Interest Rates and the Fisher Equation
ONEXYS Read-ahead

Introduction

Irving Fisher was one of the most important economists of the 20th century. He received the first Ph.D. in economics given by Yale, and is well known today for his work on interest rates, macroeconomics, and general equilibrium.

Figure 1: Irving Fisher

Figure 2: Bank of England

In this application, we will look at the Fisher Equation, which estimates the relationship between inflation and interest rates. You will cover this topic in greater detail in Econ 111 or Econ 116 (introductory macroeconomics) at Yale, if you choose to take one of these courses.

Instructions

After reading through the *Interest Rates and the Fisher Equation* context and questions below, you should complete the application reflection in Canvas. Note: *you will have a chance to talk further with your coach before answering the questions below in detail*. The point of the read-ahead is to "prime the pump" for further conversations with your coaches.

Interest Rates and the Fisher Equation

Before introducing the Fisher equation, we need a few basic definitions.

- The *inflation rate* is the increase in prices of goods over time. If your salary is fixed, for example, and inflation increases, you will be able to purchase less and less over time. We will represent the inflation rate by π.

- The *nominal interest rate* is the interest rate that banks offer to you. If you open an account at a bank and deposit your money, you will earn interest on your money at this rate. We will represent the nominal interest rate by i.
- The *real interest rate* is the rate of interest an investor expects to receive after allowing for inflation. We will represent the real interest rate by r.

With all of this in mind, the Fisher equation[1] says that

$$r = i - \pi.$$

Essentially, the Fisher equation draws the distinction between the interest rate the bank offers and the true economic gain (the real interest) of interest payments.

It is important to note that the Fisher equation is *not actually an equation*. In fact, it is an approximation, and should really be written $r \approx i - \pi$. We will make this distinction clear below.

Questions

1. Suppose you have \$100 in your bank account, and the annual nominal interest rate is equal to 4% ($i = 0.04$). If you don't deposit or withdraw any funds, how much will be in your account in 1 year? in 5 years? in 10 years? This is the *nominal value* of your bank account, because you are using the *nominal interest rate*.
2. Consider the same situation as in Question 1, but suppose that inflation is constant, and is equal to 2% ($\pi = 0.02$). If you don't deposit or withdraw any funds, according to the Fisher equation what will be the *real value* of your bank account in 1 year? in 5 years? in 10 years?
3. As mentioned above, the Fisher equation is an approximation. The exact relationship between r, i, and π is that $1 + i = (1 + \pi)(1 + r)$. Solve this equation for r in terms of i and π.
4. Use the expression you found in Question 3 to calculate the exact value of r when $i = 0.04$ and $\pi = 0.02$.
5. Redo Question 2 using the value of r you computed in Question 4. How do these answers compare to your answers to Question 2?
6. (Graded for completeness only.) Starting with the equation $1 + i = (1 + \pi)(1 + r)$, explain why the Fisher equation $r = i - \pi$ is a good approximation when the interest rate and the inflation rate are low. Hint: Expand the right-hand side of the equation and consider the magnitude of each term.
7. In each question above, we took interest to be compounded once per year. If continuous compounding is used for i, π and r, construct the corresponding exact relationship between e^r, e^i, and e^π. Simplify this expression to infer the corresponding Fisher's approximation. Does this continuous version of Fisher's exact equation lead to a similar approximation? Hint: Fisher's exact relationship when compounding n times a year is $\left(1 + \frac{i}{n}\right)^n = \left(1 + \frac{\pi}{n}\right)^n \left(1 + \frac{r}{n}\right)^n$

For Questions 8-10, consider Eli, who is a student in the United Kingdom. Suppose that the nominal value (in pounds) of his bank account can be represented by the equation $f(t) = 200e^{0.05t}$, where t is time (in years) and the nominal interest rate is compounded continuously. The real value of his bank account can be represented by the equation $g(t) = 200e^{0.015t}$ where the real interest rate is compounded continuously.

[1] Economics textbooks often write the equation as $i = \pi + r$.

8. When $t = 5$, what is the difference between the nominal value of Eli's account and its real value?
9. What is the real rate of interest in Eli's bank account?
10. Using the Fisher equation (and assuming that all rates remain constant), deduce the inflation rate in the United Kingdom.

Instructions, part deux

After reading and reflecting on these questions, complete the application reflection on Canvas. This will give your coach some insight on your thinking in order to best help you before you are required to formally answer these questions.