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Control of a Robot Interacting with an Uncertain Viscoelastic Environment with Adjustable Force Bounds

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Abstract— **This paper focuses on developing a neural network (NN) based force limiting controller for a robot interacting with an uncertain Hunt-Crossley viscoelastic environment. The proposed controller consists of a bounded NN term and saturated feedback which limits the control force to a known bound, which can be changed by adjusting the feedback gains. The Lyapunov-based controller, dependent only on position and velocity measurements, is shown to guarantee global uniformly ultimately bounded (GUUB) stability of the system despite uncertainties in the robot and the viscoelastic environment.**

I. Introduction

Recent advances in robotics have paved the way for robots to be used in close proximity with humans, so much so that there has been a paradigm shift from industrial robot applications to assistive, therapeutic robotics. These physically interactive robots offer great advantages to the clinician in terms of facilitating patient recovery [1], [2] e.g., gait rehabilitation for stroke patients. A primary concern in the use of these robots is that of safety when physically interacting with humans. The control development in this paper focuses on ensuring a stable contact transition while imposing user-defined bounds on the control actuation.

The problem of controlling a robot transitioning from a non-contact state to a contact state has been studied in great detail over the past two decades [3]–[10]. Capturing the phenomenon at contact in a physically consistent contact model [11]–[13] is a first step to designing an efficient controller to deal with the problem of destabilizing impact forces in an uncertain environment. In our previous works [7], [8], [14], we investigated the problem of controlling the robot contact transition with stiff environments and developed continuous controllers with guaranteed stability. The developed controllers obviated the need for force or acceleration measurements, which can be noisy and have been shown to cause contact instability [15]. Motivated by the need to control physical interaction of a robot with a human, we recently shifted our focus to examining the problem of controlling robot interaction with soft compliant environments. A continuous nonlinear contact model proposed by Hunt and Crossley [12], which was found to be suitable for modeling viscoelastic contacts [16], was used in our previous adaptive [17] and NN [9] control schemes to yield global uniform ultimately bounded stability in the presence of model uncertainties. This paper builds on our previous work [9], [17], [18], where our additional objective is to ensure that the control force remains within some prescribed limits imposed by the user.

A simplistic approach to bound the control force is to artificially saturate it when it reaches the actuator limits. However, this may lead to undesirable effects such as oscillatory transients and integral windup. A more rigorous approach is to use smooth saturation functions to saturate the terms of the controller and prove stability of the closed loop system. Most of the earlier work on designing robot controllers with bounded control inputs focussed on using the exact knowledge of the robot dynamic model. Kelly et al. [19] used energy shaping plus damping injection methodology to develop a general class of globally asymptotically stable regulators with bounded controls. Loria and Nijmeijer [20] extended the work by proposing an output feedback tracking controller with bounded control inputs, which used a high gain observer to increase the domain of attraction. Santibanez and Kelly [21] designed a global regulator with a saturated proportional derivative (SP-D) feedback plus gravity feedforward compensation. In [22], Colbaugh et al. showed the drawbacks of using exact model knowledge gravity compensation and full state feedback, and proposed an output feedback regulator with saturated PD and an adaptive feedforward term to ensure global convergence of the error. Laib [23] used an adaptive feedforward term to account for gravity and a hyperbolic tangent function to saturate the PD term. Dixon et al. [24] and Zergeroglu et al. [25] proposed full state feedback semi-global tracking controllers with bounded torque inputs. Recently, Ramirez [26] proved the stability of saturated PID regulators using single perturbation analysis. In [27], Dixon addressed the problem of designing controllers for amplitude limited robot manipulators in the presence of kinematic and dynamic uncertainty.

The problem addressed in this paper is two-fold: to control the robot contact transition and to limit the control force during contact. These two-fold objectives, which can often be conflicting, complicate the problem of robot contact transition. In previous saturated control literature, the objective is to control a robot to track a desired trajectory in the presence of bounded control inputs, where the desired trajectory is typically predesigned to be smooth and bounded. In contrast, the objective in this research is to control a robot to interact with its environment, an unactuated dynamic system, and to regulate it to a desired position. The desired robot

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trajectory in this case is generated online using Lyapunov based backstepping methods. Considering that the robot environment is uncertain and there is limited control force, proving stability of such a system poses an interesting control challenge. A NN feedforward term with known projection bounds and a PD feedback term, saturated with hyperbolic tangent function, is used as a force control input. Previous results on adaptive control with saturated control inputs used a feedforward adaptive term whose bounds were dependent on the desired trajectory, thus restricting the selection of desired robot trajectories to those which would ensure that the control force remain within prescribed limits. The controller designed in this paper overcomes that problem by using a feedforward NN term which has a known constant bound based on projection [28]. The PD feedback term is saturated using a hyperbolic tangent function. This allows us to ensure that the control force remains bounded outside of the stability analysis and that the bounds are known a priori and independent of the desired trajectory. A priori knowledge of the control force is useful, more so in a rehabilitation type application where limited control actuation is critical in ensuring safety. However, saturating the terms in the controller obscures a pure feedback term in the closed loop system, making it difficult to prove stability. The innovative use of an auxillary filter [29], made it possible to inject a stabilizing feedback term in the closed loop system, enabling us to prove GUUB stability of the robot and its environment.

II. Dynamic System and Properties

A rigid two-link revolute robot in contact with a compliant viscoelastic environment can be represented by a mass-spring robot (MSR) system, with the following dynamic model

$$M\left(x_r\right)\ddot{x}_r + C\left(x_r, \dot{x}_r\right)\dot{x}_r + h(x_r) + \begin{bmatrix} F_m \\ 0 \end{bmatrix} = F \quad (1)$$

$$m\ddot{x}_m + k_s(x_m - x_0) = F_m. \quad (2)$$

In (1), $x_r(t)$, $\dot{x}_r(t)$, $\ddot{x}_r(t) \in \mathbb{R}^2$ represent the planar Cartesian position, velocity, and acceleration of the robot end-effector respectively, $M(x_r) \in \mathbb{R}^{2\times 2}$ represents the uncertain inertia matrix, $C(x_r, \dot{x}_r) \in \mathbb{R}^{2\times 2}$ represents the uncertain centripetal-Coriolis matrix, $h(x_r) \in \mathbb{R}^2$ represents uncertain conservative forces (e.g., gravity), $F_m(t) \in \mathbb{R}$ denotes the interaction force between the robot and the mass during impact, and $F(t) \in \mathbb{R}^2$ represents the force control input. In (2), $x_m(t)$, $\ddot{x}_m(t) \in \mathbb{R}$ represent the displacement and acceleration of the unknown viscoelastic mass $m \in \mathbb{R}$, $x_0 \in \mathbb{R}$ represents the initial undisturbed position of the mass, and $k_s \in \mathbb{R}$ represents the unknown stiffness of the spring connected to the mass. The contact occurs when the horizontal position of the robot $x_{r1}(t)$ is greater than or equal to the position of the viscoelastic material $x_m(t)$ (i.e., when $x_{r1}(t) \geq x_m(t)$). The interaction force $F_m(t)$ denotes the Hunt-Crossley force, and is defined as [12]

$$F_m \triangleq \lambda\delta^n + b\dot{\delta}\delta^n. \quad (3)$$

In (3), $\lambda \in \mathbb{R}$ is the unknown contact stiffness of the viscoelastic mass, $b \in \mathbb{R}$ is the unknown impact damping coefficient, and $\delta(t) \in \mathbb{R}$ denotes the local deformation of the material and is defined as

$$\delta \triangleq \begin{cases} 0 & x_{r1} < x_m \\ x_{r1} - x_m & x_{r1} \geq x_m \end{cases}. \quad (4)$$

In (3), $\dot{\delta}(t)$ is the relative velocity of the contacting bodies, and $n \in \mathbb{R}$ is the unknown positive Hertzian compliance coefficient which depends on the surface geometry of contact. The model in (3) is a continuous contact force-based model with a nonlinear stiffness term, and a damping term which is dependent on the depth of penetration, causing the contact forces to increase from zero upon impact and return to zero upon separation [13]. Consequently, there are no tensile and sticking forces when the contacting bodies separate. Also, the energy dissipation during impact is a function of the damping constant which can be related to the impact velocity and the coefficient of restitution [12], thus making the model consistent with the physics of contact. The contact is considered to be direct-central and quasi-static (i.e., all the stresses are transmitted at the time of contact and sliding and friction effects during contact are negligible) where plastic deformation effects are assumed to be negligible [9], [17], [18], [30].

The following properties that will be utilized in the subsequent analysis.

Property 1: The following skew-symmetric relationship is satisfied [31]

$$\xi^T(\frac{1}{2}\dot{M}(x_r) - C(x_r, \dot{x}_r))\xi = 0 \qquad \forall\xi \in \mathbb{R}^2. \quad (5)$$

Remark 1: To aid the subsequent control design and analysis, we define the vector $Tanh(\cdot) \in \mathbb{R}^n$ as follows

$$Tanh(\delta) = [\tanh(\delta_1), ..., \tanh(\delta_n)]^T \quad (6)$$

where $\delta = [\delta_1, ..., \delta_n]^T \in \mathbb{R}^n$.

Property 2: The following inequalities are valid for all $\xi = [\xi_1, ..., \xi_n]^T \in \mathbb{R}^n$ [24]

$$\|\xi\|^2 \geq \sum_{i=1}^{n} \ln(\cosh(\xi_i)) \geq \ln(\cosh(\|\xi\|)) \quad (7)$$

$$\xi^T Tanh(\xi) \geq Tanh^T(\xi)Tanh(\xi) = \|Tanh(\xi)\|^2 \quad (8)$$

$$\|Tanh(\xi)\|_\infty \leq 1. \quad (9)$$

Assumption 1: The robot and mass spring positions, $x_r(t)$ and $x_m(t)$, and the corresponding velocities, $\dot{x}_r(t)$ and $\dot{x}_m(t)$, are measurable. Further, it is assumed that $x_r(t)$ is bounded because of the geometry of the robot.

Assumption 2: The local deformation of the viscoelastic material during contact, $\delta(x_r, x_m)$, defined in (4), is assumed to be upper bounded as

$$\delta \leq \mu, \quad (10)$$

where $\mu \in \mathbb{R}$ is a positive bounding constant.

Assumption 3: The damping constant *b*, in (3), is assumed to be upper bounded as

$$b \leq \bar{b} \quad (11)$$

where $\bar{b} \in \mathbb{R}$ denotes a known positive bounding constant.

Assumption 4: The minimum singular value of the manipulator Jacobian $J(q)$ is greater than a known, small positive constant $\epsilon > 0$, such that $\max\left\{\left\|J^{-1}(q)\right\|\right\}$ is known a priori, and hence, all kinematic singularities are always avoided [32].

Assumption 5: For a revolute robot, the inertia matrix $M(x_r)$ is symmetric, positive definite, and can be lower and upper bounded as

$$a_1 \|\xi\|^2 \leq \xi^T M \xi \leq a_2 \|\xi\|^2 \qquad \forall \xi \in \mathbb{R}^2 \tag{12}$$

where $a_1, a_2 \in \mathbb{R}$ are known positive constants, and $\|\cdot\|$ denotes the standard Euclidean norm. Although the inertia matrix is expressed in task/operation space, it can be assumed to be positive definite (instead of positive semi-definite), using Assumption 4.

A. Control Development

The control objective is to design a force limiting controller for a robot which transitions from a non-contact to a contact state with an unactuated viscoelastic mass-spring system and regulate the mass to a desired position. In the absence of a control input to the mass, a virtual control input in the form of the desired robot velocity $\dot{x}_{rd}(t)$ is designed based on Lyapunov-based backstepping methods. A robot force control input $F(t)$ is designed to track the desired robot velocity and ensure a stable transition of the robot from free motion to constrained motion. Based on their universal approximation property, the NNs are used as feedforward elements of the controller to account for both LP (linear-in-the-parameters) and non-LP uncertainties in the robot and environment model. The NN weight estimates are generated online using update laws based on Lyapunov stability analysis. Another objective of the controller is to be able to keep the control forces within a prescribed limit, thereby limiting the force at impact. To this end, control inputs with adjustable bounds are used in the design of the controller. Specifically, the feedback terms of the controller are saturated using a hyperbolic tangent function, and the feedforward NN term is bounded by projection as in [32], [33].

To quantify the control objective, the following errors are defined

$$e_r \triangleq x_{rd} - x_r, \qquad e_m \triangleq x_{md} - x_m, \tag{13}$$

where $e_r \triangleq [e_{r1}(t), e_{r2}(t)]^T \in \mathbb{R}^2$ denotes the robot tracking error, and $e_m \in \mathbb{R}$ denotes the regulation error of the mass. In (13), $x_{md} \in \mathbb{R}$ is a constant denoting the desired final position of the mass, and $x_{rd}(t) \triangleq [x_{rd1}(t),\ x_{rd2}]^T \in \mathbb{R}^2$ denotes the desired position of the robot end-effector. To facilitate the subsequent control design and stability analysis, the following filtered tracking errors for the robot and the mass-spring, denoted by $r_r(t) \in \mathbb{R}^2$ and $r_m(t) \in \mathbb{R}$ respectively, are defined as

$$r_r \triangleq \dot{e}_r + \alpha_1 e_r + \alpha_2 e_{fr}, \qquad r_m \triangleq \dot{e}_m + \gamma_1 e_m, \tag{14}$$

where $\alpha_1 \in \mathbb{R}^{2\times 2}$ is a positive definite, constant, diagonal gain matrix, $\alpha_2,\ \gamma_1 \in \mathbb{R}$ are positive constant gains, and $e_{fr}(t) \in \mathbb{R}^2$ is an auxiliary signal whose dynamics are

$$\dot{e}_{fr} = -\alpha_3 e_{fr} + \alpha_2 e_r - k_r r_r, \tag{15}$$

where $\alpha_3 \in \mathbb{R}^{2\times 2}$ is a positive definite, constant, diagonal gain matrix, $k_r \in \mathbb{R}$ is a positive constant control gain.

Remark 1: The motivation behind using the filter $e_{fr}(t)$ in (15) is to add a pure feedback term in the subsequently developed closed loop robot error system, which is not otherwise possible with saturated feedback in the control input.

B. Closed-Loop Error System

The open loop error system for the mass-spring system can be obtained by multiplying the filtered tracking error of the mass, $r_m(t)$, by m and using (2), (3), (13), and (14), as

$$m\dot{r}_m = f_1 - \lambda\delta^n - b\dot{\delta}\delta^n, \tag{16}$$

where $f_1(t) \in \mathbb{R}$, a function containing uncertain parameters of the mass-spring system $(m,\ k_s)$ is defined as

$$f_1 \triangleq k_s(x_m - x_0) + m\gamma_1 \dot{e}_m. \tag{17}$$

In the absence of control input in the mass error system (16), a virtual control input in the form of the desired robot velocity $\dot{x}_{rd1}(t)$ is inserted by adding and subtracting the term $\dot{x}_{rd1}$ to (16) as

$$m\dot{r}_m = f_1 - \lambda\delta^n - b\dot{\delta}\delta^n + \dot{x}_{rd1} - \dot{x}_{rd1}. \tag{18}$$

The nonlinear function $f_1(t)$ in (17), can be approximated by a three layer (input, hidden and output) NN as

$$f_1 = W_1^T \sigma\left(V_1^T x_1\right) + \varepsilon_m\left(x_1\right), \tag{19}$$

where the NN input $x_1(t) \in \mathbb{R}^4$ is defined as $x_1 \triangleq [1\ x_m\ e_m\ r_m]^T \in \mathbb{R}^4$, $W_1 \in \mathbb{R}^{(N_m+1)\times 1}$ and $V_1 \in \mathbb{R}^{4\times N_m}$ are ideal NN weights, and $N_m \in \mathbb{R}$ denotes the number of hidden layer neurons of the NN, $\sigma(\cdot) \in \mathbb{R}^{(N_m+1)}$ is a sigmoid activation function and $\varepsilon_m(x_1) \in \mathbb{R}$ is the functional reconstruction error of the NN. For more details on NNs see [28]. It can be seen from (18) that this is not a standard backstepping problem [34] since $\dot{x}_{r1}(t)$, the state which is backstepped on, has a non-invertible term $b\delta(t)^n$ multiplied by it. This complicates the design of the virtual control input $\dot{x}_{rd1}(t)$, warranting the need to strategically add and subtract the term $\dot{x}_{rd1}(t)$ in (18). To facilitate the subsequent backstepping-based design, the virtual control input to the unactuated mass-spring system $\dot{x}_{rd1}(t)$ is designed as

$$\dot{x}_{rd1} = \hat{f}_1 + k_m r_m + e_m, \tag{20}$$

where $k_m \in \mathbb{R}$ is a positive, constant control gain, $\hat{f}_1(t) \in \mathbb{R}$ is the estimate for $f_1(t)$ and is defined as

$$\hat{f}_1 \triangleq \hat{W}_1^T \sigma(\hat{V}_1^T x_1), \tag{21}$$

where $\hat{W}_1(t) \in \mathbb{R}^{(N_m+1)\times 1}$ and $\hat{V}_1(t) \in \mathbb{R}^{4\times N_m}$ are the estimates of the ideal weights updated as

$$\begin{aligned} \dot{\hat{W}}_1 &= proj(\Gamma_{w1}\hat{\sigma}_1 r_m - \Gamma_{w1}\hat{\sigma}_1^{'}\hat{V}_1^T x_1 r_m) \quad (22) \\ \dot{\hat{V}}_1 &= proj(\Gamma_{v1} x_1 \hat{W}_1^T \hat{\sigma}_1^{'} r_m), \end{aligned}$$

where $\Gamma_{w1} \in \mathbb{R}^{(N_m+1)\times(N_m+1)}$, $\Gamma_{v1} \in \mathbb{R}^{4\times 4}$ are constant, positive definite, diagonal gain matrices, and $proj(\cdot)$ denotes a smooth projection algorithm utilized to guarantee that the weight estimates $\hat{W}_1(t)$ and $\hat{V}_1(t)$ remain bounded [32], [33], a fact which will be exploited in the subsequent stability analysis. Also, $x_{rd2} = \eta$ where $\eta \in \mathbb{R}$ is an appropriate positive constant, selected such that the robot impacts the mass-spring system. Substituting (20) in the open loop error expression in (18), and using the definitions in (19) and (21), the closed loop error system for the mass-spring system can be written as

$$\begin{aligned} m\dot{r}_m &= W_1^T\sigma\left(V_1^T x_1\right) - \hat{W}_1^T\sigma(\hat{V}_1^T x_1) - \lambda\delta^n \quad (23) \\ &\quad -b\dot{\delta}\delta^n - e_m - k_m r_m + \dot{x}_{rd1} + \varepsilon_m\left(x_1\right). \end{aligned}$$

Adding and subtracting $W_1^T\hat{\sigma}_1 + \hat{W}_1^T\tilde{\sigma}_1$ to (23), and then using the Taylor series approximation [28],

$$\tilde{\sigma}_1 = \hat{\sigma}_1^{'}\tilde{V}_1^T x_1 + O(\tilde{V}_1^T x_1)^2,$$

the following expression for the closed loop mass error system can be obtained

$$\begin{aligned} m\dot{r}_m &= \tilde{W}_1^T\hat{\sigma}_1 + \hat{W}_1^T\hat{\sigma}_1^{'}\tilde{V}_1^T x_1 - \tilde{W}_1^T\hat{\sigma}_1^{'}\hat{V}_1^T x_1 + w_1 \\ &\quad -e_m - k_m r_m, \end{aligned} \quad (24)$$

where $\tilde{W}_1 \triangleq W_1 - \hat{W}_1 \in \mathbb{R}^{(N_m+1)\times 1}$ and $\tilde{V}_1 \triangleq V_1 - \hat{V}_1 \in \mathbb{R}^{4\times N_m}$ denote the estimate mismatch for NN weights, and $\tilde{\sigma}_1 \triangleq \sigma_1 - \hat{\sigma}_1 = \sigma(V_1^T x_1) - \sigma(\hat{V}_1^T x_1) \in \mathbb{R}^{(N_m+1)}$ denotes the mismatch for hidden layer output error, and $w_1(t) \in \mathbb{R}$ can be considered as a disturbance term defined as

$$\begin{aligned} w_1 &= \tilde{W}_1^T\hat{\sigma}_1^{'}V_1^T x_1 + W_1^T O(\tilde{V}_1^T x_1)^2 + \varepsilon_m\left(x_1\right) \\ &\quad -\lambda\delta^n - b\dot{\delta}\delta^n + b\dot{e}_{r1}\delta^n + \dot{x}_{rd1}. \end{aligned} \quad (25)$$

The term $w_1(t)$ can be shown to be bounded as [35]

$$\left|w_1\right| \leq c_{m1} + c_{m2}\left\|z\right\|, \quad (26)$$

where $c_{m1}, c_{m2} \in \mathbb{R}$ are computable constants, and $z \in \mathbb{R}^8$ is defined as

$$z \triangleq [e_m \; r_m \; e_r^T \; e_{fr}^T \; r_r^T]. \quad (27)$$

The open-loop robot error system can be obtained by taking the time derivative of $r_r(t)$, premultiplying by the robot inertia matrix $M(x_r)$, and using (1), (13)-(15) as

$$M\dot{r}_r = f_2 - Cr_r - F - M\alpha_2 k_r r_r - e_r, \quad (28)$$

where the function $f_2(t) \in \mathbb{R}^2$, contains the uncertain robot and Hunt-Crossley environment dynamics, and is defined as

$$\begin{aligned} f_2 &\triangleq M\ddot{x}_{rd} + M\alpha_1\dot{e}_r + h + C\dot{x}_{rd} + C\alpha_1 e_r + C\alpha_2 e_{fr} \\ &\quad + \begin{bmatrix} \lambda\delta^n + b\dot{\delta}\delta^n \\ 0 \end{bmatrix} - M\alpha_2\alpha_3 e_{fr} + M\alpha_2^2 e_r \quad (29) \\ &\quad + e_r. \end{aligned}$$

The function $f_2(t)$ can be represented by a NN, and the expression in (28) can be rewritten as

$$M\dot{r}_r = W_2^T\sigma\left(V_2^T x_2\right) + \varepsilon_r\left(x_2\right) - Cr_r - F - M\alpha_2 k_r r_r - e_r, \quad (30)$$

where the NN input $x_2(t)$ is defined as $x_2(t) \triangleq [1 \; x_m \; e_m \; r_m \; x_r^T \; e_r^T \; e_{fr}^T \; r_r^T \; \delta \; \delta\dot{\delta}]^T \in \mathbb{R}^{14}$, $W_2 \in \mathbb{R}^{(N_r+1)\times 2}$, and $V_2 \in \mathbb{R}^{14\times N_r}$ are ideal NN weights, and $N_r \in \mathbb{R}$ denotes the number of hidden layer neurons of the NN. Using (20), the term $\ddot{x}_{rd}(t)$ in (29) is continuous and can be expressed in a linearly parameterizable form which depends on only position and velocity measurements. Based on the subsequent stability analysis, the robot force control input $F(t)$ can be designed as

$$F = \hat{W}_2^T\sigma(\hat{V}_2^T x_2) + k_d Tanh(\psi r_r) + k_p Tanh(\omega e_r), \quad (31)$$

where $k_d, k_p \in \mathbb{R}^{2\times 2}$ are constant, positive definite, diagonal gain matrices, $\psi, \omega \in \mathbb{R}$ are positive constant gains, and $\hat{W}_2(t) \in \mathbb{R}^{(N_r+1)\times 2}$ and $\hat{V}_2(t) \in \mathbb{R}^{14\times N_r}$ are the estimates of the ideal weights, which are updated based on the subsequent stability analysis as

$$\begin{aligned} \dot{\hat{W}}_2 &= proj(\Gamma_{w2}\hat{\sigma}_2 r_r^T - \Gamma_{w2}\hat{\sigma}_2^{'}\hat{V}_2^T x_2 r_r^T) \quad (32) \\ \dot{\hat{V}}_2 &= proj(\Gamma_{v2} x_2 r_r^T \hat{W}_2^T \hat{\sigma}_2^{'}), \end{aligned}$$

where $\Gamma_{w2} \in \mathbb{R}^{(N_r+1)\times(N_r+1)}$, $\Gamma_{v2} \in \mathbb{R}^{14\times 14}$ are constant, positive definite, diagonal, gain matrices.

Remark 2: Using (9) and the NN projection bounds in (32), the control force $F(t)$ in (31) can be bounded as

$$\|F\|_\infty \leq \kappa + \lambda_{\max}\{k_d\} + \lambda_{\max}\{k_p\}, \quad (33)$$

where $\lambda_{\max}\{\cdot\}$ denotes the maximum eigenvalue of the argument. It can be seen from (33) that the bound on the force $F(t)$ is known a priori, and the feedback gains k_d and k_p can be changed to adjust the bound. Unlike previous results in the saturated control literature, the force bound in (33) does not depend on the actual or the desired trajectory. Also, the proportional and derivative gains, k_p and k_d, respectively, can be chosen such that the prescribed force limits are met. The use of hyperbolic tangent as smooth saturation functions for the PD term in (31) is motivated by the idea of energy shaping and passivity based approach first proposed by Arimoto [36]. Limiting control torques using this approach is quite common for the robot regulation/tracking problem [19], [21]. However, the use of such saturation functions to limit forces during a contact transition task is novel and poses a challenge to prove stability of the closed loop system.

Substituting (31) in the open loop expression (30) and following similar approach as in the mass error system in going from (23) to (24), the closed loop error system for the robot is obtained as

$$\begin{aligned} M\dot{r}_r &= \tilde{W}_2^T\hat{\sigma}_2 - \tilde{W}_2^T\hat{\sigma}_2^{'}\hat{V}_2^T x_2 + \hat{W}_2^T\hat{\sigma}_2^{'}\tilde{V}_2^T x_2 + w_2 \\ &\quad -Cr_r - k_d Tanh(\psi r_r) - k_p Tanh(\omega e_r) \quad (34) \\ &\quad -M\alpha_2 k_r r_r - e_r, \end{aligned}$$

where the disturbance term $w_2(t) \in \mathbb{R}^2$ is defined as

$$w_2 = \tilde{W}_2^T \hat{\sigma}_2^{'} V_2^T x_2 + W_2^T O(\tilde{V}_2^T x_2)^2 + \varepsilon_r(x_2). \quad (35)$$

It can be shown that $w_2(t)$ can be bounded as [35]

$$\|w_2\| \leq c_{r1} + c_{r2}\|z\|, \quad (36)$$

where $c_{r1}, c_{r2} \in \mathbb{R}$ are computable known positive constants.

III. Stability Analysis

Theorem 1: The controller given by (20), (22), (31), and (32) ensures uniformly ultimately bounded regulation of the MSR system in the sense that

$$|e_m(t)|, \ \|e_r(t)\| \leq \varepsilon_0 \exp(-\varepsilon_1 t) + \varepsilon_2 \quad (37)$$

provided the control gains are selected according to the sufficient gain conditions

$$\lambda_{\min}\{\alpha_3\} > \frac{k_r^2}{4k_{r3}\alpha_2 a_1} + \frac{\omega\alpha_2^2\lambda_{\max}^2\{k_p\}}{4\lambda_{\min}\{k_p\alpha_1\}} \quad (38)$$

$$\rho > \frac{c_{m2}^2}{4k_{m2}} + \frac{c_{r2}^2}{4k_{r2}\alpha_2 a_1} \quad (39)$$

where ε_0, ε_1, $\varepsilon_2 \in \mathbb{R}$ denote positive constants; the control gains α_1, α_2, α_3 are introduced in (15), c_{m2} is introduced in (26), k_{m2}, k_{r2}, k_{r3} are introduced in (44), k_p is introduced in (31), c_{r2} is introduced in (36), $\lambda_{\min}\{\cdot\}$ denotes the minimum eigenvalue of a matrix, and $\rho \in \mathbb{R}$ is a known bounding constant defined as

$$\rho = \min\left\{\gamma_1, k_{m3}, \lambda_{\min}\{\alpha_1\}, \left(\lambda_{\min}\{\alpha_3\} - \frac{k_r^2}{4k_{r3}\alpha_2 a_1} - \frac{\omega\alpha_2^2\lambda_{\max}^2\{k_p\}}{4\lambda_{\min}\{k_p\alpha_1\}}\right), \alpha_2 a_1\right\}. \quad (40)$$

Let $V(t) \in \mathbb{R}$ denote a non-negative, radially unbounded function (i.e., a Lyapunov function candidate) defined as

$$\begin{aligned} V \triangleq\ & \frac{1}{2}r_r^T M r_r + \frac{1}{2}e_r^T e_r + \frac{1}{2}tr(\tilde{W}_1^T\Gamma_{w1}^{-1}\tilde{W}_1) \quad (41) \\ & + \frac{1}{2}tr(\tilde{V}_1^T\Gamma_{v1}^{-1}\tilde{V}_1) + \frac{1}{2}tr(\tilde{W}_2^T\Gamma_{w2}^{-1}\tilde{W}_2) \\ & + \frac{1}{2}tr(\tilde{V}_2^T\Gamma_{v2}^{-1}\tilde{V}_2) + \frac{1}{2}mr_m^2 + \frac{1}{2}e_{fr}^T e_{fr} \\ & + \frac{1}{2}e_m^2 + \frac{k_{p1}}{\omega}\ln(\cosh(\omega e_{r1})) \\ & + \frac{k_{p2}}{\omega}\ln(\cosh(\omega e_{r2})), \end{aligned}$$

where $tr(\cdot)$ denotes the trace of the argument. Using (12), (7), and NN properties in [28], it can be shown that $V(t)$ can be upper and lower bounded as

$$\lambda_1\|z\|^2 \leq V(t) \leq \lambda_2\|z\|^2 + \theta, \quad (42)$$

where $\lambda_1, \lambda_2, \theta \in \mathbb{R}$ are known positive constants. Taking the time derivative of (41), and using (14), (15), (22), (24), (32), (34), and canceling similar terms, the following expression is obtained

$$\begin{aligned} \dot{V} =\ & -r_r^T k_d Tanh(\psi r_r) + r_r^T w_2 - k_r\alpha_2 r_r^T M r_r \\ & - e_r^T\alpha_1 e_r - \gamma_1 e_m^2 + r_m w_1 - k_m r_m^2 \quad (43) \\ & - e_{fr}^T\alpha_3 e_{fr} - k_r e_{fr}^T r_r - Tanh^T(\omega e_r)k_p\alpha_1 e_r \\ & - \alpha_2 Tanh^T(\omega e_r)k_p e_{fr}. \end{aligned}$$

The nonlinear damping gains, k_m and k_r, introduced in (15) and (20) respectively, can now be defined as

$$\begin{aligned} k_m &\triangleq k_{m1} + k_{m2} + k_{m3}, \quad (44) \\ k_r &\triangleq 1 + k_{r1} + k_{r2} + k_{r3}, \end{aligned}$$

where $k_{mi}, k_{ri} \in \mathbb{R}$, $(i = 1, 2, 3)$ are positive constants. Using (12), (8), (26), (36), and (44), the expression in (43) can be upper bounded as

$$\begin{aligned} \dot{V} \leq\ & -\lambda_{\min}\{\alpha_1\}\|e_r\|^2 - k_{m3}r_m^2 - \gamma_1 e_m^2 - \alpha_2 a_1\|r_r\|^2 \\ & -\lambda_{\min}\{\alpha_3\}\|e_{fr}\|^2 - [k_{m1}r_m^2 - c_{m1}|r_m|] \quad (45) \\ & -[k_{m2}r_m^2 - c_{m2}\|z\||r_m|] \\ & -[k_{r1}\alpha_2 a_1\|r_r\|^2 - c_{r1}\|r_r\|] \\ & -[k_{r2}\alpha_2 a_1\|r_r\|^2 - c_{r2}\|z\|\|r_r\|] \\ & -[k_{r3}\alpha_2 a_1\|r_r\|^2 - k_r\|e_{fr}\|\|r_r\|] \\ & -[\frac{\lambda_{\min}\{k_p\alpha_1\}}{\omega}\|Tanh(\omega e_r)\|^2 \\ & -\alpha_2\lambda_{\max}\{k_p\}\|Tanh(\omega e_r)\|\|e_{fr}\|]. \end{aligned}$$

Completing the squares on the terms in the bracket in (45), $\dot{V}(t)$ can be further upper bounded as

$$\begin{aligned} \dot{V} \leq\ & -\lambda_{\min}\{\alpha_1\}\|e_r\|^2 - k_{m3}r_m^2 - \lambda_{\min}\{\alpha_3\}\|e_{fr}\|^2 \\ & -\gamma_1 e_m^2 - \alpha_2 a_1\|r_r\|^2 + \frac{c_{m1}^2}{4k_{m1}} + \frac{c_{m2}^2\|z\|^2}{4k_{m2}} \quad (46) \\ & + \frac{c_{r1}^2}{4k_{r1}\alpha_2 a_1} + \frac{c_{r2}^2\|z\|^2}{4k_{r2}\alpha_2 a_1} + \frac{k_r^2\|e_{fr}\|^2}{4k_{r3}\alpha_2 a_1} \\ & + \frac{\omega\alpha_2^2\lambda_{\max}^2\{k_p\}\|e_{fr}\|^2}{4\lambda_{\min}\{k_p\alpha_1\}}. \end{aligned}$$

Provided the gain condition in (38) is satisfied, the following expression can be obtained

$$\dot{V} \leq -\left(\rho - \frac{c_{m2}^2}{4k_{m2}} - \frac{c_{r2}^2}{4k_{r2}\alpha_2 a_1}\right)\|z\|^2 + \frac{c_{m1}^2}{4k_{m1}} + \frac{c_{r1}^2}{4k_{r1}\alpha_2 a_1}, \quad (47)$$

where ρ was defined in (40). Provided the gain condition in (39) is satisfied, (47) can be rewritten as

$$\dot{V} \leq -\beta\|z\|^2 + \frac{c_{m1}^2}{4k_{m1}} + \frac{c_{r1}^2}{4k_{r1}\alpha_2 a_1}, \quad (48)$$

where $\beta \in \mathbb{R}$ is a positive constant. The expression in (48) can be used to show that $\dot{V}(t)$ is negative whenever $z(t)$ lies outside the compact set $\Omega_z \triangleq \left\{z : \|z\| \leq \sqrt{\frac{\frac{c_{m1}^2}{4k_{m1}} + \frac{c_{r1}^2}{4k_{r1}\alpha_2 a_1}}{\beta}}\right\}$, and hence, $\|z(t)\|$ is UUB [34]. The size of Ω_z can be made arbitrarily small by increasing the control gains k_m and k_r. Using the definition of $z(t)$ in (27), it can be shown that $r_r(t)$, $e_r(t)$, $e_{fr}(t)$, $r_m(t)$, $e_m(t) \in \mathcal{L}_\infty$. Since $r_m(t)$, $e_m(t) \in \mathcal{L}_\infty$, it can be shown from (14) that $\dot{e}_m(t) \in \mathcal{L}_\infty$. Similarly, the fact that $r_r(t)$, $e_r(t)$, $e_{fr}(t) \in \mathcal{L}_\infty$ can be used to show that $\dot{e}_r(t) \in \mathcal{L}_\infty$. Given that $\hat{W}_1(t)$, $\sigma(\cdot)$, $e_m(t)$, $r_m(t) \in \mathcal{L}_\infty$, (20) can be used to show that the desired robot velocity $\dot{x}_{rd}(t) \in \mathcal{L}_\infty$. Using (33), it can be shown that the force control input to the robot $F(t) \in \mathcal{L}_\infty$.

IV. Conclusion

This paper presents the development of a bounded NN based controller for the purpose of controlling a robot undergoing a non-contact to contact transition with an uncertain viscoelastic environment. The bounds on the control actuation are known a priori and can be adjusted by changing the feedback gains. The control development in this paper builds on our previous work in [9] by considering an additional objective of constraining the control force, and thereby limiting the force at impact. The saturated controller, composed of a bounded NN term and a saturated feedback term, is shown to guarantee GUUB stability result despite uncertainties in the robot and the environment, and without the use of acceleration or force measurements. The work in this paper is an attempt towards developing better force limiting controllers for physical human-robot interaction, which can prove useful for rehabilitation type applications.

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