RESEARCH ARTICLE | JUNE 01 1968

Charged-Particle Motion in Large-Amplitude Electromagnetic Fields

H. R. Jory; A. W. Trivelpiece

J. Appl. Phys. 39, 3053–3060 (1968)
https://doi.org/10.1063/1.1656732

Charged-Particle Motion in Large-Amplitude Electromagnetic Fields*

H. R. JORY
Varian Associates, Palo Alto, California
AND
A. W. TRIVELPIECE†
Department of Physics and Astronomy, University of Maryland, College Park, Maryland

(Received 29 January 1968)

The relativistic motion of a charged particle in large amplitude, inhomogeneous electromagnetic fields is studied for several situations. This study involved the numerical integration of the equations of motion including the relativistic mass correction and the nonlinear rf **v×B** force term. The cases studied include particle motion in linearly and circularly polarized, homogeneous plane waves with a homogeneous steady magnetic field parallel to the direction of propagation; particle motion in linearly and circularly polarized TE-11 modes in waveguides of circular cross section immersed in a uniform steady axial magnetic field; and particle motion in cavity resonators of circular cross section (TE-111 mode) immersed in a nonuniform steady axial magnetic field. This study is concerned with the maximum energy gains possible in the above situations with field amplitudes appropriate to high-power microwave experiments. To verify the features of the particle motion as predicted in the numerical study, and to investigate effects which could not be easily included in the analysis, such as space charge from the electron beams, beam-induced cavity fields, and ionization of residual gas in a cavity resonator, certain experiments were performed. These involved the injection of 1–100-mA electron beams at 100–1000 V on the axis of circular cross-section cavity resonators (10 GHz and 1.0 GHz) immersed in a steady axial magnetic field. With pulsed operation, an energy increase of 460 keV was obtained for a drive power of 46 kW at an over-all efficiency of acceleration of approximately 10%. Higher-efficiency pulsed operation and continuous operation at lower power levels were also investigated experimentally.

I. INTRODUCTION

Roberts and Buchsbaum[1] investigated the motion of a charged particle in the field of a homogeneous plane electromagnetic wave propagating along a constant homogeneous magnetic field. Their analysis included the relativistic mass correction in the equation of motion and the magnetic field of the electromagnetic wave in the force equation. For the case of a charged particle started from rest in the field of a circularly polarized plane wave whose frequency is equal to the rest mass cyclotron frequency ($\omega=\omega_{c0}=qB_0/m_0$), they found a "synchronous" solution in which the charged particle could gain energy indefinitely. This "synchronous" solution occurs because the particle gains energy parallel to, as well as perpendicular to, the direction of propagation of the circularly polarized plane wave. The increase in perpendicular energy lowers the cyclotron frequency of the particle [$\omega_c=qB_0/\gamma m_0$, $\gamma=1/(1-v^2/c^2)^{1/2}$], while the increase in parallel energy changes the velocity of the particle which results in a Doppler shift to a lower frequency as seen by the particle. For this special case, the Doppler shift to the lower frequency and the reduction in the cyclotron frequency are equal and the particle remains "synchronously" in cyclotron-resonance condition. That such an effect can exist was first realized by Davydovskii.[2]

Krasovitsky and Kurilko[3,4] have studied the problem of the limitations imposed by radiation reaction on the maximum energy which a synchronously driven particle can have, and also the effects of space-charge fields on the acceleration mechanism when a collection of charged particles are involved. With respect to the space-charge problem, they found that the energy of a typical particle is periodic and that a particle which initially had zero energy will again come to a condition of zero energy, neglecting radiation reaction forces.

The energy gained by a particle as a function of distance along the direction of propagation can be obtained from the work of Roberts and Buchsbaum[1] by eliminating time from the equations for energy gain and distance. The result for a particle starting from rest in a plane wave of amplitude E and wavelength λ for the synchronous condition is

$$W_c/m_0c^2=[(3/\sqrt{2})AZ]^{2/3}, \tag{1}$$

where $A=E\lambda q/2\pi m_0c^2$ and $Z=2\pi z/\lambda$ are the dimensionless amplitude and distance variables. In terms of the same variables, the energy gained by a particle in a

* Part of the experimental work reported here was supported under a contract with the U.S. Army Electronics Command, Fort Monmouth, N.J.

† Consultant to Varian Associates.

[1] C. S. Roberts and S. J. Buchsbaum, Phys. Rev. **135**, A381 (1964).

[2] V. Ya. Davydovskii, Zh. Eksper. i Teor. Fiz. **43**, 886 (1962). [English transl: Sov. Phys.—JETP **16**, 629 (1963).]

[3] V. B. Krasovitsky and V. I. Kurilko, Zh. Eksper. i Teor. Fiz. **48**, 353 (1965). [English transl: Sov. Phys.—JETP **21**, 232 (1965).]

[4] V. B. Krasovitsky and V. I. Kurilko, Vuzov-Radio Physics, News of the Higher Schools **7**, 1193 (1964).

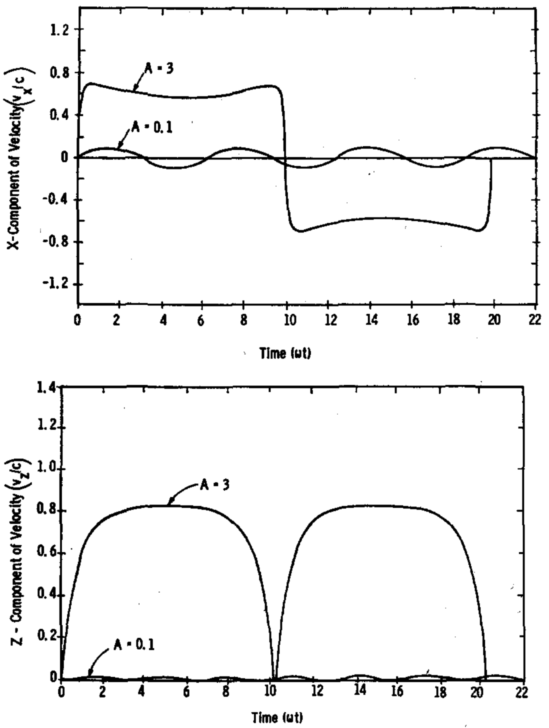

FIG. 1. The x and z components of velocity of charged particle started from rest in a homogeneous plane electromagnetic wave propagating in the z direction and polarized in the x direction, $E_x = E_0 \exp[i(\omega t - kz)]$. $A = E_0 \lambda q / 2\pi m_0 c^2$.

linear accelerator is the product of axial electric field and axial distance:

$$W_L/m_0c^2 = AZ. \tag{2}$$

This illustrates that at the same field strength, a linear accelerator is more effective in accelerating charged particles to high energy, but for lower-energy beams the transverse acceleration is more effective. For example, $W_c > W_L$ for $AZ < 4.5$. For electrons, $AZ = 4.5$ corresponds to a voltage of 2.3 MV and, for example, to a field of 10^7 V/m and a distance of 0.23 m.

The synchronous solution discussed by Roberts and Buchsbaum[1] occurs for the case of a circularly polarized, homogeneous plane wave propagating in the same direction as a homogeneous steady magnetic field.

In this paper we account for spatial variations of the electromagnetic wave field and the steady magnetic field. These studies were done to determine the maximum energy gain for a charged particle in a section of waveguide or a cavity resonator immersed in a magnetic field as a function of the various parameters such as field amplitude, initial conditions, polarization, and cavity or waveguide mode. This work was done in conjunction with an experiment involving the injection of low-energy electrons into a high Q cavity resonator to measure the energy gain as a function of the various parameters. The experiment permitted the investigation of other problems such as the effect of cavity loading by the electron beam and the effect of beam space charge on the acceleration mechanism. The first part of the paper contains results for a variety of cases which were chosen to illustrate the various effects which resulted from the nonlinear interaction with the large-amplitude electromagnetic field. The second part of the paper describes the experiments to investigate the features of the nonlinear interaction.

II. EQUATIONS OF MOTION

To study the motion of charged particles in large-amplitude electromagnetic fields, the relativistic equations of motion were solved numerically for specific cases. The relativistic force equation for a particle having mass m, charge q, position $\mathbf{r}$, and velocity $\mathbf{v}$ in an electric field $\mathbf{E}$ and magnetic field $\mathbf{B}$ is (mks units)

$$(d/dt)[m\mathbf{v}(\mathbf{r},t)] = q[\mathbf{E}(\mathbf{r},t) + \mathbf{v}(\mathbf{r},t) \times \mathbf{B}(\mathbf{r},t)]. \tag{3}$$

By writing the relativistic mass in terms of rest mass and velocity

$$m = m_0/(1 - v^2/c^2)^{1/2},$$

and by performing considerable algebraic manipulation, we can put Eq. (3) into the following form:

$$(d/dt)\mathbf{v}(r,t) = (q/\gamma m_0)\{\mathbf{E}(r,t) + \mathbf{v}(r,t) \times \mathbf{B}(r,t) - [\mathbf{v}(r,t)/c^2][\mathbf{v}(r,t) \cdot \mathbf{E}(r,t)]\}. \tag{4}$$

This form is more convenient for numerical integration.

The rest mass is denoted by m_0 and the relativistic factor $1/(1 - v^2/c^2)^{1/2}$ is denoted by γ. The position of the particle is given by $\mathbf{r}$, where

$$(d/dt)\mathbf{r} = \mathbf{v}(\mathbf{r},t). \tag{5}$$

III. CALCULATED RESULTS

The nonlinear differential equations of motion were solved numerically for a number of situations. In most cases the numerical-integration step was chosen to give an error per step of less than one part in 10^6. The cumulative error in the results which follow is estimated to be less than one part in 10^4.

A. Homogeneous Plane Wave

1. No Steady Magnetic Field—Linearly Polarized, Traveling Wave

Figure 1 displays plots of the x and z velocities of a charged particle initially at rest in a homogeneous, linearly polarized plane wave propagating in the z direction with its electric vector in x direction. There is no steady magnetic field and the particle moves in the x–z plane. This case illustrates that a charged particle which gains a significant fraction of rest mass energy in a quarter cycle of the wave will also acquire a velocity in the direction of propagation that is a significant fraction of the velocity of light. The z velocity arises from the vector product of transverse velocity and transverse rf magnetic field. It is consistent with conservation of energy and momentum between the particle and the wave. It is approximately true that $c\Delta mv_{\|}=\Delta mv_{\perp}^2$, where parallel and perpendicular refer to the direction of propagation. It is convenient to use a dimensionless parameter to characterize the amplitude of the electromagnetic wave. The results are then applicable for electromagnetic waves of any wavelength much greater than the dimensions of the electron. This amplitude parameter is defined by

$$A=E\lambda q/2\pi m_0c^2.$$

For values of $A\ll 0.1$, the nonlinear effects are small and can generally be neglected.

For values of $A=0.1$ and greater, the nonlinear effects due to the Lorentz force and the relativistic mass are not negligible. Figure 1 shows that a particle can move a large fraction of a wavelength in one cycle, and thus that the usual assumption that a particle remains at a fixed value of z is not valid.

2. *Uniform Steady Magnetic Field*

(*a*) *Circularly polarized, traveling wave:* Figure 2 is a plot of the kinetic energy gained as function of distance by a particle initially at rest in a homogeneous, circularly polarized plane wave propagating in the z direction with a uniform steady magnetic field in the z direction. The energy gain is measured in units of rest mass energy

$$W/m_0c^2=1/(1-v^2/c^2)^{1/2}-1=\gamma-1,$$

and the distance in the z direction is measured in terms of AZ, the product of normalized distance and normalized wave amplitude. For the synchronous case

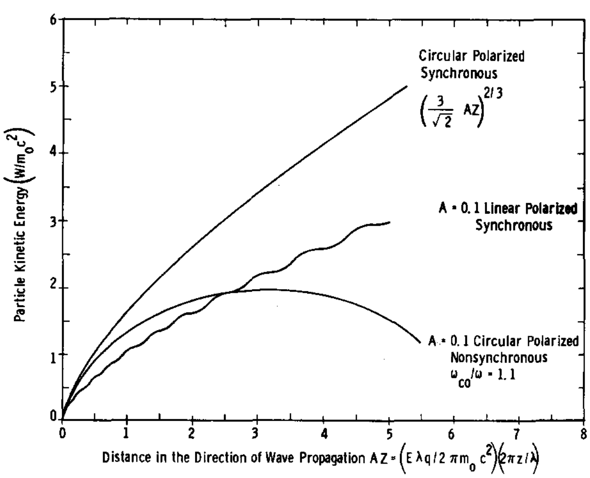

FIG. 2. Energy as a function of distance for a charged particle initially at rest in a homogeneous plane electromagnetic wave propagating in the z direction along a uniform steady magnetic field.

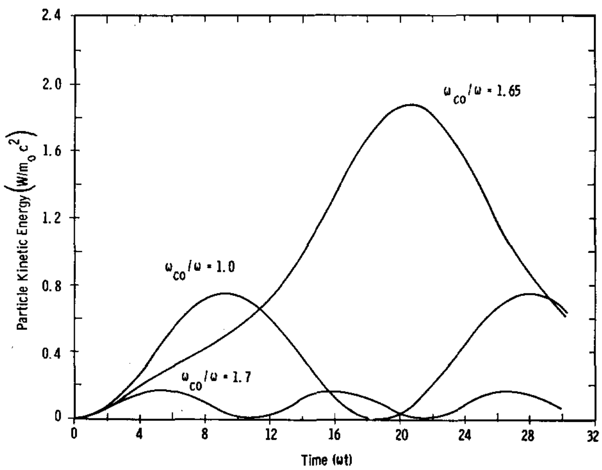

FIG. 3. Energy as a function of time for a charged particle initially at rest at the plane of maximum electric field in a pure standing wave caused by homogeneous, plane, circularly polarized, electromagnetic waves propagating in the $\pm z$ direction; $A=0.2$ where A is related to peak cavity field.

where the rest mass cyclotron frequency is equal to wave frequency, the computer result should agree with the analytic result of Roberts and Buchsbaum,[1] where the energy gain is proportional to $(AZ)^{2/3}$. In this case the results do agree with a difference of less than one part in 10^4. For a nonsynchronous situation, the numerically computed energy gain is a periodic function of distance, with the maximum energy gain always less than the $(AZ)^{2/3}$ synchronous case.

(*b*) *Linearly polarized, traveling wave:* Figure 2 also shows the energy gain as a function of distance for a linearly polarized synchronous case. This illustrates that for the same peak field amplitude the rate of energy gain as a function of distance is about half and that there is a slight interaction with the other circularly polarized component which causes a perturbation on the energy. More important is the fact that the synchronous solution is still possible in a linearly polarized wave, which is generally easier to set up in the laboratory.

(*c*) *Circularly polarized, standing wave:* This case was studied to investigate effects similar to those that might occur in a cavity resonator, but without the complication of the spatial variation of the field in the transverse direction and without the complication of an rf field component in the direction of propagation.

The first situation considered is that of charged particle started from rest in a pure standing wave at the plane of maximum electric field ($z=0$) for various values of the ratio of cyclotron frequency to wave frequency. Figure 3 is a plot of energy as a function of time for a particular electric field amplitude and for cases where the rest mass cyclotron frequencies, ω_{c0}, is taken both greater than and equal to the rf frequency. These results agree with those obtained by Hakkenberg

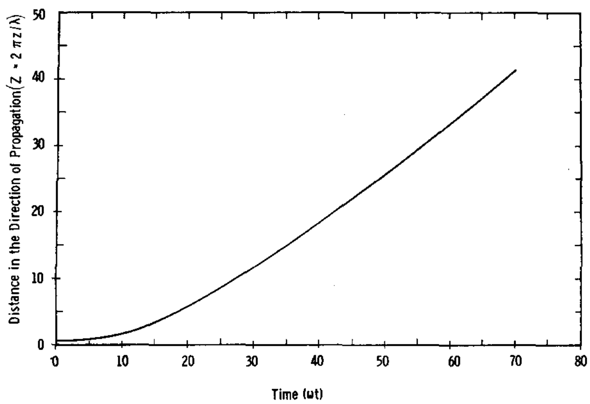

FIG. 4. The z position as a function of time for a charged particle initially at rest at the plane $Z=\pi/4$ (i.e., one-eighth of a wavelength from the plane of maximum electric field) in a pure standing wave caused by homogeneous, plane, circularly polarized, electromagnetic waves propagating in the $\pm z$ direction each having amplitude $A=0.1$, with a z-directed steady magnetic field having the synchronous value.

and Weenijk[5] and show that the optimum situation (i.e., maximum energy gain) does not occur exactly at resonance. The case $\omega_{c0}=1.65\omega$ is near optimum for this electric-field amplitude.

If a charged particle is started at rest from some other position, not at the plane of maximum electric field or at the plane of maximum magnetic field, the particle is initially accelerated in the z direction toward the plane of minimum electric field. Under some conditions, the particle will become synchronous with one of the traveling wave fields (sometimes after several oscillations about the plane of minimum electric field) and will essentially ignore the other traveling wave and accelerate at about the same rate as occurs for the ideal case of a circularly polarized traveling wave. Figures 4 and 5 illustrate this situation by showing the z position and kinetic energy of a particle started from rest at a position $Z=\pi/4$ (midway between electric- and magnetic-field maxima) in pure standing wave of peak amplitude $A=0.2$. (Forward and backward waves each have an amplitude of $A=0.1$.)

B. Transverse Electric Modes in Circular-Cross-Section Waveguides

1. No Steady Magnetic Field—Linearly Polarized, Traveling Wave

It is not easy to generate large-amplitude, homogeneous-plane electromagnetic waves in the laboratory. However, it is possible to generate large-amplitude ($A>0.1$) electromagnetic waves in waveguides of rectangular and circular cross section. To investigate the consequences of the spatial variation of the rf fields and the presence of the z component of the rf magnetic field on the acceleration of charged particles in waveguide and cavity resonators, the equations of motion were solved for a number of cases. Although the same acceleration mechanism operates in waveguide fields, there does not appear to be "synchronous" solution where a charged particle can gain energy indefinitely, as is the case with the circularly polarized homogeneous plane wave situation. The investigation was limited to transverse electric modes for the reason that these are the dominant waveguide modes, and near the axis the field is nearly the same as for a plane wave.

Figure 6 shows the projection of the x and y position on the x–y plane for a charged particle started at rest off the axis of a circular cross-section waveguide propagating a linearly polarized TE-11 mode in the positive z direction with electric field in the x direction. As can be seen, particles move rapidly out to the wall. There is no trapping of the particles and there is no significant energy gain over that possible by a one-half cycle interaction with the wave. Similar behavior applies for all particles regardless of initial phase or transverse position, except that particles starting at $y=0$ positions will remain at $y=0$ and merely oscillate in x.

2. Uniform Steady Magnetic Field—Circularly Polarized, Traveling Wave

The closest laboratory realization of the idealized homogeneous, plane-wave synchronous solution is achieved with a circular cross-section waveguide propagating a TE-11 mode in the z direction immersed in a uniform steady magnetic field also in the z direction. Figure 7 shows a plot of the gain in energy as a function of distance for a charged particle started from rest on the axis in such a situation. The initial energy gain is more rapid than that for the ideal plane-wave situation, but eventually the rate of energy gain becomes

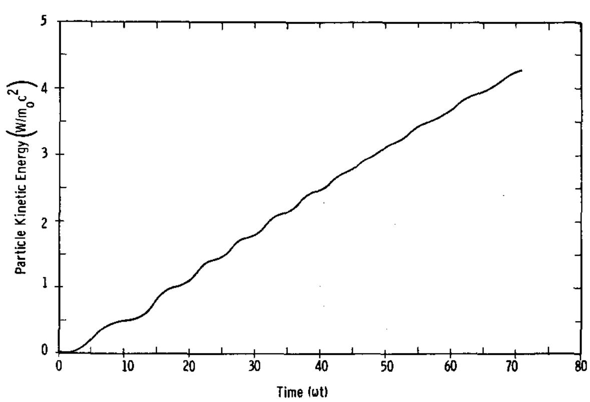

FIG. 5. Energy as a function of time for a charged particle initially at rest at the plane $Z=\pi/4$ (i.e., one-eighth of a wavelength from the plane of maximum electric field) in a pure standing wave caused by homogeneous, plane, circularly polarized, electromagnetic waves propagating in the $\pm z$ direction each having amplitude $A=0.1$, with a z-directed steady magnetic field having the synchronous value.

[5] A. Hakkenberg and M. P. H. Weenijk, Physica **30**, 2147 (1964).

less due to phase slipping and transverse field variations. The more rapid initial energy gain is caused by the fact that the transverse rf magnetic field in the waveguide is less than that for the plane wave, and therefore the z velocity is also less. If energy is plotted vs time, the initial energy gains are identical. Figure 7 also shows the energy for a case where $\omega_{c0} > \omega$. This situation was investigated to see if there was a "synchronous" solution in waveguide fields which occurred at some other magnetic field than the value which gave rest mass cyclotron resonance with wave frequency. Some optimization is achieved; however, there does not appear to be a solution where a particle can remain in cyclotron resonance indefinitely. In any event, the particle would eventually intercept the waveguide wall, unless the steady magnetic field is increased as a function of distance. Calculations were made with the steady magnetic field increasing in the z direction and with a radial component of magnetic field to satisfy the divergence condition ($\nabla \cdot \mathbf{B} = 0$). The radial magnetic field causes a reduction in z velocity, which tends to put the particle out of synchronism.

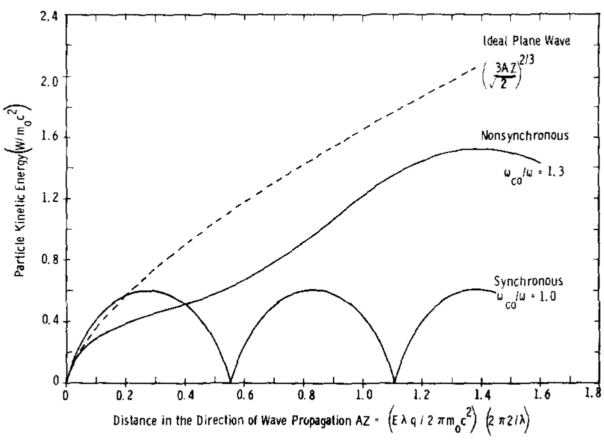

FIG. 7. Energy as a function of distance for a charged particle started from rest on the axis of a circular cross-section waveguide propagating a TE-11 mode in the z direction. The waveguide is immersed in a uniform axial steady magnetic field, in one case equal to the synchronous value, and in the other case greater than the synchronous value. Electric field amplitude is $A = 0.1$. The ratio of wavelength to cutoff wavelength is 0.707.

3. *No Steady Magnetic Field—Charge Column on Axis—Circular Polarization, Traveling Wave*

rf breakdown of an evacuated waveguide is sometimes accompanied by the emission of x rays whose energies are considerably in excess of that expected on the basis of the electric field strength at breakdown powerflow. A proposed mechanism to account for these x rays is based on the assumption that the breakdown produces a positive-charge column on the axis of the waveguide. The electric fields from the positive-charge column can cause electrons to orbit about the guide axis as though a steady axial magnetic field were present. No attempt to justify this model is made other than the result of the computation which is based on the assumption of a waveguide of circular cross section filled with a uniform-density positive charge and propagating a circularly polarized TE-11 mode. The radial electric force replaces the Lorentz force of the magnetic field, and electrons launched properly will gain energy in axis-encircling orbits in a manner similar to that obtained with the synchronous solution.

The numerical solutions show that an electron could gain 4 MeV in 1-m length of waveguide that was carrying a power flow of 200 MW at 10 cm wavelength, and which had a charge density of positive ions of $n_i = 10^{11}/\text{cm}^3$. Figure 8 shows the energy gain as a function of distance. It is seen that the rate of energy gain is $\sim 25\%$ slower than in the corresponding magnetic field case (Fig. 2). In this case the value of the positive ion density was chosen arbitrarily to realize a large energy gain.

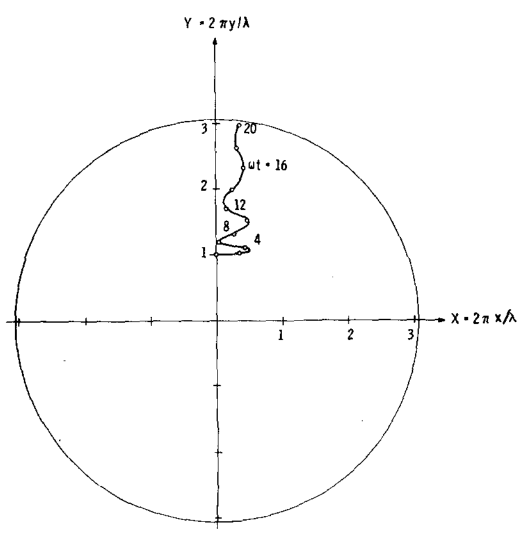

FIG. 6. Projection of particle trajectory on the x–y plane for a particle started at rest off the axis of a circular cross-section waveguide propagating a linearly plarized TE-11 mode in the z direction with amplitude $A = 0.3$. No steady magnetic field is present. The ratio of wavelength to cutoff wavelength is 0.6.

C. Transverse Electric Modes in Circular-Cross-Section Cavity Resonators—Uniform Steady Magnetic Field—Circularly Polarized, Standing Wave

As a practical matter, one method of achieving the large-amplitude electromagnetic field required to produce significant acceleration in a short physical length is to use a cavity resonator. Two experimental cavity-resonator systems were built to study the large-amplitude effects discussed in this paper. Cases which approximated the experimental situations were studied for a wide variety of initial conditions and steady magnetic-field configurations. Space does not permit inclusion of all these data, so results for only a few of the

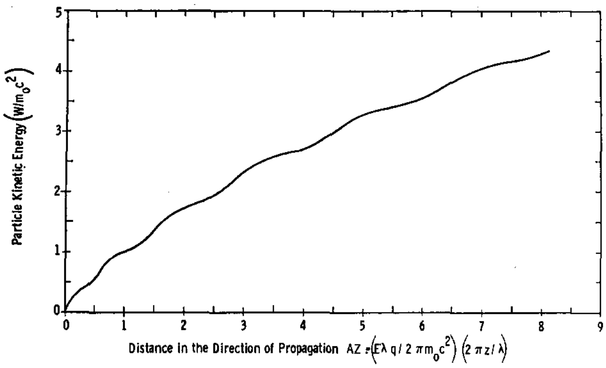

FIG. 8. Energy of a negatively charged particle in circular cross-section waveguide propagating a circularly polarized TE-11 mode in the z direction and filled with a uniform density of stationary positive charges. The electric field amplitude is $A=0.3$, and the ratio of wavelength to cutoff wavelength is 0.6. For the positive charges, the ratio of plasma frequency to signal frequency is 1.18.

most interesting cases are given. Figure 9 displays a plot of the x and y positions of the particle projected on the x–y plane for a particle injected into the cavity on axis with conditions specified (see caption) at $z=0$, the end of the cavity.

Figure 10 shows the z velocity as a function of position along the cavity, and Fig. 11 shows the energy gain as a function of position through the cavity for the same case. Results are given for three values of steady magnetic field. As can be seen, a rather large energy gain can be realized in a short distance for particle energies up to 0.5 MeV in this case. The particle behavior is sensitive to steady magnetic field and rf field amplitudes. The cavity length-to-diameter ratio is not a particularly sensitive parameter, nor is the particle initial velocity. The z velocity is strongly affected by rf forces even though there is no z component of electric field. Both the second and third terms in Eq. (4) contribute to changing the z velocity in a rather complicated manner.

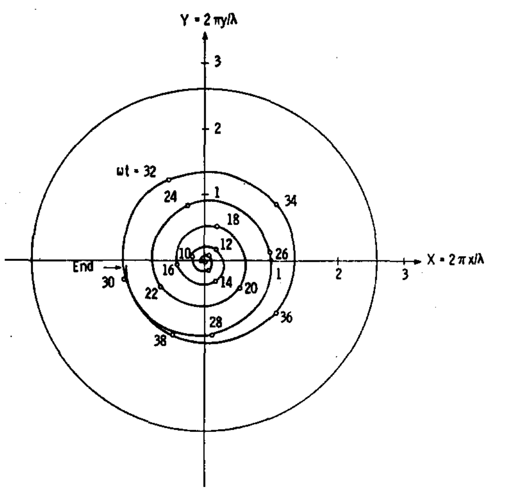

FIG. 9. The projection of the motion of a charged particle on the x–y plane for a particle injected on axis into the fields of a circular cross-section resonator oscillating in the TE-11 mode, circularly polarized. The cavity is immersed in a uniform steady magnetic field aligned along the cavity axis with $\omega_{c0}/\omega=1.5$. The peak electric field in the cavity has an amplitude $A=0.2$, and the initial velocity of the particle is $v_z/c=0.05$. The cavity length to diameter ratio is 0.85.

IV. EXPERIMENT

The experiments to study the interaction of electrons with large-amplitude electromagnetic fields were done in the device depicted schematically in Fig. 12. Two experimental devices were constructed, one at 30-cm wavelength, and one at 3-cm wavelength; however, the same schematic drawing applies for both.

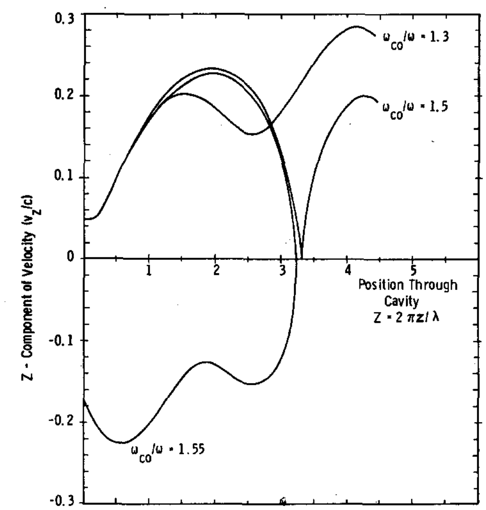

FIG. 10. The z component of the velocity of a charged particle injected on axis into the fields of a circular cross-section cavity resonator oscillating in the TE-11 mode, circularly polarized, as a function of position through the cavity. The cavity is immersed in a uniform steady magnetic field aligned along the cavity axis. The peak electric field amplitude is $A=0.2$, the cavity length to diameter ratio is 0.85, and the initial velocity of the particle is $v_z/c=0.05$.

An experiment consists of injecting a low-energy (100–1000 V), low-current (1–100 mA) electron beam through the end wall of a circular cross-section cavity resonator oscillating in the TE-111 mode. The rf drive is arranged so the cavity can be excited with either circular or linear polarization.

For the 30-cm wavelength experiments, the cavity resonator used was of 10.8-in. diam and 9.0 in. long. A 25 kW cw magnetron was used to excite the cavity. The axial magnetic field could be increased to twice the rest mass cyclotron frequency for cavity resonance. The cw electron beam was injected on the cavity axis a distance of 1.3 in. in from the end wall. As the beam accelerates through the cavity, it describes a spiral path. To measure the energy gain of the beam, the energy spectrum of target bremsstrahling x rays was measured by means of a NaI(Tl) crystal and pulse-height analyzer system. The theoretical value for A for this experiment at 10 kW drive power is 0.084. The computed energy gain for $A=0.084$ is 580 keV. Based on the x-ray bremsstrahlung and on absorber measurements, the experimental beam energy is in the range of 400–800 keV for 10 kW power input. Although internal multipactor breakdown occurred in the cavity for various values of steady magnetic field, the breakdown did not occur at values corresponding to desired operating conditions. Both space-charge effects and cavity loading might be expected to influence the acceleration mechanism; however, these were not readily observable in the experiment, where, at most,

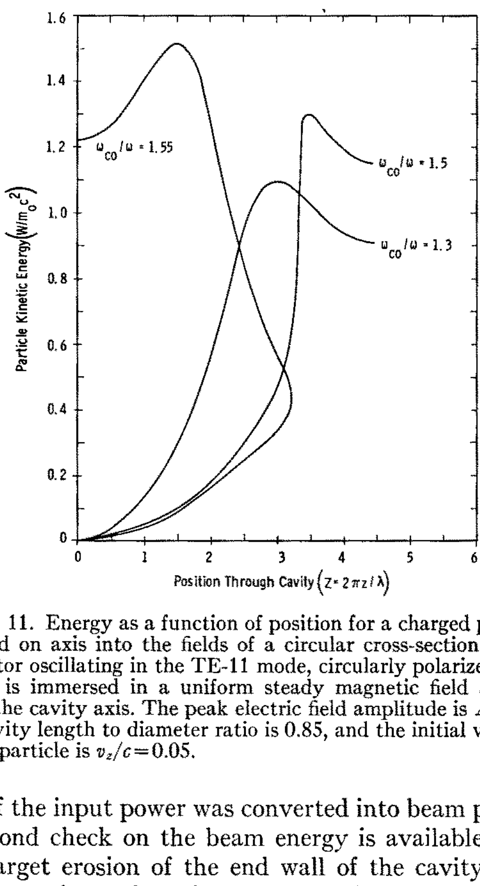

FIG. 11. Energy as a function of position for a charged particle injected on axis into the fields of a circular cross-section cavity resonator oscillating in the TE-11 mode, circularly polarized. The cavity is immersed in a uniform steady magnetic field aligned along the cavity axis. The peak electric field amplitude is $A=0.2$, the cavity length to diameter ratio is 0.85, and the initial velocity of the particle is $v_z/c=0.05$.

5% of the input power was converted into beam power. A second check on the beam energy is available from the target erosion of the end wall of the cavity. The diameter of the ring of erosion would predict energies in the range of 370–570 keV.

For the 3-cm-wavelength experiments, the cavity resonator was 1.05 in. in diam and 0.77 in. long. The axial magnetic field could be increased to 1.5 times the rest mass cyclotron frequency for cavity resonance. A 75-kW pulsed source was used to drive the cavity, and a pulsed electron beam was injected through the end wall on the cavity axis. This experiment had the added feature of an open end wall which was connected to a section of circular waveguide beyond cutoff at the cavity resonance frequency. At the end of this drift

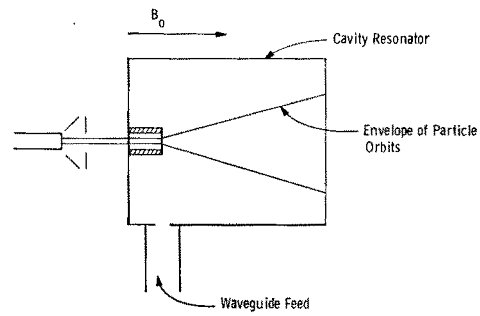

FIG. 12. Schematic drawing of cavity resonator together with electron-beam injection system and rf drive.

tube, a flat of aluminized uranium glass was located perpendicular to the cavity axis as a beam target. The fluorescence of the glass was used to measure the diameter of the accelerated beam orbit. Figure 13 shows a photograph of the target taken for a 31-mA and 800-V injected beam and 25-kW pulse-cavity drive power. The grid overlay has a 1-mm spacing. The magnetic field for the measurement was essentially uniform with a value of 4200 G. On the basis of the diameter of the beam track on the target, the beam energy is approximately 300 keV and the beam power is 9.3 kW. This gives an efficiency of 37% based on input rf power.

Assuming that 37% of the input power is carried out of the cavity by the accelerated beam, the resulting value of the cavity electric field at 25 kW input power corresponds to $A=0.051$. This value of A predicts an energy gain of 350 keV, which is in reasonable agreement with the measured value of 300 keV. At a higher power level of 46 kW a beam energy of 460 keV was measured with an injected beam current of 10 mA and a corresponding efficiency of 10%.

The width of the beam track is essentially the same

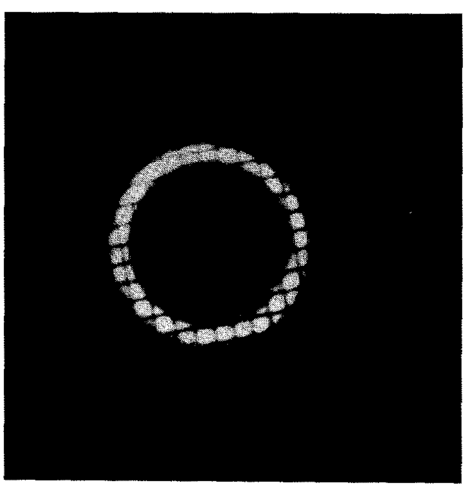

FIG. 13. Photograph of accelerated beam trace on uranium glass target situated perpendicular to the cavity axis in the 3 cm wavelength accelerator. Mean diameter of the ring is 9.6 mm and the width of the ring is 1 mm.

as the cathode diameter, which is an indication that the spread in beam energy of the accelerated beam is not large. A weakness in both sets of experiments is the difficulty in measuring the spread in beam energy.

V. CONCLUSIONS

The relativistic motion of a charged particle in combined inhomogeneous, large-amplitude, oscillating electromagnetic fields and steady magnetic fields has been investigated theoretically and experimentally. It was found that large energy gains could be obtained for conditions different from the idealized situation studied by Roberts and Buchsbaum.[1] The experiments confirmed the analysis and showed that the effects of space charge, beam diameter, cavity loading, multipactor breakdown, and background gas ionization are not limitations at the power levels investigated. The 30-cm wavelength experiment demonstrated that continuous operation at the experimental power levels is possible.

ACKNOWLEDGMENTS

The authors acknowledge many fruitful discussions with Dr. J. Feinstein and considerable help in preparation of the manuscript by Dr. R. L. Jepsen. We also gratefully acknowledge the excellent programming and numerical analysis provided by Mrs. A. Wehlau.

JOURNAL OF APPLIED PHYSICS VOLUME 39, NUMBER 7 JUNE 1968

Linewidth and Relaxation Processes for the Main Resonance in the Spin-Wave Spectra of Ni–Fe Alloy Films*

CARL E. PATTON†

California Institute of Technology, Pasadena, California

(Received 5 December 1967; in final form 12 February 1968)

Measurements of the position and linewidth of the main resonance in the spin-wave spectra of 80% Ni–20% Fe evaporated films 150 to 3200 Å thick at frequencies from 0.8 to 4.0 GHz, at room temperature and with the static magnetic field perpendicular to the film plane have been performed. The center field for resonance was independent of thickness and consistent with the Kittel resonance condition for the uniform precession. The linewidth was about 30 Oe (measured as the field separation of inflection points on the absorption curve), independent of film thickness and frequency. The linewidth data have been compared to previous data for resonance with the static field parallel to the film plane and interpreted in terms of relaxation processes for the uniform precession. In general, frequency-swept linewidths, not field-swept linewidths, are proportional to the relaxation rate and are the quantities which should be compared. The frequency-swept linewidth for perpendicular resonance is equal to that obtained for parallel resonance in films thinner than 500 Å. This result indicates that two-magnon scattering between the uniform precession mode and spin-wave states for which the exchange energy term is large (wavenumber k appreciably different from zero) does not contribute to the perpendicular resonance linewidth and is the origin of the linewidth increase with thickness for parallel resonance. It appears that scattering to states with $k\approx 0$ and exchange conductivity broadening are the most reasonable sources for the residual 30 Oe linewidth.

INTRODUCTION

Spin-wave resonance in thin ferromagnetic films was initially predicted by Kittel[1] and observed experimentally by Seavey and Tannenwald[2] shortly thereafter. Following this initial work, a large variety of spin-wave spectra have been reported and several models have been evoked to explain these different spectra.[3] In most of these studies, the emphasis has been on explaining the spacing and intensities of the various spin-wave resonances which occur at fields lower than that for the main resonance. Very little attention has been given to the properties of the main resonance. In this study, attention was focused exclusively on the linewidth and the position of the main resonance for 80% Ni–20% Fe thin films. The position of the resonance was observed to be independent of film thickness from 150 to 3200 Å. The linewidth, defined as the field separation between inflection points on the absorption curve, was about 30 Oe independent of thickness (150–3200 Å) and frequency (0.8–4.0 GHz).

EXPERIMENTAL

The nickel–iron alloy films were vacuum evaporated from a melt of 80% Ni and 20% Fe onto hot glass substrates at 300°C in a moderate vacuum of 10^{-6} Torr and in the presence of a uniform magnetic field parallel

* This work was done during the tenure of a National Aeronautics and Space Administration Fellowship.

† Present address: Raytheon Research Division, Waltham, Massachusetts.

[1] C. Kittel, Phys. Rev. **110**, 1295 (1958).

[2] M. H. Seavey and P. E. Tannenwald, Phys. Rev. Letters **1**, 168 (1958).

[3] An excellent review of this work has been given by J. W. Hartwell, Interim Tech. Rept., Department of Electrical Engineering, Duke University, Durham, North Carolina, April, 1967, and J. W. Hartwell, Proc. IEEE **56**, 23 (1968).