Inhibitor-Induced Wavetrains and Spiral Waves in an Extended FitzHugh–Nagumo Model

Proposal for MATH3888 project

Albert Stark, Calvin Ming En Kok, Kyle George, Tim Lin

University of Sydney

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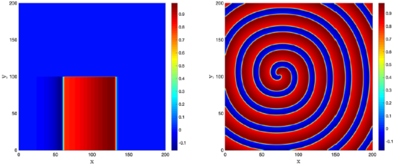

Albert Stark (SID: 540657726)
Calvin (SID: 530263092)
Kyle George (SID: 520417425)
Tim Lin (SID: 530532035)

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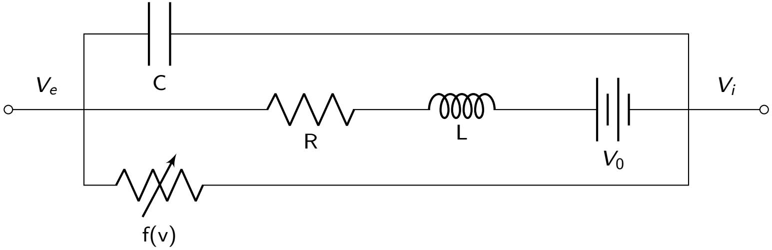
The FitzHugh-Nagumo model

$$\varepsilon \frac{dv}{dt} = f(v) - w + I_{\text{app}} \qquad \text{(cubic)}$$
$$\frac{dw}{dt} = v - \gamma w \qquad \text{(linear)}$$

where

$$f(v) = v(1 - v)(v - \alpha)$$

for typical values within $0 < \alpha < 1, \varepsilon \ll 1, \gamma \approx 0.5$ [GKO22][LM14]. This simplifies the four Hodgkin-Huxley equations to two: one fast and one slow. The circuit diagram below has equivalent function [KS09].



Biology

- Excitability: is a ability of a cell, particularly nerve and muscle cells, to respond to a stimulus by generating an electrical signal called an action potential.
- It is an intrinsic membrane property that relies on rapid ion fluxes across the cell membrane to change the membrane potential to a "threshold potential," triggering the signal; where threshold potential is the minimum potential change required to trigger an action potential.
- The model explains how small inputs decay, but once threshold is crossed, a large spike occurs (like a real neuron). After a spike, the recovery variable w prevents immediate re-excitation (models absolute/relative refractory period).

Experimental Data

In general we are restricted to a certain class of functions $G_b(x)$ that satisfy the following conditions:

$$G_b(x) = \frac{G(bx)}{b}, b \in \mathbb{R}^+ \quad G(x) \text{ satisfies } \begin{cases} G(x) = 0 \iff x = 0 \\ G(x) > 0 \iff x > 0, \quad G(x) < 0 \iff x < 0 \\ \lim_{x\to-\infty}(G(x) - x) = 0, \quad \lim_{x\to\infty} G(x) \geq 0 \end{cases}$$

$$\text{examples: } G_b(x) := \frac{xe^{-x}}{e^x + e^{-x}} \text{ (non-monotone)} \quad G_b(x) := \frac{x - \sqrt{x^2 - 4}}{2} + 1 \text{ (monotone)}$$

The paper demonstrates experimentally (via computational simulation) the existence of spiral waves in both the examples provided. The wavetrains were identified with the use of AUTO in the paper, this is possible for us, although python alternatives such as PyCoBi [Gas25] and scFates [Fau+22] [Fau25] are also of interest.
The potential of characterising in general the monotone or non-monotone functions that generate these spiral waves is of interest, as is the physicality of these functions and their solutions.

We anticipate that we will perform wavetrain and spiral-wave analysis. As well as bifurcation and limit-cycle analysis

Conclusion + bib

[Fau+22] Louis Faure et al. "scFates: a scalable python package for advanced pseudotime and bifurcation analysis from single-cell data". In: *Bioinformatics* 39.1 (Nov. 2022), btac746. ISSN: 1367-4811. DOI: 10.1093/bioinformatics/btac746. eprint: https://academic.oup.com/bioinformatics/article-pdf/39/1/btac746/48448680/btac746.pdf. URL: https://doi.org/10.1093/bioinformatics/btac746.

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