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Balanced Containment Control and Cooperative Timing of a Multi-Agent System

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***Abstract*—A multi-agent system is considered that is tasked with the objective of approaching a predetermined target from a desired region to minimize detection, and then simultaneously converging at the target. Only a subset of agents is informed of the bearing to the target and the desired common arrival time; other agents only rely on local communication with neighboring agents for coordination. A balanced containment control is achieved that restricts the motion of the group within a desired region (i.e., a circular sector) while evenly spacing the agents' orientations. To enable simultaneous arrival, a consensus algorithm is designed for agents to coordinate their arrival time by achieving consensus with the informed agents. Only local feedback (i.e., orientation information and estimated arrival time) from neighboring agents is used to navigate the group, and no position information (either absolute or relative position between agents) is required, allowing the multi-agent system to operate in a GPS denied area. Simulation results demonstrate the performance of the developed approach.**

I. INTRODUCTION

Control of multi-agent systems have attracted significant attention for various applications, including flocking and formation control [1]–[4], distributed coverage and deployment [5]–[8], and cooperative timing in [9] and [10]. In most of these applications, each agent communicates its position and/or velocity information with other team members to coordinate movements when performing collective tasks. The availability of the position information of the agents is always assumed in navigation and coordination. However, for some applications, position information may not be available, e.g., autonomous vehicles operating in GPS denied environments. Developing a cooperative controller to perform collective tasks for a multi-agent system without using position information can be challenging.

A multi-agent cooperative timing problem is considered in this paper when position information is not available. Example applications of cooperative timing include launching munitions from different locations to attack a target simultaneously, or flying UAVs to reach the boundary of a radar detection area at the same time to maximize the element of surprise [11]. To perform cooperative timing, the agents are tasked with simultaneously arriving at a predefined destination, ideally using limited information from the environment and team members. Rendezvous is a particular application of cooperative timing, and several results are reported in [12]–[20]. In [12]–[14], convergence to a common point for a group of autonomous mobile agents is studied, where synchronized and unsynchronized strategies are developed by only using position feedback from the sensing regions. In [15], the rendezvous problem is considered for nonholonomic mobile agents. To enable constant communication among agents, the results developed in [16] and [17] ensure rendezvous with the constraint of preserving network connectivity. A hybrid dynamic rendezvous protocol is designed in [18] to address finite-time rendezvous problems. However, the agents in [12]–[18] can only converge to a common setpoint determined by the initial deployment of the group, rather than a predefined destination. Moreover, cooperative timing is not considered in [12]–[18], which implies that the agents may not arrive at the common setpoint at the same time. To achieve simultaneous arrival at the desired destination, a distributed potential field-based approach is developed for a leader-follower network in [19] and [21] by requiring followers to achieve consensus with leaders, where leaders are the only agents with knowledge of the destination. However, the availability of the position information within agents is required in [19] and [21].

Containment control is a particular class of consensus problems (see [22] and [23] for a comprehensive review of consensus problems), in which follower agents are under the influence of leaders through local interactions in a leader-follower network. Distributed containment control algorithms are developed for the multi-agent systems in [24]–[27], where a group of followers is driven to a convex hull spanned by multiple leaders' states under an undirected, directed or switching topology. In our recent result in [28], containment control is applied to a social network to regulate the emotional states of individuals to a desired end. However, the classical containment control problem considered in [24]–[27] can only drive the agents to a desired region determined by the leaders, without considering the position or motion control of the agents within the desired region. To avoid collision with stationary/dynamic obstacles or to prevent detection, some applications may mandate that agents can be required to move within a desired region when approaching the destination. An extension of the classical containment

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control is required for such applications.

In this paper, a multi-agent system is tasked with the objective of simultaneously arriving at a predefined target, while approaching the target from a desired area (e.g., a circular sector). Agents are assumed to know their own orientations and velocities. Only a few agents (i.e., leaders) are provided with the bearing and range to the target, while, the range to the target is available for the remaining agents (i.e., followers) when the agents are in the proximity of their desired destination. Neither absolute nor relative position among agents is available. Due to the lack of complete target information, followers are required to coordinate their orientations with the leaders and move towards the target within a desired circular sector formed by the leaders. Moreover, the agents are required to be evenly deployed in the circular sector in terms of agent orientations.

Balanced deployment of agents is investigated in the recent work [29], where agents can only be equally positioned on an undirected circle graph. To achieve balanced deployment in a directed graph, a decentralized control algorithm is developed in this paper to not only ensure the agents move within the desired circular sector but also ensure their orientations are evenly spaced by using only orientation information from neighboring agents. To achieve simultaneous arrival, under the constraint that only leaders are informed of the desired common arrival time, a consensus algorithm is designed for agents to coordinate their arrival time. It is assumed that once the orientation of the agents is coordinated using the developed balanced containment control algorithm, each agent is navigated into a region where the agent can view the target and obtain its range information with respect to the target. Compared to most existing literature that considers global position information or at least inter-agent distance measurements, the bearing-only-based guidance algorithm in this work is applicable when position measurements are unavailable. Moreover, the developed controller allows the agents to simultaneously arrive at any desired destination, versus an unspecified destination without cooperative timing as in [12]–[18]. The result can also be extended to other tasks, such as flocking toward a common heading, or coverage control with agents evenly positioned in an area of interest.

II. Problem Formulation

The group of agents is tasked to simultaneously arrive at a predefined stationary destination as illustrated in Fig. 1. Consider N agents moving with the following kinematics in a polar coordinate frame $\mathcal{P}$:

$$\begin{bmatrix} \dot{\rho}_i(t) \\ \dot{\theta}_i(t) \end{bmatrix} = u_i(t), \ i = 1, \cdots, N \tag{1}$$

where $\rho_i \in \mathbb{R}^+$ and $\theta_i \in [0, 2\pi)$ denote the radius and orientation, respectively, of the agent i in $\mathcal{P}$, and $u_i(t) \triangleq \left[u_i^\rho, u_i^\theta \right]^T$ is the control input. Assume each agent knows its own orientation θ_i. No absolute or relative position information among agents is assumed to be available. Only

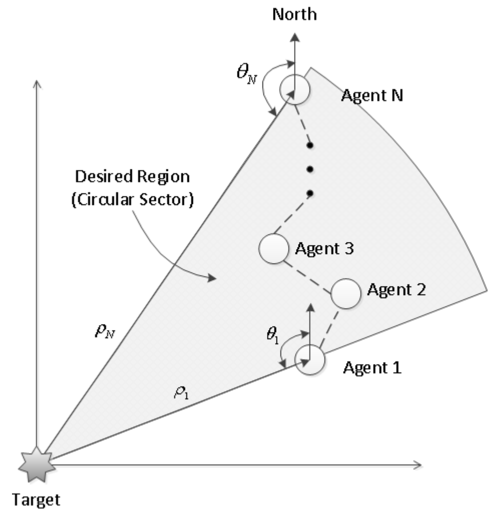

Figure 1. The problem scenario describing simultaneous arrival of N agents at a predefined target. The group of agents is required to be equally spaced and move within a desired circular sector toward the target. The interaction of agents is modeled as a directed graph, where the edges are indicated by dashed lines.

a small set of agents (i.e., leaders) are informed of both bearing and range information to target, and the range to target is not available to followers initially. Due to the lack of complete target information, followers can only use local feedback (i.e., orientation) from neighboring agents to coordinate their motion towards the target. After navigating followers to a region where the target range information can be obtained, cooperative timing among agents is performed by communicating the estimated arrival time and controlling the velocity of the agents. The set of leaders and followers are denoted as $\mathcal{V}_L$ and $\mathcal{V}_F$, respectively.

To achieve balanced containment control and simultaneous arrival, two graphs, $\mathcal{G}_\theta$ and $\mathcal{G}_\rho$, are considered. To facilitate balanced spacing of the agent orientations, the exchange of orientation information is modeled as a directed graph $\mathcal{G}_\theta = (\mathcal{V}, \mathcal{E}_\theta)$, where $\mathcal{V}$ represents agents, and $\mathcal{E}_\theta \subset \mathcal{V} \times \mathcal{V}$ indicates the orientation exchange between agents. In $\mathcal{G}_\theta$, the nodes are connected sequentially like a chain. Specifically, for each follower $i \in \mathcal{V}_F$, there exists two edges, $(i, \ i+1)$ and $(i, \ i-1)$, which implies that node i is able to access the orientation of nodes $i+1$ and $i-1$ (i.e., θ_{i-1} and θ_{i+1}) through local communication. While multiple leaders may be used, the subsequent development on the balanced containment control is focused on two leaders. Without loss of generality, assume that $\mathcal{V}_L = \{1, N\}$ and $\mathcal{V}_F = \{2, \cdots, N-1\}$, and the leaders navigate with desired orientations, θ_1 and θ_N, such that $\theta_1 < \theta_N$, which determines the desired region as shown in Fig. 1. In $\mathcal{G}_\theta$, the leader $i \in \mathcal{V}_L$ acts as the root in $\mathcal{G}_\theta$, which has directed paths to every other follower in the graph, and hence, knowledge of the destination can be delivered to all the nodes through a connected chain. One objective in this work is to not only regulate the followers' orientations within the desired circular sector (i.e., $\theta_i \in (\theta_1, \theta_N)$, for $\forall i \in \mathcal{V}_F$), but also equally space the agents in terms of orientations in the sector (i.e., $\theta_2 - \theta_1 = \ldots = \theta_N - \theta_{N-1}$).

To coordinate the arrival time, the agents communicate over a directed graph $\mathcal{G}_\rho = (\mathcal{V}, \mathcal{E}_\rho)$, where the directed edge $(i,\ j) \in \mathcal{E}_\rho$ indicates that node i is able to access the states of node j. Suppose that the leader agent 1 is informed of the desired arrival time for the group. Unlike $\mathcal{G}_\theta$ which is required to be a directed chain-like graph, $\mathcal{G}_\rho$ only requires that node 1 has directed paths to every other agent, which implies that $\mathcal{G}_\rho$ contains a directed spanning tree with node 1 as the root node. Since only node 1 knows the desired common arrival time, another objective is to develop a decentralized algorithm for all agents to achieve consensus on the arrival time of node 1 by adjusting their velocities.

To achieve the objectives of balanced containment control and simultaneous arrival, the following assumptions are made.

Assumption 1. The graph $\mathcal{G}_\theta$ is always connected in the sense that each follower can exchange orientation information with its immediate neighbors (i.e., node $i-1$ and $i+1$), and the leaders have directed paths to every follower.

Assumption 2. The graph $\mathcal{G}_\rho$ always has a connected directed spanning tree with the leader node 1 as the root, so that node 1 always has directed influence to every other agent.

Assumption 3. Assume the initial position of the agents is far from the target so that no collision between agents and target will occur before completion of orientation control.

III. Control Design

A. Balanced Containment Control

To perform balanced containment control, the orientations of the followers are required to be equally spaced in the circular sector (θ_1, θ_N). The orientation difference between two adjacent agents is defined as $e_i \triangleq \theta_{i+1} - \theta_i$ for $i = 1, \ldots, N-1$, where $\boldsymbol{e} \triangleq [e_1, \ldots, e_{N-1}] \in \mathbb{R}^{N-1}$ and $\boldsymbol{\theta} = [\theta_1, \ldots, \theta_N] \in \mathbb{R}^N$. The group orientation difference can be written in a compact form as

$$\boldsymbol{e} = D^T \boldsymbol{\theta}, \tag{2}$$

and $D \in \mathbb{R}^{N \times N-1}$ is defined as

$$D = \begin{bmatrix} -1 & 0 & \cdots & 0 \\ 1 & \ddots & \ddots & \vdots \\ 0 & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & -1 \\ 0 & \cdots & 0 & 1 \end{bmatrix}.$$

Since balanced deployment of the agents' orientations can be ensured by achieving consensus for all e_i in $\boldsymbol{e}$, i.e., $e_1 = \cdots = e_{N-1}$, inspired by the work of [29] and the Kuramoto model in [30]–[32], the controller $u^\theta = \left[u_1^\theta, \ldots, u_N^\theta\right]^T$ is designed as

$$u^\theta = K_\theta P \cos\left(D^T \boldsymbol{\theta}\right), \tag{3}$$

where $K_\theta > 0$ is the control gain, and $P = [p_{ij}]_{N \times N-1}$ is defined as

$$P = \begin{bmatrix} 0 & \cdots & \cdots & \cdots & 0 \\ 1 & -1 & 0 & \cdots & \vdots \\ 0 & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & 1 & -1 \\ 0 & \cdots & \cdots & \cdots & 0 \end{bmatrix}.$$

The matrix P in (3) is designed to achieve consensus by allowing each follower to update its own orientation by using the orientation differences from the adjacent nodes (i.e., e_{i-1} and e_i for node i). Since the leaders are informed of the desired orientation to the target, and have directed paths to the followers, the leaders can influence the followers through local communication, but not vice versa. Hence, the matrix P is designed with $p_{1i} = 0$ and $p_{Ni} = 0$ for all $i = 1, \ldots, N-1$, which indicates that θ_1 and θ_N are immutable from (3).

Note that the controller in (3) is not limited to a fixed number of agents. Due to the decentralized nature of (3), where each agent updates its orientation by communicating with the adjacent nodes, if any agents are added or removed, the remaining agents will correspondingly alter their orientations to achieve balanced spacing.

Theorem 1. *Given a directed graph $\mathcal{G}_\theta$ composed of N agents, where their initial orientations satisfy $\theta_1 \le \theta_2(0) \le \ldots \le \theta_{N-1}(0) \le \theta_N$, the controller designed in (3) yields asymptotic orientation convergence to a balanced distribution in the desired circular sector (θ_1, θ_N), in the sense that $e_1 = \cdots = e_{N-1} = \frac{\theta_N - \theta_1}{N-1}$.*

Proof: Consider a Lyapunov function candidate

$$V = \frac{1}{2} \boldsymbol{e}^T \boldsymbol{e}, \tag{4}$$

where $\boldsymbol{e}$ is defined in (2). Taking time derivative of V and using (1)-(3) yields

$$\begin{aligned} \dot{V} &= \boldsymbol{e}^T \dot{\boldsymbol{e}} \\ &= \boldsymbol{e}^T \left(D^T K_\theta P \cos\left(D^T \boldsymbol{\theta}\right)\right) \\ &= K_\theta \left(D \boldsymbol{e}\right)^T P \cos\left(D^T \boldsymbol{\theta}\right), \end{aligned} \tag{5}$$

where

$$D\boldsymbol{e} = \begin{bmatrix} -e_1, & e_1 - e_2, & \ldots, & e_{N-2} - e_{N-1}, & e_{N-1} \end{bmatrix}^T \tag{6}$$

and

$$\cos\left(D^T \boldsymbol{\theta}\right) = \begin{bmatrix} \cos(\theta_2 - \theta_1), & \ldots, & \cos(\theta_N - \theta_{N-1}) \end{bmatrix}^T. \tag{7}$$

Substituting (6) and (7) into (5) yields

$$\dot{V} = K \sum_{i=1}^{N-2} (e_i - e_{i+1}) (\cos(e_i) - \cos(e_{i+1})) . \quad (8)$$

From (8), $\dot{V} \leq 0$ if

$$(e_i - e_{i+1}) (\cos e_i - \cos e_{i+1}) \leq 0$$

for $i = 1, \ldots, N-2$, which means that if $e_i \leq e_{i+1}$ then $\cos e_i \geq \cos e_{i+1}$, or if $e_i \geq e_{i+1}$ then $\cos e_i \leq \cos e_{i+1}$. Given that $e_i \in [0, 2\pi)$ and

$$\sum_{i=1}^{N-1} e_i = \theta_N - \theta_1 \leq 2\pi \quad (9)$$

by definition, four conditions are considered.

To show the case that $\cos e_i \geq \cos e_{i+1}$ if $e_i \leq e_{i+1}$, two cases are considered

Case 1: $e_k \leq e_{k+1} < \pi$. The monotonicity of the cosine function in $[0, \pi]$ implies that $\cos e_k \geq \cos e_{k+1}$.

Case 2: $e_k \leq \pi \leq e_{k+1}$. From (9), $e_k = \theta_N - \theta_1 - e_{k+1} - \sum_{j \neq k, k+1} e_j$, which implies that

$$e_k \leq \theta_N - \theta_1 - e_{k+1} \leq 2\pi - e_{k+1} < \pi \quad (10)$$

From (10) and the fact that $\cos(2\pi - e_{k+1}) = \cos(e_{k+1})$, clearly, $\cos e_k \geq \cos e_{k+1}$ from $e_k \leq 2\pi - e_{k+1} < \pi$.

To show the case that $\cos e_i \leq \cos e_{i+1}$ if $e_i \geq e_{i+1}$, the other two cases are considered likewise.

Case 3: $e_{i+1} \leq e_i < \pi$. The monotonicity of the cosine function in $[0, \pi]$ implies that $\cos e_i \leq \cos e_{i+1}$.

Case 4: $e_{i+1} \leq \pi \leq e_i$. From (9), $e_{i+1} = \theta_N - \theta_1 - \sum_{j \neq i+1} e_j$, which implies that

$$e_{i+1} \leq \theta_N - \theta_1 - e_i \leq 2\pi - e_i < \pi. \quad (11)$$

From (11) and the fact that $\cos(2\pi - e_i) = \cos(e_i)$, clearly, $\cos e_k \leq \cos e_{k+1}$.

The above cases indicate that $\dot{V} \leq 0$. Note that $e_i - e_{i+1}$ and $\cos e_i - \cos e_{i+1}$ always have different signs, and the equality of (8) only exists when $e_1 = \cdots = e_{N-1}$. From LaSalle's invariance principle [33] and (9), the trajectory of θ_i for each agent will converge to the equilibrium point, $\theta_2 - \theta_1 = \cdots = \theta_N - \theta_{N-1}$. Since θ_1 and θ_N are desired and immutable, $\theta_{i+1} - \theta_i = \frac{\theta_N - \theta_1}{N-1}$ for $i = 1, \ldots, N-1$. ■

B. Cooperative Timing

In this section, a consensus based coordination algorithm is developed so that each agent simultaneously arrives at the prespecified target. As described in Section III-A, the orientation controller in (3) is applied to evenly space the agents in the desired sector. When each agent has converged desired orientation (i.e., $\|e_i(t+1) - e_i(t) < \delta\|$, $\delta \in \mathbb{R}^+$), which indicates that the agents are in a balanced deployment, the agents are navigated along a line towards the target. Once the agents are in a balanced deployment and in the proximity of the target, it is assumed that the target can be sensed by all agents (i.e., $L_i(t)$ is available to followers[1]), cooperative timing can be achieved by communicating the estimated arrival time with neighboring agents and controlling the velocities of the agents for simultaneous arrival. Let $\tau_i(t)$ be the estimated arrival time for agent i at t and $L_i(t)$ denote the distance to the target at t.

The available information exchange between agents on $\mathcal{G}_\rho$ is captured by the adjacency matrix $A = [a_{ij}]_{N \times N}$, where $a_{ij} = 1$ if there exists a directed edge from node j to i, and $a_{ij} = 0$ otherwise. The Laplacian matrix L of a graph is then defined as $L = \Delta - A$, where $\Delta = [\Delta_{ij}]_{N \times N}$ is a diagonal matrix with the entry $\Delta_{ii} = \sum_{j=1}^{N} a_{ij}$. Since v_1 acts as the root node in $\mathcal{G}_\rho$ and has directed paths to the other nodes, $a_{1j} = 0$ for all $j = 1, \ldots, N$.

To arrive at the target simultaneously, i.e., $\tau_i = \tau_j$ for $\forall i, j \in \mathcal{V}$, a consensus algorithm is designed as

$$\dot{\tau}_i = -k_i \sum_{j=1}^{N} a_{ij} (\tau_i - \tau_j) \quad (12)$$

$$u_i^\rho = -\frac{L_i}{\tau_i}, \quad (13)$$

where $k_i > 0$ is a control gain, and the local estimated arrival time of the agents is coordinated by adjusting the velocities.

Lemma 2. *[22] Let $\mathcal{G}$ be a directed graph and $L \in R^{n \times n}$ be the associated Laplacian matrix. Given $\dot{z}(t) \triangleq -Lz(t)$, where $z = \begin{bmatrix} z_1, & \cdots, & z_n \end{bmatrix}^T$, if $\mathcal{G}$ has a directed spanning tree, then consensus is exponentially achieved, i.e., $z_1 = \cdots = z_n$ as $t \to \infty$.*

Theorem 3. *Given a graph $\mathcal{G}_\rho$ that contains a directed spanning tree with the controller designed in (13), the agents will simultaneously arrive at the target at the desired arrival time.*

Proof: The dynamics of $\dot{\tau}_i$ in (12) can be written in a compact form as

$$\dot{\boldsymbol{\tau}} = -KL\boldsymbol{\tau}, \quad (14)$$

where $K = \text{diag}\{k_1, k_2, \ldots, k_N\}$ is a diagonal matrix, $\boldsymbol{\tau} = [\tau_1, \ldots, \tau_N] \in \mathbb{R}^N$, and L is the Laplacian matrix of the graph $\mathcal{G}_\rho$. Since Assumption 2 ensures a directed spanning tree in the graph $\mathcal{G}_\rho$, Lemma 2 indicates that the consensus is achieved exponentially, i.e., $\tau_1 = \tau_2 = \ldots = \tau_N$. Note that v_1 acts as the root node, and can not be influenced by other agents. Hence, the arrival time of all the agents will achieve consensus to the desired arrival time of τ_1. Since the desired arrival time is known to each agent from (14), the velocity of each agent can be adjusted according to (13) for simultaneous arrival. ■

In this work, only one agent is informed of the desired arrival time, and the other agents are required to arrive at the target exactly at the same arrival time of the informed agent. For scenarios where agents are required to arrive at

[1]The range information $L_i(t)$ can be estimated by using the approaches developed in [34] and [35], if each agent knows its velocity and is equipped with a passive range sensor such as a camera.

the target within a desired time interval, multiple informed agents can be considered. Following a similar analysis as in [24]–[27], containment control can be used to ensure that the arrival time of each agent is within a convex hull formed by the arrival time of the informed agents.

Coordination to a common arrival time is achieved by reaching consensus with an informed agent (i.e., v_1 in this work). If no agents are informed of a required arrival time, alternative approaches such as max-consensus or min-consensus in [36] can also be applied for simultaneous arrival, where consensus can be reached on a minimum or maximum estimated arrival time of the group rather than a predefined arrival time.

Remark 4. Since range information is not exchanged between agents, collision avoidance within agents is not considered in this work. In our recent work in [37] and [38], a potential field based approach is investigated for a multi-agent system to avoid collision with other agents and/or stationary obstacles when performing collective tasks. If provided with the range information to nearby agents or stationary obstacles, the current work could be extended for collision avoidance.

Remark 5. From Lemma 2, Theorem 3 only ensures that consensus is achieved as $t \to \infty$. Though some applications require consensus in finite time (e.g., achieving consensus before arriving at the target in rendezvous problems), the exponential nature of the consensus in Theorem 3 is practical. Future efforts will aim to improve the current approach by considering finite time consensus.

IV. Simulation

A numerical simulation is provided in this section to demonstrate the performance of the controller developed in (3) and (13) in a scenario where a group of 6 agents are tasked to simultaneously arrive at a prespecified target. The target is located at the origin, and the agents are initially deployed in $\mathcal{P}$ as

$$\begin{aligned} \boldsymbol{\rho}(0) &= \begin{bmatrix} 20.2 & 19.3 & 22.1 & 20.5 & 25 & 19.5 \end{bmatrix}, \\ \boldsymbol{\theta}(0) &= \begin{bmatrix} \frac{7\pi}{12} & \frac{13\pi}{22} & \frac{13\pi}{22} & \frac{11\pi}{18} & \frac{9\pi}{14} & \frac{5\pi}{6} \end{bmatrix}. \end{aligned}$$

Let $\theta_1 = \frac{7\pi}{12}$ and $\theta_6 = \frac{5\pi}{6}$ be the desired orientations which determine the desired circular sector. Agent 1 moves with a constant velocity of 1 m/s towards the origin. In this simulation, the target range information is assumed to be available to followers once the balanced deployment is achieved. Let the predefined threshold $\delta = 0.01$, which indicates that the cooperative timing is activated if $\|e_i(t+1) - e_i(t)\| < \delta$ for $\forall i$. Applying the controller designed in (3) and (13), the agents are navigated to the target within the desired region, and arrive at the target simultaneously as shown in Fig. 2, where the initial positions are represented by squares, and the trajectories are represented by solid lines. To show that the orientations of the group are evenly spaced in the desired sector $\left(\frac{7\pi}{12}, \frac{5\pi}{6}\right)$, the evolution of orientations for each agent is plotted in Fig. 3, which indicates that the balanced deployment of agent orientations is achieved from $t = 2.5$ s.

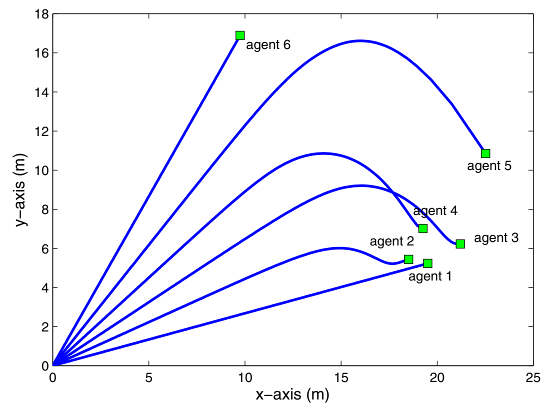

Figure 2. Plot of agent trajectories with squares indicating their initial positions.

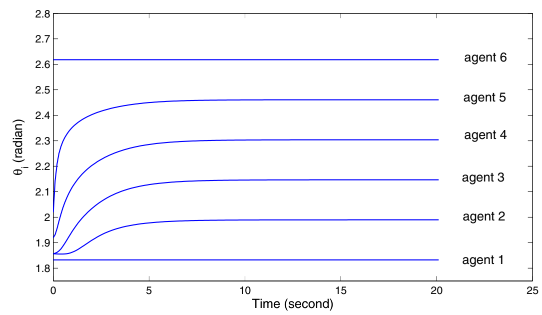

Figure 3. Evolution of agent orientations.

Fig. 4 indicates that simultaneous arrival is achieved, where all agents reach consensus on the desired arrival time of agent 1. As shown in 4, agents start to coordinate their velocities for simultaneous arrival after the completion of orientation control from $t = 2.5$ s.

V. Conclusion

This paper examines the cooperative timing problem for a multi-agent system. A balanced containment control algorithm is developed to evenly space the agents in the desired region and navigate the agents towards the target.

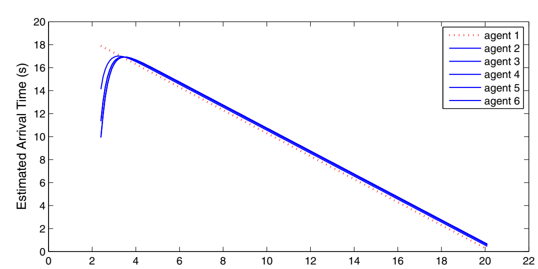

Figure 4. Evolution of estimated arrival time of each agent.

A consensus algorithm is designed for agents to coordinate their velocities by reaching consensus on the arrival time. To perform cooperative timing collectively, the agents need to communicate their expected arrival time and coordinate with other team members within a network. In the current work, the availability of continuous communication and information exchange is assumed. However, the wireless communication between agents can be interrupted by various factors such as fading and packet loss in a complex environment. Future work will examine extensions of the proposed approach on an unreliable network where the communication between agents is intermittent.

REFERENCES

[1] W. Dong and J. Farrell, "Cooperative control of multiple nonholonomic mobile agents," *IEEE Trans. Autom. Contr.*, vol. 53, no. 6, pp. 1434–1448, 2008.

[2] A. Jadbabaie, J. Lin, and A. Morse, "Coordination of groups of mobile autonomous agents using nearest neighbor rules," *IEEE Trans. Autom. Control*, vol. 48, no. 6, pp. 988–1001, June 2003.

[3] J. Fax and R. Murray, "Information flow and cooperative control of vehicle formations," *IEEE Trans. Autom. Control*, vol. 49, no. 9, pp. 1465–1476, Sept. 2004.

[4] L. Navaravong, Z. Kan, J. M. Shea, and W. E. Dixon, "Formation reconfiguration for mobile robots with network connectivity constraints," *IEEE Network*, vol. 26, no. 4, pp. 18–24, 2012.

[5] S. Martínez, F. Bullo, J. Cortés, and E. Frazzoli, "On synchronous robotic networks Part II: Time complexity of rendezvous and deployment algorithms," *IEEE Trans. Autom. Control*, vol. 52, no. 12, pp. 2214–2226, 2007.

[6] A. Howard, M. J. Matarić, and G. S. Sukhatme, "Mobile sensor network deployment using potential fields: A distributed, scalable solution to the area coverage problem," in *Distributed Autonomous Robotic Systems 5*. Springer, 2002, pp. 299–308.

[7] J. Cortes, S. Martinez, T. Karatas, and F. Bullo, "Coverage control for mobile sensing networks," *IEEE Trans. Robot. Autom.*, vol. 20, no. 2, pp. 243–255, 2004.

[8] M. Schwager, D. Rus, and J.-J. Slotine, "Decentralized, adaptive coverage control for networked robots," *Int. J. Rob. Res.*, vol. 28, no. 3, pp. 357–375, 2009.

[9] T. McLain and R. Beard, "Coordination variables, coordination functions, and cooperative-timing missions," *J. Guid. Contr. Dynam.*, vol. 28, no. 1, pp. 150–161, 2005.

[10] E. Tekin and A. Yener, "The general Gaussian multiple-access and two-way wiretap channels: Achievable rates and cooperative jamming," *IEEE Trans. Inf. Theory*, vol. 54, no. 6, pp. 2735–2751, 2008.

[11] R. W. Beard, T. W. McLain, D. B. Nelson, D. Kingston, and D. Johanson, "Decentralized cooperative aerial surveillance using fixed-wing miniature uavs," *IEEE Proc.*, vol. 94, no. 7, pp. 1306–1324, 2006.

[12] Z. Lin, M. Broucke, and B. Francis, "Local control strategies for groups of mobile autonomous agents," *IEEE Trans. Autom. Control*, vol. 49, no. 4, pp. 622–629, 2004.

[13] J. Lin, A. Morse, and B. Anderson, "The multi-agent rendezvous problem. Part 1: The synchronous case," *SIAM J. Control Optim.*, vol. 46, no. 6, pp. 2096–2119, 2007.

[14] ——, "The multi-agent rendezvous problem. part 2: The asynchronous case," *SIAM J. Control Optim.*, vol. 46, no. 6, pp. 2120–2147, 2007.

[15] D. V. Dimarogonas and K. J. Kyriakopoulos, "On the rendezvous problem for multiple nonholonomic agents," *IEEE Trans. Automat. Control*, vol. 52, no. 5, pp. 916–922, May 2007.

[16] J. Cortés, S. Martínez, and F. Bullo, "Robust rendezvous for mobile autonomous agents via proximity graphs in arbitrary dimensions," *IEEE Trans. Autom. Control*, vol. 51, no. 8, pp. 1289–1298, 2006.

[17] H. Su, X. Wang, and G. Chen, "Rendezvous of multiple mobile agents with preserved network connectivity," *Syst. Control Lett.*, vol. 59, no. 5, pp. 313–322, 2010.

[18] Q. Hui, "Finite-time rendezvous algorithms for mobile autonomous agents," *IEEE Trans. Autom. Control*, vol. 56, no. 1, pp. 207–211, 2011.

[19] T. Gustavi, D. Dimarogonas, M. Egerstedt, and X. Hu, "Sufficient conditions for connectivity maintenance and rendezvous in leader–follower networks," *Automatica*, vol. 46, no. 1, pp. 133–139, 2010.

[20] Z. Kan, J. Klotz, T.-H. Cheng, and W. E. Dixon, "Ensuring network connectivity for nonholonomic robots during decentralized rendezvous," in *Proc. Am. Control Conf.*, Montréal, Canada, June 2012, pp. 3718–3723.

[21] Z. Kan, J. Klotz, E. L. P. Jr, J. M. Shea, and W. E. Dixon. (2014) Decentralized rendezvous of nonholonomic robots with sensing and connectivity constraints. arXiv:1402.5639.

[22] W. Ren, R. W. Beard, and E. M. Atkins, "Information consensus in multivehicle cooperative control," *IEEE Contr. Syst. Mag.*, vol. 27, pp. 71–82, April 2007.

[23] R. Olfati-Saber, J. A. Fax, and R. M. Murray, "Consensus and cooperation in networked multi-agent systems," *Proc. IEEE*, vol. 95, no. 1, pp. 215 – 233, Jan. 2007.

[24] G. Notarstefano, M. Egerstedt, and M. Haque, "Containment in leader-follower networks with switching communication topologies," *Automatica*, vol. 47, no. 5, pp. 1035–1040, 2011.

[25] Y. Cao and W. Ren, "Containment control with multiple stationary or dynamic leaders under a directed interaction graph," in *Proc. IEEE Conf. Decis. Control*, 2009, pp. 3014–3019.

[26] J. Mei, W. Ren, and G. Ma, "Distributed containment control for Lagrangian networks with parametric uncertainties under a directed graph," *Automatica*, vol. 48, no. 4, pp. 653–659, 2012.

[27] Y. Cao, W. Ren, and M. Egerstedt, "Distributed containment control with multiple stationary or dynamic leaders in fixed and switching directed networks," *Automatica*, vol. 48, pp. 1586–1597, 2012.

[28] Z. Kan, J. Klotz, E. L. Pasiliao, and W. E. Dixon, "Containment control for a directed social network with state-dependent connectivity," in *Proc. Am. Control Conf.*, Washington DC, June 2013, pp. 1953–1958.

[29] Z. Xu, M. Egerstedt, G. Droge, and K. Schilling, "Balanced deployment of multiple robots using a modified Kuramoto model," in *Proc. Amer. Control Conf.*, 2013, pp. 6138–6144.

[30] Y. Kuramoto, "Self-entrainment of a population of coupled non-linear oscillators," in *Int. Symp. Math. Problems Theoret. Phys.* Springer, 1975, pp. 420–422.

[31] ——, *Chemical oscillations, waves, and turbulence*. Berlin, Germany: Springer, 1984.

[32] A. Jadbabaie, N. Motee, and M. Barahona, "On the stability of the Kuramoto model of coupled nonlinear oscillators," in *Amer. Control Conf.*, 2004, pp. 4296–4301.

[33] H. K. Khalil, *Nonlinear Systems*, 3rd ed. Prentice Hall, 2002.

[34] A. De Luca, G. Oriolo, and P. R. Giordano, "Feature depth observation for image-based visual servoing: Theory and experiments," *Int. J. Robot. Res.*, vol. 27, no. 10, pp. 1093–1116, 2008.

[35] A. Dani, N. Fischer, Z. Kan, and W. E. Dixon, "Globally exponentially stable observer for vision-based range estimation," *Mechatron.*, vol. 22, no. 4, pp. 381–389, Special Issue on Visual Servoing 2012.

[36] B. M. Nejad, S. A. Attia, and J. Raisch, "Max-consensus in a max-plus algebraic setting: The case of fixed communication topologies," in *Int. Symp. Inf. Commun. Autom. Technol*. IEEE, 2009, pp. 1–7.

[37] Z. Kan, J. M. Shea, and W. E. Dixon. (2014) Navigation function based decentralized control of a multi-agent system with network connectivity constraints. arXiv:1402.5951.

[38] Z. Kan, A. Dani, J. M. Shea, and W. E. Dixon, "Network connectivity preserving formation stabilization and obstacle avoidance via a decentralized controller," *IEEE Trans. Automat. Control*, vol. 57, no. 7, pp. 1827– 1832, 2012.