UTMC Sample Geometry Round

UTMC Committee

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1. We have two distinct, nonintersecting ovals A and B. What is the maximum number of intersections that a circle can have with the two ovals?.

2. Consider an equilateral triangle with side length 2, and a semicircle with one of the bases as its diameter (which does not intersect the triangle aside from its base). Consider the vertex opposite to said base. What is the area of the circle centered at said vertex and tangent to the semicircle?

3. Let $ABCD$ be a parallelogram with $AB = CD = 10$ and $BC = DA = 5$. Let O_1 and O_2 be the circumcenters of triangles ABC and CDA. If O_1 and O_2 are distinct points such that O_1O_2 is parallel to BC, what is the area of quadrilateral AO_1CO_2?

4. Let $ABCD$ be a parallelogram with $AB = CD = 15$, $BC = DA = 7$. Say E is a point on line BC such that EA and AB are perpendicular. If $EDAB$ is cyclic, find the length of the altitude dropped from A to CD.

5. Consider a triangle ABC, where $\angle ABC = 60^o$ and $\angle ACB = 80^o$. Let I_A be the A-excenter, and let D and E be on AC and AB respectively such that ABI_AD and ACI_AE are both cyclic. If F is the intersection of BD and CE, compute the size of angle FI_AA.