

## Trigonometric Identities

1. Pythagorean Identity:  $\cos^2 \theta + \sin^2 \theta = 1$
2. Odd/even identities
  - $\cos(-\theta) = \cos(\theta)$
  - $\sin(-\theta) = -\sin(\theta)$
3. Shifting identities:
  - $\cos(\theta - \pi/2) = \sin(\theta)$
  - $\sin(\theta + \pi/2) = \cos(\theta)$
  - $\cos(\pi/2 - \theta) = \sin(\theta)$
  - $\sin(\pi/2 - \theta) = \cos(\theta)$
4. Law of Cosines:  $a^2 + b^2 - 2ab \cos(C) = c^2$
5. Law of Sines:  $\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$ .
6. Double angle:  $\sin(2\theta) = 2 \sin(\theta) \cos(\theta)$ ,  $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$
7. Sums and Differences of Angles:
  - $\sin(\theta \pm \phi) = \sin(\theta) \cos(\phi) \pm \sin(\phi) \cos(\theta)$
  - $\cos(\theta \pm \phi) = \cos(\theta) \cos(\phi) \mp \sin(\theta) \sin(\phi)$
8. Sums and differences of sines and cosines with same amplitude. Note on next 4 identities I used  $u$  and  $v$  instead of  $\theta$  and  $\phi$  because of the way I derived the identities in the video. Watch the video and I think it will be clear why I chose these variables. Of course at the end of the day, it doesn't matter if it's  $u$  and  $v$  or  $\theta$  and  $\phi$  when we record the final result.
  - $\sin(u) + \sin(v) = 2 \sin\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right)$
  - $\cos(u) + \cos(v) = 2 \cos\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right)$
  - $\sin(u) - \sin(v) = 2 \cos\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right)$
  - $\cos(u) - \cos(v) = -2 \sin\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right)$
9. Sum of sines with same frequency:  $\alpha_1 \sin(\omega t) + \alpha_2 \sin(\omega t) = A \sin(\omega t + \phi)$  where  $A = \sqrt{\alpha_1^2 + \alpha_2^2}$  and  $\tan \phi = \frac{\alpha_2}{\alpha_1}$