Malthus and Population Growth
Read-ahead

Introduction

Thomas Robert Malthus was an 18th-century English scholar. His father, concerned about his son's education, was a friend of the philosopher David Hume, and exposed his son to a wide variety of ideas. In 1798, Malthus wrote *An Essay on the Principle of Population*, which was the most influential text of its era, impacting everyone from Charles Darwin to Karl Marx. In this application, we will investigate the hypothesis of Malthus' work, and in so doing learn more about linear and exponential functions.

Figure 1: Thomas Malthus

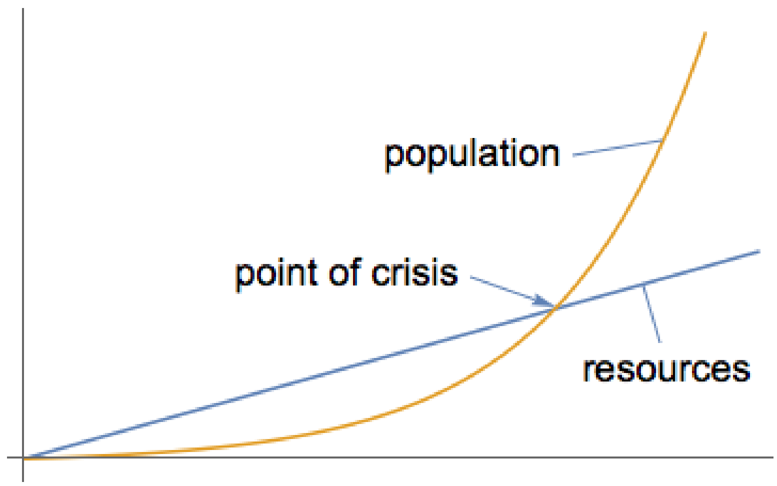

Figure 2: Malthus' basic theory

Instructions

After reading through the *Malthus and Population Growth* context and questions below, you should complete the application reflection in Canvas. Note: *you will have a chance to talk further with your coach before answering the questions below in detail*. The point of the read-ahead is to "prime the pump" for further conversations with your coaches.

Malthus and Population Growth

Malthus' central argument is summarized in Figure 2: population grows exponentially, while agricultural production grows linearly. He predicted that this mismatch will lead inevitably to food shortages, and starvation on a mass scale. (Such an event has come to be called a "Malthusian Catastrophe".) The mathematics underlying Malthus' theory is sound - exponential growth eventually outstrips linear growth - but critics have challenged his other assumptions. Population growth has slowed dramatically in developed countries, while at the same time technological advances have boosted food production in ways that Malthus did not anticipate. At the same time, we are now dealing with possible shortages in other vital resources, such as energy and water.

Questions

Suppose the population of Wholandia is currently, i.e., in the year 2018, 1.5 million people and is increasing at an annual rate of 3% per year.[1] Wholandia's food supply is currently adequate for 3 million people and is increasing at a constant rate of 50,000 people per year.

1. Let $P(t)$ be the population of Wholandia in year t, measured in millions, with $t = 0$ being the present. Write a formula for $P(t)$. Enter your answer as an equation in the form "$P(t) =$ ...".
2. Let $F(t)$ be the food supply of Wholandia in year t, measured in millions of people who can be adequately fed. Write a formula for $F(t)$. Enter your answer as an equation in the form "$F(t) =$ ...".
3. Use the graphs of $P(t)$ and $F(t)$ you found in the previous two problems to determine the calendar year in which food shortages will begin. Assume that $t = 0$ represents January 1, 2018.
4. Suppose instead that Wholandia's food supply is currently adequate for 6 million people, and is growing at the rate given above. In what calendar year do food shortages begin? Assume that $t = 0$ represents January 1, 2018.
5. Although Malthus predicted a food shortage catastrophe, it has not yet occurred. One of the potential reasons that this has not occurred is because human innovation has increased the food supply at a greater-than-expected rate. Suppose that Wholandia's food supply is currently adequate for 3 million people, but is increasing at a rate of 150,000 people per year, rather than 50,000 people per year. In what calendar year do food shortages begin? Assume that $t = 0$ represents January 1, 2018.
6. Assuming that the food supply is currently adequate for 3 million people, at what constant rate r would the food supply need to increase so that food shortages do not begin until the year 2200? *Note that the units of r should be millions of people per year.* Round your answer to three decimal places.
7. Suppose the population of Wholandia is growing at only 1.5% per year, instead of 3%. The current population is 1.5 million, and the food supply is adequate for 3 million people. At what constant rate r would the food supply need to increase so that food shortages do not begin until the year 2200? *Note that the units of r should be millions of people per year.* Round your answer to three decimal places.
8. (Graded for completeness only.) Given your answers to the above questions, what advice might you give to the Wholandians? Should they focus on reducing population growth rate? Increasing the rate of change for the food supply? Something else?

Instructions, part deux

After reading and reflecting on these questions, complete the application reflection on Canvas. This will give your coach some insight on your thinking in order to best help you before you are required to formally answer these questions.

[1] Note that there are two possible interpretations of the statement that the population is increasing at a rate of 3% per year. This is similar to the situation with compound interest. If you put P dollars in a bank account earning 3% interest compounded annually, the balance is given by $B(t) = P(1.03)^t$. If, on the other hand, the interest is compounded continuously the balance is $B(t) = Pe^{0.03t}$. These two functions are not the same, and in fact the second function grows more quickly, since $e^{0.03} \approx 1.03045 > 1.03$. In this problem, when we say that the population is growing at a rate of 3% per year, we mean in the first, i.e., non-continuous, sense.