

## Ariadne's thread

### 1 Context

Your friend, a fan of video games, asks you to help him. He noted that many games consist of moving a character in a particular environment. This character moves from place to place and may revisit a place or may not when you cross such a level of play (elevator, secret passage, etc.,...). Its objective is after playing a game for hours and after carefully numbered places encountered and the possible passages from one place to another, to establish a small map of the game, which includes only the different levels of play and opportunities to move from one level to another. Each level corresponds to a set of accessible places from other place by any path.

He provides his observation as a binary matrix  $A$  (composed of 0 and 1) of size  $n \times n$  where the indices of rows and columns correspond to different numbers of places encountered. A 1 at the location  $(i, j)$  of the matrix means that there is a direct passage from the place  $i$  to the place  $j$ . If there is no direct passage from the place  $i$  to  $j$ , then  $A(i, j) = 0$ . He asks you to provide :

- the list of the different game levels with the places inside each level
- the map of game levels, presented in a square matrix  $N$ .  $N(i, j) = k$  if there is  $k$  direct passage (possibly  $k = 0$ ) from level  $i$  to level  $j$ .
- a longest path from level including place 1 (input of the game) to the level including place  $n$  (output of the game)

### 2 Questions

1. Which classical problem of graph theory do you recognize to deal with the first point ? Write a program for this algorithm. Data may be read in a file where the matrix  $A$  is given below :

0	1	1	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	0	0	0	0	0	0	0	0
0	1	0	1	0	0	0	1	0	0	0	0	0
1	0	0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	1	0	1	0	0	0
0	0	0	0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	0	0	1	0	0	0

2. Your program should give the different game levels and the places including in each level (for instance, there are four game levels :  $\{1,2,3,4\}$ ,  $\{5,6,7\}$ ,  $\{8,9\}$ ,  $\{10,11,12,13\}$ ).
3. Your program should give the reduced matrix  $N$  with the different game levels and the number of direct passages.
4. What is the longest path that your friend asks you to find? Implement an algorithm to identify such a path.