP</u>	EOF, RP, LP
LP	<u>LP</u>	-
RP	<u>RP</u>	-
EOF	EOF	-

1	Goal ::= List
2	List ::= Pair List
3	List ::= $\epsilon$
4	Pair ::= <u>LP</u> List <u>RP</u>



<i>Rule</i>	<i>PREDICT</i>
1	EOF, LP
2	LP
3	EOF, RP
4	LP

# Building Top-down Parsers

## Building the PREDICT set

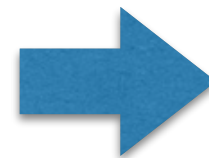
- Need a **PREDICT set** for every rule

Define  $PREDICT(A ::= \delta)$  for rule  $A ::= \delta$

- $FIRST(\delta) - \{ \epsilon \} \cup Follow(A)$ , if  $\epsilon \in FIRST(\delta)$
- $FIRST(\delta)$  otherwise

Symbol	<i>FIRST</i>	<i>FOLLOW</i>
Goal	<u>LP</u> , $\epsilon$	EOF
List	<u>LP</u> , $\epsilon$	EOF, RP
Pair	<u>LP</u>	EOF, RP, LP
LP	<u>LP</u>	-
RP	<u>RP</u>	-
EOF	EOF	-

1	Goal ::= List
2	List ::= Pair List
3	List ::= $\epsilon$
4	Pair ::= <u>LP</u> List <u>RP</u>



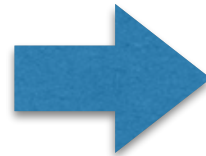
Rule	<i>PREDICT</i>
1	EOF, LP
2	LP ← FIRST(Pair List)
3	EOF, RP ← FOLLOW(List)
4	LP



# Building Top-down Parsers

Parentheses grammar

- 1 Goal ::= List
  - 2 List ::= Pair List
  - 3 List ::=  $\epsilon$
  - 4 Pair ::= LP List RP



PREDICT Sets

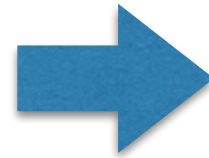
<i>Rule</i>	<i>PREDICT</i>
1	EOF, LP
2	LP
3	EOF, RP
4	LP

**Is this grammar LL(1)?**

# Building Top-down Parsers

Parentheses grammar

- 1 Goal ::= List
  - 2 List ::= Pair List
  - 3 List ::=  $\epsilon$
  - 4 Pair ::= LP List RP



PREDICT Sets

<i>Rule</i>	<i>PREDICT</i>
1	EOF, LP
2	LP
3	EOF, RP
4	LP

**Is this grammar LL(1)?**

Since only Rule 2 and Rule 3 correspond to the same non-terminal, and PREDICT(Rule 2) and PREDICT(Rule 3) are disjoint, the grammar is LL(1).

# Building Top-down Parsers

## Building the complete parse table

- Need a row for every **NT** and a column for every **T**
- Need an interpreter for the table (skeleton parser)

		<i>Rule</i>	<i>PREDICT</i>		<i>LP</i>	<i>RP</i>	<i>EOF</i>
1	Goal ::= List	<b>1</b>	<b>EOF, LP</b>	Goal			
2	List ::= Pair List	2	<b>LP</b>	List			
3	List ::= $\epsilon$	3	<b>EOF, RP</b>	Pair			
4	Pair ::= <u>LP</u> List <u>RP</u>	4	LP				

# Building Top-down Parsers

## Building the complete parse table

- Need a row for every **NT** and a column for every **T**
- Need an interpreter for the table (skeleton parser)

		<i>Rule</i>	<i>PREDICT</i>		<i>LP</i>	<i>RP</i>	<i>EOF</i>
1	Goal ::= List	<b>1</b>	<b>EOF, LP</b>	Goal	<b>1</b>		<b>1</b>
2	List ::= Pair List	2	<b>LP</b>	List			
3	List ::= $\epsilon$	3	<b>EOF, RP</b>	Pair			
4	Pair ::= <u>LP</u> List <u>RP</u>	4	<b>LP</b>				

# Building Top-down Parsers

## Building the complete parse table

- Need a row for every **NT** and a column for every **T**
- Need an interpreter for the table (skeleton parser)

		<i>Rule</i>	<i>PREDICT</i>		<i>LP</i>	<i>RP</i>	<i>EOF</i>
1	Goal ::= List	1	EOF, RP	Goal	1		1
2	List ::= Pair List	2	LP	List	2	3	3
3	List ::= $\epsilon$	3	EOF, RP				
4	Pair ::= <u>LP</u> List <u>RP</u>	4	LP	Pair			

# Building Top-down Parsers

## Building the complete parse table

- Need a row for every **NT** and a column for every **T**
- Need an interpreter for the table (skeleton parser)

		<i>Rule</i>	<i>PREDICT</i>		<i>LP</i>	<i>RP</i>	<i>EOF</i>
1	Goal ::= List	1	EOF, RP	Goal	1		1
2	List ::= Pair List	2	LP	List	2	3	3
3	$\epsilon$	3	EOF, RP	Pair	4		
4	Pair ::= <u>LP</u> List <u>RP</u>	4	LP				

# Review: Table Driven LL(1) Parsing

*Input:* a string  $w$  and a parsing table  $M$  for  $G$

push eof

push **Start** Symbol

token  $\leftarrow next\_token()$

$X \leftarrow$  top-of-stack

repeat

if  $X$  is a terminal then

if  $X == \text{token}$  then

pop  $X$

token  $\leftarrow next\_token()$

else error()

else /\*  $X$  is a non-terminal \*/

if  $M[X, \text{token}] == X \rightarrow Y_1 Y_2 \dots Y_k$  then

pop  $X$

push  $Y_k, Y_{k-1}, \dots, Y_1$

else error()

$X \leftarrow$  top-of-stack

until  $X = \text{EOF}$

if token  $\neq \text{EOF}$  then error()

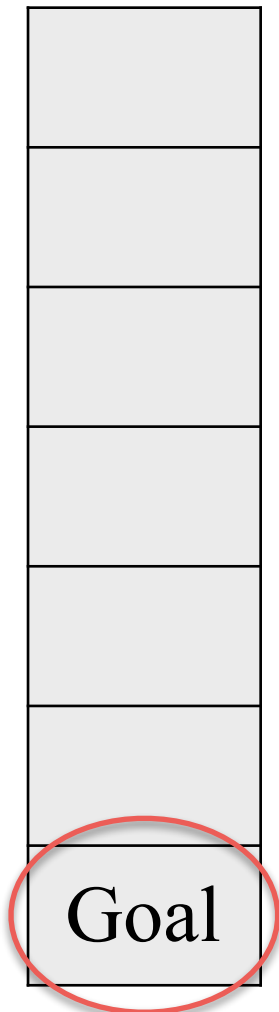
	<i>LP</i>	<i>RP</i>	<i>EOF</i>
Goal	1		1
List	2	3	3
Pair	4		

$M$  is the parse table

# LL(1) Parsing Example

- 1 Goal ::= List
- 2 List ::= Pair List
- 3       |  $\epsilon$
- 4 Pair ::= LP List RP

Goal



	<i>LP</i>	<i>RP</i>	<i>EOF</i>
Goal	1		1
List	2	3	3
Pair	4		

Remaining Input:  
LP RP LP LP RP RP

Sentential Form:  
Goal

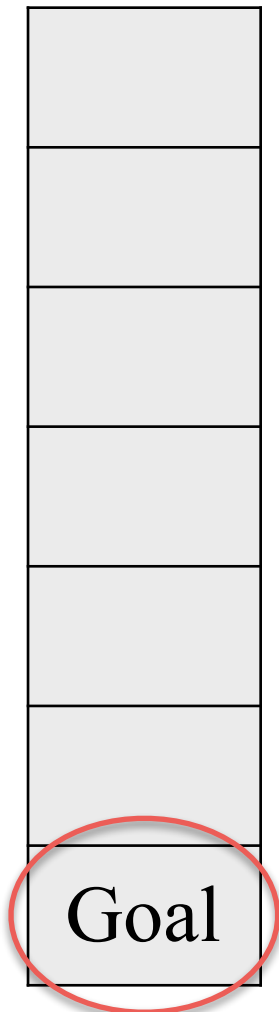
Applied Production:



# LL(1) Parsing Example

- 1 Goal ::= List
- 2 List ::= Pair List
- 3       |  $\epsilon$
- 4 Pair ::= LP List RP

Goal



	<i>LP</i>	<i>RP</i>	<i>EOF</i>
Goal	1		1
List	2	3	3
Pair	4		

Remaining Input:  
LP RP LP LP RP RP

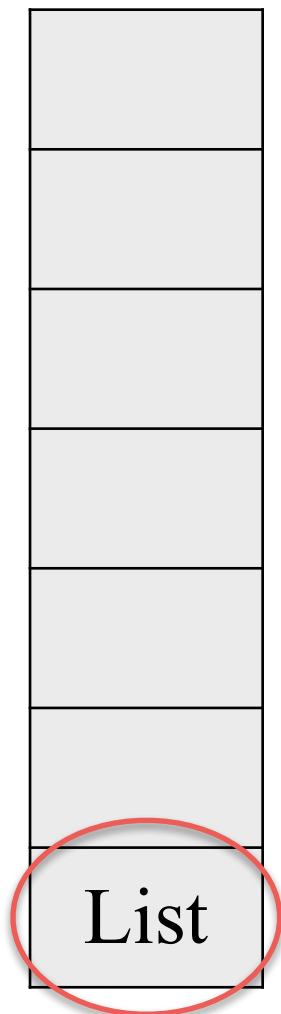
Sentential Form:  
Goal

Applied Production:

# LL(1) Parsing Example

- 1 Goal ::= List
- 2 List ::= Pair List
- 3       |  $\epsilon$
- 4 Pair ::= LP List RP

Goal  
↓  
List



	<i>LP</i>	<i>RP</i>	<i>EOF</i>
Goal	1		1
List	2	3	3
Pair	4		

Remaining Input:  
LP RP LP LP RP RP

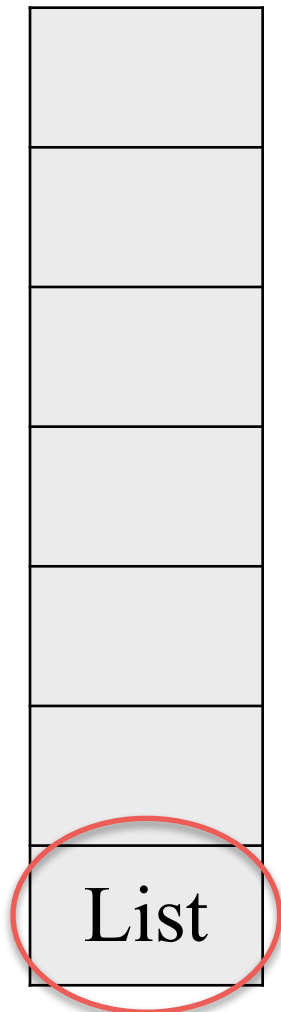
Sentential Form:  
List

Applied Production:  
1. Goal ::= List

# LL(1) Parsing Example

- 1 Goal ::= List
- 2 List ::= Pair List
- 3       |  $\epsilon$
- 4 Pair ::= LP List RP

Goal  
↓  
List



	<i>LP</i>	<i>RP</i>	<i>EOF</i>
Goal	1		1
List	2	3	3
Pair	4		

Remaining Input:  
LP RP LP LP RP RP

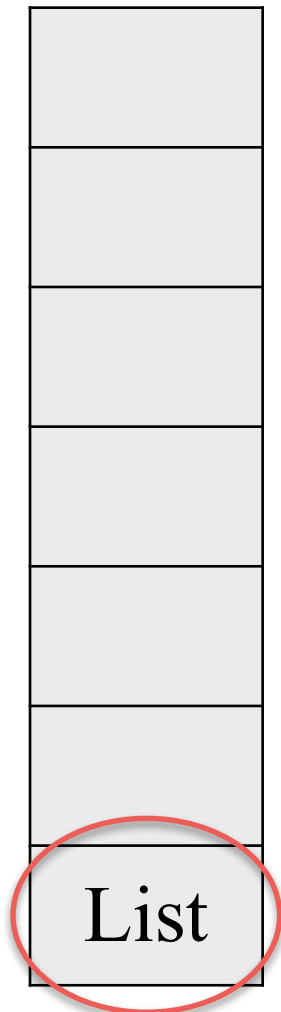
Sentential Form:  
List

Applied Production:  
1. Goal ::= List

# LL(1) Parsing Example

- 1 Goal ::= List
- 2 List ::= Pair List
- 3       |  $\epsilon$
- 4 Pair ::= LP List RP

Goal  
↓  
List



	<i>LP</i>	<i>RP</i>	<i>EOF</i>
Goal	1		1
List	2	3	3
Pair	4		

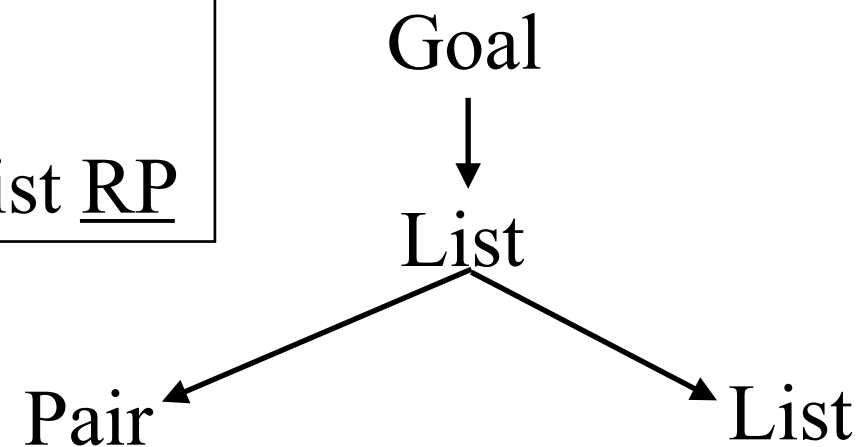
Remaining Input:  
LP RP LP LP RP RP

Sentential Form:  
List

Applied Production:  
1. Goal ::= List

# LL(1) Parsing Example

- 1 Goal ::= List
- 2 List ::= Pair List
- 3       |  $\epsilon$
- 4 Pair ::= LP List RP



	<i>LP</i>	<i>RP</i>	<i>EOF</i>
Goal	1		1
List	2	3	3
Pair	4		

Remaining Input:  
LP RP LP LP RP RP

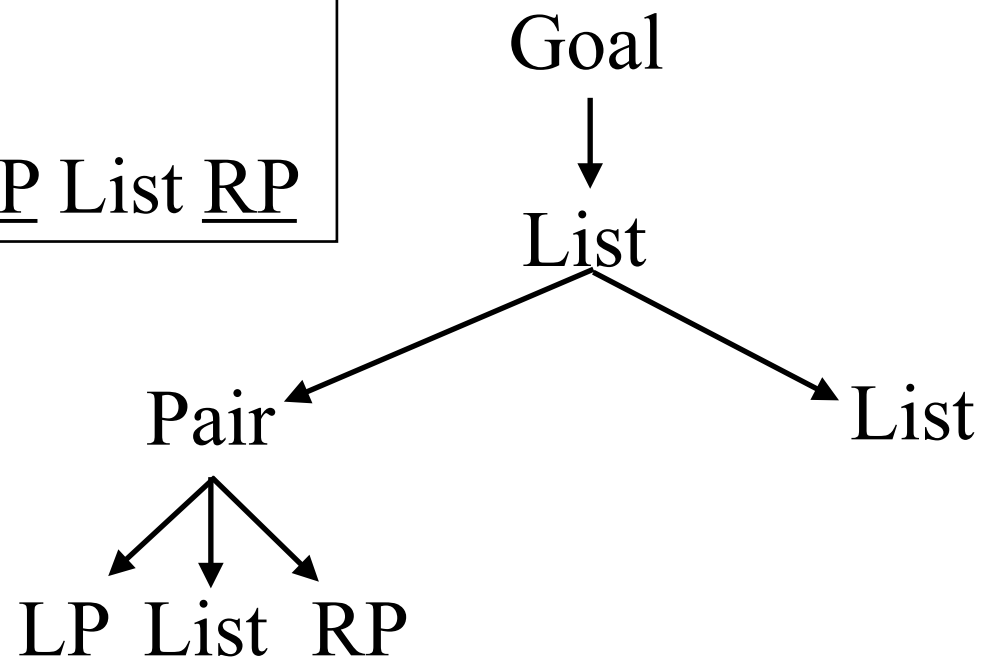
Sentential Form:  
Pair List

Applied Production:  
2. List ::= Pair List

Pair
List

# LL(1) Parsing Example

- 1 Goal ::= List
- 2 List ::= Pair List
- 3       |  $\epsilon$
- 4 Pair ::= LP List RP



LP
List
RP
List

	<i>LP</i>	<i>RP</i>	<i>EOF</i>
Goal	1		1
List	2	3	3
Pair	4		

Remaining Input:  
LP RP LP LP RP RP

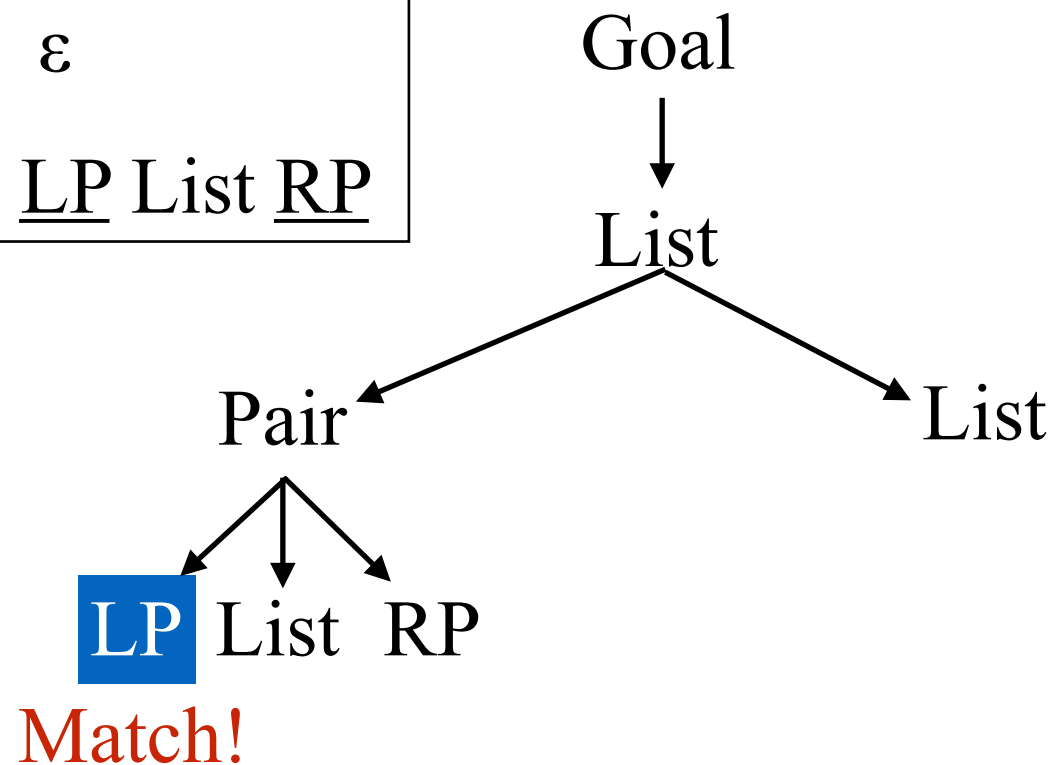
Sentential Form:  
LP List RP List

Applied Production:  
4. Pair ::= LP List RP

# LL(1) Parsing Example

- 1 Goal ::= List
- 2 List ::= Pair List
- 3       |  $\epsilon$
- 4 Pair ::= LP List RP

LP
List
RP
List



	<i>LP</i>	<i>RP</i>	<i>EOF</i>
Goal	1		1
List	2	3	3
Pair	4		

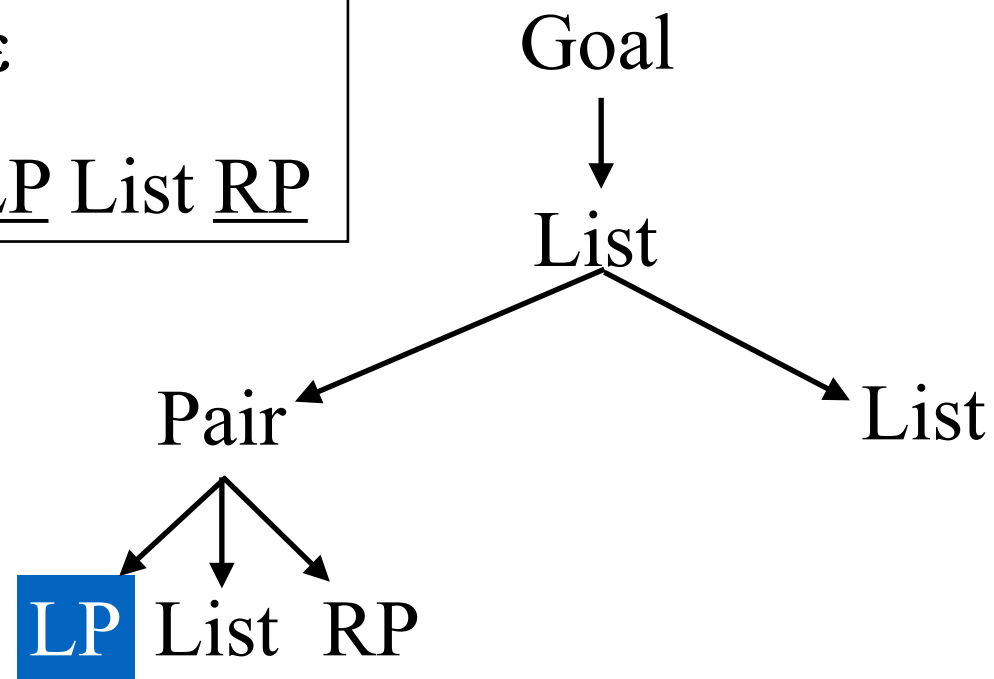
Remaining Input:  
LP RP LP LP RP RP

Sentential Form:  
LP List RP List

Applied Production:

# LL(1) Parsing Example

- 1 Goal ::= List
- 2 List ::= Pair List
- 3       |  $\epsilon$
- 4 Pair ::= LP List RP



	<i>LP</i>	<i>RP</i>	<i>EOF</i>
Goal	1		1
List	2	3	3
Pair	4		

Remaining Input:  
RP LP LP RP RP

Sentential Form:  
LP List RP List

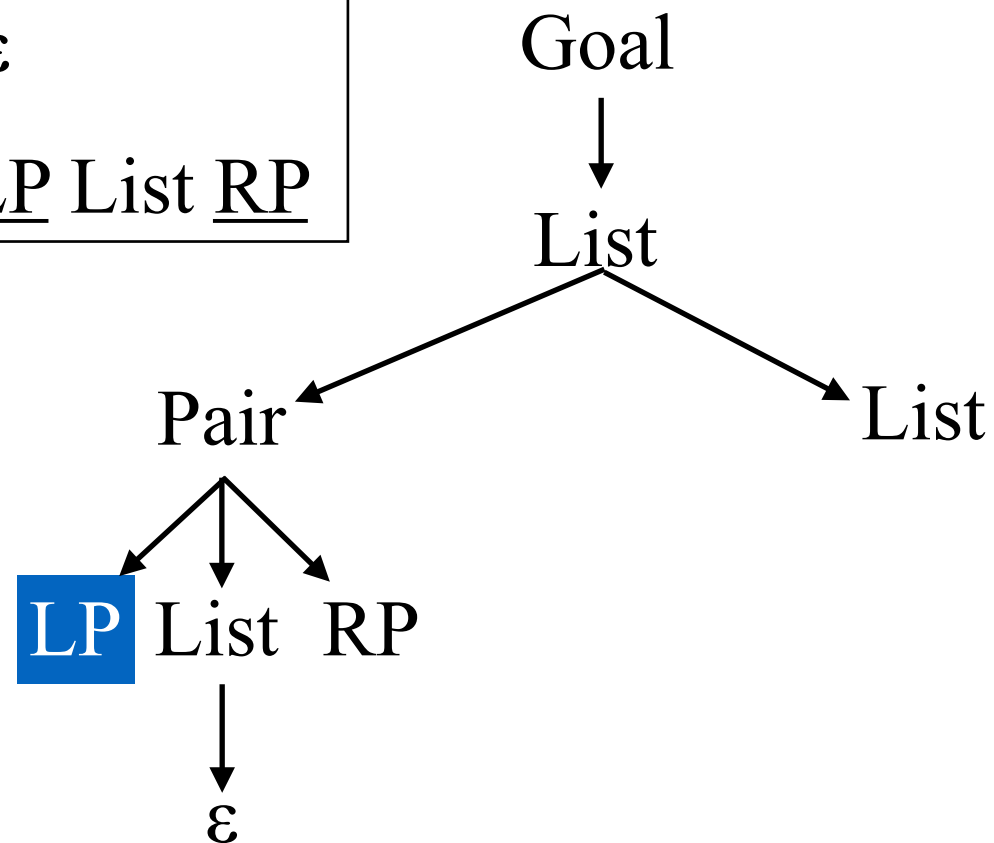
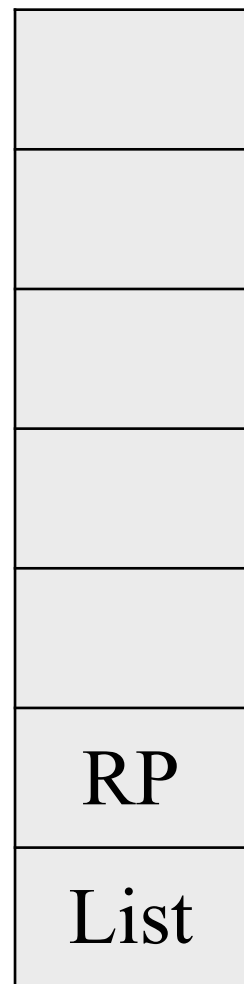
Applied Production:

List
RP
List



# LL(1) Parsing Example

- 1 Goal ::= List
- 2 List ::= Pair List
- 3       |  $\epsilon$
- 4 Pair ::= LP List RP



	<i>LP</i>	<i>RP</i>	<i>EOF</i>
Goal	1		1
List	2	3	3
Pair	4		

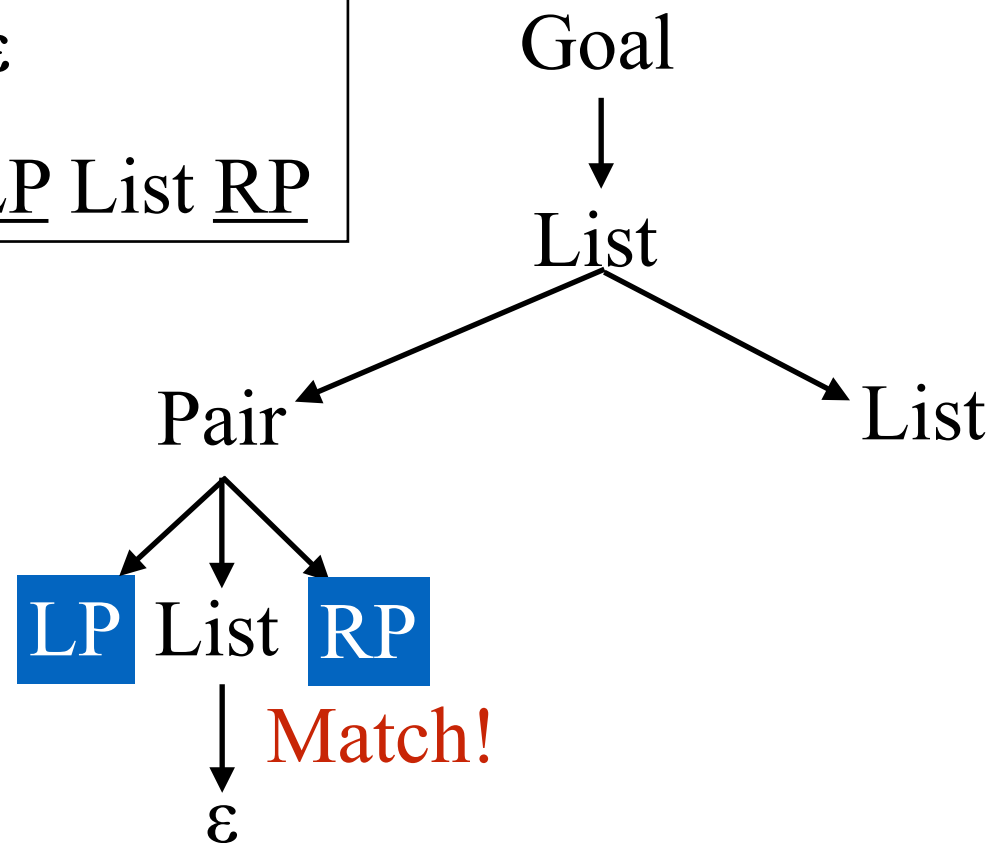
Remaining Input:  
RP LP LP RP RP

Sentential Form:  
LP RP List

Applied Production:  
3. List ::=  $\epsilon$

# LL(1) Parsing Example

- 1 Goal ::= List
- 2 List ::= Pair List
- 3       |  $\epsilon$
- 4 Pair ::= LP List RP



RP
List

	<i>LP</i>	<i>RP</i>	<i>EOF</i>
Goal	1		1
List	2	3	3
Pair	4		

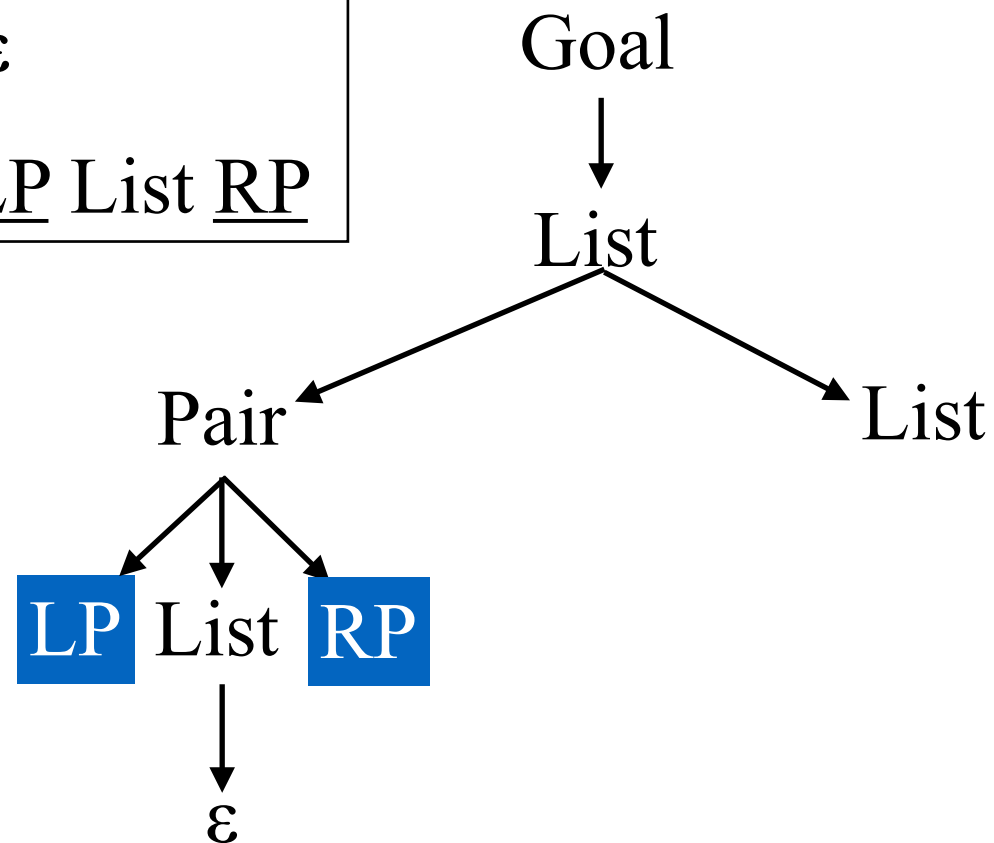
Remaining Input:  
RP LP LP RP RP

Sentential Form:  
LP RP List

Applied Production:

# LL(1) Parsing Example

- 1 Goal ::= List
- 2 List ::= Pair List
- 3       |  $\epsilon$
- 4 Pair ::= LP List RP



	<i>LP</i>	<i>RP</i>	<i>EOF</i>
Goal	1		1
List	2	3	3
Pair	4		

Remaining Input:  
LP LP RP RP

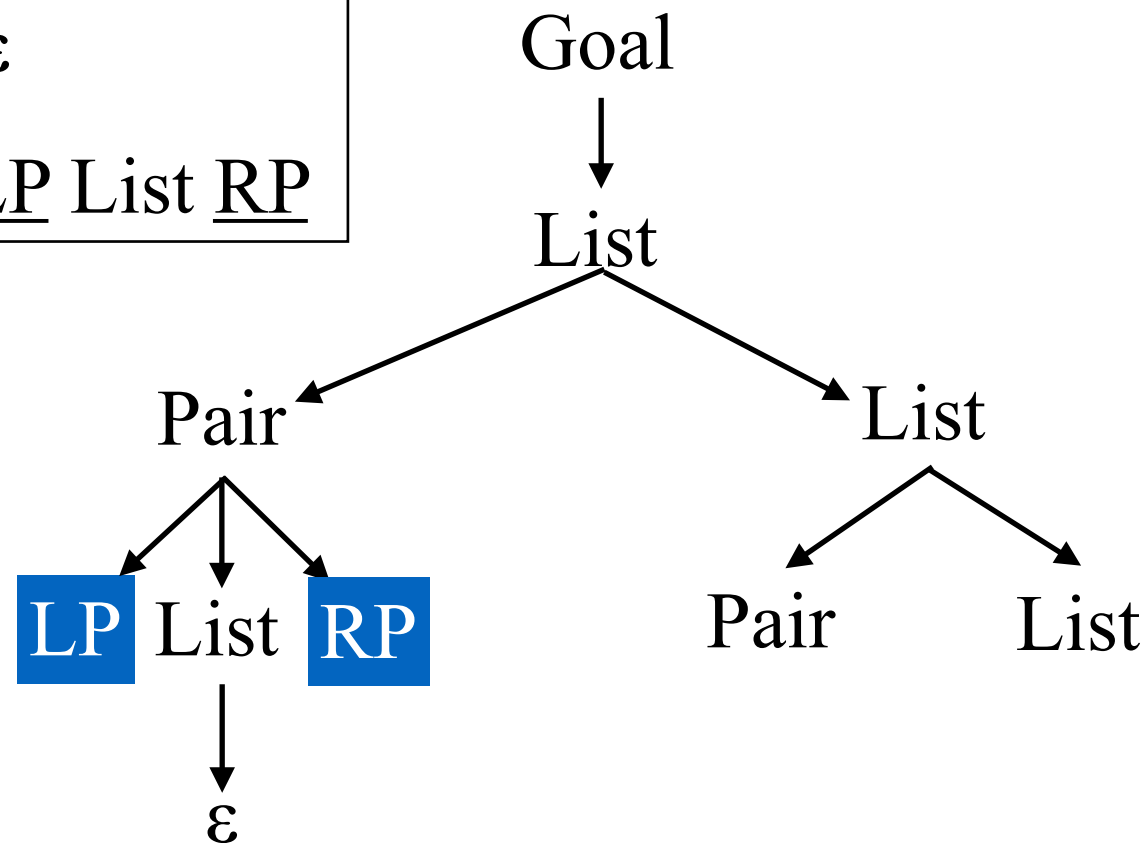
Sentential Form:  
LP RP List

Applied Production:

# LL(1) Parsing Example

- 1 Goal ::= List
- 2 List ::= Pair List
- 3       |  $\epsilon$
- 4 Pair ::= LP List RP

	<i>LP</i>	<i>RP</i>	<i>EOF</i>
Goal	1		1
List	2	3	3
Pair	4		



Remaining Input:  
LP LP RP RP

Sentential Form:  
LP RP Pair List

Applied Production:  
2. List ::= Pair List

Pair
List

Age Group	Percentage
18-24	25%
25-34	35%
35-44	20%
45-54	20%

1	Goal ::= List
2	List ::= Pair List
3	$\varepsilon$
4	Pair ::= <u>LP</u> List <u>RP</u>

1	Goal ::= List
2	List ::= Pair List
3	$\varepsilon$
4	Pair ::= <u>LP</u> List <u>RP</u>

1	Goal ::= List
2	List ::= Pair List
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1	Goal ::= List
2	List ::= Pair List
3	$\varepsilon$
4	Pair ::= <u>LP</u> List <u>RP</u>

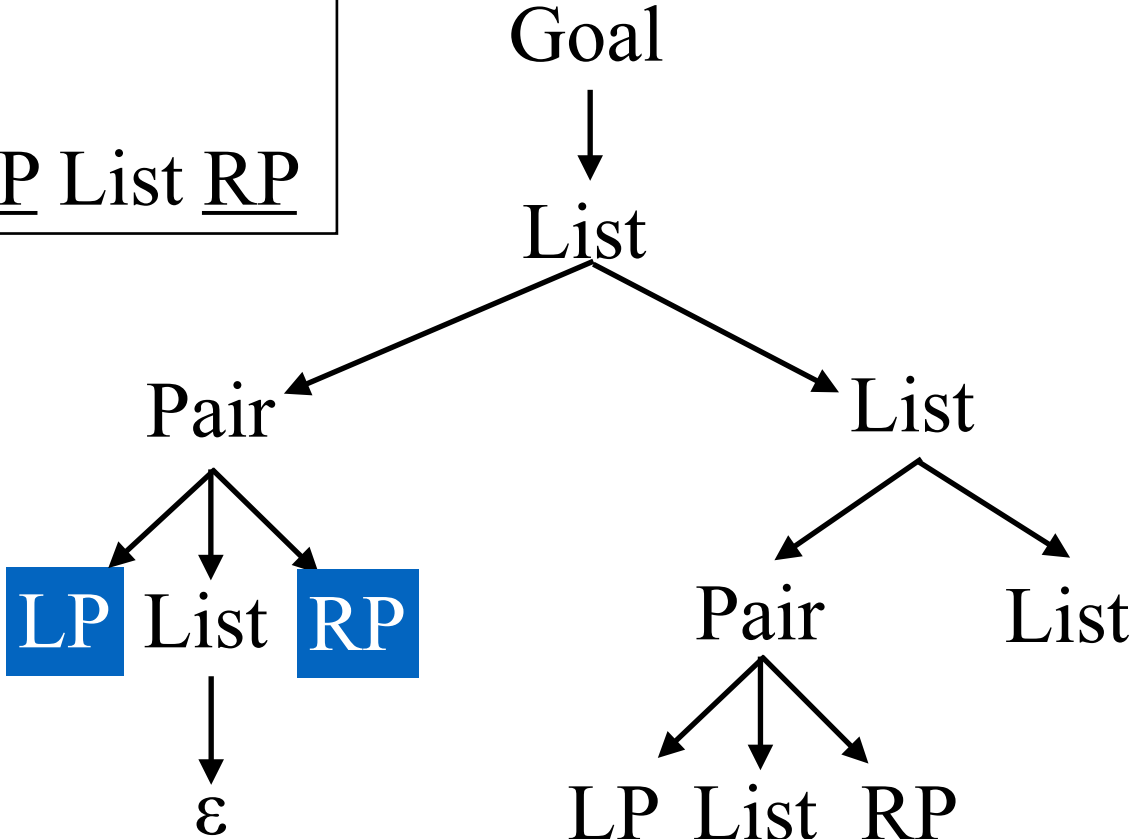
	<i>LP</i>	<i>RP</i>	<i>EOF</i>
Goal	1		1
List	2	3	3
Pair	4		

Remaining Input:  
LP LP RP RP

Sentential Form:  
LP RP LP List RP List

## Applied Production:

### 4. Pair ::= LP List RP



LP
List
RP
List

# LL(1) Parsing Example

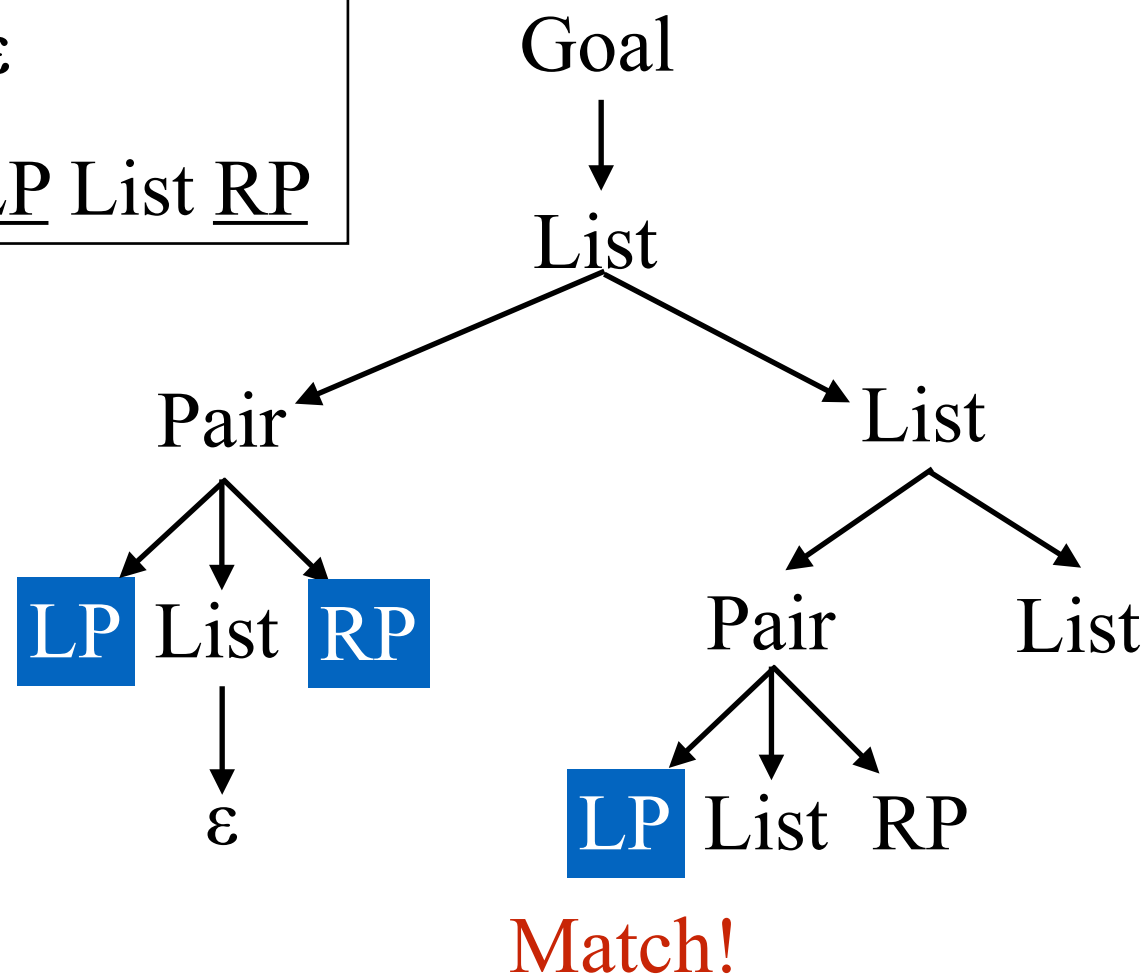
- 1 Goal ::= List
- 2 List ::= Pair List
- 3       |  $\epsilon$
- 4 Pair ::= LP List RP

	<i>LP</i>	<i>RP</i>	<i>EOF</i>
Goal	1		1
List	2	3	3
Pair	4		

Remaining Input:  
LP LP RP RP

Sentential Form:  
LP RP LP List RP List

Applied Production:



LP
List
RP
List

# LL(1) Parsing Example

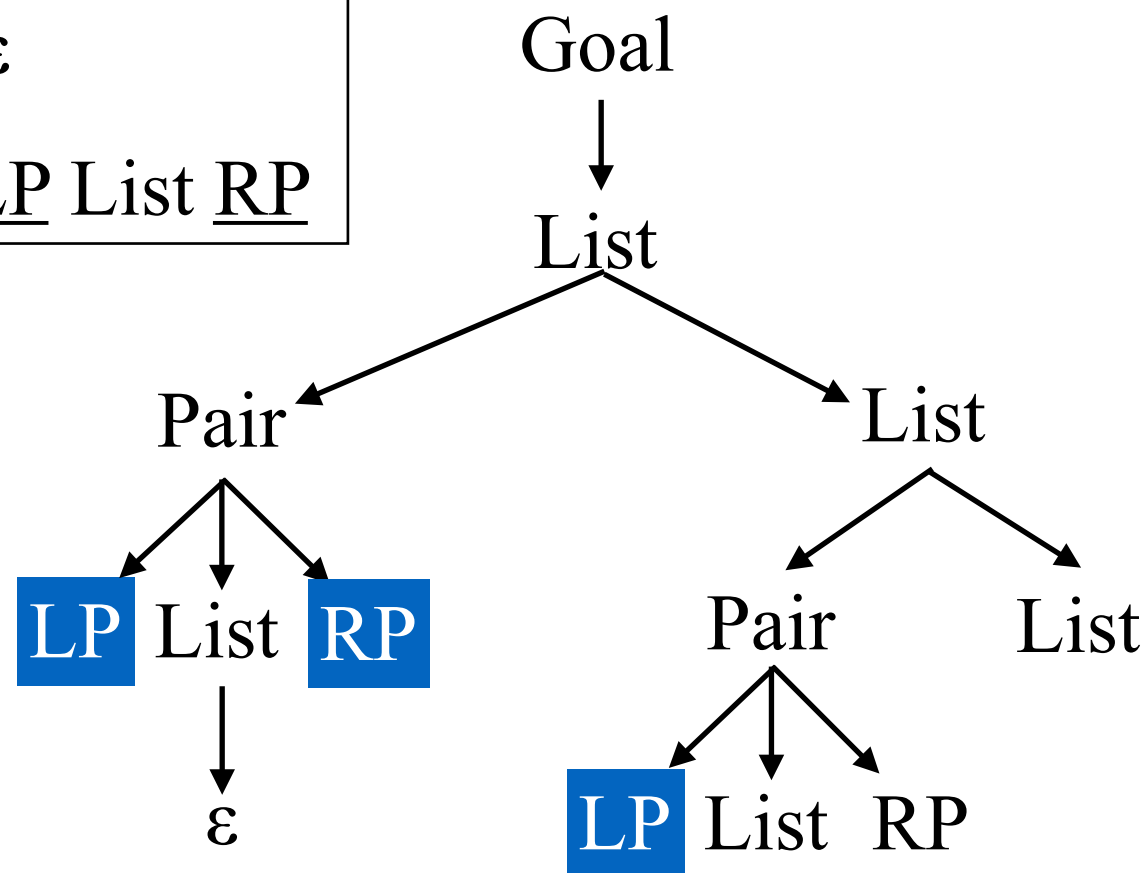
- 1 Goal ::= List
- 2 List ::= Pair List
- 3       |  $\epsilon$
- 4 Pair ::= LP List RP

	<i>LP</i>	<i>RP</i>	<i>EOF</i>
Goal	1		1
List	2	3	3
Pair	4		

Remaining Input:  
LP RP RP

Sentential Form:  
LP RP LP List RP List

Applied Production:

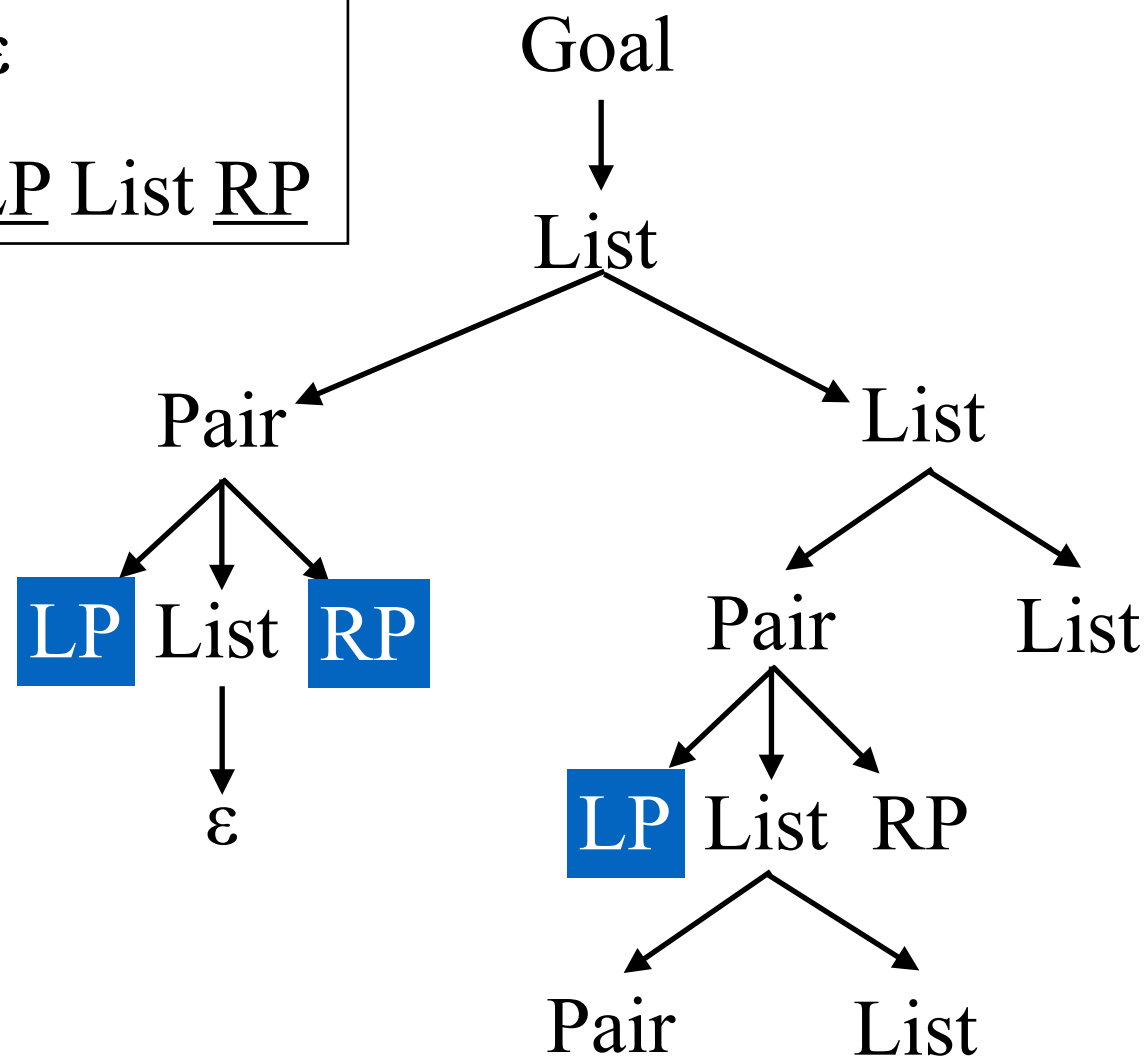


List
RP
List

# LL(1) Parsing Example

- 1 Goal ::= List
- 2 List ::= Pair List
- 3       |  $\epsilon$
- 4 Pair ::= LP List RP

	<i>LP</i>	<i>RP</i>	<i>EOF</i>
Goal	1		1
List	2	3	3
Pair	4		



Remaining Input:  
LP RP RP

Sentential Form:  
LP RP LP Pair List RP List

Applied Production:  
2. List ::= Pair List

Pair
List
RP
List



# LL(1) Parsing Example

- 1 Goal ::= List
- 2 List ::= Pair List
- 3       |  $\epsilon$
- 4 Pair ::= LP List RP

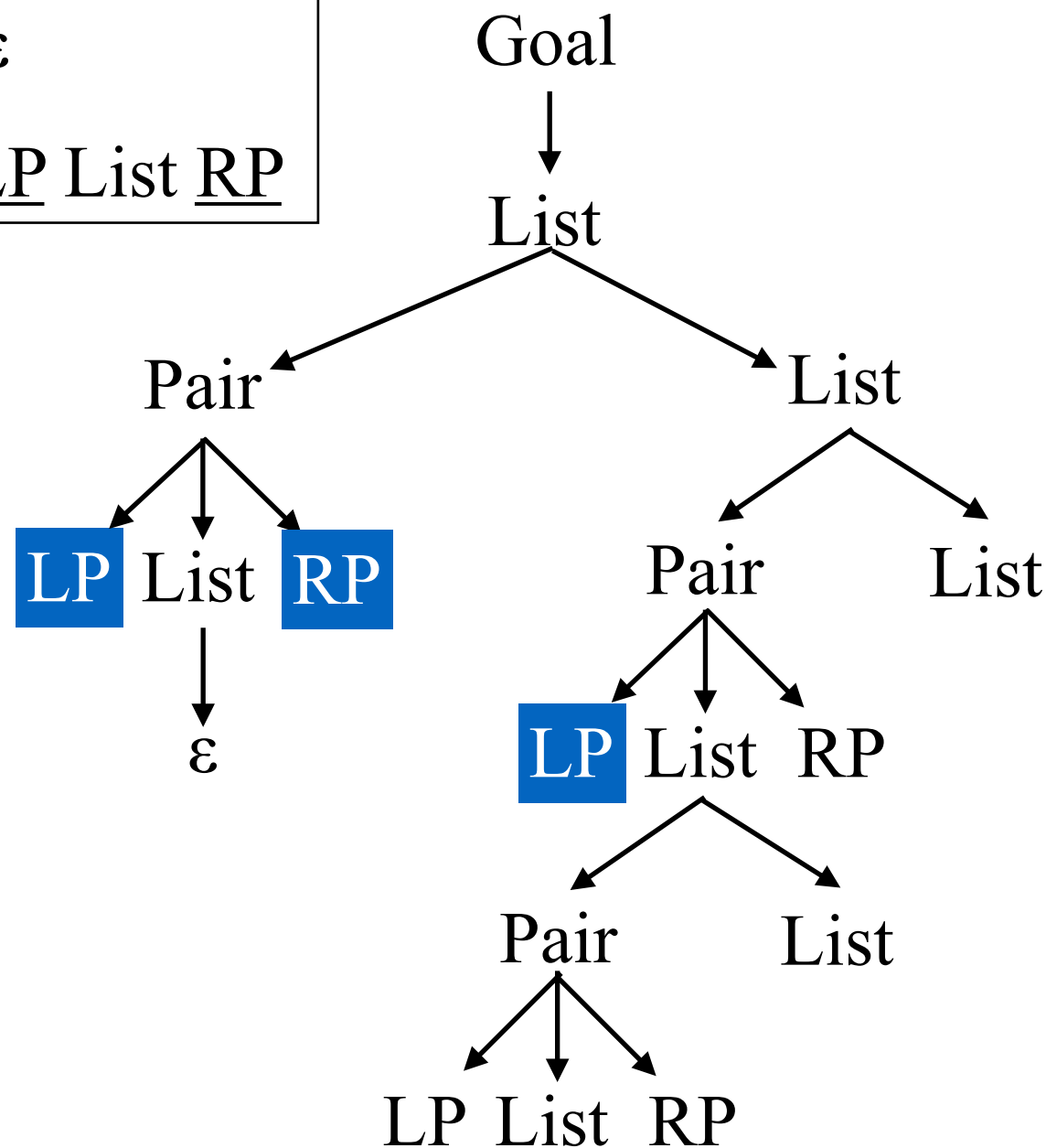
	<i>LP</i>	<i>RP</i>	<i>EOF</i>
Goal	1		1
List	2	3	3
Pair	4		

Remaining Input:  
LP RP RP

Sentential Form:  
LP RP LP  
LP List RP List RP List

Applied Production:  
4. Pair ::= LP List RP

LP
List
RP
List
RP
List





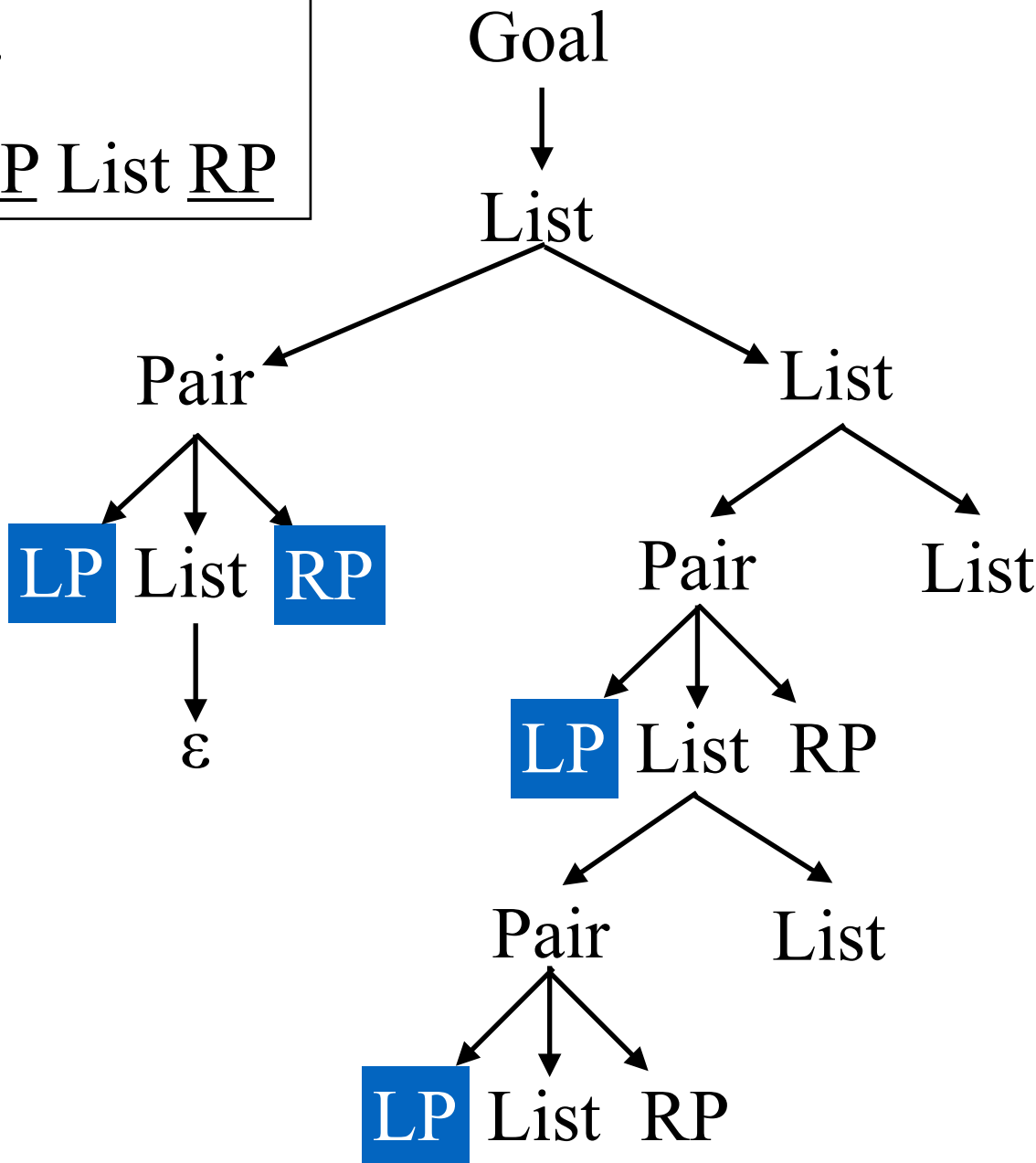
1	Goal ::= List
2	List ::= Pair List
3	$\epsilon$
4	Pair ::= <u>LP</u> List <u>RP</u>

	<i>LP</i>	<i>RP</i>	<i>EOF</i>
Goal	1		1
List	2	3	3
Pair	4		

Remaining Input:  
LP RP RP

Sentential Form:  
 LP RP LP  
 LP List RP List RP List

## Applied Production:



# Match!



# LL(1) Parsing Example

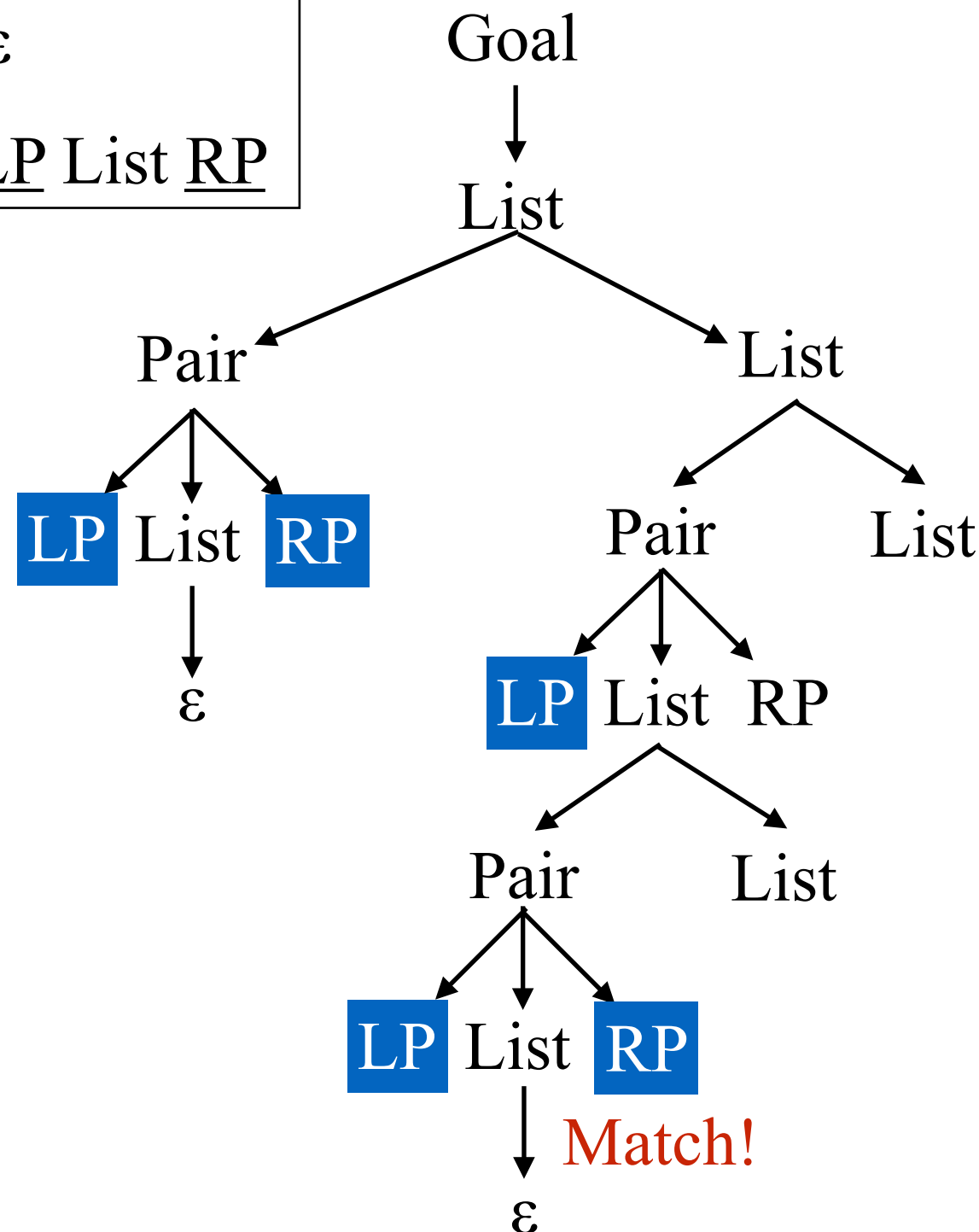
- 1 Goal ::= List
- 2 List ::= Pair List
- 3       |  $\epsilon$
- 4 Pair ::= LP List RP

	<i>LP</i>	<i>RP</i>	<i>EOF</i>
Goal	1		1
List	2	3	3
Pair	4		

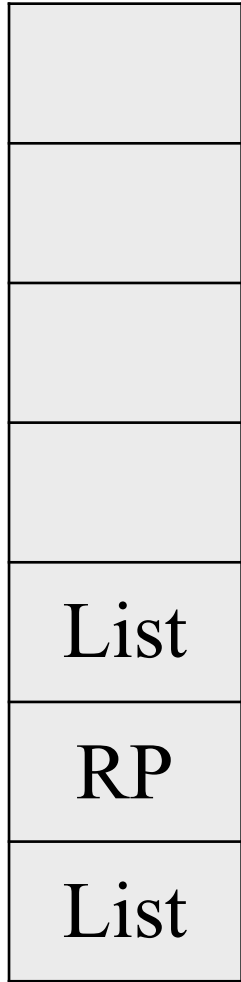
Remaining Input:  
RP RP

Sentential Form:  
LP RP LP  
LP RP List RP List

Applied Production:



RP
List
RP
List

$$\begin{array}{l} 1 \text{ Goal} ::= \text{List} \\ 2 \text{ List} ::= \text{Pair List} \\ 3 \quad \quad \quad | \quad \varepsilon \\ 4 \text{ Pair} ::= \underline{\text{LP}} \text{ List } \underline{\text{RP}} \end{array}$$
$$\begin{array}{l} 1 \text{ Goal} ::= \text{List} \\ 2 \text{ List} ::= \text{Pair List} \\ 3 \quad \quad \quad | \quad \varepsilon \\ 4 \text{ Pair} ::= \underline{\text{LP}} \text{ List } \underline{\text{RP}} \end{array}$$
$$\begin{array}{l} 1 \text{ Goal} ::= \text{List} \\ 2 \text{ List} ::= \text{Pair List} \\ 3 \quad \quad \quad | \quad \varepsilon \\ 4 \text{ Pair} ::= \underline{\text{LP}} \text{ List } \underline{\text{RP}} \end{array}$$
$$\begin{array}{l} 1 \text{ Goal} ::= \text{List} \\ 2 \text{ List} ::= \text{Pair List} \\ 3 \quad \quad \quad | \quad \varepsilon \\ 4 \text{ Pair} ::= \underline{\text{LP}} \text{ List } \underline{\text{RP}} \end{array}$$


	<i>LP</i>	<i>RP</i>	<i>EOF</i>
Goal	1		1
List	2	3	3
Pair	4		

# Remaining Input:

## RP

Sentential Form:  
LP RP LP

## Applied Production:

# LL(1) Parsing Example

- 1 Goal ::= List
- 2 List ::= Pair List
- 3       |  $\epsilon$
- 4 Pair ::= LP List RP

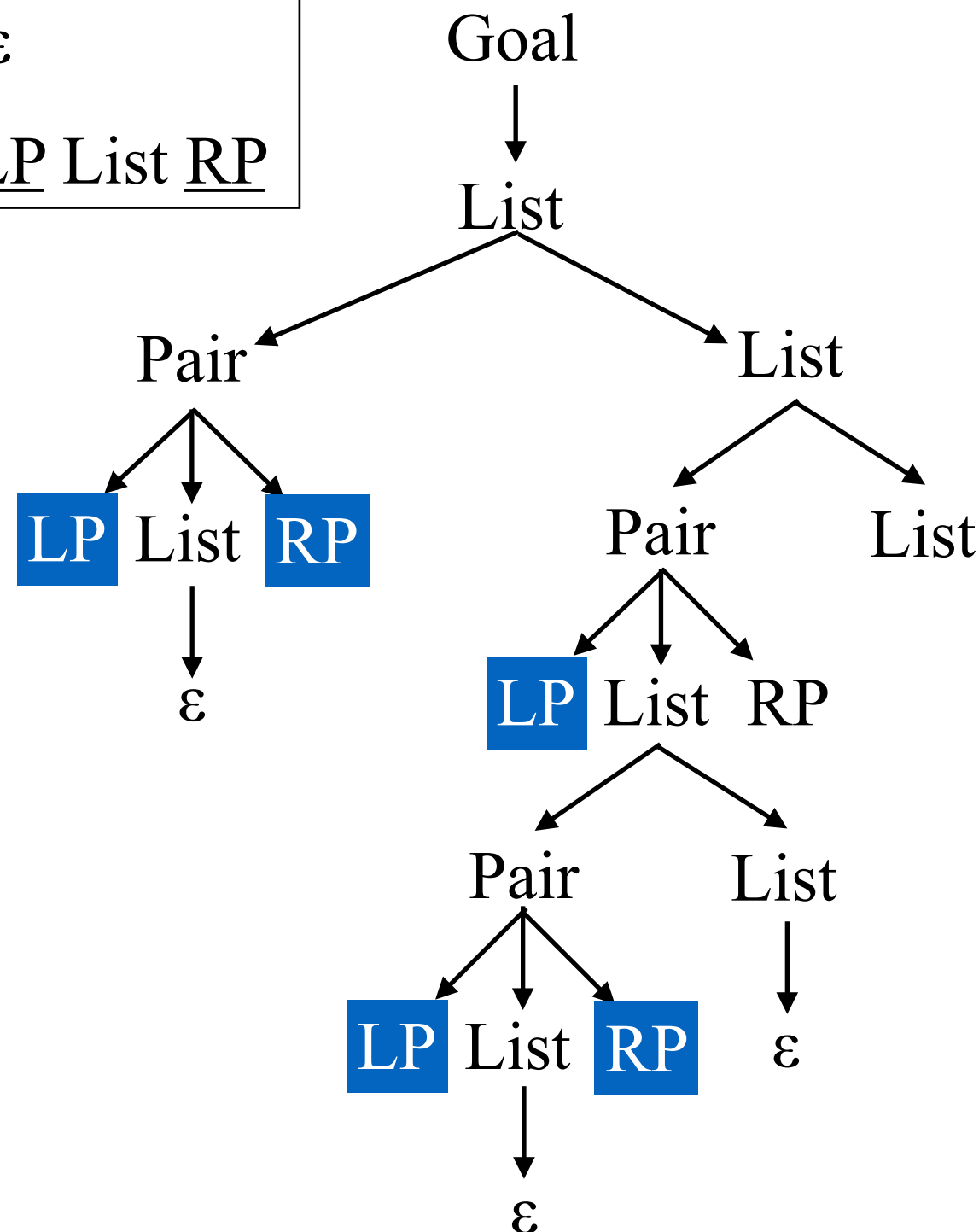
	<i>LP</i>	<i>RP</i>	<i>EOF</i>
Goal	1		1
List	2	3	3
Pair	4		

Remaining Input:  
RP

Sentential Form:  
LP RP LP  
LP RP RP List

Applied Production:  
3. List ::=  $\epsilon$

RP
List



# LL(1) Parsing Example

- 1 Goal ::= List
- 2 List ::= Pair List
- 3       |  $\epsilon$
- 4 Pair ::= LP List RP

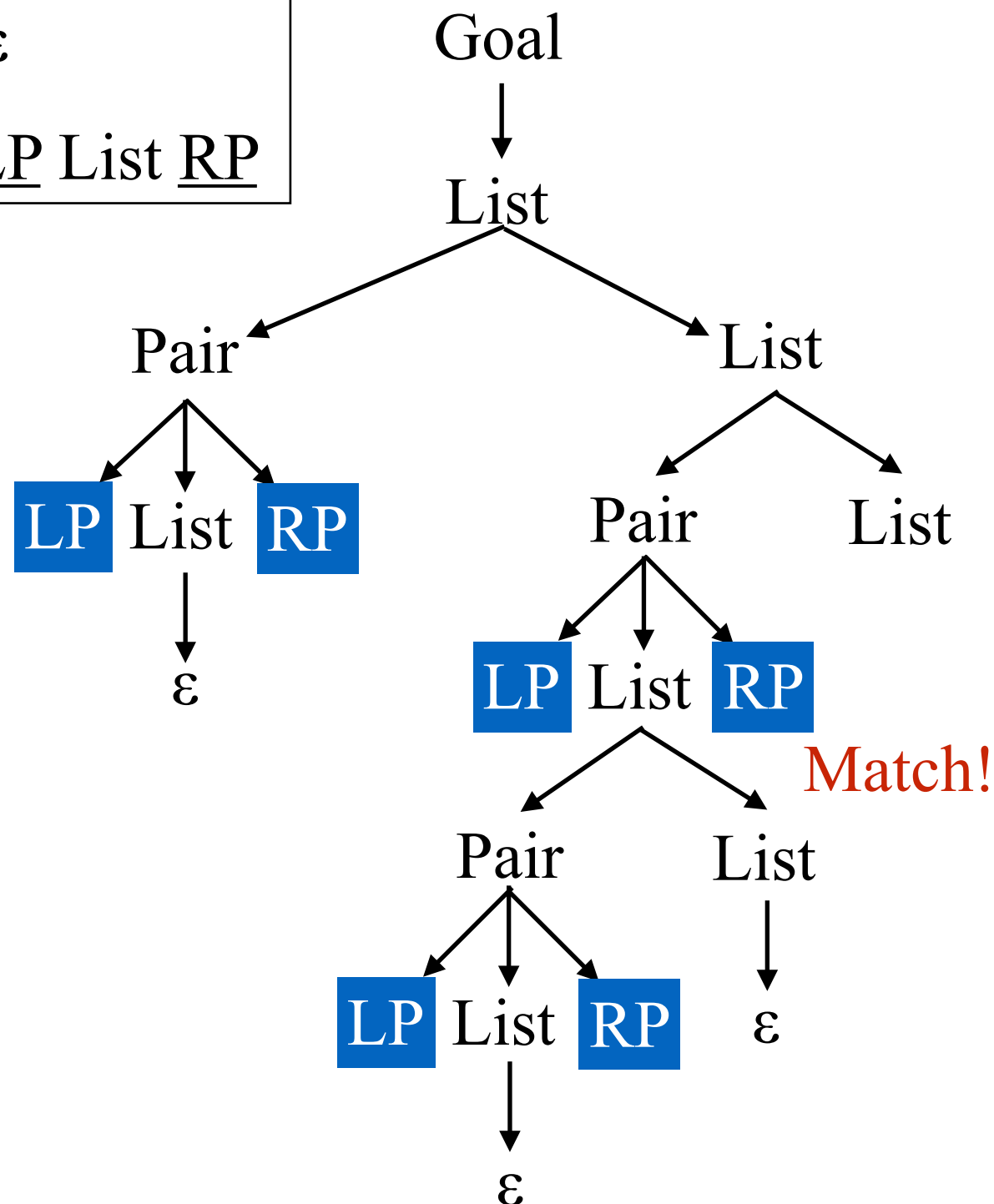
	<i>LP</i>	<i>RP</i>	<i>EOF</i>
Goal	1		1
List	2	3	3
Pair	4		

Remaining Input:  
RP

Sentential Form:  
LP RP LP  
LP RP RP List

Applied Production:

RP
List



Age Group	Percentage
18-24	25%
25-34	35%
35-44	20%
45-54	20%

1	Goal ::= List
2	List ::= Pair List
3	$\varepsilon$
4	Pair ::= <u>LP</u> List <u>RP</u>

1	Goal ::= List
2	List ::= Pair List
3	$\varepsilon$
4	Pair ::= <u>LP</u> List <u>RP</u>

1	Goal ::= List
2	List ::= Pair List
3	$\varepsilon$
4	Pair ::= <u>LP</u> List <u>RP</u>

1	Goal ::= List
2	List ::= Pair List
3	$\varepsilon$
4	Pair ::= <u>LP</u> List <u>RP</u>

	<i>LP</i>	<i>RP</i>	<i>EOF</i>
Goal	1		1
List	2	3	3
Pair	4		

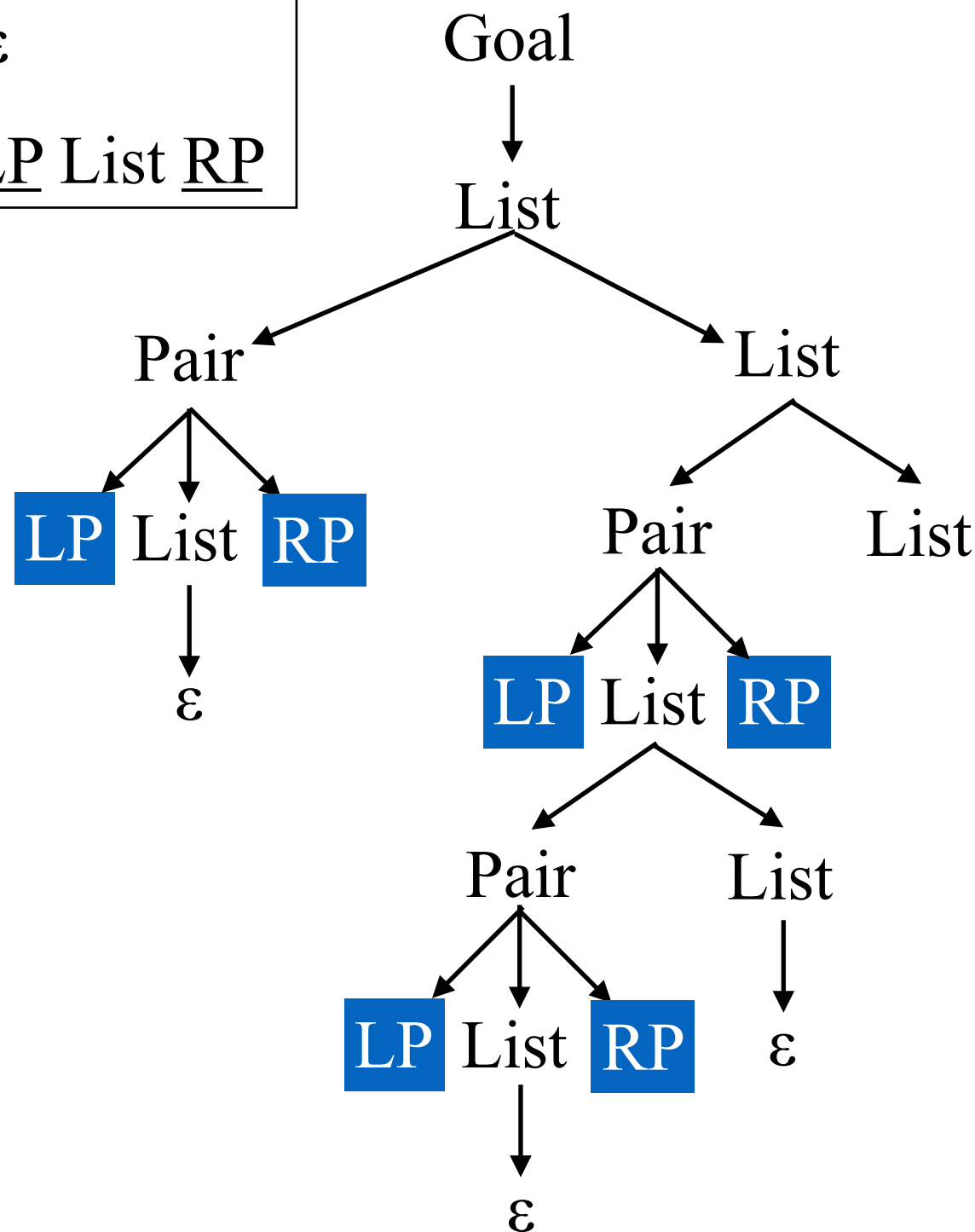
## Remaining Input:

## Sentential Form:

LP RP LP

LP RP RP List

## Applied Production:





# LL(1) Parsing Example

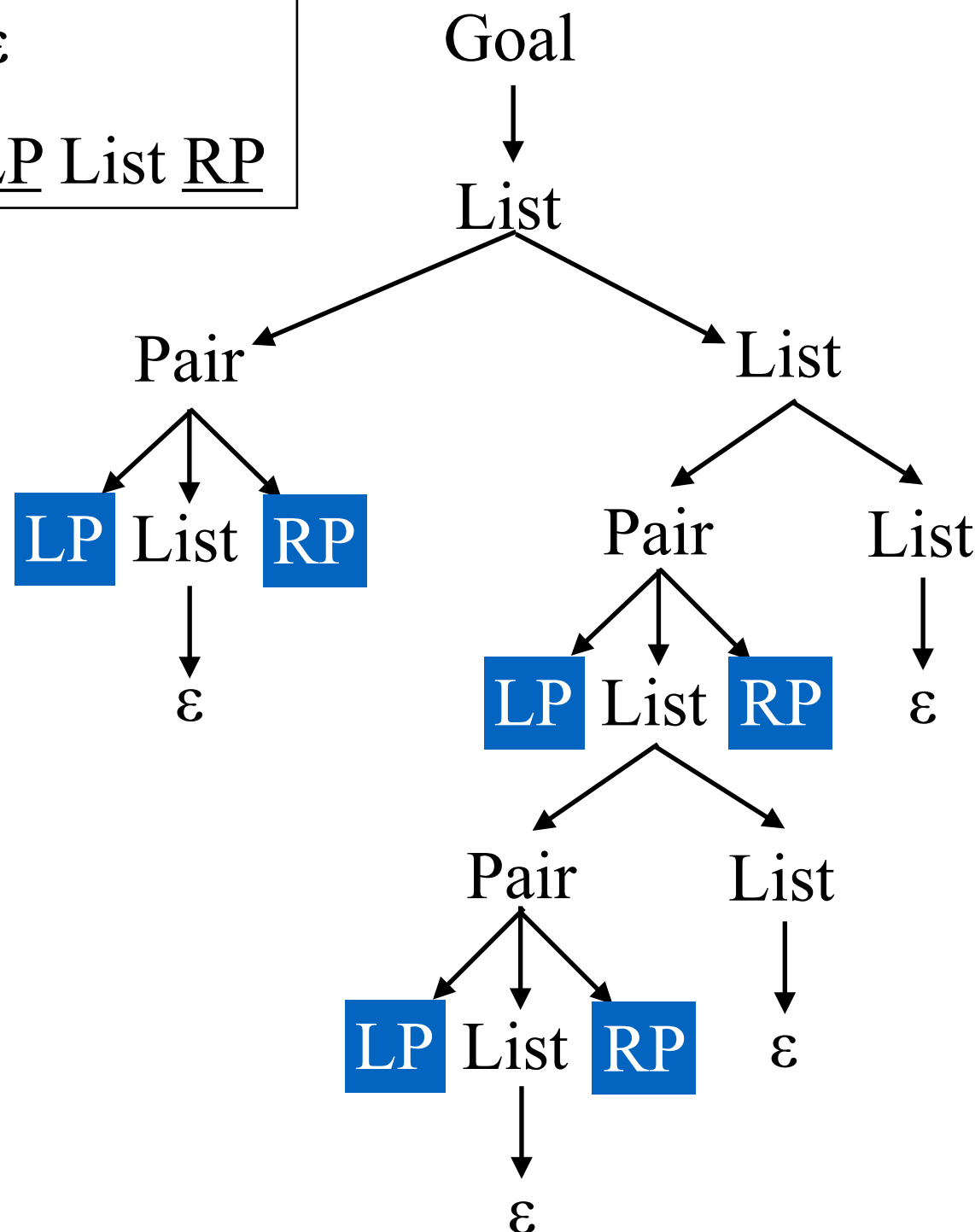
- 1 Goal ::= List
- 2 List ::= Pair List
- 3       |  $\epsilon$
- 4 Pair ::= LP List RP

	<i>LP</i>	<i>RP</i>	<i>EOF</i>
Goal	1		1
List	2	3	3
Pair	4		

Remaining Input:

Sentential Form:  
LP RP LP LP RP RP

Applied Production:  
3. List ::=  $\epsilon$



# Recursive Descent Parsing

---

## Recursive descent parser for LL(1)

- Each **non-terminal** has an associated parsing procedure that can recognize any sequence of tokens generated by that **non-terminal**
- There is a main routine to initialize all globals (e.g: the *token variable* in previous code example) and call the start symbol. On return, check whether `token==EOF`, and whether errors occurred.
- Within a parsing procedure, both **non-terminals** and **terminals** can be matched:
  - ➡ Non-terminal A: call procedure for A
  - ➡ Token t: compare t with current input token;  
if matched, **consume input**, otherwise, ERROR
- Parsing procedure may contain code that performs some useful “computations” (*syntax directed translation*)

# Recursive Descent Parsing (pseudo code)

	<i>LP</i>	<i>RP</i>	<i>EOF</i>
Goal	1		1
List	2	3	3
Pair	4		

1	Goal ::= List
2	List ::= Pair List
3	$\epsilon$
4	Pair ::= <u>LP</u> List <u>RP</u>

```
main: {  
    token := next_token( );  
    if ( List( ) and token == EOF) print “accept” else print “error”;  
}
```

# Recursive Descent Parsing (pseudo code)

	<i>LP</i>	<i>RP</i>	<i>EOF</i>
Goal	1		1
List	2	3	3
Pair	4		

```
1 Goal ::= List
2 List  ::= Pair List
3         |  $\epsilon$ 
4 Pair  ::= LP List RP
```

```
bool List( ): {
    switch token {
        case LP:
            call Pair( );
            call List( );
            break;
        case RP:
        case EOF: return true;
                break;
        default: return false;
    }
    return true;
}
```

```
bool Pair( ): {
    switch token {
        case LP: token := next_token( );
                call List( );
                if ( token == RP ) {
                    token := next_token( );
                    return true;
                }
                else
                    return false;
                break;
        default: return false;
    }
}
```

# Syntax Directed Translation

---

Examples:

- Interpreter
- Code generator
- Type checker
- Performance estimator

Use hand-written recursive descent LL(1) parser

# Example: the Original Parser

1:  $\langle \text{expr} \rangle ::= + \langle \text{expr} \rangle \langle \text{expr} \rangle \mid$   
2:  $\langle \text{digit} \rangle$   
3:  $\langle \text{digit} \rangle ::= 0 \mid 1 \mid 2 \mid 3 \mid \dots \mid 9$

	+	0...9	other
$\langle \text{expr} \rangle$	rule 1	rule 2	error
$\langle \text{digit} \rangle$	error	rule 3	error

```
bool expr( ) {  
    switch token {  
        case +:    token := next_token( );  
                   expr( );  
                   expr( ); break;  
        case 0..9: digit( ); break;  
        ...  
    }  
}  
bool digit( ) { // return value of constant  
    switch token {  
        case 1: token := next_token( ); break;  
        case 2: token := next_token( ); break;  
        ...  
    }  
}
```

# Example: the Original Parser

1:  $\langle \text{expr} \rangle ::= + \langle \text{expr} \rangle \langle \text{expr} \rangle \mid$   
2:  $\langle \text{digit} \rangle$   
3:  $\langle \text{digit} \rangle ::= 0 \mid 1 \mid 2 \mid 3 \mid \dots \mid 9$

	+	0...9	other
$\langle \text{expr} \rangle$	rule 1	rule 2	error
$\langle \text{digit} \rangle$	error	rule 3	error

call  $\langle \text{expr} \rangle$

What happens when you parse expression  
“ + 2 + 1 2 ”

```
bool expr( ): // return value of the expression

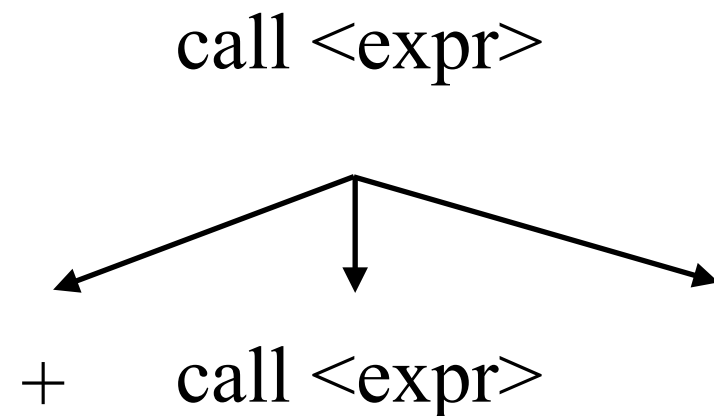
    switch token {
        case +:    token := next_token( );
                   expr( );
                   expr( ); break;
        case 0..9: digit( ); break;
        ...
    }

bool digit( ): // return value of constant
    switch token {
        case 1: token := next_token( ); break;
        case 2: token := next_token( ); break;
        ...
    }
```

# Example: the Original Parser

1:  $\langle \text{expr} \rangle ::= + \langle \text{expr} \rangle \langle \text{expr} \rangle \mid$   
2:  $\langle \text{digit} \rangle$   
3:  $\langle \text{digit} \rangle ::= 0 \mid 1 \mid 2 \mid 3 \mid \dots \mid 9$

	+	0...9	other
$\langle \text{expr} \rangle$	rule 1	rule 2	error
$\langle \text{digit} \rangle$	error	rule 3	error



What happens when you parse expression  
“+ 2 + 1 2”

```
bool expr( ): // return value of the expression

    switch token {
        case +:    token := next_token( );
                   expr( );
                   expr( ); break;
        case 0..9: digit( ); break;
        ...
    }

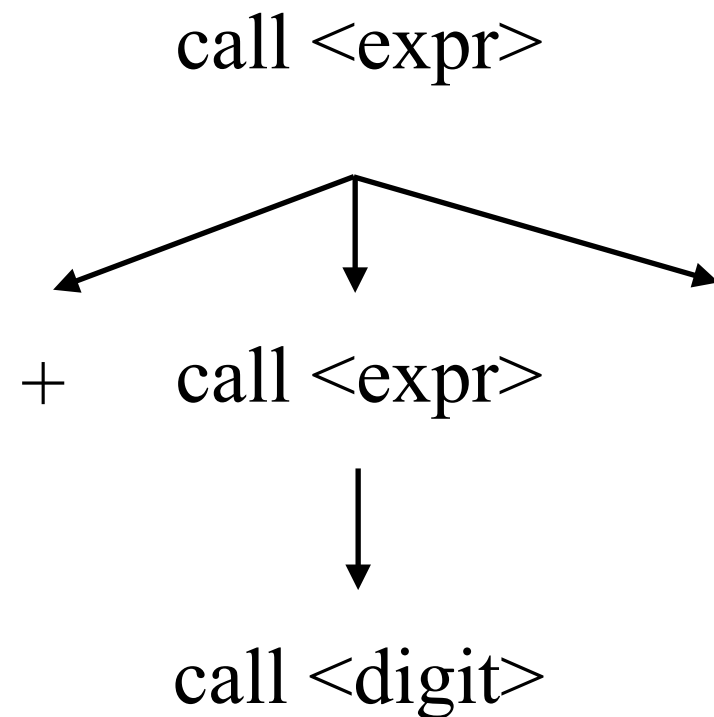
bool digit( ): // return value of constant
    switch token {
        case 1: token := next_token( ); break;
        case 2: token := next_token( ); break;
        ...
    }
```



# Example: the Original Parser

1:  $\langle \text{expr} \rangle ::= + \langle \text{expr} \rangle \langle \text{expr} \rangle \mid$   
2:  $\langle \text{digit} \rangle$   
3:  $\langle \text{digit} \rangle ::= 0 \mid 1 \mid 2 \mid 3 \mid \dots \mid 9$

	+	0...9	other
$\langle \text{expr} \rangle$	rule 1	rule 2	error
$\langle \text{digit} \rangle$	error	rule 3	error

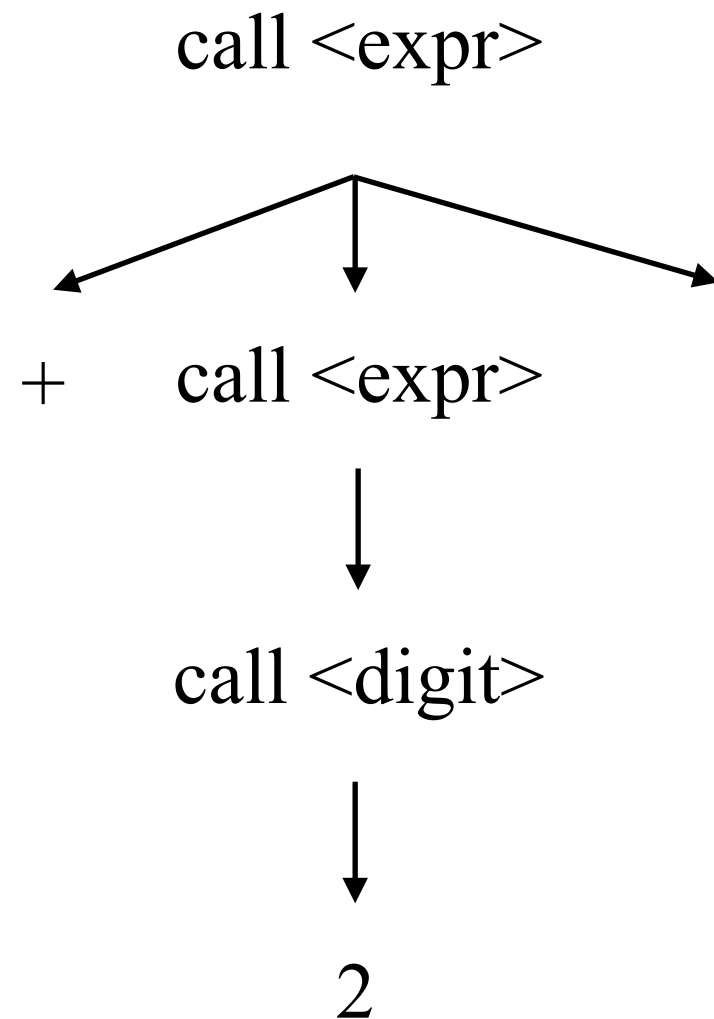


What happens when you parse expression  
“+ 2 + 1 2”

# Example: the Original Parser

1:  $\langle \text{expr} \rangle ::= + \langle \text{expr} \rangle \langle \text{expr} \rangle \mid$   
2:  $\langle \text{digit} \rangle$   
3:  $\langle \text{digit} \rangle ::= 0 \mid 1 \mid 2 \mid 3 \mid \dots \mid 9$

	+	0...9	other
$\langle \text{expr} \rangle$	rule 1	rule 2	error
$\langle \text{digit} \rangle$	error	rule 3	error

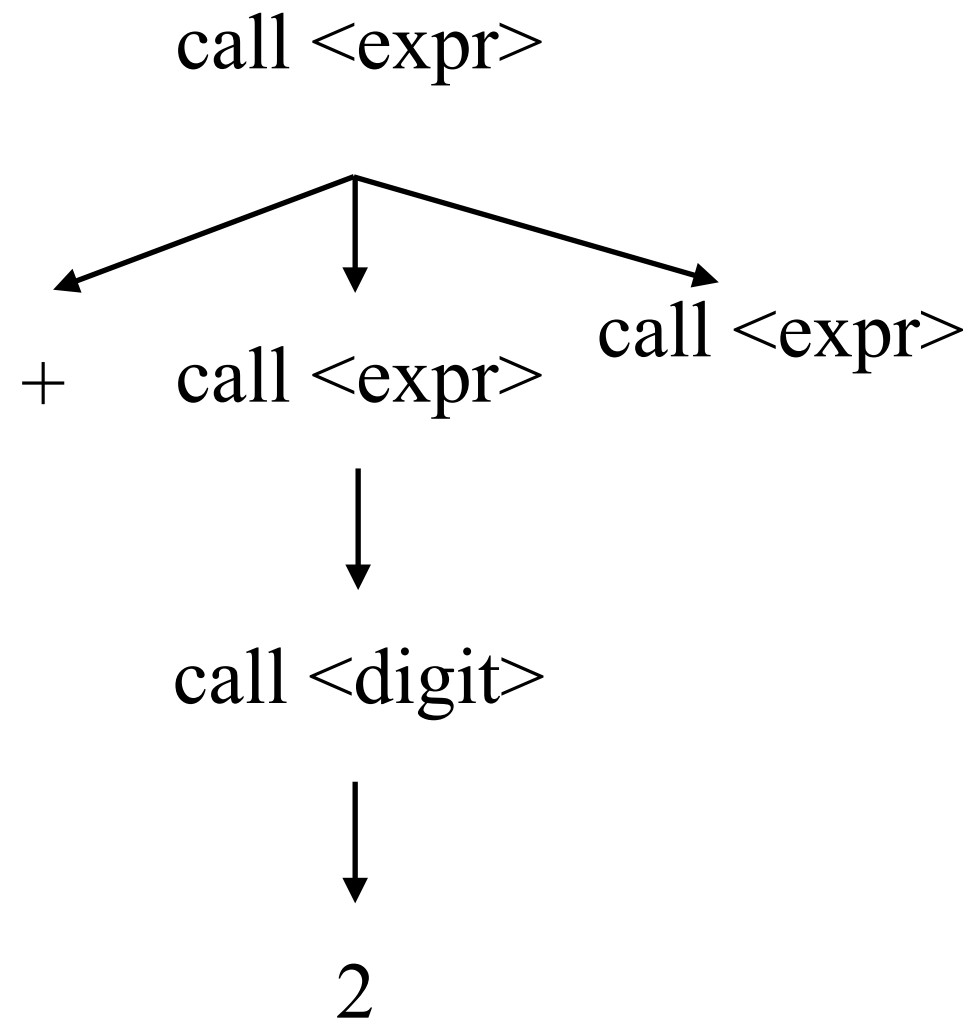


What happens when you parse expression  
“+ 2 + 1 2”

# Example: the Original Parser

1:  $\langle \text{expr} \rangle ::= + \langle \text{expr} \rangle \langle \text{expr} \rangle \mid$   
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	+	0...9	other
$\langle \text{expr} \rangle$	rule 1	rule 2	error
$\langle \text{digit} \rangle$	error	rule 3	error

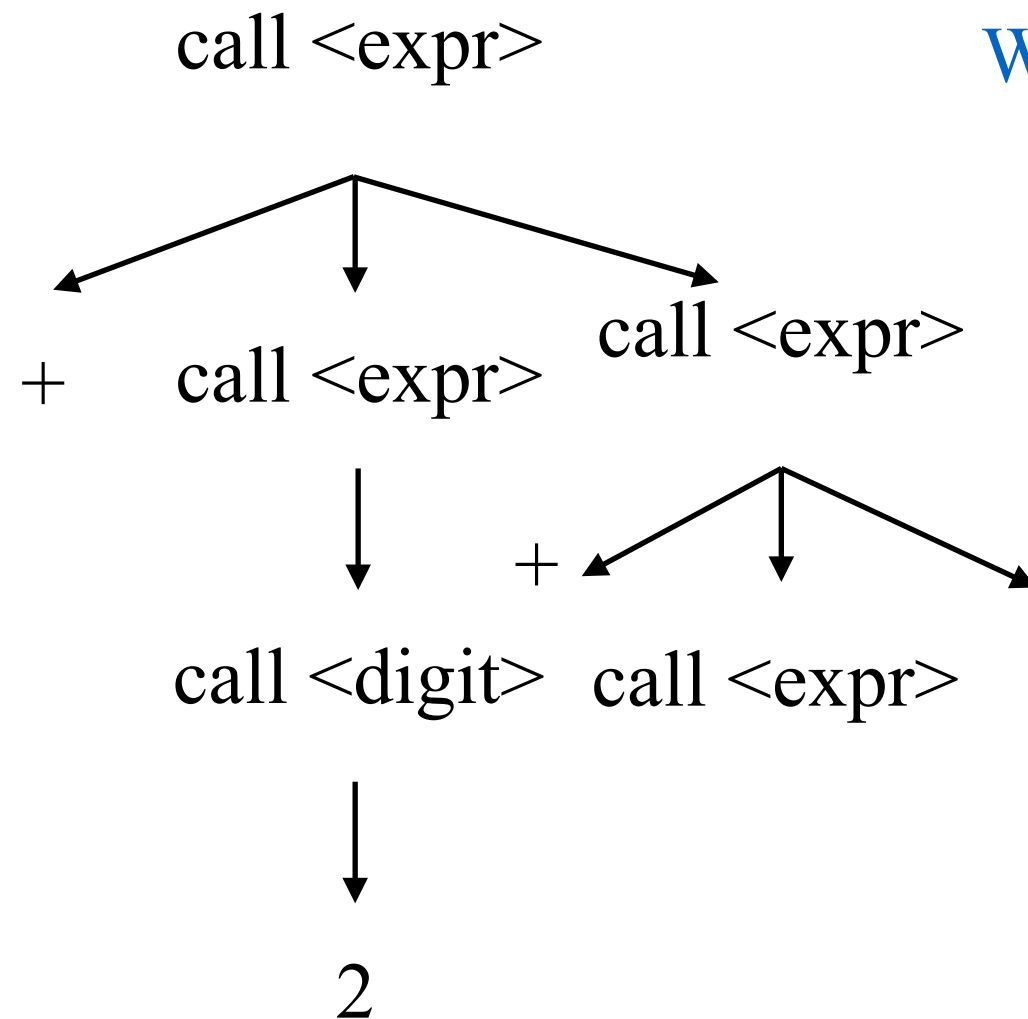


What happens when you parse expression  
“+ 2 + 1 2”

# Example: the Original Parser

1:  $\langle \text{expr} \rangle ::= + \langle \text{expr} \rangle \langle \text{expr} \rangle \mid$   
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	+	0...9	other
$\langle \text{expr} \rangle$	rule 1	rule 2	error
$\langle \text{digit} \rangle$	error	rule 3	error



What happens when you parse expression  
“+ 2 + 1 2”

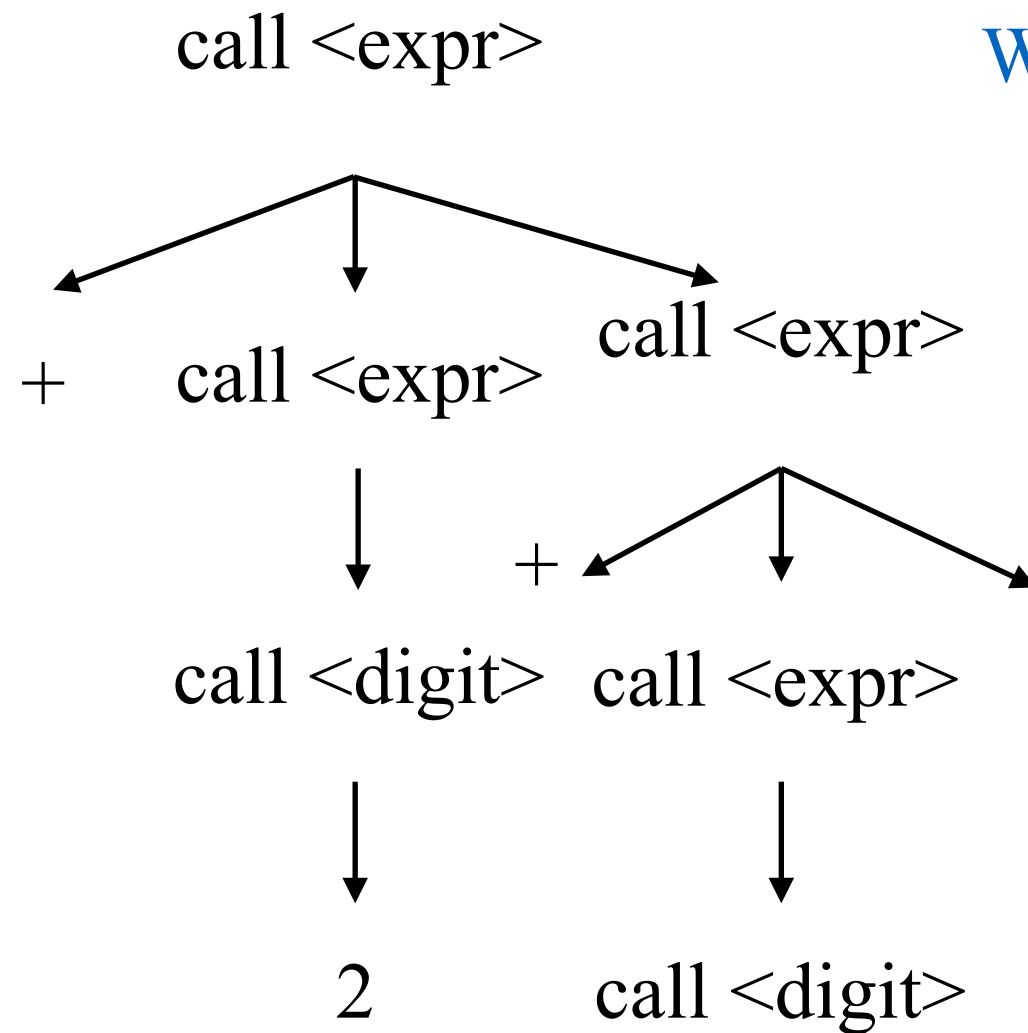
# Example: the Original Parser

1:  $\langle \text{expr} \rangle ::= + \langle \text{expr} \rangle \langle \text{expr} \rangle \mid$

2:  $\langle \text{digit} \rangle$

3:  $\langle \text{digit} \rangle ::= 0 \mid 1 \mid 2 \mid 3 \mid \dots \mid 9$

	+	0...9	other
$\langle \text{expr} \rangle$	rule 1	rule 2	error
$\langle \text{digit} \rangle$	error	rule 3	error

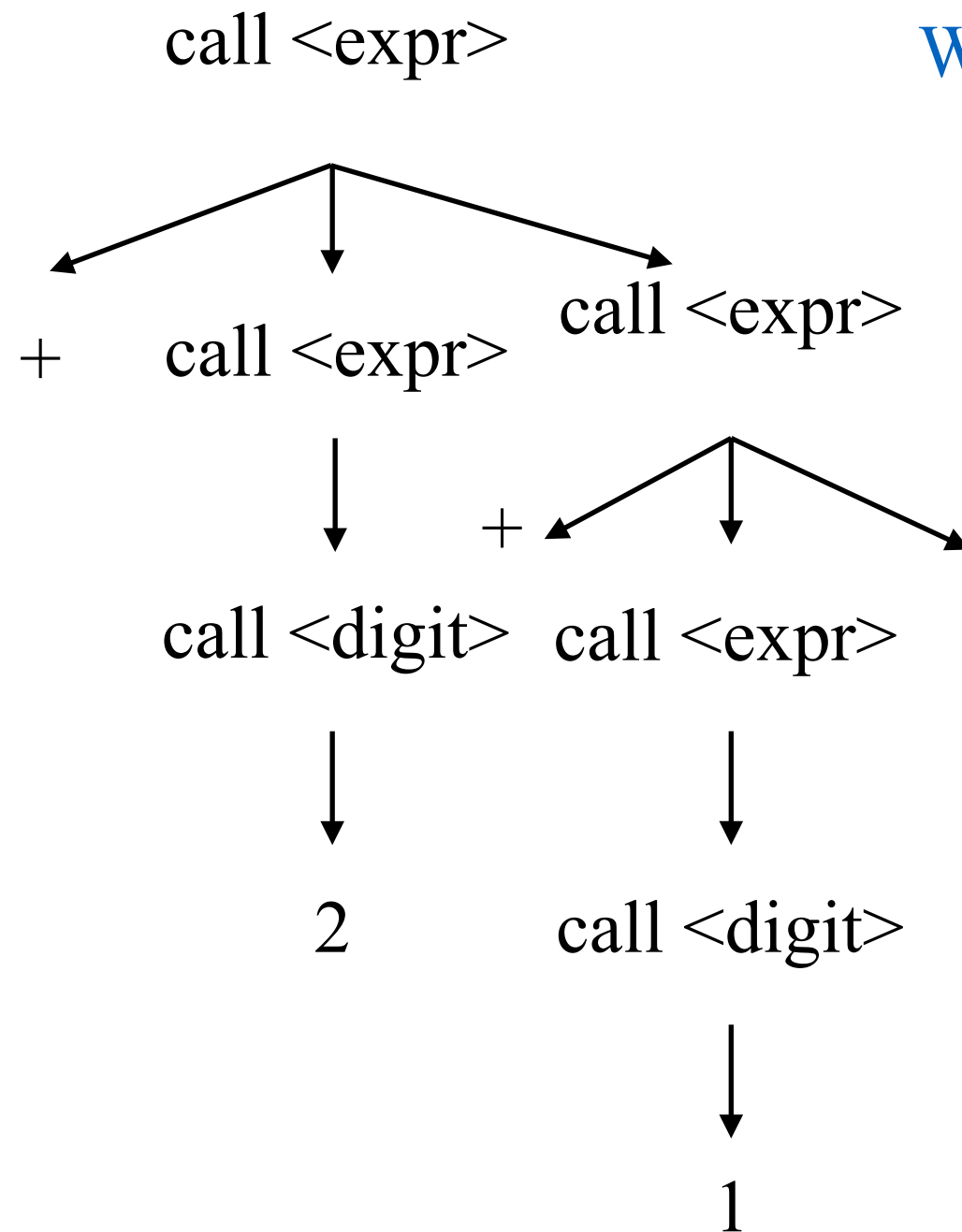


What happens when you parse expression  
“+ 2 + 1 2”

# Example: the Original Parser

1:  $\langle \text{expr} \rangle ::= + \langle \text{expr} \rangle \langle \text{expr} \rangle \mid$   
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	+	0...9	other
$\langle \text{expr} \rangle$	rule 1	rule 2	error
$\langle \text{digit} \rangle$	error	rule 3	error

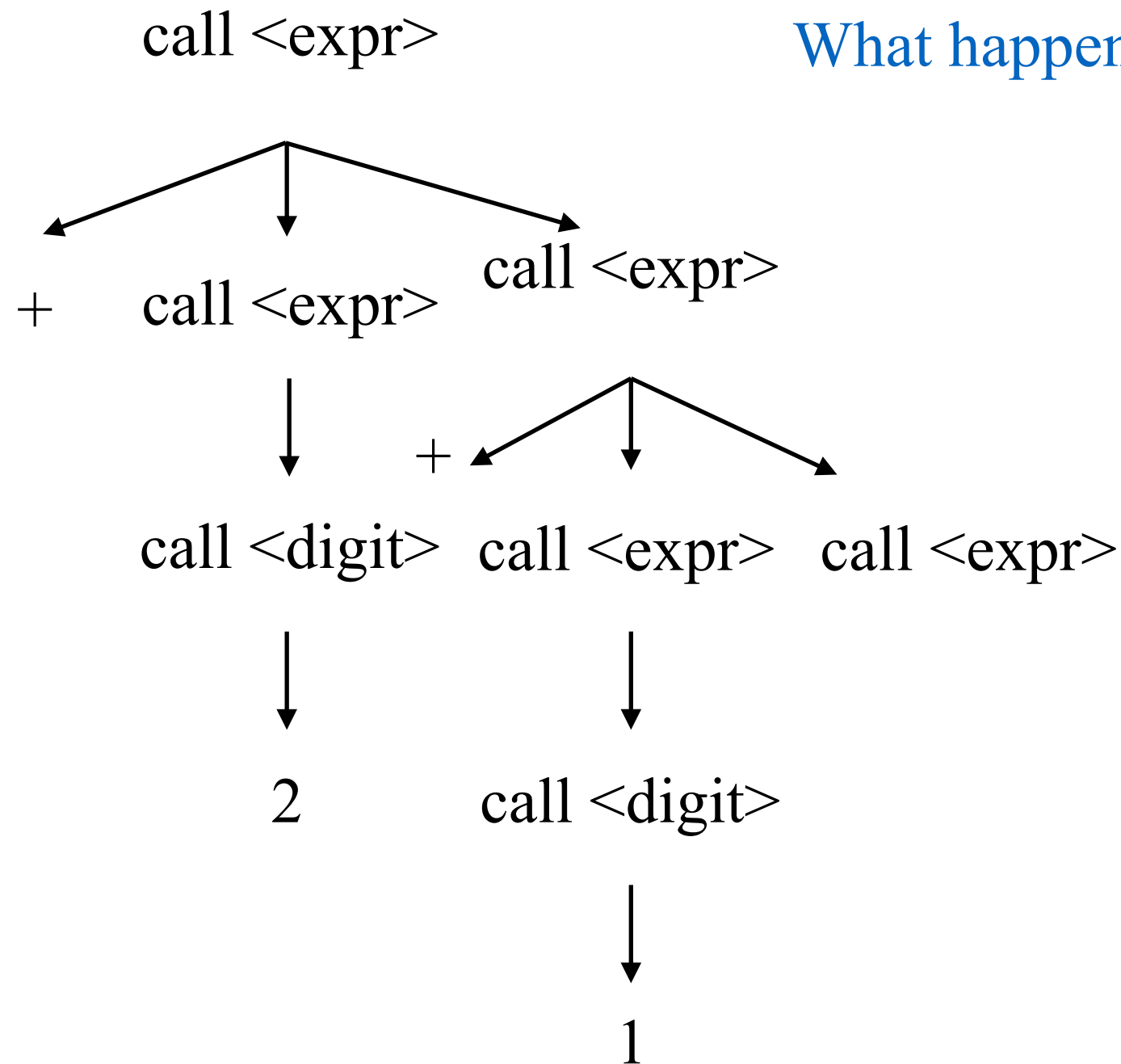


What happens when you parse expression  
“+ 2 + 1 2”

# Example: the Original Parser

1:  $\langle \text{expr} \rangle ::= + \langle \text{expr} \rangle \langle \text{expr} \rangle \mid$   
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	+	0...9	other
$\langle \text{expr} \rangle$	rule 1	rule 2	error
$\langle \text{digit} \rangle$	error	rule 3	error

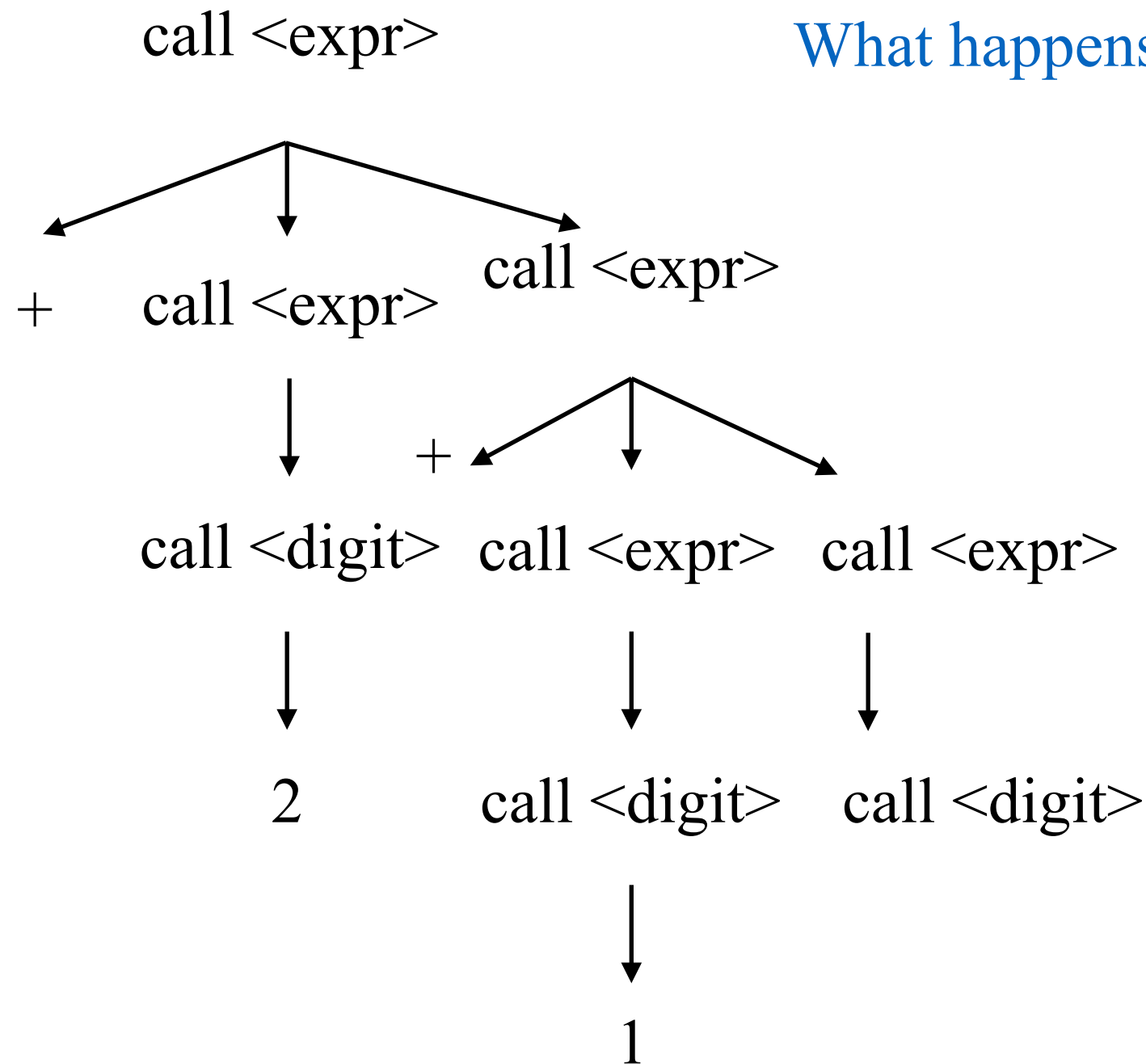


What happens when you parse expression  
“+ 2 + 1 2”

# Example: the Original Parser

1:  $\langle \text{expr} \rangle ::= + \langle \text{expr} \rangle \langle \text{expr} \rangle \mid$   
2:  $\langle \text{digit} \rangle$   
3:  $\langle \text{digit} \rangle ::= 0 \mid 1 \mid 2 \mid 3 \mid \dots \mid 9$

	+	0...9	other
$\langle \text{expr} \rangle$	rule 1	rule 2	error
$\langle \text{digit} \rangle$	error	rule 3	error



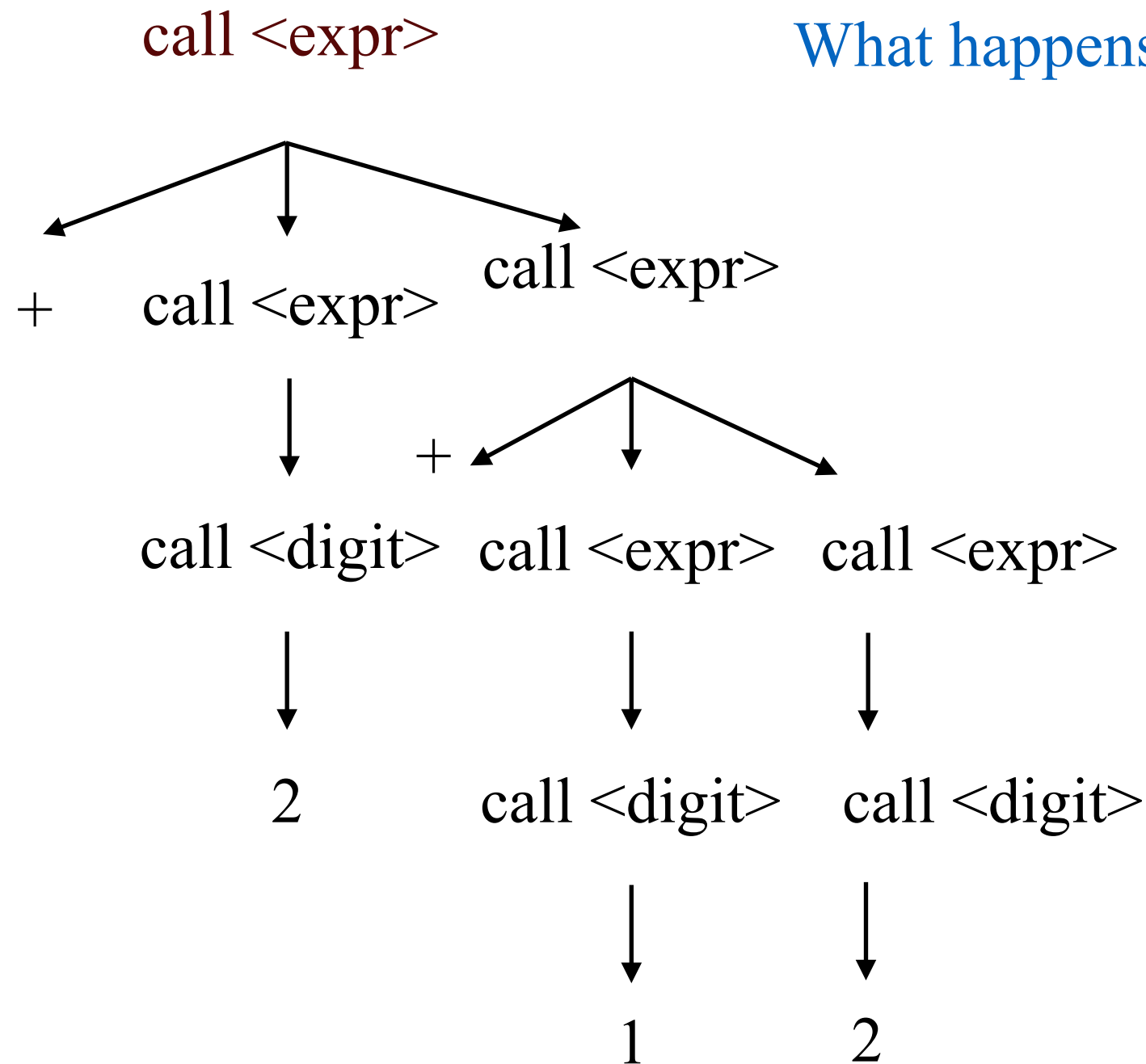
What happens when you parse expression  
“+ 2 + 1 2”



# Example: the Original Parser

1:  $\langle \text{expr} \rangle ::= + \langle \text{expr} \rangle \langle \text{expr} \rangle \mid$   
2:  $\langle \text{digit} \rangle$   
3:  $\langle \text{digit} \rangle ::= 0 \mid 1 \mid 2 \mid 3 \mid \dots \mid 9$

	+	0...9	other
$\langle \text{expr} \rangle$	rule 1	rule 2	error
$\langle \text{digit} \rangle$	error	rule 3	error



What happens when you parse expression  
“+ 2 + 1 2”

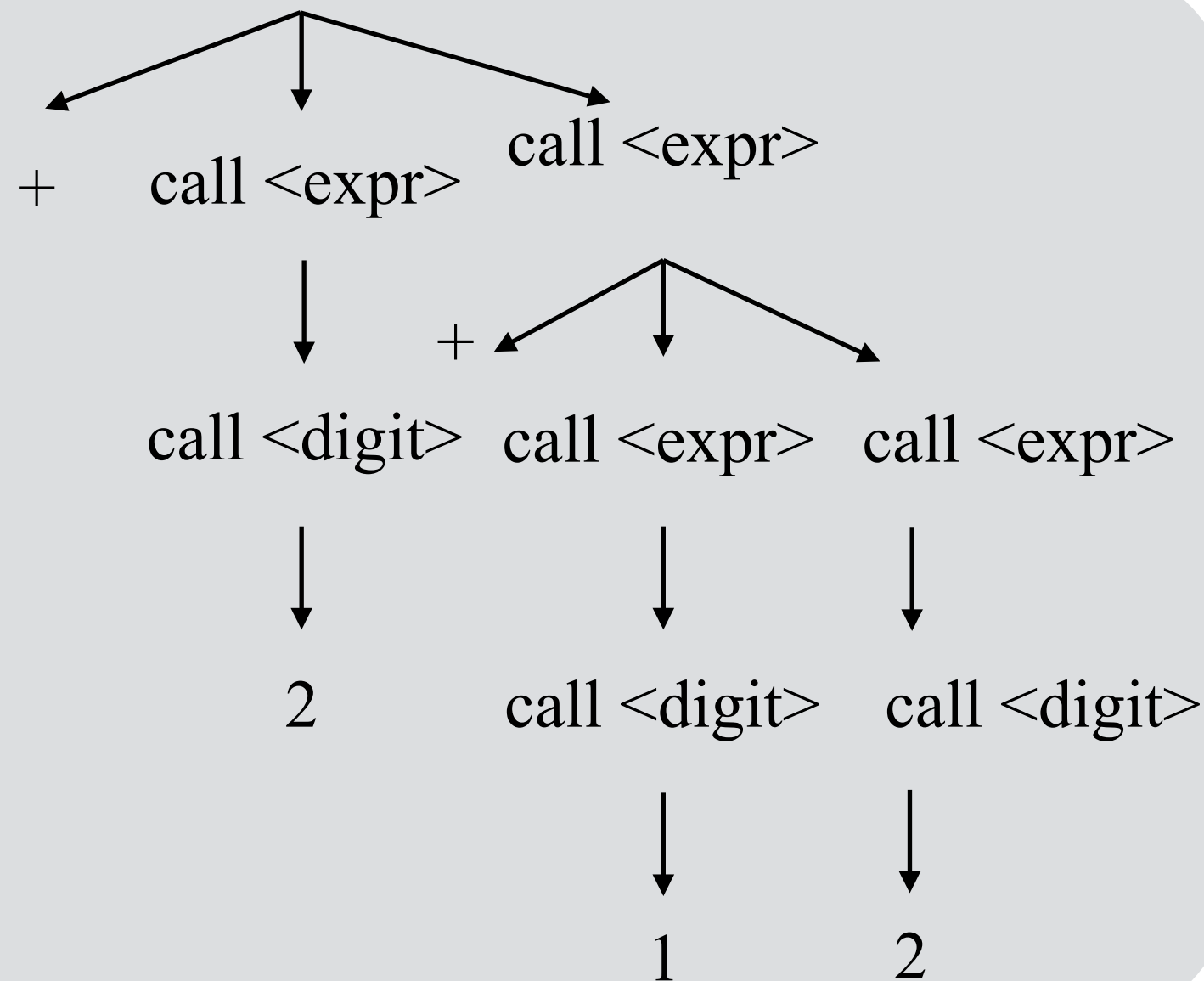
# Example: the Original Parser

1:  $\langle \text{expr} \rangle ::= + \langle \text{expr} \rangle \langle \text{expr} \rangle \mid$   
2:  $\langle \text{digit} \rangle$   
3:  $\langle \text{digit} \rangle ::= 0 \mid 1 \mid 2 \mid 3 \mid \dots \mid 9$

	+	0...9	other
$\langle \text{expr} \rangle$	rule 1	rule 2	error
$\langle \text{digit} \rangle$	error	rule 3	error

call  $\langle \text{expr} \rangle$

What happens when you parse expression  
“+ 2 + 1 2”



## Example: the Original Parser

$$1: \langle \text{expr} \rangle ::= + \langle \text{expr} \rangle \langle \text{expr} \rangle \mid$$

2:                   < digit >

$$3: \langle \text{digit} \rangle ::= 0 \mid 1 \mid 2 \mid 3 \mid \dots \mid 9$$

	+	0...9	other
< expr >	rule 1	rule 2	error
< digit >	error	rule 3	error

call <expr>

A tree diagram with a root node at the top. Three arrows point from the root to three children below. The children are labeled from left to right as: '+', 'call <expr>', and 'call <expr>'. The second 'call <expr>' is highlighted in red in the original image.

```
graph TD; A["+"] --> B["call <digit>"]; A --> C["call <expr>"]; A --> D["call <expr>"]; B --> E["2"]; C --> F["call <digit>"]; D --> G["call <digit>"]; F --> H["1"]; G --> I["2"];
```

# What happens when you parse expression “ + 2 + 1 2 ”

# Next Lecture

---

Things to do:

- Start programming in C.
- Read Scott, Chapter 3.1 - 3.3; ALSU 7.1
- Read Scott, Chapter 8.1 - 8.2; ALSU 7.1 - 7.3