

# Principles of Programming Languages

CS 314

Recitation 2



RUTGERS

Deterministic finite automaton (DFA)

Context-free grammar

Derivation

Parse tree

Ambiguous grammars

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Context-free grammar

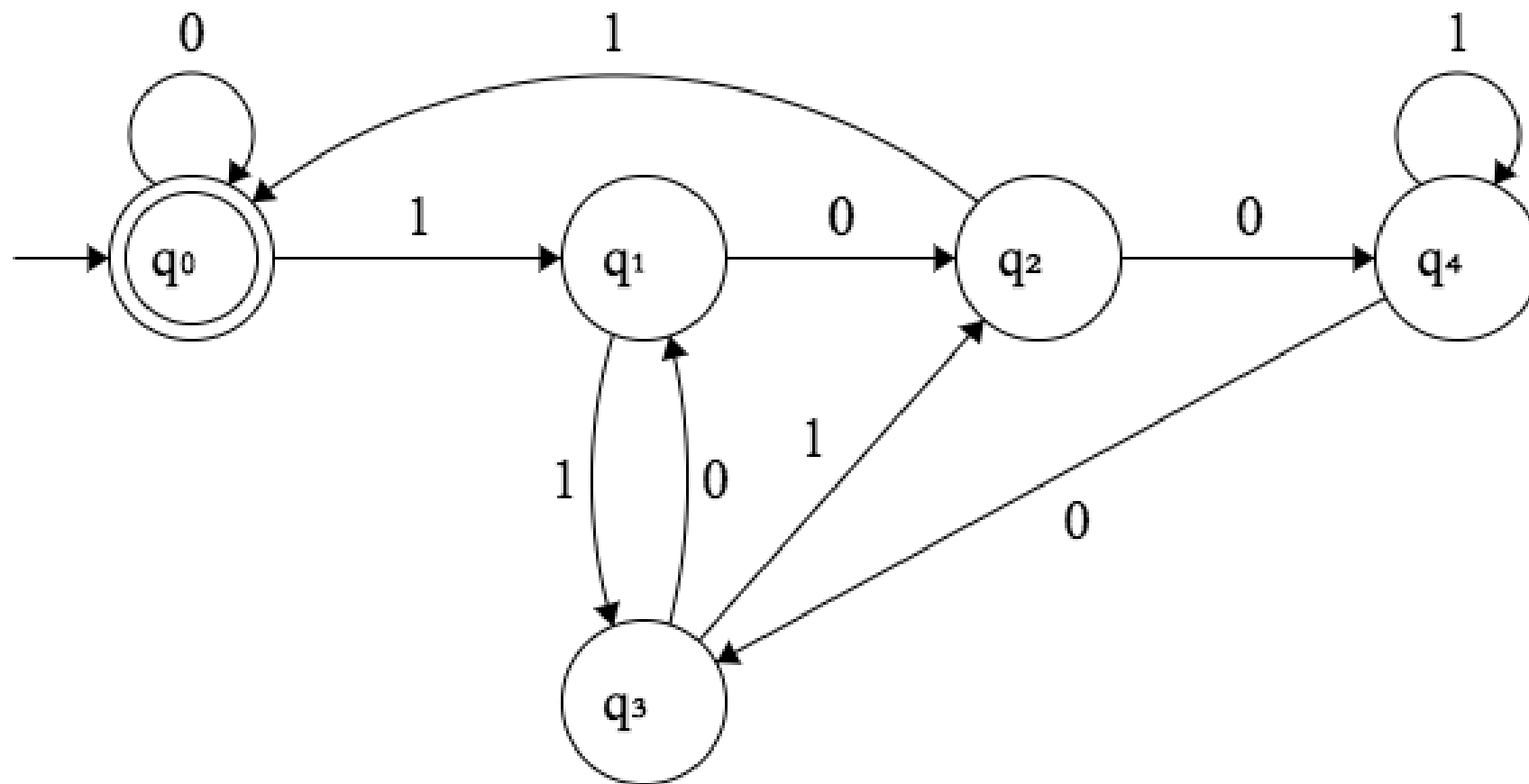
Derivation

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Specify a DFA using a transition diagram and a formal FSA specification  $\langle S, s, F, T \rangle$  that recognizes the following language:

“All strings of 0’s and 1’s that, when interpreted as a binary number, are divisible by 5. In other words,  $\text{value}(\text{binary number}) \bmod 5 = 0$ .”



Deterministic finite automaton (DFA)

**Context-free grammar**

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Specify a context-free grammar in BNF notation that generates the following language. (With alphabet  $\Sigma = \{0,1\}$  )

$\{ w \mid w \text{ has at least three } 1 \}$

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CFG:

$\langle S \rangle ::= \langle A \rangle 1 \langle A \rangle 1 \langle A \rangle 1 \langle A \rangle$

$\langle A \rangle ::= 0 \langle A \rangle \mid 1 \langle A \rangle \mid \varepsilon$



Specify a context-free grammar in BNF notation that generates the following language. (With alphabet  $\Sigma=\{0,1\}$  )

$\{ w \mid |w| \text{ is odd, and the symbol in the middle of } w \text{ is } 0 \}$

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CFG:

$\langle S \rangle ::= \langle A \rangle \langle S \rangle \langle A \rangle \mid 0$

$\langle A \rangle ::= 0 \mid 1$

Specify a context-free grammar in BNF notation that generates the following language.

$\{ a^{3n}b^{3n}c^{3n}d^{4n} \mid n \geq 0 \}$ , with alphabet  $\Sigma = \{a, b, c, d\}$

Specify a context-free grammar in BNF notation that generates the following language.

$\{ a^{3n}b^{3n}c^{3n}d^{4n} \mid n \geq 0 \}$ , with alphabet  $\Sigma = \{a, b, c, d\}$

**Not a context-free grammar!**

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Given a CFG:

$$\langle E \rangle ::= \langle E \rangle + \langle T \rangle \mid \langle E \rangle - \langle T \rangle \mid \langle T \rangle$$
$$\langle T \rangle ::= \langle T \rangle * \langle F \rangle \mid \langle T \rangle / \langle F \rangle \mid \langle F \rangle$$
$$\langle F \rangle ::= (\langle E \rangle) \mid i$$

Give a leftmost and a rightmost derivation for the sentence:  $i+i*i$

$\langle E \rangle$

$\Rightarrow_L \langle E \rangle + \langle T \rangle$

$\Rightarrow_L \langle T \rangle + \langle T \rangle$

$\Rightarrow_L \langle F \rangle + \langle T \rangle$

$\Rightarrow_L i + \langle T \rangle$

$\Rightarrow_L i + \langle T \rangle^* \langle F \rangle$

$\Rightarrow_L i + \langle F \rangle^* \langle F \rangle$

$\Rightarrow_L i + i^* \langle F \rangle$

$\Rightarrow_L i + i^* i$

$\langle E \rangle$

$\Rightarrow_R \langle E \rangle + \langle T \rangle$

$\Rightarrow_R \langle E \rangle + \langle T \rangle^* \langle F \rangle$

$\Rightarrow_R \langle E \rangle + \langle T \rangle^* i$

$\Rightarrow_R \langle E \rangle + \langle F \rangle^* i$

$\Rightarrow_R \langle E \rangle + i^* i$

$\Rightarrow_R \langle T \rangle + i^* i$

$\Rightarrow_R \langle F \rangle + i^* i$

$\Rightarrow_R i + i^* i$

Given a CFG:

$\langle E \rangle ::= \langle E \rangle + \langle T \rangle \mid \langle E \rangle - \langle T \rangle \mid \langle T \rangle$

$\langle T \rangle ::= \langle T \rangle^* \langle F \rangle \mid \langle T \rangle / \langle F \rangle \mid \langle F \rangle$

$\langle F \rangle ::= (\langle E \rangle) \mid i$

sentence:  $i + i^* i$

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Show the corresponding parse trees for the derivations we got in last example

Given a CFG:

$$\langle E \rangle ::= \langle E \rangle + \langle T \rangle \mid \langle E \rangle - \langle T \rangle \mid \langle T \rangle$$
$$\langle T \rangle ::= \langle T \rangle * \langle F \rangle \mid \langle T \rangle / \langle F \rangle \mid \langle F \rangle$$
$$\langle F \rangle ::= (\langle E \rangle) \mid i$$

sentence:  $i+i*i$

$$\langle E \rangle$$
$$\Rightarrow_L \langle E \rangle + \langle T \rangle$$
$$\Rightarrow_L \langle T \rangle + \langle T \rangle$$
$$\Rightarrow_L \langle F \rangle + \langle T \rangle$$
$$\Rightarrow_L i + \langle T \rangle$$
$$\Rightarrow_L i + \langle T \rangle * \langle F \rangle$$
$$\Rightarrow_L i + \langle F \rangle * \langle F \rangle$$
$$\Rightarrow_L i + i * \langle F \rangle$$
$$\Rightarrow_L i + i * i$$

Show the corresponding parse trees for the derivations we got in last example

Given a CFG:

$\langle E \rangle ::= \langle E \rangle + \langle T \rangle \mid \langle E \rangle - \langle T \rangle \mid \langle T \rangle$

$\langle T \rangle ::= \langle T \rangle * \langle F \rangle \mid \langle T \rangle / \langle F \rangle \mid \langle F \rangle$

$\langle F \rangle ::= (\langle E \rangle) \mid i$

sentence:  $i + i * i$

$\langle E \rangle$

$\Rightarrow_L \langle E \rangle + \langle T \rangle$

$\Rightarrow_L \langle T \rangle + \langle T \rangle$

$\Rightarrow_L \langle F \rangle + \langle T \rangle$

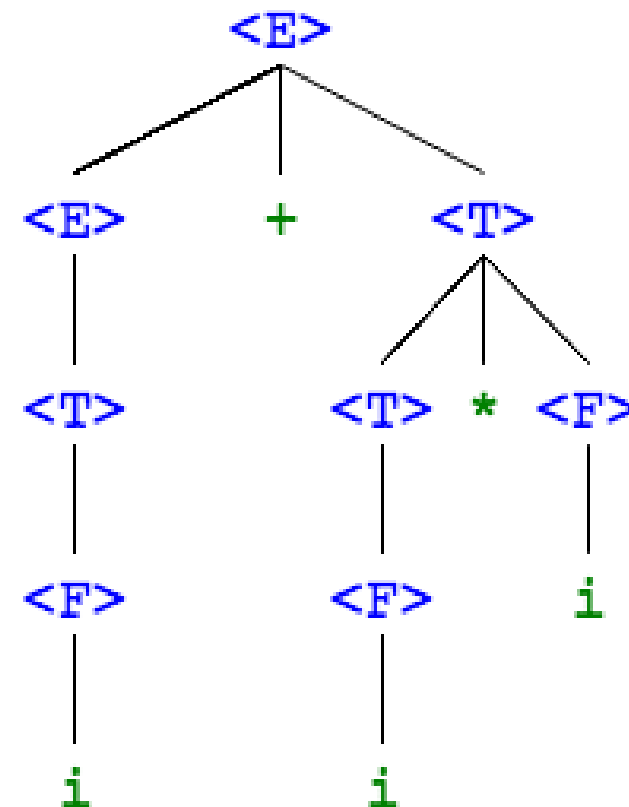
$\Rightarrow_L i + \langle T \rangle$

$\Rightarrow_L i + \langle T \rangle * \langle F \rangle$

$\Rightarrow_L i + \langle F \rangle * \langle F \rangle$

$\Rightarrow_L i + i * \langle F \rangle$

$\Rightarrow_L i + i * i$



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Given grammar:

$$\langle \text{exp} \rangle ::= \langle \text{exp} \rangle \langle \text{oper} \rangle \langle \text{exp} \rangle \mid (\langle \text{exp} \rangle) \mid i$$
$$\langle \text{oper} \rangle ::= + \mid - \mid * \mid /$$

Show that the above grammar is ambiguous.

For the sentence  $i+i*i$ , it has two distinct rightmost derivations.

So the grammar is ambiguous.

$$\begin{aligned} \langle \text{exp} \rangle &::= \langle \text{exp} \rangle \langle \text{oper} \rangle \langle \text{exp} \rangle \mid (\langle \text{exp} \rangle) \mid i \\ \langle \text{oper} \rangle &::= + \mid - \mid * \mid / \end{aligned}$$

(1)

$\langle \text{exp} \rangle$

$\Rightarrow_R \langle \text{exp} \rangle \langle \text{oper} \rangle \langle \text{exp} \rangle$

$\Rightarrow_R \langle \text{exp} \rangle \langle \text{oper} \rangle i$

$\Rightarrow_R \langle \text{exp} \rangle * i$

$\Rightarrow_R \langle \text{exp} \rangle \langle \text{oper} \rangle \langle \text{exp} \rangle * i$

$\Rightarrow_R \langle \text{exp} \rangle \langle \text{oper} \rangle i * i$

$\Rightarrow_R \langle \text{exp} \rangle + i * i$

$\Rightarrow_R i + i * i$

(2)

$\langle \text{exp} \rangle$

$\Rightarrow_R \langle \text{exp} \rangle \langle \text{oper} \rangle \langle \text{exp} \rangle$

$\Rightarrow_R \langle \text{exp} \rangle \langle \text{oper} \rangle \langle \text{exp} \rangle \langle \text{oper} \rangle \langle \text{exp} \rangle$

$\Rightarrow_R \langle \text{exp} \rangle \langle \text{oper} \rangle \langle \text{exp} \rangle \langle \text{oper} \rangle i$

$\Rightarrow_R \langle \text{exp} \rangle \langle \text{oper} \rangle \langle \text{exp} \rangle * i$

$\Rightarrow_R \langle \text{exp} \rangle \langle \text{oper} \rangle i * i$

$\Rightarrow_R \langle \text{exp} \rangle + i * i$

$\Rightarrow_R i + i * i$