Gravitational Waves
Read-ahead

Introduction

On February 11th, 2016, the Laser Interferometer Gravitational-Wave Observatory (LIGO) working with Virgo Collaboration announced the detection of gravitational waves, a monumental discovery for physics and a confirmation of Einstein's theory of general relativity. Einstein postulated that the fabric of the universe, rather than being static and unchanging, was in fact malleable. The changing topology of this fabric, called *spacetime* is credited with causing gravity. This is a notable departure from Newton's earlier theory of gravity. Whereas Newton believed the force of gravity was transported instantaneously ("force acting at a distance"), Einstein's theory implies that gravity travels at the speed of light, the universe's accepted speed limit.

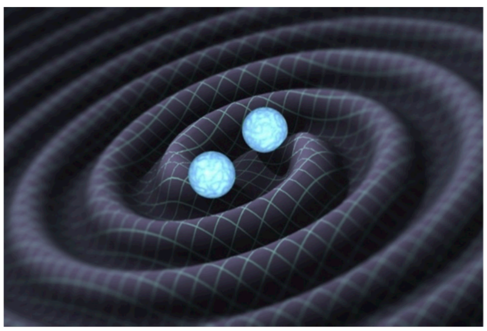

Figure 1: Gravitational waves

Figure 2: LIGO

If you imagine a blanket stretched taught to be the fabric of our universe (spacetime), then a massive object such as a black hole or a neutron star can be pictured as a weight placed on top of the blanket. The blanket would sag down, and a marble placed on the edge of the blanket would roll towards the weight. This is essentially how gravity works, although of course an actual blanket itself sags due to gravity. In reality the mere presence of mass causes distortions in spacetime by warping the existing spacetime around it.

If a larger object like a baseball were placed on the edge of the blanket and given some initial push to get it moving around the weight, it would gradually spiral in towards the center. When the baseball gets close to the weight, the weight would start to oscillate slightly up and down, sending waves across the blanket. It is these very waves, distortions of spacetime, that we call gravitational waves. In this application we will model gravitational waves using trigonometric functions.

Instructions

Your first step for this module should be to watch the video of physicist Brian Greene discussing gravitational waves that is linked from the Canvas site. After watching this video and reading through *Gravitational Waves* context and questions below, you should complete the reflection assignment in Canvas. Note: *you will have a chance to talk further with your coach before answering the questions below in detail.* The point of this read-ahead and the reflection is to "prime the pump" for further conversations with your coaches.

Gravitational Waves

Gravitational waves are incredibly small in comparison to the amount of energy necessary to produce them. The first definitive observation of gravitational waves came from the merging of two black holes over 1.3 billion light years away, resulting in the release of more than fifty times the combined power (energy per unit time) of all the stars in the observable universe. Even so, the ripples were so small that the length of the 4 km arm at LIGO only fluctuated by about one one thousandth of the width of a proton.

For the purpose of this application we will be considering wave emissions produced by a binary system, i.e. two objects, as shown in Figure 1 above. The power given off in gravitational waves by such a system is

$$P = \frac{32}{5}\frac{G^4}{c^5}\frac{m^2 n^2 (m+n)}{r^5},$$

where $G = 6.67 \times 10^{-11}$ m^3/(kg·s^2) is the gravitational constant, $c = 3 \times 10^8$ m/s is the speed of light, m and n are the masses of the two objects, and r is the distance between the centers of the two objects.

Questions

1. (Graded for completeness only.) Does the power given off in gravitational waves by a binary system increase or decrease as the two objects get closer and closer together? Explain.
2. The power given off in gravitational waves by the Earth, Sun system is 200 watts, about enough to run a typical window fan. Needless to say, this is undetectable when combined with the electromagnetic radiation of the sun, which is about 3.86×10^{26} watts. The radius of the Earth's orbit is about 1.5×10^{11} m. How small would the radius of the earth's orbit need to be for the Earth, Sun system to give off as much power in gravitational waves as the sun does in electromagnetic radiation?
3. (Graded for completeness only.) Is it possible for the radius of the Earth's orbit to be as small as the number you computed in Question 2? Explain. Hint: Look up the radius of the sun.

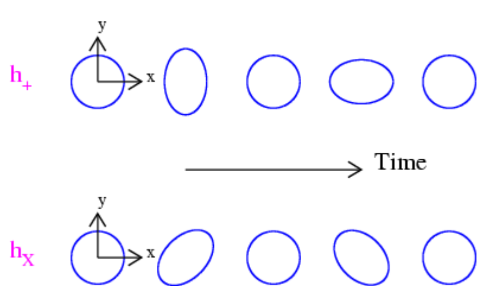

Figure 3: The polarizations + and ×

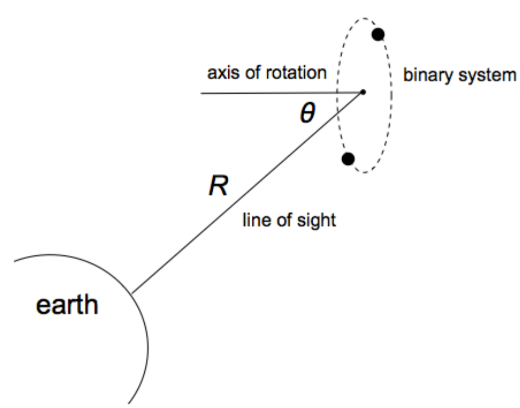

Figure 4: A binary system

Gravitational waves have two "polarizations", or directions of oscillation, denoted by $+$ and $\times$. The corresponding oscillations, pictured in Figure 3 above, can be modeled with trigonometric functions. Suppose the observer is at a distance R from the binary system, and the line of sight from the observer to the center of mass of the system makes an angle θ with the axis of rotation, as in Figure 4. Then the two polarizations of the gravitational wave are given by the following functions.

$$h_+(t) = -\frac{1}{R}\frac{G^2}{c^4}\frac{2mn}{r}(1+\cos^2\theta)\cos(2\omega(t-R/c))$$
$$h_\times(t) = -\frac{1}{R}\frac{G^2}{c^4}\frac{4mn}{r}(\cos\theta)\sin(2\omega(t-R/c))$$

where $\omega = \sqrt{G(m_1+m_2)/r^3}$.

4. What is the effect on the amplitudes of $h_+(t)$ and $h_\times(t)$ of doubling m or n?
5. What is the effect on the amplitudes of $h_+(t)$ and $h_\times(t)$ of doubling r?
6. What is the effect on the amplitudes of $h_+(t)$ and $h_\times(t)$ of doubling R?
7. What is the effect on the frequency of $h_+(t)$ and $h_\times(t)$ of simultaneously doubling both m and n?
8. What is the effect on the frequency of $h_+(t)$ and $h_\times(t)$ of doubling r?
9. What is the effect on the frequency of $h_+(t)$ and $h_\times(t)$ of doubling R?
10. (Graded for completeness only.) Give a physical interpretation of the quantity R/c and explain the role that this term plays in $h_+(t)$ and $h_\times(t)$.
11. (Graded for completeness only.) Suppose the binary system is rotating end-on to us, i.e., so that the axis of rotation is perpendicular to our line of sight. Would we be able to detect the gravitational waves from this system? Explain.
12. Although the two objects in a binary system are moving around each other, they can also be thought of as both orbiting the center of mass of the system. Note that the periods of these two orbits must be equal, so that the objects are always directly opposite each other with the center of mass in between. Suppose the object with mass m orbits the center of mass at a radius r and velocity v, and the object with mass n orbits at a radius s and velocity w. Find a relationship between r, v, s, and w. Write your answer as an equation involving these four quantities. Hint: The circumference of the orbit of the object of mass m is $2\pi r$. If the object is moving with velocity v, the time it takes to complete one orbit is $\frac{\text{distance}}{\text{velocity}} = \frac{2\pi r}{v}$.

Instructions, part deux

After reading and reflecting on these questions, complete the application reflection on Canvas. This will give your coach some insight on your thinking in order to best help you before you are required to formally answer these questions.