FIR - Filter Design

Expression :-

$$\sum_{k=0}^{N-1} a_k y(n-k) = \sum_{k=0}^{M-1} b_k x(n-k)$$

$a_0 = 1, \quad a_k = 0, \quad k = 1 \text{ to } (N-1)$

$$\boxed{y(n) = \sum_{k=0}^{M-1} b_k x(n-k)}$$

Non Recursive system
Hence o/p $y(n)$ is finite
$\therefore$ is always stable.

Take Z T

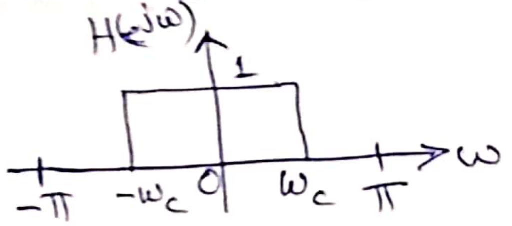
$$Y(z) = \sum_{k=0}^{M-1} b_k z^{-k} X(z)$$

$$\boxed{H(z) = \frac{Y(z)}{X(z)} = \sum_{k=0}^{M-1} b_k z^{-k}}$$

Consists only zeros
No poles
$\therefore$ always stable

Properties :-

1. This filter is always stable $h(n) < \infty$
2. Linear phase :- consider FR of Low pass digital filter



$$H(e^{j\omega}) = \begin{cases} 1 \cdot e^{-j\omega\alpha}, & |\omega| < \omega_c \\ 0, & |\omega| > \omega_c \end{cases}$$

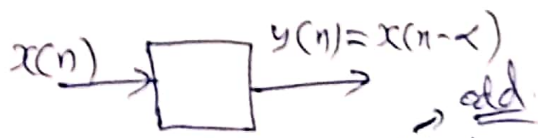
ω_c is the cutoff frequency

PB: $H(e^{j\omega}) = e^{-j\omega\alpha} = \dfrac{Y(e^{j\omega})}{X(e^{j\omega})}$

$$Y(e^{j\omega}) = X(e^{j\omega}) \cdot e^{-j\omega\alpha}$$

Take IFT

$$y(n) = x(n-\alpha)$$



$\alpha \to$ delay $= \dfrac{N-1}{2}$ odd

o/p = i/p with delay
hence o/p preserves i/p in the PB

3) $h(n) = h(N-1-n)$

if $N=5$, $n = 0$ to 4

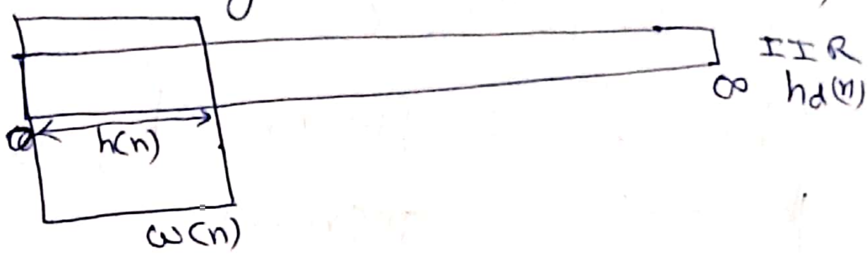
$h(0) = h(4)$
$h(1) = h(3)$
$h(2) = h(2)$
$h(3) = h(1)$
$h(4) = h(0)$

∵ Non Recursive System
→ order of filter N is very high.

impulse Response coefficients decreases

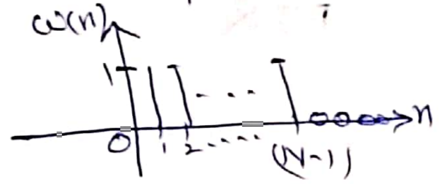
Design of FIR filters using windows

Consider IIR $h_d(n)$ and truncating it into FIR using window function $\omega(n)$



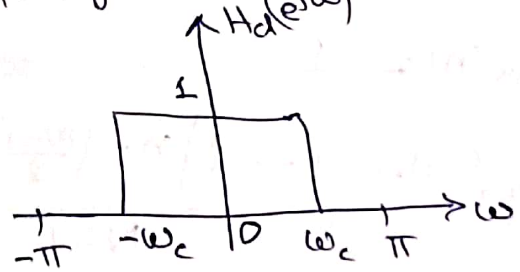
The window considered is Rectangular window

$$\omega_R(n) = \begin{cases} 1, & 0 \le n \le N-1 \\ 0, & \text{OW} \end{cases}$$



$$\boxed{h(n) = h_d(n) \cdot \omega_R(n)} \quad \text{—— (1)}$$

To get $h_d(n)$:- Consider FR of Low pass IIR digital filter



$$H_d(e^{j\omega}) = \begin{cases} 1 \cdot e^{-j\omega\alpha}, & |\omega| < \omega_c \\ 0, & \cancel{\pi < |\omega| > \omega_c} \ \omega_c < |\omega| < \pi \end{cases} \quad \text{—— (2)}$$

Take IFT on (2)

$$h_d(n) = \text{IFT}\left[H_d(e^{j\omega})\right] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega}) \cdot e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} 1 \cdot e^{-j\omega\alpha} \cdot e^{j\omega n}\, d\omega = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{j\omega(n-\alpha)}\, d\omega$$

$$= \frac{1}{2\pi} \cdot \left. \frac{e^{j\omega(n-\alpha)}}{j(n-\alpha)} \right|_{-\omega_c}^{\omega_c}$$

$$= \frac{1}{2\pi j(n-\alpha)} \left[e^{j\omega_c(n-\alpha)} - e^{-j\omega_c(n-\alpha)} \right]$$

$$= \frac{1}{\cancel{2}\pi \cancel{j}(n-\alpha)} \cdot \cancel{2}\cancel{j} \sin\omega_c(n-\alpha)$$

$$\boxed{h_d(n) = \frac{\sin[\omega_c(n-\alpha)]}{\pi(n-\alpha)}} \quad — (3)$$

Substitute (3) in (1)

$$\boxed{h(n) = \frac{\sin[\omega_c(n-\alpha)]}{\pi(n-\alpha)} \times \omega_R(n)}$$

FIR IIR window

LPF

Types of windows

window	Equation, $\omega(n)$, $0 \le n \le N-1$
Rectangular $\omega_R(n)$	$\omega_R(n) = 1$
Bartlet	$\omega_{Bar}(n) = 1 - \frac{2\left\|n - \left(\frac{N-1}{2}\right)\right\|}{N-1}$
Hanning	$\omega_{Han}(n) = 0.5 - 0.5\cos\left(\frac{2\pi n}{N-1}\right)$
Hamming	$\omega_{Ham}(n) = 0.54 - 0.46\cos\left(\frac{2\pi n}{N-1}\right)$
Blackman	$\omega_{BLK}(n) = 0.42 - 0.5\cos\left(\frac{2\pi n}{N-1}\right) + 0.08\cos\left(\frac{4\pi n}{N-1}\right)$

(5)

1) Design FIR filter using rectangular window of size 5 for the desired frequency response

$$H_d(e^{j\omega}) = \begin{cases} e^{-j2\omega}, & |\omega| < \frac{\pi}{4} \\ 0, & \frac{\pi}{4} < |\omega| \le \pi \end{cases}$$

Also find frequency, magnitude and phase responses of FIR filter

$= \quad \alpha = 2, \quad \alpha = \frac{N-1}{2} = 2 \quad \therefore N = 5$

FIR filter design:- $h(n) = ?$

$$h(n) = h_d(n) \cdot \omega_R(n) = h_d(n) \times 1, \quad 0 \le n \le 4 \quad \text{--- (1)}$$

$$h_d(n) = IFT[H_d(e^{j\omega})] = \frac{1}{2\pi} \int_{-\pi/4}^{\pi/4} H_d(e^{j\omega}) \cdot e^{j\omega n} d\omega$$

$$h_d(n) = \frac{\sin\left[\frac{\pi}{4}(n-2)\right]}{\pi(n-2)} \quad \text{--- (2)}$$

Substitute (2) in (1)

$$\therefore \text{FIR } h(n) = \frac{\sin\left[\frac{\pi}{4}(n-2)\right]}{\pi(n-2)}, \quad n \ne 2$$

$$= \frac{1}{4} = 0.25, \quad n = 2$$

$n=0, \; h(0) = 0.159$

$n=1, \; h(1) = 0.225$

$n=2, \; h(2) = 0.25$

$n=3, \; h(3) = 0.225$

$n=4 \; h(4) = 0.159$

FIR Filter Design

$$\boxed{h(n) = [0.159, 0.225, 0.25, 0.225, 0.159]}$$

Frequency Response of FIR Filter

$$H(e^{j\omega}) = FT[h(n)]$$

$$= \sum_{n=0}^{4} h(n)\, e^{-j\omega n}$$

$$= h(0)e^{-j\omega 0} + h(1)e^{-j\omega} + h(2)e^{-j2\omega}$$

$$+ h(3)e^{-j3\omega} + h(4)e^{-j4\omega}$$

$$\boxed{H(e^{j\omega}) = 0.159 + 0.225e^{-j\omega} + 0.25e^{-j2\omega} + 0.225e^{-j3\omega} + 0.159e^{-j4\omega}}$$

Magnitude Response of FIR Filter

$$H(e^{j\omega}) = e^{-j2\omega}\left[0.159e^{j2\omega} + 0.225e^{j\omega} + 0.25 \right.$$
$$\left. 0.225e^{-j\omega} + 0.159e^{-j2\omega}\right]$$

$$= e^{-j2\omega}\left[0.25 + 0.159(e^{j2\omega} + e^{-j2\omega}) + 0.225(e^{j\omega} + e^{-j\omega})\right]$$

$$= e^{-j2\omega}\left[0.25 + 0.159 \times 2\cos(2\omega) + 0.225 \times 2\cos(\omega)\right]$$

$$H(e^{j\omega}) = \left[0.25 + 0.318\cos(2\omega) + 0.45\cos(\omega)\right] e^{-j2\omega}$$

$$\boxed{|H(e^{j\omega})| = 0.25 + 0.318\cos(2\omega) + 0.45\cos(\omega)}$$

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Phase Response of FIR Filter

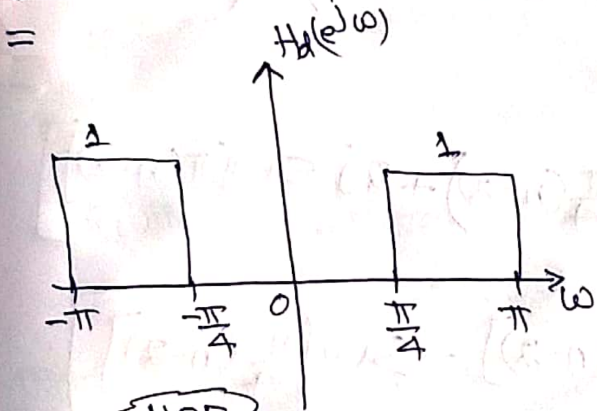
$$\boxed{\angle H(e^{j\omega}) = -2\omega}$$

Q) Design FIR filter using rectangular window of length 5 for the desired frequency response

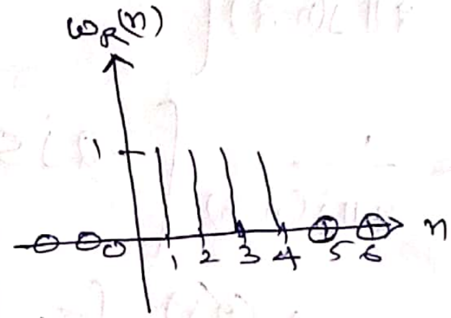
$$H_d(e^{j\omega}) = \begin{cases} e^{-j2\omega}, & \frac{\pi}{4} \le |\omega| \le \pi \\ 0, & -\frac{\pi}{4} \le \omega \le \frac{\pi}{4} \end{cases}$$

Also find frequency response, magnitude response, phase response and delay of FIR filter

=



{HPF}



FIR: $h(n) = h_d(n) \times \omega_R(n)$ —— ①

$$h_d(n) = IFT\{H_d(e^{j\omega})\} = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega}) \cdot e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{-\pi/4} e^{-j2\omega} \cdot e^{j\omega n}\, d\omega + \int_{\pi/4}^{\pi} e^{-j2\omega} \cdot e^{j\omega n}\, d\omega\right]$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{-\pi/4} e^{j\omega(n-2)}\, d\omega + \int_{\pi/4}^{\pi} e^{j\omega(n-2)}\, d\omega\right]$$

$$= \frac{1}{2\pi}\left\{ \left.\frac{e^{j\omega(n-2)}}{j(n-2)}\right|_{-\pi}^{-\pi/4} + \left.\frac{e^{j\omega(n-2)}}{j(n-2)}\right|_{\pi/4}^{\pi} \right\}$$

$$= \frac{1}{2\pi}\left\{\left(\frac{e^{j\left(-\frac{\pi}{4}\right)(n-2)} - e^{-j\pi(n-2)}}{j(n-2)}\right) + \left(\frac{e^{j\pi(n-2)} - e^{j\frac{\pi}{4}(n-2)}}{j(n-2)}\right)\right\}$$

$$\cancel{= \frac{1}{2\pi}\left\{\frac{e^{j\frac{\pi}{4}(n-2)} + e^{-j\frac{\pi}{4}(n-2)}}{j(n-2)}\right\}}$$

$$= \frac{1}{2\pi}\left\{\frac{-e^{j\frac{\pi}{4}(n-2)} + e^{-j\frac{\pi}{4}(n-2)} + e^{j\pi(n-2)} - e^{-j\pi(n-2)}}{j(n-2)}\right\}$$

$$= \frac{1}{2\pi j(n-2)}\left[-\left(e^{j\frac{\pi}{4}(n-2)} + e^{-j\frac{\pi}{4}(n-2)}\right) + \left(e^{j\pi(n-2)} - e^{-j\pi(n-2)}\right)\right]$$

$$= \frac{1}{2\pi j(n-2)}\left[-2j\sin\left(\frac{\pi}{4}(n-2)\right) + 2j\sin[\pi(n-2)]\right]$$

$$= \frac{1(\cancel{2}\cancel{j})}{\cancel{2}\pi\cancel{j}(n-2)}\left[\sin[\pi(n-2)] - \sin\left[\frac{\pi}{4}(n-2)\right]\right]$$

$$h_d(n) = \begin{cases} \frac{1}{\pi(n-2)} \times \left\{\sin[\pi(n-2)] - \sin\left[\frac{\pi}{4}(n-2)\right]\right\}, & n \neq 2 \\ 3/4, & n = 2 \end{cases}$$

$n=0$, $h_d(0) = -0.159$

$n=1$, $h_d(1) = -0.225$

$n=2$, $h_d(2) = 0.75$

$n=3$, $h_d(3) = -0.225$

$n=4$, $h_d(4) = -0.159$

FIR: $h(n) = h_d(n) \cdot w_R(n) = h_d(n) \times 1 = h_d(n)$

$\therefore h(n) = [-0.159, -0.225, 0.75, -0.225, -0.159]$

Frequency Response of FIR Filter

$$H(e^{j\omega}) = FT\{h(n)\}$$

$$= \sum_{n=0}^{4} h(n)\, e^{-j\omega n}$$

$$= h(0)\, e^{-j\omega 0} + h(1)\, e^{-j\omega 1} + h(2)\, e^{-j\omega 2} + h(3)\, e^{-j\omega 3} + h(4)\, e^{-j\omega 4}$$

$$= -0.159 - 0.225\, e^{-j\omega} + 0.75\, e^{-j2\omega} - 0.225\, e^{-j3\omega} - 0.159\, e^{-j4\omega}$$

$$= e^{-j2\omega}\left[-0.159\, e^{j2\omega} - 0.225\, e^{j\omega} + 0.75 - 0.225\, e^{-j\omega} - 0.159\, e^{-j2\omega}\right]$$

~~Magnitude Response~~

~~$|H(e^{j\omega})| = -0.159$~~

$$= e^{-j2\omega}\left[-0.159\left(e^{j2\omega} + e^{-j2\omega}\right) - 0.225\left(e^{j\omega} + e^{-j\omega}\right) + 0.75\right]$$

$$H(e^{j\omega}) = \left[-0.159 \times 2\cos(2\omega) - 0.225 \times 2\cos(\omega) + 0.75\right] e^{-j2\omega}$$

$$H(e^{j\omega}) = \left[-0.318\cos(2\omega) - 0.45\cos(\omega) + 0.75\right] e^{-j2\omega}$$

Magnitude Response $|H(e^{j\omega})|$

$$|H(e^{j\omega})| = -0.318\cos(2\omega) - 0.45\cos(\omega) + 0.75$$

Phase Response: $\angle H(e^{j\omega}) = -2\omega$

Delay: $\alpha = 2$

(3) Design FIR filter using Hamming window for the desired frequency response

$$H_d(e^{j\omega}) = \begin{cases} e^{-j3\omega}, & |\omega| < \frac{3\pi}{4} \\ 0, & \frac{3\pi}{4} \le |\omega| \le \pi \end{cases}$$

Also find frequency response

$\Rightarrow$ $h(n) = ?$ $H(e^{j\omega}) = ?$ $|H(e^{j\omega})| = ?$ $\angle H(e^{j\omega}) = ?$

Given $\alpha = 3 = \frac{N-1}{2}$, $\boxed{N = 7}$

$$h_d(n) = IFT\{H_d(e^{j\omega})\}$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}.d\omega$$

$$= \frac{1}{2\pi}\int_{-3\pi/4}^{3\pi/4} e^{-j3\omega}.e^{j\omega n}.d\omega$$

$$= \frac{1}{2\pi}\int_{-3\pi/4}^{3\pi/4} e^{j\omega(n-3)}.d\omega$$

$$h_d(n) = \frac{\sin\left[\frac{3\pi}{4}(n-3)\right]}{\pi(n-3)}, \quad n \neq 3$$

$$3/4 = 0.75 \qquad n = 3$$

$$h_d(n) = [0.075, -0.159, 0.225, 0.75, 0.225, -0.159, 0.075]$$

$$\omega_{Ham}(n) = 0.54 - 0.46\cos\left(\frac{2\pi n}{N-1}\right)$$

$$\omega_{Ham}(n) = 0.54 - 0.46\cos\left(\frac{2\pi n}{6}\right)$$

$n = 0$ to 6

$$\omega_{Ham}(n) = \{0.08, 0.31, 0.77, 1, 0.77, 0.31, 0.08]$$

FIR: $h(n) = h_d(n) \cdot \omega_{Ham}(n)$

multiply element by element of $h_d(n)$ & $\omega_{Ham}(n)$

$$h(n) = [0.006, -0.049, 0.173, 0.75, 0.173, -0.049, 0.006]$$

Frequency Response $H(e^{j\omega})$:-

$$H(e^{j\omega}) = FT[h(n)] = \sum_{n=0}^{6} h(n) \cdot e^{-j\omega n}$$

$$= h(0) + h(1)e^{-j\omega} + h(2)e^{-j2\omega} + h(3)e^{-j3\omega} + h(4)e^{-j4\omega} + h(5)e^{-j5\omega} + h(6)e^{-j6\omega}$$

$$= \{0.006 - 0.049e^{-j\omega} + 0.173e^{-j2\omega} + 0.75e^{-j3\omega} + 0.173e^{-j4\omega} - 0.049e^{-j5\omega} + 0.006e^{-j6\omega}]$$

$$= e^{-j3\omega}[0.006e^{j3\omega} - 0.049e^{j2\omega} + 0.173e^{j\omega} + 0.75 + 0.173e^{-j\omega} - 0.049e^{-j2\omega} + 0.006e^{-j3\omega}]$$

$$= e^{-j3\omega}\{+0.06(e^{j3\omega} + e^{-j3\omega}) - 0.049(e^{j2\omega} + e^{-j2\omega}) + 0.173(e^{j\omega} + e^{-j\omega}) + 0.75]$$

$$= e^{-j3\omega}[0.06 \times 2\cos(3\omega) - 0.049 \times 2\cos(2\omega) + 0.173 \times 2\cos(\omega) + 0.75]$$

$$= e^{-j3\omega}[0.12\cos(3\omega) - 0.098\cos(2\omega) + 0.346\cos(\omega) + 0.75]$$

Magnitude Response $|H(e^{j\omega})|$

$$|H(e^{j\omega})| = 0.12\cos(3\omega) - 0.098\cos(2\omega) + 0.346\cos(\omega) + 0.75$$

Phase Response $\angle H(e^{j\omega})$

$$\angle H(e^{j\omega}) = -3\omega$$

4. Design FIR filter for the following specifications using rectangular & Hanning windows

(i) Gain of a filter is 0 dB

(ii) LPF cutoff frequency 2 KHZ

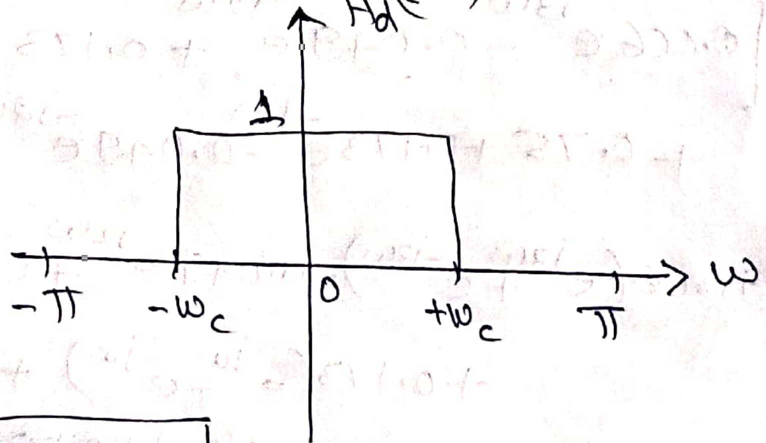
(iii) Sampling frequency = 8 KHz

(iv) length of the filter = 5

$\Rightarrow$ $h(n) = ? = hd(n) \cdot w_R(n) = ?$

$$h(n) = hd(n) \cdot w_{Han}(n) = ?$$

$$hd(n) = IFT\{H_d(e^{j\omega})\}$$



$\boxed{\omega_c = ?}$, Given $f_c = 2$ KHZ

$F = 8$ KHZ

$\therefore T = \frac{1}{F} = \frac{1}{8000}$

$$\Omega_c = 2\pi f_c = 2\pi \times 2000$$
$$= 4000\pi \text{ rad/sec.}$$

$$\omega_c = \Omega_c T$$
$$= 4000\pi \times \frac{1}{8000}$$
$$\boxed{\omega_c = \frac{\pi}{2} \text{ rad}}$$

$$\therefore H_d(e^{j\omega}) = \begin{cases} 1 \cdot e^{-j\omega\alpha}, & \frac{\pi}{2} \leq |\omega| \\ 0, & \frac{\pi}{2} \leq |\omega| \leq \pi \end{cases}$$

$$\alpha = \frac{N-1}{2} = \frac{5-1}{2} = 2$$

$$\therefore H_d(e^{j\omega}) = \begin{cases} e^{-j2\omega}, & \frac{\pi}{2} \leq |\omega| \\ 0, & \frac{\pi}{2} \leq |\omega| \leq \pi \end{cases}$$

$$h_d(n) = IFT\{H_d(e^{j\omega})\} = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega}) \cdot e^{j\omega n} d\omega$$

$$= \int_{-\pi/2}^{\pi/2} e^{-j2\omega} \cdot e^{j\omega n} d\omega$$

$$= \int_{-\pi/2}^{\pi/2} e^{j\omega(n-2)} d\omega = \frac{\sin\left[\frac{\pi}{2}(n-2)\right]}{\pi(n-2)}, \quad n \neq 2$$

$$= \frac{1}{2}, \quad n = 2$$

~~n=0~~, $h_d(0) =$

$h_d(1) =$

$h_d(2) =$

$h_d(3) =$

$h_d(4) =$

$h_d(n) = \{ \quad]$

Using Rectangular window

$\omega_R(n) = 1, \quad 0 \le n \le 4$

$\therefore h(n) = h_d(n) \cdot \omega_R(n) = h_d(n) = [$

Using Hanning window :-

$\omega_{Han}(n) = 0.5 - 0.5 \cos\left(\frac{2\pi n}{N-1}\right)$

$\omega_{Han}(n) = [\qquad]$

$h(n) = h_d(n) \cdot \omega_{Han}(n)$

$= [\qquad]$

5) Design FIR filter for the desired frequency response

$$H_d(e^{j\omega}) = \begin{cases} e^{-j3\omega}, & -\frac{3\pi}{4} \le \omega \le \frac{3\pi}{4} \\ 0, & \frac{3\pi}{4} \le |\omega| \le \pi \end{cases}$$

using (i) Rectangular window and
(ii) Hanning window
Also find frequency responses

$= h(n) = ?$ $\quad H(e^{j\omega}) = ?$ $\quad |H(e^{j\omega})| = ?$

$\angle H(e^{j\omega}) = ?$

6) Determine the coefficients of the impulse response $h(n)$ of the FIR filter for the frequency response of FIR filter

$$H(e^{j\omega}) = e^{-j3\omega}\left[1 + 1.8\cos(3\omega) + 1.2\cos(2\omega) + 0.5\cos(\omega)\right] \quad \text{—— (A)}$$

PyQ

Go through

= $h(n) = ?$

$$\alpha = 3 = \frac{N-1}{2}, \quad \therefore N = 7$$

$$H(e^{j\omega}) = \sum_{n=0}^{6} h(n)\, e^{-j\omega n}$$

$$= h(0) + h(1)e^{-j\omega} + h(2)e^{-j2\omega} + h(3)e^{-j3\omega} + h(4)e^{-j4\omega} + h(5)e^{-j5\omega} + h(6)e^{-j6\omega} \quad \text{—— (1)}$$

WKT

$$h(n) = h(N-1-n)$$

$$\left.\begin{array}{ll} n=0, & h(0) = h(6) \\ n=1, & h(1) = h(5) \\ n=2, & h(2) = h(4) \\ n=3, & h(3) = h(3) \\ n=4, & h(4) = h(2) \\ n=5, & h(5) = h(1) \\ n=6, & h(6) = h(0) \end{array}\right\} \text{—— (2)}$$

Substitute (2) in (1)

$$H(e^{j\omega}) = h(0) + h(1)e^{-j\omega} + h(2)e^{-j2\omega} + h(3)e^{-j3\omega} + h(2)e^{-j4\omega} + h(1)e^{-j5\omega} + h(0)e^{-j6\omega}$$

$$H(e^{j\omega}) = e^{-j3\omega}\left[h(0)e^{j3\omega} + h(1)e^{j2\omega} + h(2)e^{j\omega} + h(3) + h(2)e^{-j\omega} + h(1)e^{-j2\omega} + h(0)e^{-j3\omega}\right]$$

$$= e^{-j3\omega}\left[h(0)\left(e^{j3\omega} + e^{-j3\omega}\right) + h(1)\left(e^{j2\omega} + e^{-j2\omega}\right) + h(2)\left(e^{j\omega} + e^{-j\omega}\right) + h(3)\right]$$

$$= e^{-j3\omega}\left[h(0) \times 2\cos(3\omega) + h(1) \times 2\cos(2\omega) + h(2) \times 2\cos(\omega) + h(3)\right] \quad \text{— (B)}$$

Compare Equations (A) & (B)

$h(0) \times 2 = 1.8 \implies h(0) = 0.9$

$h(1) \times 2 = 1.2 \implies h(1) = 0.6$

$h(2) \times 2 = 0.5 \implies h(2) = 0.25$

$h(3) = 1$

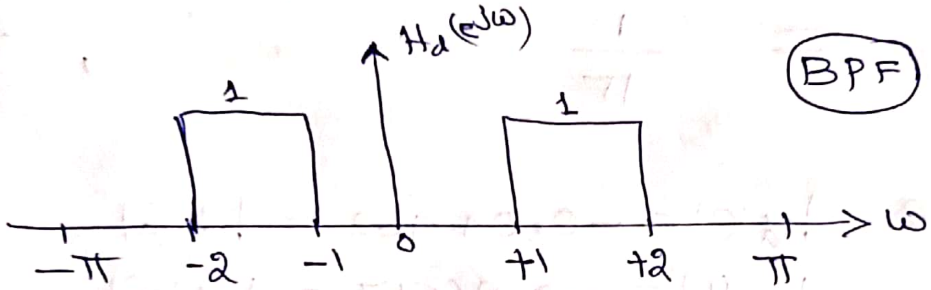
$\therefore h(n) = [0.9, 0.6, 0.25, 1, 0.25, 0.6, 0.9]$

(7) Design ~~BF~~ bandpass FIR filter for the length of 7 using Hamming window for the desired frequency response

Note: Here the Limits not in terms of Pie

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega\alpha}, & 1 \le |\omega| \le 2 \\ 0, & 2 \le |\omega| \le \pi \;\&\; -1 \le \omega \le 1 \end{cases}$$

= $h(n) = 2$ $\quad H(\omega) = 2$



(BPF)

$$\alpha = \frac{N-1}{2} = 3$$

$$h_d(n) = IFT\{H_d(e^{j\omega})\} = \frac{1}{2\pi}\left\{\int_{-\pi}^{\pi} e^{-j3\omega} \cdot e^{j\omega n}\, d\omega\right\}$$

$$= \frac{1}{2\pi}\left\{\int_{-\pi}^{\pi} e^{j\omega(n-3)}\, d\omega\right\}$$

$$= \frac{1}{2\pi}\left\{\int_{-2}^{-1} e^{j\omega(n-3)}\, d\omega + \int_{1}^{2} e^{j\omega(n-3)}\, d\omega\right\}$$

$$= \frac{1}{2\pi}\left\{\left.\frac{e^{j\omega(n-3)}}{j(n-3)}\right|_{-2}^{-1} + \left.\frac{e^{j\omega(n-3)}}{j(n-3)}\right|_{1}^{2}\right\}$$

$$= \frac{1}{2\pi j(n-3)}\left\{e^{-j(n-3)} - e^{-j2(n-3)} + e^{j2(n-3)} - e^{j(n-3)}\right\}$$

$$= \frac{1}{2\pi j(n-3)}\left\{e^{j2(n-3)} - e^{-j2(n-3)} - \left(e^{j(n-3)} - e^{-j(n-3)}\right)\right\}$$

$$= \frac{1}{2\pi j(n-3)} \left\{ 2j \sin(2(n-3)) - 2j\sin(n-3) \right\}$$

$$= \frac{\cancel{2j}}{\cancel{2}\pi \cancel{j}(n-3)} \left[\sin[2(n-3)] - \sin(n-3) \right]$$

$$h_d(n) = \frac{\sin[2(n-3)] - \sin(n-3)}{\pi(n-3)}, \quad n \neq 3$$

$$= \frac{1}{\pi}, \quad n=3$$

$h_d(0) = -0.04462 = h_d(6)$

$h_d(1) = -0.26517 = h_d(5)$

$h_d(2) = 0.02159 = h_d(4)$

$h_d(3) = 0.31831$

$\therefore h_d(n) = [-0.045, -0.265, 0.022, 0.318, 0.022, -0.265, -0.045]$

Hamming window

$$w_{Ham}(n) = \begin{cases} 0.54 + 0.46\cos\left(\frac{2\pi n}{N-1}\right), & 0 \le n \le N-1 \\ 0, & ow \end{cases}$$

$w_{Ham}(n) = [0.08, 0.31, 0.77, 1, 0.77, 0.31, 0.08]$

FIR Filter: $h(n) = h_d(n)\, w_{Ham}(n)$

(21)

$$h(n) = [-0.004, -0.082, 0.017, 0.32, 0.017, -0.082, -0.004]$$

Frequency Respons

$$H(e^{j\omega}) = FT\{h(n)\}$$

$$= \sum_{n=0}^{6} h(n)\, e^{-j\omega n} = ?$$

$$=$$

(8) Design FIR filter using Hamming window and Hanning window for desired frequency response

$$H_d(\omega) = \begin{cases} e^{-j\omega\alpha}, & |\omega| \le \frac{\pi}{6} \\ 0, & \frac{\pi}{6} \le |\omega| \le \pi \end{cases}$$

with $N = 13$

$\Rightarrow$

$$h(n) = ? \qquad \alpha = \frac{N-1}{2} = \frac{13-1}{2} = 6$$

$$h_d(n) = IFT\{H_d(\omega)\} = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(\omega) \cdot e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi/6}^{\pi/6} e^{-j\omega 6} \cdot e^{j\omega n}\, d\omega = \frac{1}{2\pi}\int_{-\pi/6}^{\pi/6} e^{j\omega(n-6)}\, d\omega$$

$$= \frac{1}{2\pi} \cdot \left. \frac{e^{j\omega(n-6)}}{j(n-6)} \right|_{-\pi/6}^{\pi/6}$$

$$h_d(n) = \begin{cases} \dfrac{\sin\left[\frac{\pi}{6}(n-6)\right]}{\pi(n-6)}, & n \neq 6 \\ 1/6, & n = 6 \end{cases}$$

$$h_d(n) = ?, \quad n = 0 \text{ to } 12$$

(i) Using Hamming window :-

$$\omega_{Ham}(n) = 0.54 - 0.46 \cos\left(\frac{2\pi n}{N-1}\right)$$

$\omega_{Ham}(n) = ?$ $\quad n = 0$ to 12

$h(n) = h_d(n) \times \omega_{Ham}(n) = ?$

(ii) Using Hanning window :-

$$\omega_{Han}(n) = 0.5 - 0.5 \cos\left(\frac{2\pi n}{N-1}\right)$$

$\omega_{Han}(n) = ?$ $\quad n = 0$ to 12

$h(n) = h_d(n) \times \omega_{Han}(n) = ?$

(9) Design FIR Low pass filter using rectangular window of length 7 for the following specifications of desire IIR filter

(i) Gain of 1

(ii) cutoff frequency is 1 KHz

(iii) Sampling frequency is 5 KHz

=

$h(n) = ?$

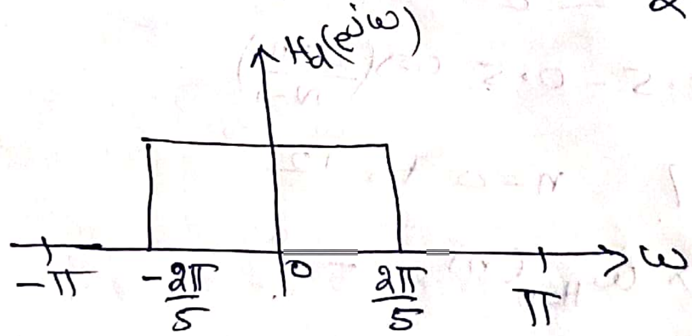
$f_c = 1000 Hz \qquad \Omega_c = 2\pi f_c = 2000\pi$ rad/sec

$F = 5000 Hz, \quad T = \frac{1}{F} = \frac{1}{5000}$ sec.

$\omega_c = \Omega_c T = \frac{2000\pi}{5000}$ rad

$\omega_c = \frac{2\pi}{5}$ rads.

$\alpha = \frac{N-1}{2} = \frac{7-1}{2} = \frac{6}{2} = 3$



$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega\alpha}, & -\frac{2\pi}{5} \le \omega \le \frac{2\pi}{5} \\ 0, & \frac{2\pi}{5} \le |\omega| \le \pi \end{cases}$$

$$h_d(n) = IFT\{H_d(e^{j\omega})\} = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\frac{2\pi}{5}}^{2\pi/5} e^{-j3\omega} \cdot e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi} \int_{-\frac{2\pi}{5}}^{2\pi/5} e^{j\omega(n-3)} \, d\omega$$

$$hd(n) = \begin{cases} \dfrac{\sin\left[\frac{2\pi}{5}(n-3)\right]}{\pi(n-3)}, & n \neq 3 \\ 2/5, & n = 3 \end{cases}$$

$h_d(n) = ?$ $\quad n = 0$ to 6

$hd(n) = [-0.06, 0.09, 0.3, 0.4, 0.3, 0.09, -0.06]$

$h(n) = h_d(n) \times \omega_R(n) = h_d(n) \times 1 = hd(n)$

$h(n) = [-0.06, 0.09, 0.3, 0.4, 0.3, 0.09, -0.06]$

(10) Design FIR differentiator for the desired frequency response

$$H_d(e^{j\omega}) = j\omega \quad , \quad -\pi \leq \omega \leq \pi$$

(or)

S.T FIR differentiator impulse response is antisymmetric, Given $N=7$, Rectangular window

$= \quad h(n) = ?$

$$h_d(n) = IFT\{H_d(e^{j\omega})\}$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega}) \cdot e^{j\omega n} \cdot d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} j\omega \cdot e^{j\omega n} \cdot d\omega$$

$$= \frac{j}{2\pi}\int_{-\pi}^{\pi} \omega \cdot e^{j\omega n}\, d\omega$$

integrate by parts

$$h_d(n) = \begin{cases} \dfrac{\cos(n\pi)}{n} & , \ n \neq 0 \\ 0 & , \ n = 0 \end{cases}$$

$$h(n) = h_d(n) \cdot \omega_R(n) = h_d(n) = \frac{\cos(n\pi)}{n}$$

$n=0$, $h(0) = 0$

$n=1$, $h(1) = -1$

$n=2$, $h(2) = 0.5$

$n=3$, $h(3) = -0.33$

$n=4$, $h(4) = 0.25$

$n=5$, $h(5) = -0.2$

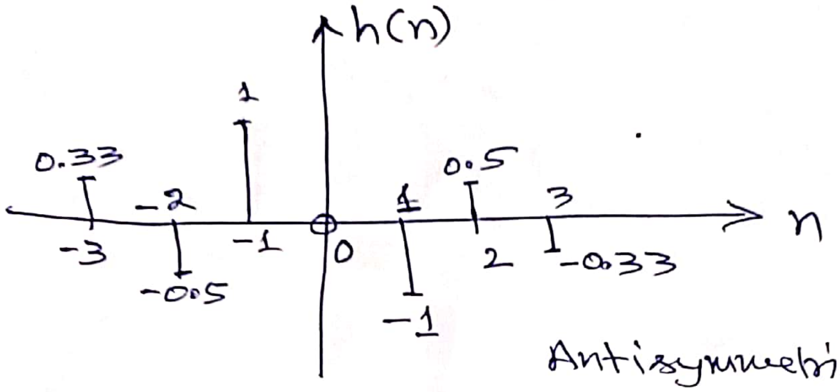
$n=6$, $h(6) = 0.17$

$h(n) = [\underset{\uparrow}{0}, -1, 0.5, -0.33, 0.25, -0.2, 0.17]$

For Antisymmetric :-

$h(n) = h_d(n)$, $n = -3$ to 3

$h(n) = [0.33, -0.5, 1, \underset{\uparrow}{0}, -1, 0.5, -0.33]$



Antisymmetric

(11) Design an ideal differentiator with frequency response for

$$H_d(e^{j\omega}) = j\omega \,, \quad -\pi \leq \omega \leq \pi$$

using $N=7$ using Hamming window

(12) Find the frequency response of rectangular window

$$\omega_R(n) = \begin{cases} 1, & -\left(\frac{N-1}{2}\right) \le n \le \left(\frac{N-1}{2}\right) \\ 0, & ow \end{cases}$$

Draw

$$= W_R(e^{j\omega}) = FT\left[\omega_R(n)\right] = \sum_{n=-\left(\frac{N-1}{2}\right)}^{\left(\frac{N-1}{2}\right)} \omega_R(n) \cdot e^{-j\omega n}$$

$$= \sum_{n=-\left(\frac{N-1}{2}\right)}^{\left(\frac{N-1}{2}\right)} (e^{-j\omega})^n$$

$$= \frac{e^{+j\omega\left(\frac{N-1}{2}\right)} - \left(e^{-j\omega}\right)^{\left(\frac{N-1}{2}+1\right)}}{1 - e^{-j\omega}}$$

$$\boxed{\sum_{n=\square}^{N-1} (a)^n = \begin{cases} \frac{a^{\square} - a^{N-1+1}}{1-a}, & a \ne 1 \\ N-1+1-\square, & a = 1 \end{cases}}$$

$$= \frac{e^{j\omega\frac{N}{2}} \cdot e^{-j\frac{\omega}{2}} - e^{-j\omega\frac{N}{2}} \cdot e^{-j\frac{\omega}{2}}}{1 - e^{-j\omega}}$$

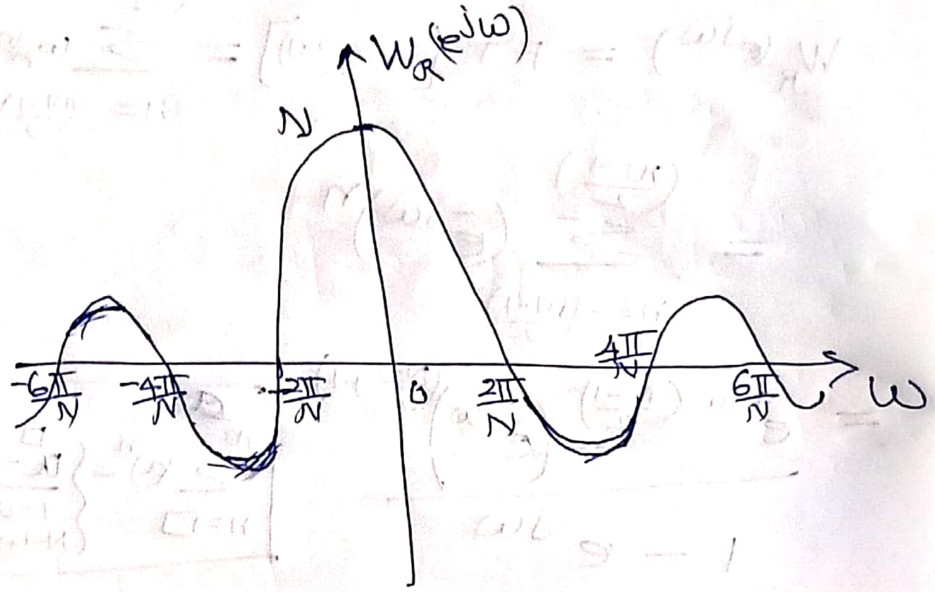
$$= \frac{e^{j\omega\frac{N}{2}} \cdot e^{-j\frac{\omega}{2}} - e^{-j\omega\frac{N}{2}} \cdot e^{-j\frac{\omega}{2}}}{1 - e^{-j\frac{\omega}{2}} \cdot e^{-j\frac{\omega}{2}}}$$

$$= \frac{\cancel{e^{-j\frac{\omega}{2}}}\left[e^{j\omega\frac{N}{2}} - e^{-j\omega\frac{N}{2}}\right]}{\cancel{e^{-j\frac{\omega}{2}}}\left[e^{j\frac{\omega}{2}} - e^{-j\frac{\omega}{2}}\right]}$$

$$= \frac{\cancel{2j} \sin\left(\omega \frac{N}{2}\right)}{\cancel{2j} \sin\left(\frac{\omega}{2}\right)}$$

$$W_R(e^{j\omega}) = \begin{cases} \dfrac{\sin\left(\omega \frac{N}{2}\right)}{\sin\left(\frac{\omega}{2}\right)}, & \omega \neq 0 \\ N, & \omega = 0 \end{cases}$$

Frequency Response $W_R(e^{j\omega})$:



1st Zero cross: $\dfrac{\sin\left(\frac{\omega N}{2}\right)}{\sin\left(\frac{\omega}{2}\right)} = 0$

$$\sin\left(\frac{\omega N}{2}\right) = 0$$

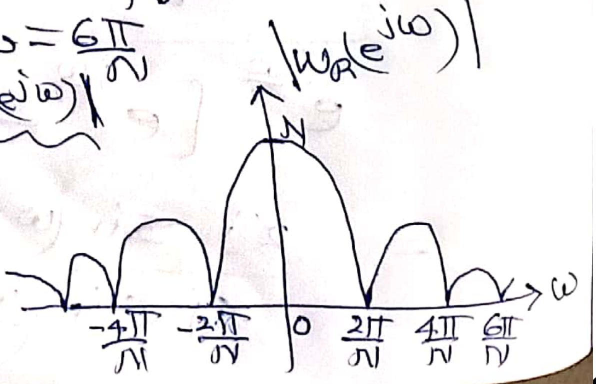
$$\frac{\omega N}{2} = \sin^{-1} 0 = 0, \pi, 2\pi, \ldots$$

$$\omega = \cancel{0}, \frac{2\pi}{N}, \frac{4\pi}{N}, \frac{6\pi}{N}, \ldots$$

2nd Zero cross: $\omega = \frac{4\pi}{N}$

3rd Zero cross: $\omega = \frac{6\pi}{N}$

Magnitude Response $|W_R(e^{j\omega})|$



(13) Find DC gain in dB of frequency response of the following with $N = 7$

(i) Rectangular window

(ii) Hanning window

(iii) Hamming window

= The frequency Response of window

$$W(e^{j\omega}) = FT\{\omega(n)\}$$

$$W(e^{j\omega}) = \sum_{n=0}^{N-1} \omega(n) e^{-j\omega n}$$

where $N \rightarrow$ window length

DC gain $\rightarrow \omega = 0$

$$\therefore W(e^{j\omega}) = \sum_{n=0}^{N-1} \omega(n) e^{-j0n}$$

$$W(e^{j\omega}) = \sum_{n=0}^{N-1} \omega(n)$$

DC gain in dB

$$\boxed{20 \log [W(e^{j\omega})] = 20 \log \sum_{n=0}^{N-1} \omega(n)}$$

(i) Rectangular window :-

$$\omega_R(n) = \begin{cases} 1, & 0 \le n \le N-1 \\ 0, & \text{ow} \end{cases}$$

$$20 \log [W(e^{j\omega})] = 20 \log \sum_{n=0}^{N-1} (1)$$

$$\boxed{20 \log [W(e^{j\omega})] = 20 \log N}$$

$$= 20 \log 7$$

$$= 16.901$$

(ii) Hanning window $\omega_{Han}(n)$

$$\omega_{Han}(n) = \begin{cases} \left(0.5 - 0.5\cos\left(\frac{2\pi n}{N-1}\right)\right) , & 0 \leq n \leq N-1 \\ 0 , & o.w. \end{cases}$$

$$20\log\left[W_{Han}(e^{j\omega})\right] = 20\log\sum_{n=0}^{N-1}\left[0.5 - 0.5\cos\left(\frac{2\pi n}{N-1}\right)\right]$$

$$= 20\log\sum_{n=0}^{N-1}0.5 - 20\log\sum_{n=0}^{N-1}0.5\cos\left(\frac{2\pi n}{N-1}\right)$$

$$= 20\log\left[0.5\sum_{n=0}^{N-1}1\right] - 20\log\left[0.5\sum_{n=0}^{N-1}\left(\frac{e^{\frac{j2\pi n}{N-1}} + e^{\frac{-j2\pi n}{N-1}}}{2}\right)\right]$$

$$= 20\log(0.5N) - 20\log\left[\frac{1}{4}\left(\sum_{n=0}^{N-1}\left(e^{\frac{j2\pi}{N-1}}\right)^n + \sum_{n=0}^{N-1}\left(e^{\frac{-j2\pi}{N-1}}\right)^n\right)\right]$$

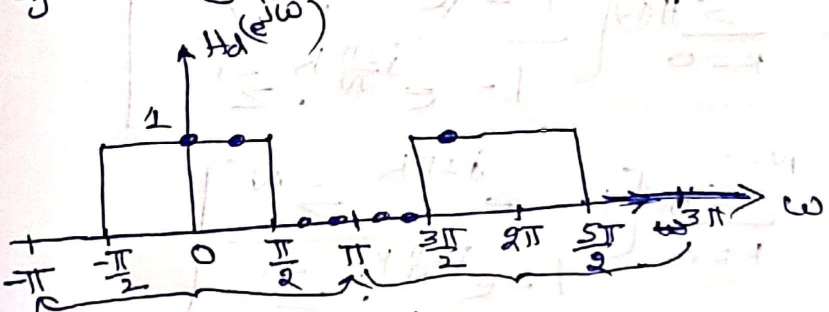
$$= 20\log(0.5N)$$

FIR Filters design by Frequency Sampling Technique

$$H_d(e^{j\omega}) \xrightarrow[\omega=\frac{2\pi}{N}K]{\text{Sampling}} H(K) \xrightarrow{\text{IDFT}} h(n) \xrightarrow{\text{ZT}} H(z)$$

$\uparrow$ Continous $\qquad$ $\uparrow$ Discrete

Consider Frequency response $H_d(e^{j\omega})$ of one cycle as shown in the Figure



$$H_d(e^{j\omega}) = \begin{cases} e^{-j3\omega}, & 0 \le |\omega| \le \frac{\pi}{2} \\ 0, & \frac{\pi}{2} \le |\omega| \le \pi \end{cases}$$

$\alpha = \frac{N-1}{2} \implies N=7$

Sample $H_d(e^{j\omega})$ to obtain $H(K)$

$$H_d(e^{j\omega}) = \sum_{n=0}^{N-1} h(n)\, e^{-j\omega n}$$

Sample $\omega = \frac{2\pi}{N} K$

$$H_d\left(e^{j\frac{2\pi}{N}K}\right) = \sum_{n=0}^{N-1} h(n)\, e^{-j\frac{2\pi}{N}Kn}$$

DFT $\boxed{H(K) = \sum_{n=0}^{N-1} h(n)\, e^{-j\frac{2\pi}{N}Kn}}$

Take IDFT

$$\boxed{h(n) = \frac{1}{N} \sum_{K=0}^{N-1} H(K) \cdot e^{j\frac{2\pi}{N}Kn}}$$

Take ZT

$$H(z) = \sum_{n=0}^{N-1} h(n)\, z^{-n}$$

$$= \sum_{n=0}^{N-1} \left[\frac{1}{N} \sum_{k=0}^{N-1} H(k)\, e^{j\frac{2\pi}{N}kn}\right] \cdot z^{-n}$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} H(k) \sum_{n=0}^{N-1} e^{j\frac{2\pi}{N}kn} \cdot z^{-n}$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} H(k) \sum_{n=0}^{N-1} \left(e^{j\frac{2\pi}{N}k} \cdot z^{-1}\right)^{n}$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} H(k) \left[\frac{1-\left(e^{j\frac{2\pi}{N}k} \cdot z^{-1}\right)^{N}}{1-e^{j\frac{2\pi}{N}k} \cdot z^{-1}}\right]$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} H(k) \left[\frac{1-e^{j2\pi k} \cdot z^{-N}}{1-e^{j\frac{2\pi}{N}k} \cdot z^{-1}}\right]$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} H(k) \left[\frac{1-z^{-N}}{1-e^{j\frac{2\pi}{N}k} z^{-1}}\right]$$

$$\boxed{H(z) = \left(\frac{1-z^{-N}}{N}\right) \sum_{k=0}^{N-1} \left[\frac{H(k)}{1-e^{j\frac{2\pi}{N}k} \cdot z^{-1}}\right]}$$

) Design FIR Filter using frequency sampling technique for the low pass filter with a desired frequency response

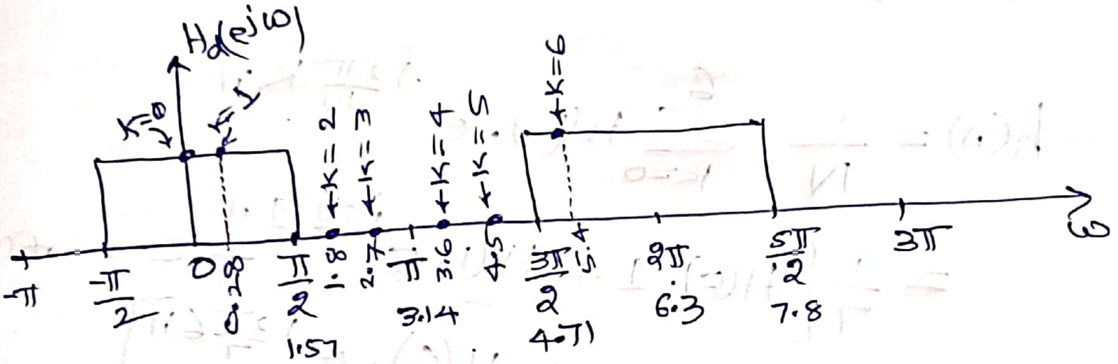
$$H_d(e^{j\omega}) = \begin{cases} e^{-j3\omega} , & 0 \le |\omega| \le \frac{\pi}{2} \\ 0 , & \frac{\pi}{2} \le |\omega| \le \pi \end{cases}$$

= $h(n) = ?$ $H(K) = ?$ $\alpha = 3 = \frac{N-1}{2}$

$\therefore N = 7$

$H(K) = H_d(e^{j\omega})\Big|_{\omega = \frac{2\pi}{N}K}$, $K = 0$ to 6

$h(n) = IDFT[H(K)]$



Magnitudes of Frequency Sampling

$\omega = \frac{2\pi}{N}K = \frac{2\pi}{7}K$, $K = 0$ to 6.

$K = 0$, $\omega = 0$, $H_d(e^{j\omega}) = 1 \cdot e^{-j3\omega}$

$K = 1$, $\omega = \frac{2\pi}{7} \times 1 = 0.28$, $H_d(e^{j\omega}) = 1 \cdot e^{-j3\omega}$

$K = 2$, $\omega = \frac{2\pi}{7} \times 2 = 1.8$, $H_d(e^{j\omega}) = 0$

$K = 3$, $\omega = \frac{2\pi}{7} \times 3 = 2.7$, $H_d(e^{j\omega}) = 0$

$K = 4$, $\omega = \frac{2\pi}{7} \times 4 = 3.6$, $H_d(e^{j\omega}) = 0$

$K = 5$, $\omega = \frac{2\pi}{7} \times 5 = 4.5$, $H_d(e^{j\omega}) = 0$

$K = 6$. $\omega = \frac{2\pi}{7} \times 6 = 5.4$, $H_d(e^{j\omega}) = 1 \cdot e^{-j3\omega}$

$$H(k) = H_d(e^{j\omega})\Big|_{\omega = \frac{2\pi}{N}\times k = \frac{2\pi}{7}k} = 1 e^{-j3\times\frac{2\pi}{7}k}$$

$k=0,\quad H(0) = 1\cdot e^{-j3\frac{2\pi}{7}\times 0} = 1$

$k=1,\quad H(1) = 1\cdot e^{-j3\frac{2\pi}{7}1}$

$k=2,\quad H(2) = 0$

$k=3,\quad H(3) = 0$

$k=4,\quad H(4) = 0$

$k=5,\quad H(5) = 0$

$k=6,\quad H(6) = 1\cdot e^{-j3\frac{2\pi}{7}\times 6}$

$$h(n) = \frac{1}{N}\sum_{k=0}^{6} H(k)\cdot e^{j\frac{2\pi}{N}kn}$$

$$= \frac{1}{7}\Big[H(0)\cdot 1 + H(1)\, e^{j\frac{2\pi}{7}\cdot 1n} + 0 + 0 + 0 + 0 + H(6)\cdot e^{j\frac{2\pi}{7}6n}\Big]$$

$$= \frac{1}{7}\Big[1 + e^{-j3\times\frac{2\pi}{7}}\cdot e^{j\frac{2\pi}{7}n} + e^{-j3\times\frac{2\pi}{7}6}\cdot e^{j\frac{2\pi}{7}6n}\Big]$$

$$= \frac{1}{7}\Big[1 + e^{j\frac{2\pi}{7}(n-3)} + e^{j\frac{2\pi}{7}\overset{(7-1)}{6}(n-3)}\Big]$$

$$= \frac{1}{7}\Big[1 + e^{j\frac{2\pi}{7}(n-3)} + e^{j\frac{2\pi}{7}\cancel{7}(n-3)}\cdot e^{j\frac{2\pi}{7}(-1)(n-3)}\Big]$$

$$= \frac{1}{7}\Big[1 + e^{j\frac{2\pi}{7}(n-3)} + e^{-j\frac{2\pi}{7}(n-3)}\Big]$$

$$\boxed{h(n) = \frac{1}{7}\Big[1 + 2\cos\Big(\frac{2\pi}{7}(n-3)\Big)\Big]}\qquad n = 0 \text{ to } 6$$

⑤

$n=0$, $h(0) = -0.11$

$n=1$, $h(1) = 0.08$

$n=2$, $h(2) = 0.32$

$n=3$, $h(3) = 0.43$

$n=4$, $h(4) = 0.32$

$n=5$, $h(5) = 0.08$

$n=6$, $h(6) = -0.11$

$h(n) = [-0.11, 0.08, 0.32, 0.43, 0.32, 0.08, -0.11]$

linear phase

(2) Design FIR bandpass filter with the following specifications for $N = 7$

Sampling frequency = 8 KHz

cutoff frequencies $f_{c_1} = 1$ KHz

& $f_{c_2} = 3$ KHz

= $h(n) = ?$ $n = 0$ to 6

$H(k) = ?$

$\omega_{c_1} = ? = \Omega_{c_1} T$, $\omega_{c_2} = ? = \Omega_{c_2} T$

$T = \frac{1}{F} = \frac{1}{8000}$ Sec.

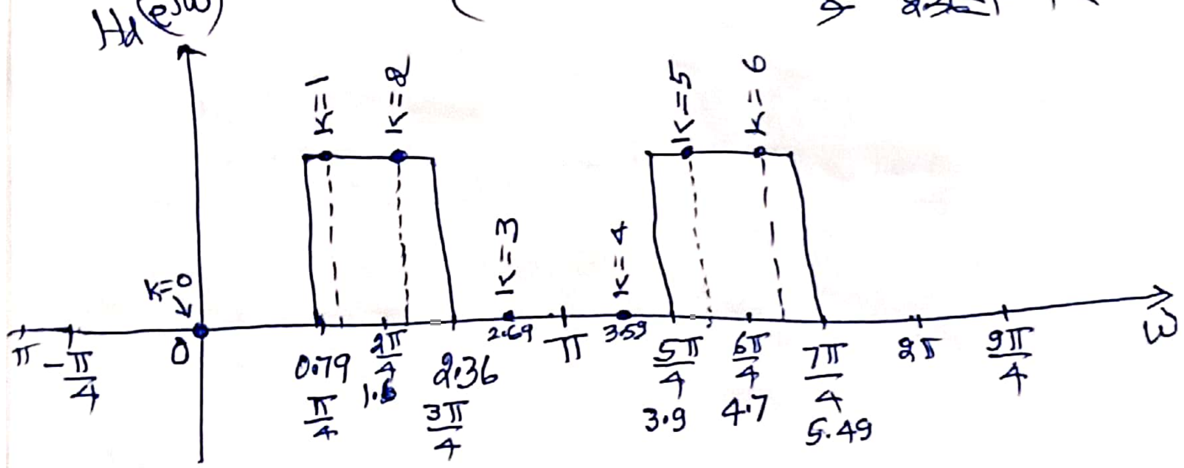
$\Omega_{c_1} = 2\pi f_{c_1} = 2\pi \times 1000 = 2000\pi$ rad/sec.

$\Omega_{c_2} = 2\pi f_{c_2} =$ 6000π rad/sec.

$\omega_{c_1} = \Omega_{c_1} T = \frac{\cancel{2000}\pi}{\cancel{8000}_4} = 0.79$ rad

$\omega_{c_2} = \Omega_{c_2} T = \frac{\cancel{6000}\pi}{\cancel{8000}} = 2.36$ rad

$$H_d(e^{j\omega}) = \begin{cases} 1 \cdot e^{-j3\omega} , & 0.79 \leq |\omega| \leq 2.36 \\ 0 , & 0 \leq |\omega| \leq 0.79 \\ & \& \ 2.36 \leq |\omega| \leq \pi \end{cases}$$



magnitudes of frequency sampling

~~k=0~~ $\omega = \frac{2\pi}{N}k = \frac{2\pi}{7}k.$

$k=0,\quad \omega = 0,\quad H_d(e^{j\omega}) = 0$

$k=1,\quad \omega = 0.89,\quad H_d(e^{j\omega}) = 1\cdot e^{-j3\omega}$

$k=2,\quad \omega = 1.79,\quad H_d(e^{j\omega}) = 1\cdot e^{-j3\omega}$

$k=3,\quad \omega = 2.69,\quad H_d(e^{j\omega}) = 0$

$k=4,\quad \omega = 3.59,\quad H_d(e^{j\omega}) = 0$

$k=5,\quad \omega = 4.48,\quad H_d(e^{j\omega}) = 1\cdot e^{-j3\omega}$

$k=6,\quad \omega = 5.38,\quad H_d(e^{j\omega}) = 1\cdot e^{-j3\omega}$

$$H(k) = H_d(e^{j\omega})\Big|_{\omega = \frac{2\pi}{N}k = \frac{2\pi}{7}k} = e^{-j3\times\frac{2\pi}{7}k}$$

$k=0,\quad H(0) = 0$

$k=1,\quad H(1) = 1\cdot e^{-j3\times\frac{2\pi}{7}\times 1}$

$k=2,\quad H(2) = 1\cdot e^{-j3\times\frac{2\pi}{7}\times 2}$

$k=3,\quad H(3) = 0$

$k=4,\quad H(4) = 0$

$k=5,\quad H(5) = 1\cdot e^{-j3\times\frac{2\pi}{7}\times 5}$

$k=6,\quad H(6) = 1\cdot e^{-j3\times\frac{2\pi}{7}\times 6}$

$$h(n) = IDFT[H(k)] = \frac{1}{N}\sum_{k=0}^{6} H(k)\, e^{j\frac{2\pi}{N}kn}$$

$$h(n) = \frac{1}{7}\sum_{k=0}^{6} H(k)\, e^{j\frac{2\pi}{7}kn}$$

$$= \frac{1}{7}\Big[0 + H(1)e^{j\frac{2\pi}{7}1\times n} + H(2)e^{+j\frac{2\pi}{7}\times 2n} + 0 + 0 + H(5)e^{j\frac{2\pi}{7}5n} + H(6)e^{j\frac{2\pi}{7}6n}\Big]$$

$$= \frac{1}{7}\left[e^{-j\,3\times\frac{2\pi}{7}} \cdot e^{j\frac{2\pi}{7}n} + e^{-j\,3\times\frac{2\pi}{7}\times 2} \cdot e^{j\frac{2\pi}{7}2n} \right.$$

$$\left. + e^{-j\,3\times\frac{2\pi}{7}\times 5} \cdot e^{j\frac{2\pi}{7}5n} + e^{-j\,3\times\frac{2\pi}{7}\times 6} \cdot e^{j\frac{2\pi}{7}6n} \right]$$

$$= \frac{1}{7}\left[e^{j\frac{2\pi}{7}(n-3)} + e^{j\frac{2\pi}{7}2(n-3)} + e^{j\frac{2\pi}{7}\overset{(7-2)}{5}(n-3)} + e^{j\frac{2\pi}{7}\overset{(7-1)}{6}(n-3)} \right]$$

$$= \frac{1}{7}\left[e^{j\frac{2\pi}{7}(n-3)} + e^{j\frac{2\pi}{7}2(n-3)} + e^{j\frac{2\pi}{7}(7-2)(n-3)} \right.$$

$$\left. + e^{j\frac{2\pi}{7}(7-1)(n-3)} \right]$$

$$= \frac{1}{7}\left[e^{j\frac{2\pi}{7}(n-3)} + e^{j\frac{2\pi}{7}2(n-3)} + e^{j\frac{2\pi}{7}(-2)(n-3)} \right.$$

$$\left. + e^{j\frac{2\pi}{7}(-1)(n-3)} \right]$$

$$= \frac{1}{7}\left[e^{j\frac{2\pi}{7}(n-3)} + e^{j\frac{2\pi}{7}2(n-3)} + e^{-j\frac{2\pi}{7}2(n-3)} \right.$$

$$\left. + e^{-j\frac{2\pi}{7}(n-3)} \right]$$

$$= \frac{1}{7}\left[2\cos\left[\frac{2\pi}{7}(n-3)\right] + 2\cos\left(\frac{2\pi}{7}2(n-3)\right) \right]$$

$$\boxed{h(n) = \frac{1}{7}\left[2\cos\left(\frac{2\pi}{7}(n-3)\right) + 2\cos\left(\frac{4\pi}{7}(n-3)\right) \right]}$$

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$n=0,\ h(0) = -0.08 = h(6)$

$n=1,\ h(1) = -0.321 = h(5)$

$n=2,\ h(2) = 0.11 = h(4)$

$n=3,\ h(3) = 0.57$

$$h(n) = [-0.08, -0.321, 0.11, 0.57, 0.11, -0.321, -0.08]$$

(3) Design band reject FIR filter using frequency sampling technique for the following specifications

Sampling frequency $F = 10\ KHz$

cutoff frequencies $f_{c1} = 2\ KHz$

& $f_{c2} = 4\ KHz$

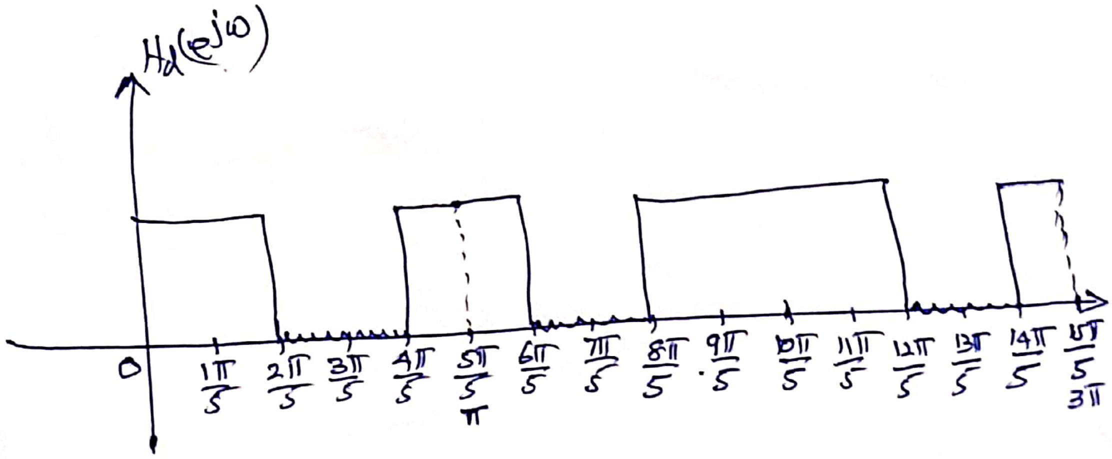
= $T = \frac{1}{F} = \frac{1}{10,000}$ sec.

$\Omega_{c1} = 2\pi f_{c1} = 4000\pi$ rad/sec.

$\Omega_{c2} = 2\pi f_{c2} = 8000\pi$ rad/sec.

$w_{c1} = \Omega_{c1} T = \frac{4000\pi}{10000} = \frac{2\pi}{5}$ 1.26 rad/sec

$w_{c2} = \Omega_{c2} T = \frac{8000\pi}{10,000} = \frac{4\pi}{5}$ 2.51 rad/sec.



FIR- Filter Structures

Realized in two forms (i) Direct form structures and (ii) linear phase structures

1) Realize the FIR filter with impulse response

$$h(n) = \delta(n) + \frac{1}{2}\delta(n-1) - \frac{1}{4}\delta(n-2) + \frac{1}{2}\delta(n-3) + \delta(n-4)$$

as (i) direct form (ii) linear phase structure.

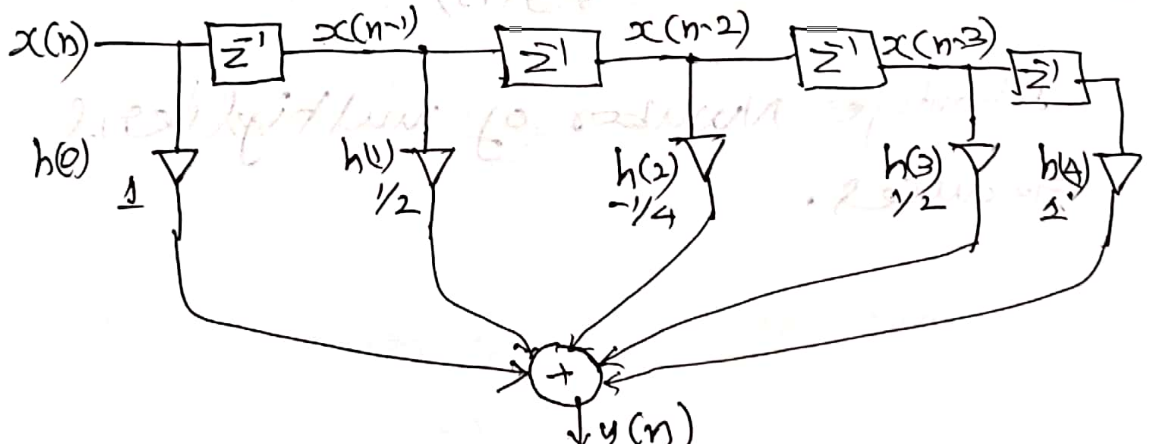
= (i) Direct Form :- $N = 5$

$h(0) = 1, \quad h(1) = \frac{1}{2}, \quad h(2) = -\frac{1}{4}, \quad h(3) = \frac{1}{2}$

$h(4) = 1.$

FIR Filter output is $y(n) = \sum_{k=0}^{N-1} h(k)\, x(n-k).$

$$y(n) = h(0)x(n) + h(1)x(n-1) + h(2)x(n-2) + h(3)x(n-3) + h(4)x(n-4) \quad \text{--- (1)}$$



(ii) Linear phase structure :-

$$h(n) = h(N-1-n)$$

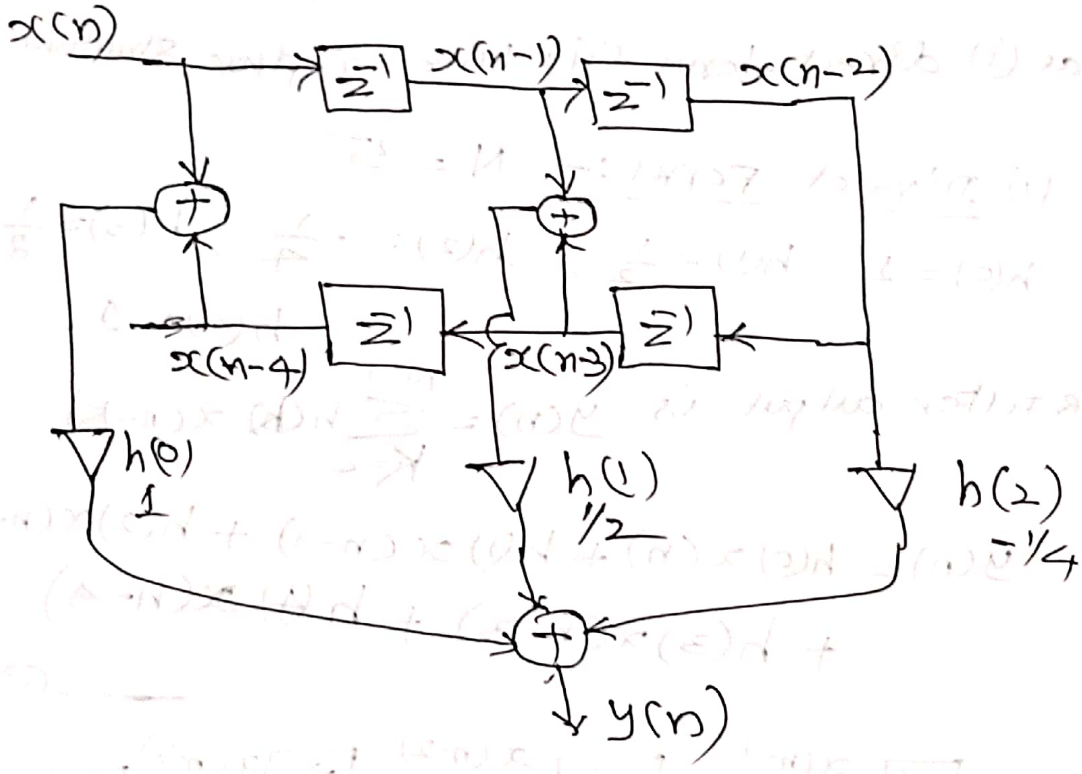
$$\left.\begin{aligned} n=0, \quad & h(0) = h(4) = 1 \\ n=1, \quad & h(1) = h(3) = \tfrac{1}{2} \\ n=2, \quad & h(2) = h(2) = -\tfrac{1}{4} \end{aligned}\right\} \text{--- (2)}$$

From Equation (1) & (2)

FIR Filter output is

$$y(n) = h(0)\,x(n) + h(1)\,x(n-1) + h(2)\,x(n-2) + h(1)\,x(n-3) + h(0)\,x(n-4)$$

$$y(n) = h(0)\left[x(n) + x(n-4)\right] + h(1)\left[x(n-1) + x(n-3)\right] + h(2)\,x(n-2)$$



Advantage Number of multipliers reduces.

2) Realize FIR filter using direct form and linear phase structures

$$h(n) = \delta(n) - \frac{1}{2}\delta(n-1) + \frac{1}{4}\delta(n-2) + \frac{1}{4}\delta(n-3) - \frac{1}{2}\delta(n-4) + \delta(n-5)$$

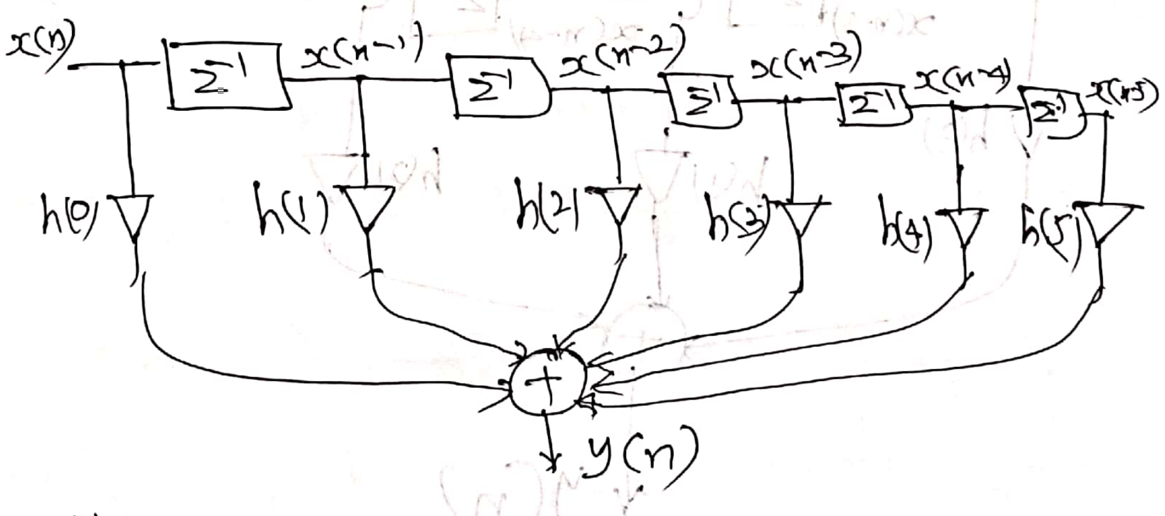
= (i) Direct form :— FIR filter output

$$y(n) = \sum_{k=0}^{5} h(k)\, x(n-k) \quad , \quad N=6$$

$$y(n) = h(0)\,x(n) + h(1)\,x(n-1) + h(2)\,x(n-2) + h(3)\,x(n-3) + h(4)\,x(n-4) + h(5)\,x(n-5) \quad (1)$$

$h(0) = 1$, $h(1) = -1/2$, $h(2) = \frac{1}{4}$, $h(3) = \frac{1}{4}$

$h(4) = -\frac{1}{2}$, $h(5) = 1$



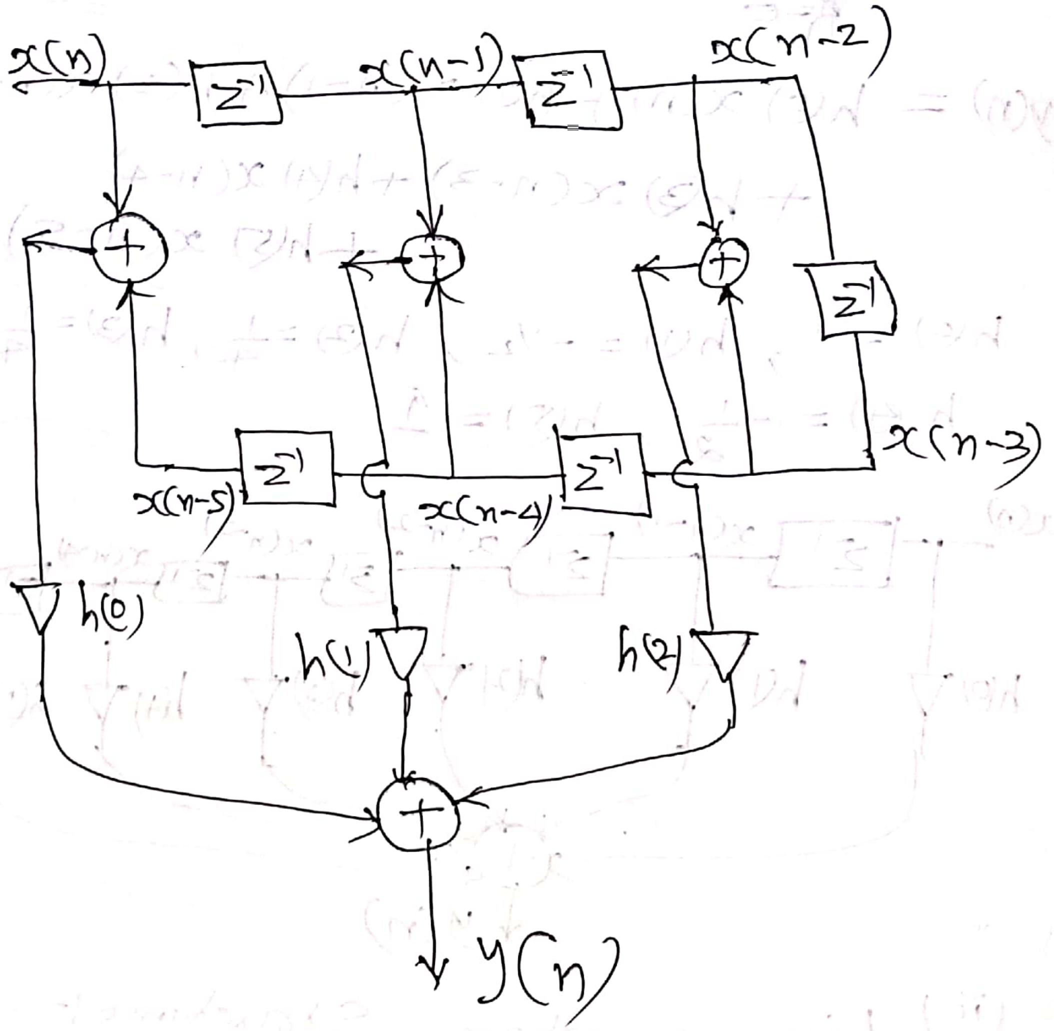
(ii) Linear phase Structure :—

$$\left.\begin{aligned} h(n) &= h(N-1-n) \\ h(0) &= h(5) = 1 \\ h(1) &= h(4) = -1/2 \\ h(2) &= h(3) = 1/4 \end{aligned}\right\} \longleftarrow (2)$$

From Equations 1 & 2,

$$y(n) = h(0)\,x(n) + h(1)\,x(n-1) + h(2)\,x(n-2) + h(2)\,x(n-3) + h(1)\,x(n-4) + h(0)\,x(n-5)$$

$$= h(0)\left[x(n) + x(n-5)\right] + h(1)\left[x(n-1) + x(n-4)\right] + h(2)\left[x(n-2) + x(n-3)\right]$$



③ Realize FIR Filter in direct and linear phase forms

$$H(z) = 1 + \frac{3}{4} z^{-1} + \frac{17}{8} z^{-2} + \frac{3}{4} z^{-3} + z^{-4}$$

= Take IZT

$$h(n) = \delta(n) + \frac{3}{4}\delta(n-1) + \frac{17}{8}\delta(n-2) + \frac{3}{4}\delta(n-3) + \delta(n-4)$$

$N = 5$

$4\delta(n) \xleftrightarrow{ZT} 4 \times 1$

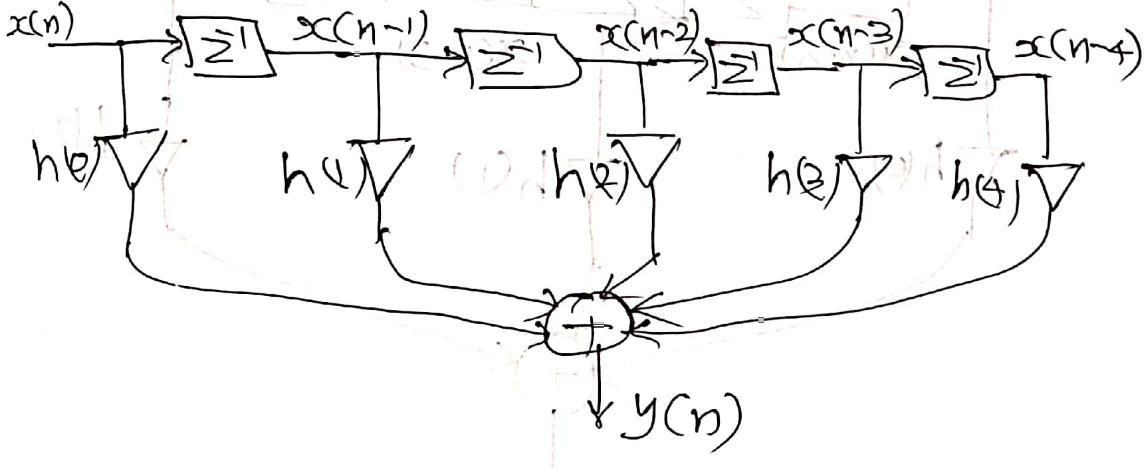
$4\,\delta(n-1) \xleftrightarrow{ZT} 4 \cdot z^{-1}$

(i) Direct Form :-

$$y(n) = \sum_{k=0}^{4} h(k)\, x(n-k)$$

$$y(n) = h(0)\,x(n) + h(1)\,x(n-1) + h(2)\,x(n-2) + h(3)\,x(n-3) + h(4)\,x(n-4)$$

$h(0) = 1$, $h(1) = \frac{3}{4}$, $h(2) = \frac{17}{8}$, $h(3) = \frac{3}{4}$, $h(4) = 1$



(ii) Linear phase structure:-

$h(n) = h(N-1-n)$

$h(0) = h(4)$

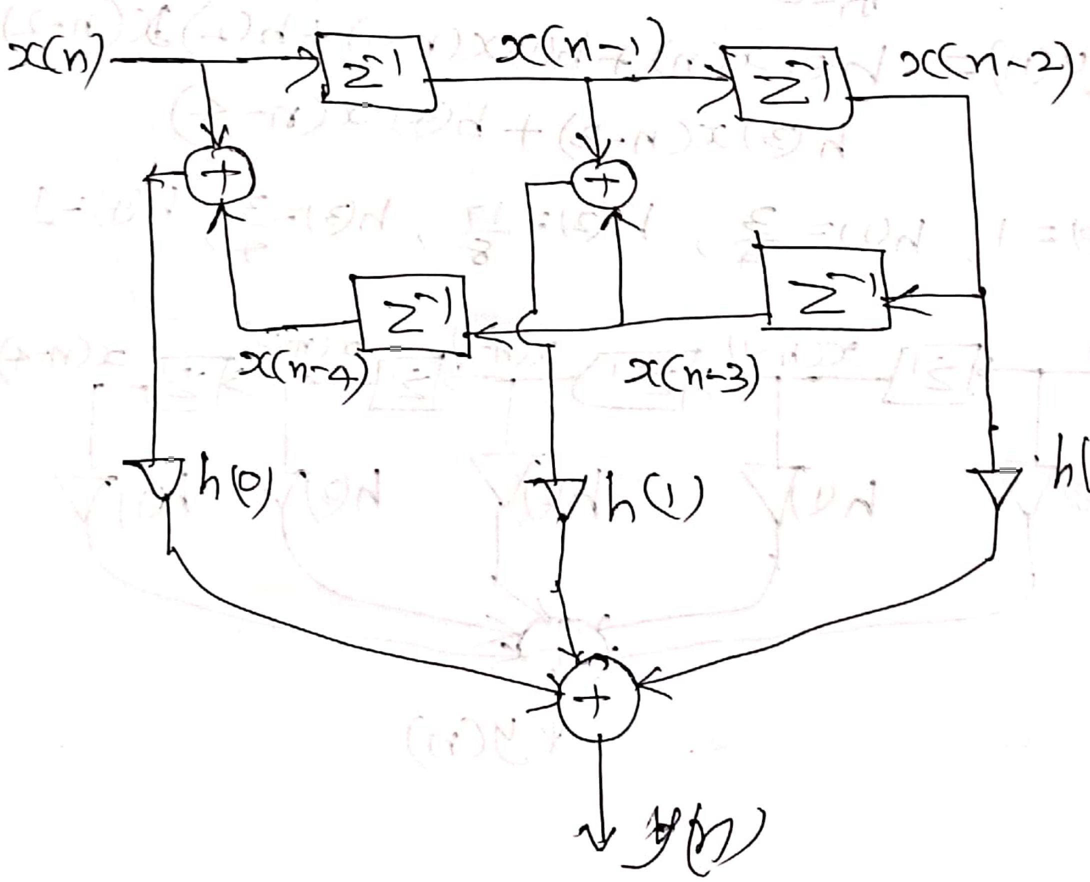
$h(1) = h(3)$

$h(2) = h(2)$

$$y(n) = h(0)x(n) + h(1)x(n-1) + h(2)x(n-2) + h(3)x(n-3) + h(4)x(n-4)$$

$$= h(0)x(n) + h(1)x(n-1) + h(2)x(n-2) + h(1)x(n-3) + h(0)x(n-4)$$

$$= h(0)[x(n)+x(n-4)] + h(1)[x(n-1)+x(n-3)] + h(2)x(n-2)$$



4 Realize FIR filter
$H(z) = 1 - 3z^{-1} + 2z^{-2} - 3z^{-3} + z^{-4}$
by direct and linear form structure.

(5) Realize FIR filter in direct and linear phase structure for

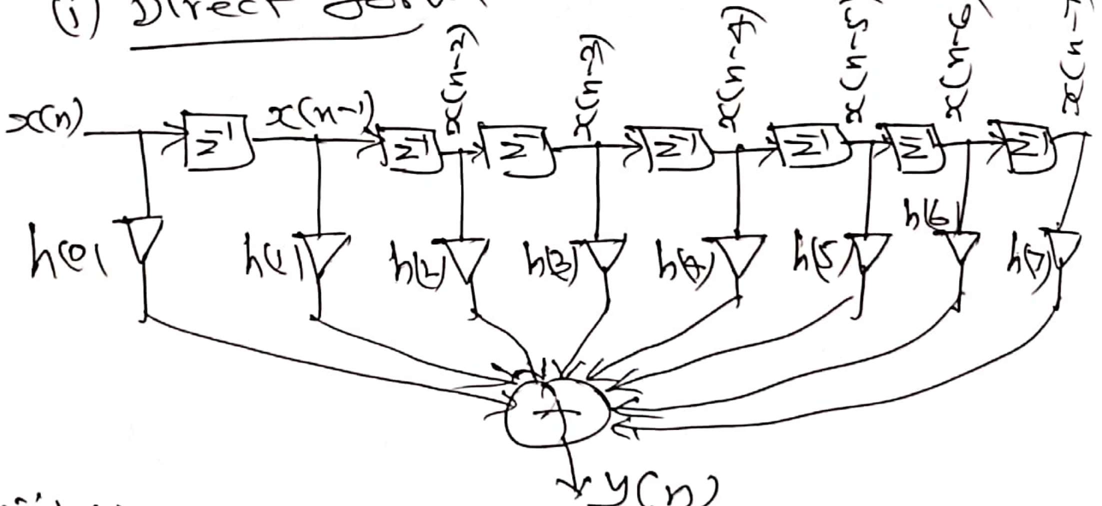
$$h(n) = \begin{cases} 2^{n}, & 0 \le n \le 3 \\ 2^{7-n}, & 4 \le n \le 7 \end{cases}$$

$$= \quad h(n) = [2^{0}, 2^{1}, 2^{2}, 2^{3}, 2^{3}, 2^{2}, 2^{1}, 2^{0}]$$

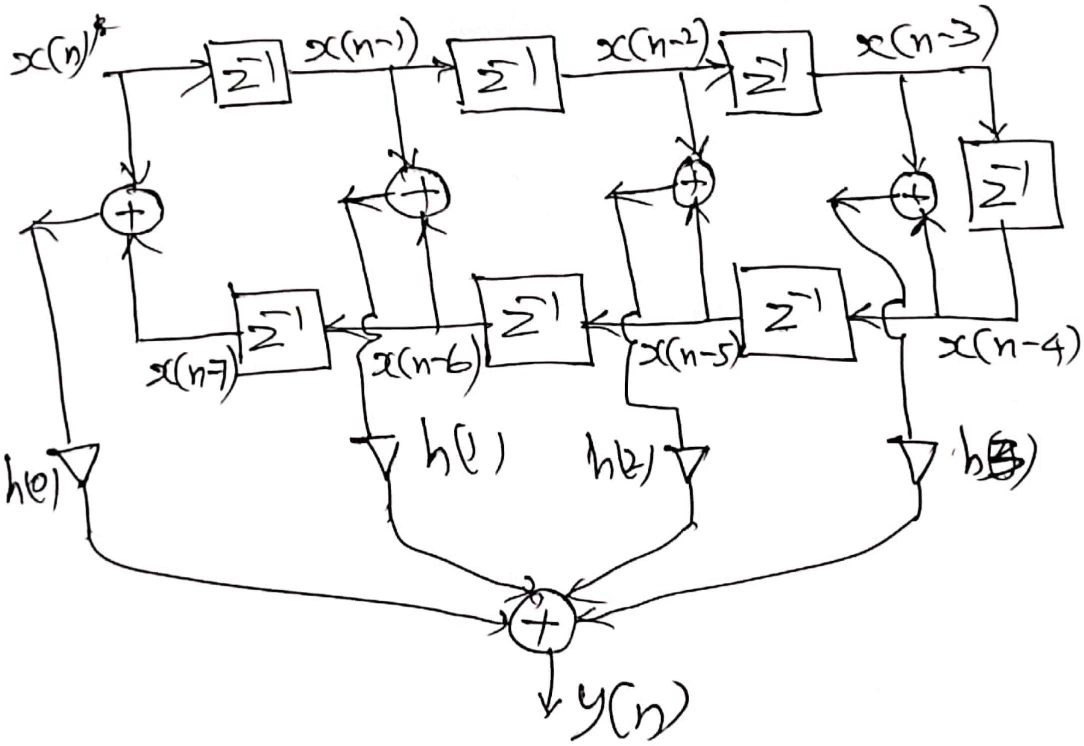
$$h(n) = [1, 2, 4, 8, 8, 4, 2, 1]$$

$$N = 8$$

(i) Direct form:-



(ii) Linear Form structure:-



(6) Realize the FIR filter

$$H(z) = 1 + \frac{3}{4} z^{-1} + \frac{17}{8} z^{-2} + \frac{3}{4} z^{-3} + z^{-4} \quad \text{—— (A)}$$

in direct form and cascade form.
and also linear form.

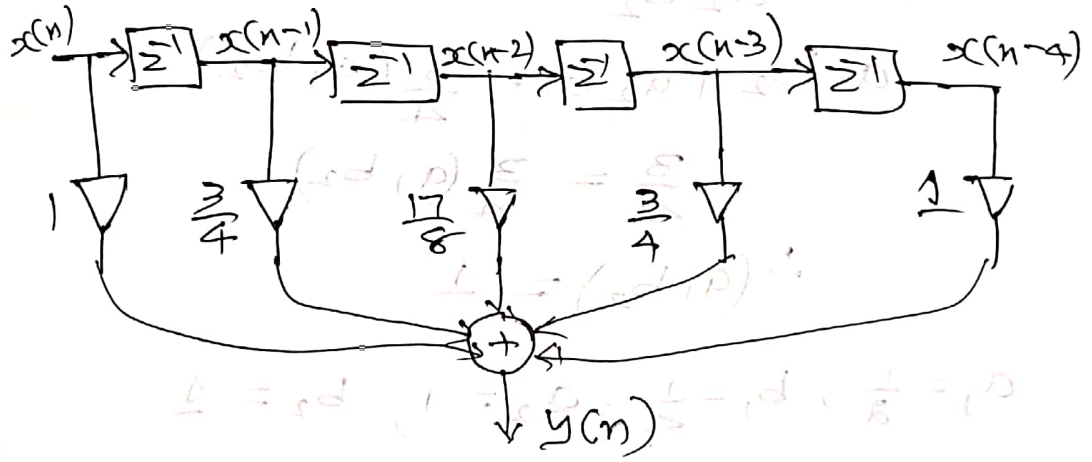
= (i) Direct form :-

$$\frac{Y(z)}{X(z)} = 1 + \frac{3}{4} z^{-1} + \frac{17}{8} z^{-2} + \frac{3}{4} z^{-3} + z^{-4}$$

$$Y(z) = X(z) + \frac{3}{4} z^{-1} X(z) + \frac{17}{8} z^{-2} X(z) + \frac{3}{4} z^{-3} X(z) + z^{-4} X(z)$$

Take IZT

$$y(n) = x(n) + \frac{3}{4} x(n-1) + \frac{17}{8} x(n-2) + \frac{3}{4} x(n-3) + x(n-4)$$



(ii) Cascade form :-

$$H(z) = H_1(z) \cdot H_2(z)$$

where $H_1(z)$ & $H_2(z)$ are Quadratic sections

Let $H_1(z) = (1 + a_1 z^{-1} + a_2 z^{-2})$

& $H_2(z) = (1 + b_1 z^{-1} + b_2 z^{-2})$

$$\therefore H_1(z) \cdot H_2(z) = (1 + a_1 z^{-1} + a_2 z^{-2})(1 + b_1 z^{-1} + b_2 z^{-2})$$

$$= 1 + b_1 z^{-1} + b_2 z^{-2} + a_1 z^{-1} + a_1 b_1 z^{-2} + a_1 b_2 z^{-3} + a_2 z^{-2} + a_2 b_1 z^{-3} + a_2 b_2 z^{-4}$$

$$\cancel{H(z)=H_1(z)\cdot H_2(z)} = 1 + (a_1+b_1)z^{-1} + (a_2+b_2+a_1b_1)z^{-2} + (a_1b_2+a_2b_1)z^{-3} + a_2b_2 z^{-4} \quad \text{—} (1)$$

compare equations (A) & (1)

$$a_1+b_1 = \frac{3}{4}, \quad a_2+b_2+a_1b_1 = \frac{17}{8}$$

$$a_1b_2+a_2b_1 = \frac{3}{4}, \quad a_2b_2 = 1$$

Solve :-

$$a_1\left(\frac{1}{a_2}\right) + \left(\frac{b_1}{b_2}\right) = \frac{3}{4}$$

$$\frac{a_1b_2+a_2b_1}{a_2b_2} = \frac{3}{4}$$

$$a_1b_2+a_2b_1 = \frac{3}{4}(a_1b_2)$$

$$\frac{3}{4} = \frac{3}{4}(a_1b_2)$$

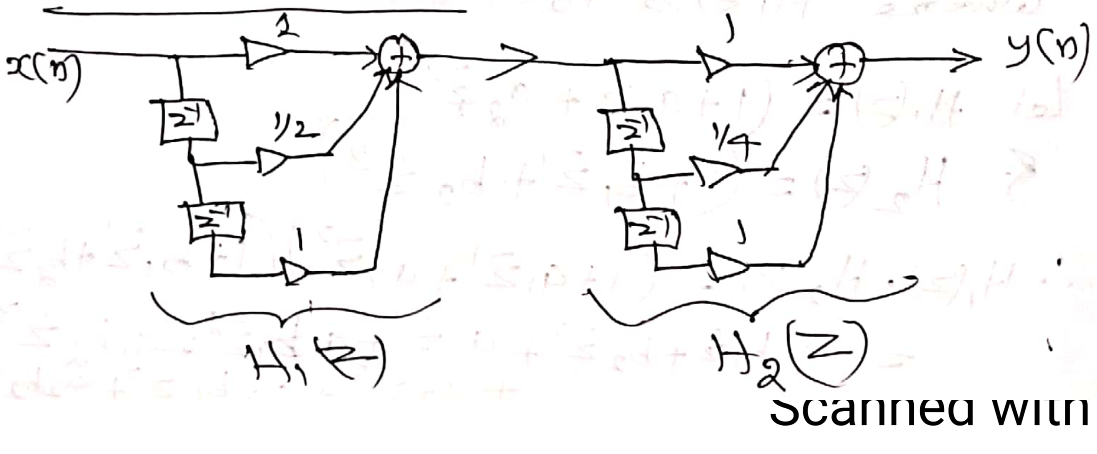
$$\therefore (a_1b_2) = 1$$

$$a_1 = \frac{1}{2}, \quad b_1 = \frac{1}{4}, \quad a_2 = 1, \quad b_2 = 1$$

$$\therefore H_1(z) = \left(1+\frac{1}{2}z^{-1}+z^{-2}\right)$$

$$H_2(z) = \left(1+\frac{1}{4}z^{-1}+z^{-2}\right)$$

Cascade structure :-

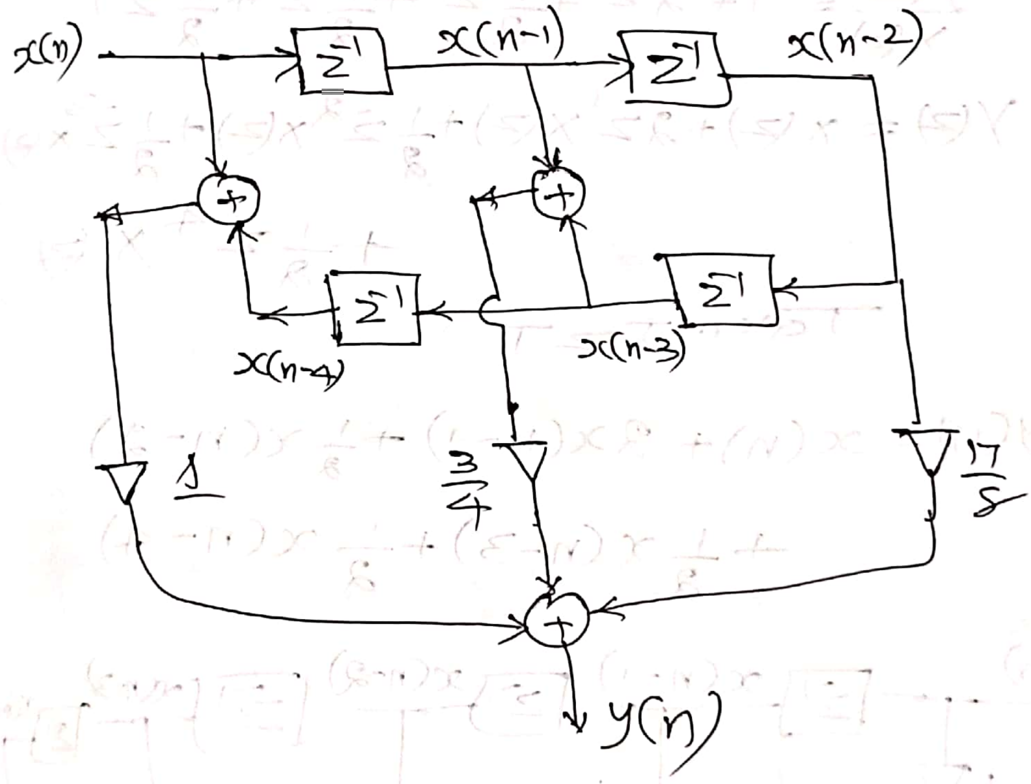


(iii) Linear phase form structure

$$y(n) = x(n) + \frac{3}{4}x(n-1) + \frac{17}{8}x(n-2) + \frac{3}{4}x(n-3) + x(n-4)$$

$$y(n) = 1\left[x(n) + x(n-4)\right] + \frac{3}{4}\left[x(n-1) + x(n-3)\right] + \frac{17}{8}x(n-2)$$

$\boxed{N=5}$



7 Realize FIR filter in direct form and cascade form

$$H(z) = 1 + 2z^{-1} + \frac{1}{2}z^{-2} + \frac{1}{2}z^{-3} + \frac{1}{2}z^{-4} \quad \text{— (A)}$$

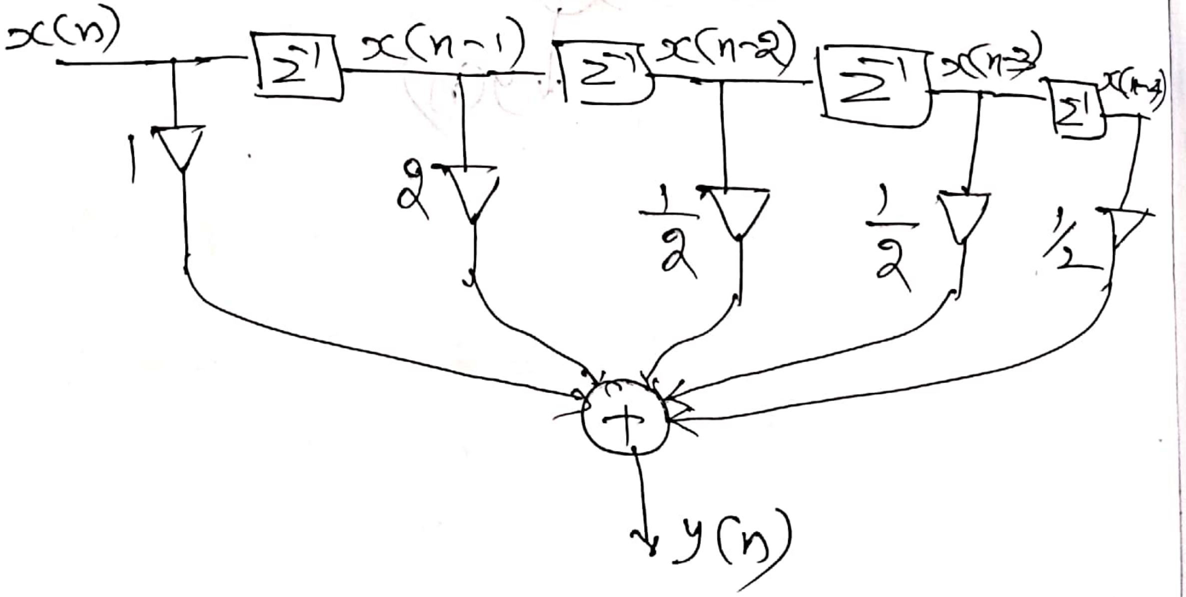
= (i) Direct form :-

$$\frac{Y(z)}{X(z)} = 1 + 2z^{-1} + \frac{1}{2}z^{-2} + \frac{1}{2}z^{-3} + \frac{1}{2}z^{-4}$$

$$Y(z) = X(z) + 2z^{-1}X(z) + \frac{1}{2}z^{-2}X(z) + \frac{1}{2}z^{-3}X(z) + \frac{1}{2}z^{-4}X(z)$$

Take IZT

$$y(n) = x(n) + 2x(n-1) + \frac{1}{2}x(n-2) + \frac{1}{2}x(n-3) + \frac{1}{2}x(n-4)$$



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(ii) cascade Structure:

$$H(z) = H_1(z) \cdot H_2(z)$$

let $H_1(z) = (1 + a_1 z^{-1} + a_2 z^{-2})$

$$H_2(z) = (1 + b_1 z^{-1} + b_2 z^{-2})$$

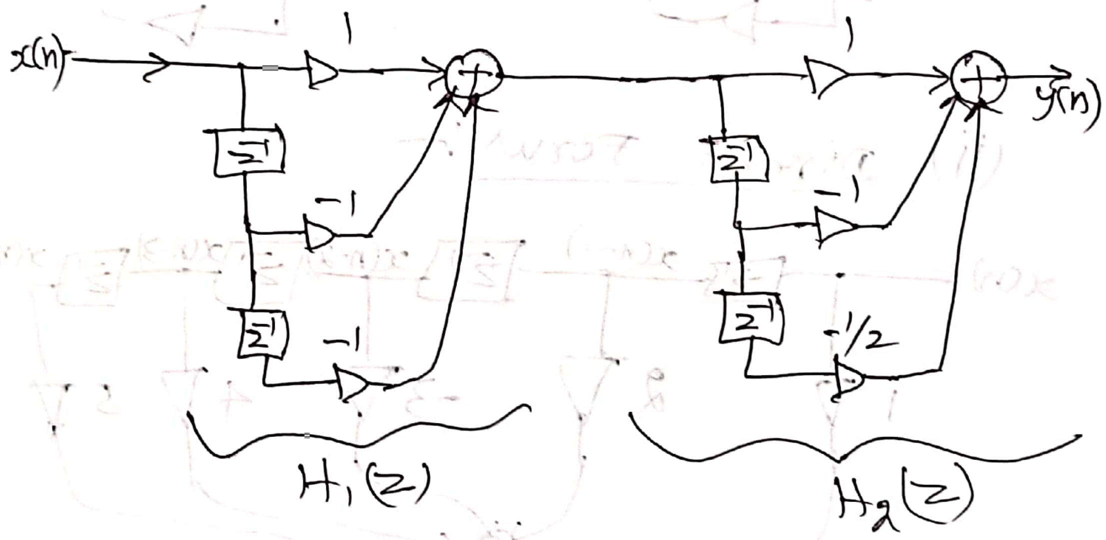
$$H_1(z) \cdot H_2(z) = 1 + (a_1 + b_1) z^{-1} + (a_2 + b_2 + a_1 b_1) z^{-2} + (a_1 b_2 + a_2 b_1) z^{-3} + a_2 b_2 z^{-4} \quad \text{—— (1)}$$

compare (A) & (1)

$a_1 = -1$, $a_2 = -1$, $b_1 = -1$, $b_2 = \frac{1}{2}$

$$H_1(z) = (1 - z^{-1} - z^{-2})$$

$$H_2(z) = (1 - z^{-1} - \tfrac{1}{2} z^{-2})$$



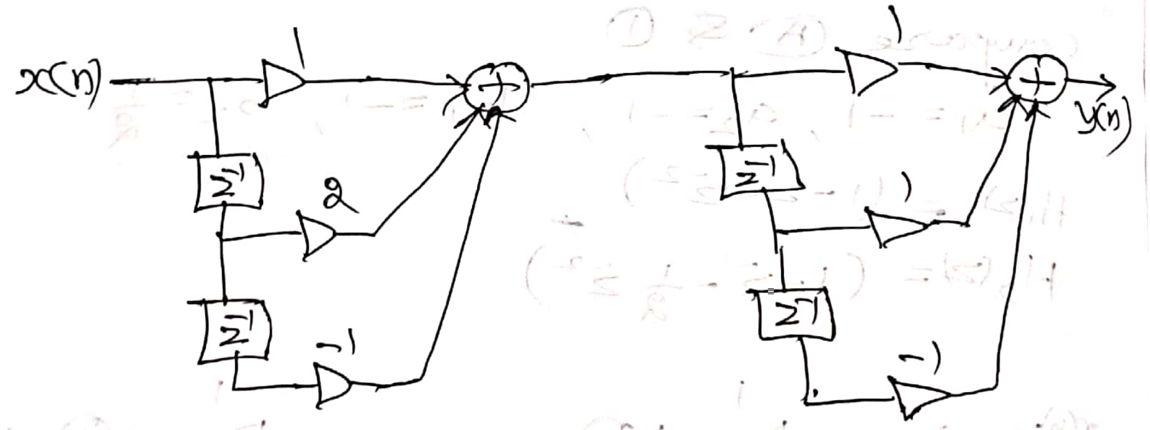
8. (i) ~~obj~~ obtain the cascade realization of FIR filter

$$H(z) = (1 + 2z^{-1} - z^{-2})(1 + z^{-1} - z^{-2})$$

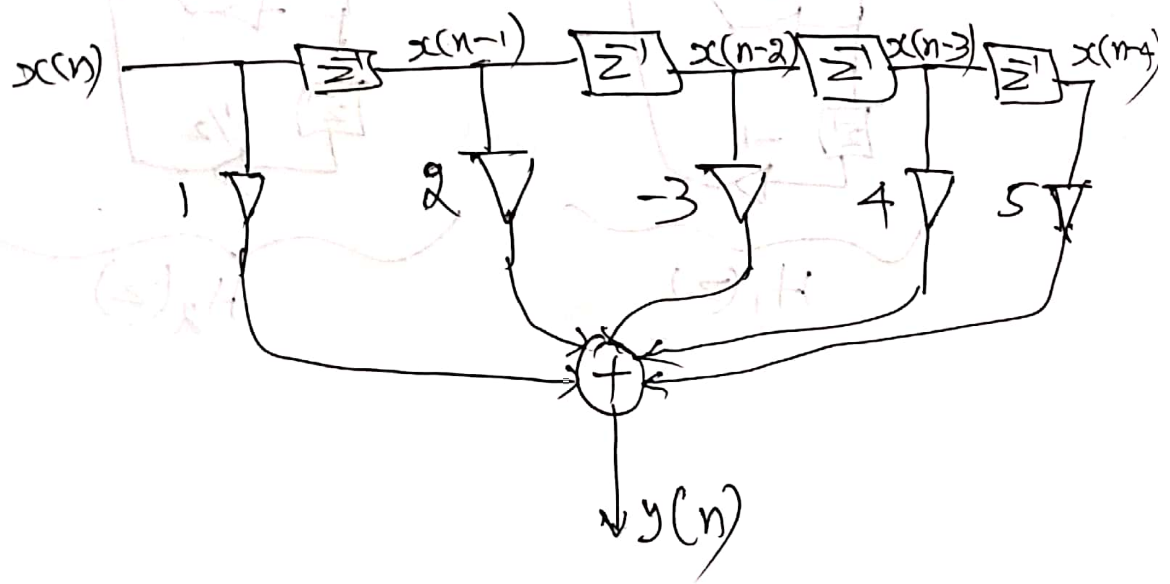
(ii) Determine the DF realization.

$$H(z) = 1 + 2z^{-1} - 3z^{-2} - 4z^{-3} + 5z^{-4}$$

= (i) Cascade Structure :-



(ii) Direct Form :-



(9) Realize a linear phase FIR filter having $h(n) = \delta(n) + \frac{1}{4}\delta(n-1) - \frac{1}{8}\delta(n-2) \cancel{+ \frac{1}{4}\delta(n-3)}$

$$+ \frac{1}{4}\delta(n-3) + \delta(n-4)$$

=

10 Realize the FIR filters

(i) $H(z) = \frac{1}{2} + \frac{z^{-1}}{3} + z^{-2} + \frac{z^{-3}}{4} + z^{-4} + \frac{z^{-5}}{3} + \frac{z^{-6}}{2}$

by DF & linear phase form

(ii) $H(z) = \left(\frac{1}{2} + z^{-1} + \frac{z^{-2}}{2}\right)\left(1 + \frac{z^{-1}}{3} + z^{-2}\right)$

by cascade form

11. Realize a linear phase FIR filter using necessary equations

$$h(n) = \delta(n) + \frac{1}{2}\delta(n-1) - \frac{1}{4}\delta(n-2) + \delta(n-4) + \frac{1}{2}\delta(n-3)$$

= Take ZT $\qquad N = 5$

$$H(z) = 1 + \frac{1}{2}z^{-1} - \frac{1}{4}z^{-2} + \frac{1}{2}z^{-3} + z^{-4}$$

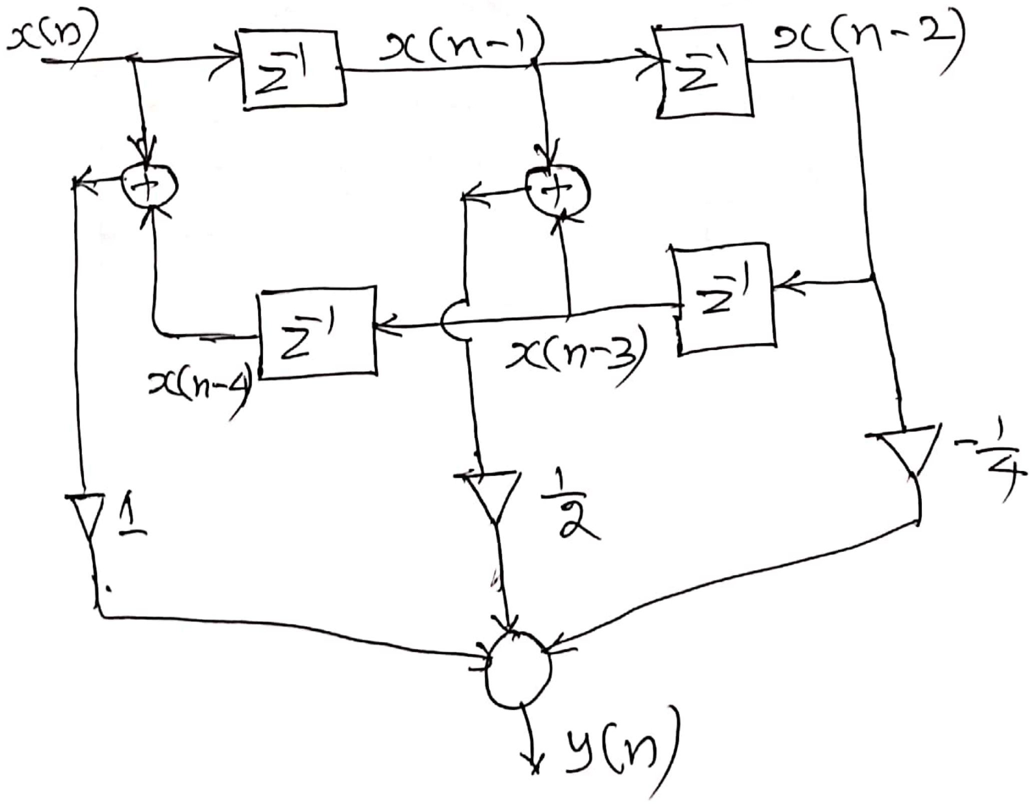
$$\frac{Y(z)}{X(z)} = 1 + \frac{1}{2}z^{-1} - \frac{1}{4}z^{-2} + \frac{1}{2}z^{-3} + z^{-4}$$

$$Y(z) = X(z) + \frac{1}{2}z^{-1}X(z) - \frac{1}{4}z^{-2}X(z) + \frac{1}{2}z^{-3}X(z) + z^{-4}X(z)$$

Take IZT

$$y(n) = x(n) + \frac{1}{2}x(n-1) - \frac{1}{4}x(n-2) + \frac{1}{2}x(n-3) + x(n-4)$$

$$y(n) = 1[x(n) + x(n-4)] + \frac{1}{2}(x(n-1) + x(n-3)) - \frac{1}{4}x(n-2)$$



12 Obtain linear phase FIR filter realization using necessary equations

$$H(z) = \cancel{K + \frac{z}{4}}\; 1 + \frac{1}{4}\left(z^{-1} + z^{-2}\right) + z^{-3}$$

$=$

Comparison of IIR and FIR Filters

SL NO	IIR Filter	FIR Filter
1.	Impulse response h(n) infinite	h(n) Finite.
2	consists of zeros & poles	consists ~~of~~ of only zeros → no poles
3.	stability is not assured	Always stable
4	Recursive system	Non Recursive System
5	Non linear phase response	linear phase response for symmetric h(n)
6.	Require lower order filter for specific task	Requir higher order filter for similar task
7	Less memory Required	More memory required