

HW 14 – Due Wednesday, August 11, 2011

1.
  - a. Not SD
  - b. In D. For any given input,  $xy$  can be defined. Thus, once the TM is fed input it should accept anything given to it. Similar to H.
  - c. In SD. Using a two state TM (affecting the structure) we can keep track of whether or not the length of a string is even. However, if the string is infinitely long, it may not be decidable.
  - d. Not SD
  - e.
2. Prove that  $TM_{REG}$  is not in SD.

By reduction of H:

$R(\langle M, w \rangle) =$

- a. Construct the description  $\langle M\# \rangle$  of  $M\#(x)$ ;
  - i. Erase the tape
  - ii. Write  $w$  on the tape.
  - iii. Run  $M$  on  $w$ .
- b. Return  $\langle M\# \rangle$ .

If Oracle exists and decides  $TM_{reg}$ , then  $C = \text{Oracle}(R(\langle M, w \rangle))$  decides H.  $R$  can be implemented as a Turing machine. And  $C$  is correct.  $M\#$  ignores its own input. It halts on everything or nothing. So:

- i.  $\langle M, w \rangle \in H$ :  $M$  halts on  $w$ , so  $M\#$  halts on all inputs. Oracle accepts.
- ii.  $\langle M, w \rangle \notin H$ :  $M$  does not halt on  $w$ , so  $M\#$  halts on nothing. Oracle rejects.

- 3.