Project 1: Assignment Part 1

1) R is both Symmetric and Asymmetric

   Suppose R is both symmetric and asymmetric. Then by symmetry for any x and y, if xRy it follows that yRx. However, by asymmetry it also follows that it is not the case that yRx. Hence, R cannot be both symmetric and asymmetric.

2) R is both Asymmetric and Reflexive

   Suppose R is both Asymmetric and Reflexive. Then by reflexivity xRx, however with antisymmetry if xRx then it is not the case that xRx; this generates a contradiction. Thus, R cannot be both Asymmetric and Reflexive

3) R is both Reflexive and Irreflexive

   Suppose R is both Reflexive and Irreflexive. Then by reflexivity xRx, however by irreflexivity ¬xRx. This generates a contradiction. Thus, R cannot be both Reflexive and Irreflexive.

4) R is both Functional and Transitive

   Suppose R is both functional and transitive. By functional property if xRy and xRz then y=z. Now let's assume yRw. Then by transitivity, since xRy and yRw then xRw. And by functional property y=w. So, if x relates multiple values through transitivity then functional property requires all of them to be equal. With this we'll get an arbitrary set of values which needs to be collapsed into a single value, this is not decidable. Hence the combination's truth is not decidable.

5) R is both Inverse Functional and Transitive

   Suppose we take a relation that follows strict ordering like greater than(>). Then it follows transitivity, but doesn't follow asymmetry. Eg; if 5 > 4 and 4 > 3 thus, 5 > 3 follows transitivity however, if 5 > 4 and 6 > 4 then it doesn't follow that 5 = 6. In this case this combination cannot hold true.
   But let's take another example relation (equality), if a=b and b = c, this means a = c, and with inverse functional property it also says a = b, which is true.
   Hence we can't decide the veracity or unsatisfiability of this combination.

6) R is both Transitive and Asymmetric

Suppose we take a relation that follows strict ordering like greater than(>). Then it follows transitivity, and also follows asymmetry. Eg; $5 > 4$ and $4 > 3$ thus, $5 > 3$. And if $5 > 3$ it also follows that $\neg 3 > 5$. In this case this combination holds true.
But let's take another example relation (equality), if $a = b$ and $b = c$, this means $a = c$, but with antisymmetry it also says $\neg c = a$ which is not correct. Hence we can't decide the veracity or unsatisfiability of this combination.

7) R is both Transitive and Irreflexive

Assume R is both transitive and irreflexive.
Suppose if xRy and yRz then xRz.
If we assume that zRx holds then by transitivity xRx. But this contradicts irreflexivity.
However we can't decide if zRx holds always.
For instance, greater than(>) relation is transitive and also is irreflexive, however equality(=) relation is transitive but not irreflexive.
So, the combination's truthfulness or unsatisfiability is undecidable.