A Quantum Algorithm for Solving Linear Differential Equations: Theory and Experiment

Background

Recent years have witnessed remarkable progress in quantum computing, with continual advancements in both hardware and software. Today, quantum computers equipped with hundreds of qubits are capable of executing algorithms with circuit depths extending to several thousand gates—while still yielding meaningful computational results. This growth has paved the way for exploring more complex quantum algorithms, yet one of the central challenges remains: the development of efficient, innovative algorithms that offer clear advantages—ideally exponential speedups—over their classical counterparts.

One notable example of such an algorithm is "A Quantum Algorithm for Solving Linear Differential Equations: Theory and Experiment", by Tao Xin, et al. (2020). This work introduces a quantum algorithm designed to solve linear differential equations, a foundational class of problems with widespread applications in science and engineering.

However, as with most quantum algorithm research, the paper presents a primarily theoretical contribution. Translating such theoretical proposals into working implementations is far from straightforward. It involves navigating a range of challenges—from abstract algorithmic design to concrete circuit synthesis and optimization—highlighting the gap between theory and practical implementations of quantum algorithms.

Detailed Project Description

The objective of this project is to apply the algorithm described in the paper to a simple, well-understood physical system: the quantum harmonic oscillator. Specifically, you will implement a quantum algorithm to solve the harmonic oscillator equation with the initial conditions:

$$y'' + \omega^2 y = 0,$$

$$y(0) = 1, \quad y'(0) = 1$$

With frequency $\omega = 1$. Once the algorithm is implemented, use the resulting quantum state to evaluate the system's kinetic and potential energies as a function of time in the interval [0,1]. Explore how varying algorithmic parameters, such as the bounds used in functions like `inplace_prepare_state()`, affects the accuracy of these energy values. Finally, analyze how resource-efficient is your implementation, by comparing circuit depth and width under different optimization settings.

Deliverables

A notebook containing:

- A quantum program that solves the harmonic oscillator equation using the algorithm from the paper.
- Computed kinetic and potential energy values as functions of time, derived from the simulated output.
- An investigation of how parameter choices (e.g., amplitude bounds in state preparation) affect energy estimations.
- A graphical analysis of circuit depth and width under different optimization settings.

Resources

Project-specific resources

1. A Quantum Algorithm for Solving Linear Differential Equations: Theory and Experiment, by Tao Xin, et al.

Classiq resources

1. Getting Started with Classiq - Classiq 101
2. Classiq Documentation
3. The Classiq Library containing implementations of algorithms and tutorials
4. Classiq Community Slack available for any technical questions