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Decentralized Event-Triggered Control for Leader-follower Consensus

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***Abstract*—A decentralized event-triggered control scheme for leader-follower consensus is developed. The approach aims to reduce inter-agent communication while ensuring asymptotic leader-follower consensus. The objective is achieved by designing a control algorithm that uses the estimates of neighbors' states for feedback and only updates the estimates through communication when events are triggered. The update events are determined by a decentralized trigger condition, designed from the stability analysis, such that the control ensures asymptotic leader-follower consensus with only intermittent communication. The effectiveness of reducing inter-agent communication is shown by developing a positive constant lower bound of the minimum inter-event interval. Since switched dynamics are considered, Zeno execution is proven to be avoided, and Lyapunov-based convergence analysis is provided to ensure the asymptotic leader-follower consensus. Simulation results are provided to demonstrate the effectiveness of the developed control strategy.**

I. Introduction

The performance of various applications can be improved by employing a group of networked agents, where agents communicate and collaborate for navigation and completion of desired tasks. Due to the limited bandwidth of communication channels, increasing the number of agents in the network can cause communication congestion, leading to failure to exchange information. To reduce bandwidth usage, decentralized control strategies have been developed using local feedback (e.g., states from neighboring agents) to achieve global control objectives [1]–[4]. However, most existing solutions (cf. [5]–[7]) require continuous state feedback from neighboring agents for decentralized implementation.

To reduce communication, real-time scheduling methods, called event-triggered approaches (cf. [8], [9]), can be applied on an as-needed basis for state feedback. In classical event-triggered control, the control task is executed when the ratio of a certain error norm to the state norm exceeds a threshold. When compared to periodic-sampling based methods, event-triggered execution yields a minimum inter-event interval. Two of the earliest event-triggered strategies applied to control a multi-agent system are in [10] and [11], where the control is held constant between update periods and thus requires no feedback. However, the potential advantages of minimizing bandwidth usage were compromised since continuous communication are required for detecting the update events.

Extended to directed and undirected graphs, the controllers and trigger functions developed in [12] and [13] were proven to be decentralized. However, a priori knowledge (i.e., Fiedler value and the final consensus value) were required for verifying the triggering condition. These requirements were relaxed in [14] and [15] by designing a trigger function using the sum of relative states from neighbors. In [16], a time-based triggering function associated with the Fiedler value was introduced for asymptotic consensus. A similar time-varying triggering condition was applied in [17] on a directed time-varying communication topology with neighbor-synchronous and asynchronous updating protocols. These applications target leaderless consensus problems, and only differ in the design of the event-triggered functions. Additionally, continuous communication is used in these results for detecting the trigger condition, and thus mitigate the benefits of event-triggered strategies. Since the leader state is usually time-varying and not available to follower agents, the approach developed in [18] is limited in application.

To reduce inter-agent communication, a decentralized controller using state-estimate feedback is developed in this paper. The estimates are updated by event-triggered communication. The events happen when the triggering conditions are satisfied, where the conditions designed from stability analysis aim to maintain leader-follower consensus with a minimized communication. Therefore, no inter-agent communication is required between any two event times. A convergence analysis shows that the controller requires only intermittent communications while still achieving consensus with asymptotic convergence. Simulation demonstrates asymptotic leader-follower consensus convergence with reduced inter-agent communication.

II. Preliminaries and problem Statement

A. Algebraic Graph Theory Preliminaries [19]

A directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ consists of a node set $\mathcal{V} \triangleq \{1, 2, \cdots, N\}$, and an edge set $\mathcal{E}$, where $\mathcal{E} \in (\mathcal{V} \times \mathcal{V})$ is a set of paired nodes. An edge, denoted as (i, j), implies that node j can obtain information from node i, but not vice versa. On the contrary, the graph $\mathcal{G}$ is undirected if $(i, j) \in \mathcal{E}$

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implies $(j,\, i) \in \mathcal{E}$, and vice versa. The neighbor set of agent i is defined as $\mathcal{N}_i \triangleq \{j \in \mathcal{V} \mid (j,\, i) \in \mathcal{E},\, j \neq i\}$.

A directed path is a sequence of edges in a graph. An undirected path of the undirected graph is defined analogously. An undirected graph is connected if there exist a undirected path between any two distinct nodes in the graph. An adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ of the directed graph is given by $a_{ij} > 0$ if $(j,\, i) \in \mathcal{E}$, and $a_{ij} = 0$ otherwise. For the undirected graph, $a_{ij} = a_{ji}$. For both the directed and undirected graph, $a_{ii} = 0$ holds, and furthermore, it is assumed that $a_{ij} = 1$ if $(j,\, i) \in \mathcal{E}$. The Laplacian matrix of the graph $\mathcal{G}$ is defined as $L = [l_{ij}] \in \mathbb{R}^{N\times N}$, where $l_{ii} = \sum_{j\neq i} a_{ij}$ and $l_{ij} = -a_{ij}$, where $i \neq j$.

B. Dynamics

Consider N follower agents with a network topology that can be modeled by an undirected graph $\mathcal{G} = (\mathcal{V},\, \mathcal{E})$. Let $\bar{\mathcal{G}}$ denote a new directed graph with the node set $\mathcal{V} \cup \{0\}$ and the edge set that contains all edges in $\mathcal{E}$ and the edges connecting leader agent 0 and follower agent $j \in \mathcal{V}$. The dynamics of the followers and the leader are described by

$$\dot{x}_i = Ax_i + Bu_i, \tag{1}$$

$$\dot{x}_0 = Ax_0, \tag{2}$$

where $x_i \in \mathbb{R}^n$ and $u_i \in \mathbb{R}^m$ denote the state and control input of follower agent $i \in \mathcal{V}$, respectively, $x_0 \in \mathbb{R}^n$ denotes the leader's state, $A \in \mathbb{R}^{n\times n}$ is a state matrix, and $B \in \mathbb{R}^{n\times m}$ is a full column rank matrix.

C. Conventional Approach and Control Objective

A decentralized continuous feedback controller for the system in (1) and (2) was developed in [5] as

$$u_i = K\sum_{j\in\mathcal{N}_i} (x_j - x_i) + Kd_i\,(x_0 - x_i)\,,\; i \in \mathcal{V}, \tag{3}$$

where $K \in \mathbb{R}^{m\times n}$ is the control gain matrix, and $d_i = 1$ if agent $i \in \mathcal{V}$ is connected to the leader, $d_i = 0$ otherwise. Note that this control implementation requires continuous state feedback from the neighboring agents.

Assumption 1. The graph $\bar{\mathcal{G}}$ is connected.

Assumption 2. The dynamics of the follower agents are controllable, or the pair $(A,\, B)$ is stabilizable.

Assumption 3. The followers that are connected to the leader can continuously receive information from the leader.

Based on Assumption 1, the matrix $H \in \mathbb{R}^{N\times N}$ defined as $H = L + D$ is positive definite [20], where $D \in \mathbb{R}^{N\times N}$ is defined as $D \triangleq \operatorname{diag}(d_1,\, d_2,\, \ldots,\, d_N)$. Based on Assumption 2, there exists a symmetric positive definite matrix $P \in \mathbb{R}^{n\times n}$ that satisfies the following Riccati inequality

$$PA + A^TP - 2\delta_{\min}PBB^TP + \delta_{\min}I_n < 0, \tag{4}$$

so the control gain in (3) can be designed as

$$K = B^TP, \tag{5}$$

where $\delta_{\min} \in \mathbb{R}^+$ denotes the minimum eigenvalue of H and is a positive constant based on Assumption 1 and [20], and I is an identity matrix with denoted dimension. In summary, the leader-follower consensus problem is achieved if the network described in (1) and (2) satisfies

$$\varepsilon_i \triangleq x_i - x_0 \to 0 \quad \text{as} \quad t \to \infty, \quad i \in \mathcal{V}, \tag{6}$$

where $\varepsilon_i \in \mathbb{R}^n$ represents the leader-follower consensus error for agent i.

III. Development of the Event-triggered Decentralized Controller

To eliminate the need for continuous communication while achieving the control objective, an event-triggered based decentralized control approach is developed.

A. Controller Design

Based on the continuous controller in (3) and subsequent convergence analysis, a decentralized event-triggered controller for agent $i \in \mathcal{V}$ is designed as

$$u_i = K\hat{z}_i, \tag{7}$$

$$\hat{z}_i = \sum_{j\in\mathcal{N}_i} (\hat{x}_j - \hat{x}_i) + d_i(x_0 - \hat{x}_i), \tag{8}$$

where K is the control gain defined in (5). In (8), the computation of $\hat{z}_i$ only requires the estimates of agent i and its neighboring followers' state estimate (i.e., $\hat{x}_{j\in\mathcal{N}_i}$), instead of using their true states $x_{j\in\mathcal{N}_i}$ via continuous communication. When the leader is a neighbor, the true state x_0 is used since the leader state is available according to Assumption 3. The estimate $\hat{x}_j$ in (8) evolves according to the dynamics

$$\dot{\hat{x}}_j\,(t) = A\hat{x}_j\,(t)\,,\; j \in \mathcal{N}_i,\; t \in \left[t_k^j,\, t_{k+1}^j\right), \tag{9}$$

$$\hat{x}_j\left(t_k^j\right) = x_j\left(t_k^j\right), \tag{10}$$

for $k = 0,\, 1,\, 2\ldots$, where $\hat{x}_j$ flows along the leader dynamics during $t \in \left[t_k^j,\, t_{k+1}^j\right)$ and is updated via x_j communicated from neighboring agent j at its discrete times t_k^j, for $j \in \mathcal{N}_i$, where t_k^j is the event-triggered time described in Section III-C. Although agent $i \in \mathcal{V}$ does not communicate the estimate $\hat{x}_i$, agent i maintains $\hat{x}_i$ for implementation in (8). The estimate $\hat{x}_i$ is updated continuously with the dynamics in (9) and discretely at time instances described in (10). Therefore, u_i is a piecewise continuous signal, where communication is required when state information is transmitted to, or received from, neighboring agents for estimate updates; otherwise, u_i flows during the inter-event intervals.

B. Dynamics of Estimate Error

Since x_i follows different dynamics from the estimate $\hat{x}_i$ computed by its neighbors, an estimate error $e_i \in \mathbb{R}^n$ characterizing this mismatch is defined as

$$e_i \triangleq \hat{x}_i - x_i,\; i \in \mathcal{V},\; t \in \left[t_k^i,\, t_{k+1}^i\right), \tag{11}$$

where e_i is reset to 0 at the event time t_k^i, $k = 0, 1, \ldots$, due to the estimate updates. Although x_i and $\hat{x}_i$ are both known for agent i; using $\hat{x}_i$ enables agent i to judge how far another $\hat{x}_i$ is away from its state x_i. Using (1), (7), and (9), the stacked form of the time-derivative of (11) can be expressed as

$$\dot{e} = (I_N \otimes A)\, e + (H \otimes BK)\, \varepsilon + (H \otimes BK)\, e, \tag{12}$$

where $e \in \mathbb{R}^{nN}$ denotes $e \triangleq \left[e_1^T, e_2^T, \ldots, e_N^T\right]^T$, $\otimes$ denotes the Kronecker product, and $\varepsilon \in \mathbb{R}^{nN}$ defined as $\varepsilon \triangleq \left[\varepsilon_1^T, \varepsilon_2^T, \ldots, \varepsilon_N^T\right]^T$ is a stacked form of ε_i.

C. Event-triggered Communication Mechanism

Fig. 1 depicts how the communication between neighboring agents proceeds during triggered events.

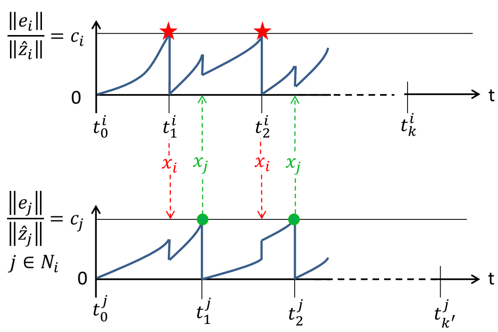

Figure 1. Inter-agent communication mechanism under an event-triggered approach. The stars and dots represent instances when decentralized triggering conditions are satisfied, and the triggered agents communicate their states over the network to update neighbors' estimates.

In Fig. 1, $\frac{\|e_i\|}{\|\hat{z}_i\|}$ is a decentralized, non-negative, and piecewise continuous signal used to verify the triggering condition. The detailed design of the trigger condition is shown in Section IV. The dots represent the event-triggered time t_k^i when $\frac{\|e_i\|}{\|\hat{z}_i\|}$ reaches a constant c_i, designed in the subsequent analysis. At t_k^i, x_i is communicated over the network to update the estimate $\hat{x}_i$, used by each neighboring agent $j \in \mathcal{N}_i$. Additionally, $\frac{\|e_i\|}{\|\hat{z}_i\|}$ is reset to zero at t_k^i since the updated estimate has no estimate error. Similarly, at neighbor agent j's event time t_k^j, x_j is communicated over the network to update the estimate $\hat{x}_j$. Since $\|\hat{z}_i\|$ is a decentralized and estimate-based function, verification of the triggering conditions requires no neighbor state information, and hence no communication is required during any inter-event interval (e.g., $\left[t_1^i, t_1^j\right)$, $\left[t_1^j, t_2^i\right)$, $\left[t_2^i, t_2^j\right)$).

D. Closed-loop error system

Using (11), a non-implementable form (to facilitate the subsequent analysis) of (7) can be expressed as

$$\begin{aligned} u_i =& K \sum_{j \in \mathcal{N}_i} \left[(x_j - x_i) + (e_j - e_i)\right] \\ &+ K d_i \left(x_0 - x_i\right) - K d_i e_i. \end{aligned} \tag{13}$$

Substituting (13) into the open-loop dynamics in (1) and using the definition in (6) yields a stacked form of the closed-loop error system

$$\dot{\varepsilon} = (I_N \otimes A)\, \varepsilon - (H \otimes BK)\, \varepsilon - (H \otimes BK)\, e. \tag{14}$$

To facilitate the subsequent analysis, a relation between ε and $\hat{z}$ is developed, where $\hat{z} \triangleq \left[\hat{z}_1, \hat{z}_2, \ldots, \hat{z}_N\right]^T \in \mathbb{R}^{nN}$ represents the stacked form of $\hat{z}_i$

$$\hat{z} \triangleq (H \otimes I_n)\left[(1_N \otimes x_0) - \hat{x}\right], \tag{15}$$

where $\hat{x} \triangleq \left[\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_N\right]^T \in \mathbb{R}^{nN}$, and 1_N is the ones vector with denoted dimension. Using the relation

$$\varepsilon_i = x_i - x_0 = (\hat{x}_i - e_i) - x_0,$$

the useful expression

$$\hat{x} - (1_N \otimes x_0) = \varepsilon + e \tag{16}$$

can be obtained. Combining (15) and (16) yields

$$\varepsilon = -\left(H^{-1} \otimes I_n\right) \hat{z} - e, \tag{17}$$

where $\hat{z}$ is governed by the dynamics

$$\dot{\hat{z}} = (I_N \otimes A)\, \hat{z}, \tag{18}$$

where (2) and (9) were used.

IV. Convergence Analysis

In this section, leader-follower consensus with the event-triggered controller designed in (7) is examined using a Lyapunov-based analysis. To facilitate subsequent convergence analysis, the event time t_k^i is explicitly defined below before proving the main theorem.

Definition 1. An event time t_k^i is defined as

$$t_k^i \triangleq \inf\left\{t > t_{k-1}^i \mid f_i(t) = 0\right\}, \; i \in \mathcal{V} \tag{19}$$

for $k = 0, 1, 2 \ldots$, where $f_i(\cdot)$, denoted as $f_i\left(e_i(\cdot), \hat{z}_i(\cdot)\right)$, is a decentralized trigger function

$$f_i\left(e_i, \hat{z}_i\right) \triangleq \|e_i\| - \sqrt{\frac{\sigma_i\left(k_1 - \frac{k_3}{\beta}\right)}{k_2 + k_3\beta}}\, \|\hat{z}_i\|, \tag{20}$$

where $\sigma_i \in \mathbb{R}_{>0}$ satisfying $0 < \sigma_i < 1$ provides flexibility in real-time scheduling, and $\beta \in \mathbb{R}_{>0}$ satisfies

$$\beta > \frac{k_3}{k_1}, \tag{21}$$

where k_i, $i = 1, 2, 3$, are positive constants defined as

$$\begin{aligned} k_1 &\triangleq \delta_1 S_{\min}\left(H^{-2}\right) \\ k_2 &\triangleq S_{\min}\left(H \otimes \left(2PBB^TP\right)\right) - \delta_1 \\ k_3 &\triangleq S_{\min}\left(I_N \otimes \left(2PBB^TP\right) - H^{-1} \otimes 2\delta_1 I_n\right), \end{aligned}$$

where $k_2 \neq 0$ or $k_3 \neq 0$, $\delta_1 \in \mathbb{R}_{>0}$ satisfies $0 < \delta_1 < \delta_{\min}$, $S_{\min}(\cdot)$ denotes the minimum singular value of a matrix argument.

Theorem 1. *The controller designed in (7) ensures that the network system achieves asymptotic leader-follower consensus in the sense that*

$$x_i - x_0 \to 0 \quad as \quad t \to \infty, \quad i \in \mathcal{V} \tag{22}$$

provided that the estimate $\hat{x}_i$ in (7) is updated at t_k^i defined in (19).

Proof: Consider a Lyapunov function candidate $V : \mathbb{R}^{nN} \to \mathbb{R}$ as

$$V \triangleq \varepsilon^T (I_N \otimes P) \varepsilon, \tag{23}$$

where P is a symmetric positive definite matrix satisfying (4). Using (5) and (14), the time derivative of (23) can be expressed as

$$\begin{aligned} \dot{V} =& \varepsilon^T \left[I_N \otimes \left(PA + A^T P \right) - H \otimes \left(2PBB^T P \right) \right] \varepsilon \\ & - e^T \left[H \otimes \left(2PBB^T P \right) \right] \varepsilon. \end{aligned} \tag{24}$$

Since H is symmetric and positive definite, (4) can be used to upper bounded (24) as

$$\dot{V} \leq -\delta_{\min} \varepsilon^T \varepsilon - e^T \left[H \otimes \left(2PBB^T P \right) \right] \varepsilon. \tag{25}$$

Using (17), (25) can be upper bounded by

$$\begin{aligned} \dot{V} \leq & - \delta_1 \hat{z}^T \left(H^{-2} \otimes I_n \right) \hat{z} - \delta_1 e^T e + 2\delta_1 e^T \left(H^{-1} \otimes I_n \right) \hat{z} \\ & - e^T \left[I_N \otimes \left(2PBB^T P \right) \right] \hat{z} + e^T \left[H \otimes \left(2PBB^T P \right) \right] e \\ & - \delta_2 \varepsilon^T \varepsilon, \end{aligned} \tag{26}$$

where $\delta_2 \in \mathbb{R}_{>0}$ satisfies $\delta_{\min} = \delta_1 + \delta_2$. By using the inequality $x^T y \leq \frac{\beta}{2} \|x\|^2 + \frac{1}{2\beta} \|y\|^2$, (26) can be upper bounded by

$$\begin{aligned} \dot{V} \leq & - k_1 \|\hat{z}\|^2 + 2k_3 \left(\frac{\beta}{2} \|e\|^2 + \frac{1}{2\beta} \|\hat{z}\|^2 \right) + k_2 \|e\|^2 \\ & - \delta_2 \varepsilon^T \varepsilon \\ \leq & - \sum_{i \in \mathcal{V}} \left[\left(k_1 - \frac{k_3}{\beta} \right) \|\hat{z}_i\|^2 - (k_2 + k_3 \beta) \|e_i\|^2 \right] \\ & - \delta_2 \varepsilon^T \varepsilon. \end{aligned} \tag{27}$$

In (27), the necessary conditions for $\dot{V}$ to be negative definite are (19)-(21). Using (19)-(21), (27) can be rewritten as

$$\dot{V} \leq -\sum_{i \in \mathcal{V}} (1 - \sigma_i) \left(k_1 - \frac{k_3}{\beta} \right) \|\hat{z}_i\|^2 - \delta_2 \varepsilon^T \varepsilon, \tag{28}$$

which is strictly negative definite as

$$\dot{V} \leq -\delta_2 \varepsilon^T \varepsilon. \tag{29}$$

Given (23) and (29),

$$\|\varepsilon(t)\| \leq \|\varepsilon(0)\| \exp(-\gamma t),$$

where $\gamma \in \mathbb{R}_{>0}$ is a positive constant. Based on (6), the exponential convergence of $\|\varepsilon\|$ implies (22). ∎

Remark 1. Based on (20), the constant c_i in Fig. 1 can be designed as $c_i = \sqrt{\frac{\eta_i \left(k_1 - \frac{k_3}{\beta} \right)}{k_2 + k_3 \beta}}$. At t_k^i, e_i will be reset to zero as agent i communicates its state x_i to all its neighboring agents to update $\hat{x}_i$, and hence $\frac{\|e_i\|}{\|\hat{z}_i\|} = 0$ (i.e., $f_i < 0$). After the update, $\|e_i\|$ grows in time until meeting the next trigger condition $\frac{\|e_i\|}{\|\hat{z}_i\|} = c_i$ (i.e., $f_i = 0$). Then, the cycle repeats.

V. Minimal inter-event interval

To show the proposed trigger functions in Theorem 1 do not lead to Zeno behavior, it is sufficient to find a positive lower bound for the inter-event interval. To facilitate subsequent analysis, two constants $\bar{c}_0, \bar{c}_1 \in \mathbb{R}_{>0}$ are defined as

$$\bar{c}_0 \triangleq S_{\max}(A) \tag{30}$$

$$\begin{aligned} \bar{c}_1 \triangleq & S_{\max}((I_N \otimes A) + (H \otimes BK)) \\ & + S_{\max}(H \otimes BK) + S_{\max}(A), \end{aligned} \tag{31}$$

where $S_{\max}(\cdot)$ denotes the maximum singular value of a matrix argument

Theorem 2. *The event time defined in (19) ensures that there exists at least one agent $h \in \mathcal{V}$ such that its minimum inter-event interval $\tau \in \mathbb{R}_{>0}$ is lower bounded by*

$$\tau \geq \frac{1}{c} \ln \left(\frac{1}{N} \sqrt{\frac{\sigma_h \left(k_1 - \frac{k_3}{\beta} \right)}{k_2 + k_3 \beta}} + 1 \right), \tag{32}$$

where $h \in \mathcal{V}$ is defined in the subsequent analysis, and $c \in \mathbb{R}_{>0}$ is a positive constant defined as

$$c \triangleq \max\{\bar{c}_0, \bar{c}_1\}. \tag{33}$$

Proof: Inspired by the proof in [10], we consider an agent $h \in \mathcal{V}$ that satisfies

$$h \triangleq \arg\max_{i \in \mathcal{V}} \|\hat{z}_i\|.$$

Since $\|e_h\| \leq \|e\|$, the following inequality holds

$$\frac{\|e_h\|}{N \|\hat{z}_h\|} \leq \frac{\|e\|}{N \|\hat{z}_h\|} \leq \frac{\|e\|}{\|\hat{z}\|},$$

which is equivalent to

$$\frac{\|e_h\|}{\|\hat{z}_h\|} \leq N \frac{\|e\|}{\|\hat{z}\|}. \tag{34}$$

For any interval $t \in \left[t_k^h, t_{k+1}^h \right)$, $\frac{\|e\|}{\|\hat{z}\|}$ is continuous. To show the inter-event interval is lower bounded as in [8], one can investigate the time derivative of $\frac{\|e\|}{\|\hat{z}\|}$ over the interval $t \in \left[t_k^h, t_{k+1}^h \right)$ as

$$\frac{d}{dt} \left(\frac{\|e\|}{\|\hat{z}\|} \right) = \frac{d}{dt} \left[\frac{\left(e^T e \right)^{\frac{1}{2}}}{\left(\hat{z}^T \hat{z} \right)^{\frac{1}{2}}} \right] \leq \frac{\|\dot{e}\|}{\|\hat{z}\|} + \frac{\|e\| \left\| \dot{\hat{z}} \right\|}{\|\hat{z}\|^2}. \tag{35}$$

Using (12), (17), (18), and applying the inequality $x^T y \leq \|x\| \|y\|$ yields

$$\frac{d}{dt}\left(\frac{\|e\|}{\|\hat{z}\|}\right)$$
$$\leq \|(I_N \otimes A) + (H \otimes BK)\| \frac{\|e\|}{\|\hat{z}\|} + \|(I_N \otimes BK)\|$$
$$+ \|(H \otimes BK)\| \frac{\|e\|}{\|\hat{z}\|} + \|(I_N \otimes A)\| \frac{\|e\|}{\|\hat{z}\|},$$

which can be further expressed as

$$\dot{y} \leq \bar{c}_0 + \bar{c}_1 y, \tag{36}$$

where $\bar{c}_0$ and $\bar{c}_1$ are defined in (30) and (31), and $y : [0, \infty) \to \mathbb{R}_{\geq 0}$ is a non-negative, piecewise continuous function, which is differentiable at the inter-event interval and is defined as

$$y\left(t - t_k^h\right) \triangleq \frac{\|e(t)\|}{\|\hat{z}(t)\|}, \text{ for } t \in \left[t_k^h, t_{k+1}^h\right) \tag{37}$$

for $k = 0, 1, 2 \ldots$. The inequality in (36) can be simply upper bounded by

$$\dot{y} \leq c(1 + y), \tag{38}$$

where c is defined in (33). Based on (38), a non-negative function $\phi : [0, \infty) \to \mathbb{R}_{\geq 0}$, satisfying

$$\dot{\phi} = c(1 + \phi), \quad \phi(0) = y_0, \tag{39}$$

can be lower bounded by y as

$$y \leq \phi, \text{ for } t \in [0, \tau), \tag{40}$$

where $\tau \triangleq t_{k+1}^h - t_k^h \in \mathbb{R}_{>0}$ is the minimum inter-event interval, and $y_0 \in \mathbb{R}_{\geq 0}$ is the initial condition of y, which is 0 since $e\left(t_k^h\right) = 0$ for $k = 0, 1, 2 \ldots$. An analytical solution to (39) with initial condition $\phi(0) = 0$ can be solved as

$$\phi(t) = \exp(ct) - 1. \tag{41}$$

Using (40) and (41) with $t \to \tau$ yields

$$\lim_{\varphi \to 0} y(\tau - \varphi) \leq \exp(c\tau) - 1. \tag{42}$$

Using (20) where $f_h\left(t_{k+1}^h\right) = 0$, (34), and $y(\tau)$ in (37) yields

$$\frac{1}{N}\sqrt{\frac{\sigma_h\left(k_1 - \frac{k_3}{\beta}\right)}{k_2 + k_3\beta}} \leq \exp(c\tau) - 1,$$

which can be solved to yield (32). ■

Remark 2. This lower bound implies that Zeno behaviors can be excluded. However, there is a trade-off between the minimum inter-event interval and the error convergence rate. The lower bound in (32) can be increased by selecting a higher σ_h, but this increase results in a slower convergence due to the fact that $\dot{V}$ in (28) becomes less negative.

VI. Simulation

In this section, a leader-follower network system consisting of 4 follower and 1 leader as depicted in Fig. 2 is simulated to validate the developed event-triggered control strategy.

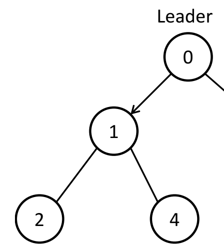

Figure 2. Network topology of the graph $\bar{\mathcal{G}}$, where the agent indexed by 0 is the leader and other agents are the followers.

The dynamics of the leader and the followers and the Laplacian matrix of the graph $\bar{\mathcal{G}}$ are

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, L = \begin{bmatrix} 2 & -1 & 0 & -1 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix}.$$

Based on the network topology in Fig. 2, the smallest nonzero eigenvalue of H is $\delta_{\min} = 0.2679$, and the solution P of (4) and the control gain $K = B^T P$ are

$$P = \begin{bmatrix} 24.82 & 10.58 \\ 10.58 & 9.13 \end{bmatrix}, K = \begin{bmatrix} 10.58 & 9.13 \end{bmatrix}.$$

The initial conditions of each agent for the simulation are $x_0 = [1,\ 1]^T$, $x_1 = [10,\ 2]^T$, $x_2 = [3,\ 7]^T$, $x_3 = [9,\ -4]^T$, and $x_4 = [6,\ 5]^T$. The consensus errors of each follower agent are shown in Fig. 3, where both plots show the leader-follower asymptotic consensus.

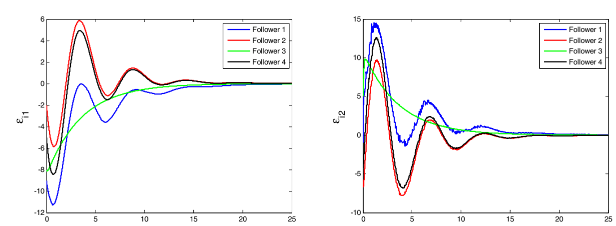

Figure 3. Consensus errors ε_{i1} and ε_{i2} of follower agents.

For ε_{i2}, the sawtooth waves reflect the jumps of the control input whenever events are triggered. Fig. 4 shows triggered events individually. The average intervals between two contiguous events within follower agents 1-4 are 44 ms, 24 ms, 80 ms, and 17 ms, respectively. These intervals imply that Zeno behavior did not occur.

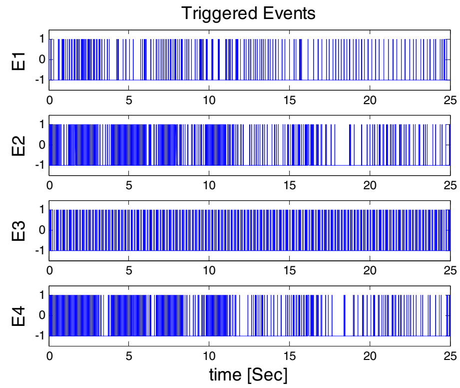

Figure 4. $E1$-$E4$ represent the occurrences of the events for all follower agents (1: triggered, -1: not triggered).

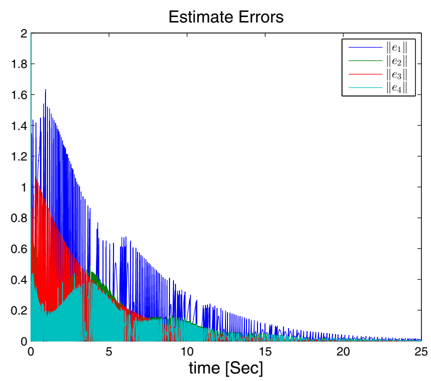

Figure 5. Norm of the estimate errors of the follower agents.

Fig. 5 shows asymptotic convergences of the estimate errors for follower agent 1-4, which implies the estimate asymptotically converges to the true states.

VII. Discussion

A decentralized event-triggered control scheme for the leader-follower network consensus is developed to reduce inter-agent communication. The controller uses intermittent feedback to reduce the inter-agent communication. A Lyapunov-based stability analysis indicates that the network system achieves asymptotic leader-follower consensus under this event-triggered control scheme. Moreover, the trigger function is proven to never exhibit Zeno behavior. Finally, simulation results are presented to support the developed theorems.

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