Homework 4: Regular Expressions and NFAs

CIS 352: Programming Languages

8 February 2018, Version 1

Administrivia

- **No teams**, this assignment is a solo effort.
- Document in the cover sheet any ideas you use from other students or other sources.
- For Part I, *legible* hand written answers are fine.
- For Part II, copy all the files in `http://www.cis.syr.edu/courses/cis352/code/RegExp2/` and use `Top2.hs` as your starter file.
- Let me know if any of my QuickCheck tests seem dodgy.
- **Turn in Part I by:** dropping the papers in the CIS 352 bin on the 4th floor of SciTech.[1] Include a paper copy of your cover sheet.
- **Turn in Part II via:** Blackboard, include *(i)* your modified versions of `Matches2.hs` and `BuildNFA2.hs` from the Reg2 directory, *(ii)* the transcripts of test runs, and *(iii)* your coversheet.

Grading Criteria

- The homework is out of 100 points.
- Each programming problem is $\approx$ 70% correctness and $\approx$ 30% testing.
- Omitting your name(s) in the source code looses you 5 points.

[1] It is next to SciTech 4-226 and the CIS 252 box and the slot has a Keep Calm, Curry On sign over it.

Part I: Problems on Paper

The languages (i.e., set of strings) considered in Problems 1, 2, and 3 are all over the alphabet $\{\,\mathbf{a}, \mathbf{b}\,\}$.

Fair Warning: Variations of Problems 1, 2, and 3 are likely to show up on quizzes. So you should practice answering such questions "by-hand".

❖ ***Problem 1 (40 points)*** ❖

Use the rules on page 5 of the *Lexical Analysis* slides to give a formal derivation of each of the following. Each part is 4 points except for (i) which is 8 points.

(a) $((\mathbf{a}|\mathbf{b})|\mathbf{c}) \Downarrow \mathbf{a}$ (d) $(\mathbf{a}(\mathbf{bc})) \Downarrow \mathbf{abc}$ (g) $((\mathbf{ab})|\mathbf{c})^* \Downarrow \mathbf{c}$

(b) $((\mathbf{a}|\mathbf{b})|\mathbf{c}) \Downarrow \mathbf{b}$ (e) $((\mathbf{ab})\mathbf{c}) \Downarrow \mathbf{abc}$ (h) $((\mathbf{ab})|\mathbf{c})^* \Downarrow \mathbf{ab}$

(c) $((\mathbf{a}|\mathbf{b})|\mathbf{c}) \Downarrow \mathbf{c}$ (f) $((\mathbf{ab})|\mathbf{c})^* \Downarrow \epsilon$ (i) $((\mathbf{ab})|\mathbf{c})^* \Downarrow \mathbf{cabc}$

❖ ***Problem 2 (16 points)*** ❖

(a) Background. Let $L_1 = \{\, w \in \{\,\mathbf{a}, \mathbf{b}\,\}^* \;:\; w \text{ ends with } \mathbf{ab}\,\}$. A regular expression for this language is: $(\mathbf{ab})^*\mathbf{ab}$ and an NFA is $M_1 = (\{\,0,1,2\,\}, \mathit{Moves}_1, 0, \{\,2\,\})$ where

$$\mathit{Moves}_1 \;=\; \{\, 0 \xrightarrow{\mathbf{a}} 1,\; 0 \xrightarrow{\mathbf{b}} 0,\; 1 \xrightarrow{\mathbf{a}} 1,\; 1 \xrightarrow{\mathbf{b}} 2,\; 2 \xrightarrow{\mathbf{a}} 1,\; 2 \xrightarrow{\mathbf{b}} 0\,\}$$

or see Figure 1 for the diagram form.

Your problem: ***(4 points)*** Give an M_1-accepting path for **aabbaab**. (See pages 18 and 19 of the *Lexical* slides.)

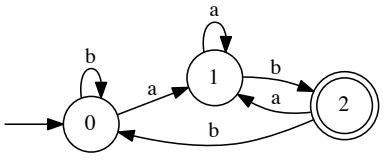


Figure 1: The diagram for M_1

(b) BACKGROUND. Let

$L_2 = \{\, w \in \{\, \mathbf{a}, \mathbf{b} \,\}^* \;:\; w \text{ has substring } \mathbf{ab} \text{ or substring } \mathbf{ba} \,\}$.

which has $((\mathbf{a}|\mathbf{b})^*\mathbf{ab}(\mathbf{a}|\mathbf{b})^*)|((\mathbf{a}|\mathbf{b})^*(\mathbf{ba})(\mathbf{a}|\mathbf{b})^*)$ as a regular expression. An NFA is $M_2 = (\{0,\ldots,6\}, Moves_2, 0, \{3,6\})$ where

$$Moves_2 = \left\{ \begin{array}{lll} 0 \xrightarrow{\epsilon} 1, & 0 \xrightarrow{\epsilon} 4, & \\ 1 \xrightarrow{\mathbf{a}} 1, & 1 \xrightarrow{\mathbf{b}} 1, & 1 \xrightarrow{\mathbf{a}} 2, \\ 2 \xrightarrow{\mathbf{b}} 3, & 3 \xrightarrow{\mathbf{a}} 3, & 3 \xrightarrow{\mathbf{b}} 3, \\ 4 \xrightarrow{\mathbf{a}} 4, & 4 \xrightarrow{\mathbf{b}} 4, & 4 \xrightarrow{\mathbf{b}} 5, \\ 5 \xrightarrow{\mathbf{a}} 6, & 6 \xrightarrow{\mathbf{a}} 6, & 6 \xrightarrow{\mathbf{b}} 6 \end{array} \right\}$$

or see Figure 2 for the diagram form.

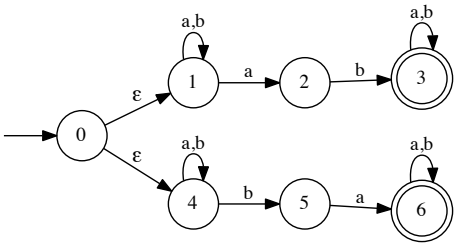


Figure 2: The diagram for M_2

YOUR PROBLEM: ***(12 points)*** Give ***four*** distinct M_2-accepting paths for **ababa**. (See pages 18 and 19 of the *Lexical* slides.)

❖ *Problem 3 (16 points)* ❖

For each of the following languages over $\{\, \mathbf{a}, \mathbf{b} \,\}$, give both (i) a regular expression and (ii) a NFA that precisely captures it.[2]

(a) Those strings in which every **b** is immediately followed by at least two **a**'s.

(b) Those strings that have both **ab** and **ba** as substrings

[2] *Hint:* It is often easier to start with the NFA and then use the NFA to help figure out the regular expression.

Part II: Programming Problems

You will need the files in `http://www.cis.syr.edu/courses/cis352/code/RegExp2/` and you will end up turning in changed versions of `Matches2.hs` and `BuildNFA2.hs`. This code is a modified version of Simon Thompson's regular expressions and automata library[3].

[3] Simon Thompson. Regular expressions and automata using Haskell. Technical report, Computing Laboratory, University of Kent at Canterbury, 2000. URL `http://www.haskellcraft.com/craft3e/Reg_exps.html`

❖ *Problem 4 (12 points)* ❖

BACKGROUND. Let w^R denote the reverse of string w.[4]

YOUR PROBLEM. In `Matches2.hs` add a Haskell function:

```
revExp :: Reg -> Reg
```

`(regExp e)`, on regular expression e, returns a new regular expression such that, for each String w:

$$e \text{ matches } w \iff \texttt{(revExp } e\texttt{)} \text{ matches } w^R. \qquad (1)$$

[4] *Example.* $(\mathbf{abcd})^R = \mathbf{dcba}$.

Example. If e represents $(\mathbf{ab}|\mathbf{cd})^*$, then `(revExp e)` represents $(\mathbf{ba}|\mathbf{dc})^*$.

Testing: `Matches2.hs` defines `Reg`'s

`re1` $\equiv \mathbf{a}(\mathbf{a}|\mathbf{b}|\mathbf{c}|\mathbf{d})^*$ `re2` $\equiv (\mathbf{a}|\mathbf{b}|\mathbf{c}|\mathbf{d})^*\mathbf{ab}(\mathbf{a}|\mathbf{b}|\mathbf{c}|\mathbf{d})^*$

Run:

```
(quickCheck (rev_prop re1))
(quickCheck (rev_prop re2))
```

Also, come up with some convincing tests of your own.

❖ ***Problem 5 (16 points)*** ❖

BACKGROUND. On page 13 Mogensen[5] defines the shorthands

$$r? \;=_{\text{def}}\; r|\epsilon \qquad\qquad r^+ \;=_{\text{def}}\; r(r^*)$$

A start at modifying Thompson's library to handle these two new forms can be found in:

`http://www.cis.syr.edu/courses/cis352/code/RegExp2/`

YOUR PROBLEMS.

(a) In `Matches2.hs` the function `matches` does not have cases for `Opt` or `Plus` expressions. Add the missing cases to `matches`.

Testing: Run `(quickCheck prop_equivA)`. Also come up with some convincing tests of your own.

(b) In `BuildNFA2.hs` the function `build` is missing cases for `Opt` or `Plus` expressions. Add the missing cases to `build`.

Testing: Run `(quickCheck prop_equivB)`. Also come up with some convincing tests of your own.

[5] Torben Ægidius Mogensen. *Introduction to Compiler Design*. Diku, 2010. URL `http://www.diku.dk/hjemmesider/ansatte/torbenm/Basics/`

Obvious hint for both parts (a) and (b): The `Opt`-case should be a variation on the `Or`-case and the `Plus`-case should be a variation on the `Star`-case.

Reference rule-sets

Rules for a big-step rules for regular expression matching

$$\epsilon\text{:}\ \frac{}{\epsilon \Downarrow \epsilon} \qquad |_1\text{:}\ \frac{r_1 \Downarrow s}{(r_1|r_2) \Downarrow s} \qquad |_2\text{:}\ \frac{r_2 \Downarrow s}{(r_1|r_2) \Downarrow s}$$

$$\textit{Lit:}\ \frac{}{x \Downarrow x} \qquad \textit{Seq:}\ \frac{r_1 \Downarrow s_1 \quad r_2 \Downarrow s_2}{(r_1 r_2) \Downarrow s}\ (s = s_1 s_2)$$

$$*_1\text{:}\ \frac{}{r^* \Downarrow \epsilon} \qquad *_2\text{:}\ \frac{r \Downarrow s_1 \quad r^* \Downarrow s_2}{r^* \Downarrow s}\ (s = s_1 s_2)$$

Example. See page 6 of the *Lexical* slides for sample derivations.

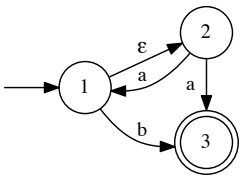
A small-step semantics for an NFA

For $M = (States, Moves, start, Final)$:

$$\frac{}{M \vdash s \xrightarrow{a} s'}\ \big((s, a, s') \in Moves\big)$$

$$\frac{}{M \vdash s \xrightarrow{\epsilon} s'}\ \big((s, \epsilon, s') \in Moves\big)$$

Example. For the NFA with diagram:



an accepting path for input **aab** is:

$$1 \xrightarrow{a} 2 \xrightarrow{\epsilon} 1 \xrightarrow{a} 2 \xrightarrow{\epsilon} 1 \xrightarrow{b} 3$$