Transformations and Modeling
ONEXYS Read-ahead

Introduction

Now that we have introduced the techniques of graph transformations, we will use our new tools to explore several examples of modeling involving sine and cosine functions. The main application will be to modeling the populations in a predator-prey relationship, and we will also revisit the sound and pendulum applications you have completed recently.

Instructions

After reading through the *Transformations and Modeling* context and questions below, you should complete the application reflection in Canvas. Note: *you will have a chance to talk further with your coach before answering the questions below in detail*. The point of the pre-read is to "prime the pump" for further conversations with your coaches.

Transformations and Modeling

Predator-prey models are mathematical models that describe interactions among species. Simply put, one species eats another for survival. These models are arguably the building blocks of the biological and and ecological systems as biomasses are grown out of their resource masses. But there are many other contexts in which these mathematical models are helpful. They can be used to describe resource-consumer relationships, plant-herbivore, parasite-host, and susceptible-infectious interactions, etc. [1]

Figure 1: Predator-Prey Relationships

Today, we're going to consider how periodic functions might be able to describe simple predator-prey relationships. In particular, we will estimate and interpret *period*, *amplitude*, and *midline* in the context of predator-prey relationships.

Rabbits and Foxes

The data in the table below describes the populations of foxes and rabbits in a national park over a 12 month period. Each value of t corresponds to the beginning of the month, with $t = 0$ corresponding to the beginning of January.

[1] Frank Hoppensteadt (2006) Predator-prey model. Scholarpedia, 1(10):1563.

t, month	0	1	2	3	4	5	6	7	8	9	10	11	12
$r(t)$, rabbits	100	750	567	500	567	750	1000	1250	1466	1500	1466	1250	1000
$f(t)$, foxes	150	143	125	100	75	57	50	57	75	100	125	146	150

This data can also be displayed graphically as in Figure 2 below.

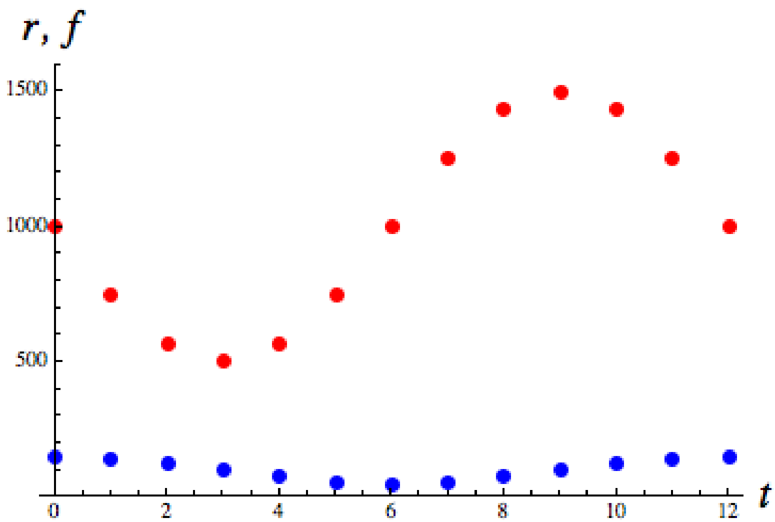

Figure 2: Fox and rabbit population.

1. Explain why it might be appropriate to model the number of rabbits and foxes as periodic functions of time.
2. Assuming the data indicates that the rabbit population is well-modeled by a periodic function, what are the period, midline, and amplitude of this function?
3. Assuming the fox population is also modeled by a periodic function, what are the period, midline, and amplitude of this function?
4. Looking at the data, it appears that one population "chases" the other, in the sense that the low values of one are quickly followed by low values of the other, and similarly for high values. Write a sentence or two explaining why we might expect this to be the case.
5. Find appropriate sinusoidal functions for $r(t)$ and $f(t)$. In both cases, write the functions so that the input for sine and/or cosine is in radians.

In the remaining questions, we revisit two earlier applications and use our new tools to explore models involving sine and cosine functions.

6. Recall in Questions 3 and 4 of the Sound application question you explored the phenomenon of noise cancellation. Consider the following sound wave.

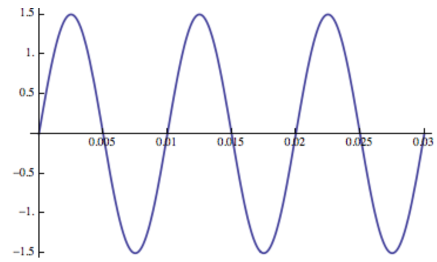

Figure 3: A sound wave.

(a) Write a sine function $g(t)$ to model this sound wave. Use the format $g(t) = A * \sin(2\pi * f * t)$ from the Sound simulation.

(b) As we have seen, noise cancellation is effected by producing a second wave $h(t)$ so that $g(t) + h(t) = 0$. This second wave has the same amplitude and frequency as the first, but has been phase-shifted so that maxima of $h(t)$ coincide with the minima of $g(t)$, and vice versa. Find the smallest positive value of k so that $h(t) = g(t + k)$ exactly cancels out the sound produced by $g(t)$.

(c) If you want to write $h(t)$ in the form $h(t) = A * \sin(2\pi * f * t + \phi)$, what should be the value of ϕ?

7. Recall from the last Pendulum question that as long as the initial angle θ_0 is relatively small and the pendulum is released from rest, the angle θ (in radians) of the pendulum, measured from vertical, will be well-approximated by the function $\theta(t) = \theta_0 \cos\left(\sqrt{\frac{g}{L}} * t\right)$, where θ_0 is the initial angle, g is the acceleration due to gravity (9.8 m/s2), L is the length of the pendulum, and t is in seconds.

    (a) Using the language of graph transformations, describe the effect on the graph of $\theta(t) = \theta_0 \cos\left(\sqrt{\frac{g}{L}} * t\right)$ of doubling the initial angle θ_0.

    (b) Using the language of graph transformations, describe the effect on the graph of $\theta(t) = \theta_0 \cos\left(\sqrt{\frac{g}{L}} * t\right)$ of doubling the length L of the pendulum.

Instructions, part deux

After reading and reflecting on these questions, complete the pre-read assignment on Canvas. This will give your coach some insight on your thinking in order to best help you before you are required to formally answer these questions.