EXCITATION AND RADIATION OF AN ACCELERATED DETECTOR AND ANOMALOUS DOPPLER EFFECT

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The excitation of a uniformly accelerated detector moving in a vacuum has been widely discussed in recent years. The aim of the present paper is to point out that such an excitation and the associated radiation are similar to those occurring in the region of anomalous Doppler effect which takes place when a detector is moving at a constant superlight velocity in a medium.

The case of detector (an oscillator, an atom, etc.) excitation occurring when it moves in a vacuum with a constant acceleration a has been rather widely discussed in literature for already ten years, beginning with ref. [1]. In the stationary (equilibrium) state the probability of detector distribution in energy levels turns out to be the same as in the field of equilibrium thermal radiation with a temperature

$$T = \hbar a/2\pi kc. \tag{1}$$

First obtained in ref. [1] for a scalar massless field interacting with a detector of special type, this result was later extended to the case of other fields [2–8] and interactions with detectors of a more general type [9–12] (see also ref. [13]). We should emphasize that this result obtained initially in an accelerated coordinate system, in which the detector is at rest, remained for some time not quite comprehensible, and this especially concerned the change in the field state which accompanied the detector excitation. The question was clarified [14] when the process was considered in the inertial reference frame (i.e. in ordinary cartesian coordinates in Minkowski space–time). In such a coordinate system the excitation of a detector (for instance, a system with two levels 1 and 2) is accompanied by radiation of a quantum of the corresponding field with which the detector interacts (this important circumstance has been ignored or misunderstood in several earlier papers). A similar process (photon emission and thermal excitation of "internal" degrees of freedom) occurs in the case when an electron is moving with uniform acceleration in a homogeneous constant electric field $\mathscr{E}$, the role of energy "levels" of the detector being played by the energy of transverse motion of an electron, and the temperature is equal to (1) with $a = e\mathscr{E}/m$ [15]. This conclusion remains valid also in the case when there exists a magnetic field $\mathscr{H}$ parallel to $\mathscr{E}$ and the energy levels of the transverse motion are quantized (one can verify this using the results of ref. [16]). Thermal distribution with the same temperature (1) is also typical of the energy level populations due to the interaction between the spin of an accelerated electron with a magnetic field [17]. As for the nature and the physical mechanism of detector radiation and excitation they seem to remain not quite clear because of unwieldiness of the calculations [1,14] or the use of the "classical electromagnetic zero-point radiation" [3–5]. In this connection it would be instructive to pay attention (see also ref. [18]) to the fact that an analogous excitation and radiation occur in the well-known [19,20] case of the anomalous Doppler effect (ADE).

Let a two-level (the lower level ℓ and the upper level t) detector moving in a medium with a refractive index $n(\omega)$ at a constant velocity $\boldsymbol{v}$ emit a photon with momentum $\hbar k^\mu = (\hbar\omega/c, \hbar\boldsymbol{k})$, $k =$

$\omega n(\omega)/c$. (For definiteness we speak hereafter about a photon while all the consideration is also valid for quanta of other massless Bose-fields). Let us write the energy–momentum conservation law for photon radiation as follows

$$p_1^\mu - \hbar k^\mu = p_2^\mu,$$
$$p_i^\mu = \left(E_i/c \equiv \left[(m_0 + m_i)^2 c^2 + \boldsymbol{p}_i^2\right]^{1/2},\ \boldsymbol{p}_i\right), \qquad (2)$$

where p_i^μ is the four-momentum of the detector before ($i = 1$) and after ($i = 2$) radiation. Squaring eq. (2) we have

$$-\Delta\epsilon(2m_0 + m_1 + m_2)$$
$$= 2(E_1/c^2)\hbar\omega[1 - (vn/c)\cos\theta]$$
$$+ \hbar^2(\omega^2/c^2)(n^2 - 1), \qquad (3)$$

where $\boldsymbol{v} = \boldsymbol{p}_1 c/E_1$, $\boldsymbol{k}\cdot\boldsymbol{v} = kv\cos\theta$ and $\Delta\epsilon \equiv (m_2 - m_1)c^2$ is the change of the detector energy in its reference frame. In the approximation, in which $m_{1,2} \ll m_0$ and the recoil is negligible ($\hbar\omega/m_0c^2 \ll 1$), eq. (3) implies that

$$\hbar\omega = -\Delta\epsilon(1 - v^2/c^2)^{1/2}[1 - (vn/c)\cos\theta]^{-1}. \qquad (4)$$

In the region of the normal Doppler effect (NDE), when $(vn/c)\cos\theta < 1$ one has $\Delta\epsilon < 0$ and, therefore, $\epsilon_1 = \epsilon_t$ and $\epsilon_2 = \epsilon_\ell < \epsilon_t$. It means that the emission is possible only when the detector transits from the upper state t to the lower state ℓ. On the other hand in the ADE region when $(vn/c)\cos\theta > 1$ (i.e. $\theta < \theta_0$, where $\cos\theta_0 = c/nv$, θ_0 is the Vavilov–Cherenkov radiation angle) then $\Delta\epsilon > 0$, $\epsilon_1 = \epsilon_\ell$, $\epsilon_2 = \epsilon_t > \epsilon_\ell$ and, therefore, the emission of a photon is accompanied by excitation of the detector. The energy necessary for this process is gained from the kinetic energy of motion of the detector. In the methodical aspect the introduction of a medium makes it possible to consider the superlight motion ($v > c/n$), and therefore the ADE, which is absent for the sublight ($v < c/n$) motion. It should also be noted that the detector excitation and photon emission is possible even in the case when the detector is moving with constant sublight ($v < c/n$) velocity in a medium provided the refractive index n changes along the trajectory of motion due to variability of n in space and (or) in time (a kind of quantum analogue of transition radiation [20]).

So, a detector moving faster than light will become excited emitting photons even if at the beginning it was at the lowest (ground) level. In a stationary (equilibrium) state the population of the levels ℓ and t (or in the general case of any detector levels) is determined by the probability of radiation in the ADE and NDE regions (see ref. [20] and the literature cited there and refs. [21,22]). In hydrodynamics the situation is similar, namely, the role of superlight motion is played by the motion at a supersonic velocity [23,24].

In a vacuum, when a source is moving at a constant velocity $v < c$ the Vavilov–Cherenkov radiation (the radiation of a charged system without internal degrees of freedom) is absent and ADE is impossible. Therefore, a detector excitation does not occur. But a charge moving with an acceleration begins to radiate, of course, already in vacuum, and a detector, i.e. a system with internal degrees of freedom (an oscillator, an atom, etc), may radiate and become excited. We see here a close similarity with the ADE.

Conservation of energy and momentum for the detector and radiation is, of course, guaranteed, but with an account of forces accelerating the detector. To make sure of this, we suppose that a two-level detector as a whole possesses an electric charge Q and is placed in a homogeneous constant electric field $\mathscr{E}$ directed along the x-axis. The energy–momentum conservation law is of the form

$$P_1^\mu = P_2^\mu + \hbar k^\mu, \qquad (5)$$

where

$$P_i^\mu = p_i^\mu + q_i^\mu,$$
$$q_{i\mu} = QF_{\mu\nu}x_i^\nu = 2Q\mathscr{E}\delta^0_{[\mu}\delta^1_{\nu]}x_i^\nu, \qquad (6)$$

k^μ is the four-momentum of the emitted photon, p_i^μ and x_i^μ are the four-momentum and the coordinates of the detector before ($i = 1$) and after ($i = 2$) emission. (The expression for p_i^μ is given by eq.

(2).) From (5) we have

$$\hbar\omega = \Big[-\Delta\epsilon(2m_0 + m_1 + m_2)c^2 + 2p_1^\mu \Delta q_\mu - \Delta q_\mu \Delta q^\mu \Big] \times \{2E_2[1-(v_2/c)\cos\theta]\}^{-1}, \tag{7}$$

where $\Delta q_\mu = q_{2\mu} - q_{1\mu}$, $\boldsymbol{v}_2 = \boldsymbol{p}_2 c/E_2$, $\boldsymbol{v}_2 \cdot \boldsymbol{k} = v_2(\omega/c)\cos\theta$, $\Delta\epsilon = (m_2 - m_1)c^2$. In order that the photon emission be accompanied by a detector excitation ($\Delta\epsilon > 0$) it is necessary that the numerator of the right-hand side of (7) be positive. The corresponding condition in the approximation $m_{1,2} \ll m_0$ and $\hbar\omega/m_0c^2 \ll 1$ has the form

$$\Delta\epsilon < p_1^\mu \Delta q_\mu (m_0 + m_1)^{-1} \equiv Q\mathscr{E}\Delta x, \tag{8}$$

where Δx is the space distance between the positions of the detector at the moments $i = 1$ and $i = 2$ measured in the rest frame of the detector. The expression (8) can, of course, be obtained immediately by considering the detector which is at first at rest and assuming that a photon is emitted after the detector has passed a distance Δx. The condition (8) shows that the energy expended for detector excitation is gained from the energy of the detector-accelerating electric field. In our case there exists an additional conservation law associated with the Killing vector field $\zeta_\tau^\mu \partial_\mu = x\partial_t + t\partial x$. This law, which can be written in the form $-\zeta_\tau^\mu(p_{2\mu} - p_{1\mu}) \equiv \Delta\epsilon/c^2 = \hbar\zeta_\tau^\mu k_\mu$ implies that the photon accompanying the detector excitation ($\Delta\epsilon > 0$) may be created only in the region $ct - x > 0$, where $\zeta_\tau^\mu k_\mu > 0$.

In the general case the detector distribution over levels (that is, the population of the detector energy levels) depends on the character of its motion and on the probabilities of corresponding transitions [25]. In this respect the motion with a constant acceleration is singled out. The corresponding stationary (equilibrium) distribution over levels turns out to be thermal with the temperature (1). In the motion with a constant superlight velocity in a medium, that is, under ADE conditions, the absence of some universal temperature of detector excitation (except $T = \infty$) is clear already from dimensionality consideration (at the same time, in particular cases, the distribution over levels may be thermal with a temperature determined by the distance between levels [21,22]). The fact that the distribution over detector levels at $\boldsymbol{a} =$ const is thermal is closely connected with the equivalence principle. In a homogeneous and constant gravitational field $\boldsymbol{g} = -\boldsymbol{a}$ the thermodynamic equilibrium takes place with an ordinary thermal distribution. The value of the temperature (1) is singled out by the condition that the state corresponding to this temperature possesses a wider symmetry group (Poincaré group) and therefore the values of local observables (for instance, $\langle T_\mu^{\ \nu} \rangle^{\text{ren}}$) in this state are finite at horizons $ct \pm x = 0$.

It should be emphasized that the excitation of a detector with discrete energy levels or a detector in a vacuum which at the beginning is in the ground state has an essentially quantum character. If the initial amplitude of the classical oscillator vanishes, it will get excited neither in the case of ADE nor in the case of acceleration of the oscillator as a whole. On the contrary, a quantum oscillator, an atom, etc. which is in the ground state and interacts with an electromagnetic or with some other quantized fields will generally get excited both in the case of ADE [20] and in the case of acceleration. So, we deal with a quantum effect (hence we have $\hbar$ in (1), in this connection see also the remarks in ref. [18]).

There exists another class of phenomena, namely, the radiation of polarizable bodies, which is closely related to radiation of a moving detector. From the point of view of the microscopical theory such bodies consist of particles which possess a dipole moment or acquire it under the action of the external field. Polarizable bodies may therefore be regarded as a special type of detectors with an extremely large number of degrees of freedom and small distances between levels. In the same conditions in which a detector may be excited and emit photons polarizable bodies generally begin to radiate. A well-known example is a quantum radiation of mirrors (which may be regarded as bodies with infinite polarizability) when they are accelerated [13]. The radiation of a uniformly moving superlight neutral polarizable particle considered in [26] is an analogue of ADE occurring due to zero-point oscillations of electric polarizability. In this connection we may mention also the quantum effect of wave generation by a rotating absorbing body [27,28], which is associated with the effect of excitation and radiation of a detector in its circular motion [25,29]. It is of interest to consider an accelerated motion of superlight sources in a medium (or near a medium, in particular, in an empty channel [20]). A combination of ADE and acceleration may change an equilibrium distribution over levels and may lead, in particular, to the appearance of inverse population of levels. An example of a system, admitting probably its complete analysis, is an electron moving in a magnetoactive plasma (with an account of electron spin and its flip-over, electron acceleration by the field and, of course, dielectric permittivity of plasma).

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