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Optical Flow

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Overview

Method to estimate apparent motion of scene points from a sequence of images

Topics:

(1) Motion Field and Optical Flow

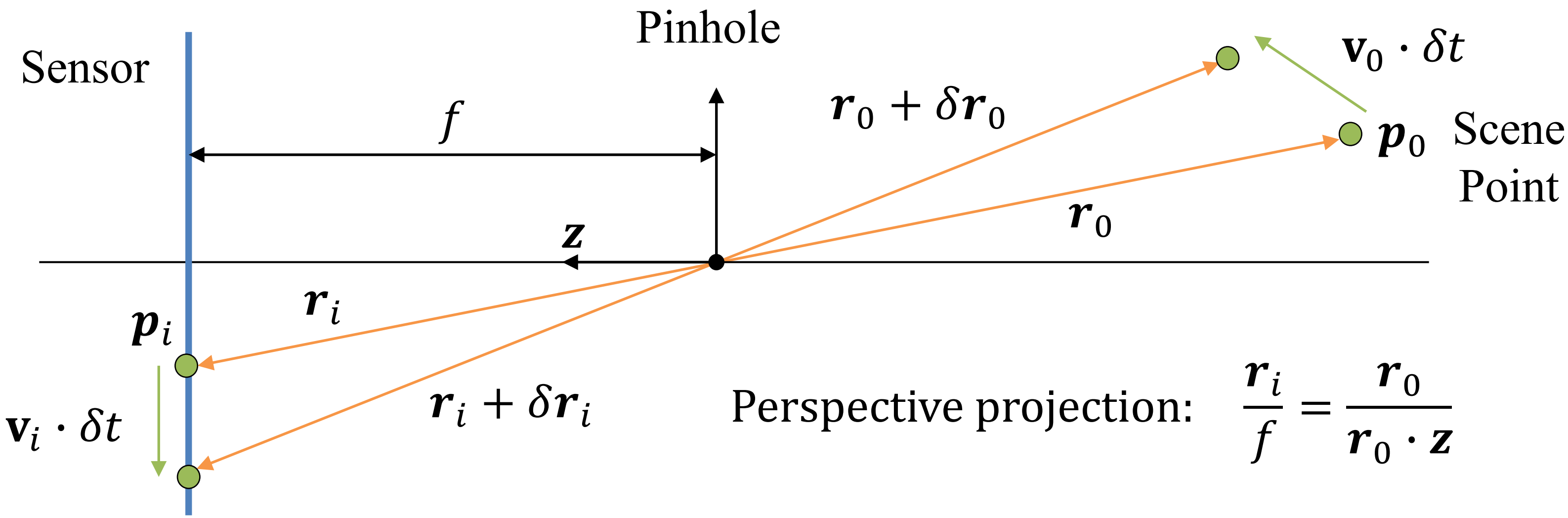
(2) Optical Flow Constraint Equation

(3) Lucas-Kanada Method

(4) Coarse-to-Fine Flow Estimation

(5) Applications of Optical Flow

Motion Field



Perspective projection: $\dfrac{\boldsymbol{r}_i}{f} = \dfrac{\boldsymbol{r}_0}{\boldsymbol{r}_0 \cdot \mathbf{z}}$

Image Point Velocity: $\mathbf{v}_i = \dfrac{d\boldsymbol{r}_i}{dt} = f \dfrac{(\boldsymbol{r}_0 \cdot \mathbf{z})\mathbf{v}_0 - (\mathbf{v}_0 \cdot \mathbf{z})\boldsymbol{r}_0}{(\boldsymbol{r}_0 \cdot \mathbf{z})^2}$

(Motion Field)

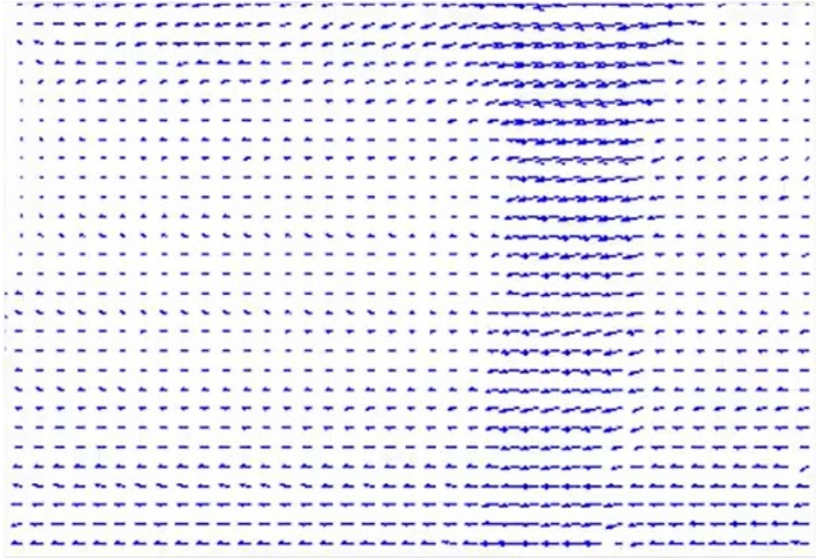
Scene Point Velocity: $\mathbf{v}_0 = \dfrac{d\boldsymbol{r}_0}{dt}$

$$\boxed{\mathbf{v}_i = \frac{(\boldsymbol{r}_0 \times \mathbf{v}_0) \times \mathbf{z}}{(\boldsymbol{r}_0 \cdot \mathbf{z})^2}}$$

Optical Flow

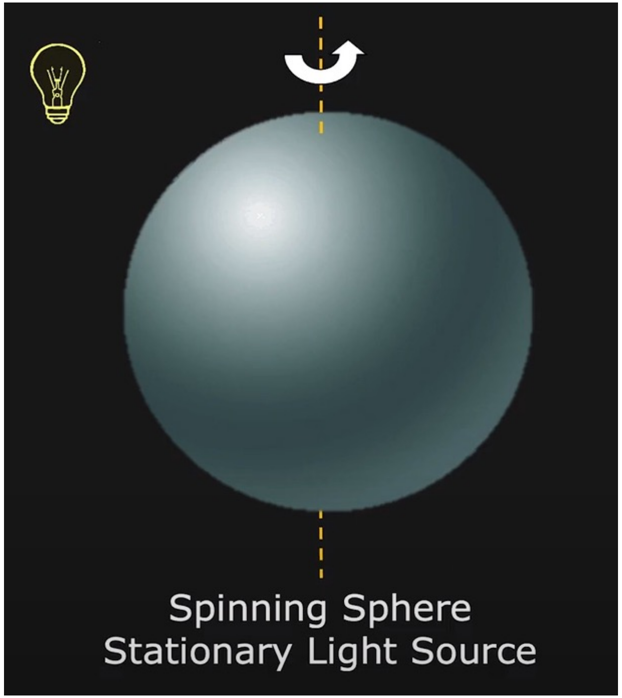
Motion of brightness patterns in the image

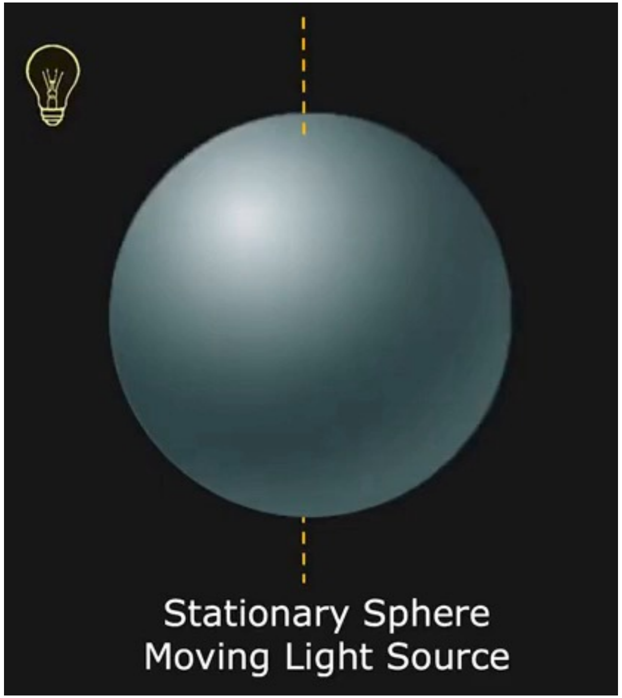
Image Sequence
(2 frames)



Optical Flow

When is Optical Flow ≠ Motion Field ?



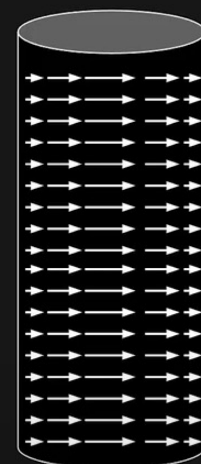


Motion Field exists
But no Optical Flow

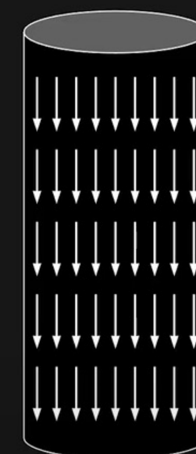
No Motion Field exists
But there is Optical Flow

When is Optical Flow ≠ Motion Field ?

Barber Pole Illusion

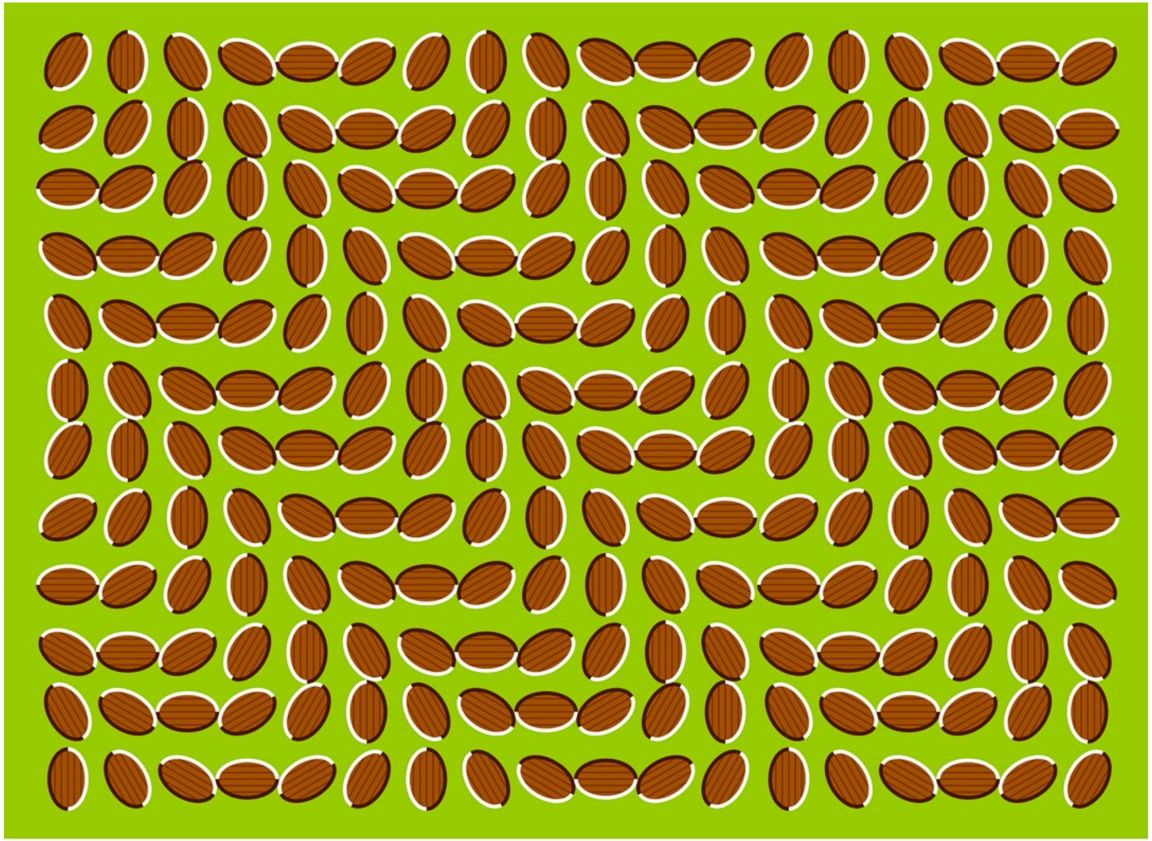


Motion Field



Optical Flow

Motion Illusions



“Dongurakokko” (Donguri wave), produced by Akiyoshi Kitaoka in 2004 as an artwork of waving demonstration of the ‘optimized’ Fraser-Wilcox illusion Type IIa. Fermüller, C., Ji, H., and Kitaoka, A. (2010). Illusory motion due to causal time filtering. Vision Research, 50, 315-329.

Optical Flow

t

(x, y)

$t + \delta t$

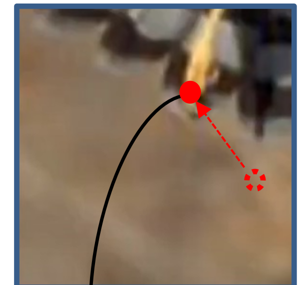
$(x + \delta x, y + \delta y)$

Displacement: $(\delta x, \delta y)$

Optical Flow: $(u, v) = (\frac{\delta x}{\delta t}, \frac{\delta y}{\delta t})$

Optical Flow Constraint Equation

$I(x, y, t)$



$I(x + \delta x, y + \delta y, t + \delta t)$

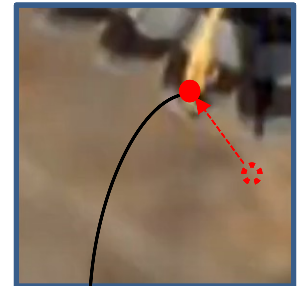
Assumption #1：

Brightness of image point remains constant over time

$$\boxed{I(x + \delta x, y + \delta y, t + \delta t) = I(x, y, t)}$$

Optical Flow Constraint Equation

$I(x, y, t)$



$I(x + \delta x, y + \delta y, t + \delta t)$

Assumption #2：

Dispacement $(\delta x, \delta y)$ and time step δt are small

$$I(x + \delta x, y + \delta y, t + \delta t) = I(x, y, t) + \frac{\partial I}{\partial x}\delta x + \frac{\partial I}{\partial y}\delta y + \frac{\partial I}{\partial t}\delta t$$

$$\boxed{I(x + \delta x, y + \delta y, t + \delta t) = I(x, y, t) + I_x\delta x + I_y\delta y + I_t\delta t}$$

Optical Flow Constraint Equation

$$I(x + \delta x, y + \delta y, t + \delta t) = I(x, y, t) \qquad \text{-------------} \ (1)$$

$$I(x + \delta x, y + \delta y, t + \delta t) = I(x, y, t) + I_x \delta x + I_y \delta y + I_t \delta t \qquad \text{-------------} \ (2)$$

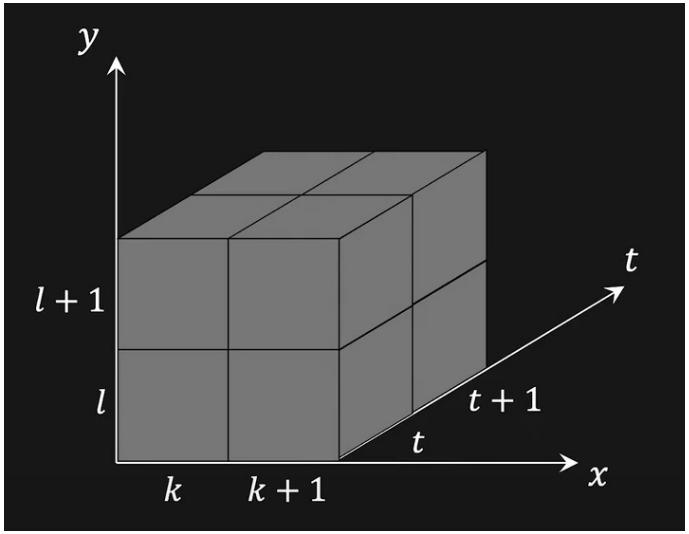
Subtract (1) from (2)： $I_x \delta x + I_y \delta y + I_t \delta t = 0$

Divide by δt and take limit as $\delta t \to 0$ ： $I_x \dfrac{\partial x}{\partial t} + I_y \dfrac{\partial y}{\partial t} + I_t = 0$

Constraint Equation： $\boxed{I_x u + I_y v + I_t = 0}$ (u, v): Optical Flow

(I_x, I_y, I_t) can be easily computed from two frames

Computing Partial Derivatives I_x, I_y, I_t



$I_x(k,l,t)$

$$= \frac{1}{4}[I(k+1,l,t) + I(k+1,l+1,t) + I(k+1,l,t+1) + I(k+1,l+1,t+1)]$$

$$- \frac{1}{4}[I(k,l,t) + I(k,l+1,t) + I(k,l,t+1) + I(k,l+1,t+1)]$$

Similarly find $I_y(k,l,t)$ and $I_t(k,l,t)$

Geometric interpretation

For any point (x, y) in the image, its optical flow (u, v) lies on the line:

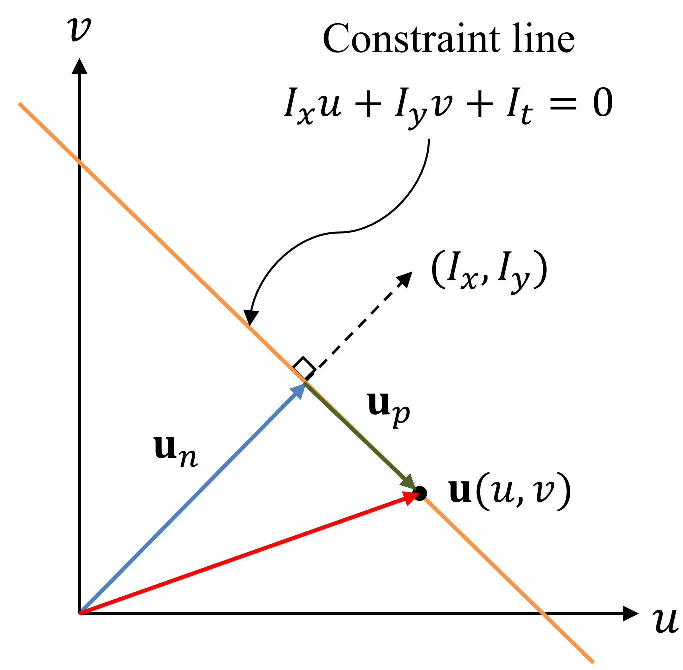
$$I_x u + I_y v + I_t = 0$$

Optical Flow can be split into two components.

$$\boldsymbol{u} = \boldsymbol{u}_n + \boldsymbol{u}_p$$

$\boldsymbol{u}_n$: Normal Flow

$\boldsymbol{u}_p$: Parallel Flow



Normal Flow

Direction of Normal Flow:

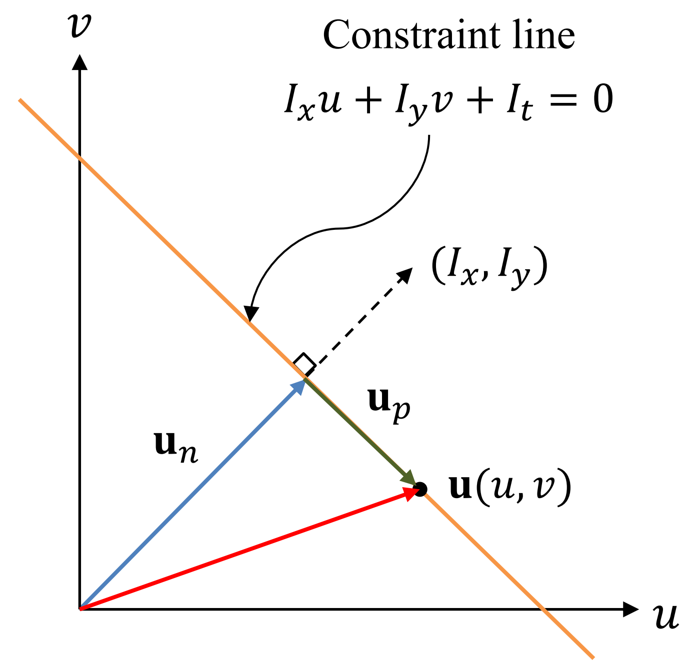
Unit vector perpendicular to the constraint line:

$$\hat{\boldsymbol{u}}_n = \frac{(I_x, I_y)}{\sqrt{I_x^2 + I_y^2}}$$

Magnitude of Normal Flow:

Distance of origin from the constant line:

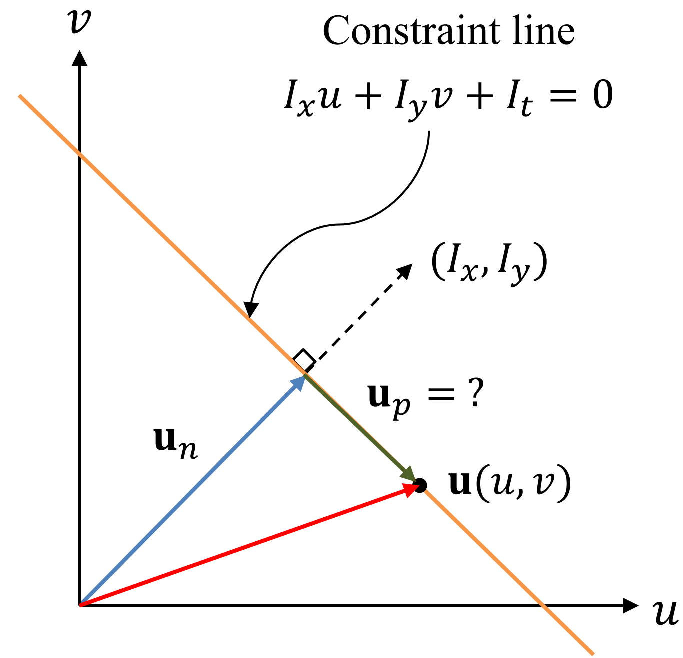
$$|\boldsymbol{u}_n| = \frac{|I_t|}{\sqrt{I_x^2 + I_y^2}}$$



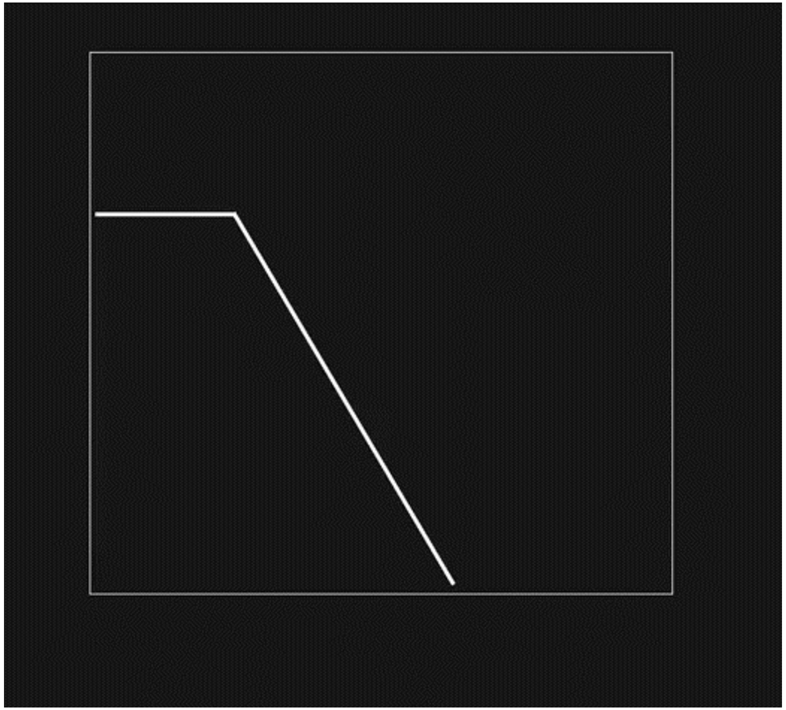
$$\boxed{\boldsymbol{u}_n = \frac{|I_t|}{(I_x^2 + I_y^2)}(I_x, I_y)}$$

Parallel Flow

We can not determine u_p, the optical flow component parallel to the constraint line.



Aperture Problem



Locally, we can only determine Normal Flow!

Optical Flow is Under constrained

Constraint Equation： $\boxed{I_x u + I_y v + I_t = 0}$

2 unknowns, 1 equation.

Lucas-Kanada Solution

Assumption： For each pixel, assume Motion Field, and hence Optical Flow (u, v), is constant within a small neighbourhood W.

W

That is for all points $(k, l) \in W$：

$$I_x(k,l)u + I_y(k,l)v + I_t(k,l) = 0$$

Lucas-Kanada Solution

For all points $(k, l) \in W$: $I_x(k, l)u + I_y(k, l)v + I_t(k, l) = 0$

Let the size of window W be $n \times n$

In matrix form:

$$\underbrace{\boxed{\begin{bmatrix} I_x(1,1) & I_y(1,1) \\ \vdots & \vdots \\ I_x(k,l) & I_y(k,l) \\ \vdots & \vdots \\ I_x(n,n) & I_y(n,n) \end{bmatrix}}}_{\substack{A \\ \text{(Known)} \\ n^2 \times 2}} \underbrace{\boxed{\begin{bmatrix} \mathrm{u} \\ v \end{bmatrix}}}_{\substack{\boldsymbol{u} \\ \text{(Unknown)} \\ 2 \times 1}} = \underbrace{\boxed{-\begin{bmatrix} I_t(1,1) \\ \vdots \\ I_t(k,l) \\ \vdots \\ I_t(n,n) \end{bmatrix}}}_{\substack{B \\ \text{(Known)} \\ n^2 \times 1}}$$

n^2 Equations, 2 Unknowns: Find Least Squares Solution

When Dose Optical Flow Estimation Work?

$$\boxed{Au = B}$$

$$\boxed{A^T Au = A^T B}$$

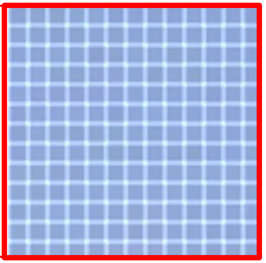
- $A^T A$ must be invertible. That is $\det(A^T A) \neq 0$
- $A^T A$ must be well-conditioned.

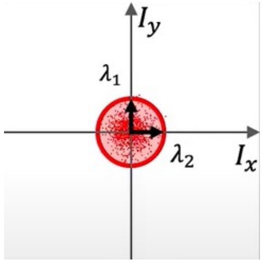
If λ_1 and λ_2 are eigen values of $A^T A$, then

$\lambda_1 > \epsilon$ and $\lambda_2 > \epsilon$

$\lambda_1 \geq \lambda_2$ but not $\lambda_1 \gg \lambda_2$

Smooth Regions (Bad)





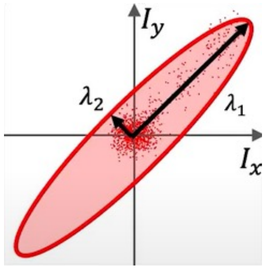
$\lambda_1 \sim \lambda_2$

Both are small

Equations for all pixels in window are both more or less the same

Cannot reliably compute flow!

Edges (Bad)



$\lambda_1 \gg \lambda_2$

Badly conditioned. Prominent gradient in one direction.

Cannot reliably compute flow!

Same as Aperture Problem.

Textured Regions (Good)

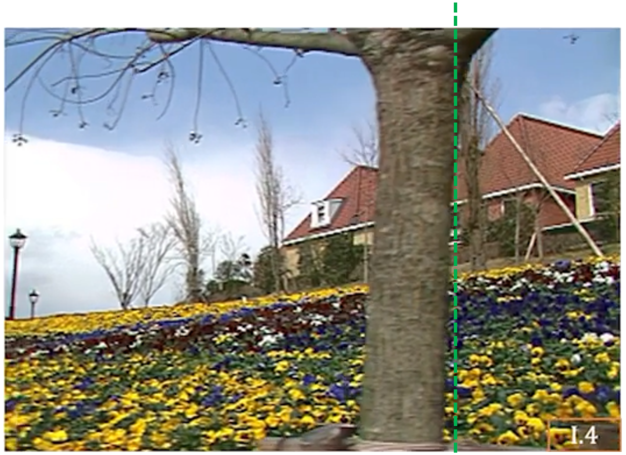
$\lambda_1 \sim \lambda_2$

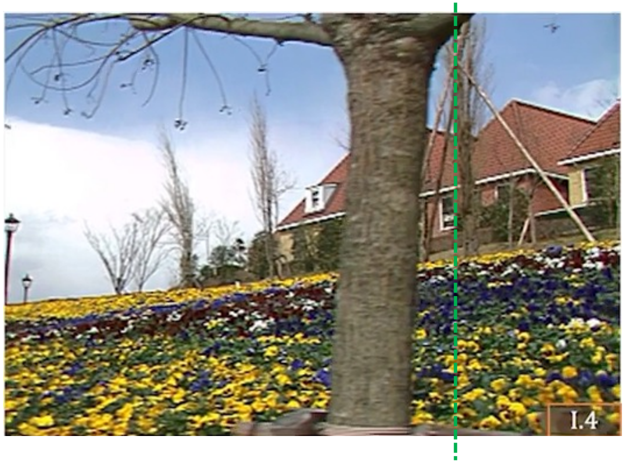
Both are Large

Well conditioned. Large and diverse gradient magnitudes.

Can reliably compute optical flow!

What if we have Large Motion?





Taylor Series approximation of

$I(x + \delta x, y + \delta y, t + \delta t)$ is not valid

Our simple linear constraint equation not valid

$$I_x u + I_y v + I_t \neq 0$$

Large Motion: Coarse-to-Fine Estimation

Resolution $N \times N$

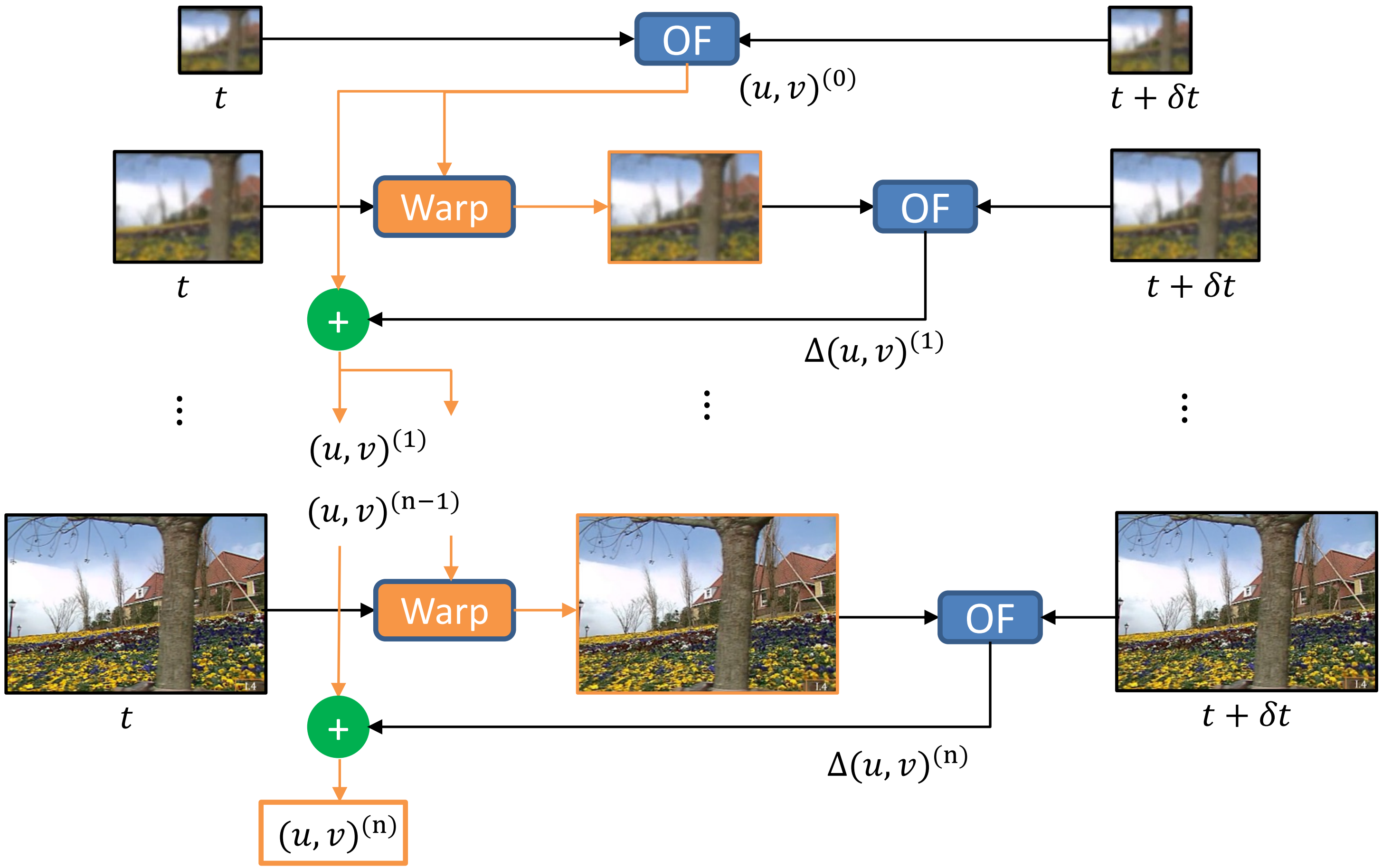
Resolution $\frac{N}{2} \times \frac{N}{2}$

Resolution $\frac{N}{4} \times \frac{N}{4}$

Resolution $\frac{N}{8} \times \frac{N}{8}$

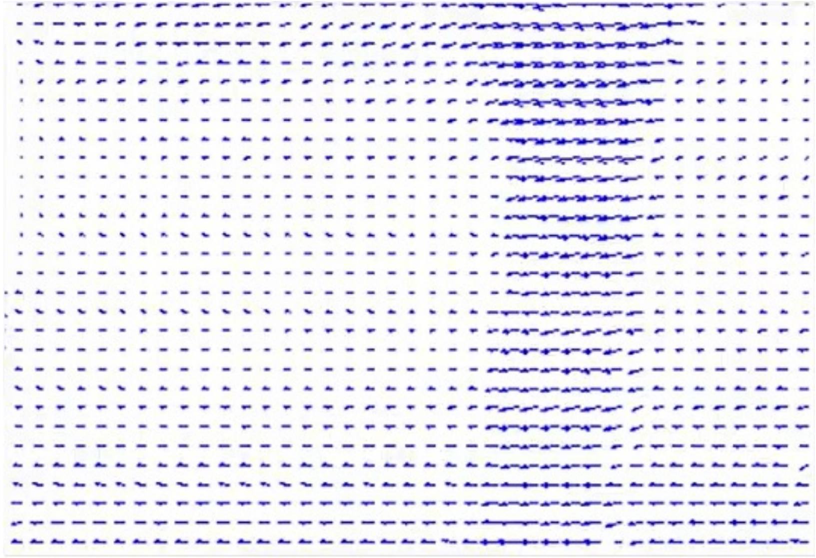
At lowest resolution, motion $\leq$ 1 pixel

Coarse-to-Fine Estimation Algorithm



Results: Tree Sequence

Image Sequence



Optical Flow

Results: Rotating Ball

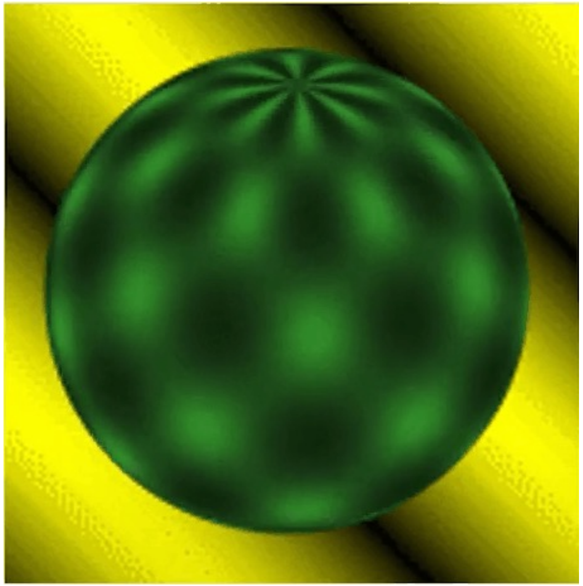
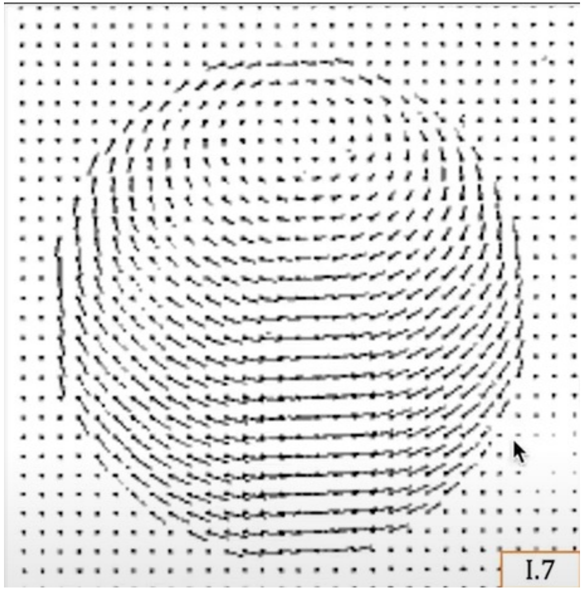


Image Sequence



Optical Flow

Alternative Approach: Template Matching

Determine Flow using Template Matching

Template window T

Image I_1 at time t

Search window S

Image I_2 at time $t + \delta t$

For each template window T in image I_1,
find the corresponding match in image I_2

Alternative Approach: Template Matching

Determine Flow using Template Matching

Template window T

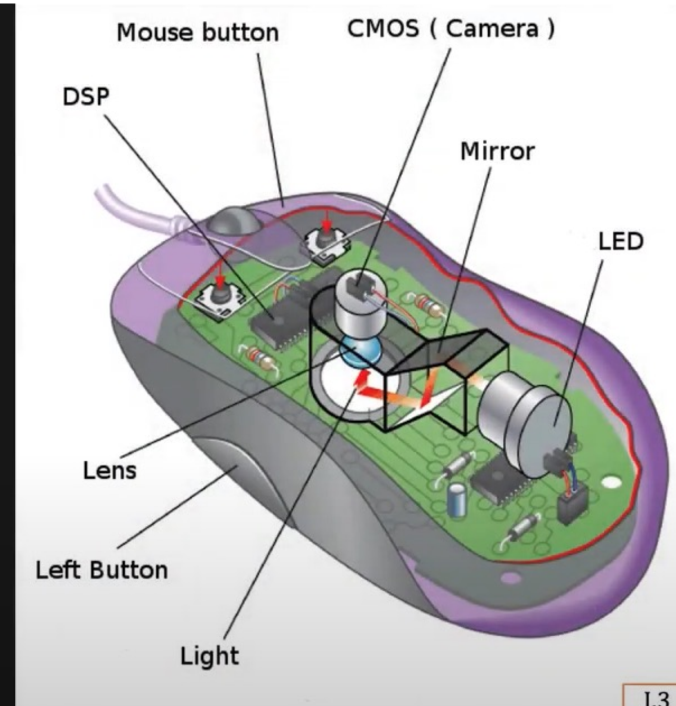
Image I_1 at time t

Search window S

Image I_2 at time $t + \delta t$

1. Template Matching is slow when search window S is large.
2. Also mismatches are possible

Applications: Optical Mouse



Estimating Mouse Movements

Applications: Traffic Monitoring

Finding Velocities of Vehicles

Applications: Video Retiming

Optical Flow is used to determine the intermediate frames to produce slow-motion effect.