Particle Acceleration by Electrostatic Waves with Spatially Varying Phase Velocities

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We present here the results of numerical calculations connected with a mechanism recently proposed [*Swift*, 1968] for the acceleration of charged particles in a strong dc magnetic field. An electrostatic plasma wave propagates nearly perpendicularly to the magnetic field, and a weak density gradient exists parallel to the magnetic field. The wave propagates into the region of increasing plasma density at constant frequency and becomes more nearly perpendicular to the field, i.e., the component of the wave vector parallel to the field, $k_\parallel$, *decreases* slowly.

If the magnetic field is arbitrarily strong, the equation of motion for the particle coordinate parallel to the magnetic field (x, say) can be written in dimensionless units as

$$\ddot{x}(t) = \epsilon \cos [k_\parallel x(t) - \omega t + \phi] \qquad (1)$$

where ϵ is proportional to the charge-to-mass ratio of the particle times the component of electric field parallel to the magnetic field, and ω and ϕ are the frequency and phase of the wave, assumed constant. Swift has suggested that when $k_\parallel$ is a slowly decreasing function of x, particle acceleration can be vastly more efficient for a given field strength ϵ than for $k_\parallel$ = constant. It is our purpose here to demonstrate numerically, independently of any specific geophysical context, that this conjecture is correct.

The solution of equation 1 for $k_\parallel$ = a constant is well known and can be expressed in terms of elliptic functions for arbitrary $x(0)$ and $\dot{x}(0)$. For small ϵ, the results may be summarized by saying that for those initial values of $\dot{x}(0)$ far from $\omega/k_\parallel$, the particle trajectory is little perturbed from its free-flight value ($\epsilon = 0$ value). For $\dot{x}(0)$ close to $\omega/k_\parallel$, the maximum increment of parallel velocity $\dot{x}(t)$ is of the order of the 'trapping width' of the wave, $4(\epsilon/k_\parallel)^{1/2}$. The particle, if affected appreciably by the wave at all, stays within a trapping width of the effective phase velocity $\omega/k_\parallel$. The hope, here confirmed numerically, is that if the effective phase velocity is allowed to increase slowly, the particles initially in the neighborhood of $\dot{x}(t) \cong \omega/k_\parallel$ will ride with the wave as the phase velocity increases. If $k_\parallel$ is a function of x, the effective 'phase velocity' seen instantaneously *by the particle* is $\omega(k_\parallel + x dk_\parallel/dx)^{-1}$. Thus if $k_\parallel$ decreases, this phase velocity increases.

The numerical integrations were performed on an IBM 360/65 according to the methods

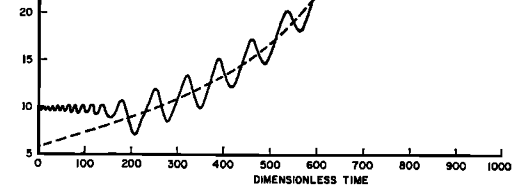

Fig. 1. Velocity as a function of time for a typical particle, with $\dot{x}(0)$ = 10.0 and $\omega/k_\parallel$ = (1.052/0.18) (1 + 0.000125 x). Note that the particle experiences little acceleration after it 'falls off' the wave at about $\dot{x}(t)$ = 40. The broken line is the effective 'phase velocity' evaluated *at the instantaneous position of the particle.*

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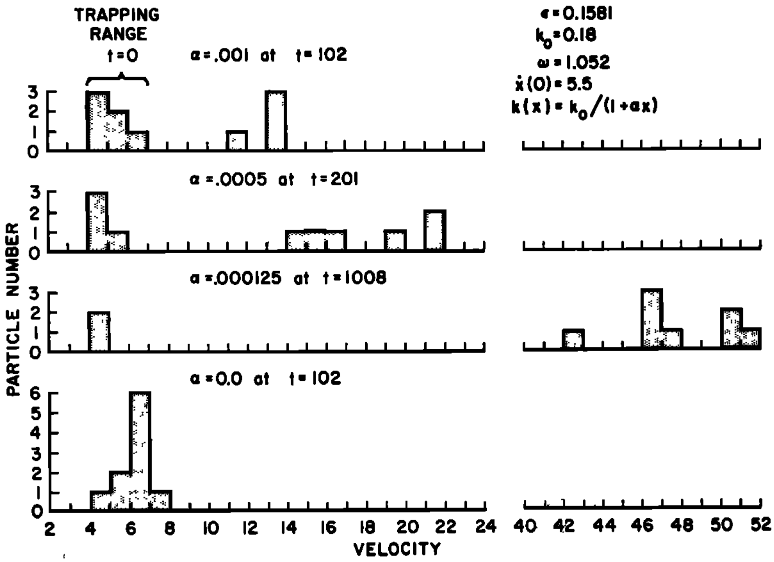

Fig. 2. Number of particles for unit velocity range at large times for $\dot{x}(0) = 5.5$, and $\omega/k_\parallel = (1.052/0.18)(1 + \alpha x)$, for $\alpha = 0$, 1.25×10^{-4}, 5×10^{-4}, and 10^{-3}. The smaller α is, the longer the particle can remain trapped, if the wave train is infinitely long and completely coherent. However, for α strictly zero, little acceleration occurs. After the times indicated, the particles have all become untrapped in the $\alpha > 0$ cases.

described elsewhere [*Gary and Montgomery*, 1968]. For definiteness, we chose $k_\parallel = (1 + \alpha x)^{-1} k_0$. We choose a fixed value of $\dot{x}(0)$ and consider an ensemble of ten phases ϕ, each one of which defines the initial coordinates of a 'particle' with respect to the wave. The gradient α is chosen $\ll 1$, and the limit $\alpha = 0$ has been checked against the accurate analytical solutions.

Computations for Finite Gradients

For $\alpha > 0$, numerical results for individual particle trajectories show that particles fall into two classes: in one group are those which become untrapped (i.e., fall outside the neighborhood of $\omega/k_\parallel \cong \dot{x}(t)$) soon after $t = 0$ and never undergo any important velocity change thereafter; particles in a second group remain trapped as the trapping region moves up in velocity space. These eventually also become untrapped, drop off the wave, and experience no significant acceleration thereafter. A typical plot of velocity versus time for particles of the second type is shown in Figure 1 as a solid line, for $\alpha = 1.25 \times 10^{-4}$, $\omega = 1.052$, $k_0 = 0.18$, $\epsilon = 0.1581$, $\dot{x}(0) = 10.0$. This particle gains a factor of 16 in energy from an electric field strength which could only yield at most a factor of $\lesssim 1.5$ in energy if $k_\parallel$ were constant. The dashed line indicates the effective phase velocity evaluated *at the instantaneous particle position.*

Velocity distribution functions for $\alpha = 0$, 1.25×10^{-4}, 5×10^{-4}, and 10^{-3} are shown as histograms in Figure 2 for large times, for the same values of ϵ, k_0, and ω, but $\dot{x}(0) = 5.5$. It will be clear that the process is more efficient the smaller α is; the gentler the increase of $\omega/k_\parallel$, the longer the particles can remain trapped. In an actual physical situation, this process would of course be limited by the coherence time of the wave, which would be bounded from above. This has not been considered here.

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