The Mathematics of Sound
Read-ahead

Introduction
Sound travels through the air in compression waves. These waves are often represented as sinusoidal plane waves, and the sound we hear depends on the frequency and the amplitude of the wave. In this module you will use a simulation to explore these basic properties, as well as the phenomenon of noise cancellation.

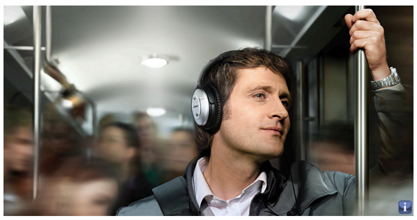


Figure 1: Ad for noise cancelling headphones.

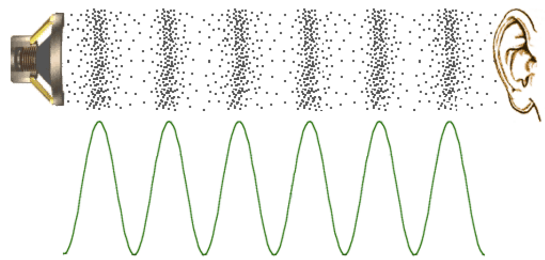


Figure 2: Compression waves.

Instructions
Your first step for this module should be to read through *The Mathematics of Sound* context and questions below, and complete the reflection assignment in Canvas. Note: *you will have a chance to talk further with your coach before answering the questions below in detail*. The point of this read-ahead and the reflection is to "prime the pump" for further conversations with your coaches.

The Mathematics of Sound
The standard note used for tuning is the fifth A on a piano, denoted A_4, which has a frequency of 440 Hz, i.e., 440 cycles per second. Equivalently, the period of A_4 is 1/440 seconds. If we model this sound wave with a function of the form $g(t) = A * \sin(\omega * t)$, where the input of the sine function is in radians, then we need to set $\omega = 2\pi * 440$. More generally, to produce a sound wave of frequency f, we need to set $\omega = 2\pi * f$. When you open the Sound simulation, you will see the default function written in this way.

Questions
You will need to use the Simulation tool *Sound* located on the Canvas course website to answer the questions below.

1. (Graded for completeness only.) Using the Sound simulation, enter the function $g(t) = \sin(2\pi * 440 * t)$ by changing the sliders to $A = 1, \varphi = 0$, and $f = 440$. When you click the "Play Sound" button, you should hear the computer play the note A_4 for 10 seconds. Adjust the A slider to a wide variety of values in order to hear the impact of the amplitude on what you hear. In particular, adjust the slider to $A = 0.5$ in order to hear $g(t) = 0.5 * \sin(2\pi * 440 * t)$ and to $A = 2$ in order to hear $g(t) = 2 * \sin(2\pi * 440 * t)$. Describe in words the differences in the sound (if any) that you hear.
2. (Graded for completeness only.) Now compare the sound made by $g(t) = \sin(2\pi * 440 * t + \varphi)$,

for different values of φ. (The easiest way to do this is to just use the slider.) Describe in words the differences in the sound (if any) that you hear.

3. (Graded for completeness only.) Enter the function $g(t) = \sin(2\pi * 440 * t) + \sin(2\pi * 440 * t + \varphi)$ into the function box. Use the slider to explore what happens to the sound (and the graph) as you change φ. Describe your observations in words. Note: You will need to type the second function into the function box. When you click in the function box, a small square containing α appears in the function box. Click on this to see a palette of Greek letters and choose φ as needed.
4. Continuing from the previous problem, what is the smallest positive value of φ for which the sound is the loudest? What is the smallest positive value of φ for which the sound is the softest? The phenomenon you are exploring here is known as *interference*. Note: This is the principle behind noise-canceling headphones, which sample the external noise and emit sound waves at the same frequency but shifted so as to maximize the destructive interference.
5. In the default function $g(t) = A * \sin(2\pi * f * t + \varphi)$, the parameter A represents the amplitude. Taking $\varphi = 0$ and $f = 440$, replace the constant amplitude A in this expression with a linear function $A(t)$ that decreases from 10 at $t = 0$ to 0 at $t = 10$. What is the resulting $g(t)$ containing this changing amplitude? Describe in words the differences in the sound (if any) that you hear.
6. So far, we have only experimented with changing the amplitude and shifting the phase. In the default function $g(t) = A * \sin(2\pi * f * t + \varphi)$, the parameter f represents the frequency. Taking $A = 1$ and $\varphi = 0$, replace f in this expression with a linear function $f(t)$ that increases from 87 at $t = 0$ (the note F_2) to 3136 at $t = 10$ (the note G_7). This is the reported vocal range of Mariah Carey. What is the resulting $g(t)$ containing this changing frequency? Describe in words the differences in the sound (if any) that you hear.

In the last part of this module we'll explore the phenomena of resonance to answer the question of whether a singer could break a wine glass with his/her voice.[1] In the first part of Question 4 above you heard what happens when the two sound waves of the same frequency line up, so that the peaks of one wave match the peaks of another. The result is a sound wave with amplitude the sum of the original amplitudes. The energy of the two waves is combined. This is called *constructive* interference.

A related phenomenon is *resonance*, which involves a transfer of energy from an external force to a system. Resonance can be illustrated through the example of a swing. When you start swinging on a swing, the system becomes a free oscillator with height of the swing seat moving in a sinusoidal fashion whose natural frequency is influenced by factors such as swing length, weight, and so on. What happens when someone starts pushing you on the swing? Common sense will tell you that the swing will swing higher and higher, at least up to a point. Physically what is happening is that the pusher is applying an external force to the system at the same frequency as the swing is swinging. The increasingly high swings that result are due to resonance. (Note that with an actual swing, energy is lost due to friction, so the external force is needed just to maintain the swing height.)

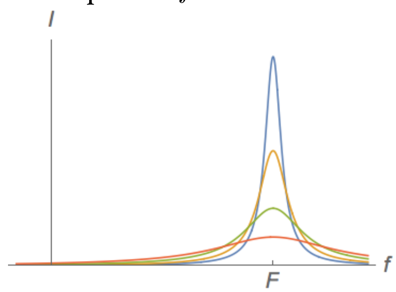
7. The natural frequency of a wine glass depends on the shape, size and physical composition of the glass, and ranges between 1 kHz and 10 kHz. (1 kHz, or *kilohertz*, is 1000 cycles per second.) For this problem we'll say that the natural frequency of your wine glass is 1.1 kHz. If you hit the glass with a knife to make a toast, what will be the wavelength of the sound produced? (Recall that the speed of sound in dry air is roughly 343 meters per second.)
8. (Graded for completeness only.) For an object with natural frequency F, the *intensity* (power per

[1] If you are wondering, the answer to this question is "yes". See the supplementary video sinked from the Canvas site.

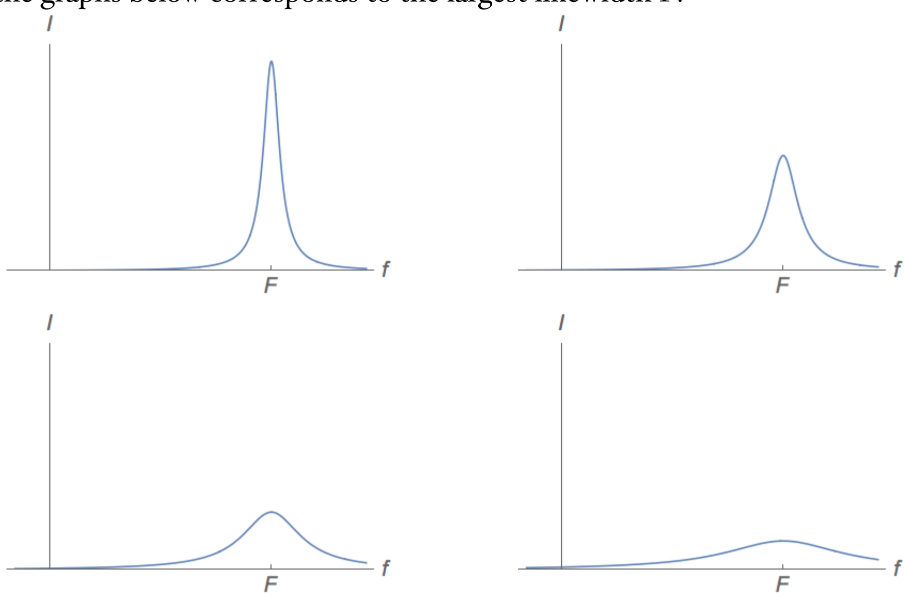
unit area) of the vibration as a function of the frequency f of the driving force can be approximated by

$$I(f) = \frac{1}{2\pi^2} \frac{\Gamma/4\pi}{(f-F)^2 + (\Gamma/4\pi)^2},$$

where Γ is the *linewidth* caused by damping. Linewidth is a measure of the noise in a signal; the smaller the linewidth, the more pure the tone. Graphs of $I(f)$ for varying linewidth are shown below. Explain why the graphs have a spike at $f = F$.



9. Which of the graphs below corresponds to the largest linewidth Γ?



10. Find an expression for the maximum intensity of the vibrations as a function of Γ.
11. Suppose your wine glass, with natural frequency 1.1 kHz, will break if the intensity of the vibrations reaches 0.01 Watts per square meter (this is 100 decibels). Assuming you can sing a note at exactly 1.1 kHz, so that $f = F$, what is the largest linewidth Γ that will break the glass?
12. Suppose you are able to sing a note with linewidth $\Gamma = 50$. (Recall that lower linewidth corresponds to a more pure note.) How close to the natural frequency 1.1 kHz do you need to get to break the glass?

Instructions, part deux

After reading and reflecting on these questions, complete the pre-read assignment on Canvas. This will give your coach some insight on your thinking in order to best help you before you are required to formally answer these questions.