Dendrochronology
Read-ahead

Introduction

When a museum or art gallery is considering acquiring a painting, particularly if it is valuable, the work must first be authenticated. This is done in three main ways: by dating the materials, by establishing the provenance (the complete list of owners), and by comparing of the subject and style of the painting with known works of the artist. For paintings on wood panels, dendrochronology, a dating method using tree rings, is an important way to identify when the painting was created.

Dendrochronology relies on the principle that trees of a given species from the same area and time period experience the same climate and accumulate the same amount of girth each year, resulting in tree rings of a specific width. The first step is to record the distance between growth rings found in a piece of wood. However, this data is not useful without some known patterns of tree ring growth with which to compare. By collecting wood samples from fossils, building timbers, and living trees, a *reference chronology* can be created, as shown in Figure 1 below. A reference chronology is a pattern of tree ring widths associated with a particular species, location and region. Numerically, reference chronologies can be expressed as graphs, where the y-value of each point represents the average or predicted tree width for the year indicated on the x axis.

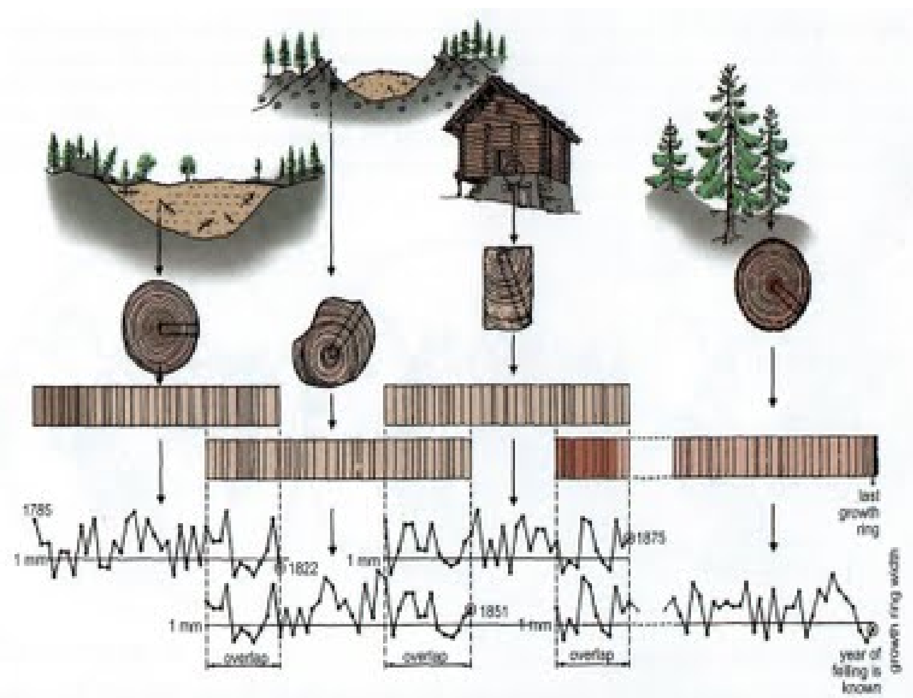


Figure 1: Building a reference chronology

The tree ring width patterns obtained from the sample are compared with the reference chronology to determine the approximate time period in which the wood from the painting originated. In this module you will explore one method for modeling the reference chronology using quadratic functions, and then try your hand at dating a painting.

Instructions

Your first step for this module should be to watch the video clip on dendrochronology and Viking longships that is linked from the Canvas site. After watching this video and reading through *Dendrochronology* context and questions below, you should complete the pre-read assignment in Canvas. Note: *you will have a chance to talk further with your coach before answering the questions below in detail*. The point of pre-read is to "prime the pump" for further conversations with your coaches.

Dendrochronology

Suppose you are a consultant for an agency that authenticates artwork. A lot of valuable art work has been lost throughout history or destroyed during wars. One particular collection from a 16th century Flemish painter is missing one out six of its works depicting the seasons. This painter, Pieter Brueghel the Elder (1535-1569), was at his peak in the Netherlands around the same time Michelangelo was painting the Sistine Chapel in Italy, and when much of the other well-known European art depicted religious imagery. Pieter Brueghel the Elder's paintings are incredibly important and valuable examples of early landscapes and secular paintings. The five paintings are shown below with their respective titles:

Hunters in the Snow (January) | The Gloomy Day (February) | Haymaking (July)

The Harvesters (August) | The Return of the Herd (November) | Unknown

Figure 2: Paintings by Pieter Brueghel the Elder

A third party claims to have found the final painting in the set of six paintings. Your job is to determine whether this painting is from the time period in which Pieter Brueghel the Elder lived using dendrochronology. Your boss provides you with a reference chronology from the Netherlands spanning from 1450 to 1650. Your first task is to manipulate the data so it is more easily comparable to the sample. One way to do this is to fit a *quadratic spline*[1] to the data; essentially, approximating the curve by joining together a series of parabolas. The result is shown in Figure 3.

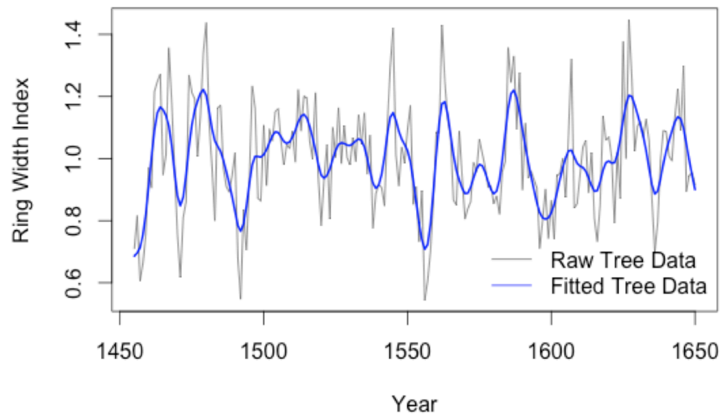


Figure 3: Reference chronology

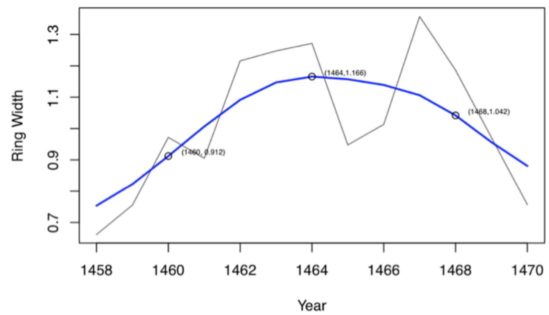
Questions

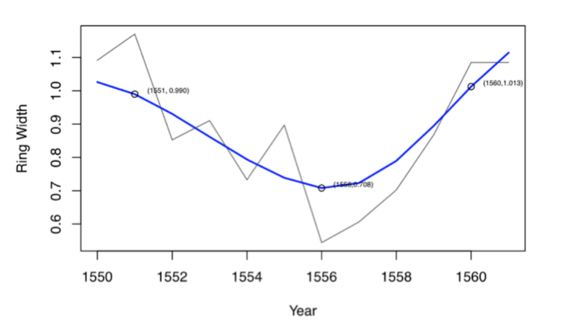
1. To construct a quadratic spline, we first need to be able to find the equation of a parabola through two or three given points. Find the equation of the parabola through the points $(-2, 3)$, $(0, 10)$, and $(2, 3)$. Write your answer in the form $y = ax^2 + bx + c$.
2. (Graded for completion only.) You will have an easier time with Question 1 if you realize that the vertex of the parabola is given to you. Explain in general what needs to be true about the points you are given for you to immediately identify the vertex.
3. Even when the vertex is not given, in some cases it is straightforward to determine the axis of symmetry of a parabola. Find the axis of symmetry of the parabola through the points $(1, 4)$, $(3, -2)$, and $(11, 4)$.
4. Find the equation of the parabola through the points $(1, 4)$, $(3, -2)$, and $(11, 4)$. Hint: Start with the vertex form of a parabola. Enter your equation as “y = ...”.
5. If only two points are given, there are infinitely many parabolas. Find the equation for the family of parabolas that go through the points $(1, 4)$ and $(3, -2)$, and sketch these parabolas. Write your answer in the form $y = ax^2 + bx + c$, where b and c are in terms of a.
6. When constructing a quadratic spline, parabolas are drawn between successive pairs of points in such a way that the resulting curve is “smooth”, i.e., so that it has no sharp corners. To do this, we need the following fact from calculus: The slope of a parabola $y = ax^2 + bx + c$ at the point (x, y) is given by $m = 2ax + b$. Find the parabola through the points $(1, 4)$ and $(3, -2)$ that has slope $m = 0$ at $x = 1$. Then, use the function grapher linked from the “Explore” section of this module to confirm your work. Write your answer in the form $y = ax^2 + bx + c$.
7. Find the slope of the parabola you found in Question 6 at the second point $(3, -2)$.
8. Find the parabola through $(3, -2)$ and $(11, 4)$ that matches the slope you found in Question 7 at the point $(3, -2)$. Then, use the function grapher to confirm that this parabola lines up correctly with the one you found in Question 6, so that they meet and have no sharp corner at the point

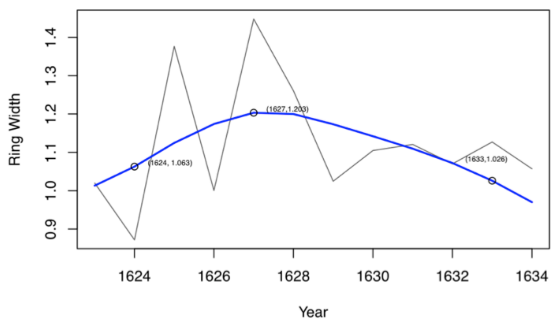
[1] To draw smooth curves through data points, drafters once used thin flexible strips of wood, hard rubber, metal or plastic called mechanical splines. To use a mechanical spline, pins were placed at a judicious points along a curve in a design, then the spline was bent so that it touched each of these pins.

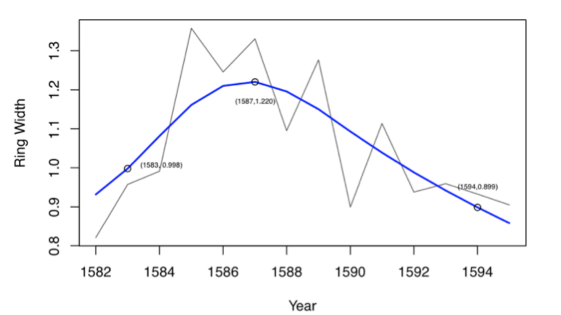
$(3, -2)$. Write your answer in the form $y = ax^2 + bx + c$. You have now found the quadratic spline through the points $(1, 4)$, $(3, -2)$, and $(11, 4)$.

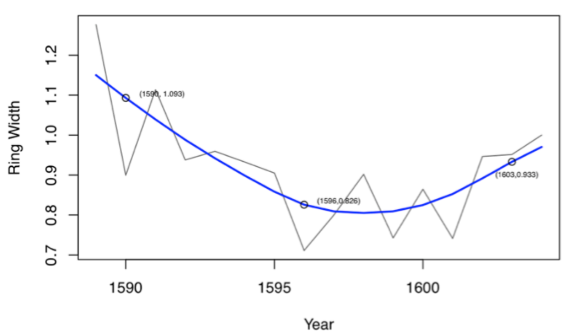
The process outlined in Questions 6-8 above can be automated on computer, and applied to a large number of data points. The result is a piecewise-defined curve like the one shown in Figure 3. Five separate pieces of this graph are shown below.











9. (Graded for completeness only.) Suppose you measure the tree ring widths on the panel and record the following set of data.

Ring Width	0.892	0.914	0.987	1.063	1.173	0.970	0.935

Can you match the data from the painting to any of the five portions of the chronological record shown above? What range of years does this wood possibly originate from? Explain your reasoning.

Instructions, part deux

After reading and reflecting on these questions, complete the pre-read assignment on Canvas. This will give your coach some insight on your thinking in order to best help you before you are required to formally answer these questions.