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Context-Aware Communication to Stabilize Bandwidth-Limited Nonlinear Networked Control Systems

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***Abstract*—In this paper, context-aware communication that stabilizes a class of nonlinear networked control systems (NCS) operating over a bandwidth-limited shared-channel is proposed to reduce network traffic. The NCS under consideration includes feedback and feedforward communication channels, and is assumed to be perturbed by an unmodeled, nonlinear disturbance. Informational value of the output states can be analyzed in the context of system stability using model-based approaches. When fused with event-based triggering, these context-aware communication policies result in aperiodic feedback and feedforward signals that guarantee uniformly ultimately bounded tracking of output states of an uncertain perturbed system along the desired time-varying trajectory. The proposed NCS is validated using extensive simulation results for nonlinear coupled MIMO systems.**

I. Introduction

The presented research focuses on a class of nonlinear networked control system (NCS), where a controller may not be collocated with sensors and the system. Therefore, sensor measurements are transmitted over a feedback communication channel to the controller, which, in turn, sends control inputs over a feedforward channel to the system. The communication channel can be shared among various systems and network devices, and for such shared-channel scenarios, effective utilization of bandwidth-limited channel resources becomes crucial to the stability of the system as well as satisfactory operation of other network services. Smart sensors that can analyze the informational value of data to be transmitted based on an objective function to determine the "worthiness" of data for transmission can significantly reduce network traffic. In [1]–[4], optimal communication policies are developed to minimize estimation error at the receiver given knowledge of the underlying process. [5]–[7] developed an event-triggered controller for uncertain linear systems to guarantee stability of the system with limited feedback. In [5], approximate model knowledge is used to run the system in open-loop until the error between the estimated and actual states exceeds a constant design parameter. [8] developed an event-based controller for linear systems in the presence of a bounded disturbance input, where it is assumed that the plant is stable and no model uncertainties occur. A context-aware feedback policy for a class of uncertain nonlinear systems is developed in our previous work in [9]. Conditions for stability of a general class of nonlinear systems are developed in [10], [11] assuming there exists an input-to-state (ISS) control law that renders the system stable with respect to measurement errors in information-limited NCS. [10] proved global asymptotic stability (GAS) if a suitable relationship holds between the number of values taken by the encoder, the sampling period, and a system parameter. [11] developed a condition to update the control input based on system stability, and a sample-hold implementation holds the control input constant between updates. Event-triggered network control system research can be found in [7], [12]–[19] and the references therein.

In this paper, we extend our results in [9] to include communication channels in both the feedback and the feedforward loop. The main contribution of the presented paper is in the development of context-aware feedback and feedforward communication policies to stabilize a class of uncertain nonlinear NCS subject to unmodeled, nonlinear disturbances while reducing network traffic. The context-aware communication policies are developed by analyzing the informational value of output states to determine whether or not to transmit output measurements and/or control inputs over the feedback and feedforward communication channels with an objective to reduce communication and to stabilize the system. The proposed model-based, event-triggered, aperiodic communication policy ensures uniformly ultimately bounded tracking of output states of an uncertain, perturbed nonlinear system along a time-varying desired trajectory.

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II. System Model

The presented work analyzes a NCS of the form shown in Fig. 1, where output measurements are communicated to a remotely located controller, which, in turn, uses a network

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channel to provide the control input to the system. The feedback path includes a decision maker at the system output, while the feedforward path includes an estimator at the controller (see Fig. 1). The role of the decision maker is to determine whether the feedback and/or the control input is required to maintain stability of the system. It is assumed that an approximate system model is available to the decision maker, which can be updated using an online adaptation scheme. The function of the rest of the components in the feedback and feedforward path can be found in [9].

Various communication scenarios are presented in Table I, where the goal is to determine context-aware feedback and feedforward policies for each communication scenario to reduce channel usage while stabilizing the NCS. A beacon signal in the form of a 2-bit policy (0-0, 0-1, 1-0, or 1-1) is broadcasted at each time t by the decision maker based on the policy selected at time t.

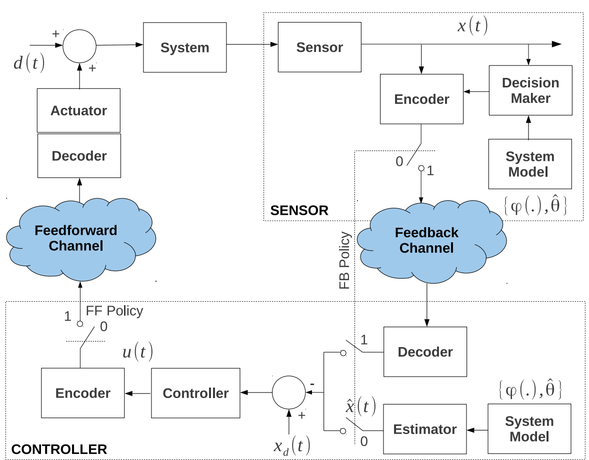

Fig. 1. Schematic of the proposed networked control system

FB channel	FF channel	Policy
0	0	0-0
0	1	0-1
1	0	1-0
1	1	1-1

TABLE I
COMMUNICATION CHANNEL USAGE SCENARIOS (FB-FEEDBACK, FF-FEEDFORWARD), WHERE 1 INDICATES THE CHANNEL IS USED AND *vice versa* FOR 0.

Consider a first order, autonomous, nonlinear system of the form given below

$$\dot{x}(t) = f(x) + g(x)u(t) + d(t) \tag{1}$$

where $x(t) \in \mathbb{R}^n$ denotes the system state; $u(t) \in \mathbb{R}^p$, $n \leq p$, is the piecewise continuous-in-time control input; $g(x) : \mathbb{R}^n \to \mathbb{R}^{n \times p}$ is a known control input gain matrix; $d(t) \in \mathbb{R}^n$ represents an unmodeled nonlinear disturbance such that $\|d(t)\| \leq \gamma_0$, where $\gamma_0 \in \mathbb{R}$ is a known positive scalar; and $f(x) : \mathbb{R}^n \to \mathbb{R}^n$ is assumed to be linearly parameterizable (LP) such that

$$\dot{x}(t) = \phi(x)\theta + g(x)u(t) + d(t) \tag{2}$$

where $\phi(x) : \mathbb{R}^n \to \mathbb{R}^{n \times m}$ is a known regression matrix of nonlinear functions of $x(t)$, and $\theta \in \mathbb{R}^m$ is a vector of constant parameters (e.g., geometric length, mass, inertia, aerodynamic coefficients).

Consider a nominal, undisturbed system of the form in (2) that is driven by the same control input $u(t)$. Due to parametric uncertainty present in the dynamics, the nominal system is defined as

$$\dot{\hat{x}}(t) \triangleq \phi(\hat{x})\hat{\theta}(t) + g(\hat{x})u(t) \tag{3}$$

where $\hat{x}(t) \in \mathbb{R}^n$ denotes the piecewise continuous state of the nominal system, and $\hat{\theta}(t) \in \mathbb{R}^m$ is an estimate of the unknown parameter vector θ.

III. CONTROL OBJECTIVE

To reduce network traffic, it is necessary to restrict access to the feedback and feedforward communication channels. In the absence of feedback, the system state $x(t)$ is not available to the controller, and only the state estimate $\hat{x}(t)$ can be obtained at the controller using (3) between feedback instances. While between feedforward instances the system does not receive the control input $u(t)$, and it is considered to be unactuated.

The control objective is to track the output $x(t)$ of the system given in (1) along a desired time-varying trajectory $x_d(t) \in \mathbb{R}^n$, i.e., $x(t) \to x_d(t)$, in the presence of unmodeled disturbances and dynamic uncertainties while reducing network channel usage. To this end, the goal of this research is twofold: 1) develop a stabilizing tracking controller to track the desired trajectory as discussed above, and 2) formulate the feedback and feedforward communication policies that reduce network channel usage (bandwidth) for the communication scenarios presented in Table I while maintaining system stability.

An open-loop tracking controller is developed in Section IV to track the piecewise continuous state $\hat{x}(t)$ of the nominal system along a desired trajectory $x_d(t)$ when the feedforward channel is used. Whenever feedback and feedforward signals are available, i.e., corresponding to a 1-1 policy, the uncertain system parameter estimates $\hat{\theta}(t)$ are updated in the system model at the output and the controller using the adaptive update law developed in Section V. Context-aware communication policies to reduce network traffic are developed in Section VI.

IV. CONTROLLER DEVELOPMENT

In this section, an open-loop tracking controller is developed for the nominal system in (3), such that $\hat{x}(t) \to x_d(t)$. Let $i \in \mathbb{N}$, $i = 1, 2, \ldots, k_1$ be the index that counts instances when feedback is available to the controller with the corresponding time $t_f = \{t_1, t_2, \ldots, t_{k1}\}$, and $j \in \mathbb{N}$, $j = 1, 2, \ldots, k_2$ denote an index that counts instances when the system is unactuated, i.e., feedforward channel is not used, with the corresponding time $t_u = \{t_1, t_2, \ldots, t_{k_2}\}$ (see Fig. 2). The open-loop tracking

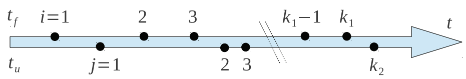

Fig. 2. An example timeline indicating instances when the state feedback is available, t_f, and when the system is unactuated, t_u.

error $e_1(t) \in \mathbb{R}^n$ over each continuous interval of $\{t \cap \{t_f \cup t_u\}^c\}$ is defined as

$$e_1(t) \triangleq \hat{x}(t) - x_d(t) \tag{4}$$

where the desired trajectory $x_d(t)$ and its first time derivative are assumed to be known and bounded (i.e., $x_d(t), \dot{x}_d(t) \in \mathcal{L}_\infty$), and $\{\cdot\}^c$ denotes the complement of the set. The error $e_1(t)$ is piecewise continuous in time t over each continuous interval of $\{t \cap \{t_f \cup t_u\}^c\}$. Thus, an open-loop error system can be obtained by taking the time-derivative of (4) over each interval of $\{t \cap \{t_f \cup t_u\}^c\}$ and substituting (3) in the resulting expression as

$$\dot{e}_1(t) = \phi(\hat{x})\hat{\theta}(t) + g(\hat{x})u(t) - \dot{x}_d(t). \tag{5}$$

Based on the error dynamics in (5) and the subsequent stability analysis, the control input $u(t)$ for the nominal system can be developed as

$$u(t) = g^+(\hat{x})\left(-ke_1(t) - \phi(\hat{x})\hat{\theta}(t) + \dot{x}_d(t) - \frac{e_1(t)\gamma_0^2}{\|e_1(t)\|\gamma_0 + \varepsilon}\right) \tag{6}$$

where $g^+(\hat{x})$ is the Moore-Penrose pseudoinverse of $g(\hat{x})$, $k \in \mathbb{R}$ denotes a positive control gain, and $\varepsilon \in \mathbb{R}$ is a known positive scalar that can be selected arbitrarily small. The robust feedback term included in (6) compensates for the unknown disturbances in (2).

Theorem 1: The control input $u(t)$ in (6) ensures the output of the nominal system globally exponentially tracks the desired trajectory $x_d(t)$ in each continuous interval of $\{t \cap \{t_f \cup t_u\}^c\}$ in the sense that

$$\|e_1(t)\| \leq \zeta_1 \exp\{-\zeta_2 t\} \tag{7}$$

where $\zeta_1, \zeta_2 \in \mathbb{R}$ denote positive bounding constants.

Proof:

Consider a positive definite function $V_1(e_1) \in \mathbb{R}$ defined over each continuous interval of $\{t \cap \{t_f \cup t_u\}^c\}$ as

$$V_1(e_1) = \frac{1}{2}e_1^T(t)e_1(t) \tag{8}$$

$$\lambda_1\|e_1(t)\|^2 \leq V_1(e_1) \leq \lambda_2\|e_1(t)\|^2 \tag{9}$$

where $\lambda_1, \lambda_2 \in \mathbb{R}$ are known positive bounding constants. Since $V_1(e_1)$ is piecewise continuous in time over each interval of $\{t \cap \{t_f \cup t_u\}^c\}$, taking the time-derivative of (8) and substituting (5), (6), and (9), the simplified expression can be obtained as

$$\dot{V}_1(e_1) \leq -e_1^T(t)ke_1(t) \leq -\frac{k}{\lambda_2}V_1(e_1). \tag{10}$$

Using (8)-(10) and the standard signal chasing arguements, the result in (7) can be proved. ■

The control input $u(t)$ in (6) drives the state $\hat{x}(t)$ of the nominal plant along $x_d(t)$ between feedback instances t_f and the instances when the system is unactuated t_u. The stabilizing control input $u(t)$ of the nominal system also drives the actual system in (2). However, at this point we have not proved the stability of the actual system, which will be discussed in the subsequent sections.

V. Adaptive Update Law

The uncertain constant parameter vector $\theta \in \mathbb{R}^m$ in (2) can be identified in a direct adaptive control framework when both the feedback and feedforward communication is available (i.e., 1-1 policy). In contrast to continuous feedback systems, $x(t)$ is available to the controller only at discrete time instances t_f, and the control input $u(t)$ drives the system during t_u^c. Therefore, estimates of an unknown parameter vector θ can be updated at $\{t_f \cap t_u^c\}$. Subsequently, the system model at the estimator and decision maker (see Fig. 1) is updated with the new parameter vector $\hat{\theta}(t)$. The updated nominal dynamics with $\hat{\theta}(t)$ determine the estimated state of the system using (3) until the next feedback is available.

Let $e_2(t) \in \mathbb{R}^n$ be the actual tracking error between $x(t)$ and $x_d(t)$ as

$$e_2(t) \triangleq x(t) - x_d(t). \tag{11}$$

Remark 1: The tracking error $e_2(t)$ is continuous and measurable at the encoder at all times, while it is measurable at the decoder only when the feedback is available, i.e., at t_f.

Taking the time derivative of (11) and substituting (2) and (6), we obtain the closed-loop error dynamics for $e_2(t)$ as

$$\dot{e}_2(t) = \phi(x)\theta + g(x)g^+(\hat{x})\left(-ke_1(t) - \phi(\hat{x})\hat{\theta}(t) + \dot{x}_d(t) - \frac{e_1(t)\gamma_0^2}{\|e_1(t)\|\gamma_0 + \varepsilon}\right) + d(t) - \dot{x}_d(t) \quad \forall t \in \mathbb{R}. \tag{12}$$

Stability of the closed-loop system in (12) with respect to the equilibrium point $e_2(t) = 0$ is analyzed by considering two cases. First, the stability is analyzed at times $\{t_f \cap t_u^c\}$ when the feedback and feedforward signals are available, where a direct adaptive parameter update law is obtained to update the system dynamics. Second, the stability at inter-feedback times $\{t_f^c \cap t_u^c\}$ can be analyzed. The first case is analyzed in the following development, and the case corresponding to stability during the inter-feedback times is considered in Section VI-B.

At time t_f, the state estimates $\hat{x}(t)$ in (12) are re-initialized to the current state $x(t)$ of the system, i.e., $\hat{x}(t) = x(t)\ \forall t \in t_f$. Therefore, the closed-loop dynamics in (12) can be written as

$$\dot{e}_2(t) = -ke_1(t) + \phi(x)\tilde{\theta}(t) - \frac{e_1(t)\gamma_0^2}{\|e_1(t)\|\gamma_0 + \varepsilon} + d(t) \tag{13}$$

where $\tilde{\theta}(t) \in \mathbb{R}^m$ is the parameter estimation error defined as

$$\tilde{\theta}(t) \triangleq \theta - \hat{\theta}(t). \tag{14}$$

Based on the closed-loop error system in (13) and the subsequent stability analysis, the adaptive update law is designed as

$$\dot{\hat{\theta}}(t) = \text{proj}(\Gamma\phi^T(x)e_2(t)) \tag{15}$$

where $\Gamma \in \mathbb{R}^{m\times m}$ is a constant, diagonal, positive definite gain matrix, and the function proj $(\cdot)$ denotes a normal projection algorithm, which ensures that elements $\hat{\theta}_i\,(t)\ \forall\ i = 1,\ldots,m$ of $\hat{\theta}(t)$ are bounded as (for further details see [20], [21])

$$\underline{\theta}_i \leq \hat{\theta}_i(t) \leq \bar{\theta}_i \tag{16}$$

where $\underline{\theta}_i$, $\bar{\theta}_i \in \mathbb{R}$ denote known, constant lower and upper bounds of $\hat{\theta}_i(t)$, respectively.

Theorem 2: The control input in (6), (15), and (16) ensures the state $x(t)$ of the actual system tracks the desired trajectory $x_d(t)$ during $\{t_f \cap t_u^c\}$ such that the tracking error is uniformly ultimately bounded in the sense that

$$\|e_2(t)\| \leq \sqrt{\zeta_3 \exp(-\zeta_4 t) + \zeta_5} \tag{17}$$

where $\zeta_3, \zeta_4, \zeta_5 \in \mathbb{R}$ denote positive bounding constants.

Proof: To prove the stability of the closed-loop system in (13) during $t \in \{t_f \cap t_u^c\}$, a positive definite Lyapunov function $V_2(t) \in \mathbb{R}$ is considered as

$$V_2(t) = \frac{1}{2}e_2^T(t)e_2(t) + \frac{1}{2}\tilde{\theta}^T(t)\Gamma^{-1}\tilde{\theta}(t). \tag{18}$$

Based on (14) and (16), the Lyapunov function in (18) can be upper and lower bounded as

$$\lambda_3 \left\|e_2(t)\right\|^2 + c_1 \leq V_2\,(t) \leq \lambda_4 \left\|e_2(t)\right\|^2 + c_2 \tag{19}$$

where λ_3, λ_4, c_1, $c_2 \in \mathbb{R}$ are known positive constants.

Taking the time derivative of (18) and substituting (13) and (15) in the resulting expression, the upper bound on the Lyapunov derivative can be obtained as

$$\dot{V}_2(t) \leq -k\|e_2(t)\|^2 + \varepsilon \tag{20}$$

The result in (17) can now be directly obtained from (20). Thus, the state $x(t)$ tracks the desired trajectory $x_d(t)$ such that the tracking error is uniformly ultimately bounded during $\{t_f \cap t_u^c\}$, where the bound on the error can be reduced by selecting an arbitrarily small ε.

Remark 2: Since $e_2(t), x(t) \in \mathcal{L}_\infty$, from (7) and the fact that $\hat{x}(t) = x(t)\ \forall t \in t_f$ the state of the nominal system $\hat{x}(t) \in \mathcal{L}_\infty\ \forall t$.

∎

Since $\tilde{\theta}(t) \in \mathcal{L}_\infty$, it is possible to evaluate an upper bound on $\tilde{\theta}(t)$. The parameter vector $\hat{\theta}(t)$ is updated using (15) and (16) at $\{t_f \cap t_u^c\}$, while at $\{t_f^c \cap t_u^c\}$ the update law is $\dot{\hat{\theta}}(t) = 0$. In practice, assuming the instantaneous update law $\dot{\hat{\theta}}(t)$ to be a zero-order hold signal with sample interval of t_s, the parameter vector $\hat{\theta}(t)$ at time t can be obtained as

$$\hat{\theta}(t) = \hat{\theta}(0) + \sum_{\forall z \in t_a} \int_z^{z+t_s} \dot{\hat{\theta}}(\eta)d\eta, \quad t_a = \{t_f \cap t_u^c\} \tag{21}$$

where $\hat{\theta}(0) \in \mathbb{R}^m$ denotes the initial guess of the constant unknown parameter vector θ. For $\tilde{\theta}(0) = \theta - \hat{\theta}(0)$, using (21), the parameter estimation error $\tilde{\theta}(t)$ at time t can be obtained as

$$\tilde{\theta}(t) = \tilde{\theta}(0) - t_s \sum_{\forall z \in t_a} \dot{\hat{\theta}}(z) \tag{22}$$

and let an upper bound on the initial parameter estimation error $\tilde{\theta}(0)$ be

$$\|\tilde{\theta}(0)\|_\infty = \max\{|\tilde{\theta}_1(0)|, |\tilde{\theta}_2(0)|, \ldots, |\tilde{\theta}_m(0)|\} \leq \zeta_0. \tag{23}$$

VI. Context-Aware Communication

The objective of context-aware communication is to stabilize a NCS while reducing the network traffic by analyzing the informational value of the signal for transmission. The decision maker at the output (see Fig. 1) determines a communication policy (0-0, 0-1, 1-0, or 1-1) at each time t that guarantees stability with reduced network traffic, and the corresponding 2-bit beacon signal governs the feedback and feedforward communication channel usage. A hierarchical approach to determine an appropriate communication policy is presented below.

A. 0-0 Policy

The 0-0 communication policy is derived based on the condition that if an unactuated system is stable with respect to an equilibrium point, then network traffic can be reduced for the system in Fig. 1 by not transmitting over the feedback and feedforward channels.

Taking the time derivative of (11) and substituting (2) along with $u(t) = 0$, the error dynamics can be obtained as

$$\dot{e}_2(t) = \phi(x)\theta + d(t) - \dot{x}_d(t). \tag{24}$$

To analyze the stability of the unactuated system in (24), consider a positive definite Lyapunov function $V_3(t) \in \mathbb{R}$ as

$$V_3(t) = \frac{1}{2}e_2^T(t)e_2(t). \tag{25}$$

Taking the time-derivative of (25), and substituting (14), (22) and (24), the upper bound on the Lyapunov derivative can be obtained as

$$\begin{aligned}\dot{V}_3(t) \leq\ & e_2^T(t)\phi(x)\hat{\theta} + \|e_2(t)\|\|\phi(x)\|\zeta_0 \\ & - e_2^T(t)\phi(x)t_s \sum_{\forall z \in t_a} \dot{\hat{\theta}}(z) + \|e_2(t)\|\gamma_0 - e_2^T\dot{x}_d \end{aligned} \tag{26}$$

where the bound defined in (23) and the fact that $\|d(t)\| \leq \gamma_0$ are used. Also, note that $\dot{\hat{\theta}}(z)\ \forall z \in t_a$ is a measurable signal, and hence can be used in designing policies. From (26), the 0-0 policy that guarantees stable operation of an unactuated system with no communication channel usage can be obtained as $\dot{V}_3(t) \leq 0$, i.e., if the RHS ≤ 0 for the inequality in (26) then a beacon signal 0-0 is broadcasted by the decision maker.

B. 0-1 Policy

In (26), when $\dot{V}_3(t) > 0$ the unactuated system can not be guaranteed to be stable, and hence a control input is required to stabilize the system. The objective of 0-1 policy is to develop a feedforward channel usage condition using the control input (6) in the absence of feedback.

Consider a positive definite Lyapunov function $V_4(t) \in \mathbb{R}$ as given in (18). Taking the time derivative of $V_4(t)$ and substituting (12) and (14), we get

$$\begin{aligned}\dot{V}_4(t) &= e_2^T(t)\phi(x)\left(\hat{\theta}+\tilde{\theta}\right) + e_2^T(t)g(x)g^{-1}(\hat{x})\Bigg(-ke_1(t) \\ &\quad -\phi(\hat{x})\hat{\theta}(t) + \dot{x}_d(t) - \frac{e_1(t)\gamma_0^2}{\|e_1(t)\|\gamma_0+\varepsilon}\Bigg) \\ &\quad + e_2^T d(t) - e_2^T\dot{x}_d(t) \qquad (27)\end{aligned}$$

where the fact that $\dot{\hat{\theta}}(t) = 0$ in the absence of feedback is used. Substituting (22) in (27), and using (23) and $\|d(t)\| \leq \gamma_0$, the following upper bound on the Lyapunov derivative can be obtained:

$$\begin{aligned}\dot{V}_4(t) &\leq e_2^T(t)\phi(x)\Bigg(\hat{\theta} - t_s\sum_{\forall z\in t_a}\dot{\hat{\theta}}(z)\Bigg) + \|e_2(t)\|\|\phi(x)\|\zeta_0 \\ &\quad + e_2^T(t)g(x)g^{-1}(\hat{x})\Bigg(-ke_1(t) - \phi(\hat{x})\hat{\theta}(t) + \dot{x}_d(t) \\ &\quad - \frac{e_1(t)\gamma_0^2}{\|e_1(t)\|\gamma_0+\varepsilon}\Bigg) + \|e_2\|\gamma_0 - e_2^T\dot{x}_d(t). \qquad (28)\end{aligned}$$

From (28), the 0-1 policy that guarantees stability of the NCS in (2) using only feedforward communication channel can be obtained as $\dot{V}_4(t) \leq 0$, i.e., if the RHS ≤ 0 for the inequality in (28) then a beacon signal 0-1 is broadcasted by the decision maker.

C. 1-1 Policy

The communication policies developed in Sections VI-A and VI-B can guarantee stability of the NCS while reducing the network traffic. However, at times when the unactuated system as well as the open-loop control system are unstable, both the state feedback and control input are required to render the NCS stable, i.e., feedback and feedforward communication is required.

Therefore, when $\dot{V}_3(t), \dot{V}_4(t) > 0$ the 1-1 policy for network usage can be obtained from (28) as

$$\begin{aligned}&e_2^T(t)\phi(x)\Bigg(\hat{\theta} - t_s\sum_{\forall z\in t_a}\dot{\hat{\theta}}(z)\Bigg) + \|e_2(t)\|\|\phi(x)\|\zeta_0 \\ &+ e_2^T(t)g(x)g^{-1}(\hat{x})\Bigg(-ke_1(t) - \phi(\hat{x})\hat{\theta}(t) + \dot{x}_d(t) \\ &- \frac{e_1(t)\gamma_0^2}{\|e_1(t)\|\gamma_0+\varepsilon}\Bigg) + \|e_2\|\gamma_0 - e_2^T\dot{x}_d(t) > 0. \qquad (29)\end{aligned}$$

Remark 3: The 1-0 policy with only the feedback channel usage is not considered in this work, since it represents sending output measurements over a feedback channel when the control input is not transmitted over the feedforward channel.

	A	B
$\|t_f\| + \|t_u^c\|$	20000	12526
RMSE	$[0.1072 \;\; 0.1221]^T$	$[0.1339 \;\; 0.1392]^T$

TABLE II
COMPARISON BETWEEN A) A CONTINUOUS CLOSED-LOOP SYSTEM AND B) THE PROPOSED NCS TRACKING CONTROLLER IN TERMS OF THE NUMBER OF COMMUNICATION INSTANCES $|t_f| + |t_u^c|$ AND THE ROOT MEAN SQUARE ERROR.

VII. SIMULATION RESULTS

The presented stabilization scheme for nonlinear systems using a context-aware feedback policy is verified using numerical simulations. In this example, we consider a MIMO nonlinear system represented by coupled differential equations as follows:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \underbrace{\begin{bmatrix} x_1\sin(x_1) & x_2^2 & 0 & 0\\ 0 & 0 & x_1^3 & x_2\end{bmatrix}}_{\phi(x)}\underbrace{\begin{bmatrix}a\\ b\\ c\\ d\end{bmatrix}}_{\theta} + \begin{bmatrix}u_1\\ u_2\end{bmatrix} \qquad (30)$$

The actual value of constant parameter vector θ is considered to be $\theta = \begin{bmatrix} 3 & 2 & 1 & 12\end{bmatrix}^T$, while the controller development assumes an initial guess $\hat{\theta}(t) = \begin{bmatrix} 1 & 0.7 & 0.3 & 10\end{bmatrix}^T$ at $t = 0$. The control gain k and adaptation gain Γ are selected to be $k = 10$ and $\Gamma = \text{diag}(240, 210, 100, 90)$, and $\varepsilon = 0.0001$. At time $t = 0$, the state of the system in (2) and (3) is assumed to be $x(t) = \begin{bmatrix} 1.5 & -1.5\end{bmatrix}^T$ and $\hat{x}(t) = \begin{bmatrix} 0 & 0\end{bmatrix}^T$, respectively. The desired trajectory $x_d(t) = \begin{bmatrix} x_{1d}(t) & x_{2d}(t)\end{bmatrix}^T$ is selected to be

$$\begin{bmatrix} x_{1d}(t)\\ x_{2d}(t)\end{bmatrix} = \begin{bmatrix} 0.4\sin(10t)\\ 0.5\cos(t)\end{bmatrix}. \qquad (31)$$

Simulation results are shown in Figs. 3-6. Fig. 3 shows the tracking error $e_1(t) = \begin{bmatrix} e_{11}(t) & e_{12}(t)\end{bmatrix}^T$ between the state $\hat{x}(t)$ of the nominal system and the desired trajectory $x_d(t)$, and Fig. 4 shows the tracking error $e_2(t) = \begin{bmatrix} e_{21}(t) & e_{22}(t)\end{bmatrix}^T$ between the state $x(t)$ of the actual system and the desired trajectory $x_d(t)$. The plot of the predicted and actual Lyapunov derivatives evaluated at time t is shown in Fig. 5. It can be seen that for the unstable system in (30), the Lyapunov derivative $\dot{V}_3(t)$ is positive for most duration, and hence the control input must be sent over the feedforward communication channel. Note that the actual Lyapunov derivative $\dot{V}_2(t) \leq 0 \;\forall t$. Fig. 6 compares the states $x(t)$ and $\hat{x}(t)$ of the actual and nominal plants, respectively, with the desired time-varying trajectory $x_d(t)$.

For the presented MIMO problem, the number of communication instances are observed to be $|t_f| + |t_u^c| = 12526$ for the total number of time steps 20000, i.e., the tracking control objective can be achieved with $\approx 37\%$ reduced communication channel usage. The results are summarized in Table II.

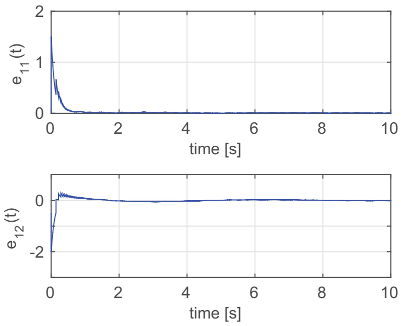

Fig. 3. Tracking error $e_1(t) = [e_{11}(t)\ e_{12}(t)]^T$ between state $\hat{x}(t)$ of the nominal system and the desired trajectory $x_d(t)$.

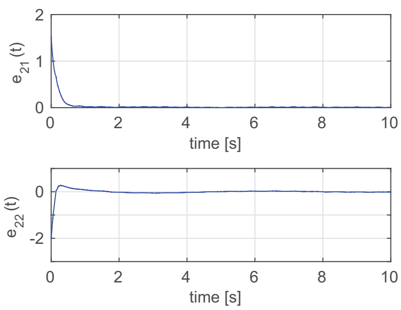

Fig. 4. Tracking error $e_2(t) = [e_{21}(t)\ e_{22}(t)]^T$ between the state $x(t)$ of the actual system and the desired trajectory $x_d(t)$.

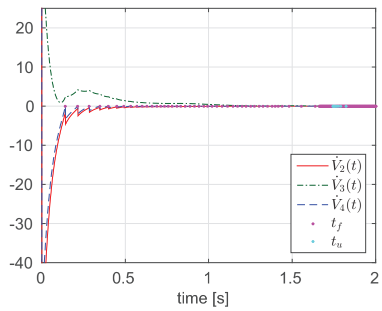

Fig. 5. Comparison between the actual Lyapunov derivative $\dot{V}_2(t)$, and the predicted Lyapunov derivatives $\dot{V}_3(t), \dot{V}_4(t)$, respectively.

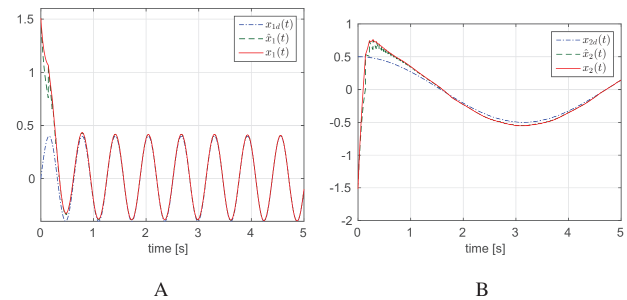

Fig. 6. Plot showing the desired trajectory, actual state, and nominal state.

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