Effective Dynamic Coverage Control for Heterogeneous Driftless Control Affine Systems

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***Abstract*—To dynamically monitor areas of interest by a group of mobile robots, an effective coverage control strategy for heterogeneous driftless control affine systems is developed in this letter. The coverage strategy guides the robot to move in the direction of the effective coverage and escape the saddle point when the loss function stops descending. In addition, the connectivity of the underlying communication network of the robots is maintained during coverage control. Simulation validates the effectiveness of our proposed coverage strategy.**

***Index Terms*—Cooperative control, distributed control, networked control systems.**

I. Introduction

COVERAGE control has been wildly used in exploration, search and rescue, wireless networks, and other engineering fields. Coverage control problems can be divided into static coverage control and dynamic coverage control. Static coverage control focuses on solving the optimal sensor placement problem [1]–[4], while dynamic coverage control aims at using a group of mobile robots to cover a given workspace over time [5]–[10].

For the optimal sensor placement problem, however, due to the limited number of sensors, there is no guarantee that sensors can completely cover the entire wireless network, especially if a large-scale network or a dense network is considered. To address this challenge, a limited number of mobile sensory robots tasked with the objective of dynamically monitoring areas of interest is considered, e.g., [11]. This kind of problem belongs to the effective dynamic coverage control problem.

Effective dynamic coverage control is a complex task that usually requires multi-robots to cooperate with each other. The single integrator model is usually adopted due to its simplicity [12], [13]. Other research adopts nonholonomic dynamic models, e.g., the unicycle model in [8], dubins dynamics for autonomous underwater vehicles (AUVs) in [14], [15]. However, among robots with different dynamics, the control strategies are not universal, that is, the control law may not be applicable to robots with different dynamics. The letter of [16], [17] considered heterogeneous agents, however; the results rely on a fully connected network, which might not be practical, especially if agents with limited communication capability are considered.

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In this letter, we generalize the aforementioned models (e.g., single integrator model, unicycle model, dubins model) into a more general heterogeneous driftless control affine model and propose an effective coverage strategy. The overall coverage strategy guides the robots to move in the direction of the effective coverage and escapes the saddle point when the robots are trapped without completing the coverage task. In addition, the connectivity of the underlying communication network of the robots is maintained during coverage control.

The saddle point is a common issue in coverage control. As stated in [18], gradient-based controllers can only guarantee the convergence to a local minimum. In case of a loss function, the local minimum could become the local saddle point. For example, [12] provides a control strategy that smoothly transitions between control modes to avoid trapping by saddle points.

The main contributions of this letter are as follows:

- Robots with general heterogeneous dynamics are considered in our proposed coverage control strategy. The control strategy is for heterogeneous driftless control affine systems, which can be extended to many dynamical systems.
- The coverage control strategy ensures the robots to dynamically monitor a workspace by making the robot move in the direction of the effective coverage and escape the saddle point until the workspace is completely covered.
- Due to the consideration of limited communication capability, the proposed coverage control strategy ensures that the network connectivity of the robots is maintained throughout the coverage task.

The remainder of the letter is organized as follows: Section II formulates the problem. Section III derives the results of effective coverage strategy and Section IV gives out the results of maintaining the inter-communication among robots. Section V shows the simulation results. Section VI summarizes this letter and discusses future directions.

II. Problem Formulation

Consider M spots (e.g., areas of interest to be monitored) deployed in a two-dimension compact workspace W. In this letter, these M spots are modeled by a node set $V_T = \{1, \ldots, M\}$, and let $p_j \in \mathbb{R}^2$, $j \in \{1, \ldots, M\}$, denote the position of the j^{th} node.

Consider N heterogeneous robots tasked to dynamically monitor the M areas of interest in the workspace. Suppose the robots are driftless control-affine systems following the dynamics [19]

$$\dot{x}_i = G_i(x_i)u_i, \; i = 1, 2, \ldots, N, \tag{1}$$

with smooth functions $G_i(x_i) : \mathbb{R}^{n_i} \to \mathbb{R}^{n_i \times m_i}$, where $x_i \in \mathbb{R}^{n_i}$ is the state of the i^{th} robot and $u_i \in \mathbb{R}^{m_i}$ is the control of the i^{th} robot. Let $p_{x,i}(t) = H_i(x_i(t))$ be the position of the i^{th} robot at time t, where $H_i(x_i) : \mathbb{R}^n \to \mathbb{R}^2$ maps $x_i \in \mathbb{R}^n$ to the Cartesian coordinate system.

In this letter, each robot is assumed to have a limited circular coverage zone S_i with a sensing radius $r \in \mathbb{R}^+$. Inspired by [20], a second-order polynomial function is adopted to describe the coverage intensity of robots

$$J_i(p_{x,i}(t), p_j) = \begin{cases} \frac{M_p}{r^4}(s_{ij}(t) - r^2)^2, & s_{ij} \le r^2, \\ 0, & s_{ij} > r^2, \end{cases} \tag{2}$$

where $s_{ij}(t) \triangleq \|p_{x,i}(t) - p_j\|^2$ and $M_p \in \mathbb{R}^+$ is the peak coverage intensity. The coverage intensity reaches the peak M_p when the robot coincides with the node and degrades as the distance between the robot and the node increases. The coverage intensity vanished when the node is out of the coverage zone.

Let $\phi_i(t; 0, x_i(0))$ denote the trajectory of the i^{th} robot within a time interval $[0, t]$ under the influence of controller u_i in (1) from an initial state $x_i(0)$. The j^{th} node's accumulated coverage intensity imposed by the i^{th} robot Q_{ij} along the trajectory $\phi_i(t; 0, x_i(0))$ over $[0, t]$ is defined as

$$Q_{ij}(\phi_i, p_j) \triangleq \int_0^t J_i(p_{x,i}(\tau)), p_j)d\tau, \; \forall i \in V, j \in V_T. \tag{3}$$

The time derivative of (3) is

$$\dot{Q}_{ij} = J_i(p_{x,i}(t)), p_j). \tag{4}$$

For the j^{th} node, its accumulated coverage imposed by all robots is simply the sum of Q_{ij} for all robots, which is

$$Q_{V,j}(\phi_1, \ldots, \phi_n, p_j) \triangleq \int_0^t \sum_{i \in V} J_i(p_{x,i}(\tau), p_j))d\tau. \tag{5}$$

The time derivative of (5) is

$$\dot{Q}_{V,j} = \sum_{i \in V} J_i(p_{x,i}(t)), p_j). \tag{6}$$

The inter-communication among robots is modeled by an undirected graph $\mathcal{G} = (V, \Sigma)$, where V represents the collection of robots and Σ represents the inter-communication among robots. It is assumed that two robots can exchange information (e.g., robot position, nodes' status) if they are within the communication radius R. Other communication constraints, such as delay and package loss, are not considered in this letter. Let $N_i = \{k \in V | (i, k) \in \Sigma\}$ be the neighbors of the i^{th} robot, where (i, j) denotes the communication link between i^{th} and j^{th} robot. The graph $\mathcal{G}$ is connected if the inter-communication among robots holds. It is assumed that the initial graph $\mathcal{G}(0)$ is connected.

Due to the locations and roles of nodes in V_T, different nodes have different importance. The expected coverage intensity $Q_j^* \in \mathbb{R}^+$ of the j^{th} node indicates how much coverage intensity the robots should impose on this node. If $Q_{V,j} \ge Q_j^*$, it means that the j^{th} node is effectively covered. Effective coverage of all nodes in V_T is accomplished if and only if $Q_{V,j} \ge Q_j^*, \; \forall j \in V_T$.

In the present work, it is assumed that $\forall j \in V_T$, p_j and its expected coverage intensity Q_j^* are globally known, that is, all robots are aware of the positions of the nodes and their expected coverage intensity. The goal of the letter is to develop distributed control laws for heterogeneous driftless control affine robots so as to dynamically cover the nodes in V_T up to a predefined coverage level over time. To accomplish this task, we design the loss function

$$e_1(t) = \sum_{j \in V_T} h(Q_j^* - Q_{V,j}), \tag{7a}$$

$$h(w) = (\max\{0, w\})^3, \tag{7b}$$

where $h(w)$ is non-negative and twice differentiable with $h'(w) = 3(\max\{0, w\})^2$ and $h''(w) = 6 \max\{0, w\}$. It reflects the gap between the accumulated coverage intensity and the expected coverage intensity at all nodes. In (7), $e_1(t) = 0$ indicates that the accumulated coverage intensity of each node has reached the expected coverage intensity Q_j^* over $[0, t]$, i.e., the coverage task is completed.

III. Effective Coverage Strategy

A. Rapid Descend

From (6) the time derivative of $e_1(t)$ is

$$\dot{e}_1(t) = -\sum_{j \in V_T} h'(Q_j^* - Q_{V,j}) \sum_{i \in V} J_i(p_{x,i}(t), p_j)), \tag{8}$$

where $h'(Q_j^* - Q_{V,j}) \ge 0$, $\sum_{i \in V} J_i(p_{x,i}(t), p_j)) \ge 0$, and thus $\dot{e}_1(t) \le 0$. In fact, as long as there is at least one uncovered node inside the robots' coverage zone, any input $U = \{u_1, u_2, \ldots, u_n\}$ can lead to the decrease of the loss function and thus effective coverage. We can see from (8) that u_i is not involved in $\dot{e}_1(t)$. We turn to study the second time derivative of $e_1(t)$

$$\begin{aligned} \ddot{e}_1(t) = &\sum_{j \in V_T} h''(Q_j^* - Q_{V,j}) \left(\sum_{i \in V} J_i(p_{x,i}(t), p_j) \right)^2 \\ &- 2 \sum_{i \in V} \sum_{j \in V_T} h'(Q_j^* - Q_{V,j}) \\ &\times \frac{\partial J_i}{\partial s_{ij}} (p_{x,i}(t) - p_j)^T \frac{dH_i}{dx_i} \dot{x}_i. \end{aligned} \tag{9}$$

Substituting the dynamics in (1) into (9), we have

$$\begin{aligned}\ddot{e}_1(t) &= \sum_{j\in V_T} h''(Q_j^* - Q_{V,j})\left(\sum_{i\in V} J_i(p_{x,i}(t), p_j)\right)^2 \\ &- 2\sum_{i\in V}\sum_{j\in V_T} h'(Q_j^* - Q_{V,j}) \\ &\times \frac{\partial J_i}{\partial s_{ij}}(p_{x,i}(t) - p_j)^T \frac{dH_i}{dx_i} G_i(x_i)u_i. \end{aligned} \tag{10}$$

To allow fast converge of e_1, based on (10), the control input is designed as

$$\begin{aligned} u_i = \alpha \sum_{j\in V_T} h'(Q_j^* - Q_{V,j})\frac{\partial J_i}{\partial s_{ij}} \\ \times \left((p_{x,i}(t) - p_j)^T \frac{dH_i}{dx_i} G_i(x_i)\right)^T, \end{aligned} \tag{11}$$

where $\alpha \in \mathbb{R}^+$. Then we have

$$\begin{aligned}\ddot{e}_1(t) &= \sum_{j\in V_T} h''(Q_j^* - Q_{V,j})\left(\sum_{i\in V} J_i(p_{x,i}(t), p_j)\right)^2 \\ &- 2\alpha \sum_{i\in V} \|u_i\|^2. \end{aligned} \tag{12}$$

B. The Saddle Point of the Loss Function

Let T be the moment when the loss function e_1 stops descending, i.e., $\dot{e}_1(T) = 0$, and $\dot{e}_1(t) < 0$, $\forall t < T$. We have,

$$\sum_{j\in V_T} h'(Q_j^* - Q_{V,j})J_i(p_{x,i}(T), p_j) = 0, \ \forall i \in V, j \in V_T, \tag{13}$$

where $h'(Q_j^* - Q_{V,j})J_i(p_{x,i}(T), p_j) \geq 0$. This indicates that each term in (13) equals 0. So, we have,

$$h'(Q_j^* - Q_{V,j})J_i(p_{x,i}(T), p_j) = 0, \ \forall i \in V, j \in V_T. \tag{14}$$

This indicates that if the j^{th} node is in the coverage zone of the i^{th} robot where $J_i(p_{x,i}(T), p_j) \neq 0$, we have $h'(Q_j^* - Q_{V,j}) = 0$, which means that the j^{th} node has been effectively covered. As a result, the nodes within the coverage zone of the robots have been effectively covered at time T. For the nodes outside of the coverage zone of the robots we have $J_i = 0$, so the value of $h'(Q_j^* - Q_{V,j})$ can not be determined, which indicates that these nodes require to be further covered. To conclude, the control input u_i in (11) can yield $\dot{e}_1 \to 0$, but it does not necessarily guarantee that $e_1 \to 0$.

It can also be inferred that $u_i = 0$, $\forall i \in V$. So, the system is now "trapped" at a saddle point: all robots remain stationary with their nearby nodes being effectively covered, while the other nodes remain to be further covered. Therefore, new inputs $\bar{U} = \{\bar{u}_1, \bar{u}_2, \dots, \bar{u}_n\}$ need to be considered. As long as the robot can cover other nodes that are not effectively covered, the loss function will continue to decrease.

The general control strategy is then designed as follows. To let $e_1(t)$ descend rapidly, the robots first adopt control strategy $U = \{u_1, u_2, \dots, u_n\}$. When e_1 stops descending, robots adopt $\bar{U} = \{\bar{u}_1, \bar{u}_2, \dots, \bar{u}_n\}$ to escape the saddle point by letting each robot find the closest not-fully-covered node and move to it. As soon as the robots move to their nearest not-fully-covered nodes, robots return to control strategy U again.

To this end, we let each robot find the closest node and move to it.

Theorem 1: Consider

$$\begin{aligned}\Psi \triangleq \{x_i \in \mathbb{R}^2, \forall i \in V | p_{x,i} = p_{d(i)}, \\ d(i) = \min_{d\in V_T} \|p_{x,i} - p_{d(i)}\|_2\}, \end{aligned} \tag{15}$$

where $p_{x,i}$ is the initial postion of the i^{th} robot, $d(i)$ is the nearest not-fully-covered node around the i^{th} robot and $p_{d(i)}$ is the position of this node. The following control strategy

$$\bar{u}_i = -\alpha_2((P_{d(i)} - p_{x,i})^T \frac{dH_i}{dx_i} G_i)^T, \tag{16}$$

can guide the i^{th} robot with driftless control affine dynamics to gradually approach Ψ, where $\alpha_2 \in \mathbb{R}^+$.

Proof: Consider a Lyapunov function candidate

$$V_c(t) \triangleq \sum_{i\in V} \frac{1}{2}(p_{d(i)} - p_{x,i})^2, \tag{17}$$

where $V_c(t) \geq 0$, and $V_c(t) = 0$ if and only if $p_{x,i} = p_{d(i)}$. Taking the time derivative of $V_c(t)$

$$\dot{V}_c(t) = \sum_{i\in V,d} (p_{d(i)} - p_{x,i})^T \frac{dH_i}{dx_i}\dot{x}_i. \tag{18}$$

Substituting the dynamics in (1) and controller $\bar{u}_i$ in (16) into (18) yields

$$\dot{V}_c(t) = -\alpha_2 \sum_{i\in V,d} \|(p_{d(i)} - p_{x,i})^T \frac{dH_i}{dx_i} G_i\|_2^2. \tag{19}$$

As a result, $\dot{V}_c(t) \leq 0$. If $\dot{V}_c(t) = 0$, it must have $p_{x,i} = p_{d(i)}$, which means that the i^{th} robot reaches the position of the $d(i)^{th}$ node, and the loss function will continue to decrease. The largest invariant set of $\dot{V}_c(t) = 0$ is Ψ in (15). According to the LaSalle's invariance principle [21], the system will converge to the set Ψ. ■

Remark 1: As long as the system converges to Ψ, it means that robots have arrived at their nearest not-fully-covered nodes $d(i)$, $\forall i \in V$. These nodes can be covered since they are inside the robots' coverage zone, ensuring the loss function continue to descend. If the closest not-fully-covered nodes happen to be the same, the collision of the robots could occur. In our previous works [22], [23], collision avoidance among robots was considered. Ongoing research considers extending the results in [22], [23] to dynamic coverage control.

IV. Communication Between Robots

While the coverage tasks can be accomplished by control strategies u_i in (11) and $\bar{u}_i$ in (16), we still need to maintain the inter-communication among robots, since our solution is based on the assumption that the communication network is connected. In this section, we will design the control strategy that can accomplish the coverage task and maintain the communication network connectivity. We need to maintain the inter-communication Σ while ensuring $e_1(t) \to 0$ as $t \to \infty$.

For the i^{th} robot, consider a circular escape region, which is defined as an outer ring with the radius R, where $r_0 - \delta < R < r_0$ and $\delta \in \mathbb{R}^+$ is the length of the predefined escape region. In order to ensure the inter-communication between the i^{th} robot and the k^{th} robot, consider the penalty function

$$C_{ik}(x_i, x_k) \triangleq \left(min\{0, \frac{d_{ik} - (R-\delta)^2}{d_{ik} - R^2}\} \right)^2, \tag{20}$$

where $d_{ik} \triangleq \|p_{x,i} - p_{x,k}\|^2$. The non-negative function $C_{ik} \to \infty$ as $d_{ik} \to R^2$ and $C_{ik} \to 0$ as $d_{ik} \to (R-\delta)^2$. Taking the partial derivation of C_{ik} yields

$$\frac{\partial C_{ik}}{\partial x_i} \triangleq \begin{cases} 0, & d_{ik} \leq (R-\delta)^2, \\ \Theta, & (R-\delta)^2 < d_{ik} < R^2, \\ undefined, & d_{ik} = R^2, \end{cases} \tag{21}$$

where

$$\Theta \triangleq \frac{4(d_{ik} - (R-\delta)^2)((R-\delta)^2 - R^2)(p_{x,i} - p_{x,k})^T \frac{dH_i}{dx_i}}{(d_{ik} - R^2)^3}. \tag{22}$$

To make $C_{ik}(x_i, x_k)$ in each time step as small as possible, the local loss function is designed as

$$l_1(t) \triangleq \sum_{i \in V} \sum_{k \in N_i} \int_0^t \left(min\{0, \frac{d_{ik} - (R-\delta)^2}{d_{ik} - R^2}\} \right)^2 dt. \tag{23}$$

According to (7) and (23), the whole loss function is defined as

$$\begin{aligned} E_1(t) &\triangleq e_1(t) + l_1(t) \\ &= \sum_{j \in V_T} h(Q_j^* - Q_{V,j}) + \sum_{i \in V} \sum_{k \in N_i} \int_0^t C_{ik}(x_i, x_k) dt, \end{aligned} \tag{24}$$

where N_i is the neighbors of the i^{th} robot in $\mathcal{G}$. According to (7) - (9) and (20) - (22), the first and second time derivative of $E_1(t)$ are

$$\begin{aligned} \dot{E}_1(t) = &- \sum_{j \in V_T} h'(Q_j^* - Q_{V,j}) \left(\sum_{i \in V} J_i(p_{x,i}(t), p_j) \right) \\ &+ \sum_{i \in V} \sum_{k \in N_i} C_{ik}(x_i, x_k), \end{aligned} \tag{25}$$

$$\begin{aligned} \ddot{E}_1(t) = &\sum_{j \in V_T} h''(\sum_{i \in V} J_i)^2 - 2 \sum_{i \in V} \sum_{j \in V_T} h' \\ &\times \frac{\partial J_i}{\partial s_{ij}} (p_{x,i}(t) - p_j)^T \frac{dH_i}{dx_i} G_i(x_i) u_i \\ &+ \sum_{i \in V} \sum_{k \in N_i} \frac{\partial C_{ik}}{\partial x_i} G_i u_i. \end{aligned} \tag{26}$$

Let

$$u_i = u_{1,i} + u_{2,i}, \tag{27}$$

$$\begin{aligned} u_{1,i} = &\alpha \sum_{j \in V_T} h'(Q_j^* - Q_{V,j}) \frac{\partial J_i}{\partial s_{ij}} \\ &\times \left((p_{x,i}(t) - p_j)^T \frac{dH_i}{dx_i} G_i(x_i) \right)^T, \end{aligned} \tag{28}$$

$$u_{2,i} = -\beta \sum_{k \in N_i} \left(\frac{\partial C_{ik}}{\partial x_i} G_i \right)^T, \tag{29}$$

where $\alpha, \beta \in \mathbb{R}^+$, $u_{1,i}$ is used to cover the spots of interests and $u_{2,i}$ is used to maintain the communication network connectivity. So, we have

$$\begin{aligned} \ddot{E}_1(t) = &\sum_{j \in V_T} h''(Q_j^* - Q_{V,j}) \left(\sum_{i \in V} J_i(p_{x,i}(t), p_j) \right)^2 \\ &- \sum_{i \in V} 2/\alpha u_{1,i}^T u_i - \sum_{i \in V} 1/\beta u_{2,i}^T u_i \\ = &\sum_{j \in V_T} h'' \left(\sum_{i \in V} J_i \right)^2 \\ &- \sum_{i \in V} (2/\alpha u_{1,i} + 1/\beta u_{2,i})^T (u_{1,i} + u_{2,i}). \end{aligned} \tag{30}$$

Provided that the tuning parameters satisfy $\alpha = 2\beta$,

$$\ddot{E}_1(t) = \sum_{j \in V_T} h'' \left(\sum_{i \in V} J_i \right)^2 - 2/\alpha \sum_{i \in V} \|u_{1,i} + u_{2,i}\|^2. \tag{31}$$

Therefore, we can make $E_1(t)$ descend rapidly and reduce as much as possible. The control strategy in (28) makes $e_1(t)$ drop rapidly, while the control strategy in (29) maintains the communication network connectivity.

When the robots escape the saddle point, conservation of the inter-communication among robots is also required.

Theorem 2: Consider

$$\Phi \triangleq \{d_{ik} < (R-\delta)^2, \forall (i,k) \in \Sigma\}.$$

The following control strategies

$$\bar{u}_i = \bar{u}_{1,i} + \bar{u}_{2,i}, \tag{32}$$

$$\bar{u}_{1,i} = \alpha_2 \left((P_{d,i} - p_{x,i})^T \frac{dH_i}{dx_i} G_i \right)^T, \tag{33}$$

$$\bar{u}_{2,i} = -2\alpha_2 \sum_{k \in N_i} \left(\frac{\partial C_{ik}}{\partial x_i} G_i \right)^T. \tag{34}$$

can make the i^{th} robot of the driftless control affine system gradually approach $\Psi \cap \Phi$, where $\alpha_2 \in \mathbb{R}^+$, $\bar{u}_{1,i}$ is used to escape the saddle point, and $\bar{u}_{2,i}$ is used to maintain the network connectivity.

Proof: Consider a Lyapunov function candidate

$$V = V_c + V_p, \tag{35}$$

where V_c is from (17) and V_p is defined as

$$V_p(x(t)) \triangleq \sum_{i \in V} \sum_{k \in N_i} C_{ik}(x_i, x_k). \tag{36}$$

According to (17) - (18) and (20) - (22), the time derivation of V is

$$\begin{aligned} \dot{V} &= \dot{V}_c + \dot{V}_p \\ &= - \sum_{i \in V, d} \frac{1}{\alpha_2} \bar{u}_{1,i}^T (\bar{u}_{1,i} + \bar{u}_{2,i}) \\ &\quad - \sum_{i \in V, d} \frac{1}{2\alpha_2} 2\bar{u}_{2,i}^T (\bar{u}_{1,i} + \bar{u}_{2,i}) \\ &= -\frac{1}{\alpha_2} \sum_i \|\bar{u}_{1,i} + \bar{u}_{2,i}\|^2. \end{aligned} \tag{37}$$

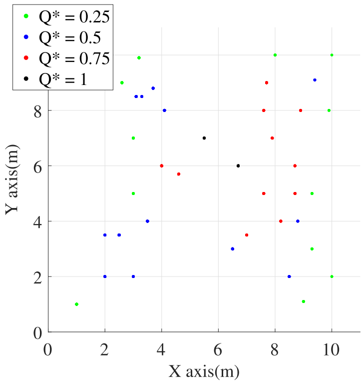


Fig. 1. Deployment of nodes in the simulation.

TABLE I
PARAMETERS

Parameters	Value
Workspace (W)	$11m \times 11m$
Coverage Radius r	$1.5m$
Peak Coverage Value M_p	0.5
Communication Radius R	$6m$
Length Of The Predefined Escape Region δ	$2m$
Time Step Δt	$0.01s$

This means that $\dot{V} \leq 0$. When the communication between the i^{th} and k^{th} robots is disconnected, $C_{ik} \to \infty$. The bounded V means that the inter-communication of $\mathcal{G}$ are maintained. $\bar{u}_{1,i} = 0$ if and only if the system converges to Ψ. $\bar{u}_{2,i} = 0$ if and only if the system converges to Φ. Therefore, the system will converge to $\Psi \cap \Phi$, that is, each robot can reach the location of the nearest uncovered node and can also maintain the inter-communication among robots. ■

Remark 2: In a nutshell, the controller for each robot adopts strategies in (27) - (29), which can ensure the loss function to descend. When the loss function is in the saddle point, the controller changes to strategies in (32) - (34), which makes the loss function escape the saddle point, then the controller return to the strategies in (27) - (29). The whole process ends when the coverage task is accomplished, i.e., $e_1(t) = 0$.

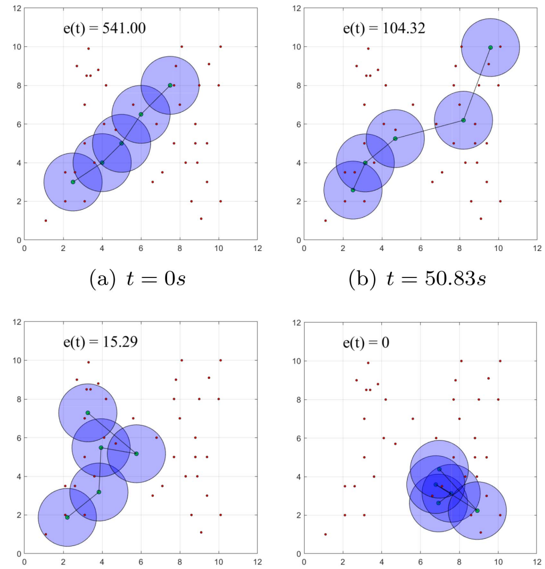
V. SIMULATION

In this section, simulations are performed to support the effectiveness of the proposed control strategy. The parameters used are listed in Table I. Five robots are tasked to monitor 38 nodes in the workspace, among which two robots adopt the single-integrator model while the other three adopt the unicycle model, that is:

$$\dot{x}_i = \dot{p}_{x,i} = u_i, i = 1, 2, \tag{38}$$

$$\dot{x}_i = \begin{bmatrix} \dot{p}_{x,i} \\ \dot{\omega}_i \end{bmatrix} = \begin{bmatrix} \cos\omega_i & 0 \\ \sin\omega_i & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_i \\ \dot{\omega}_i \end{bmatrix}, \quad i = 3, 4, 5, \tag{39}$$

where v_i and $\dot{\omega}_i$ are the linear and angular velocity of the i^{th} robot. All robots have the same coverage radius, peak coverage value and communication radius. The inter-communication among them are $\Sigma = \{(1, 2), (2, 3), (3, 4), (4, 5)\}$.



(a) $t = 0s$ (b) $t = 50.83s$ (c) $t = 101.65s$ (d) $t = 152.47s$

Fig. 2. Simluation results.

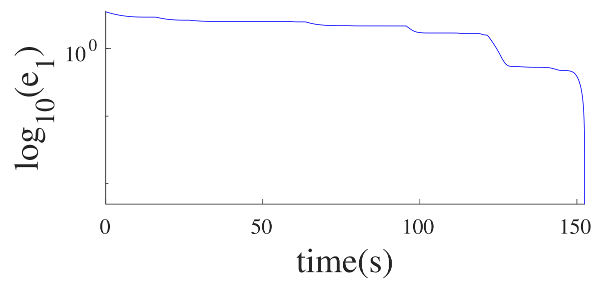


Fig. 3. The evolution of loss function $e_1(t)$.

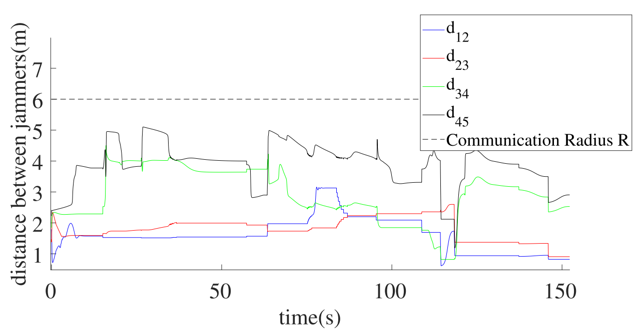


Fig. 4. The evolution of the inter-distance between connected robots.

The deployment of nodes is shown in Fig. 1, the color of the node denotes the corresponding expected coverage intensity.

Fig. 2 (a)–(d) show the snapshots of the simulation process at $t = 0s$, $t = 51s$, $t = 102s$, and $t = 152s$, respectively. The red and green dots represent the nodes to be monitored and mobile robots, respectively. The coverage zones are denoted by purple circles, and the black lines between robots are the inter-communication among robots.[1]

[1] A simulation video can be seen at: https://drive.google.com/file/d/19P3Lo4KTNMzZWgbPOJuL1ab470otSQFs/view?usp=sharing.

Fig. 3 shows the evolution of the loss function $e_1(t)$. Every time the system reaches the saddle point, the loss function $e_1(t)$ stops to descend. By adjusting the control strategy, the descend of the loss function is ensured. Finally, the loss function reaches its minimum, which means that the coverage task is complete.

In Fig. 4, the d_{ij} denotes the distance between the i^{th} and the j^{th} robot, which shows that the distances of the connected robots never go beyond the communication radius R and the inter-communication among robots is preserved.

VI. Conclusion

An effective coverage control strategy for heterogeneous driftless control affine systems is developed to dynamically monitor areas of interest. During the coverage process, by letting robots move to the direction where the loss function descends fast and repeatedly escapes the saddle point, the loss function is assured to descend to the minimum 0, which means the workspace is dynamically monitored. The control structure also guarantees to maintain the inter-communication among robots. Simulation supports the effectiveness of the proposed control strategy. Future work will consider collision avoidance during coverage control.

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