The Pendulum
CONEX Read-ahead

Introduction

Pendulums were first studied systematically by Galileo, who proved around 1602 the surprising fact that the period of a pendulum is independent of its amplitude. The first pendulum clock was developed in the 1650's by Christian Huygens; this technology remained the most accurate way to keep time for almost 300 years.

Figure 1: Foucault pendulum at the Pantheon in Paris.

You may have seen a *Foucault* pendulum at a science museum; these extra-long pendulums are used to give a simple demonstration of the rotation of the earth. The pendulum also was responsible for highly accurate estimates of gravitational acceleration, calculations of the density of the Earth, advances in geodetic surveys, and development of early seismometers to sense tremors. Today, pendulums are used as tuned mass dampers to protect high rise building from earthquakes.

In the video titled "The Pendulum: Worked Example", which is posted in the "Watch" section of the module page, we model the periodic horizontal and vertical motion of a simple pendulum. In this application, we are going to further explore the mathematics behind this motion.

Instructions

After watching the video "The Pendulum: Worked Example" and reading through *The Pendulum* context and questions below, you should complete the reflection assignment in Canvas. Note: *you will have a chance to talk further with your coach before answering the questions below in detail.* The point of this read-ahead and the reflection is to "prime the pump" for further conversations with your coaches.

The Pendulum

Consider the simple pendulum.

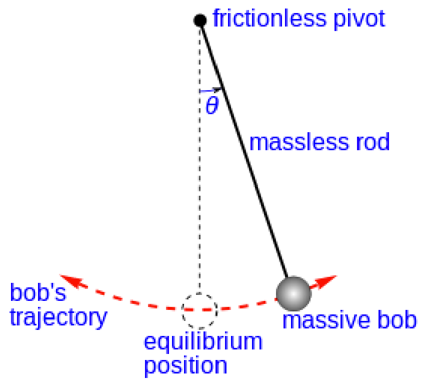

Figure 2: Simple pendulum.

The bob is connected by a massless rod to a pivot, which we will assume has no friction. We denote the angle the rod makes with the vertical by θ. Suppose the bob is pulled up to some initial angle $\theta = \alpha$ and released. Since we are ignoring friction, the pendulum will swing back and forth forever, in what is called "harmonic motion".

Questions

1. Suppose the pendulum is 5 meters long, and is released at an angle of 30 degrees from the vertical. What is the length (in meters) of one arc of the pendulum? Note that one arc is the distance the bob travels from when it is released to the time just before it begins to swing back towards the release point. Give an exact answer.
2. More generally, if a pendulum of length L meters is released from an initial angle of α, measured in radians, what is the length of one arc of the pendulum?
3. Putting the origin at the pivot on the pendulum, as in Figure 3 below, find expressions for the horizontal and vertical coordinates (x and y) of the location of the bob as a function of theta. For example, when theta is zero, the horizontal coordinate is $x = 0$ and the vertical coordinate is $y = -5$.

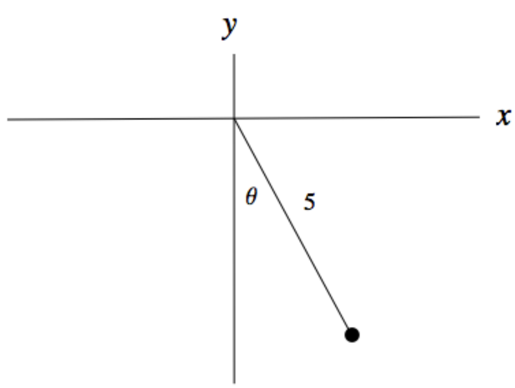

Figure 3: Pendulum drawn on coordinate axes.

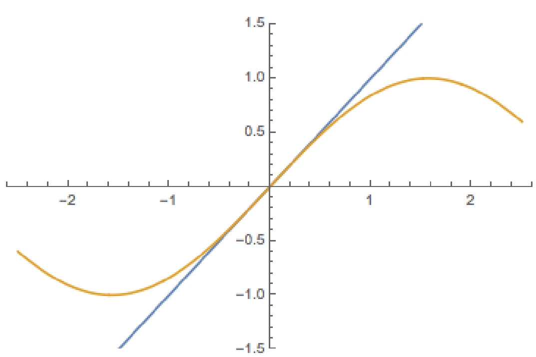

Figure 4: Graphs of $y = \theta$ and $y = \sin\theta$

4. As long as the initial angle α is relatively small and the pendulum is released from rest, θ (in

radians) will be well-approximated by the function

$$\theta(t) = \alpha \cos\left(\sqrt{\frac{g}{L}} * t\right),$$

where α is the initial angle, g is the acceleration due to gravity ($9.8\ m/s^2$), L is the length of the pendulum, and t is in seconds. This follows from the fact, displayed in Figure 4 above, that $\sin(\theta) \approx \theta$ when θ (in radians) is small. Calculate the period[1] of the pendulum described in Question 1, with $L = 5$ meters. Round your answer to two decimal places.

Instructions, part deux

After reading and reflecting on these questions, complete the pre-read assignment on Canvas. This will give your coach some insight on your thinking in order to best help you before you are required to formally answer these questions.

[1] The period of a swinging pendulum is the time it take to return to the position of its initial release, i.e., to complete two full arcs.