LEARNING AGENDA

PREREQUISITES FOR THE CHAPTER
- CONTENTS OF CHAPTER 1 (THEORY)

LEARNING OBJECTIVES
KNOWLEDGE
- TO LEARN ABOUT THE THEORY BEHIND ADDING WAVEFORMS (PHASE, CONSTRUCTIVE INTERFERENCE, DESTRUCTIVE INTERFERENCE)
- TO LEARN ABOUT THE THEORY AND USE OF BASIC ADDITIVE SYNTHESIS, USING BOTH FIXED AND VARIABLE SPECTRA TO PRODUCE BOTH HARMONIC AND NON-HARMONIC SOUNDS
- TO LEARN ABOUT THE RELATIONSHIP BETWEEN PHASE AND BEATS
- TO LEARN HOW TO USE WAVETABLES, AND HOW INTERPOLATION IS IMPLEMENTED
- TO LEARN SOME THEORY TO SUPPORT BASIC VECTOR SYNTHESIS

SKILLS
- TO BE ABLE TO DIFFERENTIATE BETWEEN HARMONIC AND NON-HARMONIC SOUNDS
- TO BE ABLE TO RECOGNIZE BEATS UPON HEARING THEM
- TO IDENTIFY THE DIFFERENT SEGMENTS OF A SOUND ENVELOPE, AND TO DESCRIBE THEIR CHARACTERISTICS

CONTENTS
- ADDITIVE SYNTHESIS USING BOTH FIXED AND VARIABLE SPECTRA
- HARMONIC AND NON-HARMONIC SOUNDS
- PHASE AND BEATS
- INTERPOLATION
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ACTIVITIES
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- FUNDAMENTAL CONCEPTS
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2.1 FIXED SPECTRUM ADDITIVE SYNTHESIS

A sound produced by an acoustic instrument, any sound at all, is a set of complex oscillations, all produced simultaneously by the instrument in question. Each oscillation contributes a piece of the overall timbre of the sound, and their sum wholly determines the resulting waveform. However, this summed set of oscillations, this complex waveform, can also be described as a group of more elementary vibrations: sine waves.
Sine waves are the basic building blocks with which it is possible to construct all other waveforms. When used in this way, we call the sine waves frequency components, and each frequency component in the composite wave has its own frequency, amplitude, and phase. The set of frequencies, amplitudes, and phases that completely define a given sound is called its **sound spectrum**. Any sound, natural or synthesized, can be decomposed into a group of frequency components. Synthesized waveforms such as we described in Section 1.2 are no exception; each has its own unique sound spectrum, and can be built up from a mixture of sine waves. (Sine waves themselves are self-describing – they contain only themselves as components!).

SPECTRUM AND WAVEFORM

Spectrum and waveform are two different ways to describe a single sound. Waveform is the graphical representation of amplitude as a function of time.[1] In figure 2.1, we consider the waveform of a complex sound in which the x-axis is time and the y-axis amplitude. We note that the waveform of this sound is bipolar, meaning that the values representing its amplitude oscillate above and below zero. A waveform graph is portrayed in the **time domain**, a representation in which instantaneous amplitudes are recorded, instant by instant, as they trace out the shape of the complex sound.

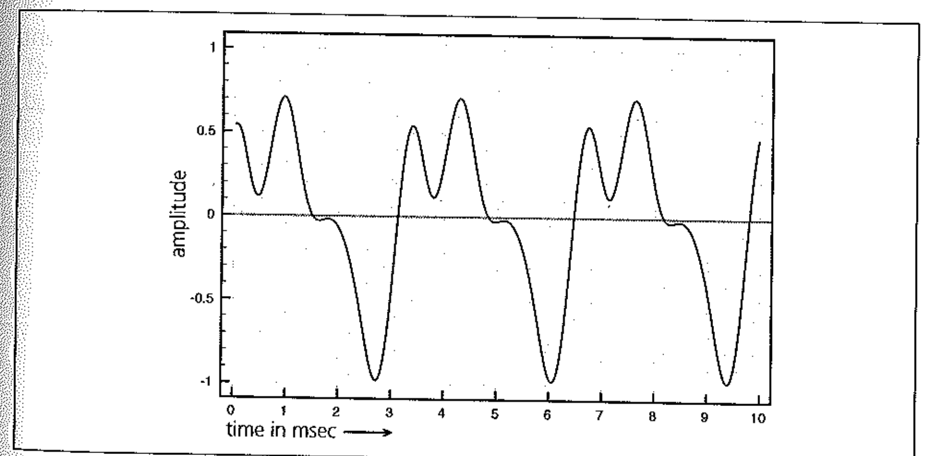


Fig. 2.1 The waveform of a complex sound

[1] In the case of periodic sounds, the waveform can be fully represented by a single cycle.

In figure 2.2, we see the same complex sound broken into frequency components. Four distinct sine waves, when their frequencies and amplitudes are summed, constitute the complex sound shown in the preceding figure.

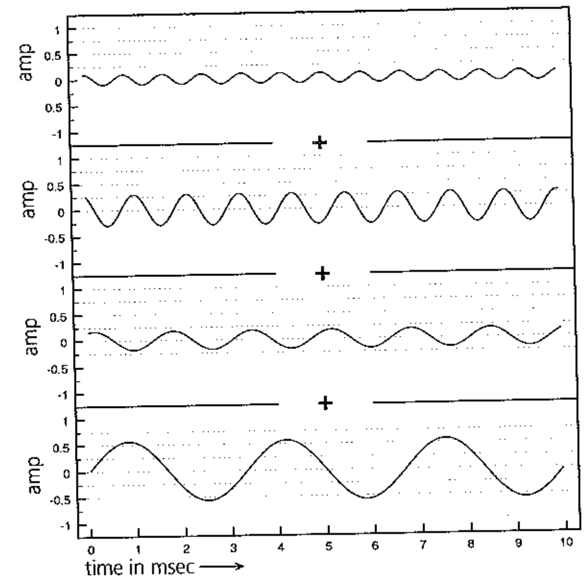
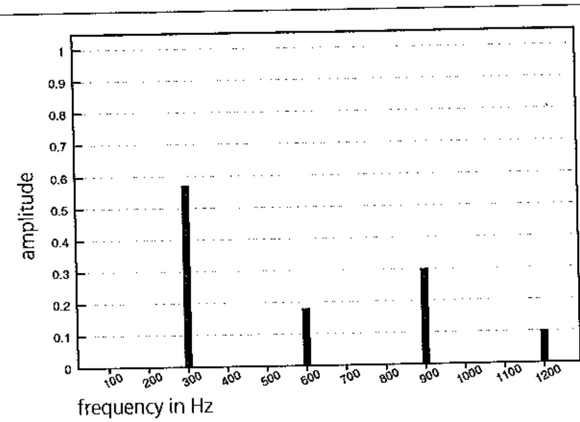


Fig.2.2 Decomposition of a complex sound into sinusoidal components

A clearer way to show a "snapshot" of a collection of frequencies and amplitudes such as this might be to use a graph in which the amplitude of the components is shown as a function of frequency, an approach known as **frequency domain** representation. Using this approach, the x-axis represents frequency values, while the y-axis represents amplitude. Figure 2.2b shows our example in this format: a graph displaying peak amplitudes for each component present in the signal.



In order to see the evolution of components over time, we can use a graph called a **sonogram** (which is also sometimes called a spectrogram), in which frequencies are shown on the y-axis and time is shown on the x-axis (as demonstrated in figure 2.2c). The lines corresponding to frequency components become darker or lighter as their amplitude changes in intensity. In this particular example, there are only four lines, since it is a sound with a simple fixed spectrum.

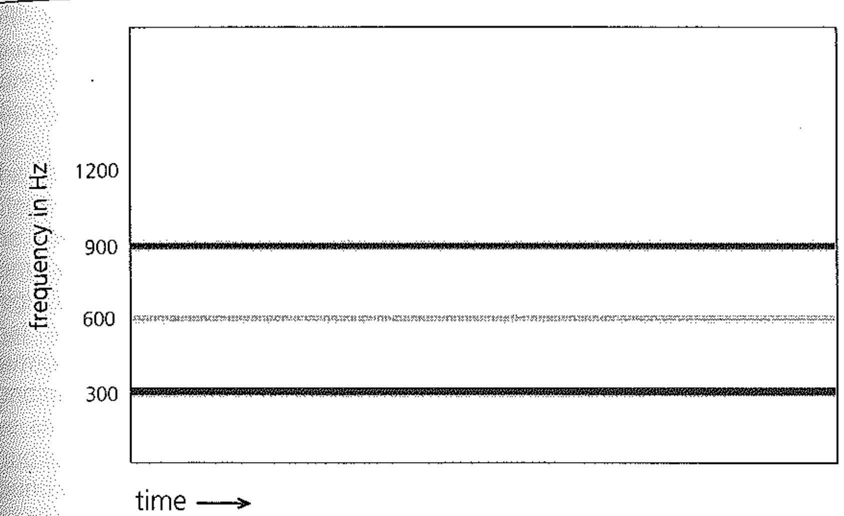


Fig. 2.2c A sonogram (also called a spectrogram)

Now we will consider a process in which, instead of decomposing a complex sound into sine waves, we aim to do the opposite: to fashion a complex sound out of a set of sine waves.

This technique, which should in theory enable us to create any waveform at all by building up a sum of sine waves, is called **additive synthesis**, and is shown in diagrammatic form in figure 2.3.

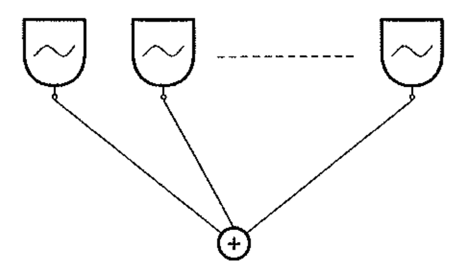


Fig. 2.3 A sum of signals output by sine wave oscillators

In figure 2.4, two waves, A and B, and their sum, C, are shown in the time

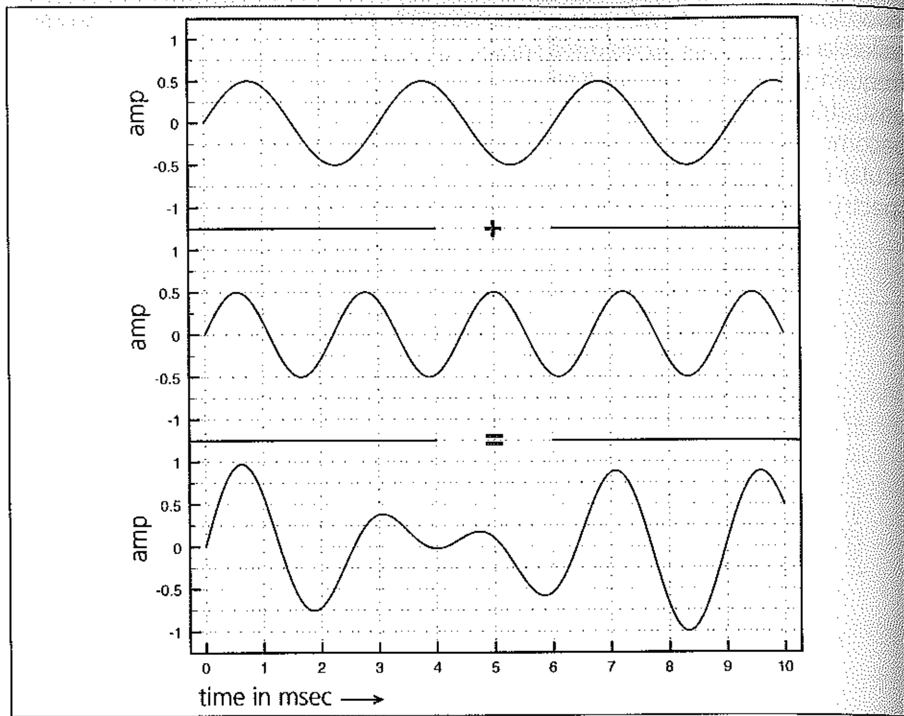


Fig.2.4 A graphical representation of a sum of sine waves

As you can easily verify by inspection, instantaneous amplitudes for wave C are obtained by summing the instantaneous amplitudes of the individual waves A and B. These amplitude values are summed point-by-point, taking their sign, positive or negative, into consideration. Whenever the amplitudes of A and B are both positive or both negative, the absolute value of the amplitude of C will be larger than that of either of the component, resulting in **constructive interference**, such as displayed by the following values:

A = -0.3
B = -0.2
C = -0.5

Whenever the amplitudes of A and B differ in their signs, one being positive and the other negative, the absolute value of their sum C will be less than either one or both components, resulting in **destructive interference**, as shown in the following example:

A = 0.3
B = -0.1
C = 0.2

"The largest part, indeed nearly the entirety, of sounds that we hear in the real world are not *pure* sounds, but rather, **complex sounds**; sounds that can be

resolved into bigger or smaller quantities of pure sound, which are then said to be the components of the complex sound. To better understand this phenomenon, we can establish an analogy with optics. It is noted that some colors are *pure*, which is to say that they cannot be further decomposed into other colors (red, orange, yellow, and down the spectrum to violet). Corresponding to each of these pure colors is a certain wavelength of light. If only one of the pure colors is present, a prism, which decomposes white light into the seven colors of the spectrum, will show only the single color component. The same thing happens with sound. A certain perceived pitch corresponds to a certain **wavelength**[2] of sound. If no other frequency is present at the same moment, the sound will be *pure*. A pure sound, as we know, has a *sine* waveform."
(Bianchini, R., Cipriani, A., 2001, pp. 69-70)

The components of a complex sound sometimes have frequencies that are integer multiples of the lowest component frequency in the sound. In this case the lowest component frequency is called the **fundamental**, and the other components are called **harmonics**. (A fundamental of 100 Hz, for example, might have harmonics at 200 Hz, 300 Hz, 400 Hz, etc.) The specific component that has a frequency that is twice that of its fundamental is called the *second harmonic*, the component that has a frequency that is three times that of the fundamental is called the *third harmonic*, and so on. When, as in the case we are illustrating, the components of a sound are integer multiples of the fundamental, the sound is called a *harmonic sound*. We note that in a harmonic sound the frequency of the fundamental represents the greatest common divisor of the frequencies of all of the components. It is, by definition, the maximum number that exactly divides all of the frequencies without leaving a remainder.

INTERACTIVE EXAMPLE 2A – *HARMONIC COMPONENTS*

If the pure sounds composing a complex sound are not integer multiples of the lowest frequency component, we have a non-harmonic sound and the components are called **non-harmonic components**, or **partials**.

INTERACTIVE EXAMPLE 2B – *NON-HARMONIC COMPONENTS*

[2] "The length of a cycle is called its **wavelength** and is measured in meters or in centimeters. This is the space that a cycle physically occupies in the air, and were sound actually visible, it would be easy to measure, for example, with a tape measure." (Bianchini, R. 2003)

TECHNICAL DETAILS – PHASE

We already pointed out, in the previous chapter, the trigonometric functions **sine** and **cosine**, which respectively generate sine waves and cosine waves. These functions are very important because they describe simple harmonic motion, which is the fundamental movement of all vibrating bodies (and therefore of all bodies that produce sounds).

In more general terms, the sine function describes the motion of an object that is subject to a force displacing it from a position of equilibrium. The magnitude of this force is proportional to the distance of the object from the point of equilibrium. Many real world phenomena trace their progress in such a sinusoidal form, including the movement of a swinging pendulum, the changes in length of the day during the course of a year, and the motion of a piston in a car engine.

From the trigonometric point of view, if we construct a unit circle (a circle with a radius of 1) whose center we place at the origin of a Cartesian coordinate system, we can define sine and cosine as a projection of an arbitrary ray onto the y-axis and onto the x-axis, respectively (as shown in figure 2.5).

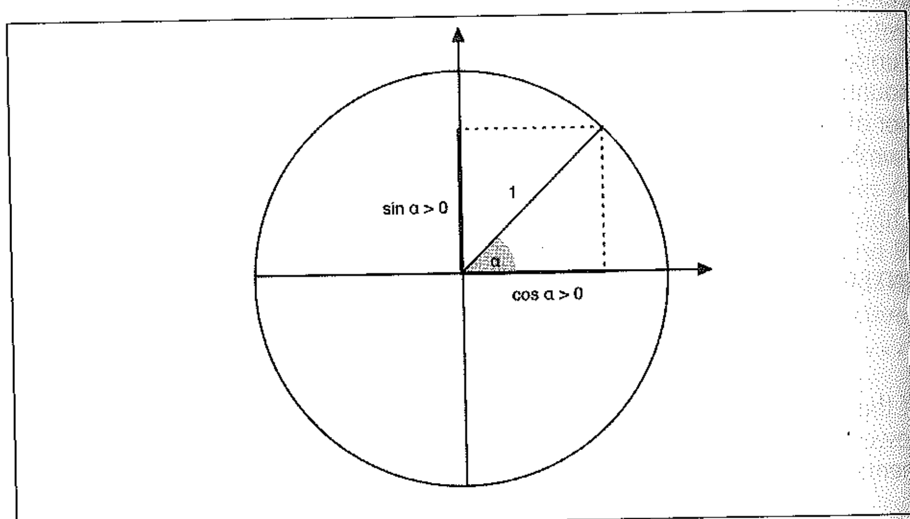


Fig. 2.5 Sine and cosine of the angle α

As you can see in figure 2.5, if we rotate the ray counterclockwise around the circumference of the circle and track the angle α formed between the ray and the x-axis, the projected lengths of the ray onto the coordinate axes will equal the values of sin(α) and cos(α). When the angle α has swept the entire circumference, forming angles of 0 to 360 degrees (or from 0 to 2π radians), we will have finished a complete cycle of the sine and cosine functions. As you can see in figure 2.6, a sine wave can be thought of as a graphical display of the changing length of the projection onto the y-axis while the value of the underlying angle varies over time.

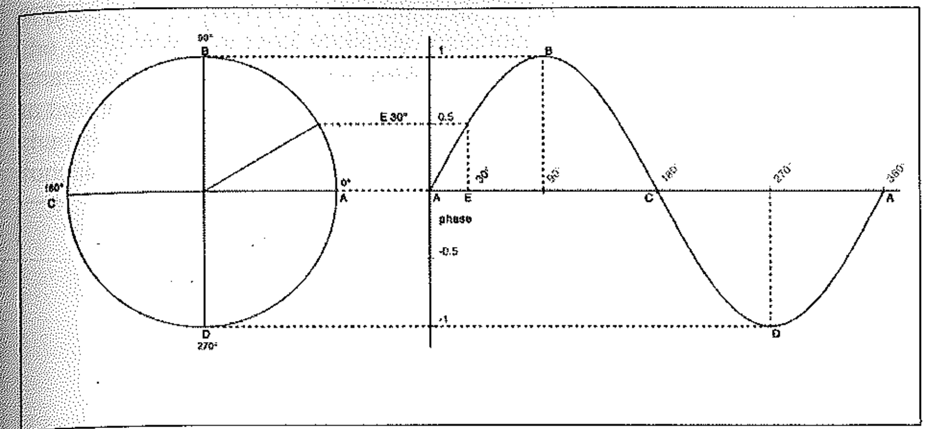


Fig. 2.6 Graphical representation of a sine wave with regard to phase

The angle α can also be defined as the phase of a waveform. As the phase passes from 0 degrees to 360 degrees (or from 0 radians to 2π radians), the waveform, as we have already seen, completes one cycle.

Often in programming languages for computer music one finds "normalized" phase: a phase value that, instead of being represented in degrees or radians, is represented as decimal values between 0 and 1. To clarify the equivalence relationships that exist for these different representations of phase, the following figure shows some common phase values in degrees, radians, and normalized decimal values, along with related values of the sine function. Verify these relationships visually by following the progression shown in figure 2.7.

PHASE			VALUE
Degrees	Radians	Normalized	
0°	(0)	0	0
45°	(π/4)	0.125	0.707
90°	(π/2)	0.25	1
135°	(3π/4)	0.375	0.707
180°	(π)	0.5	0
225°	(5π/4)	0.625	-0.707
270°	(3π/2)	0.75	-1
315°	(7π/4)	0.875	-0.707
360°	(2π)	1	0

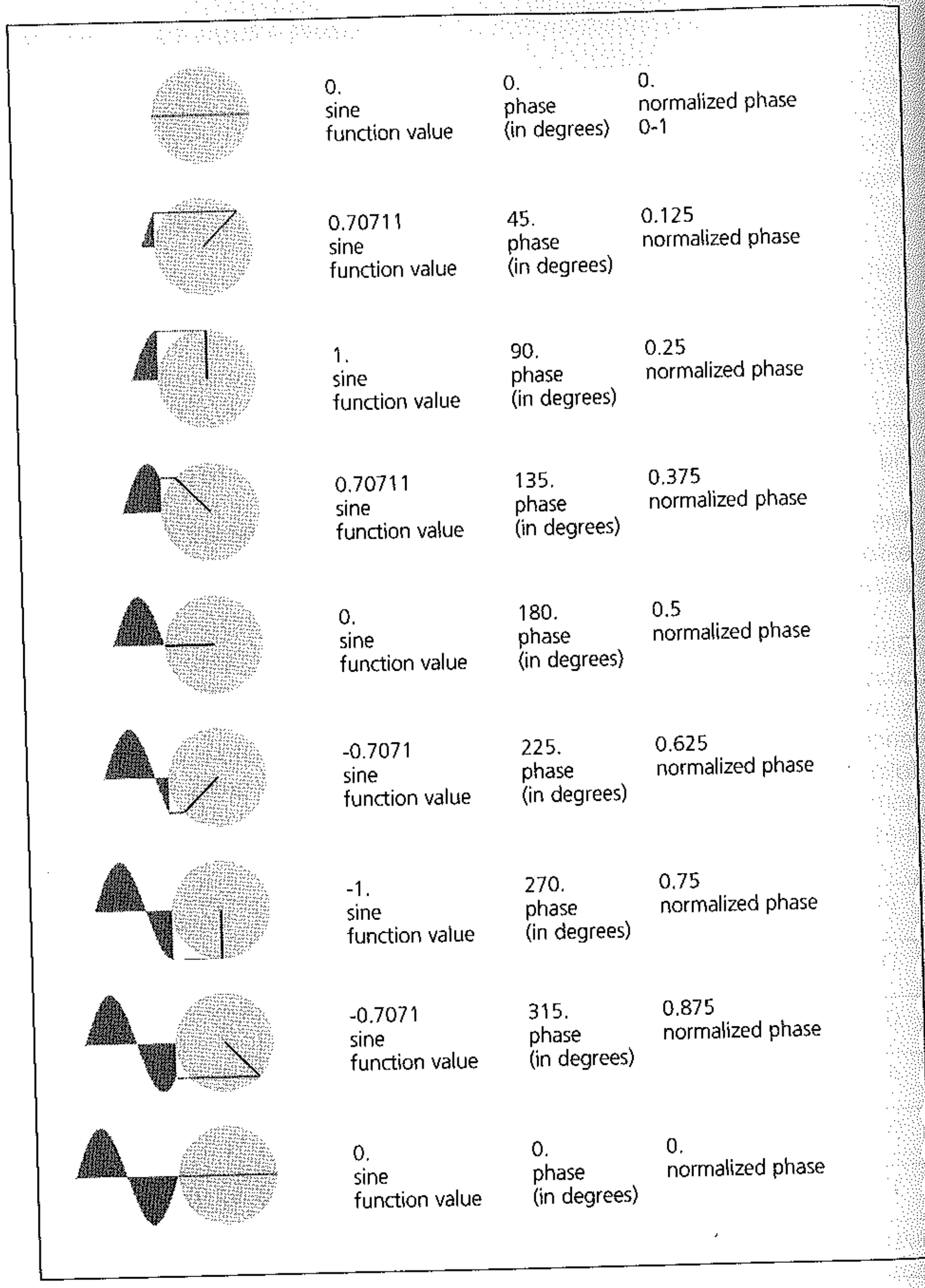


Fig. 2.7 Construction of a sine wave

INTERACTIVE EXAMPLE 2C – *PHASE AND SINE WAVES*

A phase of 360 degrees is equivalent to a phase of 0 degrees. Because of this, the cycle wraps around endlessly, beginning anew whenever we increment phase values beyond the basic range.

In figure 2.8, we have graphed the projection of our ray onto the x-axis rather than the y-axis, producing a graph that is otherwise known as the cosine function.

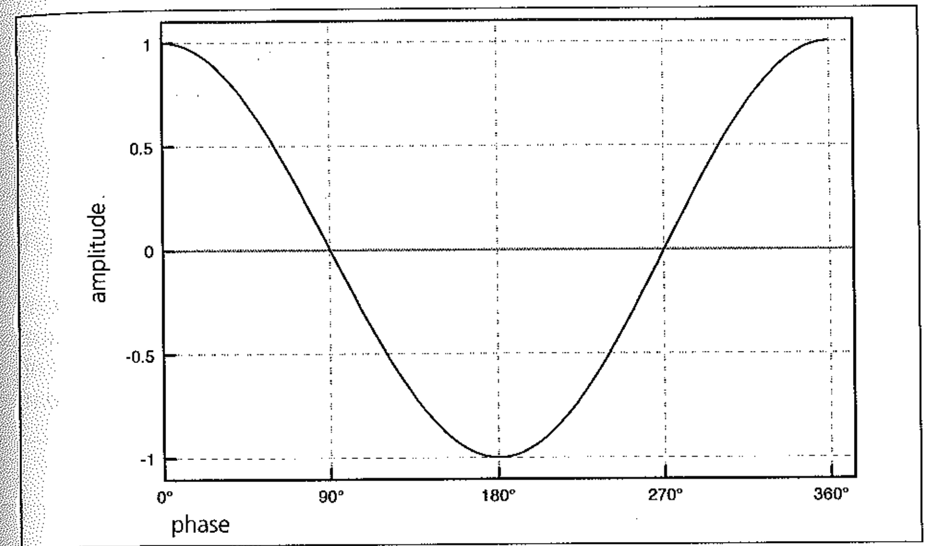


Fig. 2.8 The cosine function

The two waveforms are identical. The cosine wave is simply a sine wave whose phase is shifted by 90 degrees, as you can see in figure 2.9.

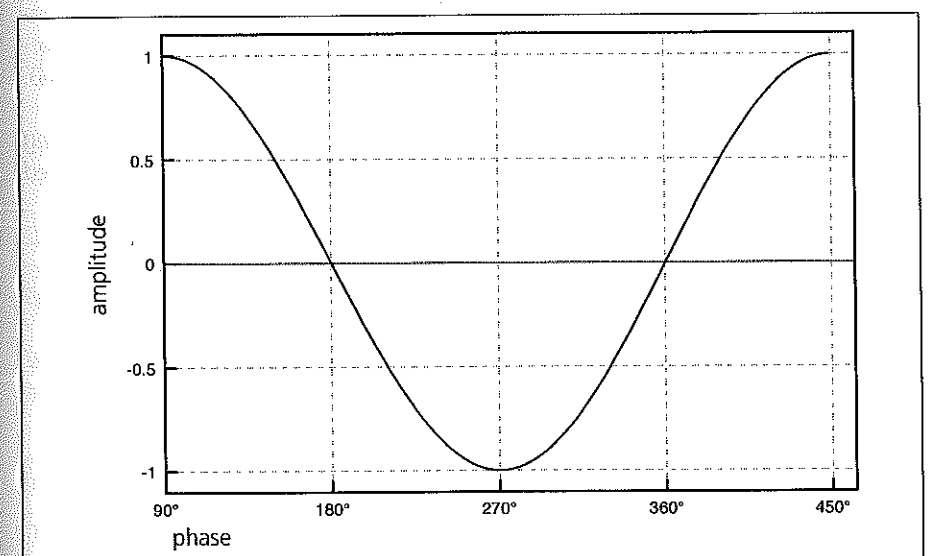


Fig. 2.9 A sine wave, phase shifted by 90 degrees

If we go further and shift the phase of a sine or a cosine wave by a full 180 degrees, we obtain a waveform that has *reversed polarity* with respect to the original waveform (which had a phase angle of 0 degrees). For every positive value in one of these waves, there is a corresponding negative value in the other. In such a case, the two waves are said to be **in antiphase** (as shown in figure 2.10).

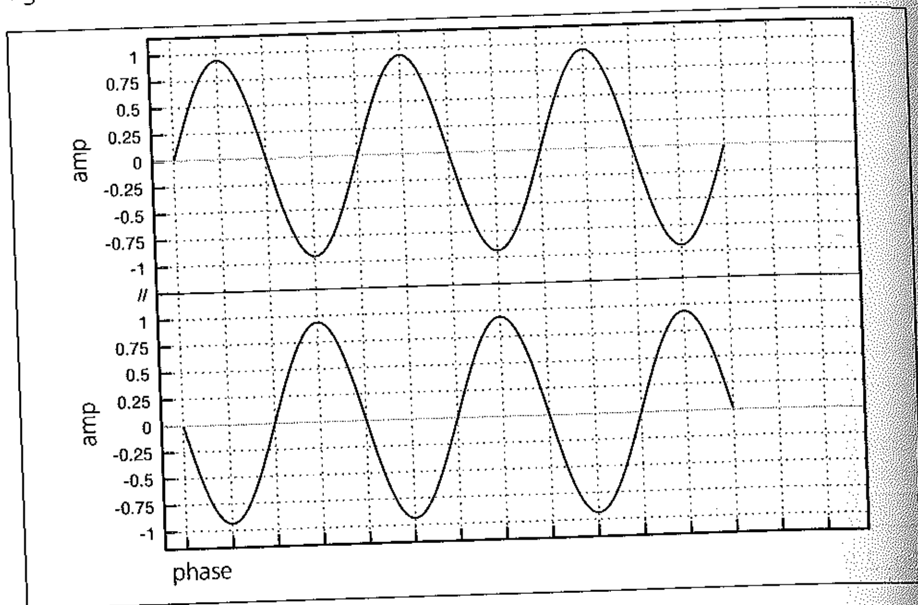


Fig. 2.10 Two sine waves in antiphase

If we sum the two waveforms in figure 2.10, destructive interference will completely eliminate the sound, since the sum of every sample from the first waveform with its corresponding sample in the second waveform will always yield 0.

An easy way to obtain a polarity-reversed waveform consists of multiplying every sample value by -1. Using this method, positive values are transformed into negative, and negative into positive.

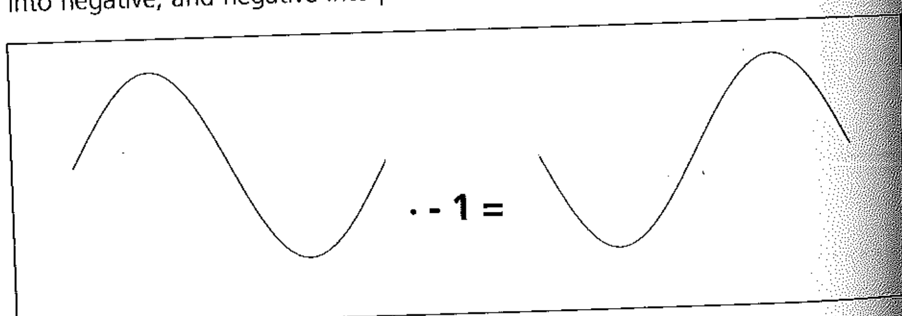


Fig. 2.11 Multiplication of a sampled signal by a negative constant

We will revisit this discussion in Section 2.2, entitled "Beats".

HARMONIC AND NON-HARMONIC SPECTRA

We can obtain sounds with either a **harmonic spectrum** or a **non-harmonic spectrum** by using additive synthesis.

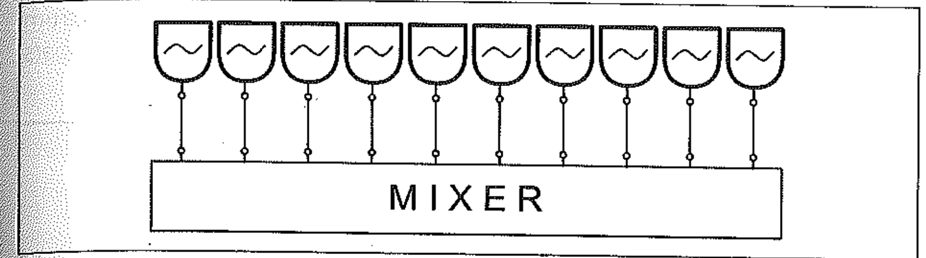
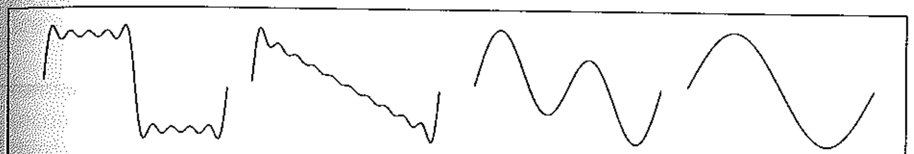


Fig. 2.12 Summing oscillators

In the diagram in figure 2.12, we see sine wave oscillators, whose outlets are summed together using a mixer.
In figure 2.13 we show four examples of additive synthesis that produce harmonic spectra.



arm	amp
1	1
2	0
3	.33
4	0
5	.2
6	0
7	.14
8	0
9	.11
10	0

arm	amp
1	1
2	.5
3	.33
4	.25
5	.2
6	.16
7	.14
8	.12
9	.11
10	.1

arm	amp
1	.5
2	1
3	0
4	0
5	0
6	0
7	0
8	0
9	0
10	0

arm	amp
1	1
2	0
3	0
4	0
5	0
6	0
7	0
8	0
9	0
10	0

Fig. 2.13 Waveforms constructed from summed sine waves

When defining the spectrum of an harmonic sound, it is sufficient to provide the amplitudes of the non-zero components, using a table such as the following:

HARMONIC	I	II	III	IV	V	VI	VII	VIII	IX	X
FREQ. (Hz)	100	200	300	400	500	600	700	800	900	1000
AMPLITUDE	1	.8	.6	.75	.4	.3	.2	.28	.26	.18

In this table, all of the upper components have an amplitude near 0. The table can be graphed as shown in figure 2.14, where we see the spectrum of a sound: the horizontal axis shows frequencies by harmonic number, and the vertical axis shows amplitude. This spectrum is clearly harmonic, since the frequencies are equidistant from each other; all of the components have a harmonic relationship with the fundamental. The line traced over the spectrum is called the **spectral envelope**: it is a curve that connects the tops of the bars that represent the amplitude of the harmonics.[3]

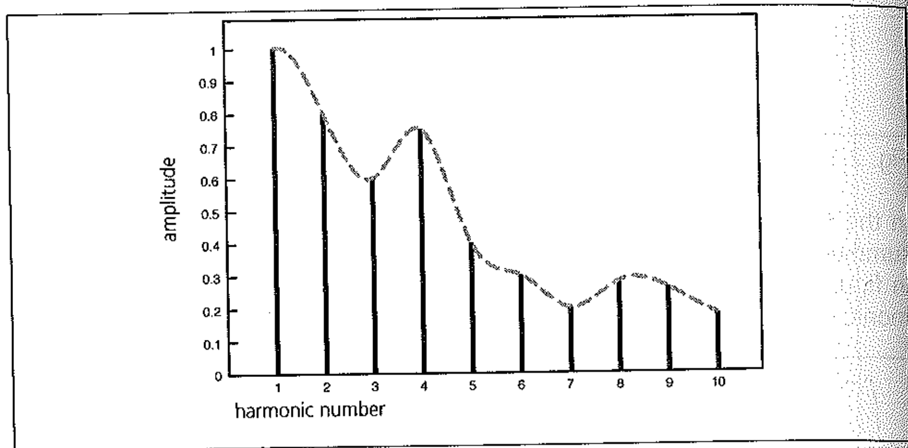


Fig. 2.14 Spectral envelope

In figure 2.15 the waveform produced by the spectrum pictured in figure 2.14 is shown. As you see, it is a periodic waveform: a wave whose shape repeats once per period without changing.

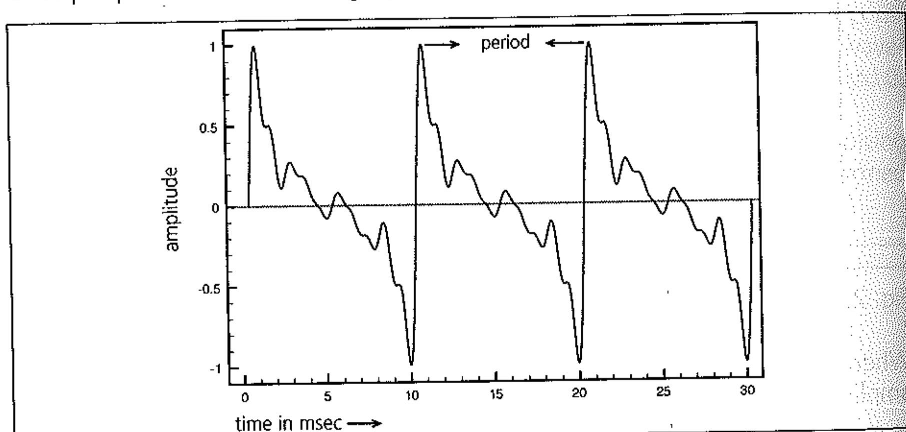


Fig. 2.15 Periodic waveform

[3] It is possible, by means of this envelope, to apply the spectral profile of one sound to another completely unrelated sound, independent of the frequency content of the components (see Section 2.4, and especially Chapter 12).

Some waveforms, very common in the production of analog and digital electronic music, can be approximated by a series of harmonics whose amplitudes are expressed by convenient formulas.

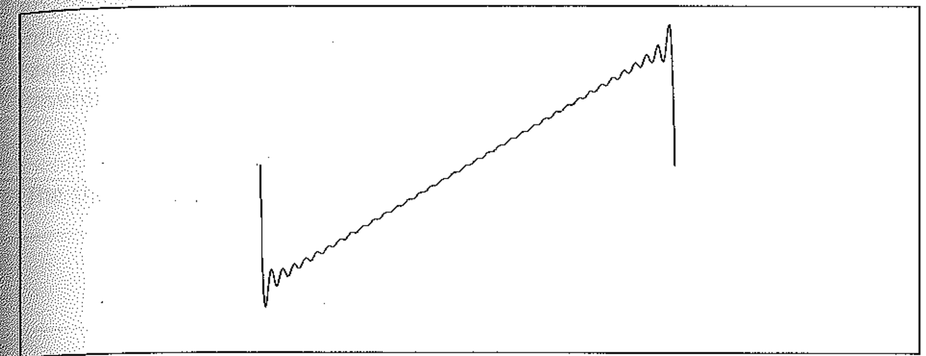


Fig. 2.16 Sawtooth wave

For example, the **sawtooth wave**, as seen in figure 2.16, can be approximated by a series of harmonically linked sine waves that all have a phase of 180 degrees (like the wave in the lower part of figure 2.10), and whose amplitudes are:

fundamental	1/1
second harmonic	1/2
third harmonic	1/3
fourth harmonic	1/4
fifth harmonic	1/5
sixth harmonic	1/6
...	

As you can see, for the nth given harmonic, the amplitude will be 1/n. The more harmonics are present, the more the resulting waveform will resemble that in figure 2.16.

Of course, a "perfect" sawtooth wave could only be produced using an infinite series of harmonics. (See figure 2.17.)

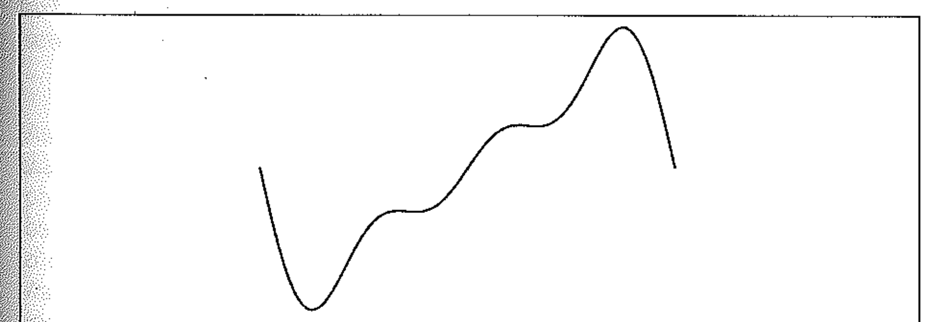


Fig. 2.17a Sawtooth wave with 3 harmonics

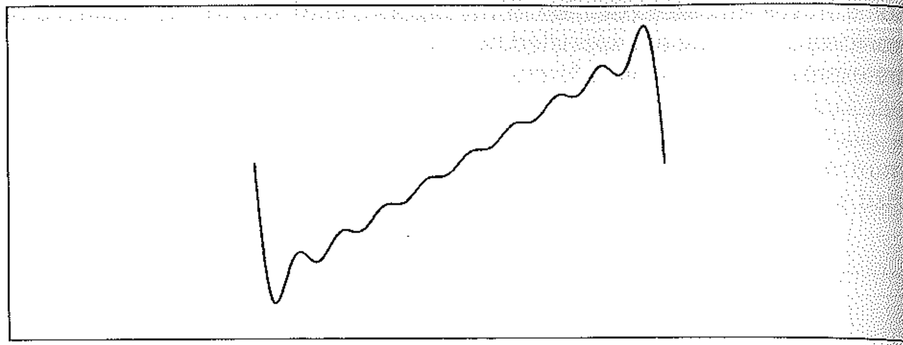


Fig. 2.17b Sawtooth wave with 9 harmonics

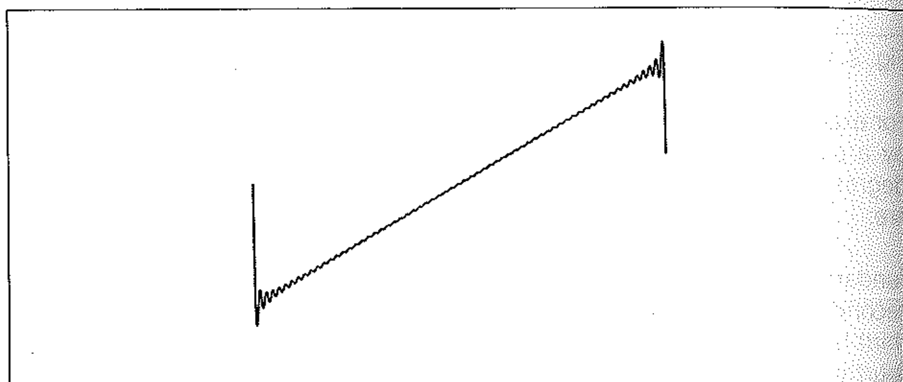


Fig. 2.17c Sawtooth wave with 64 harmonics

Another very common waveform is the **square wave**.

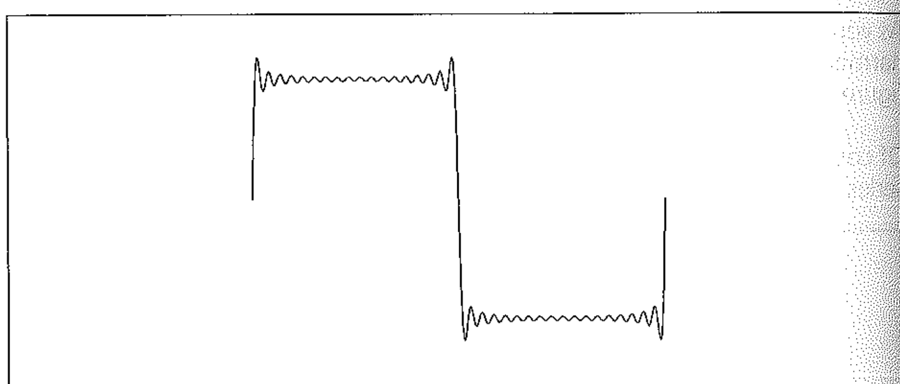


Fig. 2.18 Square wave

The series of components used for the approximation of the square wave is similar to that of the sawtooth wave, but consists of only odd-numbered components with phase of 0 degrees. The amplitude of even-numbered components in this waveform is always 0.

fundamental	1/1
second harmonic	0
third harmonic	1/3
fourth harmonic	0
fifth harmonic	1/5
sixth harmonic	0
seventh harmonic	1/7
...	

The shape of the **triangle wave** is shown in figure 2.19.

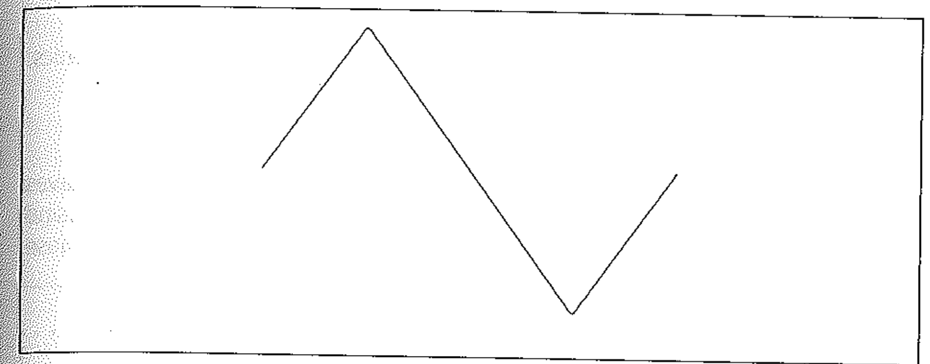


Fig. 2.19 Triangle wave

You can approximate the triangle wave by using an amplitude factor of $1/n^2$, for odd n only, and by multiplying the series of odd numbers alternately by 1 and by -1 (or by alternating the phase of the sine waves between 0 degrees and 180 degrees).

Here are the resulting factors:

fundamental	1/1
second harmonic	0
third harmonic	-1/9
fourth harmonic	0
fifth harmonic	1/25
sixth harmonic	0
seventh harmonic	-1/49
...	

As we learned in the earlier section dedicated to phase, multiplying a digital sine wave by a negative number will give you a "reversed" form of the wave, out of phase by 180 degrees. The negative values in this table should be thought of as being 180 degrees out of phase with the positive values.

As a final wave to consider, examine the **impulse** (shown in figure 2.20), which is a signal that contains energy at all frequencies (for a deeper definition of the impulse, see Section 3.9).

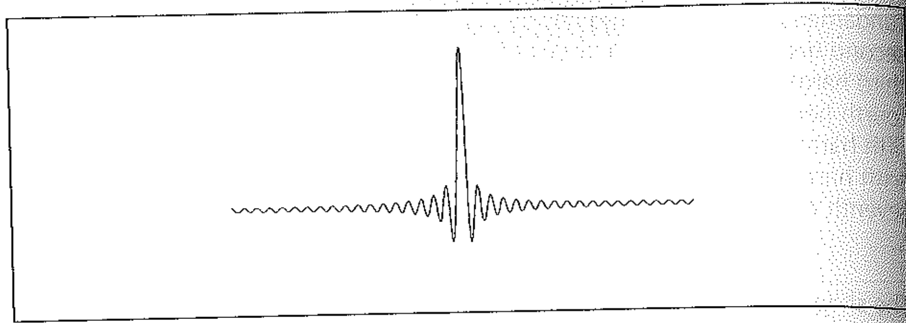


Fig. 2.20 Impulse

The impulse can be approximated by using an identical amplitude for all harmonics.

1 1 1 1 1 ...

When approximating an impulse, the components should all be cosine waves rather than sine waves. The cosine function is, as we know, identical to the sine function, but out of phase by $\pi/2$ (or 90 degrees). See the discussion of phase above for more details.

INTERACTIVE EXAMPLE 2D – *CLASSIC WAVEFORMS*

Normally, in the presence of a harmonic sound, we perceive a single pitch, which is almost always the pitch of the fundamental. There are cases, however, in which a fundamental is not present in a sound, but the sound still possesses harmonic structure. In this case, the **missing fundamental**, sometimes called the phantom fundamental, can still be identified by our brain. We encounter telephones and radios every day which have speakers that cannot accurately reproduce bass frequencies. Despite these limitations, we are able to infer the fundamental frequencies of the sounds we hear on these devices by listening to upper harmonics alone.

Let's look at two examples of this phenomenon, both of which begin with a fundamental frequency of 200 Hz:

> The first sound is composed of all harmonics from the first through the seventh. (See figure 2.21.)

> The second sound has only the fourth through seventh harmonics (800 Hz, 1000 Hz, 1200 Hz, 1400 Hz, as shown in figure 2.22).

As you can see, the period of the two waveforms is the same. The greatest common divisor (200 Hz) shared by these components is the same, and because of this, their perceived pitches will also be the same, even when the frequency of this 200 Hz component is absent.

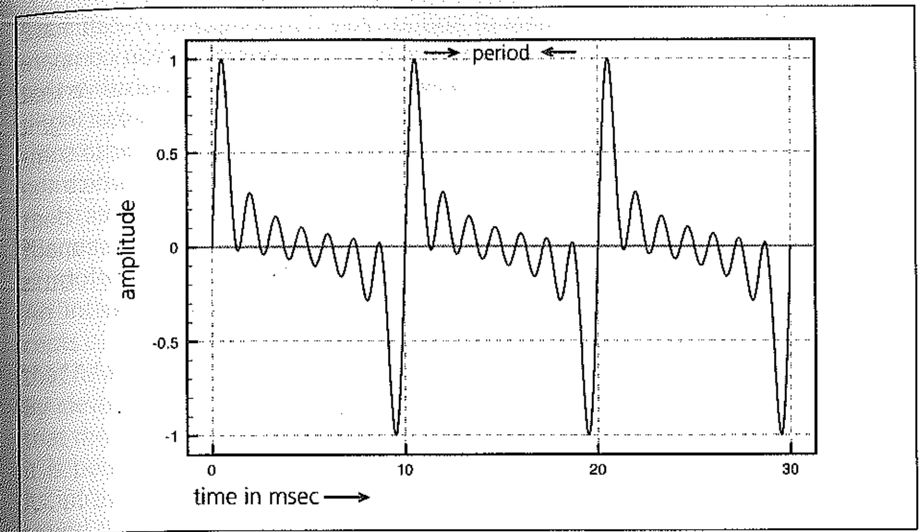


Fig. 2.21 Periodic sound composed of the first through seventh harmonics

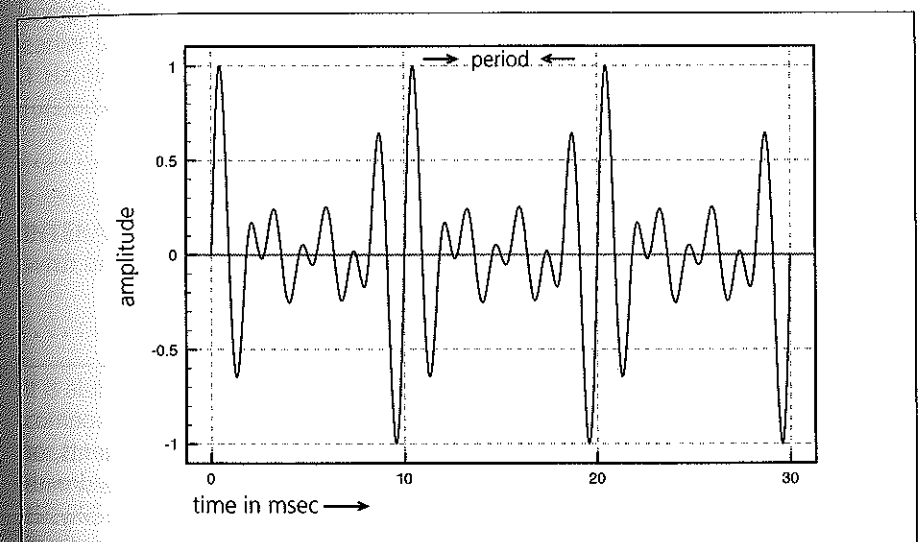


Fig. 2.22 Periodic sound composed of the fourth through seventh harmonics

If, however, too many components are missing in the lower portion of the spectrum, we can no longer perceive the fundamental, leaving such a sound devoid of definite pitch.

INTERACTIVE EXAMPLE 2E – *MISSING FUNDAMENTAL*

There is, of course, a second kind of sound: spectral components can have *non-harmonic* relationships, resulting in non-harmonic sounds. If you compare the upper portion of figure 2.23 with figure 2.14, you will quickly observe that the frequencies in figure 2.23 do not exhibit the same kind of equal spacing; it is easy to suppose that a greatest common divisor does not exist for these component frequencies. This means that a fundamental frequency, from which the frequencies for all other components can be derived as integer multiples, does not exist. In this case, the relationships between the frequencies are irrational.[4] In the lower portion of figure 2.23, we can see that the resulting waveform is non-periodic. It does not repeat cyclically.

INTERACTIVE EXAMPLE 2F – *NON-HARMONIC SOUNDS*

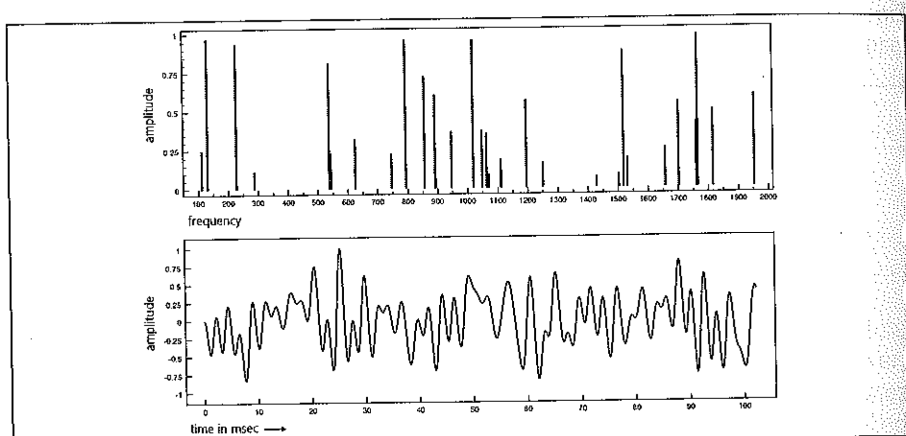


Fig. 2.23 The spectrum and waveform of a non-harmonic sound

"Periodic sounds, such as the pitched sounds made by musical instruments, or the vowel sounds of the human voice, are perceived as being equipped with a definite pitch.[5] (A better term might be quasi-periodic sounds, since physics defines periodic phenomena as being infinite.) Non-periodic sounds, such as the sounds made by musical instruments of indeterminate pitch like cymbals, gongs, and triangles, or consonants produced by the human voice, are not perceived to have definite pitch. The closest we can come to pitch identification for such sounds is to perceive a frequency band in which the density of components is thick enough to bestow a relevant amplitude."
(Bianchini, R., Cipriani, A., 2001, pp. 71-72)

[4] An irrational number is a number that cannot be expressed exactly as an integer fraction with a non-zero denominator. Some famous irrational numbers are the square root of 2 and π.

[5] Assuming that the fundamental frequency falls within the audible band of frequencies, of course.

Sounds can contain both harmonic and non-harmonic components; this is true in particular for musical instruments and naturally occurring sounds. A flute sound, for example, contains harmonic frequencies that give the note played its pitch, but it also contains non-harmonic elements that are tied to the breath of the player. Another example is the sound of a piano, which has non-harmonic components during its attack that are tied to the action of the hammers on the strings; to make the sound even more complicated, the string vibrations of the piano don't produce mathematically perfect harmonic partials, but instead produce partials that are stretched apart by a very small amount.[6]
An interesting experience, which should stimulate discussion about the concepts of harmonicity and non-harmonicity, is to try constructing periodic sounds that have fundamental frequencies that fall below the audible frequency range. By cleverly choosing a set of irregularly spaced harmonics, we can easily hear that the resulting sound is non-harmonic and has no definite pitch, even though it is periodic by definition (meaning that all of its components are related to its infrasonic fundamental via integer ratios). For example, let's hypothesize a sound that has components at 113 Hz, 151 Hz, 257 Hz, 331 Hz, 577 Hz, 811 Hz, 1009 Hz, and 1237 Hz. This sound is non-harmonic to the ear, despite the fact that it is periodic and has a fundamental of 1 Hz, of which all of the components are integral multiples. The implied fundamental of which we spoke earlier cannot be identified, not only because the ear can't hear a frequency of 1 Hz, but also because the components in question are very distant from their fundamental, being the 113th harmonic, the 151st, and so on.

INTERACTIVE EXAMPLE 2G – *PRESET 1 – Non-harmonic periodic sound with an infrasonic fundamental*

It is also possible to construct a non-harmonic, yet periodic, sound that possesses components whose frequencies form integer ratios with some theoretically audible fundamental, but that does not cause our brains to hear its fundamental. For example, a fundamental of 35 Hz forms a basis for harmonic frequencies of 455 Hz, 665 Hz, 735 Hz, 945 Hz, 1085 Hz, 1295 Hz, 1695 Hz, and 1995 Hz, and yet it is simply not possible for our brain to re-create the lost fundamental and to attach a definite pitch to the sound since so many of the lower harmonics are lacking.

INTERACTIVE EXAMPLE 2G – *PRESET 2 – Non-harmonic periodic sound with a missing fundamental of 35 Hz*

[6] Piano tuners know this phenomenon well, since they need to tune slightly low in the bass octaves and then gradually higher as they move towards the upper range to ensure that the higher harmonics of the low notes are in tune with the fundamentals of high notes.

PERIODIC VERSUS APERIODIC, AND HARMONIC VERSUS NON-HARMONIC

Let's systematize and clarify the concepts that we have introduced in this chapter.

The cycle is the smallest portion of a wave that repeats over time.

The fundamental of a harmonic sound is the component with the lowest frequency, and generally also the highest amplitude. The frequencies of all of the other components are integer multiples of this frequency.
If the lowest component in a harmonic sound is missing, but the immediately succeeding components are present, the sound will be heard as having a pitch equal to the missing fundamental. One way of understanding this is that the period of the waveform (the duration of the wave portion that repeats periodically) corresponds to the inverse of the frequency of the missing fundamental.

If, in addition to the fundamental, we start to remove other lower harmonics, we hear the gradual loss of its harmonicity, because at some point, our brain is no longer able to reconstruct the fundamental. A sound thus obtained is non-harmonic, but at the same time periodic, because its period is still the inverse of the frequency of its "virtual" fundamental.

The fundamental of a **periodic sound** is the frequency of which all components are integral multiples. It follows that the fundamental frequency of such a sound, whether real or virtual, is the greatest common divisor of the component frequencies that are present. A non-harmonic sound composed of partials at 100, 205, 290, 425, and 460 hertz, for example, has a fundamental of 5 Hz, and is therefore a periodic sound that repeats 5 time per second (although it is not possible to hear this fundamental).

A **non-periodic sound** is, by contrast, a sound for which it is not possible to identify a portion that repeats in time. An example of non-periodic sound is white noise,[7] or the sound made by a percussion instrument that has no definite pitch such as a cymbal.

We've seen that you can have a periodic sound that has no definite pitch. On the other hand, is it possible to have a non-periodic sound that does have a definite pitch? Certainly. It is enough to have components whose frequencies are close to being integral multiples of some audible fundamental. The partials 110, 220.009, 329.999, 439.991, and 550.007 hertz, for example can be heard unequivocally as an A, even though their period doesn't correspond to the perceived fundamental, but rather to the greatest common divisor of the components, which is 0.001 hertz. To be precise about this, the period of this waveform is 1000 seconds, and a sound with a period as long as this is psychoacoustically equivalent to a non-periodic sound.

[7] For a definition of white noise, see Section 3.1.

To have a truly non-periodic sound with a fixed spectrum, the spectrum must include irrational frequencies, such as the following: 100, $200+\pi/5$, $300+\pi/4$, $400+\pi/3$, $500+\pi/2$. A sound such as this would be impossible to reproduce on a computer, of course, because of numerical compromises that are imposed by the CPU when performing calculations. Nonetheless, for the example cited, we could still obtain a sound with such a long period that we could consider it, for all practical purposes, to be non-periodic.

We define sounds such as those found in the last two examples, which have a perceptible pitch and a series of components that are almost multiples of the fundamental, as **quasi-harmonic sounds**.

Sounds with variable spectra (which we will speak more about in Section 2.4) can easily be non-periodic. (Sounds, for example, in which components slide irregularly from one frequency to another.) Real-world, non-electronic, sounds are almost never periodic. It is simply impossible, even for the most precise clarinettist in the world, to emit a sound in which successive periods of the waveform are perfectly identical (not counting the fact that the poor clarinettist would need to produce such a sound for an infinite amount of time...)

In the case of natural sounds that are harmonic and have definite pitch (such as the clarinet), we speak of such sound as being "quasi-periodic" or "pseudo-periodic".

The fundamental
It is exceedingly difficult to give a definition for the **fundamental** that admits all cases. For a periodic sound with a given pitch, the fundamental is easily defined as the frequency that forms integer ratios with all other components. The pitch that we perceive for such a sound corresponds to the pitch of this fundamental. We can also say that in the case of the "quasi-harmonic" sounds that we saw above, the fundamental ought to be the lowest component, since this component also happens to be the frequency that we perceive to be the pitch of such sounds.

But what about the case of non-harmonic sounds? If a sound is still periodic, we could say that the fundamental is the component that corresponds to the greatest common divisor of the components, such as in the case that we examined above that had an implied fundamental of 5 Hz. But such a "fundamental" is not audible, since its frequency is well below the audio band. Likewise, we could designate the lowest component of a non-harmonic sound as its fundamental (which would be the 100 Hz component in the example above), but such a component turns out to be not very important. In many percussive sounds, for example, the lowest components turn out to be almost inaudible, while those with higher frequencies almost completely characterize the timbre; removing the "fundamental" from such a sound doesn't change its timbre in the least. Because of this, in the cases of non-harmonic sounds, we will simply choose to say that no fundamental exists. We will use the term only for sounds that have definite pitch, whether periodic, quasi-periodic, harmonic, or quasi-harmonic.

INTERPOLATION

In the preceding section we spoke of **digital oscillators** without clarifying how they function. Referring to figure 2.24, we construct a **wavetable** (or array[8]) of 20 elements, and fill it with values taken from a sine wave moving between amplitudes of -1 and 1.

indices	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
values	0	.31	.60	.83	.97	1	.92	.74	.48	.17	-.16	-.47	-.73	-.91	-1	-.97	-.84	-.61	-.32	0

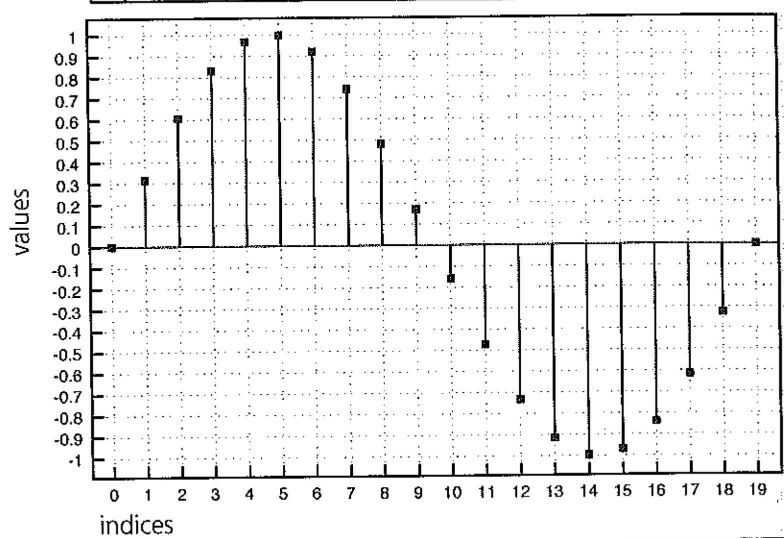


Fig. 2.24 A wavetable containing values taken from a sine wave

Every element is identified with an index or phase,[9] which are numbered from 0 to 19 in the example. Each index refers to a value, which is indicated in the upper portion of the figure. The set of values in this example wavetable sketch the outline of a sine wave when taken consecutively (or more accurately, a few points that roughly approximate a sine wave).

To generate a sine wave, you need to do nothing more than read one value after another from the wavetable and, when you have reached the end, start again from the beginning. Don't be deceived, however: it almost never works to simply read the values that *exactly* correspond to the indices of the wavetable. In the case shown, for example, reading the 20 elements one after the other at a sampling frequency of 44,100 Hz would result in reading the table 2,205 times in one second, generating 2,205 cycles of a sine wave (since 44,100/20 = 2,205).

[8] We will define an **array** as an ordered series of values in which each value is assigned a numerical index, drawn from the consecutive integers beginning with 0. For example, the index 0 in the array (2, 3.5, 6, 1, -12) refers to the value 2, the index 1 refers to the value 3.5, the index 2 refers to 6, and so on.

[9] See the "phase" section above. The concept here is similar, but in this case the indices or phase values indicate a position within the cycle of a waveform.

If we want to produce a frequency different than 2,205 Hz, say perhaps 441 Hz, every cycle would need to be 100 elements long (since 44,100/100 = 441). This means that, given a wavetable of 20 elements, we would need to read more intermediate elements than actually exist. The indices to consider would not be 0, 1, 2, 3, 4, etc., as in the last example, but rather 0, 0.2, 0.4, 0.6, 0.8, 1.0, etc. Such fractional indices might serve to help us arrive at 100 elements, but what values would they take on? We might, for example, use the value 0 (which is the value associated with the index 0 in figure 2.24) for *any* fractional index lying on the interval between 0 and 1. For the indices between 1 and 2, we might use the value 0.31 (associated with index 1), and so forth. Another possibility would be to *round* the index values, assigning the value 0 (the value for index 0) to 0.2 and 0.4, while assigning 0.31 (the value for index 1) to the indices 0.6 and 0.8. Both cases, however, would result in distortion, due to the stepped nature of the signal (shown in figure 2.25).

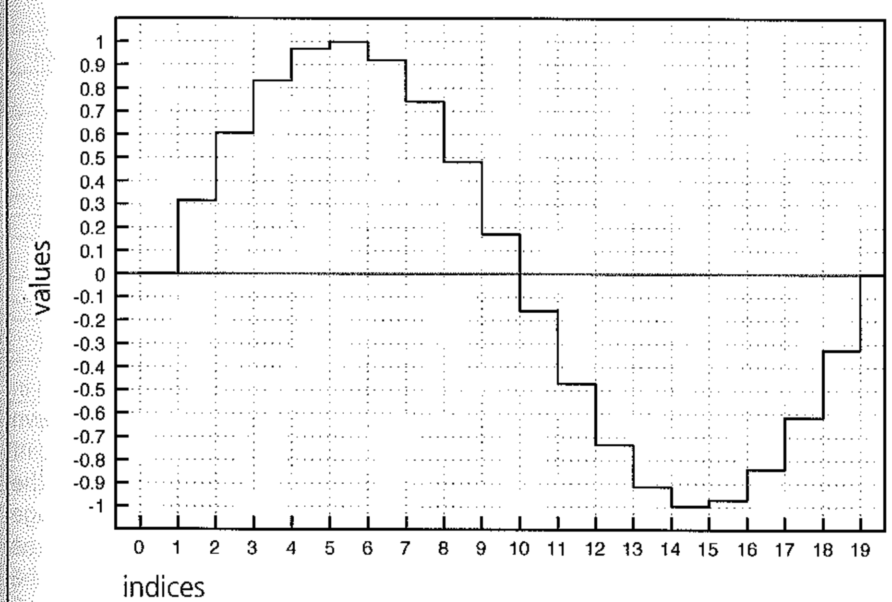


Fig. 2.25 A stepped waveform

The distortion problems in this example are caused by the differences between the values that ought to be found in a sine wave and those obtained by truncating decimal places using the methods that we have been experimenting with. The amount of error contained in the truncated values varies continuously, resulting in an imprecise digital signal whose stepped appearance will be heard as distortion. To obtain better results, we could instead use a technique for defining intermediate values called **interpolation**, which consists of estimating intermediate values between points in a wavetable using a calculation. The example of this technique shown in figure 2.26 is called **linear interpolation**; the figure shows how an intermediate value could be calculated between a point, designated as k, and its successor, designated as k+1.

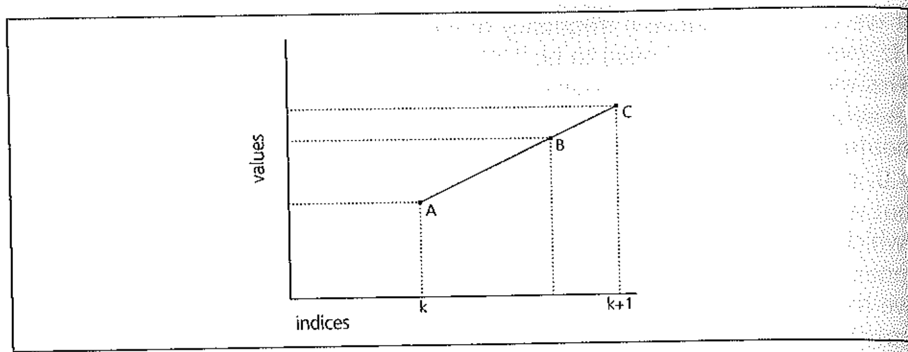


Fig. 2.26a Linear interpolation

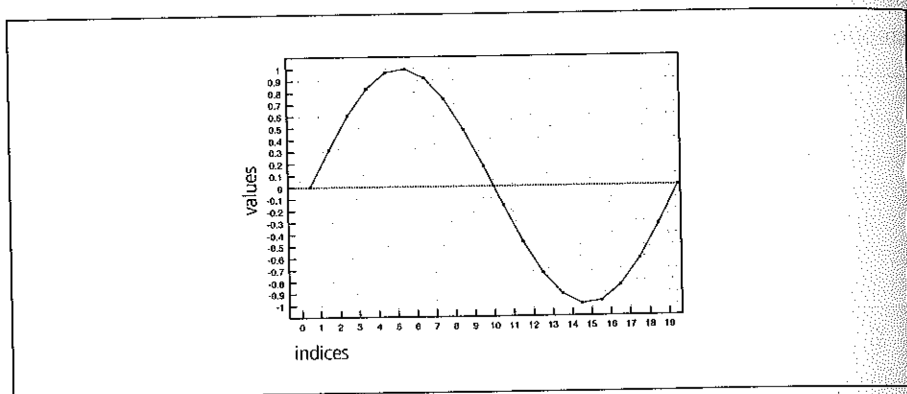


Fig. 2.26b Graph of an interpolated waveform

Using linear interpolation, the value for an intermediate index, designated as B in the figure, is calculated as lying on a line between the values for the two known indices A and C. Using such interpolation to insert values between every point in the table enables us to appreciably reduce the distortion associated with the previous jagged waveform. The improvement in terms of sound quality is huge (due to a big reduction in harmonic distortion[10]), but there is a tradeoff: the time needed to calculate every value is now much greater. The technique yields the following values (graphed in figure 2.26b) for the fractional indices between 0 and 1 of the sine wave shown in figure 2.24.

Indices	Values
0.0	0.0
0.2	0.062
0.4	0.124
0.6	0.186
0.8	0.248
1.0	0.31

10 Harmonic distortion is changes in a signal's spectrum that can be attributed to alterations of its waveform. See Section 5.1 for more details.

There are other kinds of interpolation. Besides the very simple, but not very efficient, linear interpolation, polynomial interpolation (quadratic, cubic, or even higher degrees) is widely used, and allows for a major reduction in harmonic distortion. It is, however, also computationally expensive. Given its complexity, we will not expand upon its internals, but be aware that many programming languages for sound synthesis make it possible for you to use polynomial interpolation in their oscillators.

Computer music programming languages often use "normalized" indices or phases when working with wavetables (see the discussion of the term *phase* above), in which the indices are always a decimal number between 0 and 1, independent of how many elements are in the wavetable. 0 corresponds to the first value in the table, 0.5 to the central element, and so forth. In the case of the table shown in figure 2.24, for example, the relationship between integer indices and normalized indices would be as follows:

Integer Index	Normalized Index
0	0
1	0.05
2	0.1
3	0.15
4	0.2
...	...
10	0.5
...	...
18	0.9
19	0.95

As you can see, the normalized value 1 never occurs, because it coincides with the beginning of a new cycle (just as a phase of 360 degrees is equivalent to 0 degrees).

When we want to construct a sound using **fixed spectrum additive synthesis**, in which neither the frequency nor the amplitudes of the components vary over time, we can work in one of two ways. The more typical and simpler method is to sum the outputs of multiple sine wave oscillators, each of which generates a frequency component of the sound. The second, more efficient method consists of summing all of the sine wave components directly within a single wavetable, which contains as a result one complete cycle of the waveform. We then use a single oscillator to read this wavetable and generate the complex sound. This second method optimizes the use of computer resources; to realize the sound shown in figure 2.14, for example, we would need only a single oscillator, rather than the 10 sine wave oscillators that would be needed for the first method.

Technically, we shouldn't call the second method additive synthesis, since there is no realtime addition of signals going on. You might instead think of it as a simple form of wavetable synthesis that loads a complete wave cycle, containing all of the necessary sine wave components, into an oscillator for playback.[11] However you think of it, the results obtained using either method, as well as the principles that stand behind the techniques, are the same.

TEST WITH SHORT ANSWERS *(30 words maximum)*

1) What can you do using additive synthesis?

2) When does one encounter constructive interference? Destructive interference?

3) What relationship must the fundamental have with one of its components to be defined as harmonic?

4) If a sound has a harmonic type of spectrum, does this imply a periodic or a non-periodic wave form? Why?

5) If it is impossible to hear a definite pitch for a sound, is that sound harmonic or non-harmonic?

2.2 BEATS

If two waves have the same frequency, but not necessarily the same amplitude, and if their positive and negative peaks coincide, we say that they are **in phase**. Summing waves that are in phase will never result in destructive interference, and the amplitude of the summed wave can be calculated as a point-by-point sum of the two waves. Figure 2.27a shows two waves with the same frequency that are in phase. Due to the constructive interference that is always present when waves are in phase, it is easy to see that their sum is simply the sum of the amplitudes.
On the other hand, if the two waves are in antiphase (out of phase by 180 degrees, as shown in figure 2.27b), the point-by-point sum will yield a wave that has a peak amplitude that is equal to the difference between the peak amplitudes of the two constituent waves. At the limit, if two waves are out of phase and their amplitudes are equal, as shown in figure 2.27c, the result of summing them will be 0 at all points; the two waves will cancel themselves out. In example 2.27d, two waves of equal frequency and amplitude are shown out of phase by 90 degrees (in the same relationship that sine waves have to cosine waves). In this case, positive and negative peaks no longer coincide, and because of this, we observe both constructive and destructive interference in action. The resulting sine wave has the same frequency as the original waves, but its phase and amplitude differ from those of the originals.

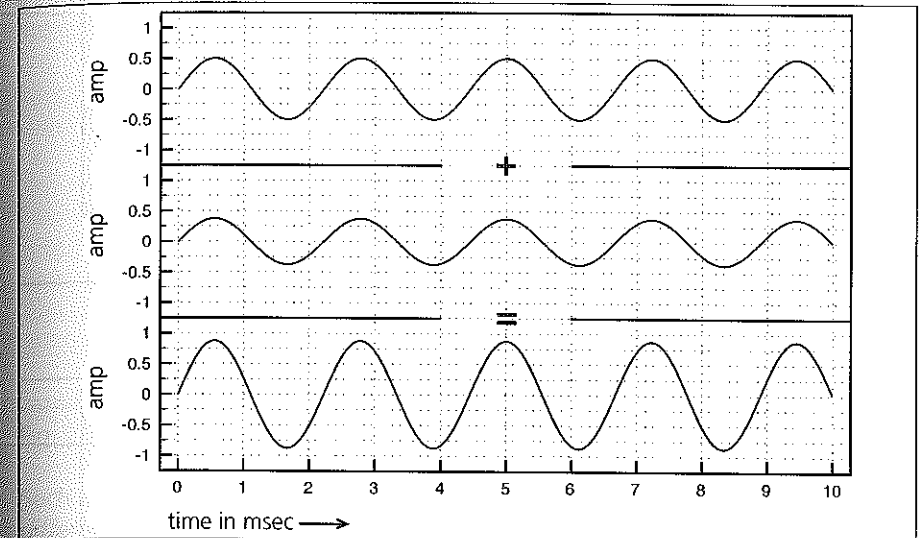


Fig. 2.27a Sum of two in-phase waves

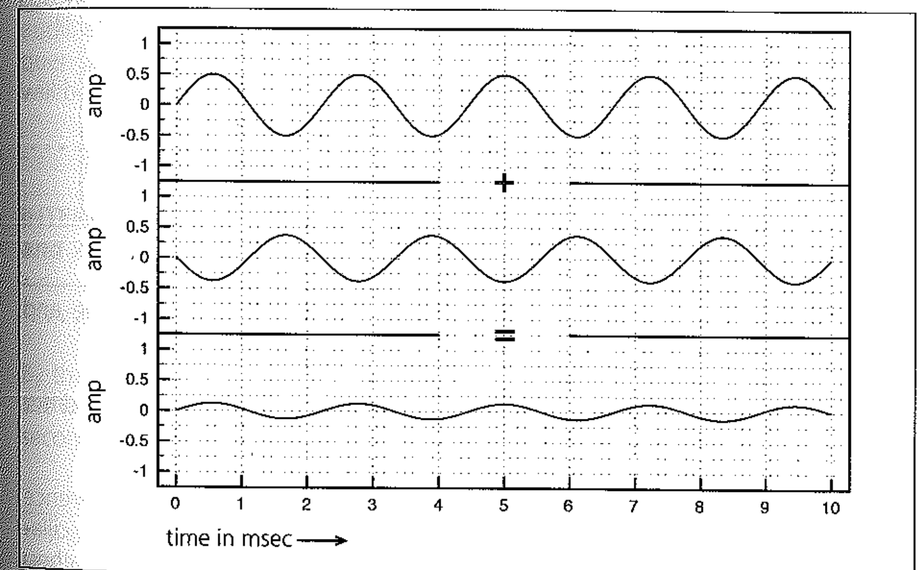


Fig. 2.27b Sum of two waves in antiphase

11 The wavetable synthesis that is used in many commercial synthesizers almost always uses multiple wavetables, along with a mechanism for morphing from one table to the other in order to obtain complex sounds that evolve in time. By contrast, the simple fixed spectrum case that we illustrate here requires only a single table. In Section 2.3, we will examine a variation on this technique.

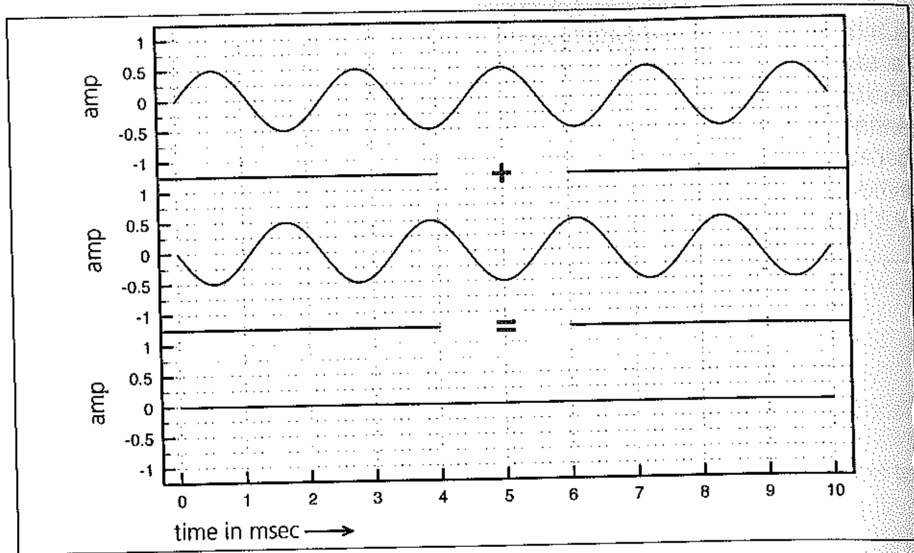


Fig. 2.27c Sum of two waves of equal amplitude in antiphase

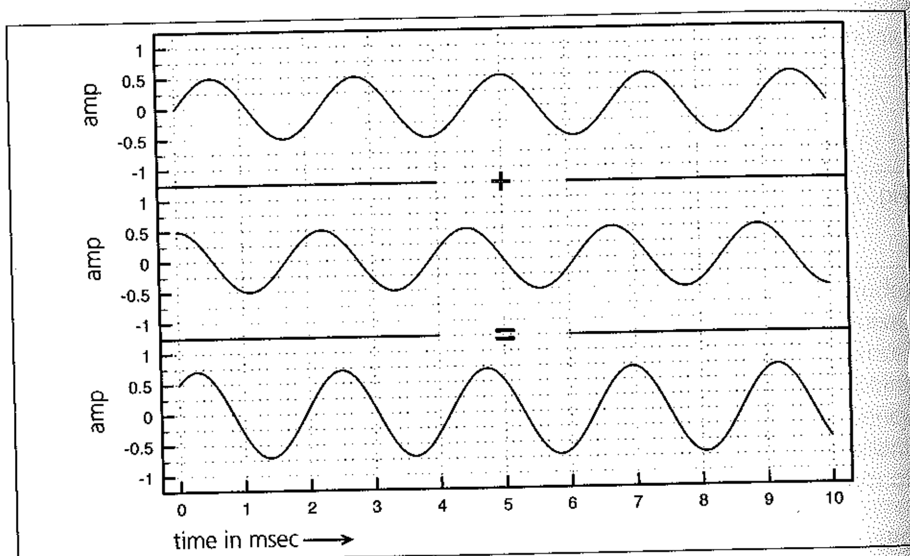


Fig. 2.27d Sum of two waves whose phase differs by 90 degrees (sine and cosine waves)

Now let's consider the sum of two sine waves with frequencies that differ slightly from each other, as shown in figure 2.28.

The two waves start out in phase, with the resulting constructive interference causing the overall amplitude to grow, but after a certain number of oscillations, they gradually shift to being maximally out of phase with each other, which causes the overall amplitude to be reduced.

After a certain number of oscillations, the waves are once again in phase, and the cycle repeats. The amplitude of the resulting wave will alternately rise and fall, causing a phenomenon known as **beats**.

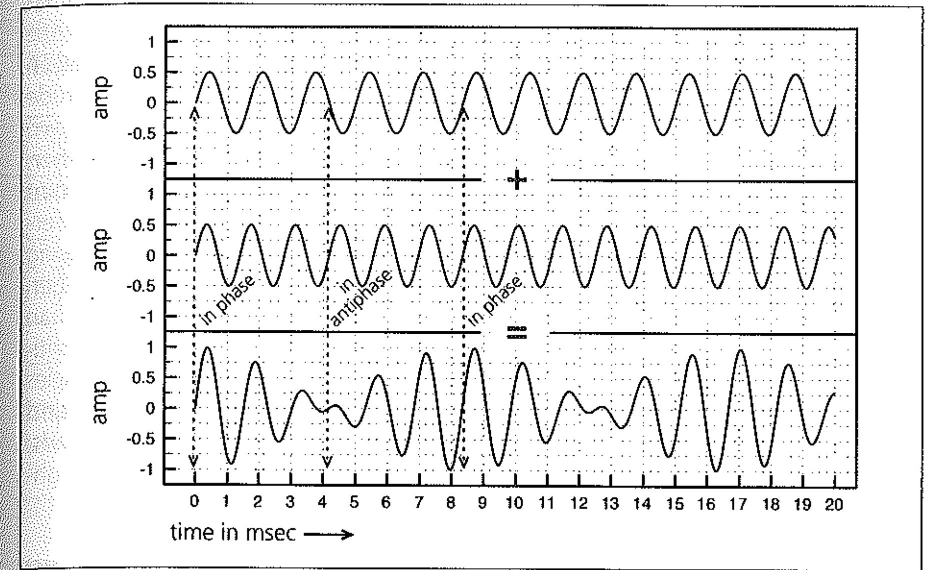


Fig. 2.28 Sum of two waves with slightly different frequencies

Such a periodic oscillation in amplitude will itself have a frequency equal to the *difference in frequency* between the two interfering waves. For example, if the frequency of the first sine wave is 220 Hz and the second is 222 Hz, the oscillation in amplitude that will be heard in the resulting wave will be 2 cycles per second. (We will say "2 beats per second"). Besides this beat frequency, the audible frequency of the resulting wave will lie between the two original frequencies; if we represent the frequencies as f1 and f2, the perceived frequency will be (f1 + f2) / 2, which, in the example given, would be 221 Hz.

INTERACTIVE EXAMPLE 2H – *PRESET 1 – Beats caused by sounds of 220 and 222 Hz*

By the same method, if the frequency of the first wave were to be 439 Hz and the frequency of the second 435 Hz, the resulting wave would have 4 beats per second, and its frequency would be heard as 437 Hz.[12]

[12] If the difference between the two frequencies becomes larger than the minimum audible frequency (around 30 Hz), beats become inaudible and give rise under certain conditions to so-called *difference tones* or *Tartini tones*.

INTERACTIVE EXAMPLE 2H – *PRESET 2 – Beats caused by sounds of 439 and 435 Hz*

The beats phenomenon is caused by the small difference in frequency between the two summed waves, which results in the cyclical variation in the amplitude of the resulting wave. If we slowly increase the difference between the two frequencies, the frequency of the beats will increase as well. As we go farther, continuing to increase the difference between the frequencies, at some point we will no longer perceive single beats, but instead we will hear the interference between the two waves as a sort of "roughness" in the sound. Increasing the difference even further, we will ultimately hear two distinct sounds. When two interfering sounds create beats or the sensation of roughness rather than two distinct sounds, we say that the respective frequencies lie within a "critical band"; when we hear two distinct frequencies instead, these frequencies are said to fall outside of the critical band. In other words, the **critical band** delineates boundaries within which our ears cannot separate individual sounds; instead, we resolve the complexity of such a signal with a sensation of roughness or by hearing beats.

The width of the critical band, established experimentally, varies with changes in the intermediate frequency between the two frequencies in play. Above 200 Hz, the width of the critical band increases as the intermediate frequency increases, corresponding to an interval that lies somewhere between a whole tone and a minor third. This interval may explain why we hear the intervals of a whole tone and semitone as dissonant, while intervals larger than the critical band seem more consonant.[13] Below 200 Hz, on the other hand, the width of the critical band remains a constant width, and because of this, it occupies an interval (in semitones) that steadily increases as the pitch descends. This means that at lower frequencies, only very wide intervals are perceived of as consonant. This phenomenon is well known to composers of all periods – examine any score, perhaps one written for piano, for example, and it is probable that the intervals between notes sounding simultaneously in the low register will be wider than the intervals between notes in the middle and higher registers.[14]

13 We are speaking of pure sounds, devoid of harmonics. In more complex sounds, such as the sounds of acoustic musical instruments, we need to also consider the strongest components of the spectrum, which are in general the first harmonics. An interval of a seventh, formed by complex sounds, will be heard as dissonant, because the second harmonic of the lower sound and the fundamental of the higher sound, fall within the critical band. The intervals considered dissonant can be heard as such by a musician even if they are played with pure sounds, because an educated musical ear is habituated to associate certain intervals with the sensation of dissonance. On the other hand, it is definitely possible for a non-musician to perceive any interval outside of the critical band as being consonant, or non-rough.

14 Unless, of course, the composer is purposefully exploiting dissonance by using small intervals in the lower range!

In the following table (taken from Rossing, T. 1990, p. 74) we see the relationship between intermediate frequencies and the width of the critical band. As you can see, when the intermediate frequency is around 100 Hz, the critical band is 90 Hz, or 90% of the intermediate frequency. At the extreme opposite, for an intermediate frequency of 10,000 Hz, we find a critical band of 1200 Hz, which is equal to only 12% of the intermediate frequency.

Intermediate Frequency	Width of the Critical Band
100	90
200	90
500	110
1,000	150
2,000	280
5,000	700
10,000	1,200

INTERACTIVE EXAMPLE 2H – *PRESET 3 – Gradual passage from beats to two distinct tones*

When we sum three or more sine waves whose frequencies differ only slightly, beats result between each pair of waves. For example, if we have three sine waves with frequencies of 200, 201, and 202.5 Hz, we will have the following combinations:

1) 200 Hz + 201 Hz 1 beat per second
2) 201 Hz + 202.5 Hz 1.5 beats per second
3) 200 Hz + 202.5 Hz 2.5 beats per second

Besides the beats produced by each of the three combinations, the beats themselves will also interact in a periodic fashion, both in pairs and as three sounds together, resulting in accented beats that repeat regularly. In our case, the rhythmic cycle will repeat itself every 2 seconds. In 2 seconds, the first pair of sounds will have produced 2 complete beats, the second pair 3 complete beats, and the third pair 5 complete beats.

INTERACTIVE EXAMPLE 2H – *PRESET 4 – Multiple beats: combinations of three sine waves*

Dodge and Jerse (1985, p. 37) assert that the phenomenon of beats can also be heard, although in a less marked way, between two frequencies that nearly form an octave, such as the frequencies 220 and 443 Hz. These are beats of the second order, with a ratio of 2:1, and the beat frequency in this case is equal to the difference between the frequency of the higher sound minus the frequency of the lower sound transposed upwards by exactly an octave (443 - 440 = 3 beats per second, in our example). Beats of this type vanish above about 1500 Hz.[15]

INTERACTIVE EXAMPLE 2H – *PRESET 5 – Beats among tones forming a near-octave*

If instead of summing two sine waves, we sum two complex sounds whose frequencies differ by only a small amount, we can create beats between the upper components of the first sound and the upper components of the second. For example, if we have a first sound that contains frequency components at 100, 202.5, and 750 Hz and a second sound that contains components at 101, 220.5, and 753.5 Hz, we will obtain 1 beat per second between the components of 100 and 101 Hz, 2 beats per second between 200.5 and 202.5 Hz, and 3 beats and a half per second between 750 and 753.5 Hz (ignoring any beats at the octave which in this case will be almost imperceptible). In figure 2.29, we present the sonogram of this combination. You can see the beats appear as dark zones at regular intervals.

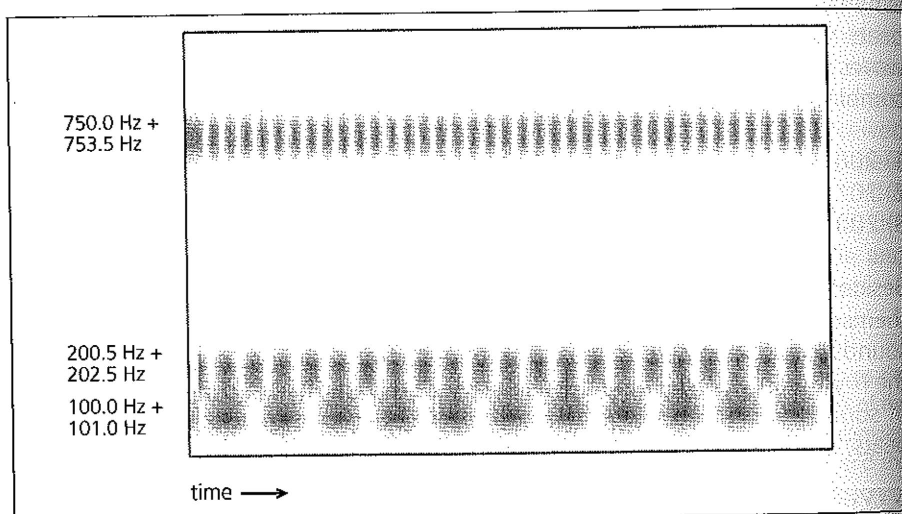


Fig. 2.29 Beats between two complex sounds

[15] In the text of Dodge and Jerse, beats at the fourth and at the fifth are also cited, although they are scarcely audible.

INTERACTIVE EXAMPLE 2H – *PRESET 6 – Multiple beats: combinations of two complex sounds*

If a number of oscillators generate complex sounds by sharing the same wavetable, and if the frequencies of these sounds differ only slightly from each other, it is possible to obtain interesting pulsing patterns that are produced by the beats between the harmonics of the sounds. It is often possible to hear "internal melodies" or even glissandi between harmonics in these patterns. To achieve this, it is necessary to ensure that the beat rates differ for each harmonic. Two complex sounds that share the same waveform, with fundamentals of 110 and 111 Hz, for example, will pulsate once per second at the fundamental, twice per second at the second harmonic (where the frequencies are 220 and 222 Hz), three times per second at the third, and so on. The interactions between these pulses create a movement within the sound. If after this we add a third sound at 112 Hz, there will now be 2 pulses per second at the fundamental, 4 per second at the second harmonic, 6 at the third, etc., and these pulsations will be more or less strong depending upon whether all three components, or only two, are in phase at a given moment. Increasing the number of oscillators will increase the complexity of the rhythmic pulsations. In figure 2.30 we show the sonogram of a sum of 7 sounds (each of which has 24 harmonics) whose frequencies fall near 110 Hz. The frequencies of the individual oscillators for this example are spaced at intervals of 0.07 Hz (110.07, 110.14, etc.)

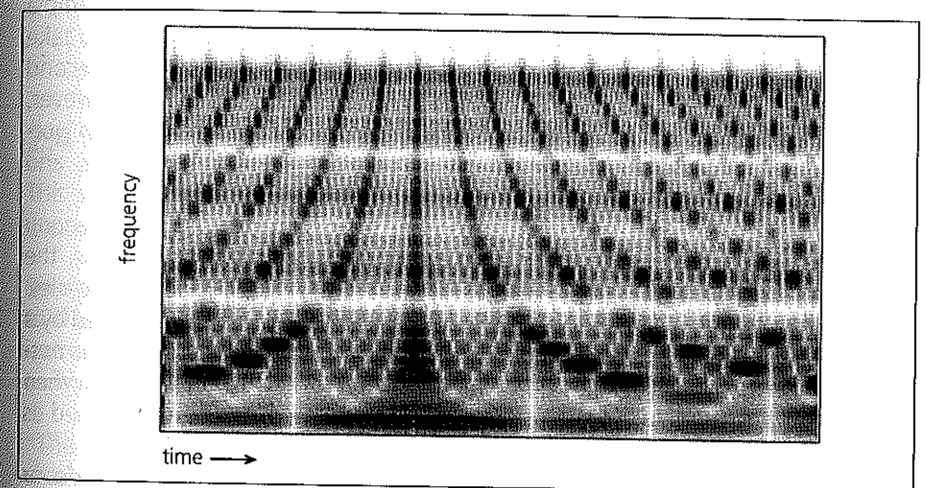


Fig. 2.30 Beats between 7 complex sounds each of which has 24 harmonics

INTERACTIVE EXAMPLE 2I – *Multiple beats: combination of 7 complex sounds*

2.3 CROSSFADING BETWEEN WAVETABLES: VECTOR SYNTHESIS

The sounds that we have created to this point through the use of wavetables have all been obtained using one table alone. In this section, we will relax this limitation and discuss a technique that involves crossfading between tables, which will enable us to generate sounds whose spectra vary. We have thus far used single timbres, contained in single tables, but if we build several tables, each containing a different timbre, we can crossfade between them in time, dissolving from one timbre to another and thereby changing the spectrum. For example, we might use the attack of a lute to model the beginning of a sound, and move from the lute to the decay of an electric guitar at the end of the same sound.
For our current discussion we will limit ourselves to using tables that contain single wave cycles, each differing from the others purely in terms of waveform (since we haven't yet touched on other synthesis techniques that might be relevant). During the course of a musical event (a single note), we will generate a sound that changes in time simply by dissolving from one table to another. You may hear this technique described using many different names – wavetable crossfading, **vector synthesis**, and linear algorithmic synthesis are all valid.
The implementation details of this type of synthesis are rather simple, since one can sum any number of different sounds by using the correct envelopes. You need only to organize a few things in the right way, including the sounds to be used, their durations, the length of the crossfades, and so forth. In the simple case of two sounds, for example, we would use one envelope segment to take the first sound from the maximum amplitude to 0, while taking the second sound from 0 to the maximum using a second envelope. Generally, however, such a simple case can be implemented using a single segment for both purposes, by applying the envelope directly to the first sound's amplitude, and reversing it[16] to control the second sound.

We see in figure 2.31 an example of a crossfade: given two waveforms, the mixed signal passes from the first to the second timbre over a span of 10 cycles. For two sound sources, the passage from one to the other can be described by a line segment, but when there are more than two sources the movement is better described by using a geometric plane or a three-dimensional space. When vector synthesis is implemented in hardware, in fact, a joystick is often used as the controller for mixing the different sources.
It is interesting to note that a sound effect known as *infinite glissando* or **Shepard tone** can be generated by using vector synthesis. The sound generated by this particular effect seems to slide endlessly, without ever arriving at a final pitch or passing beyond hearing. The effect can be obtained by crossfading sounds in which glissandi are slightly out-of-phase in time between themselves, and we will examine how to do this in more detail in Interlude B of this book.

16 We will see in the practical portion of this chapter how to actually accomplish this.

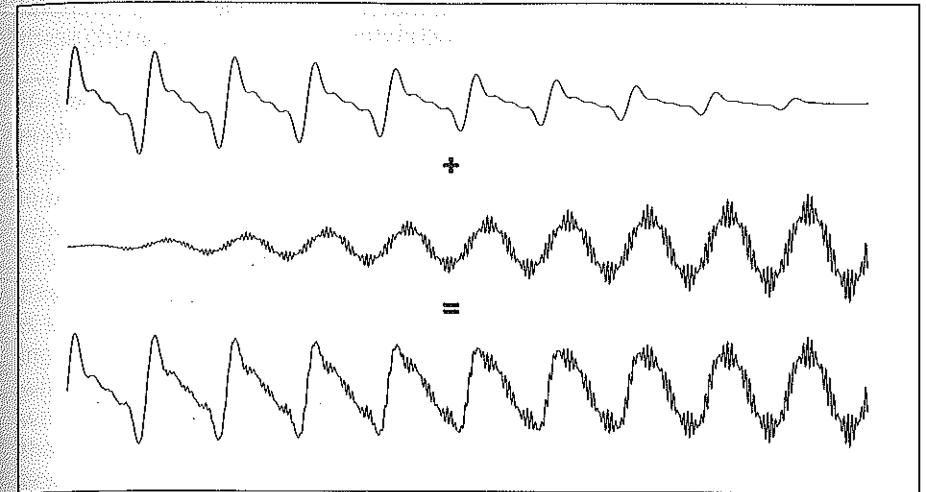


Fig. 2.31 Crossfading between two waveforms

INTERACTIVE EXAMPLE 2J – *VECTOR SYNTHESIS ON A SEGMENT*
INTERACTIVE EXAMPLE 2K – *VECTOR SYNTHESIS ON A PLANE*
INTERACTIVE EXAMPLE 2L – *INFINITE GLISSANDO, OR SHEPARD TONE*

2.4 VARIABLE SPECTRUM ADDITIVE SYNTHESIS

This section will discuss how to overcome some of the synthesis limitations imposed by fixed spectra.

Sounds, of course, are not static phenomena; they are dynamic and variable to the extreme. Over the evolutionary course of a note played by an acoustic instrument, for example, all three of the basic parameters (frequency, amplitude, and spectrum) vary continuously.

"The independent temporal evolution of all frequency components is a basic characteristic of natural sounds, and for this reason it is important to learn how to manage such changes in order to produce a 'living' sound." (Ibid, p. 55.)
Not only the amplitude and the frequency of the fundamental of a particular complex sound possess "signature" variations that make them unique to us. The amplitudes and frequencies of each and every spectral component also change over time. From this, we might deduce that additive synthesis using a fixed spectrum, as demonstrated in Section 2.1, is unlikely to be very interesting. Let's search, therefore, in order to find a way to implement a richer model that includes the possibility for spectral variation.

In figure 2.32, we see how a spectrum of a sound can be made to vary by applying various envelopes to each of the sound's components.

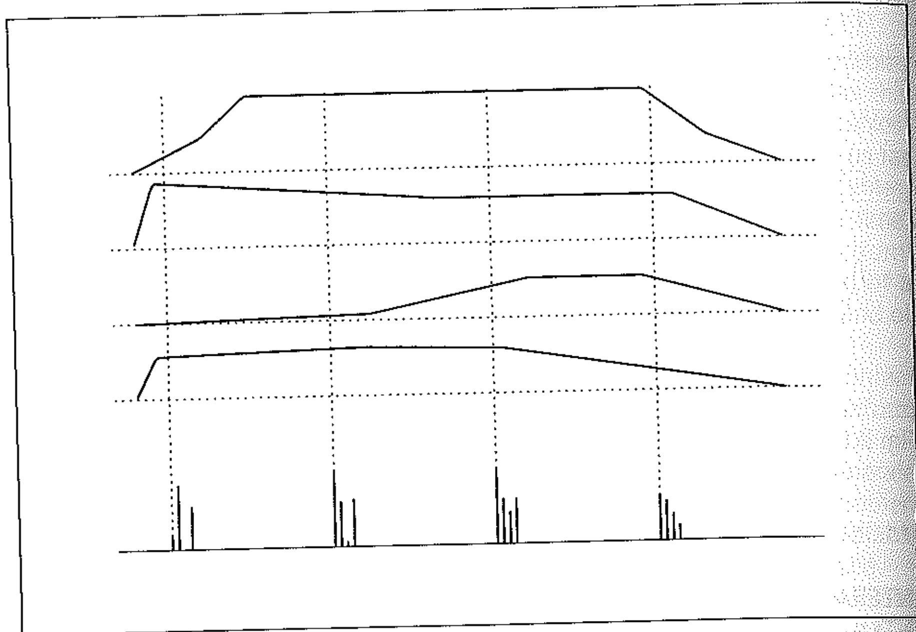


Fig. 2.32 Variable spectrum

During a single event, every single component can vary in both amplitude and frequency. In the practical part of this chapter, we will create components that are initially harmonic and that change to being non-harmonic (and vice versa), keeping their fundamentals fixed while modifying the frequencies of their components by using glissandi. This method will enable us to generate variable spectra, or sounds whose timbre changes in time, by modeling changing frequency and/or amplitude ratios between components.

From what we have said, it should be evident that the spectrum parameter is very different from the parameters of frequency and amplitude. The latter parameters are one dimensional quantities (which is a way to say that a single number can completely capture their state – the values of those parameters can be represented as points on a line). Because these units are one dimensional, it is easy to compare them. We can confirm, for example, that a certain frequency's relationship to another by using simple operations such as "less than" or "greater than" to compare the numbers.

On the other hand, timbre (or even better, spectrum), is a multi-dimensional quantity, whose definition requires capturing a whole series of numbers expressing the amplitude, the frequency, and the phase for each component. Because of this, it is not possible to define an absolute scale for timbre. (It is not possible to say that one timbre is "higher" than another, for example.) It certainly *is* possible to identify characteristics of timbre, such as brilliance,

harmonicity, roughness, or denseness, that can be used to order sounds, but these characteristics are difficult to characterize with numerical precision.[17]

Variable spectrum additive synthesis is clearly a very powerful technique, enabling variations in time for every single component of a sound. Using it, it would be theoretically possible to completely describe any timbre, or any sound! In practice, however, creating such descriptions would not be easy. Every sound would be defined by dozens or even hundreds of components, and for each of these components it would be necessary to specify separate amplitude and frequency envelopes (or perhaps a series of glissandi from one to another). It is easy to imagine that each sound in a score of medium complexity (containing hundreds of sounds) would require summing dozens of components along with numerous other parameter specifications. From a practical point of view, we might hypothesize that when the components of a sound are few (no more than 8, or perhaps 16) it ought to be possible to fully create such a specification in a relatively efficient way, but that sounds beyond that level of complexity ought to be realized using a different strategy.

An example of such an alternate strategy might be the use of the so-called **masking** method, which is a method that consists of defining a "mask" of two envelopes that trace the progress of the lowest and the highest frequencies, and automatically reconstructing all frequencies for the other components by subdividing the space between the two extremes into equal or unequal parts, as shown in figure 2.33.[18]

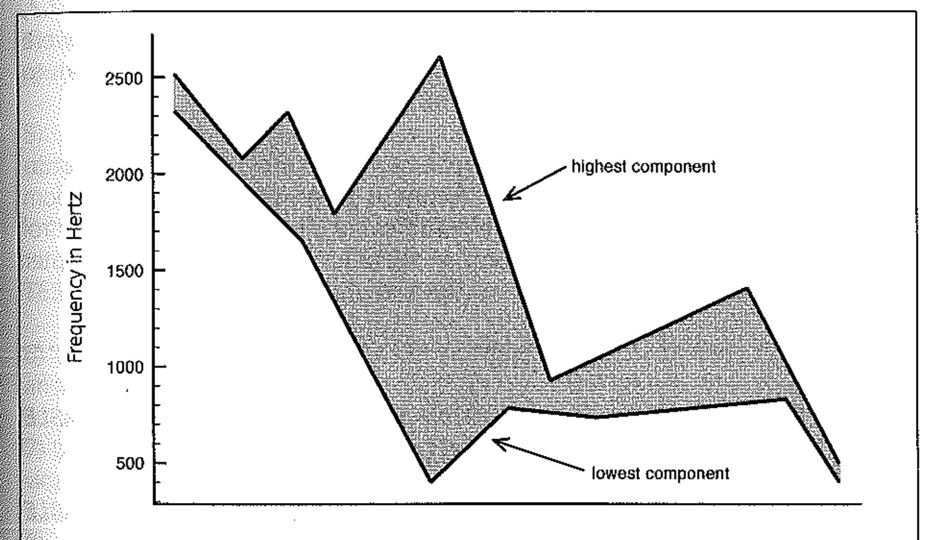


Fig. 2.33 Frequency masking for a complex sound

[17] Some interesting articles with regard to the multi-dimensional representation of sound timbre were written by J. Grey. (1975 and 1977)

[18] The masking technique can also be applied, obviously, to other parameters than frequency.

In figure 2.34 we see the sonogram (partial) resulting from the masking operation of figure 2.33, applied to a sound with 10 partials.

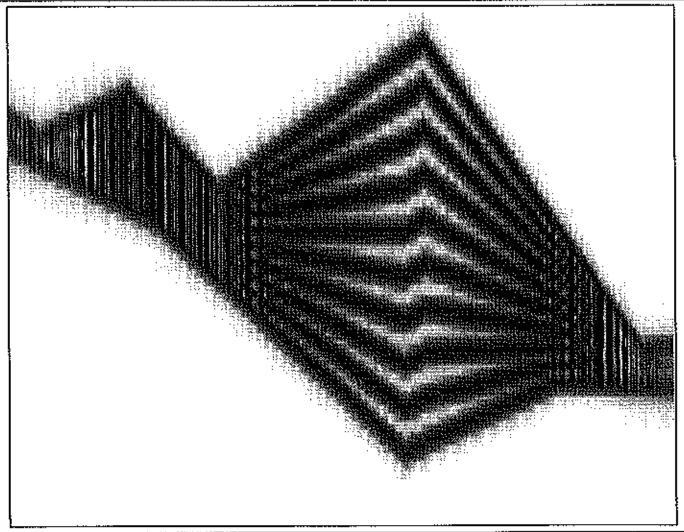


Fig. 2.34 Spectrum resulting from a masking operation

Another possibility would be to define two or more spectral envelopes and to interpolate from one to the other during the evolution of a sound. Such interpolation would be similar to vector synthesis, which we discussed in Section 2.3, but in the case of vector synthesis, the frequencies of the components were fixed. Here, the frequencies of the components might differ, enabling them to slide from one spectral envelope to another.

Other possibilities include defining control functions, either graphically or mathematically, that govern the behavior of synthesis parameters, or to use random numbers, or to build special-purpose algorithms for the generation and control of the components in a complex sound. In Section 2.4 of the practical chapter we will see how to implement some of these techniques.

INTERACTIVE EXAMPLE 2M – *ADDITIVE SYNTHESIS USING A VARIABLE SPECTRUM – EFFICIENT CONTROL*

INTERACTIVE EXAMPLE 2N – *ADDITIVE SYNTHESIS USING A VARIABLE SPECTRUM CONTROLLED – MASKING*

The most practical method for managing large numbers of timbral parameters is to perform *analysis* of a pre-existing sound followed by its *re-synthesis* through additive synthesis with a variable spectrum. So far, we have constructed

complex sounds using sums of simple sounds; we have developed a *synthesis* process. But it is also possible to take the other road, that of *analysis*, by decomposing a complex sound into its simple components. The **Fourier theorem** (developed by the French mathematician and physicist Joseph Fourier, who was active in the eighteenth and early nineteenth centuries) confirms that "any periodic waveform can be represented by a series of harmonics, or sine wave components, each with a particular amplitude and phase. It is therefore possible to reconstruct the *spectrum* of any periodic sound." (Ibid, p. 72.) Systems for the analysis of sound, even non-periodic sound, are manifold. We will speak of them in Chapter 12, where we will discuss the concepts of re-synthesis in detail. For now, it is simply useful to know that the data needed to specify the evolution of amplitude and frequency, which we have already referred to repeatedly, can also be reconstructed from the analysis of any complex sound.

TESTING – *TEST WITH SHORT ANSWERS*

1) Under what conditions can we say that two sounds are in phase?

2) Under what conditions can we say that two sounds are in antiphase?

3) If two sine waves are summed, one of 5,600 Hz and the other of 5,595 Hz, how many beats per second will they produce?

4) What is the difference between fixed spectrum additive synthesis and variable spectrum additive synthesis?

5) If you modify the frequencies and the amplitudes of the components of a complex sound, will you encounter variations of timbre?

TESTING – *TEST WITH LISTENING AND ANALYSIS*

6) Is the sound in Example AA2.1 harmonic or non-harmonic? Explain your answer.

7) Does the sound in Example AA2.2 move from harmonic to non-harmonic, or vice versa? Explain your answer.

8) Is the attack of the sound in Example AA2.3 harmonic or non-harmonic? Explain your answer.

9) Is the decay of the sound in Example AA2.4 harmonic or non-harmonic? Explain your answer.

FUNDAMENTAL CONCEPTS

1) In additive synthesis, the amplitude values of a set of different waves are added together, sample by sample, taking into account the signs of the samples.

2) In a complex sound **harmonic components** are those whose frequencies form integer ratios with some fundamental. In the absence of such ratios, we speak of **non-harmonic components**.

3) If two waves have the same frequency, and their positive and negative peaks coincide, we speak of them as being **in phase**. If the waves are exactly half of a period out of phase, we speak of them as being in **antiphase**.

4) The phenomenon known as **beats** is caused by a slight difference in frequency between two summed waves. It has the effect of periodically varying the amplitude of the resulting wave. If instead of summing pure sine waves, we sum two complex sounds of slightly varying frequency, beats are not only created between the fundamentals, but also between every component of the first sound and the corresponding component of the second sound.

5) In natural sounds, the values of all basic musical parameters change in time.

GLOSSARY

Array
(See *Wavetable*)

Beats
Cyclic variations in the amplitude of a sound, caused by summing two sounds whose frequencies differ slightly.

Complex sound
A sound formed by frequency components (sine waves).

Constructive interference
A condition determined by summing two wave whose amplitudes, at a given instant, are either both positive or both negative.

Destructive interference
A condition determined by summing two waves whose amplitudes, at a given instant, are of opposite sign.

Fixed spectrum additive synthesis
Additive synthesis in which the summed components vary neither their amplitudes nor their frequencies in time.

Fourier theorem
A theorem proposed by Joseph Fourier that states that every periodic waveform is representable as the sum of sine waves.

Frequency domain (representation in the)
Graphical representation of the amplitudes of all of the components in a sound, in which frequency values are shown on the x-axis, and amplitude values are shown on the y-axis.

Fundamental
The lowest frequency component of a harmonic spectrum.

Harmonic components
The higher components in a complex sound whose frequencies are integer multiples of the frequency of the fundamental.

In antiphase
A condition in which two waves of the same frequency are combined with each other 180 degrees out of phase, causing destructive interference.

In phase
A condition in which two waves of the same frequency, whose cycles exactly coincide, are combined with each other, causing constructive interference.

Infinite glissando
See *Shepard tone*

Masking
A technique for establishing parameter boundaries by limiting the range of an upper and a lower parameter as they change over time. The upper and lower limits of the mask can be either constant values and/or envelopes.

Missing fundamental
A sonic phenomenon in which a sound possesses definite pitch, but the pitch, which by definition is the fundamental of the sound, is missing as a component, having an amplitude of 0.

Non-harmonic components
The higher components of a complex sound whose frequencies do not form integer ratios with the lowest frequency in the sound.

Partials
See *Components* (of various types)

Periodic wave
A waveform that repeats itself exactly, once per period.

Periodic sound / non-periodic sound
A periodic sound is a sound produced by a wave form that repeats periodically. A non-periodic sound, on the other hand, is a sound for which it is not possible to differentiate a wave form that repeats cyclically in time.

Phase
The relative position that the wave cycle of a sound occupies at a given instant in time.

Quasi-harmonic components
The higher components of a complex sound whose frequencies come very close to forming integer ratios with the fundamental.

Shepard tone
An effect in which a sound seems to slide endlessly without concluding. It can be obtained by crossfading between instances of a wavetable that are slightly out of phase with each other.

Sonogram (Spectrogram)
A representation of spectral evolution as a function of time. The frequencies of the components are shown using one axis (usually the y-axis) and time is indicated using the other axis. The amplitude of every frequency component is shown, quantized by the units of the chromatic scale.

Sound spectrum
A representation of the amplitudes of the components of a sound as a function of frequency. (See *Frequency domain*.)

Spectral envelope
Considering the graph of a spectrum, a spectral envelope is a curve that outlines the components shown in the graph, pulling the amplitude bars of all of the components into a single entity.

Time domain (representation in the)
A graphical representation of instantaneous changes in the amplitude of a sound, in which time is shown on the x-axis and amplitude is shown on the y-axis.

Variable spectrum additive synthesis
Additive synthesis in which the summed components vary their amplitudes and/or their frequencies in time.

Vector
(See *Wavetable*)

Vector synthesis (Wavetable crossfading)
Crossfading between wavetables, used as a method for generating a variable spectrum.

Wavetable
A wavetable, or vector, or array, is an ordered series of values, each of which is associated with a numerical index value.

Wavetable synthesis
A synthesis technique that in its simplest form consists of playing a table that contains a single cycle of a complex waveform with an oscillator. In addition to this simple definition, there is also a table-based technique that goes by the same name, which involves dynamically modifying a complex waveform in realtime by

evolving multiple wavetables (in a manner similar to vector synthesis).