Constraining Electron's Parallel Energy in Electrostatic Field through Anomalous Doppler Effect Induced by External Electromagnetic Waves

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In this study, the interaction between free electrons and electromagnetic waves (EMW) in the presence of magnetic and electrostatic fields is simulated using the Volume-Preserving algorithm. We found that when the electric field of the EMW (which includes a left-hand polarization component) exceeds a critical value, it can continuously transfer the electron's parallel energy into rotational energy via the Anomalous Doppler Effect (ADE). This mechanism converts the work by the electric field along the background magnetic field into perpendicular kinetic energy. As a result, the electron's parallel kinetic energy saturates, while its perpendicular kinetic energy increases continuously. This energy, momentum, and angular momentum model provides a clear explanation of the contribution of left-hand polarization to the anomalous Doppler effect. This general model can be directly applied to interpret the anomalous Doppler effect in (A, B, C). To saturate the parallel energy of runaway electrons in magnetically confined plasmas, we propose an extraordinary beam injection method based on the understanding of mechanism. It offers an innovative solution for runaway electron mitigation in Tokamaks by launching extraordinary beam from the high-field side.

I. INTRODUCTION

During the current ramp-up phase in tokamak, disruptions or magnetohydrodynamic (MHD) instabilities can induce quasi-static toroidal electric field, which accelerates electrons to energies on the order of several tens of MeV.This acceleration occurs when the force from the quasi-static electric field exceeds the drag from radiation and collisions. The accelerated electrons, known as runaway electrons, can cause significant damage to the interior walls of the device, thereby reducing its operational lifespan. It is worth considering the possibility of converting the acceleration of electrons from quasi-static electric field into rotational energy within magnetic field. This conversion not only suppresses the energy of runaway electrons, mitigating their detrimental effects on the device, but also enhances discharge performance by reducing the consumption of ohmic field energy . The transport of parallel energy in electrons to rational energy primarily occurs through three mechanisms: electron avalanche processes[1], collisionless pitch-angle scattering[2], and the Anomalous Doppler effect(ADE)[3]. The first two methods to suppress runaway processes, such as gas injection[4] and enhancing magnetic perturbations[5], usually have side effects and alter the discharge environment. In contrast, the latter method can be an clean process, making it particularly attractive for investigation.

When electrons move in static magnetic field and interact with external electromagnetic wave (EMW) with frequency ω and wave vector $\vec{k}$,they experience a scattering phenomenon under the resonant condition$\omega - \vec{k}\cdot\vec{v} = n\omega_{ce}$,where $n > 0$ and $\omega_{ce} > 0$.The result of this scattering is a transfer of momentum from parallel motion to rotational motion, a phenomenon known as the Anomalous Doppler Effect (ADE). This effect was thoroughly described in the classic works of Ginzburg and Frank[3,6,7].Recently ADE has raised increasing attentions in space radiation[8] ,runaway electron instability[9] and materials science[10],it is believed that ADE can explain the phenomena as whistler turbulence in flare loops[8],the Electron Cyclotron Emission (ECE) with step-like structure in Tokamak[11–13]and the microwave bursts during Edge Localized Modes(ELM)[14] . Additionally, ADE holds potential for suppressing runaway electron energy in Tokamak discharges.This has been demonstrated by F. Santini, who found that high-energy runaways can be significantly reduced through ADE during Lower Hybrid Waves (LHW) heating in the Frascati Tokamak[15]. However, it is important to note that the high power of LHW also causes an increase in nonthermal electrons through Landau resonance. This, in turn, leads to an increase in runaway electrons after shutdown of LHW,which is a side effect of using LHW to suppress runaway electrons.

Despite the Anomalous Doppler Effect (ADE) having been described in numerous experiments and theories[16–24], previous descriptions of the single-particle ADE have mainly relied on either quantum theory or classical theory, disregarding the electrostatic field[15,25,26]. Further exploration of the ADE in the presence of electrostatic field is essential,this investigation is crucial for understanding the physics of pitch-angle scattering of runaway electrons by electromagnetic waves in Tokamak discharges. Moreover, the nature of ADE in classical electrodynamics fields remains unclear due to the complexity of formulas in previous classical theories. For instance, it is still difficult to understand why parallel kinetic energy can convert to transverse internal energy during ADE resonance, or what kind of EMW can trigger ADE. These questions require further investigation.

This paper directly simulates the full orbit electron motion in uniform magnetic field with accelerating electrostatic field and electromagnetic field using the Volume-Preserving Algorithm (VPA)[27]. Compared to conventional algorithms,such as

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Boris, VPA can acquire long-term accuracy and conservativeness via a systematic splitting method, which is an excellent method for nonlinear electron dynamic simulation. To directly demonstrate the phenomenon of the ADE, we place an electron in uniform magnetic field B_0 along with electrostatic field E_0 that is oriented in the opposite direction to B_0. As a result, the electron can be accelerated parallel to B_0. Additionally, during the simulation, the slow electromagnetic wave with its phase velocity smaller than that of light in vacuum is set up as induced wave. This induced wave allows us to directly observe the effects when the electron's velocity reaches the resonant condition for the ADE. We explore the resonance with three types of polarization waves including linear polarization, left-hand circular polarization and right-hand circular polarization in the simulation. We find only the wave with left-hand circular polarization can have ADE for runaway electron. The critical energy of wave when the electron parallel velocity can be constrained and consistently transport the parallel energy from electrostatic field to transverse rotation energy is also found in the simulation. Plus,the self-consistency between the quantum theory and direct simulation about ADE is examined.Finally, after analysing dispersion, polarization, and resonant moment , we have determined that the extraordinary wave is the most suitable for triggering ADE in plasma. Based on these findings, we propose a promising method for controlling runaway electrons.

II. QUANTUM THEORY OF ADE

This extraordinary phenomenon has already been discussed from energy conservation in the works of V.L.Ginzburg[23], I. Tamm[28] ,Nezlin[7]and I.M.Frank[29].Here we give an analysis based on the conservation of angular momentum.

As shown in Figure 1,When charged particles move through a medium faster than the speed of light in the medium, induced currents are generated. These currents stimulate secondary waves that interfere with the electromagnetic field of the particles in motion, thus giving rise to Cherenkov radiation. The direction of Cherenkov radiation can only be along the Cherenkov radiation angle $\theta_0 = \arccos(\frac{c'}{v})$,where c' is the speed of light in the medium and v is the velocity of charge particles. Now lets substitute the charge particle with a system that has internal energy (for example a oscillator or a cyclotron electron in magnetic field).The system moving faster than the speed of light $(v > c')$ emits photons with angular frequency ω and wavevector k at direction θ,where direction of the photon performs not depend on the interference of the secondary waves and can be in any direction , as shown in Figure 2.According to energy conservation and moment conservation ,we have

$$T_1 + U_1 = \hbar\omega + T_2 + U_2 \quad (1)$$

$$\boldsymbol{p}_1 = \boldsymbol{p}_2 + \hbar\boldsymbol{k} \quad (2)$$

Where $T_1$and U_1represent the kinetic energy and internal energy of system before emitting photon,T_2 and U_2 represent energy of system after emitting photon. Considering the photons energy far less than the initial kinetic energy T_1,the loses

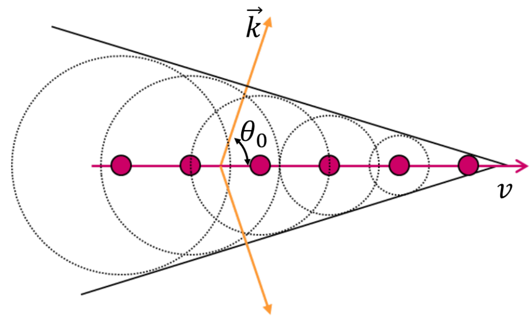

FIG. 1. Schematic diagram Cherenkov Radiation,the red point is the snapshot of the electron at different times

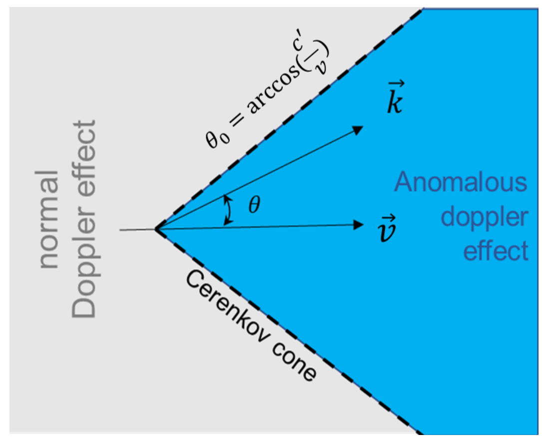

FIG. 2. The region of ADE ,Normal Doppler Effect (NDE) and Cerenkov cone,where the blue region refer to ADE,gray region refer to NDE while the dot line refer to the Cerenkov cone[10]

of kinetic energy after emitting a photon can be expressed as $\Delta T_{12} = T_1 - T_2 = \Delta\boldsymbol{p}\cdot\boldsymbol{v}$,where $\boldsymbol{v}$ is the velocity of system before emitting photon, also $\Delta p = \boldsymbol{p}_1 - \boldsymbol{p}_2 = \hbar\boldsymbol{k}$.According to Eq. (1-2),we have

$$\begin{aligned}\Delta U_{21} &= \Delta T_{12} - \hbar\omega \\ &= \hbar\vec{k}\cdot\vec{v} - \hbar\omega \\ &= \hbar\omega(\frac{kvcos\theta}{\omega} - 1)\end{aligned} \quad (3)$$

Here $\omega/k = c'$,$\Delta U_{21} = U_2 - U_1$.While the system velocity is more than the speed of light in the medium$(v > c')$,according to the sign of ΔU_{21} ,we can divide radiation into three regions , as shown in Figure 1 :for $\theta > \theta_0(\cos\theta_0 = \frac{c'}{v})$,$\Delta U_{21} < 0$, The system produces photons by consuming its own internal and kinetic energy,this region refer to Normal Doppler Effect (NDE);for $\theta < \theta_0$,$\Delta U_{21} > 0$, this region refer to Anomalous Doppler Effect (ADE),where the system gains internal energy after emitting photons .It means the loss of kinetic energy is converted to photons and the system's internal energy; for $\theta = \theta_0$,$\Delta U_{21} = 0$,The loss of kinetic energy by the system is completely converted into photon energy,this line refers to Cerenkov Effect(CE). All three effects are possible when the system velocity exceeds the speed of light ($v > c'$).While the system velocity is less than the speed of light($v < c'$), only NDE exit.As we can see ,we can judge the type of phenomena based on the change of internal energy after emoting photons.

Now consider the particular case in which the system is a electron moving freely with velocity v_z in the direction of external magnetic field $B = B_z$ and having small transverse ($\perp B$)velocity component $v_\perp$,than the kinetic energy of the cyclotron electron could be expressed as $T = \frac{1}{2}m_e v_z^2$ and the internal energy represents as $U = \frac{1}{2}m_e v_\perp^2$,which is the equal to rotational energy of the electron as shown in Figure 3.In order to calculate the change of rotational energy ,we should combine with another conservation :the angular momentum conservation ,which is

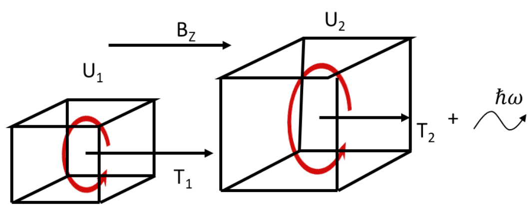

FIG. 3. Schematic diagram of electron cyclotron system,here $U_2 > U_1, T_2 > T_1$

$$L_1 = L_2 + n\hbar \tag{4}$$

Here we consider that the emitted photon contains angular moment with $n\hbar$ and the angular momentum of cyclotron electron before and after emitting photon is L_1 and L_2,since the magnetic field is along the z direction , L should be equal to L_z.From the quantum theory ,the electron wave in static magnetic field can be expressed as

$$\Psi = \Psi_0 e^{\frac{i}{\hbar}(\boldsymbol{p}-e\boldsymbol{A})\cdot\boldsymbol{s}} \tag{5}$$

where the Ψ_0 is normalized coefficient , A is vector potential and s is the position.The intrinsic equation of angular moment in z direction is

$$-i\hbar\frac{\partial}{\partial\varphi}\Psi = (\boldsymbol{p}_\varphi - e\boldsymbol{A}_\varphi)r\Psi \tag{6}$$

the eigenvalue of L_z is

$$L_z = (\boldsymbol{p}_\varphi - e\boldsymbol{A}_\varphi)r \tag{7}$$

consider $p_\varphi = m_e v_\perp$,$A_\varphi = \frac{rB_0}{2}$ and $r = \frac{m_e v_\perp}{B_0 e}$,we have

$$L_z = \frac{1}{2}\frac{m_e v_\perp^2}{\omega_{ce}} = \frac{U}{\omega_{ce}} \tag{8}$$

The variation in the angular momentum of the electron along z is

$$\Delta L_{21} = L_{z2} - L_{z1} = \frac{U_2 - U_1}{\omega_{ce}} = m\hbar \tag{9}$$

where m is the number of photon's angular momentum in z direction,than we have

$$\Delta U_{21} = m\hbar\omega_{ce}, \omega_{ce} = \frac{eB}{m_e\gamma} = \frac{\omega_0}{\gamma} \tag{10}$$

where m_e is the mass of electron ,γ is Lorentz factor,ω_0 is electron cyclotron frequency in rest frame . According to Eq. (3) and Eq. (10), we have

$$\hbar\vec{k}\cdot\vec{v} = \hbar\omega + m\hbar\omega_{ce} \tag{11}$$

or

$$\omega = k_z v_z - m\omega_{ce} \tag{12}$$

Here, $\hbar\vec{k}\cdot\vec{v}$ represents the loss of kinetic energy ΔT_{21}, $\hbar\omega$ represents the energy of the photon, and $m\hbar\omega_{ce}$ represents the change in electron cyclotron energy ΔU_{21} (also called the change in internal energy). There are three situations for ΔU_{21}:

1. When $m < 0$, $\Delta U_{21} < 0$,the cyclotron electron internal energy decreases after emitting photon, and the emitted photon will have right-hand circular polarization with angular momentum $m\hbar$ to maintain angular momentum conservation. This process is called the Normal Doppler Effect (NDE).

2. For $m = 0$, $\Delta U_{21} = 0$,the Cherenkov Effect occurs, where the emitted photon does not cause any change in the internal energy of the cyclotron electron.

3. When $m > 0$, $\Delta U_{21} > 0$,the Anomalous Doppler Effect (ADE) occurs,resulting in an increase in the internal energy of the cyclotron electron and the emission of left-hand circular polarization with angular momentum $-m\hbar$

As a result, the resonant condition is strongly associated with the wave's angular momentum. For a plane wave, the wave angular moment number includes only $m = \pm 1$. while for $|m| > 1$, it indicates that the resonant wave possesses a helicon structure. Based on above discussion,there are three kinds of resonant for system when electron moves along the uniform magnetic field with velocity v under external EMW ,the resonant frequencies are normal Doppler frequency(ω_{NDE}), Cerenkov frequency($\omega_{Cerenkov}$) and anomalous Doppler frequency(ω_{ADE}).We only include $m = 0, \pm 1$ assuming they are the most dominant resonances,than these frequencies are respectively

$$\omega_{NDE} = kv_z\cos\theta + \omega_{ce} \tag{13}$$

$$\omega_{Cerenkov} = kv_z\cos\theta \tag{14}$$

$$\omega_{ADE} = kv_z\cos\theta - \omega_{ce} \tag{15}$$

where θ is the angle between B and k. This equations are quite common resonant conditions for the kinetic equation of plasma ,what is intriguing is how the motion of electrons differs under various resonant conditions with electrostatic field. However, this aspect has been far less researched in recent years.

III. SIMULATION FRAMEWORK

Considering uniform magnetic field B_0 in the z direction, the electron is accelerated in the B field by electrostatic field E_0 that has opposite direction to B_0 as shown in Figure 4. To analyze the dynamics of electrons during interactions with electromagnetic field, we establish plane EMW characterized by frequency ω and wavevector $\vec{k}$. The angle between $\vec{k}$ and B_0 is denoted as θ, the equation of motion of the electron is

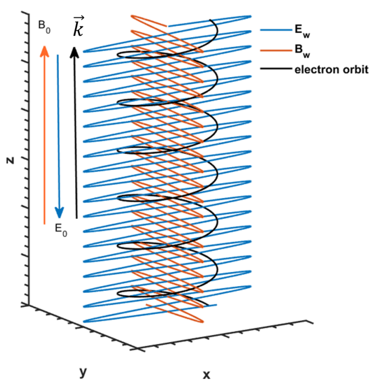

FIG. 4. Scheme of simulation setup

$$\begin{aligned} \frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} &= \frac{\mathbf{p}}{\sqrt{m_0^2+\mathbf{p}^2/c^2}} \\ \frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} &= q\left(\mathbf{E}(\boldsymbol{x},\boldsymbol{t}) + \frac{\mathbf{p}}{\sqrt{m_0^2+\mathbf{p}^2/c^2}} \times \mathbf{B}(\boldsymbol{x},\boldsymbol{t})\right) \end{aligned} \tag{16}$$

Here $\vec{E}$ and $\vec{B}$ is the sum of static field and EMW field. By using VPA algorithm[27,30,31], the discrete structure of differential Eq. (16) can be written as

$$\begin{cases} \mathbf{x}^*_{k+\frac{1}{2}} = \mathbf{x}^*_k + \frac{\Delta t^*}{2}\frac{\mathbf{p}^*_k}{\gamma_k}, \\ \mathbf{p}^{*-} = \mathbf{p}^*_k + \frac{\Delta t^*}{2}\mathbf{E}^*_{k+\frac{1}{2}}, \\ \mathbf{p}^{*+} = \text{Cay}\left(\frac{\Delta t^* \widehat{B}^*}{2\gamma^{*-}}\right)\mathbf{p}^{*-} \\ \mathbf{p}^*_{k+1} = \mathbf{p}^{*+} + \frac{\Delta t^*}{2}\mathbf{E}^*_{k+\frac{1}{2}}, \\ \mathbf{x}^*_{k+1} = \mathbf{x}^*_{k+\frac{1}{2}} + \frac{\Delta t^*}{2}\frac{\mathbf{p}^*_{k+1}}{\gamma_{k+1}}, \end{cases} \tag{17}$$

The operator Cay(A) denotes the Cayley transform of matrix A[27], $\hat{B}^*$ is

$$\hat{B}^* = \begin{pmatrix} 0 & B^*_z & -B^*_y \\ -B^*_z & 0 & B^*_x \\ B^*_y & -B^*_x & 0 \end{pmatrix} \tag{18}$$

The dimensionless parameters are momentum p^*, magnetic field B^* ,total electric field E^*,time step Δt^* , and position x^*, which are $p^* = \frac{p}{m_e c}, B^* = B/\frac{m_e}{e\tau_{ce}}, E^* = E/\frac{m_e c}{\tau_{ce} e}, \Delta t^* = \frac{\Delta t}{\tau_{ce}}, x^* = \frac{x}{\tau_{ce} c}$ respectively, where the τ_{ce} is electron cyclotron period and $\gamma^* = \sqrt{1+p^{*2}}$.

IV. RESONANCE OF ELECTRONS WITH WAVES: A COMPREHENSIVE STUDY

We start with linearly polarized wave with direction $\theta = 0$. To reduce the simulation time, B_0 is set to $2\times 10^{-2} T$. The wave angle frequency is $\omega_s = 1.5\omega_0$, where $\omega_0 = \frac{eB_0}{m_e}$. The wavevector $\vec{k}$ is parallel to z and $k = 10^5/\text{m}$. The amplitude of the electric field of the electromagnetic wave is $E_S = 9$ V/m and the electrostatic field is $E_0 = -2.5$ V .All this parameters are set without reality consideration ,merely for quick simulation . The time step always satisfies $\Delta t = \min\left(\frac{2\pi}{50(\vec{k}\cdot\vec{v})}, \frac{2\pi}{50\omega}, \frac{2\pi}{50\omega_0}\right)$ to ensure the accuracy of the simulation. The electron starts off stationary but gradually gains speed. Simultaneously, the resonant frequency increases according to Eq. (13-15). Figure 5 shows the evolutions of the electron's orbit and velocity phase during acceleration. The details of the electron's motion are given in Fig. 6. The resonant frequencies increase simultaneously (Fig. 6(b)) during electron acceleration in the electrostatic field (Fig. 6 (a)). When the normal doppler frequency equals to induced wave at about $23\tau_{ce}$, the perpendicular velocity (or rotational velocity) $v_\perp$ quickly grow up (Fig. 6(d)). The parallel velocity v_z caused by EMW also increases simultaneously ,as shown in Figure6(c),which can be calculated from $\Delta v_z = v_z - v_{zE0}$, where v_z is the parallel velocity in EMW and electrostatic field while v_{zE0} is only the result of electrostatic field . This phenomenon represents NDE where the resonant velocity $v_{NDE} = (\omega - \omega_{ce})/k_z < c'$ is "subluminal". The absorption of induced waves by the cyclotron electron causes an increase of velocity in parallel and perpendicular directions which can be viewed as a reverse process for system emitting photon as discussed before (The NDE process is widely used for in current drive[32] and plasma heating[33] in Tokamak,although it is generally believed that the current drive by electromagnetic waves follows the Fisch mechanism[34] due to the limited toroidal moment injected by the waves). However, the resonant condition is quickly broken as the parallel velocity keep increasing until it reaches $v_{ADE} = \frac{\omega+\omega_{ce}}{k_z}$, at which point the ADE starts to come into view. When time arrives at $113\tau_{ce}$,the system

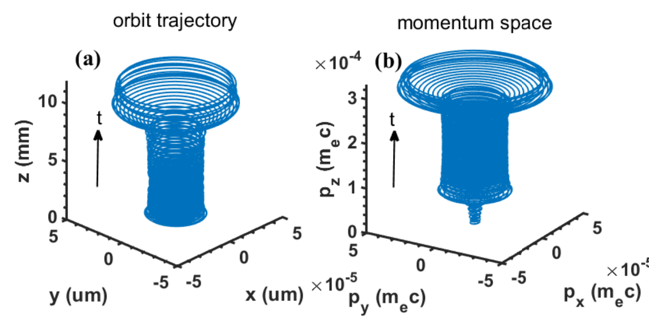

FIG. 5. (a)Orbit trajectory of electron motion(b)Momentum phase space of electron motion

starts to resonant with induced wave through ADE, where the $\omega_{ADE} = \omega$ as shown in Figure 6(b), the parallel velocity begins to scatter to perpendicular direction as we can see the decrease of Δv_z and the increase of $v_\perp$ shown in Fig.6(c)and Fig.6(d). The resonant condition quickly disappears as the parallel velocity surpasses the resonant region. To determine which type of EMW is responsible for the NDE and ADE separately, we separate the linear polarized wave into left-handed circular polarization and right-handed circular polarization. We observe that the right-hand circular polarized wave is responsible for the NDE, while the left-hand circular polarized wave induces the ADE, as illustrated in Fig.6(e) which agree well with our previous analysis. We can understand this phenomenon through the conservation of angular momentum and momentum: Because electron have a right-hand spin orbital angular momentum in a magnetic field, when electron absorb right-handed electromagnetic waves propagating in the parallel direction, according to the conservation of momentum and angular momentum, the parallel momentum and rotational energy of the electron will also increase, this process corresponds to NDE. When the electron emit left-hand circular electromagnetic waves propagating in the parallel direction, the conservation of momentum results in a decrease in the electron's parallel momentum, while the conservation of angular momentum requires an increase in the electron's rotational energy,this process corresponds to ADE.It is worth noting that there is no response during the Cerenkov resonance for the electromagnetic wave (EMW), as the Cerenkov effect primarily pertains to electrostatic waves.The ADE mechanism from classical theory is analysised in appendix.

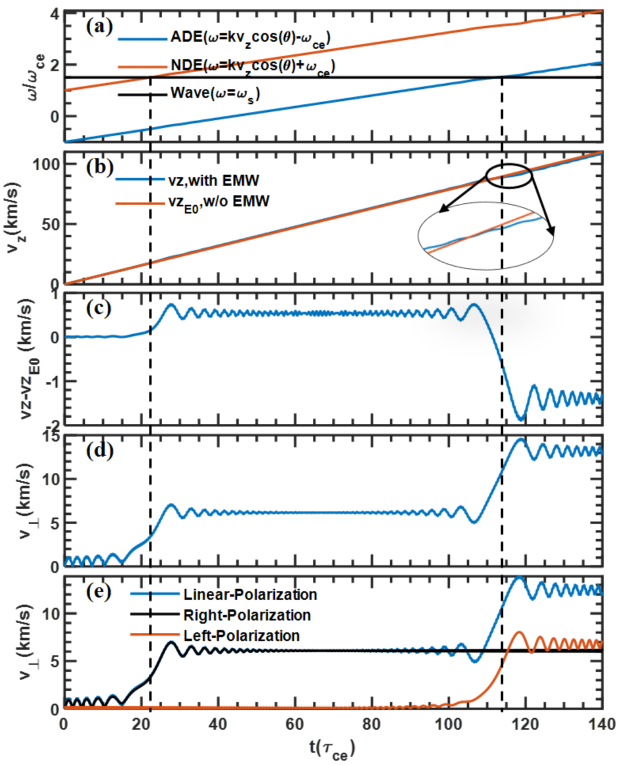

FIG. 6. Kinetic evolution of electron in magnetic field with EMW during acceleration. (a) Translational velocity v_z. (b)Wave frequencies of anomalous Doppler frequency, normal Doppler frequency and induced wave frequency. (c)The change of parallel velocity caused by EMW. (d)Cyclotron velocity $v_\perp$.(e)Interaction with Linear ,Right-hand circular ,Left-hand circular polarization .

V. CRITICAL TRAPPING THRESHOLD OF ADE

We found that the ADE is equivalent to an effective damping force, hindering the electron acceleration process. Therefore, if the intensity of the electromagnetic wave is increased, it is theoretically possible to achieve a balance with the electrostatic field force, causing the electron to no longer be accelerated by the electrostatic field.Next,we will demonstrate the existence of this equilibrium by varying the electromagnetic wave field intensity .

Let us consider an EMW with only left-hand polarized circular wave, where the wave-vector k and frequency ω of the EMW are set as $\mathrm{k} = 10^5/\mathrm{m}$ and $\omega = 1.5\omega_0$. Here $\omega_0 = (eB_0)/m_e$ and k is parallel to the static magnetic field. The electrostatic field and the static magnetic field are set as $E_0 = -2.5$ V and $B_0 = 2\times10^{-2}$ T respectively. As shown in Figure 7, when we increase the energy of EMW, the parallel velocity will be trapped in the resonant condition and stops increasing while the perpendicular velocity increase continuously when the ratio $E_w/E0$ exceeds a certain threshold. The trapped electron's orbit and moment are shown in Figure. 8.

The threshold field E can be obtained by modifying the EMW intensity using a dichotomy control approach based on the final parallel velocity with a long enough time. The critical rations of $E_c/E0$ with dimensionless parameters $\frac{\omega^2}{kc\omega_0}$ are shown in Figure 9 (here the angles between B_0 and wavevector k are $\theta = 0, 15^o, 30^o$).As observed, a larger angle between k and B_0 results in a less critical ratio for trapping electrons. This possibility could be attributed to the increase in the electric field along the parallel direction as the angle increases, which leads to an increase in the halting force along that direction. It is apparent that with only small intensity of left-hand circular polarized wave we can halt the increase of the electron's parallel momentum and transfer energy from electrostatic field to rotational energy by ADE . For example, in Tokamak the toroidal electric field is about 0.2 V/m , the threthod electric field for left-hand circular polarized wave to trap electron is about Ep = 2 V/m in the plasma (the corresponding energy flux is about 0.04 $\mathrm{W/m^2}$ for $\mathrm{k} = 2\times10^3/\mathrm{m}$ and $\omega = \omega_{ce}$ with $B_0 = 2T$) .

VI. SIMULATION WITH TOKAMAK PARAMETERS

In order to check the validity in high magnetic fields and the angle dependence of ADE, we choose uniform field of $\mathrm{B} = 2$ T and $\mathrm{E0} = -0.2\ \mathrm{V\cdot m^{-1}}$, which are typical parameters of a Tokamak during startup[9]. For a plane left-hand circularly polarized wave, $f = 56\mathrm{GHz}, E_p = 40$ V/m, and k =

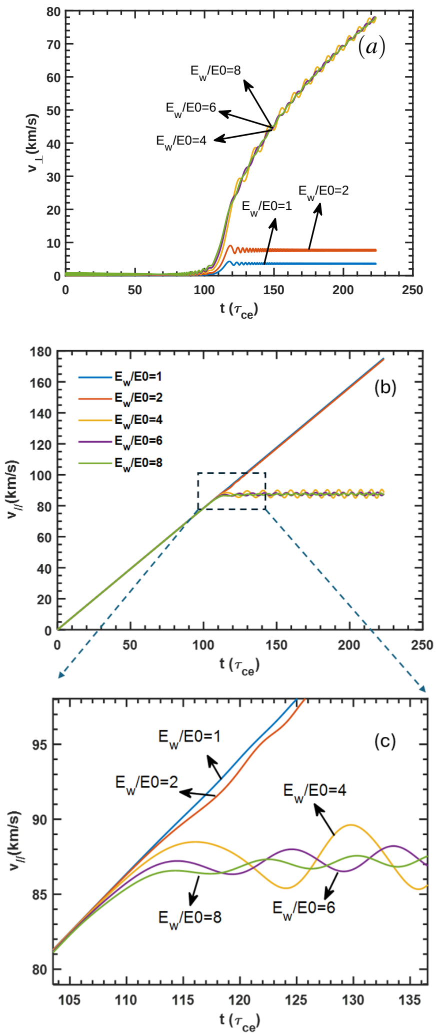

FIG. 7. Time trace of velocity under different ratio of $E_p/E0$. (a)vertical velocity (b)parallel velocity (c) Zoom in parallel velocity.

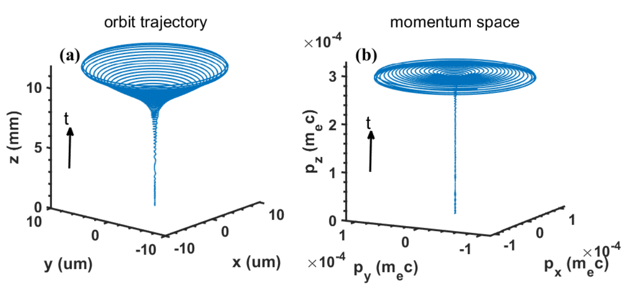

FIG. 8. The trapped electrons orbit and momentum

$2.6\times10^3/\mathrm{m}$,the energy flux of the wave is about 9 $\mathrm{W}\cdot\mathrm{m}^{-2}$. Using the powerful parallelism of supercomputer, We can simultaneously calculate the interaction of 500 electrons with a left-handed circularly polarized wave at 500 different incident angles θin the range of 0 to 90 degrees, demonstrating the dependence of the ADE effect on the angle. The time

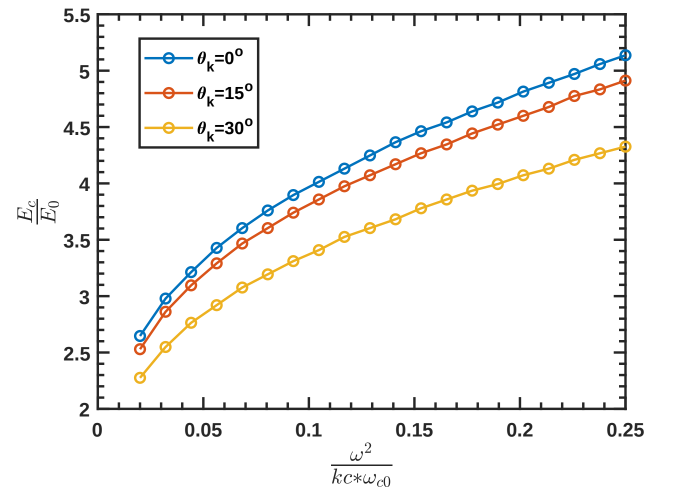

FIG. 9. Critical ratio of $\mathrm{E_c/E0}$ with normalized parameter $\frac{\omega^2}{kc\omega_0}$,where $E_0=-2.5$ V , $B_0=2\times10^{-2}$ T,$\omega=1.5\omega_0$ and the refractive index ranges from 4 to 50.

step in the simulation is set at $\Delta t=1\times10^{-14}$ s to ensure the convergence of the results. The results are shown in Fig. 10, where the dashed yellow line represents the earliest resonant time that satisfies the relation $\omega-\vec{k}\cdot\vec{v}+\omega_{ce}=0$ .It is evident that as the angle θ increases, the onset of resonance is delayed, and the requisite speed for resonance is heightened. Following the initiation of resonance, the rotational velocity $v_\perp$ witnesses a swift augment, whilst the parallel velocity gets ensnared within the resonant region, thereby ceasing any further increment. These results indicate that even with a small energy of left-hand polarized waves, we can suppress runaway electrons and transfer the acceleration from the electrostatic field to rotational energy in Tokamak parameters.

VII. THE CANDIDATE WAVE FOR ADE IN MAGNETIC PLASMA

It is well known that runaway electrons (RE) are detrimental to Tokamak devices, where their high energy can cause melting when they impact plasma-facing components (PFCs)[35] . Finding an effective way to avoid damage from runaway electrons is quite important. By studying the ADE effect, a question arises: Can the ADE effect be used in Tokamak to control the energy of runaway electrons? The controllable resonant conditions have three requirements:**1**. the existence of a left-hand polarized wave component when electrons move in the direction of the magnetic field;**2**. the phase velocity of EMW lower than the speed of light in vacuum and **3**. enough energy of the wave to stop the increasing velocity of the electron.

Consider waves in cold magnetized plasmas where $\Omega_{pe}/\Omega_{ce}=0.5$,from Appleton-Hartree formula(It is commonly believed that the cold plasma dispersion is generally adequate for calculating wave propagation when the phase ve-

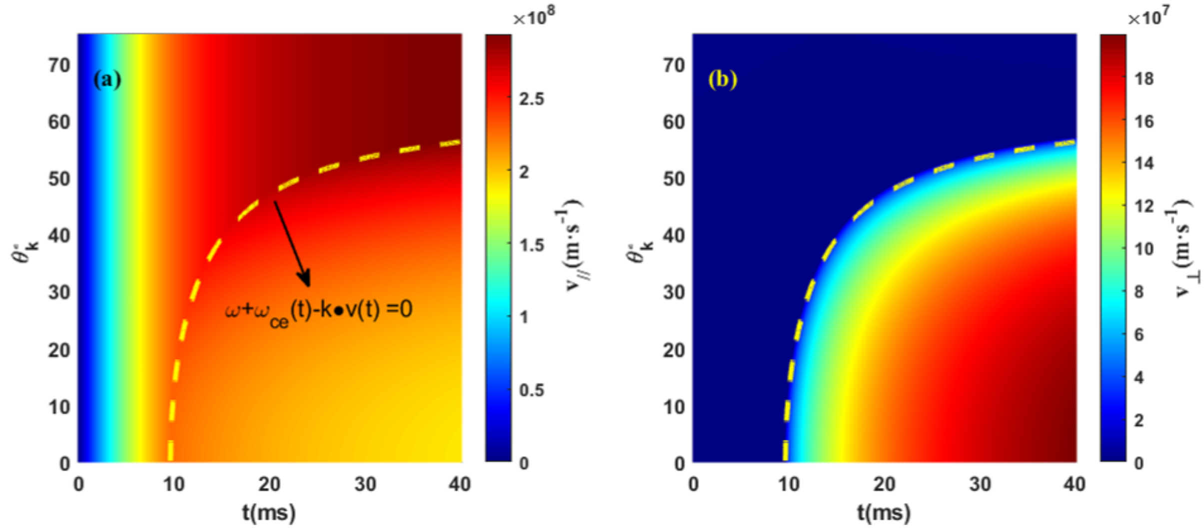

FIG. 10. Time evolution of $v_\perp$ and $v_\parallel$ by emw with different incident angle θ_k^o

locity significantly exceeds the thermal velocity of electrons in the plasma.)

$$
\begin{aligned}
&\omega^{10}-\omega^8\left(2k^2c^2+\omega_{ce}^2+\omega_{ci}^2+3\omega_{pe}^2\right)+\omega^6\left[k^4c^4+\left(2k^2c^2+\omega_{pe}^2\right)\left(\omega_{ce}^2+\omega_{ci}^2+2\omega_{pe}^2\right)+\left(\omega_{pe}^2+\omega_{ce}\omega_{ci}\right)^2\right]\\
&\quad-\omega^4\left[k^4c^4\left(\omega_{ce}^2+\omega_{ci}^2+\omega_{pe}^2\right)+2k^2c^2\left(\omega_p^2+\omega_{ce}\omega_{ci}\right)^2+k^2c^2\omega_{pe}^2\left(\omega_{ce}^2+\omega_{ci}^2-\omega_{ce}\omega_{ci}\right)\left(1+\cos^2\theta\right)+\omega_{pe}^2\left(\omega_{pe}^2+\omega_{ce}\omega_{ci}\right)^2\right]\\
&\quad+\omega^2\left\{k^4c^4\left[\omega_{pe}^2\left(\omega_{ce}^2+\omega_{ci}^2-\omega_{ce}\omega_{ci}\right)\cos^2\theta+\omega_{ce}\omega_{ci}\left(\omega_{pe}^2+\omega_{ce}\omega_{ci}\right)\right]+k^2c^2\omega_{pe}^2\omega_{ce}\omega_{ci}\left(\omega_{pe}^2+\omega_{ce}\omega_{ci}\right)\left(1+\cos^2\theta\right)\right\}\\
&\quad-k^4c^4\omega_{ce}^2\omega_{ci}^2\omega_{pe}^2\cos^2\theta=0
\end{aligned}
\tag{19}
$$

Where ω is the wave circular frequency and k represents the wavevector .The ω and k are both depend on the electron cyclotron frequency(ω_{ce}) , the plasma frequency (ω_{pe}) and the ion cyclotron frequency(ω_{ci}).Here θ is the angle between $\vec{k}$ and static magnetic field $\vec{B}$.The dispersion relationship can be illustrated in Fig. 11, where the blue color represents the wave-vector $\vec{k}\|\vec{B}(\theta=0)$, while the red color represents $\vec{k}\perp\vec{B}\left(\theta=\frac{\pi}{2}\right)$. The black line refers to the wave in a vacuum. As a result, only waves in regions A and B have the ability to cause ADE, where the phase velocity $v_p=\frac{w}{k}<c$ in these regions. The polarization vector of the wave can be expressed as[36]

$$
(e_x,e_y,e_z)=\left(1, i\frac{\frac{\omega_{pe}^2\omega_{ce}}{\omega}}{\omega^2-k^2c^2-\omega_{ce}^2-\omega_{pe}^2+\frac{k^2c^2\omega_{ce}^2}{\omega^2}}, \frac{k_\parallel k_\perp c^2}{\omega_{pe}^2+k_\perp^2c^2-\omega^2}\right) \tag{20}
$$

The electric field of the wave can be written as $\vec{E}=E_0\left(e_x+e_y+e_z\right)\exp(i(kr-wt))$ .When the imaginary part of e_y is positive, the main component of the wave is a right-hand polarized wave. Otherwise, it is predominantly left-hand polarized wave. The black dashed line represents the boundary between two different types of waves where $e_y=\infty$. Between the pair of dashed lines, the wave predominantly exhibits left-hand polarization. Conversely, beyond these demarcated lines, the wave is mainly right hand polarized. As depicted in Fig. 11, region B comprises whistlers and magnetized electron plasma wave, where all electromagnetic waves (EMW) mainly exhibit right-hand polarization.Conversely, in Region A, Extraordinary (X) waves display left-hand polarization when θ is in close proximity to $\pi/2$.

Consider the resonant condition

$$
\omega+n\omega_{ce}-\vec{k}\cdot\vec{v}=0 \tag{21}
$$

When the runaway electron's moment match the resonant

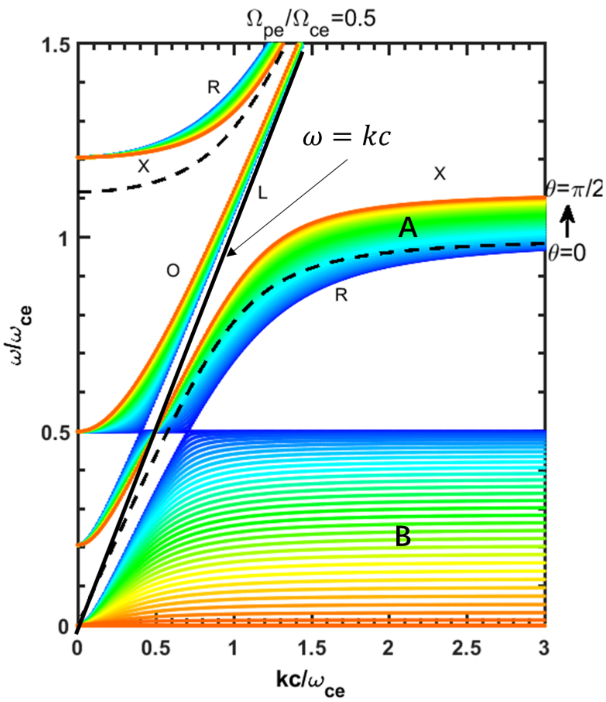

FIG. 11. Dispersion relationship in cold magnetized plasma

condition, it is possible to stimulate the intrinsic waves in plasma. Here we choose $n = 1$ for ADE and n = 0 for Landau resonance.The term n=-1 is disregarded in this context since NDE is not important in region A due to its requires right-hand polarized wave .By combining Eq. (19)and Eq. (21), we establish the relationship between the wave and resonance moment, as illustrated in Fig. 12. In region A , for ADE , resonance curves with dimensionless resonance momentum greater than 1 are converged to right bottom region shown in Fig. 12(c), which corresponds to the X wave with frequency range $\left(\omega_{ce}, \sqrt{\omega_{ce}+\omega_{pe}}\right)$. This explains why X wave near the frequencies of $\left(\omega_{ce}, \sqrt{\omega_{ce}+\omega_{pe}}\right)$ are excited when electron scattering occurs in magnetized devices[14,19]. Furthermore, the dimensionless Landau resonance momentum for most of region A is greater than 1, as shown in figure.12(a). This suggests less attenuation of waves by background thermal electrons and makes wave formation in the A region easier. For the B region, When the energy of high-energy runaway electrons exceeds 10MeV (reduced momentum p $>$ 20), the resonance curves of electromagnetic waves excited by the ADE effect almost always pass through left top region depicted in Fig. 12(d). This region corresponds closely to the whistler waves zone where whistler waves propagate parallel to the magnetic field. Therefore, in Tokamak experiments, the observation of whistler waves is typically associated with the detection of high energy electrons with energy exceeding 10MeV. The dimensionless Landau resonance momentum for region B is less than 1, as depicted in Fig. 12(b). This indicates a higher degree of wave attenuation by background thermal electrons compared to region A, making wave formation in region B more challenging.

Based on the above discussion, electromagnetic waves in region A are more prone to exciting Anomalous Doppler Resonance due to polarization and damping, while waves in region B are better suited for heating background electrons through Landau resonance, such as low-hybrid wave heating. Experiments show that runaway electrons can stimulate X waves with frequencies in the range ω_{ce} to ω_{uh} through ADE and transport their parallel energy to rotational energy[14,37]. It is natural to consider utilizing the reverse process by injecting X waves to suppress runaway electrons.

VIII. INJECT X WAVE TO SUPPRESS THE RUNAWAY ELECTRONS

The characteristic frequency with $\vec{k} \perp \vec{B}$ in Tokamak plasma is shown in figure.13 under condition where the intensity on the axis toroidal B is $B_0 = 2T, n_{e0} = 1 \times 10^{19}/\mathrm{m}^3$ and the density along the minor radius has profile as $n_e = n_{e0}\left(1 - r^2\right)$, r is the normalized minor radius. The X-Wave will slow down near f_{UH} layer and reflect at f_R layer. Therefore, it is necessary to input EX wave from high field side of Tokamak, whereas it will be reflected at the f_R layer when input from the lower-field side.Different frequency corresponds to different position. We can adjust the frequency of the X-wave to align with the region where runaway events are more likely to occur (such as in the core of Tokamak). Since the power requirement for trapping runaway electrons is not high, it is worth noting that precise frequency adjustments to match the core of the Tokamak based on real-time plasma density diagnosis is possible to achieve.

IX. SUMMARY

This study presents a simulation of the interaction between electrons and plane EMW within uniform magnetic and electrostatic field. The simulation demonstrates that the parallel velocity of electrons can be rapidly converted into rotational velocity during the ADE, with only left-hand circular polarization waves contributing to the ADE. When the electric field surpasses the critical field , the EMW can capture the parallel momentum of electrons and perpetually transfer energy from the parallel electrostatic field to rotational energy and resonance waves. The proportion between the loss of parallel energy and the augmentation in rotational energy aligns with both quantum theory and the simulation results. Furthermore, this study also delves into the fundamental physics of ADE from the basic energy-moment-angular moment conservation. We ascertain that the X wave, situated near the upper-hybrid resonance, emerges as the most apt candidate wave for mitigating runaway electrons. By injecting the X wave from the high-field side of the Tokamak, we can potentially inhibit the runaway electrons at the Tokamak core. It is possible to suppress runaway electrons at the core of the Tokamak based on real-time plasma density diagnosis by dynamically adjusting wave frequencies ,where the field intensity needed to achieve this suppression is only a few orders of magnitude greater than

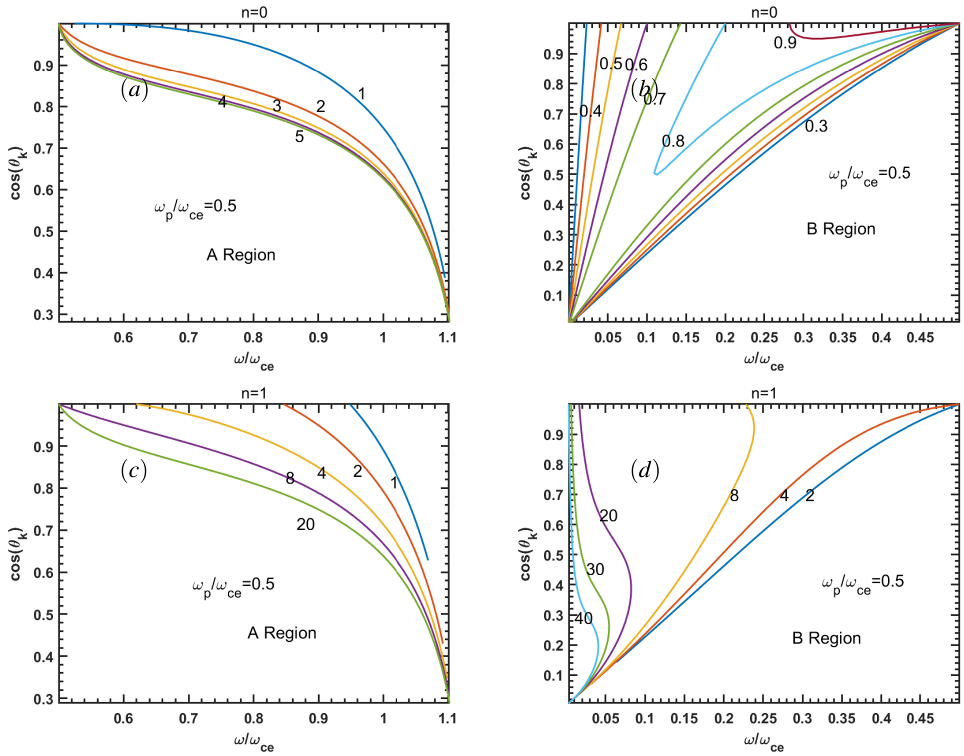

FIG. 12. In region A, the lower left boundary corresponds to the up-hybridized resonance frequency for different angles. In region B, the lower right boundary corresponds to the low-hybridized resonance frequency for different angles. (a) (b)dimensionless moment of landau resonance in region A and B. (c) (d)dimensionless moment of ADE in region A and B

the electrostatic field E_0 .

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X. APPENDIXES

Further investigation is necessary to comprehend why left-handed circular polarization is responsible for the ADE and what causes the scattering of the electron's parallel momentum to rotational moment. In this Appendixes, our objective is to understand this phenomenon using classical electrodynamic theory.

A. EMW polarization

During the resonance with the induced wave, the rotation of the electromagnetic field and electron motion become synchronized. The wave frequency is equal to the electron cyclotron frequency in the frame of a gyro-center moving electron. The frequency and wavevector in the two frames, namely the laboratory frame and the moving gyro-center electron frame, can be directly linked through the Lorentz transformation[10]:

$$\begin{bmatrix} \vec{k} \\ \frac{\omega}{c} \end{bmatrix} = \begin{bmatrix} \bar{\bar{\alpha}} & +\gamma\vec{\beta} \\ +\gamma\vec{\beta} & \gamma \end{bmatrix} \begin{bmatrix} \vec{k}' \\ \frac{\omega'}{c} \end{bmatrix} \tag{22}$$

where $\vec{k} = k_x\overrightarrow{e_x} + k_y\overrightarrow{e_y} + k_z\overrightarrow{e_z}$ $\left(\overrightarrow{k'} = k'_x\overrightarrow{e_x} + k'_y\overrightarrow{e_y} + k'_z\overrightarrow{e_z}\right)$ and $\omega\,(\omega' = \omega_0)$ are wavevector and the frequency in the laboratory frame (the moving gyro-center electron frame) respectively. $\vec{v} = v\overrightarrow{e_z}$ is the parallel velocity of the electron with normalized form being $\vec{\beta} = \frac{\vec{v}}{c}$, $\gamma = \left(1-\beta^2\right)^{-\frac{1}{2}}$ is the Lorentz factor. $\bar{\bar{\alpha}} = I + (\gamma-1)\frac{\vec{\beta}\vec{\beta}}{\beta^2}$ where $\mathbf{I}$ is unit tensor. From relationship (13),

$$\omega = \gamma\omega_0 + \gamma v {k'}_z \tag{23}$$

$$k_z = \gamma k'_z + \gamma\frac{v}{c}\frac{\omega_0}{c} \tag{24}$$

Substitute k_z with $\frac{\omega}{c'}\cos(\theta)$ and combine with (14)(15), we get the frequency in laboratory frame

$$\omega = \frac{\omega_{ce}}{1 - \frac{v}{c'}\cos(\theta)} \tag{25}$$

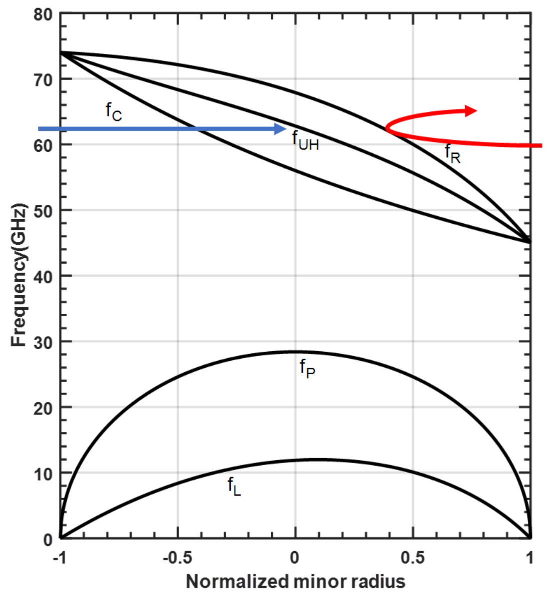

FIG. 13. Characteristic frequency distribution in plasma.f_c is electron cyclotron frequency,$f_{UH}$is upper hybrid frequency,f_p is plasma frequency and f_L is left-handed cutoff frequency, f_R is Right-handed cutoff frequency.

where ω_{ce} representing the right-hand rotation for an electron in magnetic field. In the case of the NDE,it is essential to note that the resonant velocity, denoted as v_{NDE}/c', must satisfy the condition $v_{NDE}/c' < 1$. This condition ensures that both angular frequencies, ω and ω_{ce} (where $\omega_{ce} = \frac{\omega_0}{\gamma}$), share the same sign. Consequently, it proves only waves exhibiting right-hand polarization can induce the NDE. In the case of the ADE, a different scenario arises. The resonant velocity is expressed as $v_{ADE} = \frac{\omega+\omega_{ce}}{k_z}$. For ADE to occur, the condition $\frac{v_{ADE}}{c'}\cos(\theta) = 1 + \left|\frac{\omega_{ce}}{\omega}\right| > 1$ must be satisfied. Consequently, it is evident that ω and ω_{ce} have opposite signs in Eq.(25), indicating that only left hand circular polarization waves are responsible for inducing the ADE .This feature has been rarely considered in previous theories of ADE. It is worth noting that the requirement of left-hand circular polarization for ADE performs does not mean that the wave should be completely left-hand circularly polarized. As long as there exists a component of left-hand circular polarization in the direction of the cyclotron electron's motion,it is possible to trigger the ADE , as observed with whistle waves in Tokamak systems[20].

B. Scattering

The scattering results shown in Figure 6 can be interpreted from classical electrodynamics as follows: Considering the circular polarization of EMW has the same frequency as ω_0 during resonance in the moving gyro-center electron frame, as shown in Figure 14, where the wavevector k parallel to the z-axis ,the perpendicular forces exerted by the magnetic field ((B)) and electric field ((E)) of EMW can be described as follows:

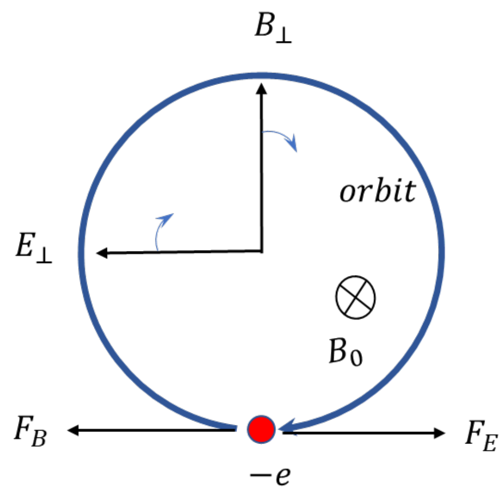

FIG. 14. Scheme of electron movement in uniform magnetic field with EMW,here B_0 is static magnetic field ,$E_\perp$ and $B_\perp$ are refer to electric field and magnetic field of EMW respectively.F_B represents magnetic force of EMW, F_E represents electric force of EMW

$$\vec{F}_{B\perp} = (-e)\overrightarrow{v_\parallel} \times \overrightarrow{B_\perp} = \frac{ev_\parallel k_z}{\omega}\overrightarrow{E_\perp} \tag{26}$$

$$\vec{F}_{E\perp} = (-e)\overrightarrow{E_\perp} \tag{27}$$

the forces $\vec{F}_{B\perp}$ and $\vec{F}_{E\perp}$ have opposite directions. The net force on electron in perpendicular direction is

$$\vec{F}_\perp = \vec{F}_{B\perp} + \vec{F}_{E\perp} = e\vec{E}_\perp\left(\frac{v_\parallel}{c'} - 1\right) = \vec{F}_{E\perp}\left(1 - \frac{v_\parallel}{c'}\right) \tag{28}$$

where $c' = \omega/k$,$k_z = k$.The electron velocity $v_\perp$ has tendency to parallel to $\vec{F}_\perp$ by changing the rotation phase of electron during resonant with EMW, as shown in Figure 14.The force from EMW paralleled to z is:

$$\vec{F}_\parallel = -e\left(\vec{v}_\perp \times \vec{B}_\perp\right) = \frac{-\mathrm{e}\vec{v}_\perp \cdot \vec{E}_\perp}{c'} = \frac{\vec{v}_\perp \cdot \vec{F}_{E\perp}}{c'} \tag{29}$$

the total work by EMW is

$$P = \vec{F}_\parallel \cdot \vec{v}_\parallel + \vec{F}_\perp \cdot \vec{v}_\perp = \vec{v}_\perp \cdot \vec{F}_{E\perp} \tag{30}$$

For NDE, $\frac{v_\parallel}{c'} < 1, |\vec{F}_{B\perp}| < |\vec{F}_{E\perp}|$, the rotational velocity $v_\perp$tend to parallel to $\vec{F}_{E\perp}$ under external EMW,then $\vec{v}_\perp \cdot \vec{F}_{E\perp} > 0$,$\vec{F}_\parallel > 0, \vec{F}_\parallel \cdot \vec{v}_\parallel < 0, \vec{F}_\perp \cdot \vec{v}_\perp > 0, P > 0$,EMW will heat and exert a parallel force, causing the electron to accelerate along the z-axis.

For the Cerenkov Effect(CE), where $\frac{v_|}{c'} = 1$ and $\vec{F}_\perp = 0$, the absence of a vertical force from EMW results in a random phase angle between $\vec{v}_\perp$ and $\vec{F}_{E\perp}$,so the average of $<\overrightarrow{F_\parallel}>= 0$, $<\overrightarrow{F_\perp}\cdot\overrightarrow{v_\perp}>= 0$, which means that the EMW doesn't take part in Cerenkov effect. The longitudinal wave is more relevant to Cerenkov resonant, which also known as landau resonant, where the electron rides on the crest of the wave, being exposed to a force thus continually absorbs energy from the wave until its velocity increases and drops out of phase with the wave. such as lower hybrid waves in plasma[15] .

For ADE, $\frac{v_\parallel}{c'} > 1, |\vec{F}_{B\perp}| > |\vec{F}_{E\perp}|$, the rotational velocity $v_\perp$tend to parallel to $\vec{F}_{B\perp}$ under external EMW,then $\vec{v}_\perp \cdot \vec{F}_{E\perp} < 0, \vec{F}_\parallel < 0, \vec{F}_\parallel \cdot \vec{v}_\parallel < 0, \vec{F}_\perp \cdot \vec{v}_\perp > 0, P < 0$.As a result,the electron will convert its kinetic energy into rotational energy and emit EMW. It is interesting to note that the magnetic field of the EMW performs positive work on the electron in the perpendicular direction and negative work in the parallel direction. The behavior of magnetic field in the EMW acts like a bridge connecting the parallel and perpendicular directions, transferring energy from kinetic energy to rotational internal energy. This fundamental physics process helps us understand why parallel kinetic energy can be converted into rotational energy.

C. Consistency between quantum theory and simulation

This section focuses on analyzing the energy flow during the Anomalous Doppler Resonance of electron excited by a left-hand circularly polarized wave with wave vector k parallel to the magnetic field B_0. In the vertical direction, the electron is only subjected to work by the electromagnetic wave, which can be divided into two components: $W_{B\perp}$, representing the work done by the magnetic field of the electromagnetic wave, and $W_{E\perp}$, representing the work done by the electric field of the electromagnetic wave. Here, $W_{B\perp} = \int \vec{F}_{B\perp} \cdot \vec{v_\perp} dt$ and $W_{E\perp} = \int \vec{F}_{E\perp} \cdot \vec{v_\perp} dt$. The total work of the EMW in the vertical direction is $W_{\perp emw} = W_{B\perp} + W_{E\perp}$. In the parallel direction, both the electrostatic field and the electromagnetic wave can exert work on the electron. This includes W_{E0}, representing the work done by the electrostatic field, and $W_{\parallel emw}$, representing the work done by the electromagnetic wave. Specifically, $W_{E0} = \int \vec{F}_{E0} \cdot \vec{v}_\parallel dt$ and $W_{\parallel emw} = \int \vec{F}_\parallel \cdot \vec{v}_\parallel dt$,where $\vec{F}_\parallel$ is Eq.(29). The total work in the parallel direction is given by $W_\parallel = W_{E0} + W_{\parallel emw}$. All this work can be calculated numerically. Here, we use the same setup as above with $\mathrm{E_c/E_0} = 6$, where the electron can be trapped by the EMW. Numerical results are shown in Figure 15, where $T_{k\perp}$ and $T_{k\parallel}$ are rotational energy and parallel kinetic energy of electron respectively. Figure 15(a) shows that EMW performs negative work on the electron in the parallel direction while positive work in the vertical direction,the total work of EMW on electron is negative ,which means the electron transfers energy to EMW . In the parallel direction, as shown in Fig. 15(b) ,the electromagnetic wave performs negative work, which is balanced by the positive work from the electrostatic field. This equilibrium causes the electrons velocity to stop increasing.In the vertical direction(Fig. 15(c)),the magnetic field of the EMW performs positive work, while the electric field of the EMW performs negative work. The total work in the vertical direction done by the EMW is equal to the change in kinetic energy of the electron, denoted as $T_{k\perp}$. This observation aligns well with the previous analysis(X B).

Let's consider the energy transformation coefficient.The decreases energy of the electron in the parallel direction is $\Delta T_{k\parallel} = -W_{\parallel emw}$, while the increase energy of the electron in the vertical direction is $\Delta T_{k\perp} = W_{\perp emw}$,the increased energy of EMW is $W_{\mathrm{emw}} = -\left(W_{\parallel emw} + W_{\perp emw}\right)$ .The ratio of the loss of parallel energy of the electron to the vertical energy is represented by $\eta_p = \frac{\Delta T_{k\perp}}{\Delta T_{k\parallel}}$, while to EMW is $\eta_w = \frac{W_{emw}}{\Delta T_{k\parallel}}$ .The numerical results of η_p and η_w are depicted in Fig.15(d) and .According to quantum theory, the decreases energy of the electron in the parallel direction is $\Delta T = \Delta \vec{p} \cdot \vec{v} = \hbar k v \cos\theta$. The increase in rotational energy is $\Delta U = n\hbar\omega_{ce}$, where we choose n = 1 for the anomalous Doppler effect at here, the photon energy is $\hbar\omega$. According to the simulation setup with the following parameters: $k = 10^5/\mathrm{m}$, $\omega = -1.5\omega_0$, $\theta = 0$, and $B = 0.02, T$, the resonant velocity is approximately $v = 86.8, \mathrm{km/s}$. Consequently, the ratio of $\eta_p = \frac{\Delta U}{\Delta T} = \frac{\omega_{ce}}{kv} = 0.4$, and the ratio of $\eta_\omega = \frac{\omega}{kv} = 0.6$. These values align well with classical electrodynamics calculations as shown in Fig.15(d).

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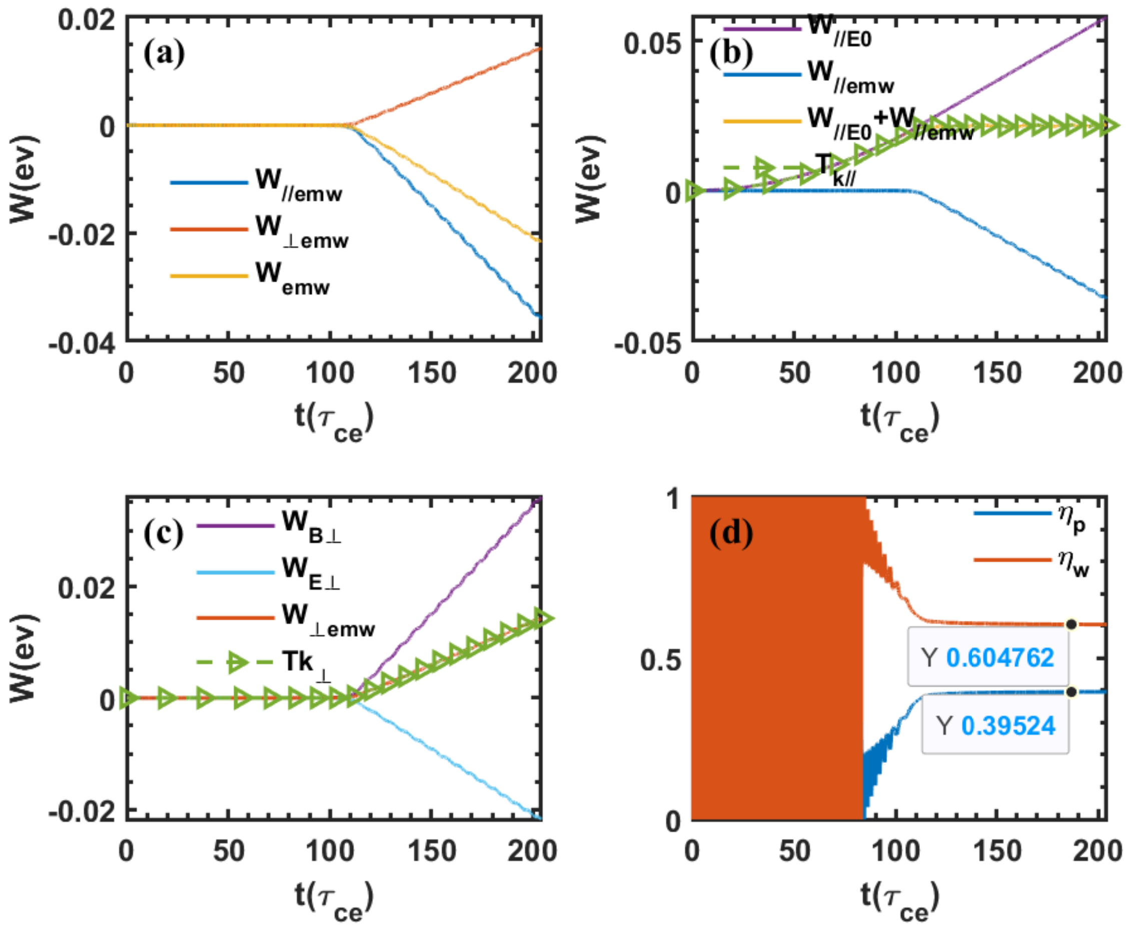

FIG. 15. Energy analysis of EMW and electrostatic on the resonant electron during ADR(Anomalous Doppler Resonance). (a) EMW's energy analysis for both direction (b) parallel direction. (c)vertical direction (d) conversion ratio of electrostatic work to EMW (η_w) and to internal energy of electron (η_p)

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