Bearing-only cooperative geo-localization using Unmanned Aerial Vehicles

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Bearing-only Cooperative Geo-Localization using Unmanned Aerial Vehicles

Rajnikant Sharma, Randal W. Beard, Clark N. Taylor, and Daniel Pack

Abstract— **In this paper, we present a cooperative approach to geo-localize a ground target using bearing-only localization of Unmanned Aerial Vehicles (UAVs). We design a distributed path planning algorithm using receding horizon control, which improves the localization accuracy of the target and of all of the air vehicles simultaneously while satisfying the observability conditions. We show that the cooperative bearing-only localization removes the constraint of having global position and heading information to all of the vehicles, as long as the sensor network is connected and at least one of the vehicle has GPS measurements. We include simulation results for bearing-only cooperative target geo-localization of a stationary and a mobile ground target demonstrating the feasibility of our proposed methods.**

I. Introduction

Recently, there has been an increase in the use of Unmanned Aerial Vehicles (UAVs) in several military and civil application that are considered dangerous for human pilots. These applications include surveillance [1], reconnaissance [2], search [3], and fire monitoring [4]. Among the suite of possible sensors, a video camera is inexpensive, lightweight, fits the physical requirements of small UAVs, and has a high information to weight ratio. One of the important applications of camera equipped UAVs is determining the location of a ground target when imaged from the UAV. The target is geo-localized using the pixel location of the target in the image plane, the position and attitude of the air vehicles, the camera's pose angles, and knowledge of the terrain elevation. Previous target localization work using a camera equipped UAV is reported in [5], [6], [7], [8] and references therein. Barber *et al.* [6] used a camera, mounted on a fixed-wing UAV, to geo-localize a stationary target. They discussed recursive least square (RLS) filtering, bais estimation, flight path selection, and wind estimation to reduce the localization errors. Pachter *at el.* [5] developed a vision-based target geo-location technique that uses camera equipped unmanned air vehicles. They jointly estimate the target's position and the vehicles's attitude errors using linear regression resulting in improved target geo-localization. A salient feature of target geo-localization using bearing and range based sensors is the dependence of the measurement uncertainty on the position of the sensor relative to the target. Therefore, the influence of input parameters on nonlinear estimation problems, can be exploited to derive the optimal geometric configurations of a team of sensing platforms. However, maintenance of optimal configurations is not feasible given constraints on the kinematics of typical fixed wing aircraft. Frew [7] evaluated the sensitivity of target geo-localization to orbit coordination, which enables the design of cooperative line of sight controllers that are robust to variations in the sensor measurement uncertainty and the dynamics of the target tracked. While the existing work on vision based geo-localization successfully demonstrates the target localization concept and provides several techniques to improve the accuracy of geo-localization, the limitations associated with geo-localizing a target in urban environments are not addressed. All of the existing methods require the UAV's position and attitude to geo-localize a ground based target. The standard method for estimating position and attitude is to fuse measurements from a global positioning system (GPS) receiver and an inertial measurement unit (IMU). However, in many environments of interest, the GPS signals are unavailable or unreliable, for example, indoors, underwater, or in urban canyons. Additionally, the accuracy of pose estimates based on GPS may be insufficient for accurate target localization. Therefore, it is important to develop methods for localization in the absence of or in addition to GPS. Furthermore, a camera is a line of sight (LOS) sensor and there may exist many occlusions from buildings, trees, etc., in urban environments that can lead to unreliable tracking of the target.

In this paper, we address the aforementioned limitations by using cooperative localization to jointly localize a team of air vehicles and geo-localize a ground target. In cooperative localization a group of vehicles exchange relative position measurements like range and bearing, from their exteroceptive sensors like a camera or a laser and their motion information, velocity and angular rate, from interoceptive sensors like an IMU or encoders, to collectively estimate their states. For ground robots, cooperative localization has been an active area of research because it provides several potential advantages, including increased localization accuracy, sensor coverage, robustness, efficiency, and flexibility. In our previous work [9], for bearing-only localization of robots, we used a graph based approach to show that if each robot has a path to two different known landmarks then the system is completely observable and the position and heading of all of the robots can be estimated with bounded uncertainty. This result [9] can be extended and it can be

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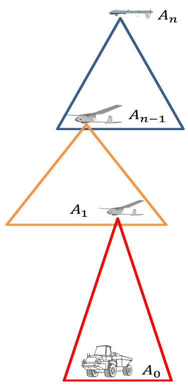


Fig. 1. Bearing-only cooperative geo-localization. A_0 is the target that is to be cooperatively geo-localized by a team of n aerial vehicles $A_1, \cdots, A_n$ that are equipped with gimballed cameras. Each vehicle flies at a constant altitude in descending order and measure bearing from each other and the target. A_n flies at the highest altitude, and is assumed to have access to GPS while the smaller vehicles flying at lower altitude do not have GPS

shown that if the graph is connected and at least one of the robots measures its position and heading from a GPS receiver, than the system is completely observable.

In this work, we develop a bearing-only cooperative localization technique for UAVs to geo-localize a ground target. In bearing-only localization the vehicles measure bearing to vehicles and targets in their field of view and that are in their sensor range. Each air vehicle flies at a constant altitude, as shown Figure. 1, such that the target is in the field-of-view of one of the agents and at least one vehicle, the larger vehicle which is flying at a higher altitude, receives signals from a sufficient number of GPS satellites to localize its position [9]. This overcomes the limitation of requiring a low-flying smaller UAVs to maintain line-of-sight while flying high enough to maintain GPS lock. Also, we design a distributed path planning algorithm using receding horizon control that improves the localization accuracy of the target and of all of the agents while satisfying the observability conditions.

The paper is organized as follows. In the next section, we formulate the problem and discuss the bearing-only cooperative localization for unmanned aerial vehicles. In Section III, we develop a distributed path planner to improve the accuracy of geo-localization. In Section IV, we present simulation results. In Section V, we give our conclusions.

II. Problem Formulation

Consider a mobile ground target A_0 moving in an urban terrain as shown in Figure 1. Next, consider a team of n aerial vehicles $(A_1, \cdots, A_n)$ flying at different constant altitudes in descending order. A_n flies at the highest altitude where a sufficient number of GPS satellites are available for its localization and A_1 flies at the lowest altitude to keep the target in its field-of-view as shown in Figure 1. Each UAV is equipped with an inertial measurement unit (IMU) and a downward facing gimballed camera. The IMU measures angular rates and accelerations in the body frame and the camera of the i^{th} agent measures the bearing from the $(i-1)^{th}$ agent. The objective is to cooperatively geo-localize, i.e., find the global position and heading of the ground target. For geo-localization, the vehicles must have accurate estimates of their position and attitude and the target must be in the sensor range of at least one of the vehicles. Since only A_n has the ability to sense its global position and heading through GPS, the global position and attitude of the other $n-1$ agents must be cooperatively estimated. This is accomplished by the agents exchanging information about their exteroceptive bearing measurements and interoceptive IMU measurements with other vehicles. This overcomes the limitation of requiring a low-flying smaller air vehicle to maintain line of sight while simultaneously flying high enough to maintain GPS lock.

There are two problems that need to be solved for cooperative target geo-localization. First, fuse the bearing-only measurements and IMU measurements from all of the agents and the GPS measurement from A_n to cooperative estimate position and heading of all of the air vehicles and the target. The second problem is to design a distributed guidance law that improves the accuracy of localization and keeps the target in the field-of-view if at least one vehicle. In the next subsection, we discuss bearing-only cooperative localization for unmanned aerial vehicles.

A. *Bearing-only cooperative localization*

In this paper, we use the following simplified guidance model for the i^{th} UAV

$$\begin{pmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{z}_i \\ \dot{\psi}_i \end{pmatrix} = f_i(X_i, u_i) = \begin{pmatrix} V_i \cos\theta_i \cos\psi_i \\ V_i \cos\theta_i \sin\psi_i \\ -V_i \sin\theta_i \\ \omega_i \end{pmatrix}, \tag{1}$$

where V_i is the airspeed, $[x_i, y_i, z_i]^\top$ is the 3-D position, $\omega_i = \frac{g}{V_i} \tan\phi_i$ is the turn rate, $u_i = [V_i, \omega_i]^\top$ is the control input, g is the gravitational acceleration constant, and $[\phi_i, \theta_i, \psi_i]^\top$ are roll, pitch, and yaw angles.

In this paper, we assume that there is no wind and that the airspeed V_i can be estimated from the differential pressure sensor [10]. Also, we assume that roll angle ϕ_i and the pitch angle θ_i are available. For small UAVs the roll and pitch angles can be estimated using gyro and accelerometer measurements [10]. The camera measures bearing from other UAVs, and landmarks that are in its sensor range. The bearing in azimuth and elevation measured from the i^{th} vehicle to the j^{th} vehicle are written as

$$\eta_{ij}^a = \tan^{-1}\left(\frac{y_j - y_i}{x_j - x_i}\right) - \psi_i + \upsilon^a \tag{2}$$

$$\eta_{ij}^e = \tan^{-1}\left(\frac{z_j - z_i}{R_{ij}}\right) - \theta_i + \upsilon^e \tag{3}$$

where $R_{ij} = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2}$ and $\upsilon^a \sim N(0, \sigma_\eta^2)$ and $\upsilon^a \sim N(0, \sigma_\eta^2)$ are Gaussian random processes with zero mean and variance σ_η^2. We assume that each camera can be gimballed to keep the other UAVs, or the target, in it's field-of-view.

Due to space constraint we omit the proof in this paper. ■

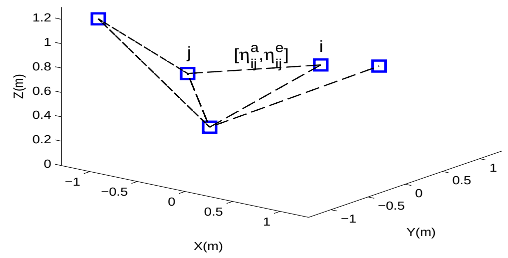


Fig. 2. Relative position measurement graph (RPMG). The nodes of an RPMG (blue squares are UAVs) represent vehicle states and the edges represent bearing measurements between nodes.

The GPS measurement for A_n is given as

$$z_{gps} = \begin{pmatrix} x_n \\ y_n \\ z_n \\ \psi_n \end{pmatrix} + \begin{pmatrix} \upsilon_x \\ \upsilon_y \\ \upsilon_z \\ \upsilon_\psi \end{pmatrix}, \tag{4}$$

where $\upsilon_x \sim N(0, \sigma_x^2)$, $\upsilon_y \sim N(0, \sigma_y^2)$, $\upsilon_z \sim N(0, \sigma_z^2)$, $\upsilon_\psi \sim N(0, \sigma_\psi^2)$ are zero mean Gaussian processes that model the measurement error in position and heading.

For cooperative localization, each vehicle exchanges its local sensor measurements including velocity, angular rates, and bearing measurements. Let N_i^M be the set of neighbors for which i^{th} UAV can obtain bearing measurements, and let N_i^C be the set of neighbors with which the i^{th} UAV can communicate. In this paper, we assume that $N_i^M = N_i^C$ and we will therefore denote the set of neighbors as N_i. To represent the connection topology of the UAVs we use a relative position measurement graph (RPMG)[11] which is defined as follows.

Definition 1: An RPMG for n nodes performing cooperative localization is a directed graph $G_n \triangleq \{\mathcal{V}_n, \mathcal{E}_n\}$, where $\mathcal{V}_n = \{1, \cdots, n\}$ is the node set consisting of n vehicle nodes and $\mathcal{E}_n(t) \subset \{\mathcal{V}_n \times \mathcal{V}_n\}$ is the edge set of m bearing measurements represented by $\{\eta_{ij}^a, \eta_{ij}^e\}$, $i, j \in \mathcal{V}_n$. Index $p \in \mathcal{E}_n$ represents the p^{th} measurement. An example RPMG G_5 is shown in Fig. 2.

The objective of cooperative localization is to estimate the combined state of all of the UAVs and the target. We use an extended information filter (EIF) [12] to implement bearing-only cooperative localization. The EIF is the dual of the EKF and the EKF is a quasi-local asymptotic observer for nonlinear systems where a necessary condition for its convergence and boundedness are that the system is fully observable[13]. Following theorem provides the conditions for the complete observability of the system.

Theorem 1: Given an RPMG G_n, if it is proper, connected, and one of the vehicle has its position and heading measurement from GPS then the system is completely observable, i.e., the rank of the observability matrix is $4n$.

Proof: The proof of this theorem is similar to the proof for the cooperative localization of ground robots given in [9].

III. Controller for Geo-Localization

Since in bearing-only cooperative localization both the motion model and measurement model are nonlinear, the uncertainty in estimates is path dependant. In this section, we develop a a distributed path planning algorithm that minimizes the uncertainty (maximizes the information) in the localization of all of the agents and the target while satisfying the observability constraints. For the target geo-localization problem, we restrict the motion of all of the vehicles in a horizontal plane at a constant altitude. Also, we assume that the vehicles and the target are moving with constant velocity, and that these velocities are in increasing order with the altitude i.e., $V_0 < V_1 < \cdots < V_n$. For developing a controller we use the equation of motion of an i^{th} UAV flying at constant altitude, $\theta = 0$, and performing a coordinated turns:

$$\begin{pmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{z}_i \\ \dot{\psi}_i \end{pmatrix} = \begin{pmatrix} V_i \cos\psi \\ V_i \sin\psi \\ 0 \\ \frac{g}{V_i}\tan\phi_i \end{pmatrix}. \tag{5}$$

Since the target moves on the ground, its equation of motion is similar to an UAV flying at constant altitude. For this work, we assume that the target moves in a straight line, i.e., $\dot{\psi}_1 = 0$, however, a maneuvering target can also be geo-localized using the scheme described in this section.

The information matrix is the inverse of the sensor measurement uncertainty and it contains all of the information about the accuracy of the sensor measurement. The information matrix for bearing measurements in the horizontal plane can be written as

$$I(t) = H^\top(t) R_{meas}^{-1} H(t) \tag{6}$$

where H is the measurement Jacobian and R_{meas} is the covariance matrix of the measurement noise. Since the bearing measurement is a nonlinear function of the states, the information matrix depends on the states of the vehicles. We can consider any number of cost functions derived from the information matrix, but in this paper we will focus on the trace of $I(t)$, which is a scalar function and it captures the quality of the estimates obtained from the set of bearing measurements.

While designing the optimization algorithm we should keep in mind the constraints imposed by the observability of the system that are given in Theorem 1. The first constraint that the global position and heading is measured by GPS is available to at least one of the vehicles is easily satisfied by flying A_n at a high enough altitude. The second constraint requires that the RPMG, including the target, be connected. To satisfy the connectivity constraint, we define a fixed RPMG topology in which agent A_i should measures the bearing to vehicle A_{i-1} at the next lower altitude. In other words, each vehicle should always keep the vehicle that is at the next lower altitude in its field-of-view. For the RPMG to be connected, the following condition should be satisfied.

$$R_i < R_{sensor}, \ \forall \ i \ \in \mathcal{V}_n \tag{7}$$

where $R_i \triangleq R_{i,i-i} = \sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2}$ is the horizontal separation between two nodes and R_{sensor} is the horizontal sensor range of the camera.

Remark 1: In this paper, we assume that there are no occlusions.

In order to improve the localization accuracy consider the following T step receding horizon control problem:

$$\max_{\phi_1(t:t+T),\cdots,\phi_n(t:t+T)} \sum_{\tau=0:T} trace(I(t+\tau)) \tag{8}$$

subject to

$$R_i < R_{sensor}, \ \forall i \in \mathcal{V}_n \tag{9}$$

$$|\phi_i(t)| < \phi_{max}, \forall i \in \mathcal{V}_n. \tag{10}$$

The condition $|\phi_i(t)| < \phi_{max}$ imposes the physical control constrains on the air vehicle. To solve the problem (8)$-$ (10), we use the distributed receding horizon control approach developed by Dunbar and Murray [14]. For the target geo-localization controller we only consider azimuth bearing measurement between two nodes. The edge set of the chain RPMG is given by

$$\mathcal{E}_n = \{\eta^a_{10}, \eta^a_{21}, \cdots, \eta^a_{n,n-1}\}. \tag{11}$$

For simplicity, we assume that $R_{meas} = \mathbf{I}$, which results in a simple cost function given by

$$trace(I(t)) = trace\left(\sum_{p\in\mathcal{E}_n} H_p^\top H_p\right) \tag{12}$$

$$= \frac{1}{R^2_{10}}(t) + \frac{1}{R^2_{21}(t)} + \cdots + \frac{1}{R^2_{n,n-1}(t)}, \tag{13}$$

where H_p is the Jacobian of the p^{th} bearing measurement. We can rewrite the distributed controller for the i^{th} UAV as

$$\max_{\phi_i(t:t+T)} \sum_{\tau=0:T} \frac{1}{R^2_{i,i-1}(t+\tau)} \tag{14}$$

subject to

$$R_i < R_{sensor} \tag{15}$$

$$|\phi_i(t)| < \phi_{max}. \tag{16}$$

We solve the above optimization problem using single agent dynamic programming [15].

IV. Results

In this section, we present the simulation results for the bearing-only cooperative geo-localization controller for stationary and mobile ground target. We consider four unmanned air vehicles that are flying at different constant altitudes and a ground mobile vehicle that needs to be geo-localized. The 4^{th} vehicle is flying at the highest altitude and receives its position and heading measurements from its GPS receiver. The other three agents do not have access to GPS receiver. The forward velocity of the target is $V_0 = 2 \ m/s$

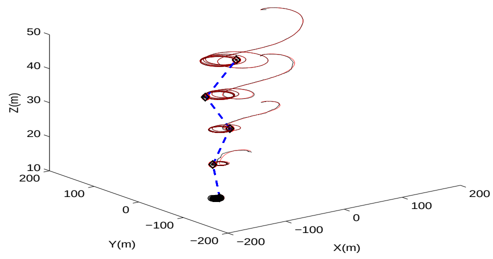


Fig. 3. Bearing-only cooperative geo-localization for a stationary ground target. This figure shows the trajectories (solid red curve is actual, dashed black curve is estimated) and covariance (black ellipse) of all of the agents. The top most vehicle has its position and heading measurement from GPS. The blue dashed curve represents the bearing measurement between two nodes . The velocity and the turning radius of each vehicle increase with increase in the altitude.

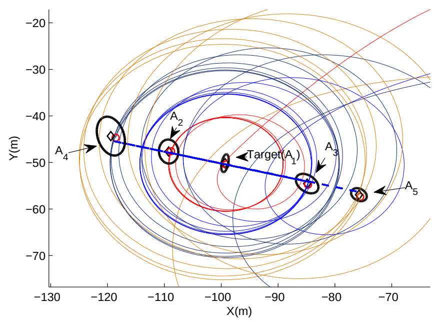
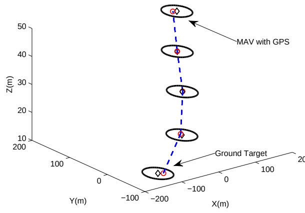
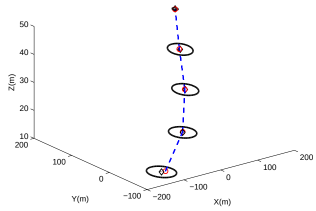


Fig. 4. Bearing-only cooperative geo-localization for a stationary ground target. This figure shows the the top view of the trajectories of all of the air vehicles. The black diamonds represents the estimated position and the red circles represents the true position. The black ellipses represents the 3σ uncertainties in the estimates. All of the vehicles settle on circular orbits around the target. Also, all of the air vehicles settle on a line joining them with the target.

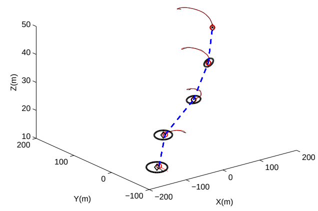
and the velocities of the other vehicles are $V_1 = 5m/s, \ V_2 = 8m/s, \ V_3 = 10m/s, \ V_4 = 12m/s$ and their respective altitudes are $z_1 = 20m, \ z_1 = 30_m, \ z_1 = 40m, \ z_4 = 50m$. The standard deviation for bearing measurement noise is $\sigma_n = 0.1 \ rad$. The sampling time is $T_s = 0.1s$ and we use a 3-step receding horizon for the optimization algorithm. First, we simulate the cooperative target geo-localization problem for a stationary target. Figure 3 shows the trajectories of all of the four air vehicles that are geo-localizing a stationary ground target. It can be seen that the estimated trajectories of all of the vehicles are close to their actual trajectories, demonstrating the effectiveness of this approach. A top-down view of the trajectories is shown in Figure 4. It can be seen that these trajectories converge on circular orbits around the target on a line joining all of the air vehicles and the target. The trajectory of A_4, which is at the highest altitude and moving at the fastest speed, converges to the outer most orbit and the trajectory of A_1, which is at the lowest altitude and moving at the slowest speed, converges to the inner most orbit. For clarity, we zoom in on Figure 4 to show the position estimates of the target as shown in Figure 5. It can



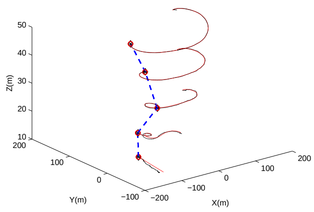
(a) t=0s



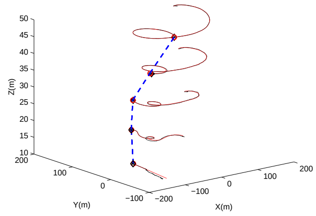
(b) t=0.1s



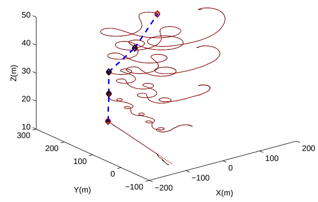
(c) t=3s



(d) t=7s



(e) t=10s



(f) t=100s

Fig. 8. Bearing-only cooperative geo-localization for mobile ground target. This figure shows the snap shots of the trajectories (solid red curve is actual, dashed black curve is estimated) and covariance (black ellipse) of all of the vehicles at different time intervals. The top most UAV has its position and heading measurement from the GPS. The blue dashed curve represent the bearing measurement between two nodes. The velocity of air vehicles decrease with the altitude.

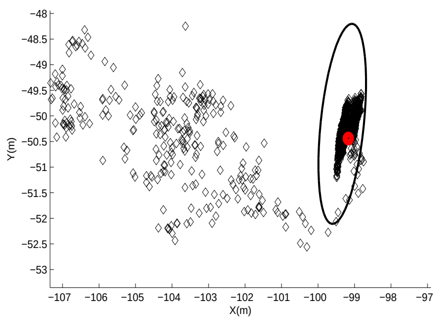


Fig. 5. The zoomed view, from Figure 4, of the position estimates of the ground target. The position estimates of the stationary ground target is represented by black diamonds). The red circle represents the true position of the ground target, and the black ellipse represents the 3σ position uncertainty.

be seen that the position estimates converge to approximately the actual position and the uncertainty reduces to a small value. The trace of the covariance matrix of the joint states of all of the vehicles are shown in the Figure 6. It can be seen that the uncertainty (trace) is minimized using the distributed receding horizon controller. Figure 7 shows the error plots of the position and heading of all of the vehicles that do not have GPS and the ground stationary target. It can be seen that uncertainties in the state estimation are minimized and they remain bounded. Next, we simulate the bearing-only cooperative target geo-localization for a mobile ground target. Figure 8 shows the snapshots of the trajectories and uncertainty in 2-D position of all of the vehicles and the ground target taken at different time intervals. The initial RPMG is shown in Figure 8(a), and the Figure 8(b) shows the RPMG at $t = 0.1s$ when the 4^{th} UAV measures its

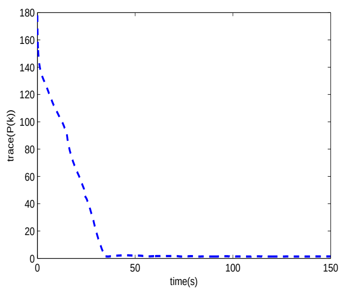
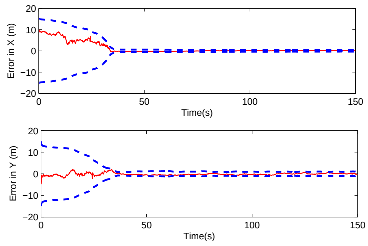


Fig. 6. Bearing-only cooperative geo-localization of a stationary ground target. This figure shows the trace of the combined state error covariance matrix $P(k)$.

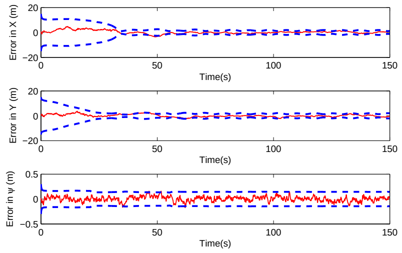
position and heading for the first time from its GPS receiver. From Figure 8, we can say that the localization information flows from top to bottom and the control information flows from bottom to top. Also, the localization errors and the uncertainty in all of the air vehicles and the ground target decrease and remain bounded. The trace of the covariance matrix of the joint states of all of the vehicles and the mobile ground target is shown in the Figure 9. It can be seen that the joint uncertainty is minimized using the distributed receding horizon controller. Figure 10 shows the error plots of the position and heading of all of the vehicles that do not have GPS and the ground target. It can be seen that uncertainties in the state estimation are minimized and that remain bounded.

V. Conclusion

In this paper, we develop a cooperative approach to geo-localize a ground moving target using bearing-only localization of Miniature Air Vehicles (MAVs). We have developed



(a) Target



(b) UAV 4

Fig. 7. Bearing-only cooperative geo-localization of a stationary ground target. This figure shows the error plots of position of the target and error plots of position and heading of the UAV 4, which do not have GPS. The blue dashed curve is the 3σ error variance and red solid curve is the estimation error.

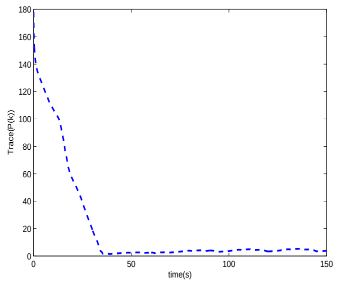


Fig. 9. Bearing-only cooperative geo-localization of a mobile ground target. This figure shows the trace of the combined state error covariance matrix $P(k)$.

a distributed path planning algorithm using receding horizon control, which improves the localization accuracy of the target and all of the MAVs while satisfying the observability conditions.

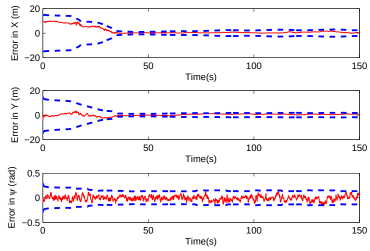
References

[1] M. Quigley, M. A. Goodrich, S. Griffiths, A. Eldredge, and R. W. Beard, "Target acquisition, localization, and surveillance using a fixed-wing mini-UAV and gimbaled camera," in *Proc. IEEE Int. Conf. Robotics and Automation ICRA 2005*, 2005, pp. 2600–2605.

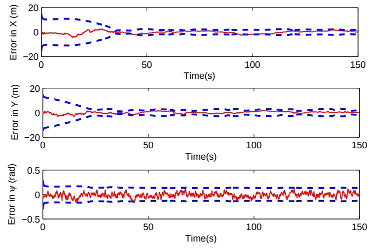
[2] A. Ayyagari, J. P. Harrang, and S. Ray, "Airborne information and reconnaissance network," in *Proc. IEEE Military Communications Conf. MILCOM '96*, vol. 1, 1996, pp. 230–234.

[3] R. W. Beard and T. W. McLain, "Multiple UAV cooperative search under collision avoidance and limited range communication constraints," in *Proc. 42nd IEEE Conference on Decision and Control*, vol. 1, 9–12 Dec. 2003, pp. 25–30.

[4] D. W. Casbeer, R. W. Beard, T. W. McLain, S.-M. Li, and R. K. Mehra, "Forest fire monitoring with multiple small UAVs," in *Proc. American Control Conf the 2005*, 2005, pp. 3530–3535.



(a) Target



(b) UAV 4

Fig. 10. Bearing-only cooperative geo-localization of a mobile ground target. This figure shows the estimation estimation error plots of the target states (position and heading) and error plots of position and heading of the UAV 4, which do not have GPS. The blue dashed curve is the 3σ error variance and red solid curve is the estimation error.

[5] M. Pachter, N. Ceccarelli, and P. R. Chandler, "Vision-based target geo-location using camera equipped MAVs," in *Proc. 46th IEEE Conf. Decision and Control*, 2007, pp. 2333–2338.

[6] D. B. Barber, J. D. Redding, T. W. McLain, R. W. Beard, and C. N. Taylor, "Vision-based target geo-location using a fixed-wing miniature air vehicle," *Journal of Intelligent and Robotic Systems*, vol. vol 47, no. 4, December, 2006.

[7] E. W. Frew, "Sensitivity of cooperative target geolocalization to orbit coordination," *Journal of Guidance, Control, and Dynamics*, vol. vol 31, no. 4, August, 2008.

[8] J. A. Ross, B. R. Geiger, G. L. Sinsley, J. F. Horn, L. N. Long, and A. F. Niessner, "Vision-based target geolocation and optimal surveillance on an unmanned aerial vehicle," in *AIAA, Guidance, Navigation, and Control Confrence, Honolulu, Hawaii*, Aug. 2008.

[9] R. Sharma, R. Beard, C. N. Taylor, and S. Quebe, "Graph-based observability analysis of bearing-only cooperative localization," *IEEE Transaction On Robotics*, vol. 28, 2, 2011.

[10] R. Beard and T. W. McLain, *Small Unmanned Aircraft: Theory and practice*. Princeton university press, 2011.

[11] A. Mourikis and S. Roumeliotis, "Performance analysis of multirobot cooperative localization," *IEEE Transaction for Robotics and Autonomus Systems*, vol. 22, no. 4, pp. 666–681, 2006.

[12] D. W. Casbeer and R. Beard, "Distributed information filtering using consensus filters," in *Proc. ACC '09. American Control Conference*, Jun. 10–12, 2009, pp. 1882–1887.

[13] Y. Song and J. W. Grizzle, "The extended kalman filter as a local asymptotic observer for descrete -time nonlinear systems," *Journal of Mathemetical Systems, Estimation, and Control*, vol. 5, pp. 59–78, 1995.

[14] W. B. Dunbar and R. M. Murray, "Receding horizon control of multi-vehicle formations: a distributed implementation," in *Proc. CDC Decision and Control 43rd IEEE Conf*, vol. 2, 2004, pp. 1995–2002.

[15] H. Yu, R. Sharma, R. W. Beard, and C. N. Taylor, "Observability-based local path planning and collision avoidance for micro air vehicles using bearing-only measurements," in *Proc. American Control Conf. (ACC)*, 2011, pp. 4649–4654.