Proposal for MATH3888 project

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The FitzHugh-Nagumo model

Biology

Experimental Data

In general we are restricted to a certain class of functions $G_b(x)$ that satisfy the following conditions:

$$G_b(x) = \frac{G(bx)}{b}, b \in \mathbb{R}^+ \quad G(x) \text{ satisfies } \begin{cases} G(x) = 0 \iff x = 0 \\ G(x) > 0 \iff x > 0, \quad G(x) < 0 \iff x < 0 \\ \lim_{x \to -\infty}(G(x) - x) = 0, \quad \lim_{x \to \infty} G(x) \geq 0 \end{cases}$$

examples: $G_b(x) := \frac{xe^{-x}}{e^x + e^{-x}}$ (non-monotone) $\quad G_b(x) := \frac{x - \sqrt{x^2 - 4}}{2} + 1$ (monotone)

The paper demonstrates experimentally (via computational simulation) the existence of spiral waves in both the examples provided. The wavetrains were identified with the use of AUTO in the paper, this is possible for us, although python alternatives such as PyCoBi [Gas25] and scFates [Fau+22] [Fau25] are also of interest.

The potential of characterising in general the monotone or non-monotone functions that generate these spiral waves is of interest, as is the physicality of these functions and their solutions.

We anticipate that we will perform wavetrain and spiral-wave analysis. As well as bifurcation and limit-cycle analysis

Conclusion + bib

[Fau+22] Louis Faure et al. "scFates: a scalable python package for advanced pseudotime and bifurcation analysis from single-cell data". In: *Bioinformatics* 39.1 (Nov. 2022), btac746. ISSN: 1367-4811. DOI: 10.1093/bioinformatics/btac746. eprint: https://academic.oup.com/bioinformatics/article-pdf/39/1/btac746/48448680/btac746.pdf. URL: https://doi.org/10.1093/bioinformatics/btac746.

[Fau25] Louis Faure. *scFates*. "Retrieved 2025-09-03 at 1400AEST". 2025. URL: https://github.com/LouisFaure/scFates/.

[Gas25] Richard Gast. *PyCoBi*. "Retrieved 2025-09-03 at 1405AEST". 2025. URL: https://pypi.org/project/pycobi/.