Performance of Distributed CFAR Tests in Nonhomogeneous Background*

C. H. Gowda and R. Viswanathan
Department of Electrical Engineering
Southern Illinois University at Carbondale
Carbondale, IL 62901
Ph: 618-453-7037
Fax: 618-453-7972
e-mail: viswa@engr.siu.edu

Abstract - We evaluate the performances of several distributed CFAR tests operating in nonhomogeneous background conditions. The analysis considers the detection of Rayleigh target in Rayleigh clutter with the possibility of differing clutter power levels in the test cells of distributed radars. The tests considered include the previously defined maximum order statistic detector (MOS), belonging to a class of signal-plus- order statistic (S+OS) detectors, a new normalized test statistic (NTS), also belonging to the S+OS class, the OR and the AND fusion rules. Numerical results studied for a two radar system show how the false alarm rate of the MOS test changes with differences in the clutter power levels of the test cells. Results also indicate that, with differing test cells' power levels, the OR fusion rule can be quite competitive to more complex tests, viz. NTS and MOS.

I. Introduction

For the past several years a considerable amount of work [1-4] on single sensor (for example, radar) constant false alarm rate (CFAR) signal detection has been done. The detection of signals becomes complex when radar returns are from nonstationary background noise (or noise plus clutter). The probability of false alarm increases intolerably when a detection scheme employing a fixed threshold is used. Therefore, adaptive threshold techniques are required in order to maintain a nearly constant false alarm rate. Because of the diversity of the radar search environment (multiple target, abrupt changes in clutter, etc.) there exists no universal CFAR scheme. Typically the adaptive threshold of a CFAR scheme is the product of two terms, one is a fixed scaling factor to adjust the probability of false alarm, and the other is an estimate of the total unknown noise (plus clutter) power of the test cell. The sample in the test cell is compared to this threshold in order to decide the presence or the absence of a target. A variety of CFAR techniques are developed according to the logic used to estimate the unknown noise power level. Some examples are, Cell Averaging CFAR (CA-CFAR), Ordered Statistics CFAR (OS-CFAR), Greatest Of CFAR, Smallest Of CFAR [3], and Selection and Estimation test [4].

Distributed signal detection schemes are needed when system performance factors such as speed, reliability, and constraint over the communication bandwidth are taken into account. In distributed detection techniques, each sensor sends either a binary decision or a condensed form of information (statistics) about the observations available at the sensor to the fusion center, where a final decision about the presence of a target is made. Such techniques have been applied to CA-CFAR, adaptive CA-CFAR, and OS-CFAR. Barkat and Varshney [5] considered CA-CFAR detection using multiple sensors and data fusion. In their approach, each CA-CFAR detector transmits a binary decision to the fusion center where a final decision based on the AND or the OR counting rule is obtained. They have also addressed the adaptive CA-CFAR detector problem for parallel and tandem distributed networks [6]. Distributed OS-CFAR detectors with the AND or the OR fusion rule is considered by Uner and Varshney [7].

The authors proposed a new distributed CFAR detection scheme, called signal-plus-order statistic CFAR (S+OS), in [8]. Instead of a binary decision, each sensor transmits the sample from the test cell and a designated order statistic from the available set of reference observations surrounding the test cell to the fusion center. At the fusion center, the sum of the test samples is compared to an adaptive threshold obtained

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by the product of a fixed scaling factor and a function of the received order statistics, to decide the presence/absence of a target. The estimate of the noise power level of the test cells is provided by this function. Some examples of this function are: minimum of, maximum of, linear combination of, or in the case of a large number of sensors, an order statistics of the variables. The S+OS test that uses the maximum order statistic is called the MOS detector. It was shown in [8] that MOS provides a considerable performance gain over OR or AND fusion rules. In deriving the above test, the problem formulation assumes that the test cells of different sensors all have statistically identical noise (clutter), and that if a target is present in the surveillance regions, all the test cells have statistically identical target returns [8]. What happens if this assumption is violated? We therefore examine in section II how the false alarm probability of MOS changes when power levels of clutter at test cells of sensors become different. In section III, we propose a new test, called normalized test statistic (NTS), also in the class of S+OS, but which maintains a constant false alarm rate independent of the clutter power variations of the test cells. Section IV examines the detection performances of various tests. In order to make the comparison reasonable, the MOS test is designed so that its test threshold corresponds to a value that guarantees the worst case false alarm probability (with respect to changes in the clutter power levels of the test cells) to be less than or equal to a desired value.

II. MOS Test and False Alarm Rate Change

Consider a collection of n distributed sensors, each looking at a search volume consisting of m_i+1 cells, i = 1,2,...n. The leading $m_i/2$ cells and the lagging $m_i/2$ cells form the reference window around the test cell of the i^{th} sensor. We assume that the samples in the test cells to be $i.i.d$ exponential with mean $\lambda_{1i}, i=1,..n$ under the target hypothesis H_1 and exponential with mean $\lambda_{0i}, i=1,..n$ under no target hypothesis H_0 (Rayleigh target and Rayleigh clutter models). Denote the random samples from the reference cells samples as $Y_{i1},....,Y_{im_i}$ and the test samples as X_i, $i=1,2..,n$. In the case of homogeneous background, $Y_{i1},....,Y_{im_i}$ are $i.i.d$ as an exponential with mean λ_{0i}. In the case of a nonhomogeneous background, the above random variables are still independent and exponentially distributed but with a mean value of either λ_{0i} or $\lambda_{0i}(1+CNR_i)$, or $\lambda_{0i}(1+INR_i)$, depending on whether a sample Y_{ij} is from a noise only region, or from a noise plus clutter region, or from an interfering target, respectively. Above, for i^{th} sensor, CNR_i denotes the clutter to noise power ratio and INR_i denotes the interfering signal strength to noise ratio.

By denoting the mean of the test sample X_i as λ_i, we have

$$\lambda_i = \begin{cases} \lambda_{0i} \text{ or } \lambda_{0i}(1 + CNR_i), & \text{under } H_0 \\ \lambda_{1i} = \lambda_{0i}(1 + SNR_i) \text{ or} & \\ \lambda_{0i}(1 + CNR_i + SNR_i), & \text{under } H_1 \end{cases} \quad (1)$$

where SNR_i denotes the signal to noise power ratio of the i^{th} sensor. If we assume that λ_{0i} is the same for all i, then the MOS test defined below is a CFAR test [8]:

$$\sum_{i=1}^{n} X_i \underset{H0}{\overset{H1}{\gtrless}} t \max\left(Y_{(k_i)}\, i = 1,2,..,n\right) \quad (2)$$

where $Y_{(k_i)}$ is the k_i^{th} order statistic of the reference samples $Y_{i1},....,Y_{im_i}$ of the i^{th} sensor. For a two sensor system, let

$$a = \frac{\lambda_{01}}{\lambda_{02}} \quad (3)$$

Therefore, the changes in false alarm probability of (2), when t is fixed assuming a = 1 and a desired false alarm rate of α, as a changes, can be investigated. The numerical calculation of the false alarm probability shows that for

$\alpha=10^{-6}$, $m_1 = 11$, $m_2 = 13, k_1 = 8, and\ k_2 = 9$,

the probability can increase up to its largest value of $\approx 10^{-5}$, and that this largest increase occurs for a being close to 0.1 or 10. Also, the greatest change in the false alarm probability occurs as a is varied from 0.1 through 10, which can be seen in Fig. 5. Unfortunately, this means that the false alarm rate of (2) is sensitive to small variations in a. Also, the maximum of the values of false alarm probabilities corresponding to $a = 0\ and\ a = \infty$ is close to 10^{-5}. If the worst case increase is to be at 10^{-6} and not at 10^{-5}, then the t value in (2) can be appropriately chosen so as to achieve this condition. This is how the MOS test threshold is computed while comparing its performance against other schemes (see section IV). If a is close to 1, then the MOS test performs much better than the OR and the AND fusion rules [8].

III. Normalized Test Statistic and Other Tests

Assume that the data model of the previous section holds. For the sake of simplicity, the following derivation is based on a two sensor system. Applying a likelihood ratio test to the test samples yields

$$\Lambda = \frac{p(X_1 | H_1)p(X_2 | H_1)}{p(X_1 | H_0)p(X_2 | H_0)} \underset{H_0}{\overset{H_1}{\gtrless}} T_L \tag{4}$$

where T_L is an appropriate threshold. Eqn. (4) can be simplified to yield

$$\left(\frac{1}{\lambda_{01}} - \frac{1}{\lambda_{11}}\right)X_1 + \left(\frac{1}{\lambda_{02}} - \frac{1}{\lambda_{12}}\right)X_2 \underset{H_0}{\overset{H_1}{\gtrless}} T' \tag{5}$$

Assuming a homogeneous reference window for each sensor (notice that sensor to sensor homogeneity is not needed, i.e. λ_{0i} need not be identical for all i), but with identical SNR_i's, (5) reduces to

$$\frac{X_1}{\lambda_{01}} + \frac{X_2}{\lambda_{02}} \underset{H_0}{\overset{H_1}{\gtrless}} T^* \tag{6}$$

where T^* is an appropriate threshold.

However, (6) cannot be realized since λ_{01} and λ_{02} are unknown. A CFAR test is obtained by replacing λ_{01} and λ_{02} by their estimates. Using the order statistic of the reference cells of each sensor as the estimates, we obtain the normalized test statistic

$$Z = \frac{X_1}{Y_{(k_1)}} + \frac{X_2}{Y_{(k_2)}} \underset{H_0}{\overset{H_1}{\gtrless}} t_1 \tag{7}$$

where t_1 is the threshold which can be adjusted to yield a desired false alarm rate under homogeneous background noise.

In order to assess the performance under nonhomogeneous background conditions involving multiple interferers or clutter power transitions within the reference cells [3], let us define

$$S_i = \frac{Y_{(k_i)}}{\lambda_{0i}} \tag{8}$$

Using [9],

$$f_{S_i}(s_i) = \sum_{h=k_i}^{m_i} \sum_{j=\max(0,h-b_i)}^{\min(h,m_i-b_i)} \sum_{v=0}^{j} \sum_{w=0}^{h-j} \binom{m_i-b_i}{j}\binom{b_i}{h-j}\binom{j}{v}\binom{h-j}{w}(-1)^{v+w+1} \left[v + (m_i - b_i - j) + (w + b_i - h + j)/c_i\right] \exp\{-s_i[(v + m_i - b_i - j) + (w + b_i - h + j)/c_i]\}, \tag{9}$$

where b_i is the number of interfering targets in the i^{th} sensor reference window and $c_i = \lambda_{1i}/\lambda_{0i}$. Hence,

$$F_Z(z) = \sum_{h_1=k_1}^{m_1} \sum_{h_2=k_2}^{m_2} \sum_{i=\max(0,h_1-b_1)}^{\min(h_1,m_1-b_1)} \sum_{j=\max(0,h_2-b_2)}^{\min(h_2,m_2-b_2)} \sum_{v_1=0}^{i} \sum_{v_2=0}^{j} \sum_{w_1=0}^{h_1-i} \sum_{w_2=0}^{h_2-j}$$
$$\binom{m_1-b_1}{i}\binom{m_2-b_2}{j}\binom{b_1}{h_1-i}\binom{b_2}{h_2-j} \binom{i}{v_1}\binom{j}{v_2}\binom{h_1-i}{w_1}\binom{h_2-j}{w_2}(-1)^{v_1+v_2+w_1+w_2}$$
$$\beta_1\beta_2\left[\left\{\frac{1}{\beta_1} - \frac{1}{z+\beta_1+\beta_2}\right\}\left\{\frac{1}{\beta_2} - \frac{1}{z+\beta_2}\right\} + \frac{1}{(z+\beta_1+\beta_2)^2}\log\frac{\beta_1\beta_2}{(z+\beta_1)(z+\beta_2)}\right] \tag{10}$$

where

$\beta_1 = (v_1 + m_1 - b_1 - i) + (w_1 + b_1 - h_1 + i)/c_1$,

$\beta_2 = (v_2 + m_2 - b_2 - j) + (w_2 + b_2 - h_2 + j)/c_2$

The probability of false alarm in homogeneous background is given by

$$P_F = 1 - F_Z(t_1). \tag{11}$$

The probability of detection P_D is obtained by replacing t_1 with $\frac{t_1}{(1+SNR)}$ in (11). The probability of false alarm under homogeneous background can be obtained by setting $b_i = 0$ in (10).

Since $Y_{(k_i)}$ is not an unbiased estimator of λ_{0i} [9], one can substitute a proportionality factor (that corrects for the bias) in each of the estimates in (7) and obtain an unbiased version of the NTS test:

$$Z = \frac{X_1}{Y_{(k_1)}} + w\frac{X_2}{Y_{(k_2)}} \underset{H_0}{\overset{H_1}{\gtrless}} t_1 \tag{12}$$

where $w = \frac{E(Y_{(k_1)})}{E(Y_{(k_2)})}$. Therefore, (12) and (7) are the biased and unbiased versions, respectively, of NTS.

Two other tests belonging to the S+OS family are the MAX and MIN tests defined below.

$$\text{MAX: } \max\left(\frac{X_1}{Y_{(k_1)}}, \frac{X_2}{Y_{(k_2)}}\right) \underset{H_0}{\overset{H_1}{\gtrless}} t \tag{13}$$

$$\text{MIN: } \min\left(\frac{X_1}{Y_{(k_1)}}, \frac{X_2}{Y_{(k_2)}}\right) \underset{H_0}{\overset{H_1}{\gtrless}} t \tag{14}$$

In the OR (AND) fusion rule [8], each sensor is assumed to employ an OS-CFAR detector of the type

$$\frac{X_i}{Y_{(k_i)}} \underset{H_0}{\overset{H_1}{\gtrless}} t_i \tag{15}$$

The individual sensor decision are combined using the OR (AND) Boolean rule. The probability expressions for the OR (AND) rule can be found in [5]. All the tests discussed in this section maintain a constant false alarm even if the λ_{0i}s are not identical for i =1,2.

IV. Performance Comparison

For a two sensor network, the following parameters are used in our numerical analysis: $m_1 = 8, m_2 = 16, k_1 = 6,$ and $k_2 = 12$. In (11), t_1 was solved through a numerical search to satisfy the constraint $P_f = 10^{-6}$. Similarly, for the OR rule, the two sensor thresholds t_1 and t_2 are solved so that the individual sensor false alarms are given by $P_{f1} = P_{f2} = 5.0\text{x}10^{-7}$. This gives an overall false alarm rate of 10^{-6}. For the AND rule, the two sensor thresholds are chosen so that $P_{f1} = P_{f2} = 10^{-3}$. Similarly, appropriate thresholds for MAX and MIN are found so as to achieve a false alarm rate of 10^{-6} in the homogeneous background condition. The threshold for the MOS test is fixed as per the discussion at the end of section II. In Fig. 1 the probability of detection is plotted against *SNR*, for homogeneous noise background, and in Figs. 2 and 3, the probability of detection is shown for two interfering target cases. Fig. 4 shows the probability of false alarm swing when a clutter transition occurs in the middle of reference cells and the test cell is in the clutter region.

In these figures, the curves marked biased and unbiased, correspond to the two forms of NTS discussed earlier. From these figures, we observe that the OR rule is competitive with the normalized test statistic. In homogeneous background (Fig. 1), the probability of detection of the OR rule is close to that of NTS (biased or unbiased). In situation corresponding to Fig.2, the NTS performs slightly better than the OR rule, whereas in the interfering target situation corresponding to Fig. 3, the OR rule even outperforms the biased and the unbiased NTS, for $b_2 \leq 5$. Therefore, considering that the normalized test requires each sensor to send two real numbers, a test cell sample and an order statistic, whereas the OR rule requires each sensor to send only a decision to the fusion center, it can be said that the OR rule provides a competitive and acceptable performance at a low cost. The MOS detector performance, in interfering target case, is poor as compared to OR (Figs. 2,3). The only drawback of NTS and OR is the occurrence of a large increase in false alarm rate during a clutter transition in the middle of the reference window (Fig. 4). If the homogeneous background noise power in all the sensors are nearly identical, then the MOS test provides a much better performance than the OR rule (and the NTS test) [8].

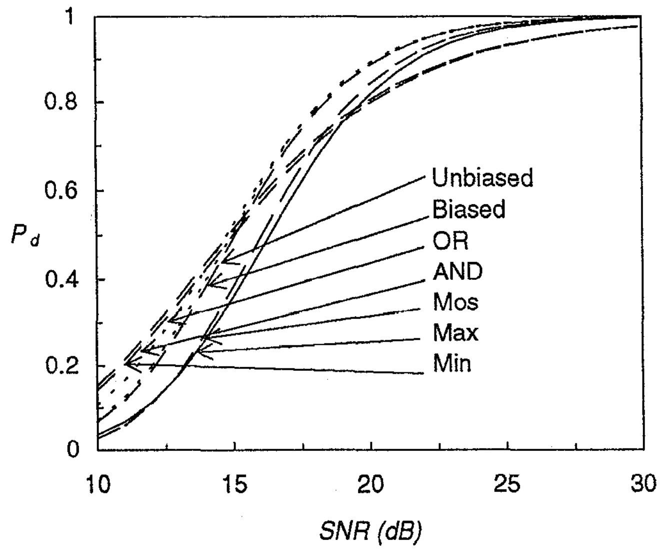

Fig. 1 Probability of Detection versus SNR when Background Noise is Homogeneous.

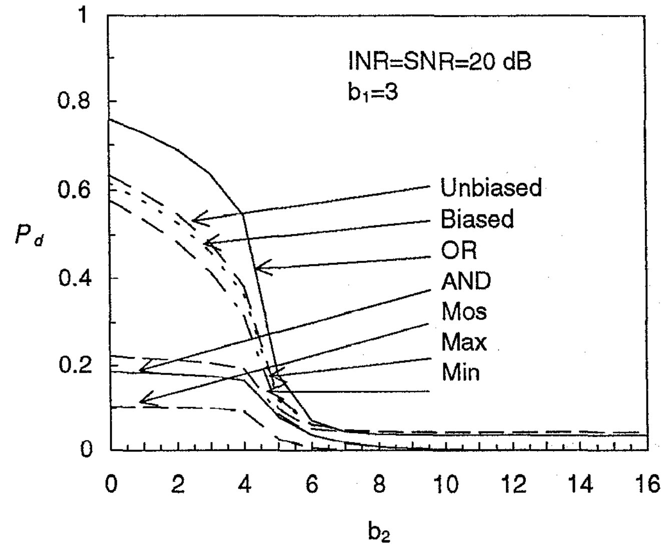

Fig. 3 Probability of Detection versus b_2 when b_1=3.

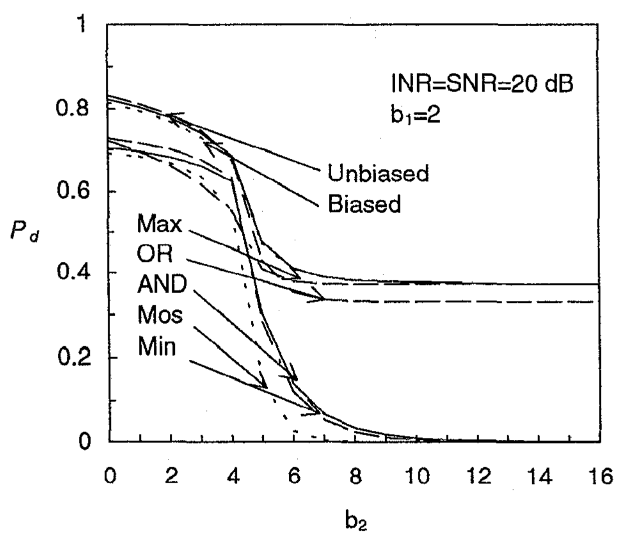

Fig. 2 Probability of Detection versus b_2 when b_1=2.

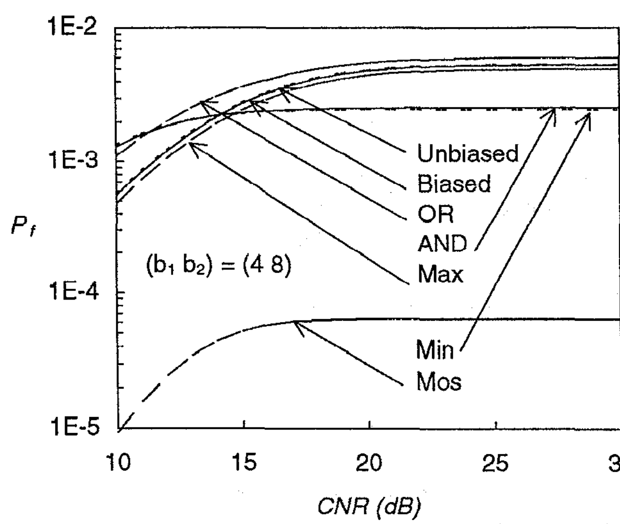

Fig. 4 The False Alarm Performance when Test Cells are in the Clutter Region.

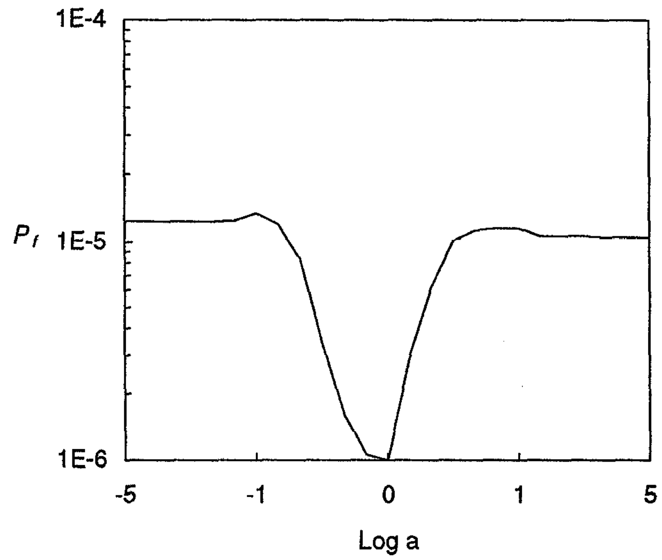

Fig. 5 The False Alarm Performance for various values of a.

V. CONCLUSION

We evaluated the performances of several two sensor distributed CFAR tests operating in nonhomogeneous environment. A somewhat surprising result is the competitive performance of OR rule as compared to some of the detectors in the class of signal-plus-order statistic tests. Further investigation is necessary to find out if a member of S+OS can significantly outperform rules based on decision fusion, such as the OR rule.

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