Bond Angles
Read-ahead

Introduction
The chemical properties of a given molecule depend in a large part on the geometry of the molecule. One of the simplest molecules is methane, CH4 , which is the main component of natural gas. Each of the four hydrogen atoms is bonded to a single, central, carbon atom, forming a tetrahedron, as seen in Figure 1 below. In this set of problems, we determine the bond angle between any pair of hydrogen atoms.

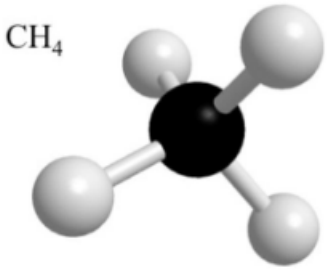

Figure 1: Methane molecule

Figure 2: A natural source of methane.

Instructions
After reading through *Bond Angles* context and questions below, you should complete the reflection assignment in Canvas. Note: *you will have a chance to talk further with your coach before answering the questions below in detail.* The point of this read-ahead and the reflection is to "prime the pump" for further conversations with your coaches.

Bond Angles
Draw a perpendicular line from the carbon atom (C) to the plane containing three of the hydrogen atoms. Let Q represent the foot of this perpendicular line (see figures below), and let y represent the distance between the carbon atom and any of the hydrogen atoms. Let a represent the distance from Q to one of the hydrogen atoms, and let x represent the measure of the required bond angle.

Questions

1. Consider the angle HQH in Figure 4 below. Find the measure of this angle, in degrees.
2. Find an expression for the distance a (the length of the line segment $\overline{\text{HQ}}$) in terms of the distance y and the angle x. Write your answer as an equation in the form "$a = ...$".
3. Now, examine the triangle formed by two hydrogen atoms and point Q, as shown in Figure 5 below. The altitude from point Q divides the triangle HQH into two congruent triangles. So, the vertex angle HQH is divided into two angles whose measures are each $60°$. In terms of a, what is the length of the line segment $\overline{\text{HT}}$? Write your answer as an expression involving a, not as an equation.
4. Figure 6 below represents the triangle formed by the carbon atom and two hydrogen atoms.

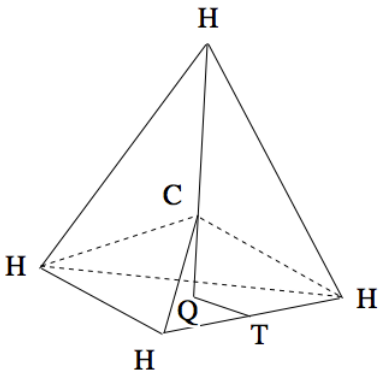

Figure 3

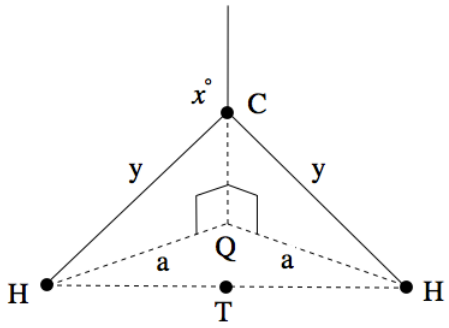

Figure 4

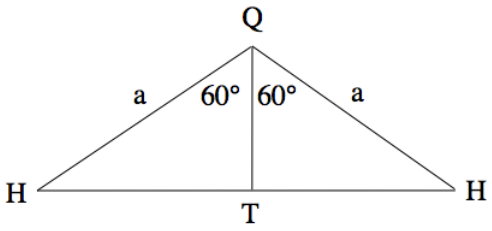

Figure 5

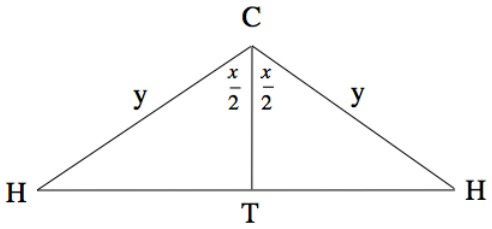

Figure 6

From this figure, we see that $\sin\left(\frac{x}{2}\right) = \frac{a\sqrt{3}}{2y}$. Use the double angle formula $\sin^2(\theta) = \frac{1-\cos(2\theta)}{2}$ to rewrite the left-hand side of this equation in terms of $\cos(x)$, then combine the resulting equation with the expression for a you found in problem 2 above, and solve for $\cos(x)$. Note that there are two solutions, one positive, one negative. Round your answers to three decimal places.

5. Note that only one of the two values for $\cos(x)$ you found in the previous part makes sense. Find x to the nearest degree.

Instructions, part deux

After reading and reflecting on these questions, complete the pre-read assignment on Canvas. This will give your coach some insight on your thinking in order to best help you before you are required to formally answer these questions.