

# Principles of Programming Languages

CS 314

Recitation 3



RUTGERS

LL(1) Parsing

Recursive Descent Parsing

## LL(1) Parsing

## Recursive Descent Parsing

Consider the following grammar:

$$\begin{aligned} \langle \text{decl} \rangle &::= \langle \text{ID} \rangle \langle \text{decl\_tail} \rangle \\ \langle \text{decl\_tail} \rangle &::= , \langle \text{decl} \rangle \mid = \langle \text{ID} \rangle ; \\ \langle \text{ID} \rangle &::= a \mid b \mid c \end{aligned}$$

Is this grammar LL(1)?

Make the FIRST sets (and if necessary, the FOLLOW sets) for this grammar, then construct the LL(1) parse table.

The grammar:

$$\langle \text{decl} \rangle ::= \langle \text{ID} \rangle \langle \text{decl\_tail} \rangle$$
$$\langle \text{decl\_tail} \rangle ::= , \langle \text{decl} \rangle \mid = \langle \text{ID} \rangle ;$$
$$\langle \text{ID} \rangle ::= a \mid b \mid c$$

has the following FIRST sets:

$$\text{FIRST}(\langle \text{ID} \rangle \langle \text{decl\_tail} \rangle) = \{ "a", "b", "c" \}$$
$$\text{FIRST}(, \langle \text{decl} \rangle) = \{ ", " \}$$
$$\text{FIRST}(= \langle \text{ID} \rangle ;) = \{ "=", " \}$$
$$\text{FIRST}(a) = \{ "a" \}$$
$$\text{FIRST}(b) = \{ "b" \}$$
$$\text{FIRST}(c) = \{ "c" \}$$

Recall that  $\text{PREDICT}(A ::= \delta) = (\text{FIRST}(\delta) - \{\epsilon\}) \cup \text{FOLLOW}(A)$  if  $\epsilon$  is an element of  $\text{FIRST}(\delta)$ .

Otherwise,  $\text{PREDICT}(A ::= \delta) = \text{FIRST}(\delta)$ .

The grammar:

$$\begin{aligned}\langle \text{decl} \rangle &::= \langle \text{ID} \rangle \langle \text{decl\_tail} \rangle \\ \langle \text{decl\_tail} \rangle &::= , \langle \text{decl} \rangle \mid = \langle \text{ID} \rangle ; \\ \langle \text{ID} \rangle &::= a \mid b \mid c\end{aligned}$$

There is no  $\epsilon$  in any of the FIRST sets, so  $\text{PREDICT}(A ::= \delta) = \text{FIRST}(\delta)$ . We have the following:

$$\text{PREDICT}(\langle \text{decl} \rangle ::= \langle \text{ID} \rangle \langle \text{decl\_tail} \rangle) = \text{FIRST}(\langle \text{ID} \rangle \langle \text{decl\_tail} \rangle) = \{“a”, “b”, “c”\}$$
$$\text{PREDICT}(\langle \text{decl\_tail} \rangle ::= , \langle \text{decl} \rangle) = \text{FIRST}(, \langle \text{decl} \rangle) = \{“,”\}$$
$$\text{PREDICT}(\langle \text{decl\_tail} \rangle ::= = \langle \text{ID} \rangle ;) = \text{FIRST}(= \langle \text{ID} \rangle ;) = \{“=”\}$$
$$\text{PREDICT}(\langle \text{ID} \rangle ::= a) = \text{FIRST}(a) = \{“a”\}$$
$$\text{PREDICT}(\langle \text{ID} \rangle ::= b) = \text{FIRST}(b) = \{“b”\}$$
$$\text{PREDICT}(\langle \text{ID} \rangle ::= c) = \text{FIRST}(c) = \{“c”\}$$

Since no two production rules for the same non-terminal have overlap in their PREDICT sets, the grammar is LL(1). Now, for the parsing table.

## Grammar:

$$\langle \text{decl} \rangle ::= \langle \text{ID} \rangle \langle \text{decl\_tail} \rangle$$
$$\langle \text{decl\_tail} \rangle ::= , \langle \text{decl} \rangle \mid = \langle \text{ID} \rangle ;$$
$$\langle \text{ID} \rangle ::= a \mid b \mid c$$

Since  $\text{FIRST}(\langle \text{ID} \rangle \langle \text{decl\_tail} \rangle) = \{“a”, “b”, “c”\}$

	,	=	a	b	c
$\langle \text{decl} \rangle$			$\langle \text{ID} \rangle \langle \text{decl\_tail} \rangle$	$\langle \text{ID} \rangle \langle \text{decl\_tail} \rangle$	$\langle \text{ID} \rangle \langle \text{decl\_tail} \rangle$
$\langle \text{decl\_tail} \rangle$					
$\langle \text{ID} \rangle$					

## Grammar:

$\langle \text{decl} \rangle ::= \langle \text{ID} \rangle \langle \text{decl\_tail} \rangle$   
 $\langle \text{decl\_tail} \rangle ::= , \langle \text{decl} \rangle \mid = \langle \text{ID} \rangle ;$   
 $\langle \text{ID} \rangle ::= a \mid b \mid c$

If the derivation of  $\langle \text{decl\_tail} \rangle$  starts with “,”, the grammar follows the rule:  $\langle \text{decl\_tail} \rangle ::= , \langle \text{decl} \rangle$

	,	=	a	b	c
$\langle \text{decl} \rangle$			$\langle \text{ID} \rangle \langle \text{decl\_tail} \rangle$	$\langle \text{ID} \rangle \langle \text{decl\_tail} \rangle$	$\langle \text{ID} \rangle \langle \text{decl\_tail} \rangle$
$\langle \text{decl\_tail} \rangle$	, $\langle \text{decl} \rangle$				
$\langle \text{ID} \rangle$					



## Grammar:

$$\langle \text{decl} \rangle ::= \langle \text{ID} \rangle \langle \text{decl\_tail} \rangle$$

$$\langle \text{decl\_tail} \rangle ::= , \langle \text{decl} \rangle \mid = \langle \text{ID} \rangle ;$$

$$\langle \text{ID} \rangle ::= a \mid b \mid c$$

If the derivation of  $\langle \text{decl\_tail} \rangle$  starts with “=”, the grammar follows the rule  $\langle \text{decl\_tail} \rangle ::= = \langle \text{ID} \rangle ;$

	,	=	a	b	c
$\langle \text{decl} \rangle$			$\langle \text{ID} \rangle \langle \text{decl\_tail} \rangle$	$\langle \text{ID} \rangle \langle \text{decl\_tail} \rangle$	$\langle \text{ID} \rangle \langle \text{decl\_tail} \rangle$
$\langle \text{decl\_tail} \rangle$	$, \langle \text{decl} \rangle$	$= \langle \text{ID} \rangle ;$			
$\langle \text{ID} \rangle$					

## Grammar:

$\langle \text{decl} \rangle ::= \langle \text{ID} \rangle \langle \text{decl\_tail} \rangle$   
 $\langle \text{decl\_tail} \rangle ::= , \langle \text{decl} \rangle \mid = \langle \text{ID} \rangle ;$   
 $\langle \text{ID} \rangle ::= a \mid b \mid c$

Following the grammar, the derivation of  $\langle \text{ID} \rangle$  can become either a, b, or c.

	,	=	a	b	c
$\langle \text{decl} \rangle$			$\langle \text{ID} \rangle \langle \text{decl\_tail} \rangle$	$\langle \text{ID} \rangle \langle \text{decl\_tail} \rangle$	$\langle \text{ID} \rangle \langle \text{decl\_tail} \rangle$
$\langle \text{decl\_tail} \rangle$	$, \langle \text{decl} \rangle$	$= \langle \text{ID} \rangle ;$			
$\langle \text{ID} \rangle$			a	b	c

## Grammar:

$$\langle \text{decl} \rangle ::= \langle \text{ID} \rangle \langle \text{decl\_tail} \rangle$$

$$\langle \text{decl\_tail} \rangle ::= , \langle \text{decl} \rangle \mid = \langle \text{ID} \rangle ;$$

$$\langle \text{ID} \rangle ::= a \mid b \mid c$$

Note that the LL(1) parse table has no conflicts (i.e., each cell in the table only has one entry).

	,	=	a	b	c
$\langle \text{decl} \rangle$			$\langle \text{ID} \rangle \langle \text{decl\_tail} \rangle$	$\langle \text{ID} \rangle \langle \text{decl\_tail} \rangle$	$\langle \text{ID} \rangle \langle \text{decl\_tail} \rangle$
$\langle \text{decl\_tail} \rangle$	$, \langle \text{decl} \rangle$	$= \langle \text{ID} \rangle ;$			
$\langle \text{ID} \rangle$			a	b	c

Consider the following modified grammar:

$\langle \text{decl} \rangle ::= \langle \text{ID} \rangle \langle \text{mid} \rangle \langle \text{tail} \rangle$

$\langle \text{mid} \rangle ::= \langle \text{mid} \rangle , \langle \text{ID} \rangle \mid \epsilon$

$\langle \text{tail} \rangle ::= : \langle \text{ID} \rangle ;$

$\langle \text{id} \rangle ::= a \mid b \mid c$

Where the rules for  $\langle \text{mid} \rangle$  are:  $\langle \text{mid} \rangle ::= \langle \text{mid} \rangle , \langle \text{ID} \rangle$  and  $\langle \text{mid} \rangle ::= \epsilon$

Is this grammar still LL(1)?

Make the FIRST sets (and if necessary, FOLLOW sets) for this grammar, then construct the LL(1) parse table.

The grammar:

$\langle \text{decl} \rangle ::= \langle \text{ID} \rangle \langle \text{mid} \rangle \langle \text{tail} \rangle$

$\langle \text{mid} \rangle ::= \langle \text{mid} \rangle , \langle \text{ID} \rangle \mid \epsilon$

$\langle \text{tail} \rangle ::= : \langle \text{ID} \rangle ;$

$\langle \text{ID} \rangle ::= a \mid b \mid c$

has the following FIRST sets:

$\text{FIRST}(a) = \{a\}$

$\text{FIRST}(b) = \{b\}$

$\text{FIRST}(c) = \{c\}$

$\text{FIRST}(: \langle \text{ID} \rangle ;) = \{:\}$

$\text{FIRST}(\langle \text{ID} \rangle \langle \text{mid} \rangle \langle \text{tail} \rangle) = \{a, b, c\}$

But what about FIRST sets for  $\langle \text{mid} \rangle$ 's rules?

Since  $\langle \text{mid} \rangle ::= \epsilon$  is a rule,

$$\text{FIRST}(\langle \text{mid} \rangle) = \{\epsilon, ", "\}$$

$$\text{FIRST}(\epsilon) = \{\epsilon\}$$

Recall that  $\text{PREDICT}(A ::= \delta) = (\text{FIRST}(\delta) - \{\epsilon\}) \cup \text{FOLLOW}(A)$  if  $\epsilon$  is an element of  $\text{FIRST}(\delta)$ .  
Otherwise,  $\text{PREDICT}(A ::= \delta) = \text{FIRST}(\delta)$ .

Since  $\epsilon$  is an element of  $\text{FIRST}(\epsilon)$ ,  $\text{PREDICT}(\langle \text{mid} \rangle ::= \epsilon) = (\text{FIRST}(\epsilon) - \{\epsilon\}) \cup \text{FOLLOW}(\langle \text{mid} \rangle)$ .

So, for the rule  $\langle \text{mid} \rangle ::= \epsilon$ :

We'll need to know what  $\text{FOLLOW}(\langle \text{mid} \rangle)$  is in order to compute this rule's  $\text{PREDICT}$  set.

For the grammar:

$\langle \text{decl} \rangle ::= \langle \text{ID} \rangle \langle \text{mid} \rangle \langle \text{tail} \rangle$

$\langle \text{mid} \rangle ::= \langle \text{mid} \rangle , \langle \text{ID} \rangle \mid \epsilon$

$\langle \text{tail} \rangle ::= : \langle \text{ID} \rangle ;$

$\langle \text{ID} \rangle ::= a \mid b \mid c$

$\langle \text{mid} \rangle$  appears in the right side of two rules:

- $\langle \text{decl} \rangle ::= \langle \text{ID} \rangle \langle \text{mid} \rangle \langle \text{tail} \rangle$
- $\langle \text{mid} \rangle ::= \langle \text{mid} \rangle , \langle \text{ID} \rangle$

So, the FOLLOW set for  $\langle \text{mid} \rangle$  would be:

$$\begin{aligned}\text{FOLLOW}(\langle \text{mid} \rangle) &= \text{FIRST}(\langle \text{tail} \rangle) \cup \{“, ”\} \\ &= \{:, “, ”\}\end{aligned}$$

Now we can find the PREDICT set:

$$\text{PREDICT}(\langle \text{mid} \rangle ::= \epsilon) = (\text{FIRST}(\epsilon) - \{\epsilon\}) \cup \{:, “, ”\} = \{:, “, ”\}$$

For the grammar:

$\langle \text{decl} \rangle ::= \langle \text{ID} \rangle \langle \text{mid} \rangle \langle \text{tail} \rangle$

$\langle \text{mid} \rangle ::= \langle \text{mid} \rangle , \langle \text{ID} \rangle \mid \epsilon$

$\langle \text{tail} \rangle ::= : \langle \text{ID} \rangle ;$

$\langle \text{ID} \rangle ::= a \mid b \mid c$

Since  $\text{FIRST}(\langle \text{mid} \rangle , \langle \text{ID} \rangle) = \{“,”\}$ ,  $\epsilon$  is not in  $\text{FIRST}(\langle \text{mid} \rangle , \langle \text{ID} \rangle)$ , so

$\text{PREDICT}(\langle \text{mid} \rangle ::= \langle \text{mid} \rangle , \langle \text{ID} \rangle) = \text{FIRST}(\langle \text{mid} \rangle , \langle \text{ID} \rangle) = \{“,”\}$

Thus, the grammar is not LL(1), since there is overlap between the PREDICT sets of the rules  $\langle \text{mid} \rangle ::= \epsilon$  and  $\langle \text{mid} \rangle ::= \langle \text{mid} \rangle , \langle \text{ID} \rangle$

$\text{PREDICT}(\langle \text{mid} \rangle ::= \epsilon) = (\text{FIRST}(\epsilon) - \{\epsilon\}) \cup \{:, “,”\} = \{:, “,”\}$

$\text{PREDICT}(\langle \text{mid} \rangle ::= \langle \text{mid} \rangle , \langle \text{ID} \rangle) = \text{FIRST}(\langle \text{mid} \rangle , \langle \text{ID} \rangle) = \{“,”\}$

The overlap is the element “,”.



## The modified Grammar:

$\langle \text{decl} \rangle ::= \langle \text{ID} \rangle \langle \text{mid} \rangle \langle \text{tail} \rangle$

$\langle \text{mid} \rangle ::= \langle \text{mid} \rangle , \langle \text{ID} \rangle \mid \epsilon$

$\langle \text{tail} \rangle ::= : \langle \text{ID} \rangle ;$

$\langle \text{ID} \rangle ::= a \mid b \mid c$

Since  $\text{FIRST}(\langle \text{ID} \rangle \langle \text{mid} \rangle \langle \text{tail} \rangle) = \{a, b, c\}$ :

	,	:	a	b	c
$\langle \text{decl} \rangle$			$\langle \text{ID} \rangle \langle \text{mid} \rangle \langle \text{tail} \rangle$	$\langle \text{ID} \rangle \langle \text{mid} \rangle \langle \text{tail} \rangle$	$\langle \text{ID} \rangle \langle \text{mid} \rangle \langle \text{tail} \rangle$
$\langle \text{mid} \rangle$					
$\langle \text{tail} \rangle$					
$\langle \text{ID} \rangle$					

## The modified Grammar:

$\langle \text{decl} \rangle ::= \langle \text{ID} \rangle \langle \text{mid} \rangle \langle \text{tail} \rangle$

$\langle \text{mid} \rangle ::= \langle \text{mid} \rangle , \langle \text{ID} \rangle \mid \epsilon$

$\langle \text{tail} \rangle ::= : \langle \text{ID} \rangle ;$

$\langle \text{ID} \rangle ::= a \mid b \mid c$

Since  $\text{FIRST}(\langle \text{mid} \rangle , \langle \text{ID} \rangle) = \{“,”\}$  and  $\langle \text{mid} \rangle ::= \langle \text{mid} \rangle , \langle \text{ID} \rangle$

	,	:	a	b	c
$\langle \text{decl} \rangle$			$\langle \text{ID} \rangle \langle \text{mid} \rangle \langle \text{tail} \rangle$	$\langle \text{ID} \rangle \langle \text{mid} \rangle \langle \text{tail} \rangle$	$\langle \text{ID} \rangle \langle \text{mid} \rangle \langle \text{tail} \rangle$
$\langle \text{mid} \rangle$	$\langle \text{mid} \rangle , \langle \text{ID} \rangle$				
$\langle \text{tail} \rangle$					
$\langle \text{ID} \rangle$					

## The modified Grammar:

$\langle \text{decl} \rangle ::= \langle \text{ID} \rangle \langle \text{mid} \rangle \langle \text{tail} \rangle$

$\langle \text{mid} \rangle ::= \langle \text{mid} \rangle , \langle \text{ID} \rangle \mid \epsilon$

$\langle \text{tail} \rangle ::= : \langle \text{ID} \rangle ;$

$\langle \text{ID} \rangle ::= a \mid b \mid c$

Since  $\text{FIRST}(\epsilon) = \{\epsilon\}$ ,  $\text{PREDICT}(\langle \text{mid} \rangle ::= \epsilon) = \{:, ", "\}$  and  $\langle \text{mid} \rangle ::= \epsilon$

	,	:	a	b	c
$\langle \text{decl} \rangle$			$\langle \text{ID} \rangle \langle \text{mid} \rangle \langle \text{tail} \rangle$	$\langle \text{ID} \rangle \langle \text{mid} \rangle \langle \text{tail} \rangle$	$\langle \text{ID} \rangle \langle \text{mid} \rangle \langle \text{tail} \rangle$
$\langle \text{mid} \rangle$	$\langle \text{mid} \rangle , \langle \text{ID} \rangle$ $\epsilon$				
$\langle \text{tail} \rangle$					
$\langle \text{ID} \rangle$					

## The modified Grammar:

$\langle \text{decl} \rangle ::= \langle \text{ID} \rangle \langle \text{mid} \rangle \langle \text{tail} \rangle$

$\langle \text{mid} \rangle ::= \langle \text{mid} \rangle , \langle \text{ID} \rangle \mid \epsilon$

$\langle \text{tail} \rangle ::= : \langle \text{ID} \rangle ;$

$\langle \text{ID} \rangle ::= a \mid b \mid c$

Since  $\text{PREDICT}(\langle \text{mid} \rangle ::= \epsilon) = \{ ":", ", " \}$ , if the symbol read is  $":"$ , then  $\langle \text{mid} \rangle ::= \epsilon$  applies.

	,	:	a	b	c
$\langle \text{decl} \rangle$			$\langle \text{ID} \rangle \langle \text{mid} \rangle \langle \text{tail} \rangle$	$\langle \text{ID} \rangle \langle \text{mid} \rangle \langle \text{tail} \rangle$	$\langle \text{ID} \rangle \langle \text{mid} \rangle \langle \text{tail} \rangle$
$\langle \text{mid} \rangle$	$\langle \text{mid} \rangle , \langle \text{ID} \rangle$ $\epsilon$	$\epsilon$			
$\langle \text{tail} \rangle$					
$\langle \text{ID} \rangle$					

## The modified Grammar:

$$\langle \text{decl} \rangle ::= \langle \text{ID} \rangle \langle \text{mid} \rangle \langle \text{tail} \rangle$$

$$\langle \text{mid} \rangle ::= \langle \text{mid} \rangle , \langle \text{ID} \rangle \mid \epsilon$$

$$\langle \text{tail} \rangle ::= : \langle \text{ID} \rangle ;$$

$$\langle \text{ID} \rangle ::= a \mid b \mid c$$

Since  $\text{FIRST}(a) = \{a\}$ ,  $\text{FIRST}(b) = \{b\}$ , and  $\text{FIRST}(c) = \{c\}$ ,

	,	:	a	b	c
$\langle \text{decl} \rangle$			$\langle \text{ID} \rangle \langle \text{mid} \rangle \langle \text{tail} \rangle$	$\langle \text{ID} \rangle \langle \text{mid} \rangle \langle \text{tail} \rangle$	$\langle \text{ID} \rangle \langle \text{mid} \rangle \langle \text{tail} \rangle$
$\langle \text{mid} \rangle$	$\langle \text{mid} \rangle , \langle \text{ID} \rangle$ $\epsilon$	$\epsilon$			
$\langle \text{tail} \rangle$		$: \langle \text{ID} \rangle ;$			
$\langle \text{ID} \rangle$			a	b	c

Notice that the parse table has a cell with two entries.

There's two choices that can be made in the circled area: "<mid>, <ID>" and  $\epsilon$ .

So, 1 symbol of look-ahead is not enough to decide between these two rules:

<mid> ::= <mid> , <ID>

<mid> ::=  $\epsilon$

	,	:	a	b	c
<decl>			<ID> <mid> <tail>	<ID> <mid> <tail>	<ID> <mid> <tail>
<mid>	<div style="border: 1px solid red; border-radius: 50%; padding: 5px; display: inline-block;">           &lt;mid&gt; , &lt;ID&gt;  <math>\epsilon</math> </div>	$\epsilon$			
<tail>		: <ID> ;			
<ID>			a	b	c

LL(1) Parsing

Recursive Descent Parsing

Consider the following grammar from Example 1 again:

$\langle \text{decl} \rangle ::= \langle \text{ID} \rangle \langle \text{decl\_tail} \rangle$   
 $\langle \text{decl\_tail} \rangle ::= , \langle \text{decl} \rangle \mid = \langle \text{ID} \rangle ;$   
 $\langle \text{ID} \rangle ::= a \mid b \mid c$

Recall that this grammar had the following LL(1) parse table:

	,	=	a	b	c
$\langle \text{decl} \rangle$			$\langle \text{ID} \rangle \langle \text{decl\_tail} \rangle$	$\langle \text{ID} \rangle \langle \text{decl\_tail} \rangle$	$\langle \text{ID} \rangle \langle \text{decl\_tail} \rangle$
$\langle \text{decl\_tail} \rangle$	, $\langle \text{decl} \rangle$	= $\langle \text{ID} \rangle ;$			
$\langle \text{ID} \rangle$			a	b	c

Now we construct a recursive descent parser for this grammar, with routines for each non-terminal. To handle the look-ahead symbol, the global variable 'token' and function 'next\_token()' will be used.



Since there's three non-terminals:

<decl>

<decl\_tail>

<ID>

	,	=	a	b	c
<decl>			<ID> <decl_tail>		
<decl_tail>	, <decl>	= <ID> ;			
<ID>			a	b	c

Each of these non-terminals will have its own method, reading the next token until parsing ends or it reaches an error, and returns **true** only if parsing was successful.

The main program:

```
void program() {
    token = next_token() //initialize starting token
    if (decl() && token == eof())
        //if parsing <decl> returns true and we reached the end of string
        print("Parsed successfully.")
    else
        print("Error!")
}
```

```
bool decl() {  
    if(token != "a" && token != "b" && token != "c")  
        //If starting token is not a, b, or c, it does not start with <ID>, so return failure  
        return false  
    if(!ID())  
        //If parsing <ID> has failed, then return failure  
        return false  
    if(!decl_tail())  
        //Once <ID> is done, the method parses <decl_tail>. If it failed, then return failure  
        return false  
    return true  
}
```

	,	=	a	b	c
<decl>			<ID> <decl_tail>		
<decl_tail>	, <decl>	= <ID> ;			
<ID>			a	b	c

```

bool decl_tail() {
    if(token == ",")
        //parsing should be successful for ", <decl>"
        token = next_token()
        if(!decl()) //if subsequent parsing for <decl> fails, return failure
            return false
    else if(token == "=") //subsequent parsing should be successful for "= <ID> ;"
        token = next_token()
        if(!ID()) //if subsequent parsing for <ID> fails, return failure
            return false
        if(token != ";") //parsing this part should end with ;
            return false
        token = next_token()
    else //neither ", <decl>" nor "= <ID> ;" was parsed, return failure
        return false
    return true
}

```

	,	=	a	b	c
<decl>			<ID> <decl_tail>		
<decl_tail>	, <decl>	= <ID> ;			
<ID>			a	b	c

	,	=	a	b	c
<decl>			<ID> <decl_tail>		
<decl_tail>	, <decl>	= <ID> ;			
<ID>			a	b	c

```

bool ID() {
    if(token == "a" || token == "b" || token == "c")
        //if the current is either a, b, or c, parsing was successful
        token = next_token()
    else
        //if the current token is not a, b, or c, it does not follow the ID rule, return failure
        return false
    return true
}

```