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Nonsingular Terminal Sliding Mode Control with Unknown Control Direction

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Abstract—**This paper considers a nonsingular sliding mode controller for a second order system without a priori knowledge of the control direction. A novel nonsingular terminal sliding hypersurface is presented to adapt to the challenges of unknown control direction and avoid singularity issues at the origin. In contrast to the Nussbaum gain, where the equilibrium point is reached asymptotically, or the classical linear hypersurface, where the states are reached exponentially, the control structure in this paper guarantees that the states are reached in finite time. Additionally, the control algorithm is bounded and globally finite time stable in the presence of input matrix uncertainty and exogenous disturbance. Simulation results are provided to demonstrate the robustness of the developed control algorithm.**

I. Introduction

Variable structure approaches have been popular due to their features of finite time convergence, robustness to parameters uncertainty, and disturbance rejection. An important aspect of variable structure control is the sliding surface, where the trajectories of the states are restricted on the desired manifolds. For linear surface design, the states converge asymptotic or exponential, and the sliding surface converges in finite time. Interestingly, nonlinear surface design allows both the states and the sliding surface to converge in finite time. The seminal work on finite time convergence, also known as terminal sliding mode control (TSMC), is introduced in [1]. This nonlinear surface allows the states to reach the origin in finite time, but a singularity exists at the origin, which requires large control effort to maintain the sliding motion. To overcome the singularity issue, Feng [2] designs a discontinuous nonsingular TSMC algorithm that allows the state to reach the origin in finite time. The algorithm is subsquently extended to a continuous control structure in the presence of uncertain input matrix in [3]. In addition to finite time, effort has also been made to achieve a fixed convergence time regardless of the initial conditions [4]. In [5], Sanchez-Torres and Loukianov develop a finite time convergence sliding mode observer algorithm for a second order system. In [6], Levant shows that any finite-time homogeneous sliding mode algorithm can be converted into fixed-time. Despite advances in finite/fixed-time convergence, TSMC with unknown control direction has to be addressed.

Innovative methods have been contrived to cope with the challenges of sign uncertainty in the control input matrix. The Nussbaum gain is one of the proposed solutions that can compensate for the sign uncertain in various systems. In [7], Du and Guo combined adaptive backstepping with the Nussbaum gain for ship course tracking. In [8], the Nussbaum gain is extended to multi-input multi-output nonlinear systems without a priori knowledge of the control direction. Neural network [9] and adaptive predictive control [10] techniques are also used in conjunction with the Nussbaum framework for discrete systems. In [11], Matko *et al.* apply the Nussbaum gain to the stabilization of lateral motion of the balloon gondola, which is confirmed in simulation and experiment. In [12], Liu and Tao consider multivariable model reference adaptive control for aircraft systems with uncertainty, where the sign changes in the control direction are considered to be caused by abrupt asymmetric damages, such as aircraft mass, aerodynamic forces, and moments due to damaged wings. In addition to the classic Nussbaum gain approach, other techniques have also been modified to adapt to the uncertain control direction problem. In [13], Oliveira *et al.* consider output feedback with monitoring functions for multivariable systems. The sign uncertainty is detected by using the monitoring function, and the algorithm is tested on a visual servoing system with an uncalibrated camera. The monitoring functions are also used in uncertain systems with arbitrary relative degree in [14]. In [15], Scheinker designs an extremum seeking algorithm for unknown sign time-varying system.

Robust control techniques have also been extended to deal with the challenges of unknown control direction. Bartolini and Pisano [16] develop a suboptimal SMC algorithm for systems with relative degree two and sign uncertainty. Kaloust and Qu [17] design a continuous robust controller via Lyapunov method that observes the changes in the control direction with the first and second derivatives. Applying SMC and using the idea of periodic switching function, Drakunov [18] divides the sliding surfaces into cells with fixed control. This allows the states to alternate the control values along the cells, resulting in a set of stability points. Then, the steady state error can be removed by using a dynamic compensator to force the sliding surface converge to origin in finite time.

Although TSMC has been able to achieve finite time stability in the presence of uncertainty and exogenous disturbance, the problem of finite time convergence in the presence of unknown control direction remains a challenging problem. In contrast to previous work, where the states converge to the origin asymptotically or exponentially [9], [19], [20], the contribution of this paper focuses on nonsingular finite time convergence of the states in the presence of uncertain control direction. In [19], for a second order system, the control gain has to be increased proportionally with the velocity to guarantee that

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sliding mode occurs. In this work, a novel surface is designed to scale the velocity term, thereby, reducing the control input effort. The developed control algorithm is computation cheap, requires no monitoring functions, function approximators, or online adapt laws. Moreover, the designed algorithm is bounded, nonsingular, and is globally stable.

II. Background

The basic principles of finite time convergence and controller design in the presence of unknown control direction, used in this paper, can be briefly summarized in the lemmas given below.

Lemma 1: [2] Consider the first order system given as $\dot{x} = -\beta x^{\gamma_1/\gamma_2}$, where $\beta \in \mathbb{R}^+$, $0 < \gamma_1/\gamma_2 < 1$, γ_1 and γ_2 are odd integers. The state $x(t)$ goes to zero in finite time.

Proof. Integrating $\dot{x}(t)$ with respect to time, the finite time interval t_x that it takes from $x(0) \neq 0$ to $x(t_x) = 0$ is given as

$$t_x = -\beta^{-1} \int_{x(0)}^{0} \frac{dx}{x^{\gamma_1/\gamma_2}} = \frac{\gamma_2}{\beta(\gamma_2 - \gamma_1)} |x(0)|^{1-\gamma_1/\gamma_2}.$$

This implies that $x(t)$ converges to zero in finite time, and this time interval depends on the initial condition and the selected constants $\gamma_1, \gamma_2, \beta$. ■

Lemma 2: [19] Let the first order system be defined as

$$\dot{x} = f(x,t) + bu$$

where $f(x,t) \in \mathbb{R}$ is the disturbance, $b \in \mathbb{R}$ is the input gain with unknown sign, and the control input $u(t)$ is designed as

$$u = M sgn \sin \frac{\tilde{s}}{\varepsilon}$$

where $\varepsilon \in \mathbb{R}^+$ is a constant, $M \in \mathbb{R}^+$ is a constant or a positive function, the hypersurface $\tilde{s}(t)$ is defined as

$$\tilde{s} = s + \lambda \int sgn(s) dt \tag{1}$$

with $s(t) = x$ is defined as the sliding surface, and $\lambda \in \mathbb{R}^+$. When the gain M is designed to satisfy the inequality $|Mb| > |f| + \lambda$, then the surface $s(t)$ goes to zero in finite time.

Proof. Taking time derivative of the hypersurface $\tilde{s}(t)$

$$\dot{\tilde{s}} = f(x,t) + bM sgn \sin \frac{\tilde{s}}{\varepsilon} + \lambda sgn(s). \tag{2}$$

When the gain M is designed such that $|Mb| > |f| + \lambda$, it is obvious that the sign of $bM sgn \sin \frac{\tilde{s}}{\varepsilon}$ is dominant in (2). In the neighborhoods of the point where

$$\tilde{s} = k\varepsilon \tag{3}$$

for $k = 0, \pm 2, \pm 4, \ldots$, the following is obtained:

$$sign \left[\sin\left(\tilde{s}\right)\right] = sign(\tilde{s} - k\varepsilon)$$

and for $k = \pm 1, \pm 3, \ldots$, the following is obtained:

$$sign \left[\sin\left(\tilde{s}\right)\right] = -sign(\tilde{s} - k\varepsilon).$$

Thus, sliding mode will occur on one of the manifolds in (3) for any sign of bM. In fact, sliding mode occurs where $\tilde{s} = constant$ after some moment of time, and after differentiating (1) yields

$$\dot{s} = -\lambda sign(s). \tag{4}$$

Thus, (4) guarantees that the manifold $s(x) = 0$ is reached in finite time. ■

III. Problem Formulation

Consider an uncertain system modeled as a double integrator and subjected to nonvanishing disturbances as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f(x,t) + bu \end{aligned} \tag{5}$$

where $x = [x_1\ x_2]^T \in \mathbb{R}^2$ is the state of the system, $b \in \mathbb{R}$ is the constant control gain with unknown sign, $f(x,t) \in \mathbb{R}$ is the nonvanishing disturbance, and $u(t) \in \mathbb{R}$ is the control input.

Assumption 1: The nonvanishing disturbance $f(x,t)$ is upper bounded by a constant $\bar{f} \in \mathbb{R}^+$ as

$$|f(x,t)| \leq \bar{f}.$$

Assumption 2: The sign of the control gain b is unknown, i.e., the control direction is unknown, but magnitude of b is known.

To facilitate the subsequent analysis, four quadrants $Q_i\ \forall i = 1, 2, 3, 4$ are defined using the following inequalities:

$\mathcal{Q}_1$: $\{x_1 \geq 0, x_2 \geq 0\}$ $\mathcal{Q}_2$: $\{x_1 < 0, x_2 > 0\}$
$\mathcal{Q}_3$: $\{x_1 \leq 0, x_2 \leq 0\}$ $\mathcal{Q}_4$: $\{x_1 > 0, x_2 < 0\}$

The objective of this paper is to design a robust controller that ensures finite time convergence of the system in (5) in the presence of sign uncertainty in the control input and nonvanishing disturbances.

IV. Sliding Surface Design

Based on the nonsingular terminal sliding mode control, the surface $s(t)$ is designed as [2]

$$s = e^{c_2 x_2^2/2} x_2^{m_2/m_1} + c x_1 \tag{6}$$

where $m_1, m_2 \in \mathbb{Z}^+$ are odd integers, $0 < m_1/m_2 < 1$, $(m_2 + m_1)/m_1 > 1, (m_2 - m_1)/m_1 < 1$, and $c \in \mathbb{R}^+$ is a constant. When $s = 0 : x_2 = (-cx_1)^{m_1/m_2} e^{-m_1 c_2 x_2^2/(2m_2)}$, it is similar to Lemma 1, x_1 and x_2 go to the equilibrium point in finite time. To compensate for the sign uncertainty in b, the augmented sliding surface $\tilde{s}(t)$ is designed as

$$\tilde{s} = s + \lambda \int \sigma^{p_1/p_2}(s(\tau)) d\tau \tag{7}$$

where $\lambda \in \mathbb{R}^+$ is a constant, $p_1, p_2 \in \mathbb{Z}^+$ are odd integers that satisfy $0 < p_1/p_2 < 1$, and $\sigma(s)$ is a saturation function with the maximum magnitude of $M_s \in \mathbb{R}^+$ and $\sigma(s) = s(t)$ for $|s(t)| \leq L_s \in \mathbb{R}^+$. Taking time derivative of (6) and (7) along (5), the rate of change of the surfaces $s(t)$ and $\tilde{s}(t)$ can be obtained as

$$\dot{s} = g\dot{x}_2 + cx_2 \tag{8}$$

$$\dot{\tilde{s}} = \dot{s} + \lambda \sigma^{p_1/p_2}(s) \tag{9}$$

$$g = \left(c_2 x_2^{(m_1+m_2)/m_1} + \frac{m_2}{m_1} x_2^{(m_2-m_1)/m_1} \right) e^{c_2 x_2^2/2} \tag{10}$$

where $g(x) \in \mathbb{R}$ is a positive semi-definite function. Substituting (5) and (8) into (9), the open-loop system can be expressed as

$$\dot{\tilde{s}} = g\left(f + bu\right) + cx_2 + \lambda \sigma^{p_1/p_2}(s). \tag{11}$$

The surface in $\dot{\tilde{s}}(t)$ is partitioned into four parts, and the control is designed to compensate for each of the terms in (11).

V. Controller Development

To facilitate the subsequent analysis, the control input $u(t)$ in (5) is segregated into four terms as

$$u = u_1 + u_2 + u_3 + u_4. \tag{12}$$

Substituting (12) into (11), the open-loop error dynamics can also be subdivided into four parts

$$\dot{\tilde{s}} = P_1(x) + P_2(x) + P_3(x) + P_4(x) \tag{13}$$

where the functions $P_1(x), P_2(x), P_3(x), P_4(x) \in \mathbb{R}$ are given as

$$P_1(x) = gbu_1 + cx_2 \tag{14}$$
$$P_2(x) = g\left(f + bu_2\right) \tag{15}$$
$$P_3(x) = gbu_3 + \lambda\sigma^{p_1/p_2}(s) \tag{16}$$
$$P_4(x) = gbu_4. \tag{17}$$

For notational simplicity, consider the auxiliary functions $\Psi(t)$ and $\Psi_s(t)$ defined as

$$\Psi = sgn\left(\sin\frac{\pi}{\varepsilon}\tilde{s}\right) \quad \Psi_s = sgn(b\Psi)$$

where $sgn(\cdot)$ denotes the sign of $(\cdot)$, and $\varepsilon \in \mathbb{R}^+$ is a constant.

Based on the subsequent analysis, the control input $u(t)$ can be designed as

$$u_1 = M_1\Psi \tag{18}$$
$$u_2 = M_2\Psi \tag{19}$$
$$u_3 = |\sigma(s)|^{p_1/p_2} M_3\Psi. \tag{20}$$
$$u_4 = \frac{m_1}{m_2}|\sigma(s)|^{q_1/q_2} M_4\Psi \tag{21}$$

where $M_1, M_2, M_3, M_4, q_1, q_2 \in \mathbb{R}^+$ are constants.

Lemma 3: The control input $u_1(t)$ and $u_2(t)$ designed in (18) and (19), respectively, allow the sign of $\Psi_s(\tilde{s})$ to dominate in (14) and (15).

Lemma 4: Define the set $x_{2a} \subset \mathbb{R}$ as

$$x_{2a} = \left\{x_2 | \Upsilon_1 + \Upsilon_2 - \frac{\lambda}{|bM_3|} = 0, x_2 > 0\right\}$$

where $\Upsilon_1 = c_2 x_2^{(m_1+m_2)/m_1}$ and $\Upsilon_2 = \frac{m_2}{m_1} x_2^{(m_2-m_1)/m_1}$. For the region $|s(t)| > L_s$ and the velocity $x_2(t)$ that satisfies

$$|x_2(t)| \geq x_{2m}, \tag{22}$$

the sign of Ψ_s is dominant in $P_3(t)$.
In (22), $x_{2m} \in \mathbb{R}^+$ is a constant defined as

$$x_{2m} = \text{minima}(x_{2a}) \tag{23}$$

that can be adjusted by tuning the control gain M_3.

To facilitate the control development, the four regions $\mathcal{R}_i$ for $i = 1, 2, 3, 4$ are defined as

$$\mathcal{R}_1 = \{(x_2, s) | |x_2| < x_{2m}, |s| > L_s\}$$
$$\mathcal{R}_2 = \{(x_2, s) | |x_2| \leq x_{2m}, |s| \leq L_s\}$$
$$\mathcal{R}_3 = \{(x_2) | |x_2| \leq x_{2m}\}$$
$$\mathcal{R}_4 = \{(x_2) | |x_2| > x_{2m}\}.$$

Suppose that $x(t) \in \mathcal{Q}_1 \cap \mathcal{R}_4$, sliding mode has not occurred and the control input $u(t)$ is causing $x_2(t)$ to increase, therefore $x_1(t)$ also increases. Since $x_2(t) \in \mathcal{R}_4$, from Lemmas 3 and 4, equation (13) is dominated by the sign of Ψ_s and the state $x(t)$ is attracted to one of the manifolds $\tilde{s}(t) = \beta$, where $\beta \in \mathbb{R}^+$, in finite time. Thus, based on (5) and (6), the magnitude $|x_2(t)|$ decreases until it enters the region $\mathcal{R}_3$. Based on (5), the same argument can be made when $x(t) \in \mathcal{Q}_3 \cap \mathcal{R}_4$ to show that $x_2(t)$ starts in $\mathcal{R}_4$ and enters $\mathcal{R}_3$.

Lemmas 3 and 4 guarantee the existence of the sliding mode when $x_2(t) \in \mathcal{R}_4$. The following Lemmas prove that even though sliding mode does not occur when $x_2(t) \in \mathcal{R}_1$, the control input $u(t)$ guarantees that $\mathcal{R}_1$ is not an attractive set and hence sliding mode occurs when $x_2(t) \notin \mathcal{R}_1$.

Lemma 5: Let sliding mode occur before the state $x(t)$ enters the set $\mathcal{R}_3$. The signs of the control input $bu(t)$ are defined as in Figure 1 and Figure 2 with $\Psi_s = +1$ on the left and $\Psi_s = -1$ on the right side of $\tilde{s} = \beta$, and the state $x_2(t)$ enters the set $\mathcal{R}_3$ with $\Psi_s = -1$ and $|s(t)| > L_s$. When M_3 is designed to satisfy the inequality

$$\frac{2x_{2ma}}{(\varepsilon - 2M_s^{p_1/p_2}\bar{\varepsilon}_{\tilde{s}})M_s^{p_1/p_2}} < M_3^{m_1/(m_2-m_1)+1} \tag{24}$$
$$x_{2ma} = \left|\frac{m_1\lambda M_s}{m_2 b}\right|^{m_1/(m_2-m_1)}$$

where $\bar{\varepsilon}_{\tilde{s}} \in \mathbb{R}^+$, then the state $x_2(t)$ leaves the set $\mathcal{R}_3$ and enters the set $\mathcal{R}_4$ in finite time.

In Lemma 5, the state $x_2(t)$ can also enter the set $\mathcal{R}_3$ with $\Psi_s = +1$ and $sgn(\Psi_s) = sgn(s)$. Since $sgn(s) = sgn(\Psi_s)$, the function $\sigma^{p_1/p_2}(s)$ is an aiding term, forcing the state $x(t)$ to be attractive to the manifold $\tilde{s}(t) = \beta$. This pushes $x(t)$ to the other side of the manifold with $\Psi_s = -1$, where sliding mode no longer occurs because $|\dot{s}| < \lambda\sigma^{p_1/p_2}(s)$ and the sign of Ψ_s no longer dominates equation (11). Then Lemma 5 can be used to show that $x_2(t)$ leaves the set $\mathcal{R}_3$.

Similar to [2], Lemma 5 proves that $x_2(t)$ crosses the boundary $x_2(t) = 0$. If the initial conditions are given as $x_2(t) = 0$ and $\Psi_s = 0$ but $s(t) \neq 0$, the surface in (11) is not zero, therefore Ψ_s will change to ± 1; then the velocity $x_2(t)$, in the worst case scenario, leaves the set $\mathcal{R}_3$, attaches itself to a sliding manifold and re-enters $\mathcal{R}_3$. Similarly, using Lemmas 3-5, once $x_2(t) \in \mathcal{R}_3$, it is shown that the velocity crosses the boundary $x_2(t) = 0$ in finite time, and $x_2(t) = 0$ is not an attractive set.

Lemma 6: Let the surface $s(t)$ be in the saturated region where $|s(t)| > L_s$, $x_2(t) \in \mathcal{R}_4$, $x(t) \in \mathcal{Q}_2$ or $x(t) \in \mathcal{Q}_4$, and sliding mode has occurred, then the state $x(t)$ converges to the origin with an average constant velocity $x_{2avg} = -\lambda M_s^{p_1/p_2} sgn(s)/c$.

Lemmas 3-5 show that sliding mode occurs in $\mathcal{R}_4$ and the velocity $x_2(t)$ enters and exits the set $\mathcal{R}_3$ from $\mathcal{Q}_1$ to $\mathcal{Q}_4$ or from $\mathcal{Q}_3$ to $\mathcal{Q}_2$. However, in order for $x(t)$ to reach the origin in finite time, sliding mode must also occur in the set $\mathcal{R}_2$. Given the right parameters, the following Lemma demonstrates that sliding mode continues to occur as the state $x(t)$ enters the set $\mathcal{R}_2$.

Lemma 7: When sliding mode occurs as $x(t)$ enters the set $\mathcal{R}_2$, the control input $u_4(t)$ designed in (21) allows the sign of

Ψ_s to be dominance in (11) and sliding mode occurs in the set $\mathcal{R}_2$.

From (5), when sliding mode occurs on one of the manifolds in $\mathcal{Q}_2$ or $\mathcal{Q}_4$, it is possible that the state $x(t)$ stays within $\mathcal{Q}_2$ or $\mathcal{Q}_4$, respectively, while converging to the origin, i.e., $sgn(\dot{x}_1) = -sgn(x_1)$. Additional analysis for $x(t) \in \mathcal{Q}_2$ and $x(t) \in \mathcal{Q}_4$ is trivial if $x(t)$ remains in $\mathcal{Q}_2$ or $\mathcal{Q}_4$ until $x(t)$ reaches the origin since $x_2(t)$ will not enter the set R_3 until it reaches the set R_2 due to Lemma 6. If the initial velocity $x_2(t_0)$ is in $\mathcal{Q}_2$ or $\mathcal{Q}_4$ and crosses over to $\mathcal{Q}_1$ or $\mathcal{Q}_3$, respectively, then Lemma 3-7 can be used to show finite time convergence of $x(t)$. Moreover, from Lemma 6, once sliding mode occurs in $\mathcal{Q}_2$ or $\mathcal{Q}_4$, the velocity $x_2(t)$ approaches the origin with an average velocity of $x_{2avg} = -M_s^{p_1/p_2} sgn(s)$ for $|s(t)| > L_s$, and the constants λ, M_s, and c are designed such that the inequality

$$\lambda \frac{M_s^{p_1/p_2}}{c} > x_{2m}$$

is satisfied, then it can be ensured that $x_2(t)$ does not re-enter into the set $\mathcal{R}_3$ until $x(t) \in \mathcal{R}_2$.

Lemma 7 shows that when sliding mode occurs before $x(t) \in \mathcal{R}_2$, sliding mode continues to occur as $x(t)$ is in $\mathcal{R}_2$, where the set $\mathcal{R}_2$ contains the origin. One possible trajectory of $x(t)$ is where $x(t)$ starts in $\mathcal{Q}_1$, enters and exits the set $\mathcal{R}_3$ to the set $\{\mathcal{Q}_4 \backslash \mathcal{R}_3\}$, travels with an average constant velocity of $x_2 = -M_s^{p_1/p_2} \lambda sgn(s)/c$, then enters the set $\mathcal{R}_2$ and converges to the origin. Similarly, if $x(t)$ starts in $\mathcal{Q}_3$, it enters and exits the set $\mathcal{R}_3$ to the set $\{\mathcal{Q}_2 \backslash \mathcal{R}_3\}$, travels with an average constant velocity of $x_2 = -M_s^{p_1/p_2} \lambda sgn(s)/c$, then enters the set $\mathcal{R}_2$ and converges to the origin.

Theorem 1: For the system in (5), where the sign of the input matrix b is unknown, the control input $u(t)$ in (18)-(21) ensures that the surface $s(t)$ is reached in finite time, and the state $x(t)$ is also reached in finite time.

Proof. Based on Lemma 3-7, the sign of Ψ_s is dominant in $\dot{\tilde{s}}(t)$ almost everywhere, the surface $\tilde{s}(t)$ is reached in finite time, and the state $x(t)$ reaches the origin in finite time. ■

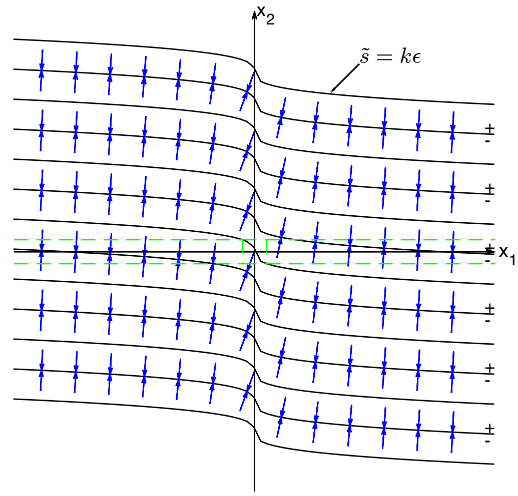

Fig. 1: Multiple Equilibrium Surface and the boundary x_{2m} shown in dotted green line.

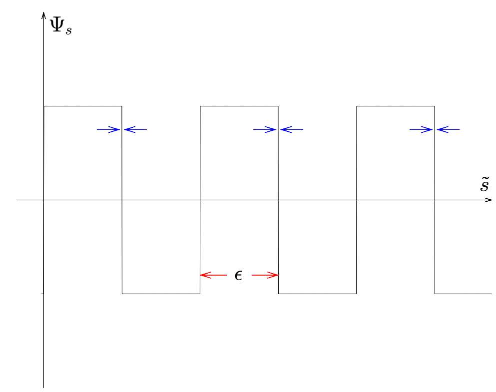

Fig. 2: The sign of the function $\Psi_s(t)$

VI. Simulation Results

Simulation for a second order system in (5) is performed. The parameters were chosen to be

$$\begin{array}{lll} M_{10} = 1.1 & M_{11} = 1.5 & M_2 = 1 \\ M_3 = 2 & M_4 = 1.1 & M_s = 1 \\ m_1 = 3 & p_1 = 3 & q_1 = 0.8 \\ m_2 = 5 & p_2 = 5 & q_2 = 5.0 \\ \lambda = 1 & c = 1 & d = 0.1 \\ b = 1 & L_s = 1 & M_{x_2} = 1. \end{array}$$

The nonvanishing disturbance $f(x,t)$ was considered as

$$f(x,t) = \cos(x_1)\sin(t).$$

The initial conditions were given as

$$x_1(0) = 30 \quad x_2(0) = 3.$$

Figure 4 shows the position $x_1(t)$ and velocity $x_2(t)$, and Figure 5 shows the surface $\tilde{s}(t)$ and the control input $u(t)$ over time. It can be seen that $x(t)$ begins in $\mathcal{Q}_1$, the velocity $x_2(t)$ decreases into $\mathcal{R}_3$ and leaves $\mathcal{R}_3$ between the third and fourth second, $x(t)$ enters $\mathcal{Q}_4$ and travels along the average velocity of $x_{2m} = -1$ until it reaches the set $\mathcal{R}_2$ and decreases to zero in finite time. It can be seen in Figure 5 the surface $\tilde{s}(t)$ reaches a constant after a brief moment and that the control input remains bounded at all times.

VII. Conclusion

This paper presents a nonsingular terminal sliding mode controller for a second order system without a priori knowledge of the control direction and in the presence non-vanishing disturbances. The proposed approach achieves finite time convergence, and ensures that the control signals remain bounded at all times. Simulation results are provided to show the efficacy of the proposed controller.

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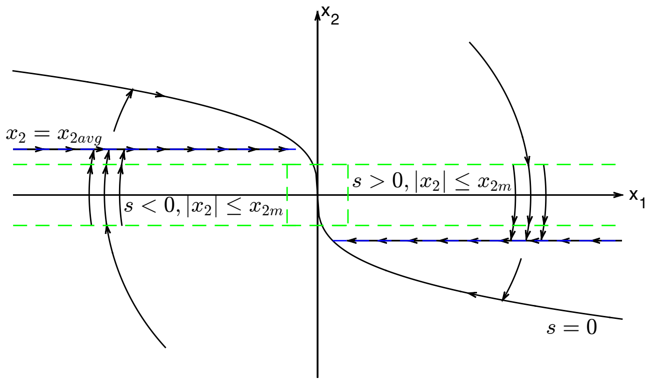

Fig. 3: The phase plot of the system.

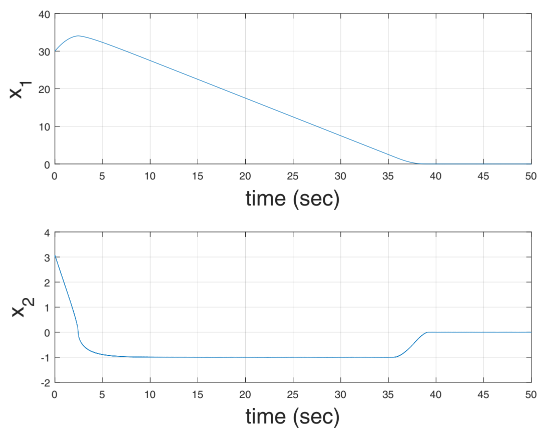

Fig. 4: Position $x_1(t)$ and velocity $x_2(t)$ versus time

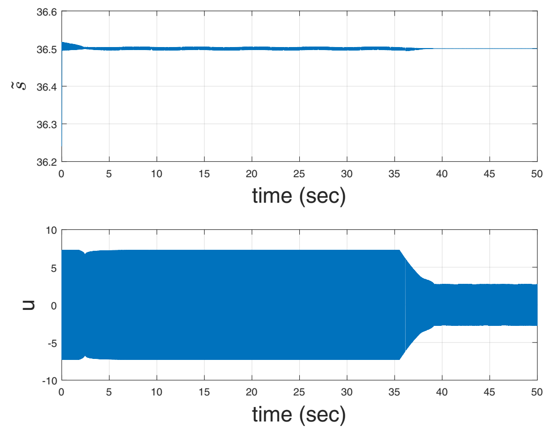

Fig. 5: Control input $u(t)$ versus time

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