AIRCRAFT ALLOCATION PROBLEM

Reference

G. Dantzig: *Linear Programming and Extensions*, Princeton University Press, 1963, pp. 572-597.

This is the classic example of a stochastic program with simple recourse.

An airline wishes to allocate airplanes of various types among its routes to satisfy an uncertain passenger demand, in such a way as to minimize operating costs plus the lost revenue from passengers turned away.

This problem will be available on the stochastic programming computer tape distributed by IIASA.

Stochastic program with simple recourse.

Choose $x_j (j = 1, \ldots, 17)$ to *minimize*

$$\sum_{j=1}^{17} c_j x_j + E\Big\{\sum_{k=1}^{5} q_k \mathbf{v}_k^-\Big\}$$

subject to

$$\begin{aligned}
&x_1 + x_2 + x_3 + x_4 + x_5 \le b_1\\
&x_6 + x_7 + x_8 + x_9 \quad \le b_2\\
&x_{10} + x_{11} + x_{12} \qquad \le b_3\\
&x_{13} + x_{14} + x_{15} + x_{16} + x_{17} \le b_4\\
&x_j \ge 0 \quad j = 1, \ldots, 17\\
&\mathbf{v}_k^+ \ge 0, \quad \mathbf{v}_k^- \ge 0\\
&\mathbf{v}_1^+ - \mathbf{v}_1^- = t_1 x_1 + t_{13} x_{13} - b_1\\
&\mathbf{v}_2^+ - \mathbf{v}_2^- = t_2 x_2 + t_6 x_6 + t_{10} x_{10} + t_{14} x_{14} - \mathbf{h}_2\\
&\mathbf{v}_3^+ - \mathbf{v}_3^- = t_3 x_3 + t_7 x_7 + t_{15} x_{15} - \mathbf{h}_3\\
&\mathbf{v}_4^+ - \mathbf{v}_4^- = t_4 x_4 + t_8 x_8 + t_{11} x_{11} + t_{16} x_{16} - \mathbf{h}_4\\
&\mathbf{v}_5^+ - \mathbf{v}_5^- = t_5 x_5 + t_9 x_9 + t_{12} x_{12} + t_{17} x_{17} - \mathbf{h}_5
\end{aligned}$$

$x_1, \ldots, x_5$:	type 1 aircraft assigned to routes $1, \ldots, 5$
$x_6, \ldots, x_9$:	type 2 aircraft assigned to routes $2, \ldots, 5$
x_{10}, x_{11}, x_{12}:	type 3 aircraft assigned to routes 2, 4, 5
$x_{13}, \ldots, x_{17}$:	type 4 aircraft assigned to routes $1, \ldots, 5$
b_i:	number of aircraft available of type $i = 1, \ldots, 4$
c_j:	cost of operating aircraft/route $j = 1, \ldots, 17$
q_k:	revenue lost per passenger turned away on route $k = 1, \ldots, 5$
$\mathbf{v}_k^+$:	empty seats on route k
$\mathbf{v}_k^-$:	passengers turned away on route k
t_j:	passenger capacity on aircraft/route j
$\mathbf{h}_k$:	passenger demand for route k.

Data:

$c =$ [18, 21, 18, 16, 10, 15, 16, 14, 9, 10, 9, 6, 17, 16, 17, 15, 10]

$q =$ [13, 13, 7, 7, 1]

$b =$ [10, 19, 25, 15]

$t =$ [16, 15, 28, 23, 81, 10, 14, 15, 57, 5, 7, 29, 9, 11, 22, 17, 55]

$\mathbf{h}_k$ are discretely distributed as follows

$\mathbf{h}_1 \sim$ [200, 220, 250, 270, 300] w.p. (0.2, 0.05, 0.35, 0.2, 0.2)
$\mathbf{h}_2 \sim$ [50, 150] w.p. (0.3, 0.7)
$\mathbf{h}_3 \sim$ [140, 160, 180, 200, 220] w.p. (0.1, 0.2, 0.4, 0.2, 0.1)
$\mathbf{h}_4 \sim$ [10, 50, 80, 100, 340] w.p. (0.2, 0.2, 0.3, 0.2, 0.1)
$\mathbf{h}_5 =$ [580, 600, 620] w.p. (0.1, 0.8, 0.1)

Solution:

Calculated to one decimal place accuracy

Aircraft Type Route	1	2	3	4
1	$x_1 = 10$	*	*	$x_{13} = 7.4$
2	$x_2 = 0$	$x_6 = 12.8$	$x_{10} = 4.3$	$x_{14} = 0.0$
3	$x_3 = 0$	$x_7 = 0.9$	*	$x_{15} = 7.6$
4	$x_4 = 0$	$x_8 = 5.3$	$x_{11} = 0$	$x_{16} = 0$
5	$x_5 = 0$	$x_9 = 0$	$x_{12} = 20.7$	$x_{17} = 0$