Team notebook

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1 Algorithms

1.1 Longest Increasing Subsequence

1.2 Merge Sort

```
// O(n * lg(n))
void merge_sort(vector<int> &v, int 1, int r) {
       if (1 + 1 == r) return;
       int m = (1 + r) / 2;
       merge_sort(v, 1, m);
       merge_sort(v, m, r);
       vector<int> tmp(r - 1);
       for (int i = 1, j = m, k = 0; i < m or j < r; k++) {
               if (j == r || (i < m && v[i] <= v[j])) {</pre>
                      tmp[k] = v[i];
                      i++;
               } else {
                      tmp[k] = v[j];
                      j++;
               }
       for (int i = 0; i < (int)tmp.size(); i++)</pre>
              v[l + i] = tmp[i];
```

1.3 Mo's Algorithm

```
// O(n * sqrt(n))
const int ROOT = 200;

struct Query {
    int id;
    int L, R;
    Query() {}
    Query(int _id, int _L, int _R) : id(_id), L(_L), R(_R) {}
```

```
};
bool cmp(Query a, Query b) {
       if (a.L / ROOT == b.L / ROOT)
               return a.R < b.R;</pre>
       return a.L / ROOT < b.L / ROOT;</pre>
}
void mo(vi &a, vector<Query> &qs, vi &ans) {
       sort(qs.begin(), qs.end(), cmp);
       int currL = 0, currR = 0;
       int currSum = 0;
       fori(i, 0, SZ(qs)) {
               int L = qs[i].L, R = qs[i].R;
               while (currL < L) {</pre>
                       currSum -= a[currL];
                      currL++;
               }
               while (currL > L) {
                      currSum += a[currL - 1];
                       currL--;
               while (currR <= R) {</pre>
                       currSum += a[currR];
                       currR++;
               while (currR > R + 1) {
                       currSum -= a[currR - 1];
                       currR--;
               ans[qs[i].id] = currSum;
       }
```

1.4 Quick Sort

```
// O(n ** 2) | Theta(n * lg(n))
void quick_sort(vector<int> &v, int l, int r) {
    int key = v[l + rand() % (r - l + 1)];
    int i = l, j = r;
    do {
        while (v[i] < key) i++;
        while (v[j] > key) j--;
    }
```

1.5 Ternary Search

```
// O(log n) Integers
int ternary_search(vector<int> &v) {
       int n = v.size();
       int lo = 0, hi = n - 1;
       while (hi - lo > 1) {
               int mid = (lo + hi) >> 1;
               if (v[mid] < v[mid + 1])</pre>
                      lo = mid;
               else
                      hi = mid;
       return lo + 1;
}
// O(log n) Floatings
double ternary_search(args) {
       double lo = 0.0, hi = oo;
       for (int i = 0; i < num_iterations; i++) {</pre>
               double mid1 = (lo * 2 + hi) / 3; // 1 + (r - 1) / 3
               double mid2 = (1o + 2 * hi) / 3; // r - (r - 1) / 3
               if (f(mid1, args) < f(mid2, args))</pre>
                      lo = mid1;
               else
                      hi = mid2;
       }
       return lo;
```

2 Data Structures

2.1 Fenwick Tree

```
struct FenwickTree {
    int n;
    vi data;
    FenwickTree(int _n) : n(_n), data(vi(n)) {}
    void update(int at, int by) {
        while (at < n) data[at] += by, at |= at + 1;
    }
    int query(int at) {
        int res = 0;
        while (at >= 0) res += data[at], at = (at & (at + 1)) - 1;
        return res;
    }
    int rsq(int a, int b) { return query(b) - query(a - 1); }
};
```

2.2 Segment Tree

```
struct SegmentTree {
       int n;
       vi A;
       vi tree;
       SegmentTree() {}
       SegmentTree(vi &_A) {
              A = _A;
              int n = SZ(A);
              tree.assign(4 * n, 0);
              build(1, 0, n - 1);
       }
       build(int p, int i, int j) {
              if (i == j) {
                     tree[p] = A[i];
              } else {
                     int mid = (i + j) / 2;
                     build(2 * p, i, mid);
                     build(2 * p + 1, mid + 1, j);
```

```
tree[p] = min(tree[2 * p], tree[2 * p + 1]);
              }
       }
       int query(int p, int i, int j, int L, int R) {
              if (j < L \mid | i > R) return oo;
              if (L <= i && j <= R) return tree[p];</pre>
              int mid = (i + j) / 2;
              int q1 = query(2 * p, i, mid, L, R);
              int q2 = query(2 * p + 1, mid + 1, j, L, R);
              return min(q1, q2);
       }
       int query(int i, int j) { return query(1, 0, n - 1, i, j); }
       void update(int p, int i, int j, int at, int by) {
              if (j < at || i > at) return;
              if (i == j) {
                      A[i] = by;
                      tree[p] = bv;
                      return;
              int mid = (i + j) / 2;
              update(2 * p, i, mid, at, by);
              update(2 * p + 1, mid + 1, j, at, by);
              tree[p] = min(tree[2 * p], tree[2 * p + 1]);
       }
       void update(int at, int by) { return update(1, 0, n - 1, at, by); }
};
// Segment Tree Lazy Propagation
struct SegmentTree {
       int n;
       vi A;
       vi tree;
       vi lazy;
       SegmentTree(vi &_A) {
              A = A:
              n = A.size();
              tree.assign(4 * n, 0);
              lazy.assign(4 * n, 0);
```

```
build(1, 0, n - 1);
}
void build(int p, int i, int j) {
       if (i == j) {
              tree[p] = A[i];
       } else {
              int mid = (i + j) / 2;
              build(2 * p, i, mid);
              build(2 * p + 1, mid + 1, j);
              tree[p] = tree[2 * p] + tree[2 * p + 1];
       }
}
int query(int p, int i, int j, int L, int R) {
       if (j < L || R < i) return 0;
       if (lazy[p] != 0) {
              tree[p] += (j - i + 1) * lazy[p];
              if (i < j) {</pre>
                      lazv[2 * p] += lazv[p];
                      lazy[2 * p + 1] += lazy[p];
              lazy[p] = 0;
       }
       if (L <= i && j <= R) return tree[p];</pre>
       int mid = (i + j) / 2;
       int q1 = query(2 * p, i, mid, L, R);
       int q2 = query(2 * p + 1, mid + 1, j, L, R);
       return q1 + q2;
}
int query(int i, int j) { return query(1, 0, n - 1, i, j); }
void update(int p, int i, int j, int L, int R, int new_value) {
       if (lazy[p] != 0) {
              tree[p] += (j - i + 1) * lazy[p];
              if (i < j) {</pre>
                      lazy[2 * p] += lazy[p];
                      lazy[2 * p] += lazy[p];
              lazy[p] = 0;
       }
       if (j < L || R < i) return;
       if (L <= i && j <= R) {</pre>
              tree[p] += (j - i + 1) * new_value;
```

2.3 Sparse Table

```
// RMQ
const int MN = 100000;
const int ML = 18;
int T[MN][ML];
int ln[MN];
void build(vi &v) {
       // log2 O(1)
       ln[2] = ln[3] = 1;
       fori(i, 4, MN) ln[i] = 2 * ln[i / 2];
       // Sparse table
       int n = v.size();
       for (int i = 0; i < n; i++)</pre>
              T[i][0] = v[i];
       for (int j = 1, p = 2; p \le n; j++, p \le 1)
              for (int i = 0; i + p - 1 < n; i++)
                     T[i][j] = min(T[i][j-1], T[i+(p>>1)][j-1]);
}
int query(int 1, int r) {
       int k = ln[r - l + 1]:
       return min(T[1][k], T[r + 1 - (1 << k)][k]);
}
```

2.4 Trie

```
// Solve the problem of find v such that v <= \lim and v ^ x is maximum
struct Node {
       int best;
       Node* children[2];
       Node() {
              best = oo;
              children[0] = children[1] = NULL;
       }
};
Node* root;
void insert(int x, int k) {
       Node* u = root;
       u->best = min(u->best, x);
       for (int i = 16; i >= 0; i--) {
              int c = (x >> i) & 1;
              if (u->children[c] == NULL)
                      u->children[c] = new Node();
              u = u->children[c];
              u->best = min(u->best, x);
       }
}
int query(int x, int lim) {
       Node* u = root;
       int ans = 0;
       for (int i = 16; i >= 0; i--) {
              int c = (x >> i) & 1;
              Node* a = u->children[1 - c];
              Node* b = u->children[c];
              if (a != NULL && a->best <= lim) {</pre>
                      u = a;
                      ans |= (1 - c) << i;
              } else {
                      u = b;
                      ans |= c << i;
              }
       }
       return ans;
}
void init() {
```

```
root = new Node();
}
```

2.5 Union Find

```
struct UnionFind {
    int num;
    vi p, size;
    UnionFind(int n) : p(n, -1), size(n, 1), num(n) {}
    int findSet(int i) { return p[i] < 0 ? i : (p[i] = findSet(p[i]));
        }
    void unionSet(int i, int j) { // p[findSet(i)] = findSet(j);
        int a = findSet(j), b = findSet(i);
        if (a != b) { p[a] = b; size[b] += size[a]; num--; }
    }
    int numSets() { return num; }
    int sizeSet(int i) { return size[findSet(i)]; }
};</pre>
```

3 Graphs

3.1 Bellman Ford

```
for (int i = 0; i < V - 1; i++) {</pre>
        for (int u = 0; u < V; u++) {</pre>
               if (dist[u] == INF) continue;
                for (int j = 0; j < g[u].size(); j++) {</pre>
                       ii v = g[u][j];
                       if (dist[v.first] > dist[u] + v.second)
                               dist[v.first] = max(-INF, dist[u] +
                                    v.second):
       }
}
neg.assign(V, false);
for (int u = 0; u < V; u++) {</pre>
        for (int i = 0; i < g[u].size(); i++) {</pre>
               ii v = g[u][i];
               if (!neg[v.first] && dist[u] < INF && dist[v.first]</pre>
                    > dist[u] + v.second)
                       dfs(v.first);
       }
}
```

3.2 Edmonds Karp Max Flow

```
// O(n * m^2) = O(n^3 * m)
int n, res[MN][MN], p[MN], f, s, t;

void augment(int v, int minEdge) {
    if (v == s) { f = minEdge; return; }
    else if (p[v] != -1) {
        augment(p[v], min(minEdge, res[p[v]][v]));
        res[p[v]][v] -= f; res[v][p[v]] += f;
    }
}
int maxFlow() {
    int mf = 0;
    while (true) {
        f = 0;
        vi dist(MN, oo); dist[s] = 0;
        queue<int> q; q.push(s);
```

```
memset(p, -1, sizeof p);
              while (!q.empty()) {
                     int u = q.front(); q.pop();
                     if (u == t) break;
                     fori(v, 0, n) {
                             if (res[u][v] > 0 && dist[v] == oo) {
                                    dist[v] = dist[u] + 1;
                                    q.push(v);
                                    p[v] = u;
                            }
                     }
              }
              augment(t, oo);
              if (f == 0) break;
              mf += f;
       }
       return mf;
}
// Vertex capacities
int in(int x) { return 2 * x; }
int out(int x) { return 2 * x + 1; }
```

3.3 Euler Path

```
int g[MN][MN];
stack<int> s;

void dfs(int u) {
        fori(v, 0, n) if (g[u][v]) {
            if (used[u][v]) continue;
            used[u][v] = 1;
            dfs(v);
        }
        s.push(u);
}
```

3.4 Hamiltonian Path

```
// O(n * 2 ** n)
int n;
```

3.5 LCA Binary Lifting

```
const int MN = 200000;
const int ML = 18;
vi g[MN];
int h[MN];
int par[MN][ML]; // initialize -1
void dfs(int u, int p) {
       par[u][0] = p;
       if (p != -1) h[u] = h[p] + 1;
       for (int i = 1; i < ML; i++)</pre>
               if (par[u][i - 1] != -1)
                      par[u][i] = par[par[u][i - 1]][i - 1];
       for (int v : g[u]) if (v != p)
              dfs(v, u);
}
int lca(int u, int v) {
       if (h[u] < h[v])</pre>
               swap(u, v);
       for (int i = ML - 1; i >= 0; i--)
               if (par[u][i] != -1 && h[par[u][i]] >= h[v])
                      u = par[u][i];
       // h[u] = h[v]
```

3.6 Max Cardinality Bipartite Matching

```
// O(n * m)
vi g[MN];
int match[MN], vis[MN];
int aug(int u) {
       if (vis[u]) return 0;
       vis[u] = 1;
       for (int v : g[u]) {
              if (match[v] == -1 || aug(match[v])) {
                     match[v] = u;
                     return 1:
              }
       }
       return 0;
}
int main() {
       /* ... */
       int mcbm = 0:
       memset(match, -1, sizeof match);
       fori(u, 0, n) {
              memset(vis, 0, sizeof vis);
              mcbm += aug(u);
       }
       /* ... */
```

3.7 Minimum Spanning Tree

```
int par[MN];
int to[MN], from[MN], weight[MN];
bool cmp(int i, int j) { return weight[i] < weight[j]; }</pre>
int findSet(int i) { return par[i] < 0 ? i : par[i] = findSet(par[i]); }</pre>
void unionSet(int i, int j) { par[findSet(i)] = findSet(j); }
int MST() {
       11 cost = 0;
       vi edges(m);
       fori(i, 0, m) edges[i] = i;
       sort(edges.begin(), edges.end(), cmp);
       memset(par, -1, sizeof par);
       fori(i, 0, m) {
              int e = edges[i];
              if (findSet(to[e]) != findSet(from[e])) {
                      cost += weight[e];
                      unionSet(to[e], from[e]);
              }
       }
       return cost;
```

3.8 Strongly Connected Components

```
while (true) {
        int v = st.top(); st.pop();
        vis[v] = 0;
        compOf[v] = nComps;
        if (u == v) break;
        }
        nComps++;
    }
}
int main() {
    cnt = 0, nComps = 0;
    memset(num, -1, sizeof num);
    fori(i, 0, n) if (num[i] == -1)
        scc(i);
}
```

4 Mathematics

4.1 Combinatoric Formulas

$$\binom{n}{k} = \binom{n}{n-k} = \frac{n!}{k!(n-k)!}$$
$$\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$$
$$(x+y)^n = \sum_{k=0}^n \binom{n}{k} x^k y^{n-k}$$

when x = y = 1:

$$2^{n} = \sum_{k=0}^{n} \binom{n}{k}$$
$$(\frac{n}{k})^{k} \le \binom{n}{k} \le (\frac{en}{k})^{k}$$

Given an integer n, how many ways can k non-negative integers less than or equal to n add up to n

$$\binom{n+k-1}{k-1}$$

4.2 Combinatoric

4.3 Cycle Finding

```
int f(int x) { return ; }

ii cycleFinding(int xo) {
    int tortoise = f(xo), hare = f(f(xo));
    while (tortoise != hare) { tortoise = f(tortoise); hare =
        f(f(hare)); }
    int mu = 0; hare = xo;
    while (tortoise != hare) { tortoise = f(tortoise); hare = f(hare);
        mu++; }
    int lambda = 1; hare = f(hare);
    while (tortoise != hare) { hare = f(hare); lambda++; }
    return ii(mu, lambda);
}
```

4.4 Diophantine Equations

Let a and b be integers with d = gcd(a, b). The equation ax + by = c has infinitely many integral solutions if $d \mid c$ is true.

Let $k = \frac{c}{d}$ and (x_1, y_1) the solution found with the Extended Euclidean Algorithm, then:

$$x = kx_1 + \lambda(\frac{b}{d}); y = ky_1 - \lambda(\frac{a}{d})$$

4.5 Extended Euclidean Algorithm

```
void extendedEuclidean(ll a, ll b, ll &d, ll &x, ll &y) {
```

```
if (b == 0) {
              d = a;
              x = 1;
              v = 0;
       } else {
              extendedEuclidean(b, a % b, d, x, y);
              11 x1 = x, y1 = y;
              x = y1;
              y = x1 - (a / b) * y1;
       }
}
ll invModular(ll c, ll m) {
       11 d, x, y;
       extendedEuclidean(c, m, d, x, y);
       return (x + m) \% m;
}
```

4.6 Linear Recurrence Equations

$$f(n) = af(n-1) + bf(n-2) + cf(n-3); n \ge 3$$

$$\begin{vmatrix} f(n) \\ f(n-1) \\ f(n-2) \end{vmatrix} = \begin{vmatrix} a & b & c \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} * \begin{vmatrix} f(n-1) \\ f(n-2) \\ f(n-3) \end{vmatrix}$$

$$\begin{vmatrix} f(n) \\ f(n-1) \\ f(n-2) \end{vmatrix} = \begin{vmatrix} a & b & c \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} * \begin{vmatrix} f(2) \\ f(1) \\ f(0) \end{vmatrix}$$

4.7 Pillai Function

```
/* Native solution

int pillai(int x) {
    int ans = 0;
    for (int i = 1; i <= x; i++)
        ans += gcd(i, x);
    return ans;
}</pre>
```

5 Strings

5.1 KMP Algorithm

```
void kmp(string &T, string &P) {
       int n = T.size(), m = P.size();
       vector\langle int \rangle b(m + 1); b[0] = -1;
       // Preprocess P
       int i = 0, j = -1;
       while (i < m) {</pre>
               while (j \ge 0 \&\& P[i] != P[j]) j = b[j];
               i++; j++;
               b[i] = j;
       }
       // Search T
       i = 0, j = 0;
       while (i < n) {</pre>
               while (j \ge 0 \&\& T[i] != P[j]) j = b[j];
               i++; j++;
               if (j == m) {
                       cout << "P is found at index " << i - j << endl;</pre>
                       j = b[j];
               }
       }
```

5.2 Rolling Hashing

```
const 11 k = 28759543;
```

5.3 Split Line

```
vector<string> splitLine(string line) {
    istringstream iss(line);
    vector<string> tokens;
    copy(istream_iterator<string>(iss), istream_iterator<string>(),
        back_inserter(tokens));
    return tokens;
}
```

5.4 Suffix Array

```
struct SuffixArray {
    string T;
    vi SA, RA, LCP;

void computeLCP() {
        int n = T.size();
        LCP.assign(n, 0);
        vi Phi(n), PLCP(n);
        Phi[SA[0]] = -1;
}
```

```
for (int i = 1; i < n; i++)</pre>
               Phi[SA[i]] = SA[i - 1];
       for (int i = 0, L = 0; i < n; i++) {
               if (Phi[i] == -1) { PLCP[i] = 0; continue; }
               while (T[i + L] == T[Phi[i] + L]) L++;
               PLCP[i] = L;
               L = \max(L - 1, 0);
       for (int i = 0; i < n; i++)</pre>
               LCP[i] = PLCP[SA[i]];
}
void countingSort(int k) {
       int n = T.size(), maxi = max(300, n);
       vi c(maxi);
       for (int i = 0; i < n; i++)</pre>
               c[i + k < n ? RA[i + k] : 0]++;
       for (int i = 0, sum = 0; i < maxi; i++) {</pre>
               int t = c[i]:
               c[i] = sum;
               sum += t;
       }
       vi tempSA(n);
       for (int i = 0; i < n; i++)</pre>
               tempSA[c[SA[i] + k < n ? RA[SA[i] + k] : 0]++] =
                   SA[i]:
       for (int i = 0; i < n; i++)</pre>
               SA[i] = tempSA[i];
}
SuffixArray(string _T) { T = _T;
       int n = T.size();
       for (int i = 0; i < n; i++) RA.push_back(T[i]);</pre>
       for (int i = 0; i < n; i++) SA.push_back(i);</pre>
       for (int k = 1; k < n; k <<= 1) {</pre>
               countingSort(k);
               countingSort(0);
               vi tempRA(n);
               int r = 0;
               tempRA[SA[0]] = 0;
               for (int i = 1; i < n; i++)</pre>
                       tempRA[SA[i]] = (RA[SA[i]] == RA[SA[i-1]]
                           && RA[SA[i] + k] == RA[SA[i - 1] + k])?
                           r : ++r;
               for (int i = 0; i < n; i++)</pre>
```

```
RA[i] = tempRA[i];
                      if (RA[SA[n-1]] == n-1) break;
              }
              computeLCP();
       }
};
// String Matching
vi stringMatching(string T, string P) {
       int n = T.size(), m = P.size(), start, end;
       SuffixArray sa(T);
       int lo = -1, hi = n;
       while (hi - lo > 1) {
              int mid = (hi + lo) / 2;
              if (P > T.substr(sa.SA[mid], m))
                      lo = mid;
              else
                      hi = mid:
       if (hi == n || P != T.substr(sa.SA[hi], m)) return vi();
       start = hi;
       lo = 0, hi = n;
       while (hi - lo > 1) {
              int mid = (hi + lo) / 2;
              if (P < T.substr(sa.SA[mid], m))</pre>
                      hi = mid:
              else
                      lo = mid;
       }
       end = lo;
       vi ans:
       for (int i = end; i >= start; i--)
              ans.push_back(sa.SA[i]);
       return ans;
}
// Longest Repeated Substring
ii LRS(string s) {
       int n = s.size();
       SuffixArray sa(s);
       int idx = 0, maxLCP = -1;
       for (int i = 1; i < n; i++)</pre>
              if (sa.LCP[i] > maxLCP)
                      maxLCP = sa.LCP[i], idx = i;
```

5.5 Z Algorithm

```
vi compute_z(string &s) {
       int n = s.size();
       vi z(n);
       int 1 = 0, r = 0;
       fori(i, 1, n) {
               if (i > r) {
                      1 = r = i:
                       while (r < n \text{ and } s[r - 1] == s[r]) r++;
                      z[i] = r - 1; r--;
               } else {
                      int k = i - 1;
                      if (z[k] < r - i + 1) {
                              z[i] = z[k];
                      } else {
                              1 = i;
                              while (r < n \text{ and } s[r - 1] == s[r]) r++;
                              z[i] = r - 1; r--;
               }
       }
       return z;
```

6 Template

6.1 Main

```
#include <bits/stdc++.h>
#define endl '\n'
#define debug(X) cout << #X << " = " << X << endl
#define fori(i,b,e) for (int i = (b); i < (e); ++i)
#define SZ(v) (int)v.size()

using namespace std;

typedef long long ll;
typedef vector<int> vi;
typedef pair<int, int> ii;

const int oo = 1e9;

int main() {
    ios_base::sync_with_stdio(false); cin.tie(0);
    /* happy coding :D */
    return 0;
}
```