The Mathematics of Sound
Read-ahead

Introduction

In this module we will revisit the Sound simulation, and explore the phenomenon of "beating", whereby two tones with approximately, but not exactly, equal frequencies combine to produce a pulsing sound. This phenomena is part of what produces the unique sound of Scottish bagpipes.

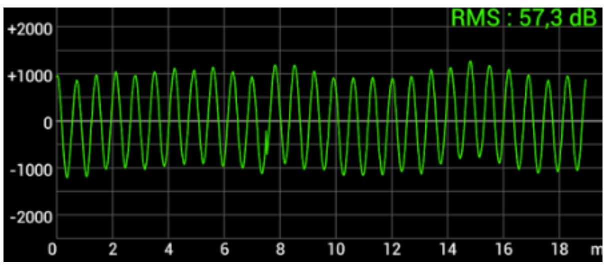

Figure 1: Sound waves displayed in an oscilloscope

Figure 2: Scottish bagpipes.

Instructions

After reading through *The Mathematics of Sound* context and questions below, you should complete the reflection assignment in Canvas. Note: *you will have a chance to talk further with your coach before answering the questions below in detail*. The point of the reflection is to "prime the pump" for further conversations with your coaches.

The Mathematics of Sound

Recall that in the simulation, entering the function $g(t) = A * \sin(2\pi * f * t)$ will produce a sound with amplitude A and frequency f, played for a duration of 10 seconds. In the first module, among other things you explored the effect of varying the frequency. Since most humans cannot detect very slight changes in frequency, one might expect that two tones with similar frequency would combine to produce a sound very similar to that of either played separately. In fact, as you will find out below, we get something quite different!

Questions

1. Using the Sound simulation, first play the sound modeled by the function $g_1(t) = \sin(2\pi * 440 * t)$, and then play the sound modeled by $g_2(t) = \sin(2\pi * 442 * t)$. Do you hear a difference in the two tones? Using the language of graph transformations, explain in words the differences between the graphs of $g_1(t) = \sin(2\pi * 440 * t)$ and $g_2(t) = \sin(2\pi * 442 * t)$.
2. (Graded for completeness only.) Now try the function $g(t) = g_1(t) + g_2(t) = \sin(2\pi * 440 * t) + \sin(2\pi * 442 * t)$ and describe what you hear.

3. Use a trig identity to rewrite $\sin(2\pi * 440 * t) + \sin(2\pi * 442 * t)$ as a product of a sine function and a cosine function.
4. In the previous part, you wrote the combination of a 440 Hz tone and a 442 Hz tone as a product. What are the frequencies of the sine and cosine factors in this product?
5. (Graded for completeness only.) Listen again carefully to the sound produced by the function $g(t) = g_1(t) + g_2(t) = \sin(2\pi * 440 * t) + \sin(2\pi * 442 * t)$. How many beats do you hear in the 10 seconds that the sound is played? Can you explain why you hear double the number of beats that you might expect from your answer to the previous problem?
6. In the previous example, you are hearing a tone whose frequency is the average of the two tones, and whose amplitude is defined by the envelope that is twice a cosine curve whose frequency is half the difference between the two tones. Try playing a 440 Hz tone together with a 330 Hz tone. Explain why you don't hear the beats in this case.

Instructions, part deux

After reading and reflecting on these questions, complete the pre-read assignment on Canvas. This will give your coach some insight on your thinking in order to best help you before you are required to formally answer these questions.