

# Probability 2020 Quiz 1

系級 資工二

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- Let  $P_n$ ,  $n = 0, 1, 2, \dots$ , be the probability that an automobile policyholder will file for  $n$  claims in a five-year period. The actuary involved makes the assumption that  $P_{n+1} = (1/4)P_n$ . What is the probability that the holder will file two or more claims during this period?
- Suppose that a school has 20 classes: 16 with 25 students in each, three with 100 students in each, and one with 300 students, for a total of 1000 students.
  - What is the average class size?
  - Select a student randomly out of the 1000 students. Let the random variable  $X$  equal the size of the class to which this student belongs, and define the pmf of  $X$ .
  - Find  $E(X)$ , the expected value of  $X$ .
- Each of the 12 students in a class is given a fair 12-sided die. In addition, each student is numbered from 1 to 12. (結果可以算式表達, 不需算出數字)
  - If the students roll their dice, what is the probability that there is at least one "match" (e.g., student 4 rolls a 4)?
  - If you are a member of this class, what is the probability that at least one of the other 11 students rolls the same number as you do?
- Let  $X$  equal the number of flips of a fair coin that are required to observe heads-tails on consecutive flips.
  - Find the pmf of  $X$ . Hint: Draw a tree diagram.
  - Find the values of (i)  $P(X \leq 3)$ , (ii)  $P(X \geq 5)$ , and (iii)  $P(X = 3)$ .
- A fair four-sided die has two faces numbered 0 and two faces numbered 2. Another fair four-sided die has its faces numbered 0, 1, 4, and 5. The two dice are rolled. Let  $X$  and  $Y$  be the respective outcomes of the roll. Let  $W = X + Y$ .
  - Determine the pmf of  $W$ .
  - Draw a probability histogram of the pmf of  $W$ .

(b) (i)  $P(X \leq 3) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$  #

(ii)  $P(X \geq 5) = 1 - P(X \leq 4) = 1 - (\frac{1}{4} + \frac{1}{4} + \frac{1}{8}) = \frac{3}{8}$  #

- 3 r 3 w                      4 r 3 w
6. Bowl  $A$  contains three red and two white chips, and bowl  $B$  contains four red and three white chips. A chip is drawn at random from bowl  $A$  and transferred to bowl  $B$ . Compute the probability of then drawing a red chip from bowl  $B$ .
  7. Let  $X$  equal the number of people selected at random that you must ask in order to find someone with the same birthday as yours. Assume that each day of the year is equally likely, and ignore February 29. (結果可以算式表達, 不需算出數字)
    - (a) What is the pmf of  $X$ ?
    - (b) Give the values of the mean, variance, and standard deviation of  $X$ .
    - (c) Find  $P(X > 400)$  and  $P(X < 300)$ .
  8. Your stockbroker is free to take your calls about 60% of the time; otherwise, he is talking to another client or is out of the office. You call him at five random times during a given month. (Assume independence.)
    - (a) What is the probability that he will take every one of the five calls?
    - (b) What is the probability that he will accept exactly three of your five calls?
    - (c) What is the probability that he will accept at least one of the calls?
  9. Two processes of a company produce rolls of materials: The rolls of Process I are 3% defective and the rolls of Process II are 1% defective. Process I produces 60% of the company's output, Process II 40%. A roll is selected at random from the total output. Given that this roll is defective, what is the conditional probability that it is from Process I?
  10. In 2012, Red Rose tea randomly began placing 1 of 12 English porcelain miniature figurines in a 100-bag box of the tea, selecting from 12 nautical figurines. (結果可以算式表達, 不需算出數字)
    - (a) On the average, how many boxes of tea must be purchased by a customer to obtain a complete collection consisting of the 12 nautical figurines?
    - (b) If the customer uses one tea bag per day, how long can a customer expect to take, on the average, to obtain a complete collection?