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Chapter 1
Introduction to Signals

A signal is a physical quantity that conveys information and varies with time, space or any other independent variables.

Examples:-

1. one dimensional signal :-

The physical quantity of a signal depends on only one independent variable (time). The speech signal is an example, as shown in Figure 1. The amplitude of the signal varies with one independent variable time

$x(n) = 5n \quad , \quad 0 \leq n \leq 5$

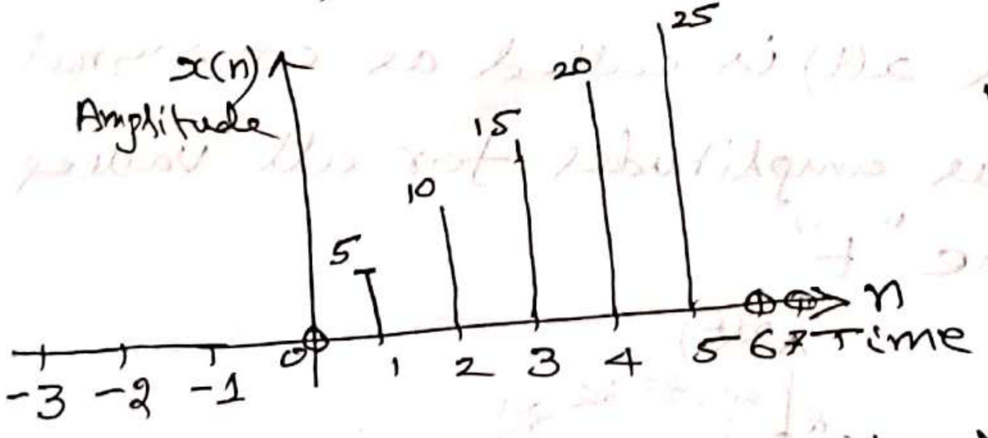


Fig. 1

2. Two dimensional Signal :-

The physical quantity of a signal depends on two independent variables. The image is an example, as shown in Figure 2.

$f(x, y) = 4x + 3xy$

where x & y are independent variables

$f(x,y)$ is an amplitude

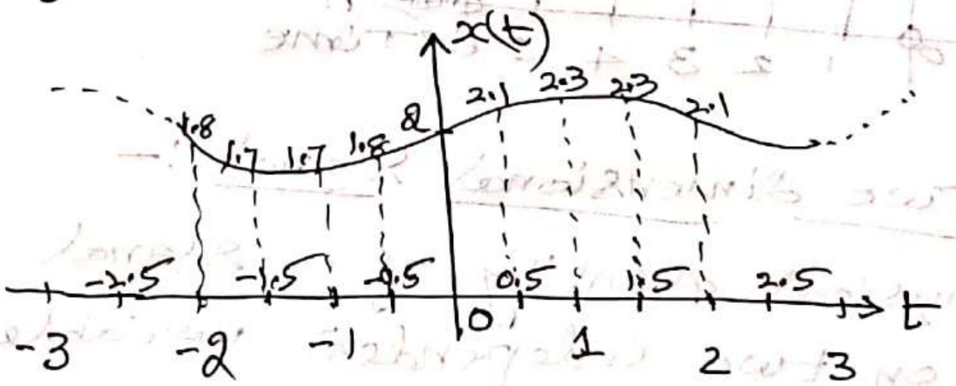
$$f(x,y) = \begin{bmatrix} 7 & 10 & 13 \\ 14 & 20 & 26 \\ 21 & 30 & 39 \end{bmatrix}$$

$f(1,1) = 7$, $f(1,2) = 10$, $f(1,3) = 13$
$f(2,1) = 14$, $f(2,2) = 20$, $f(2,3) = 26$
$f(3,1) = 21$, $f(3,2) = 30$, $f(3,3) = 39$

Types of Signals:- Basically there are two types of signals viz., continous time and discrete time signals

(i) Continous Time Signal (CT Signal)

A signal $x(t)$ is called as CT signal if it has amplitudes for all values of time "t"

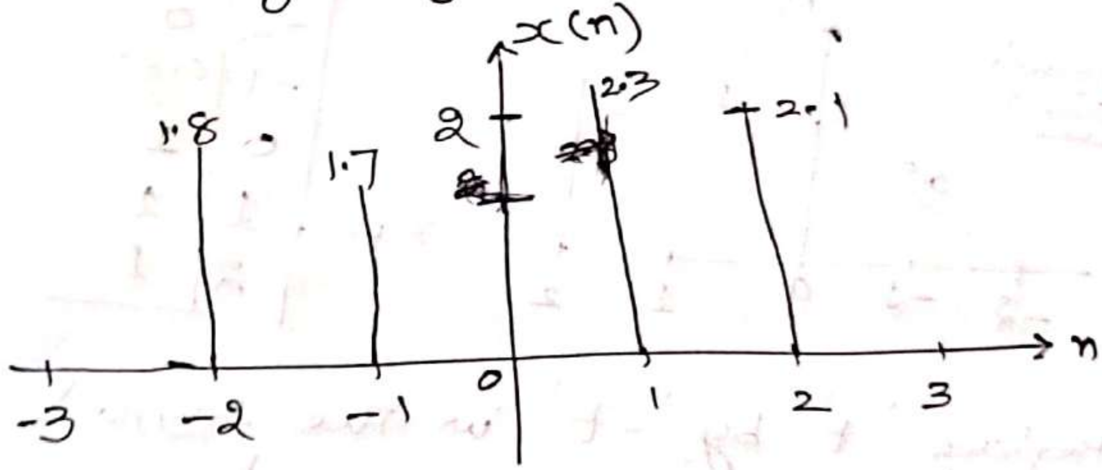


$t \rightarrow$ independent variable
$x(t) \rightarrow$ Amplitude and is dependent variable

$x(0)=2$, $x(-0.5)=1.8$, $x(-1)=1.7$, $x(-1.5)=1.7$
$x(-2)=1.8$, $x(0.5)=2.1$, $x(1)=2.3$, $x(1.5)=2.3$

(ii) Discrete Time signal (DT signal)

A signal $x(n)$ is called as DT signal, if it has amplitudes only for discrete values of time "n". The signal amplitudes are existing only for integer values of time "n".



$n \rightarrow$ Time $\rightarrow$ independent variable
$x(n) \rightarrow$ Signal $\rightarrow$ Amplitude and is dependent variable

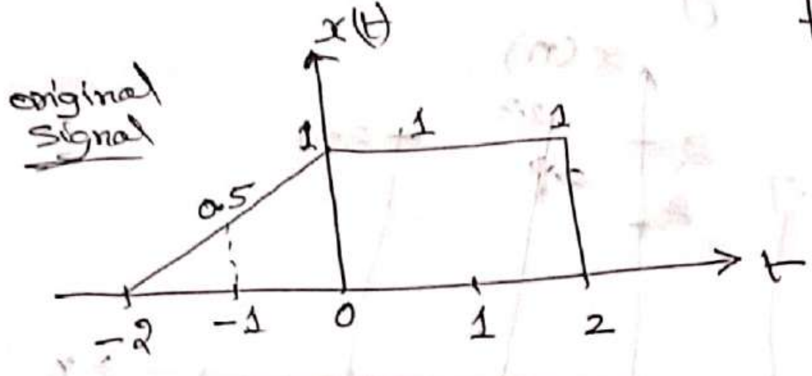
$x(0)=2$, $x(-1)=1.7$, $x(-2)=1.8$,
$x(1)=2.3$, $x(2)=2.1$.

Basic operations on signals

I. operations on independent variables:-

1. **Time Reversal**:- The time 't' or 'n' is replaced by '-t' or '-n'

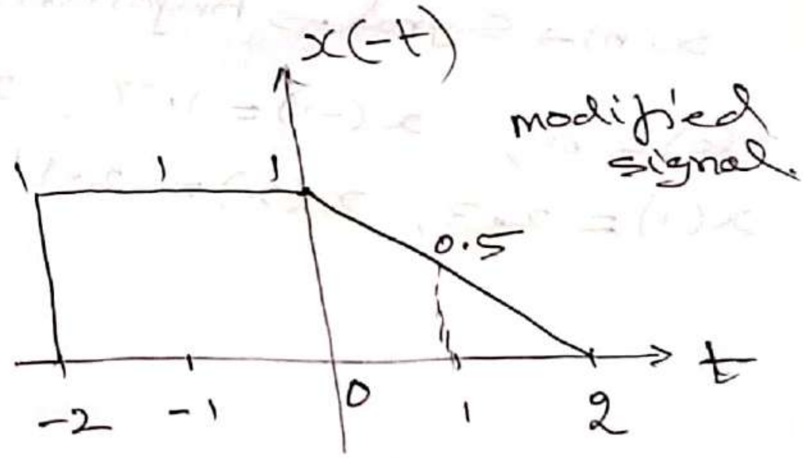
Example 1:- consider CT signal $x(t)$



t	x(t)
-2	0
-1	0.5
0	1
1	1
2	1

Replace 't' by '-t' in the signal and find amplitudes of $x(-t)$ signal

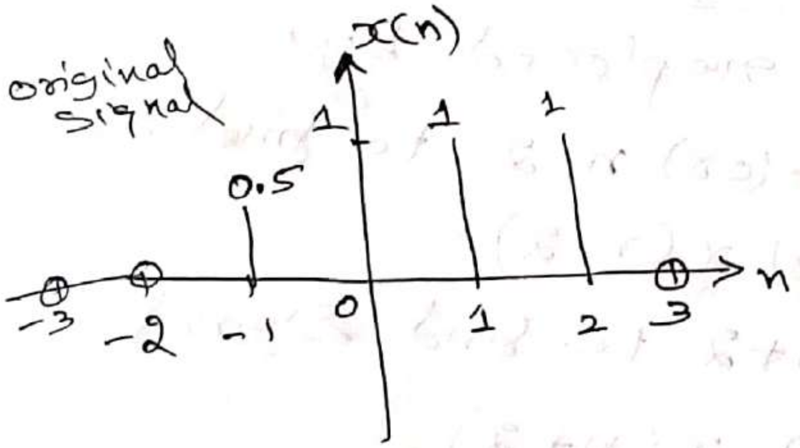
t	x(-t)
-2	$x(2)=1$
-1	$x(1)=1$
0	$x(0)=1$
1	$x(-1)=0.5$
2	$x(-2)=0$



Folded signal about the vertical axis at $t=0$

$\boxed{\text{Folded version of original signal}}$

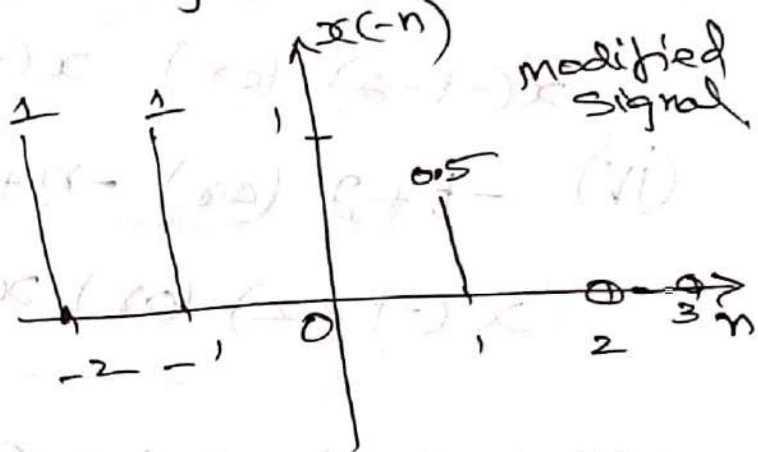
Example 2 :- consider DT signal $x(n)$



n	$x(n)$
-2	0
-1	0.5
0	1
1	1
2	1

Replace 'n' by '-n' in the signal and find the amplitude of $x(-n)$ signal.

n	$x(-n)$
-2	$x(2)=1$
-1	$x(1)=1$
0	$x(0)=1$
1	$x(-1)=0.5$
2	$x(-2)=0$



Folded signal about the vertical axis at $n=0$

$\boxed{\text{Folded version of original signal}}$

~~E~~2. Time Shifting :— The time 't' or 'n' are replaced by

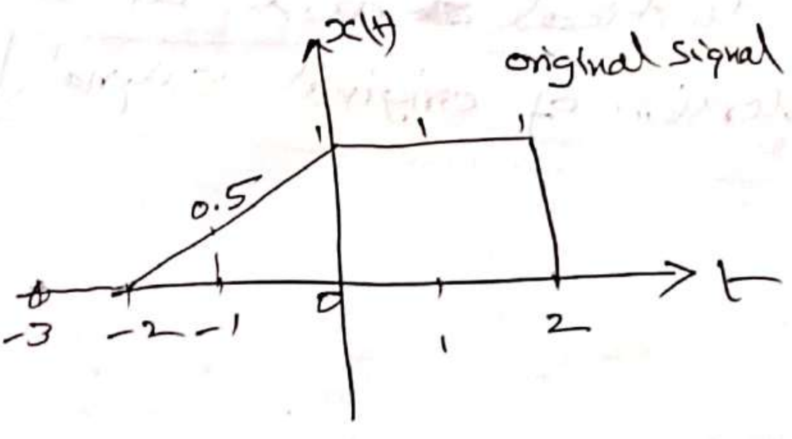
(i) $t-2$ (or) $n-2$ to find signal $x(t-2)$ (or) $x(n-2)$

(ii) $t+2$ (or) $n+2$ to find signal $x(t+2)$ (or) $x(n+2)$

(iii) $-t-2$ (or) $-n-2$ to find signal $x(-t-2)$ (or) $x(-n-2)$

(iv) $-t+2$ (or) $-n+2$ to find signals $x(-t+2)$ (or) $x(-n+2)$

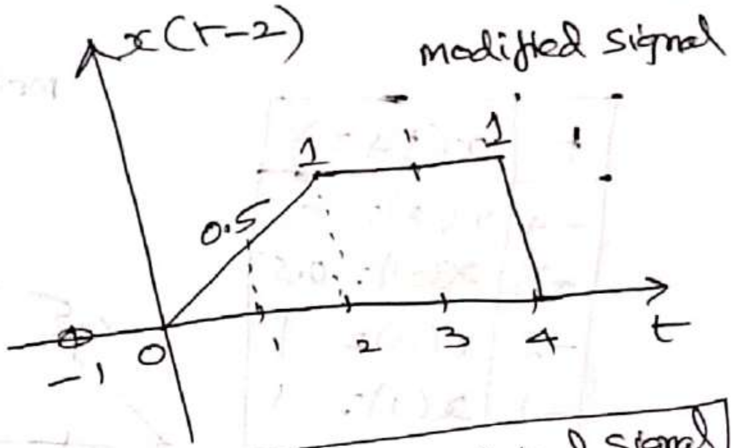
(i) Example 1 :- consider CT signal $x(t)$



t	x(t)
-2	0
-1	0.5
0	1
1	1
2	1

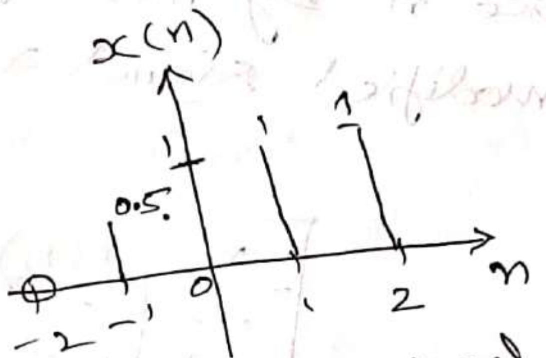
Replace 't' by '(t-2)' in the signal and find amplitude of modified signal $x(t-2)$

t	$x(t-2)$
-1	$x(-3)=0$
0	$x(-2)=0$
1	$x(-1)=0.5$
2	$x(0)=1$
3	$x(1)=1$
4	$x(2)=1$

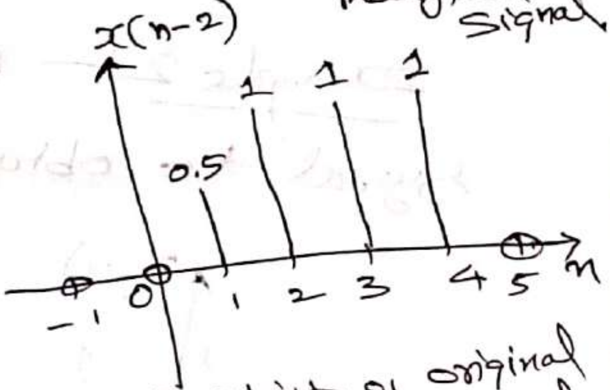


Right shift of original signal

Example 2 :- consider DT signal $x(n)$



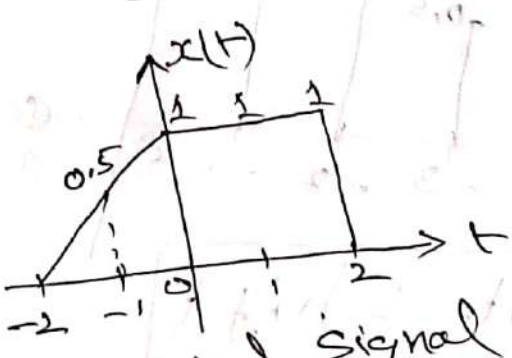
original signal



Right shift of original signal

(ii) Replace 't' or 'n' by '(t+2)' (or) (n+2) in the signal and find amplitudes of modified signal $x(t+2)$ or $x(n+2)$

Example 1: Replace 't' by $(t+2)$ to obtain modified signal $x(t+2)$

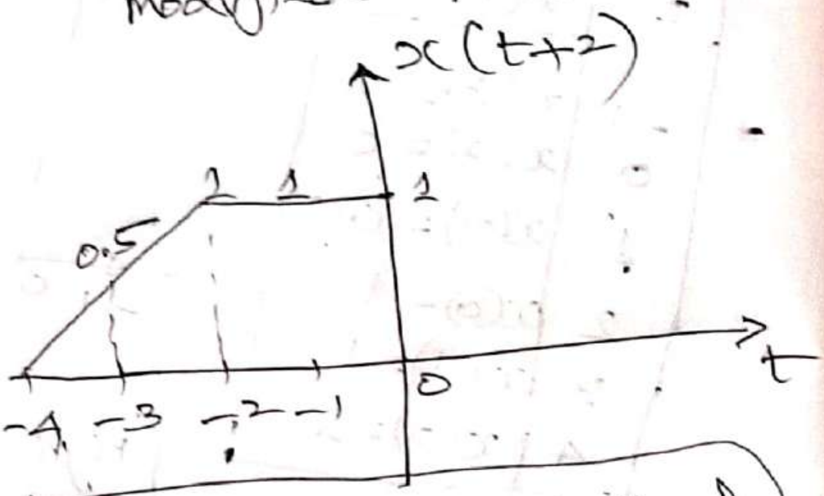


original signal.

t	$x(t)$
-2	0
-1	0.5
0	1
1	1
2	1

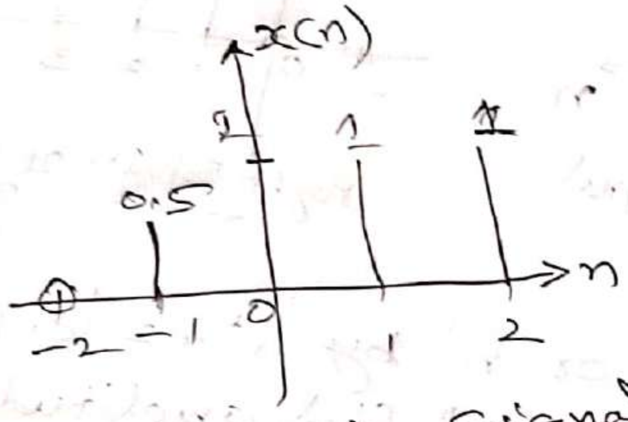
t	$x(t+2)$
-4	$x(-2) = 0$
-3	$x(-1) = 0.5$
-2	$x(0) = 1$
-1	$x(1) = 1$
0	$x(2) = 1$
1	$x(3) = 0$

modified signal :



Left shift of original signal

Example 2 :- Replace 'n' by 'n+2' in the signal to obtain modified signal $x(n+2)$

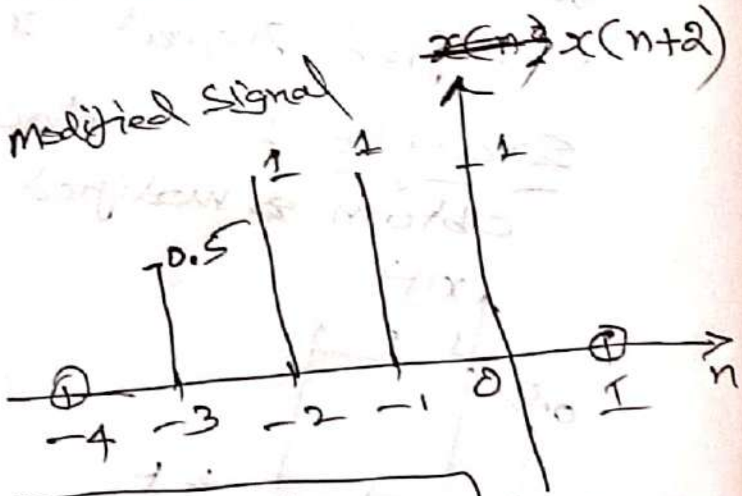


original signal

n	$x(n)$
-2	0
-1	0.5
0	1
1	1
2	1

n	$x(n+2)$
-4	$x(-2) = 0$
-3	$x(-1) = 0.5$
-2	$x(0) = 1$
-1	$x(1) = 1$
0	$x(2) = 1$
1	$x(3) = 0$

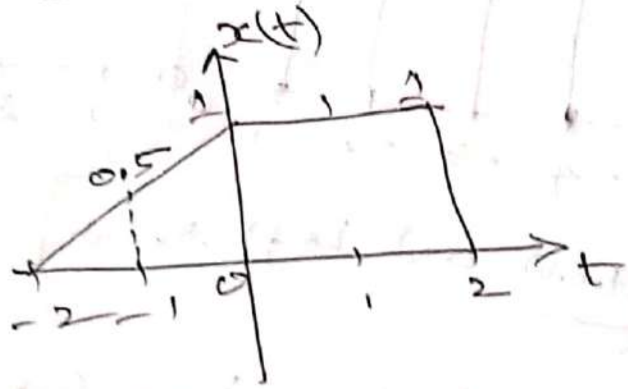
modified signal



Left shift

(iii) Replace 't' or 'n' by $(-t-2)$ (or) $(-n-2)$ in the original signal and find amplitudes of modified signal $x(-t-2)$ (or) $x(-n-2)$.

<u>Example 1</u>:– consider CT signal.

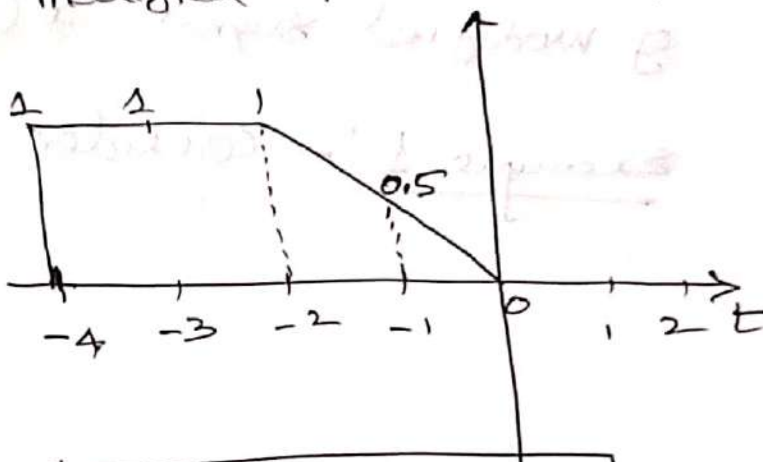


t	$x(t)$
-2	0
-1	0.5
0	1
1	1
2	1

Replace 't' by '$(-t-2)$' to obtain modified signal $x(-t-2)$

t	$x(-t-2)$
-4	$x(2) = 1$
-3	$x(1) = 1$
-2	$x(0) = 1$
-1	$x(-1) = 0.5$
0	$x(-2) = 0$
1	$x(-3) = 0$

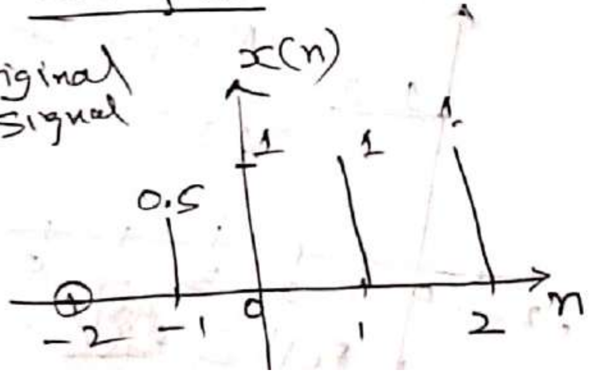
modified signal $x(-t-2)$



Fold and Left Shift

<u>Example 2</u>:– consider DT signal.

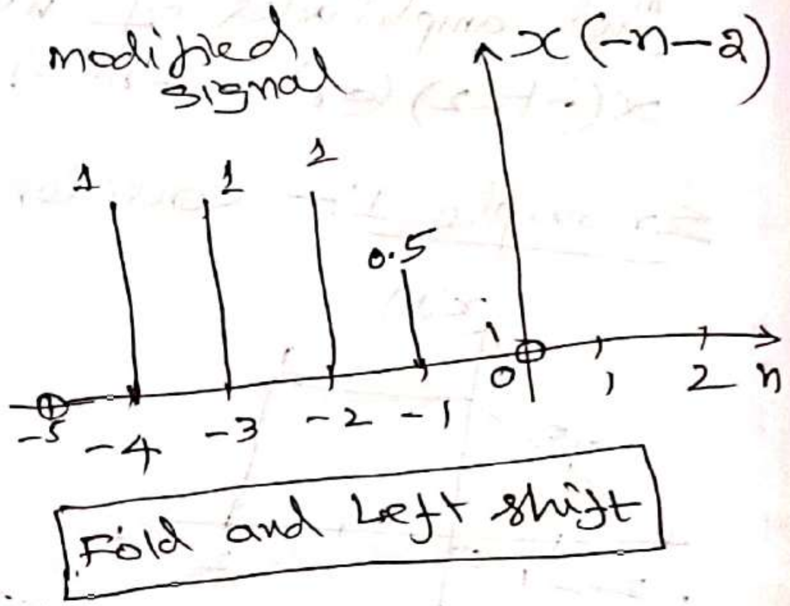
original signal



n	$x(n)$
-2	0
-1	0.5
0	1
1	1
2	1

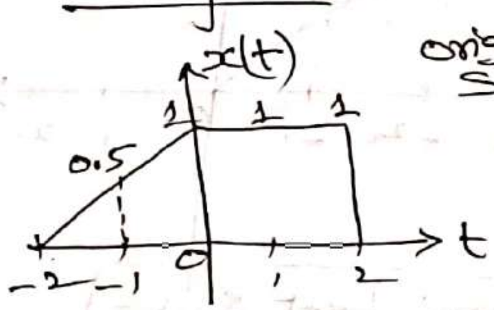
Replace 'n' by '(−n−2)' to obtain modified signal x(−n−2).

n	x(−n−2)
−4	x(2) = 1
−3	x(1) = 1
−2	x(0) = 1
−1	x(−1) = 0.5
0	x(−2) = 0
1	x(−3) = 0



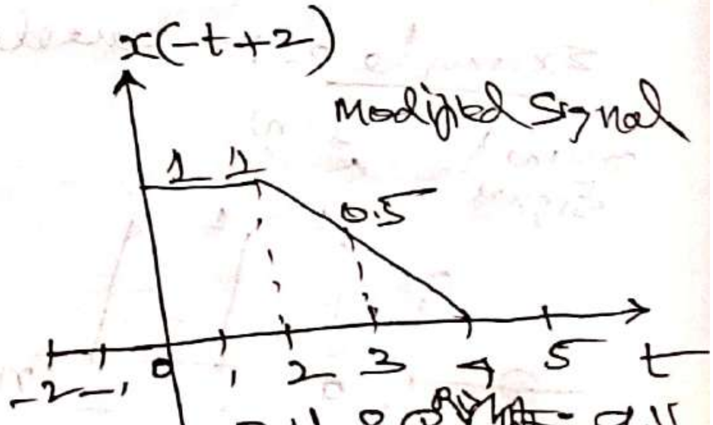
(iv) Replace 't' or 'n' by (−t+2) or (−n+2) in the original signal and find amplitudes of modified signal x(−t+2) or x(−n+2)

Example 1 :- consider CT signal



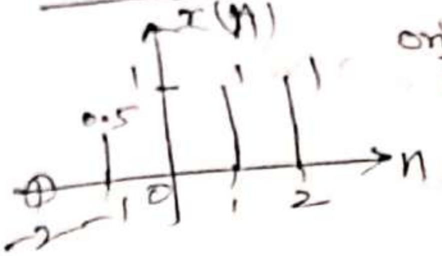
t	x(t)
−2	0
−1	0.5
0	1
1	1
2	1

t	x(−t+2)
−2	x(4) = 0
−1	x(3) = 0
0	x(2) = 1
1	x(1) = 1
2	x(0) = 1
3	x(−1) = 0.5
4	x(−2) = 0



Fold & Right Shift

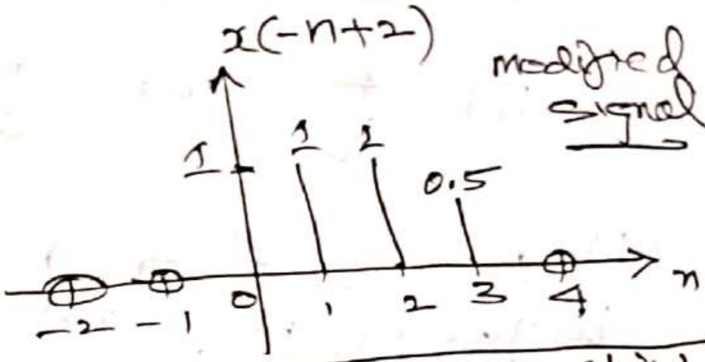
Example 2 :- consider DT signal :-



original Signal.

n	x(n)
-2	0
-1	0.5
0	1
1	1
2	1

n	$x(-n+2)$
-2	$x(4)=0$
-1	$x(3)=0$
0	$x(2)=1$
1	$x(1)=1$
2	$x(0)=1$
3	$x(-1)=0.5$
4	$x(-2)=0$



modified signal

$\boxed{\text{Fold and Right Shift}}$

Summary :-

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$$x(t-2) \longrightarrow RS$$
$$x(t+2) \longrightarrow LS$$
$$x(-t-2) \longrightarrow \text{Fold \& LS}$$
$$x(-t+2) \longrightarrow \text{Fold \& RS}$$

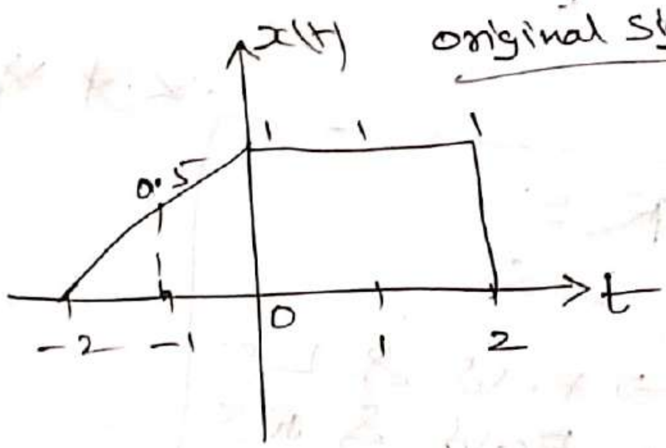
3. Time Scaling :- The time 't' or 'n' is replaced by

(i) '2t' or '2n'

(ii) $\frac{t}{2}$ or $\frac{n}{2}$

(i) Replace 't' or 'n' by 2t or 2n in the original signal and find amplitude of modified signal.

Example 1:- Consider CT Signal

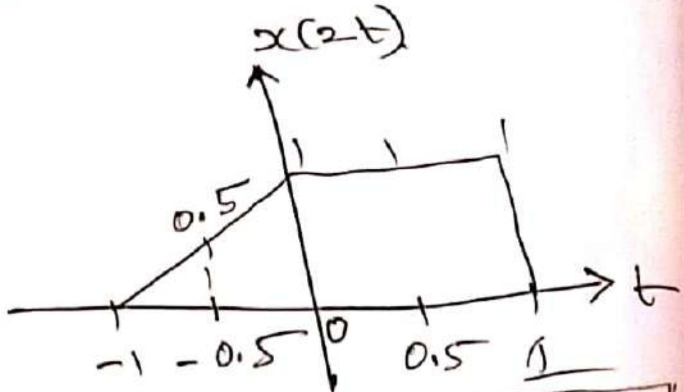


t	x(t)
-2	0
-1	0.5
0	1
1	1
2	1

Replace 't' by '2t' and find amplitude of modified signal x(2t)

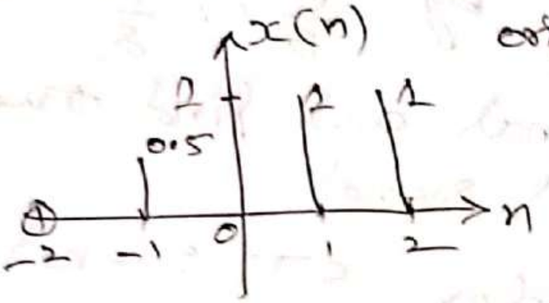
→ Divide x-axis of original Signal by 2

t	x(2t)
-1	x(-2)=0
-0.5	x(-1)=0.5
0	x(0) = 1
0.5	x(1) = 1
1	x(2) = 1



Time is compressed by 2

Example 2:- consider DT signal.



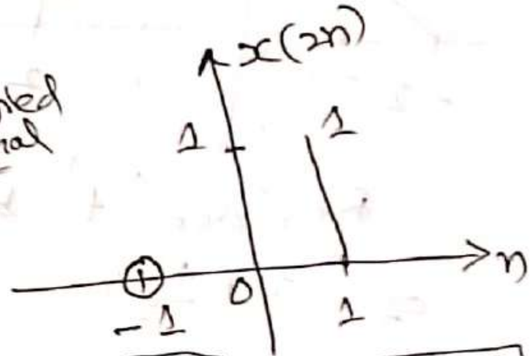
original signal

n	x(n)
-2	0
-1	0.5
0	1
1	1
2	1

Replace 'n' by '2n' and find amplitude of modified signal x(2n).

→ Divide x-axis ~~by~~ of original signal by 2

modified signal



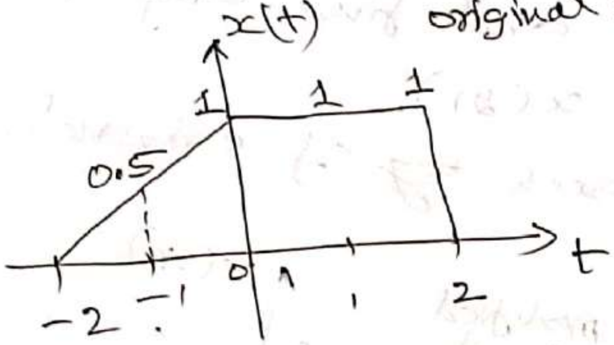
n	x(2n)
-1	x(-2) = 0
-0.5	x(-1) = 0.5
0	x(0) = 1
0.5	x(1) = 1
1	x(2) = 1

Time is compressed by 2

∵ 'n' can't be fraction
i.e., 'n' must be always an integer

(ii) Replace 't' or 'n' by $\frac{t}{2}$ or $\frac{n}{2}$ in the original signal and find the amplitude of modified signal

Example 1:- consider CT signal

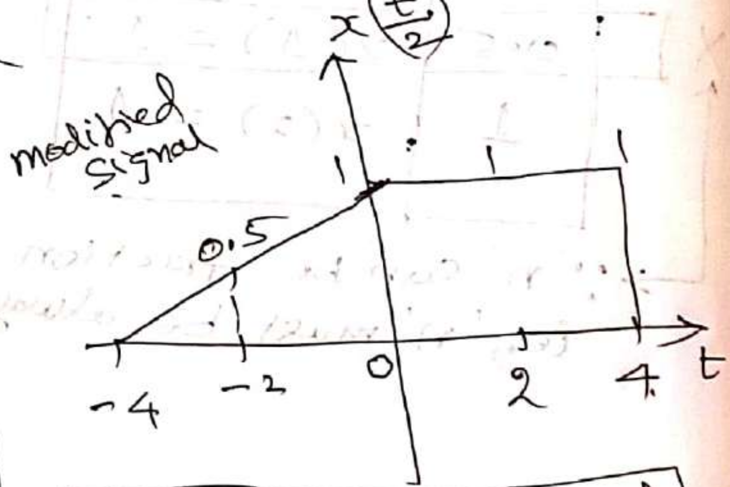


t	x(t)
-2	0
-1	0.5
0	1
1	1
2	1

Replace 't' by $\frac{t}{2}$ and find amplitude of $x\left(\frac{t}{2}\right)$

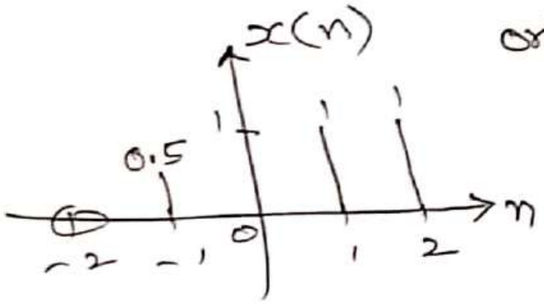
→ multiply x-axis of original signal by 2

t	$x\left(\frac{t}{2}\right)$
-4	$x(-2)=0$
-2	$x(-1)=0.5$
0	$x(0)=1$
2	$x(1)=1$
4	$x(2)=1$



Time is expanded by 2

Example 2 :- consider DT signal



original signal

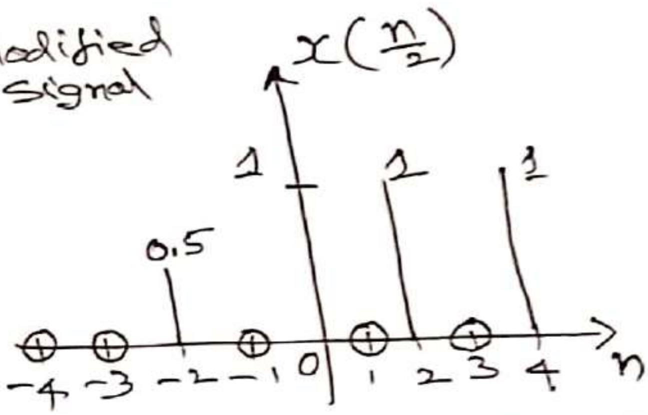
n	x(n)
-2	0
-1	0.5
0	1
1	1
2	1

Replace 'n' by ~~2~~ '$\frac{n}{2}$' and find amplitude of modified signal $x\left(\frac{n}{2}\right)$

→ multiply x-axis of original signal by 2

n	$x(\frac{n}{2})$
-4	$x(-2) = 0$
-2	$x(-1) = 0.5$
0	$x(0) = 1$
2	$x(1) = 1$
4	$x(2) = 1$

Modified signal



Time is expanded by 2

Elementary (or) Basic Signals

These signals are fundamentals for constructing more complex signals. Many available signals in the nature can be modelled using these elementary signals.

1. Unit Impulse Function ~~$\delta(t)$~~

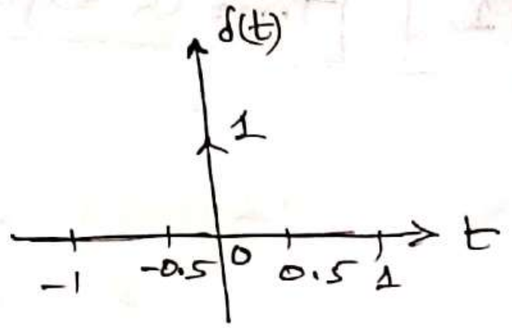
(i) CT - Unit Impulse Function $\delta(t)$:-

It is defined as

$$\delta(t) = \begin{cases} 1, & t = 0 \\ 0, & t \neq 0 \end{cases}$$

and the area is unity i.e., $\int_{-\infty}^{\infty} \delta(t)\,dt = 1$

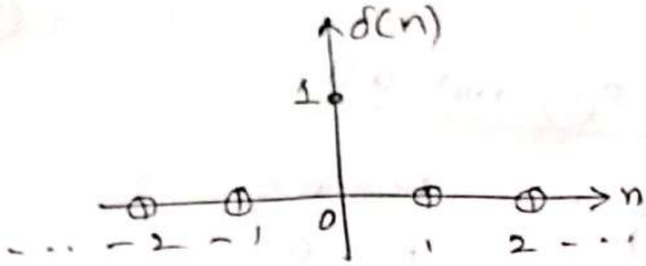
$\delta(t)$ is called as "Dirac-delta"



(ii) DT - Unit Impulse Function $\delta(n)$

It is defined as

$$\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$



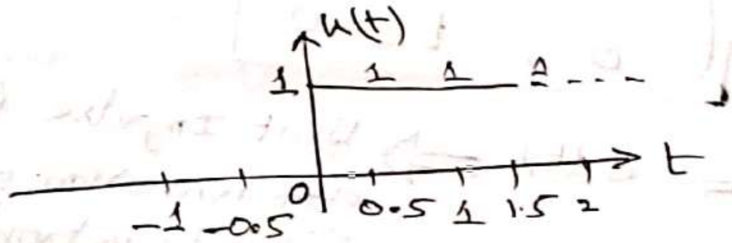
$\delta(0) = 1$
$\delta(-1)=0,\ \delta(-2)=0$
$\delta(1)=0,\ \delta(2)=0$

2. Unit Step Function

(i) CT – unit step function $u(t)$

It is defined as

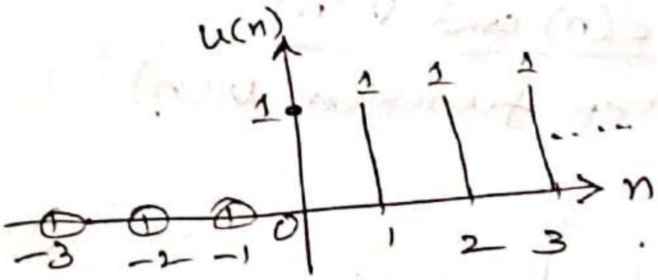
$$u(t) = \begin{cases} 1, & 0 \le t \\ 0, & t < 0 \end{cases}$$



(ii) DT – unit step function $u(n)$

It is defined as

$$u(n) = \begin{cases} 1, & n \ge 0 \\ 0, & n < 0 \end{cases}$$



$u(0)=1,\ u(1)=1$
$u(2)=1,\ u(3)=1$
$u(-1)=0,\ u(-2)=0$

Relation between $\delta(t)$ and $u(t)$

Consider $u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$

~~D.w.r.t~~ $\cancel{\frac{du(t)}{dt}} =$ $u(t) = \begin{cases} 0, & t < 0 \\ \boxed{1, \quad t = 0} & \\ 1, & t > 0 \end{cases}$ changes from 0 to 1

D.w.r.t t

$$\frac{du(t)}{dt} = \boxed{\begin{cases} 0, & t < 0 \\ 1, & t = 0 \\ 0, & t > 0 \end{cases}}$$

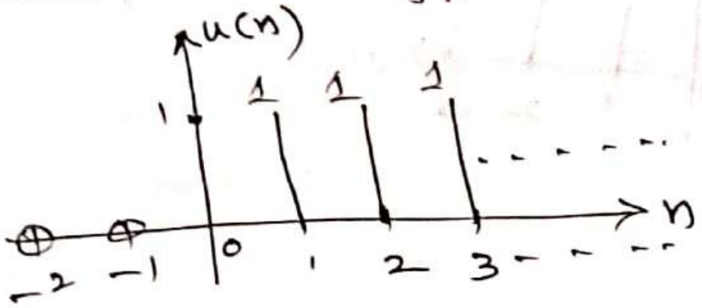
$\frac{du(t)}{dt} = \delta(t) \Rightarrow$ Unit Impulse is the differentiation of unit step Function

Integrate on both sides

$u(t) = \int \delta(t)\, dt \Rightarrow$ Unit step is the integration of Unit Impulse

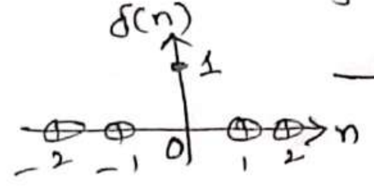
Relation between $\delta(n)$ and $u(n)$

Consider Unit Step function $u(n)$



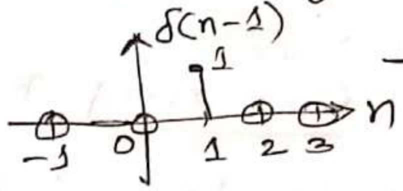
— (1)

Consider Unit Impulse function $\delta(n)$



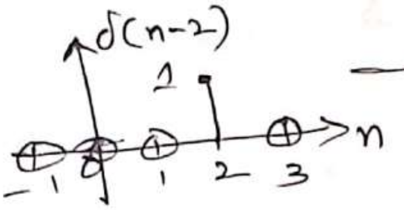
— ②

Shift Right by 1 unit



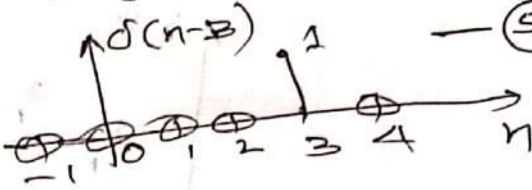
— ③

Shift Right by 2 units



— ④

Shift Right by 3 units

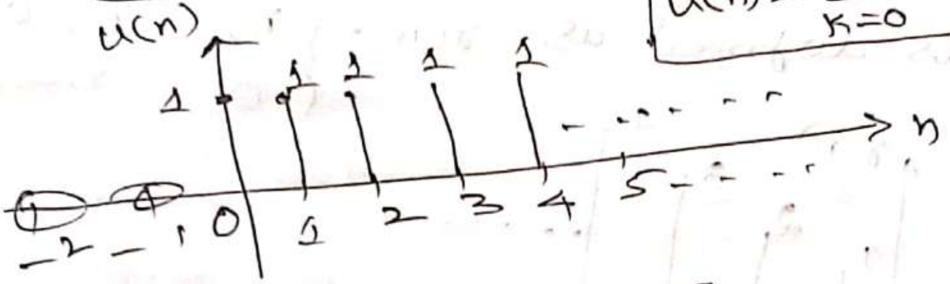


— ⑤

Add Figures ②, ③, ④, ⑤ and so on

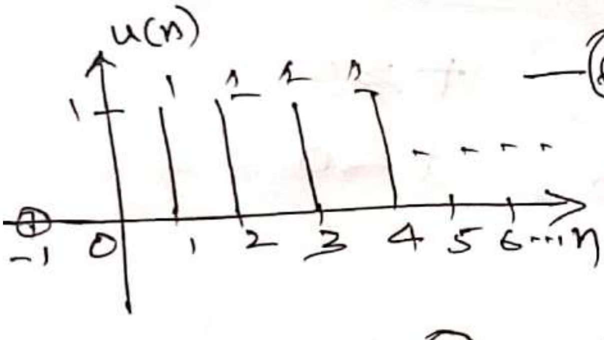
$$\boxed{u(n) = \delta(n) + \delta(n-1) + \delta(n-2) + \delta(n-3) + \cdots\cdots}$$

$$\boxed{u(n) = \sum_{k=0}^{\infty} \delta(n-k)}$$



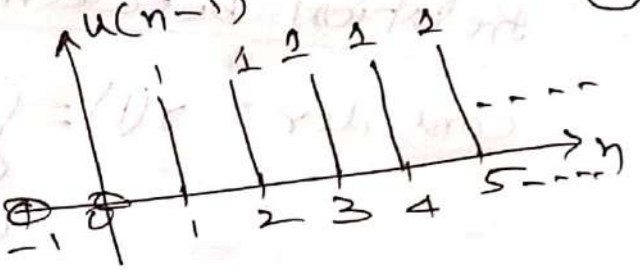
—— x ——

Consider $u(n)$



— ⑥

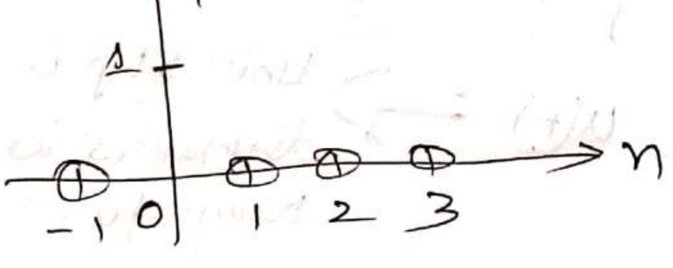
Consider $u(n-1)$



— ⑦

Do Fig. ⑥ − Fig ⑦ i.e., $u(n) - u(n-1)$

$[u(n) - u(n-1)] = \delta(n)$



$$\boxed{\delta(n) = u(n) - u(n-1)}$$

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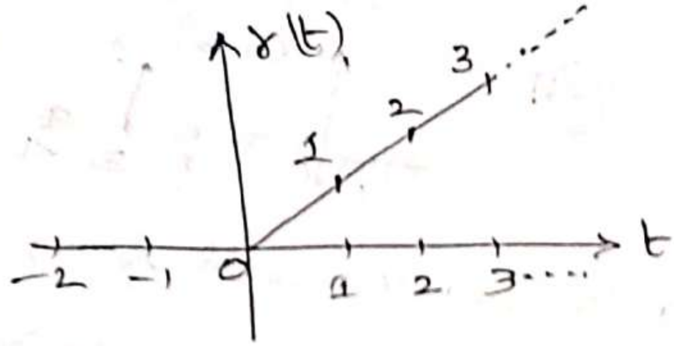
(3) Ramp Function

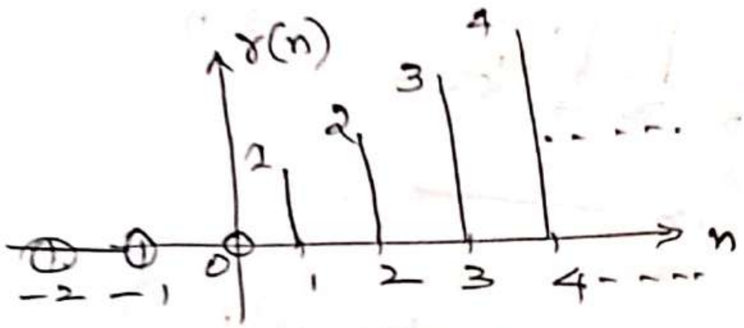
(i) CT – Ramp Function $r(t)$

It is defined as $r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$



(ii) DT – Ramp Function $r(n)$

It is defined as $r(n) = \begin{cases} n, & n \geq 0 \\ 0, & n < 0 \end{cases}$



⟹ **Relation between $r(t)$ and $u(t)$:—**

consider $r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$

D.w.r.t t

$$\frac{dr(t)}{dt} = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

$$\frac{dr(t)}{dt} = u(t)$$ ⟹ Unit Step is the differentiation of Ramp function

Integrate on both sides

$$r(t) = \int u(t)\,dt \Rightarrow$$ Ramp is the integration of unit step Function.

⇒ Relation between $r(n)$ and $u(n)$:-

$$\boxed{r(n) = n \cdot u(n)}$$

$$\boxed{u(n) = r(n) - n + 1}$$

(4) Exponential signals

(i) CT- Exponential signal $x(t)$:-

It is defined as

$$x(t) = A \cdot e^{at}$$

where $A \rightarrow$ Amplitude of the signal

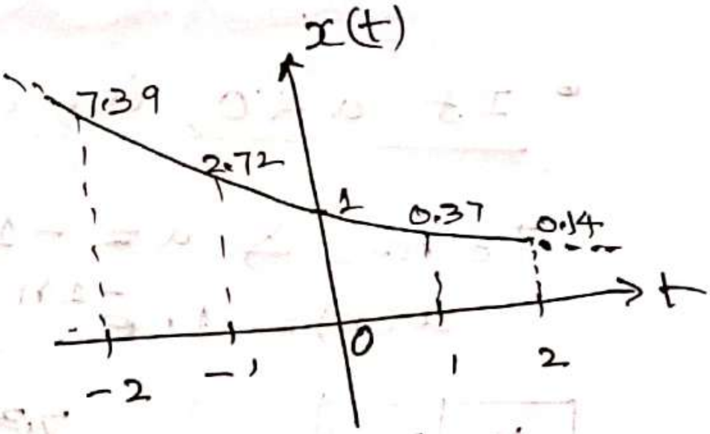
$a \rightarrow$ constant real value

• If $a < 0$, ie., $a \rightarrow -ve \Rightarrow$ Decaying exponential

If $A = 1$, & $a = -1$

$$x(t) = 1 \cdot e^{-1t}$$

t	~~$x(t)$~~	$x(t)$
-2	~~0.14~~	7.39
-1	~~0.37~~	2.72
0	~~1~~	1
1	~~2.72~~	0.37
2	~~7.39~~	0.14

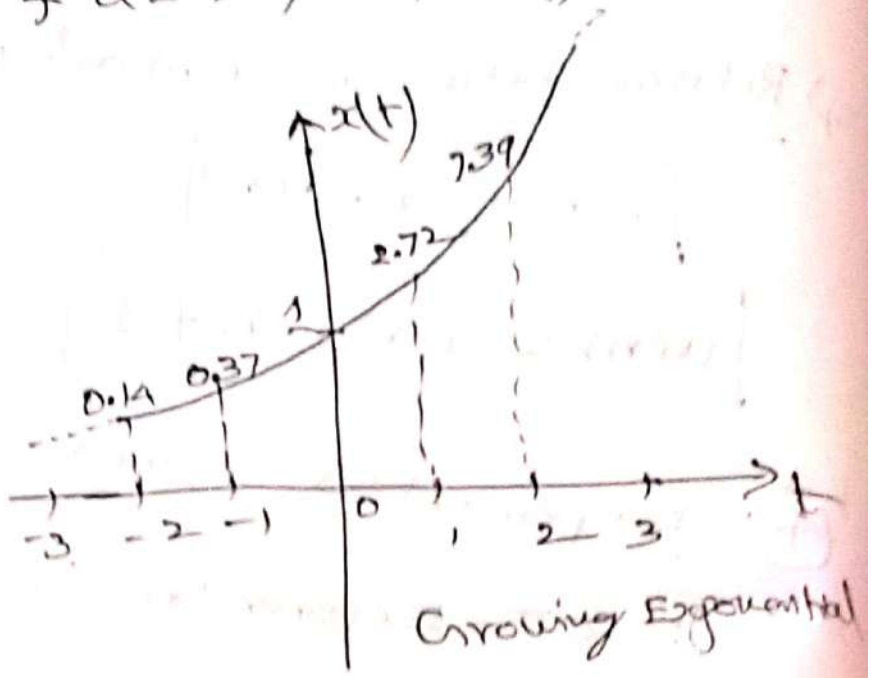


Decaying exponential

- **If $a > 0$, $a \to +ve \Rightarrow$ Growing Exponential**

If $A = 1$, & $a = 1$, Then $x(t) = e^{t}$

t	$x(t)$
-2	0.14
-1	0.37
0	1
1	2.72
2	7.39



Growing Exponential

(ii) DT- Exponential Signal $x(n)$:-

It is defined as $x(n) = A \cdot e^{an}$

where $A \to$ Amplitude of the signal

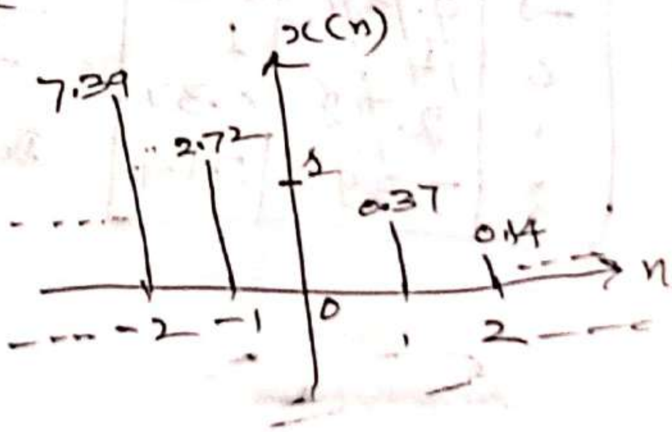
$a \to$ constant real value

- **If $a < 0$, ie., $a \to -ve \Rightarrow$ Decaying Exponential**

If $A = 1$ & $a = -1$

$x(n) = 1 \cdot e^{-1n}$

n	$x(n)$
-2	7.39
-1	2.72
0	1
1	0.37
2	0.14

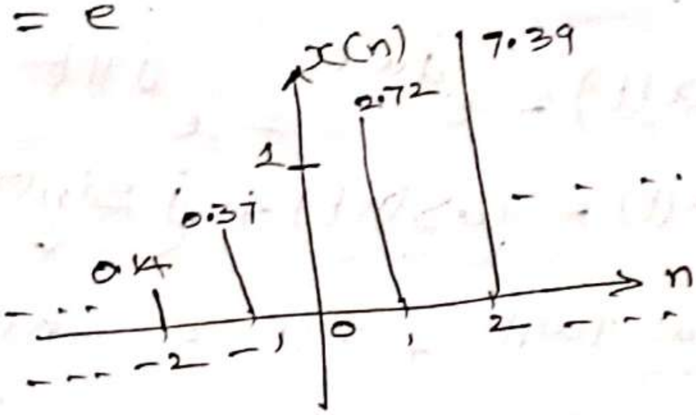


- If $a>0$, $a \rightarrow +ve \Rightarrow$ Growing Exponential.

If $A=+1$ & $a=+1$

$$x(n) = e^{n}$$

n	x(n)
-2	0.14
-1	0.37
0	1
1	2.72
2	7.39

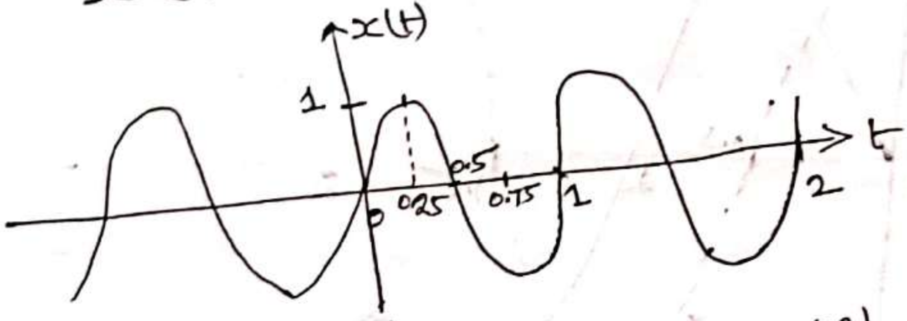


(5) Sinusoidal Signal :-

(i) CT Sinusoidal signal $x(t)$

$$x(t) = \sin(2\pi t)$$

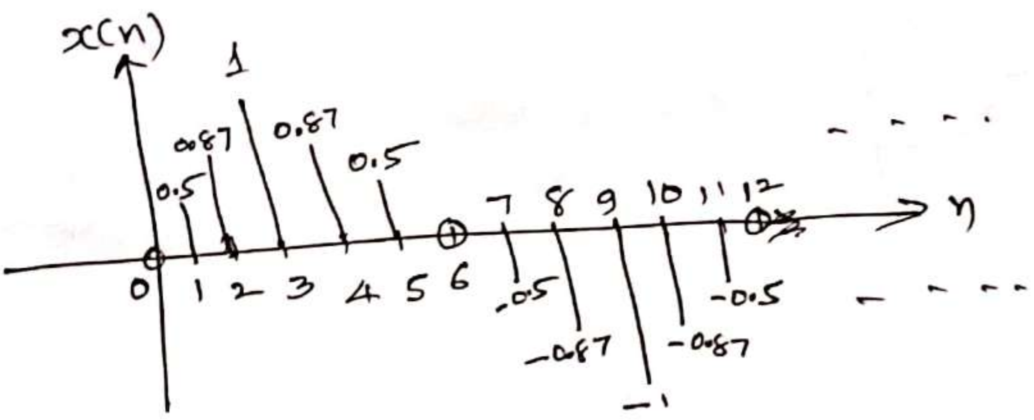
$T = 1$ sec



(ii) DT Sinusoidal Signal $x(n)$

$$x(n) = \sin\left(\frac{\pi}{6} n\right)$$

$N = 12$ Samples



(6) Complex Exponential Signal

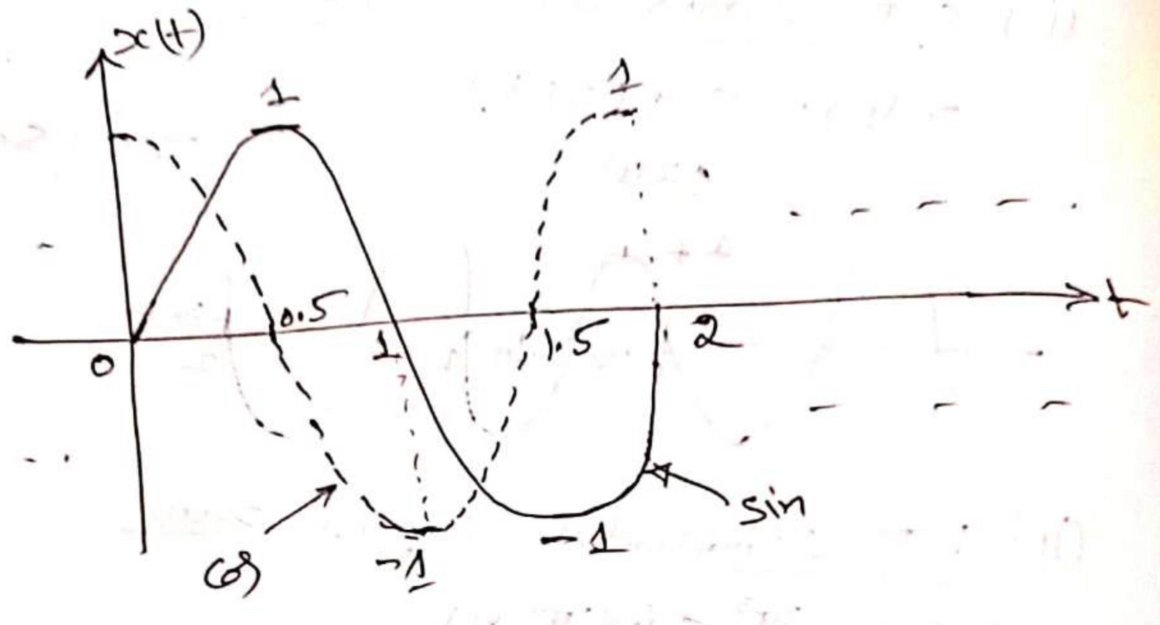
$$x(t) = e^{j\omega t} = e^{j\pi t}$$

$$x(t) = \underbrace{\cos(\pi t)} + \underbrace{j\sin(\pi t)}$$

Real part of $x(t) = \cos(\pi t)$

Imaginary part of $x(t) = j\sin(\pi t)$

Fundamental period $T = 2$ Sec.



Problems.

1. Sketch the following signals for a given triangular pulse signal $x(t)$ shown in Figure

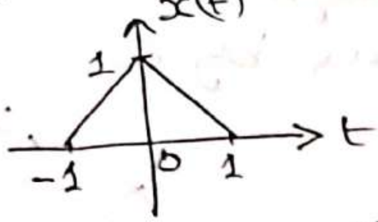


Fig 1

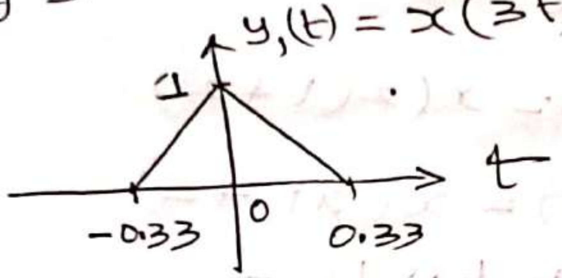
(i) $y_1(t) = x(3t)$ (ii) $y_2(t) = x(3t+2)$

(iii) $y_3(t) = x(-2t-1)$ (iv) $y_4(t) = x(2(t+2))$

(v) $y_5(t) = x(2(t-2))$ ~~(vi) $y_6(t) = x(3$~~

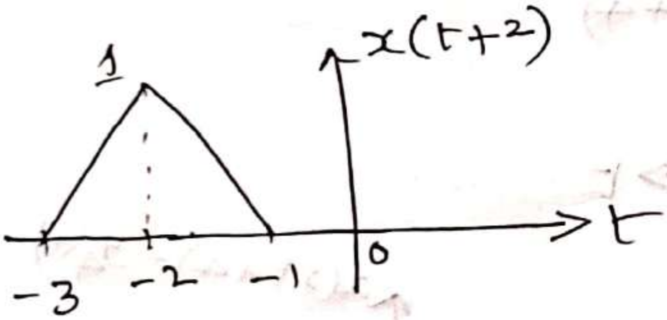
(vi) $y_6(t) = x(3t) + x(3t+2)$

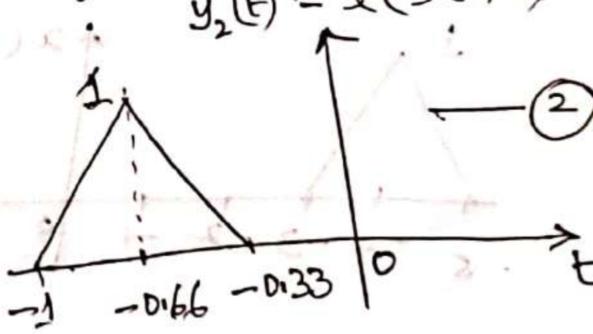
= (i) compression by 3, ie., Divide x-axis by 3



—— ①

(ii) → Shift Left by 2,
→ Devide x-axis by 3

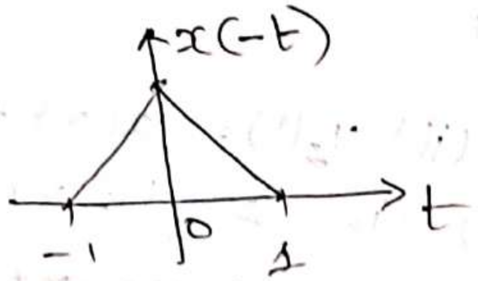


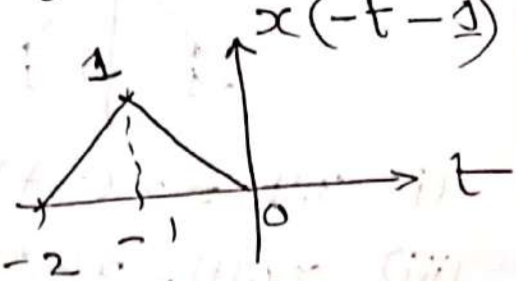


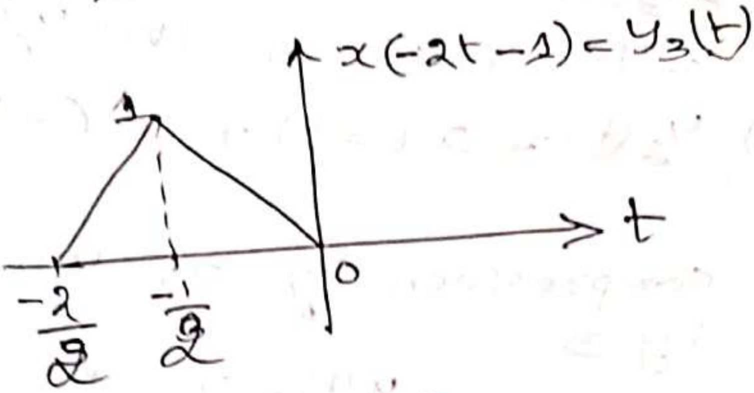
—— ②

(iii) $y_3(t) = x(-2t-1)$

→ Time Reverse on Folded version i.e,
→ Time shift ~~Right by one~~ Left shift by one
→ Divide x-axis by 2.



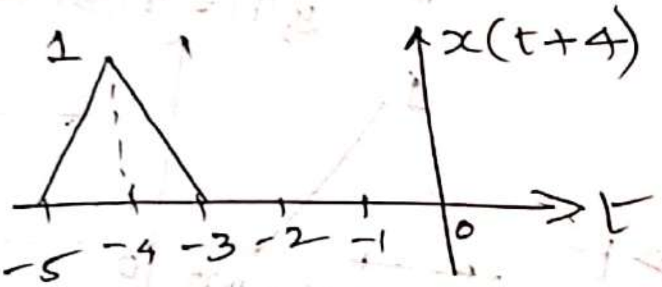


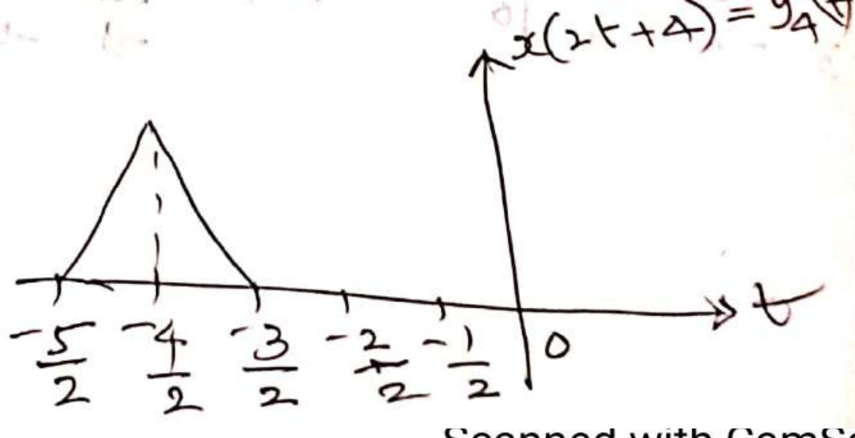


(iv) $y_4(t) = x(2(t+2))$

$y_4(t) = x(2t+4)$

→ Shift left by 4
→ Devide x-axis by 2



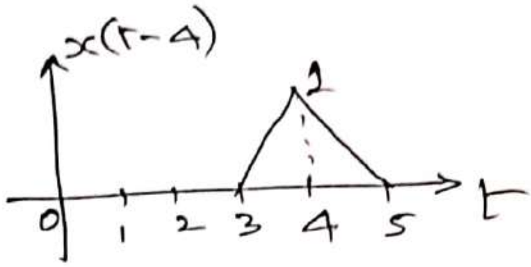


(v) $y_5(t) = x(2(t-2))$

$= x(2t-4)$

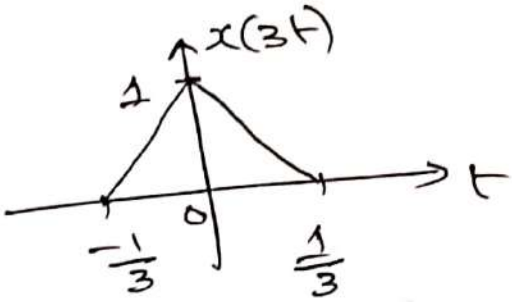
→ Shift Right by 4

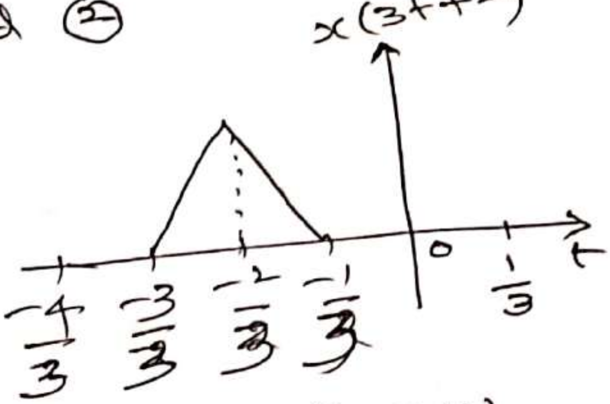
→ Divide x-axis by 2

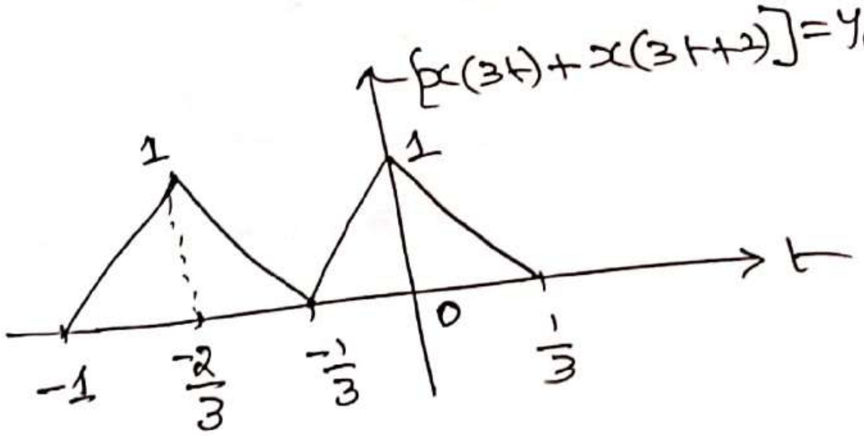


(vi) $y_6(t) = x(3t) + x(3t+2)$

Add Figures ① and ②







2. Sketch the following signals

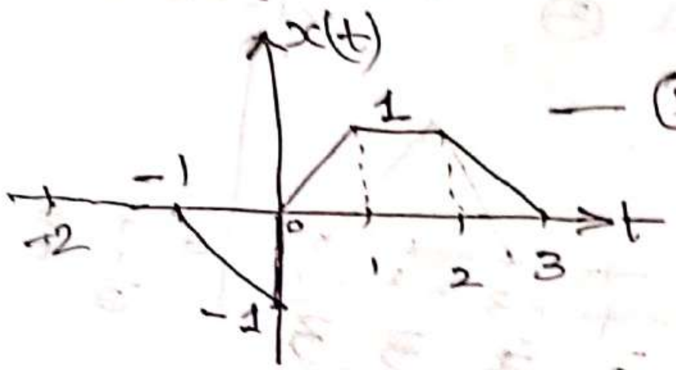
(i) $x(t)\, y(t-1)$ (ii) $x(t-1)\, y(-t)$

(iii) $x(t+1)\, y(t-2)$ (iv) $x(t)\, y(-1-t)$

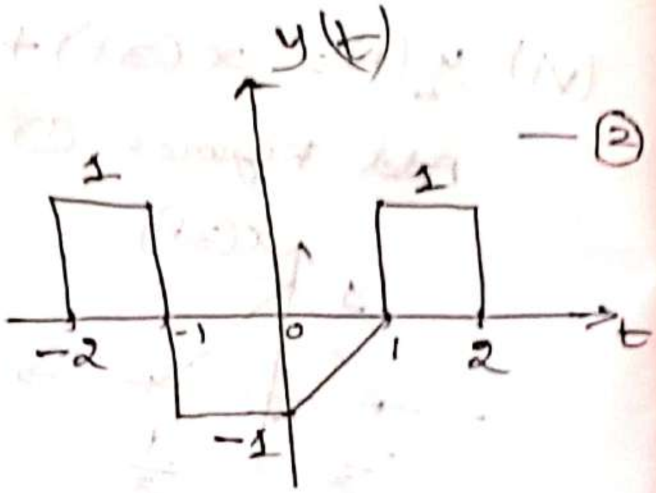
(v) $x(t)\, y(2-t)$ (vi) $x(2t)\cdot y(\frac{t}{2}+1)$

(vii) $x(4-t)\, y(t)$

Given

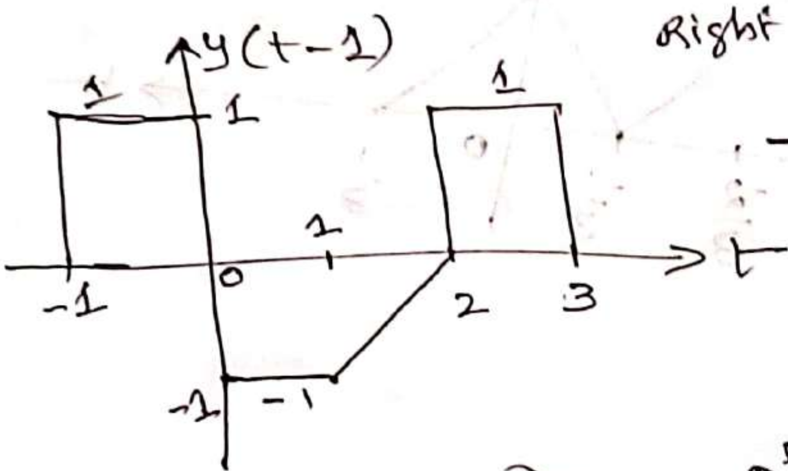


— ①



— ②

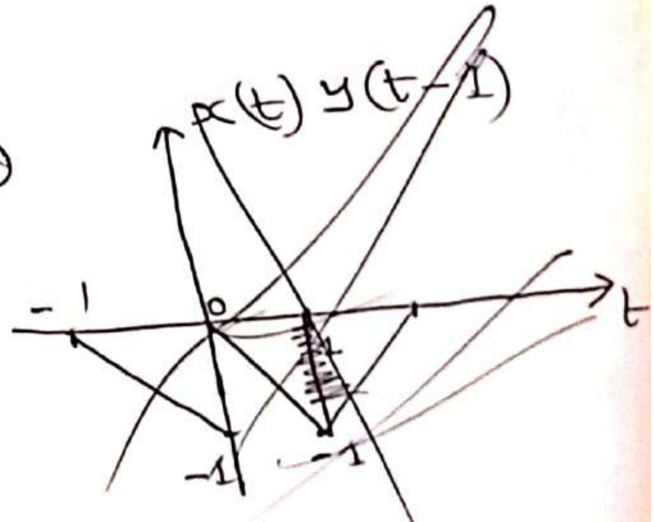
(i) $x(t)\cdot y(t-1)$



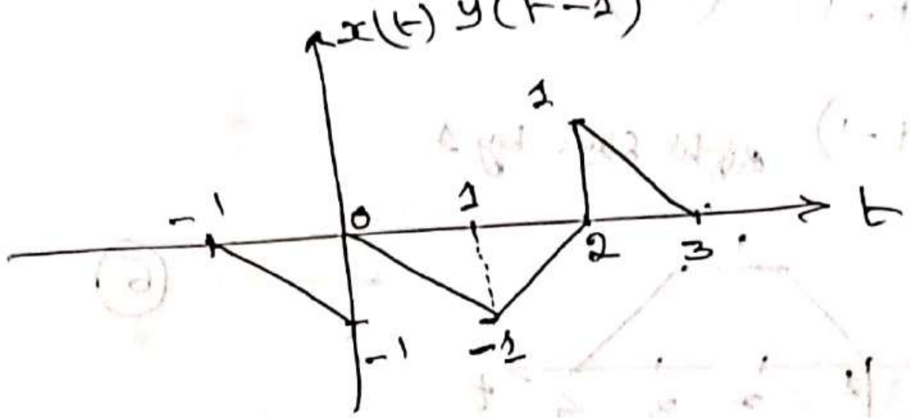
Right shift

— ③

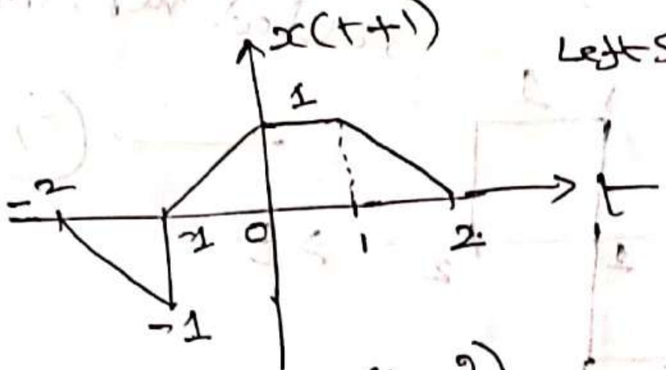
~~Do Fig ① × Fig ③~~



Do, Fig 1 × Fig 3

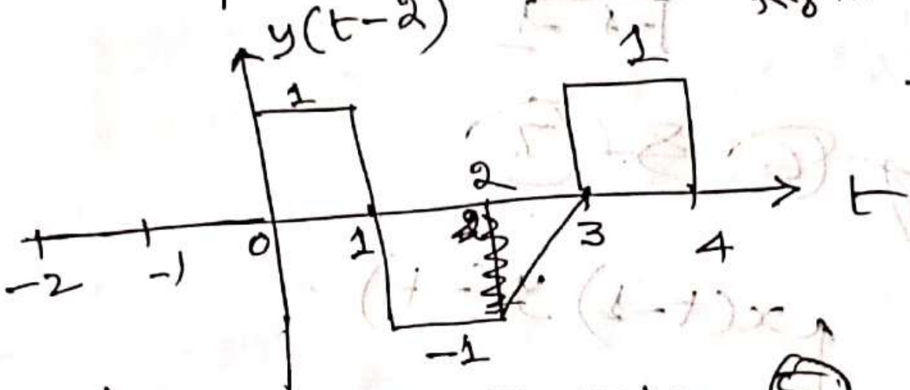


(iii) $x(t+1)\,y(t-2)$

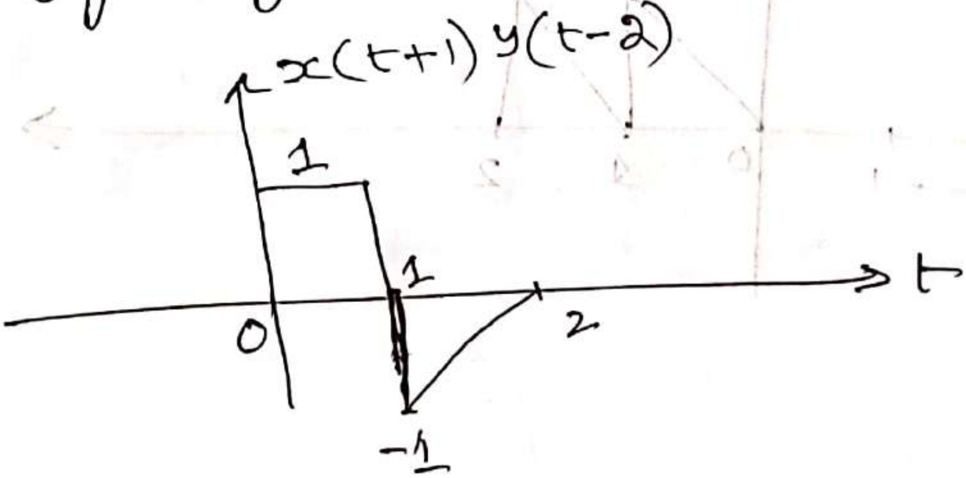


Left Shift by 1 —— (4)

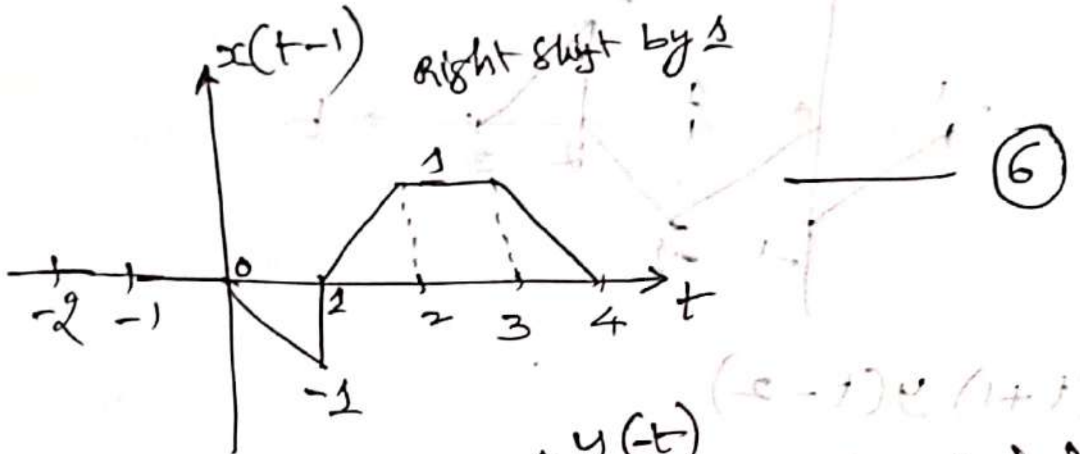
Right Shift by 2 —— (5)

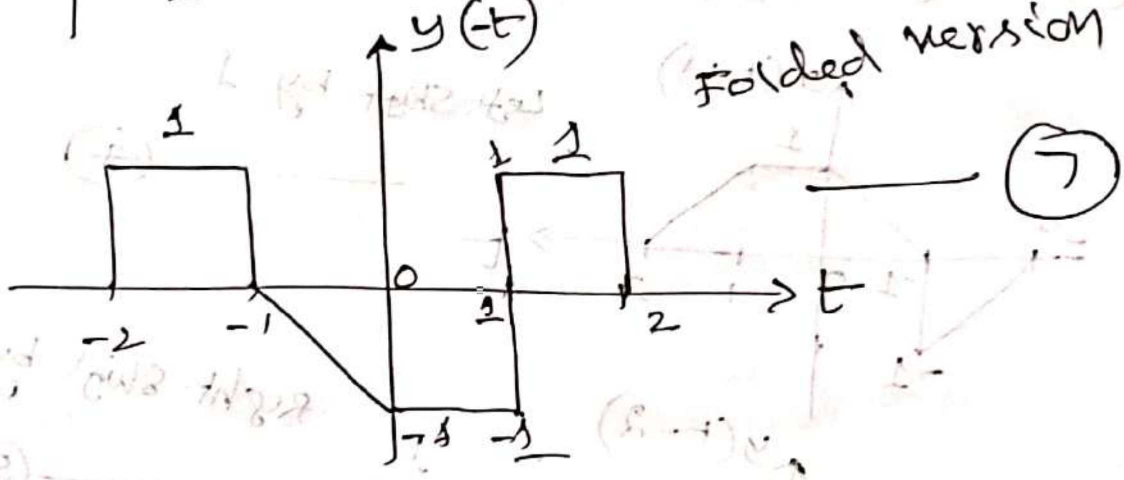


Multiply Fig (4) & Fig (5)

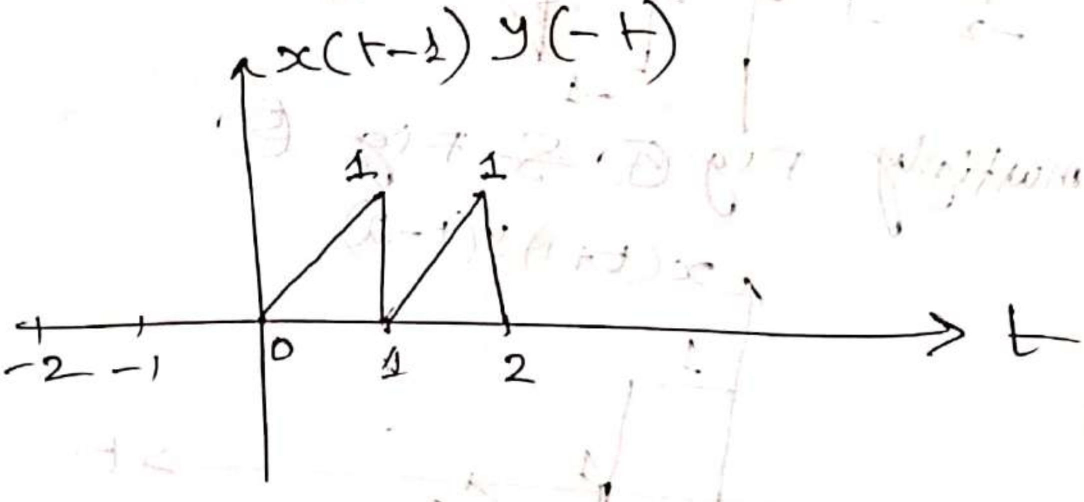


(ii) $x(t-1)\,y(-t)$



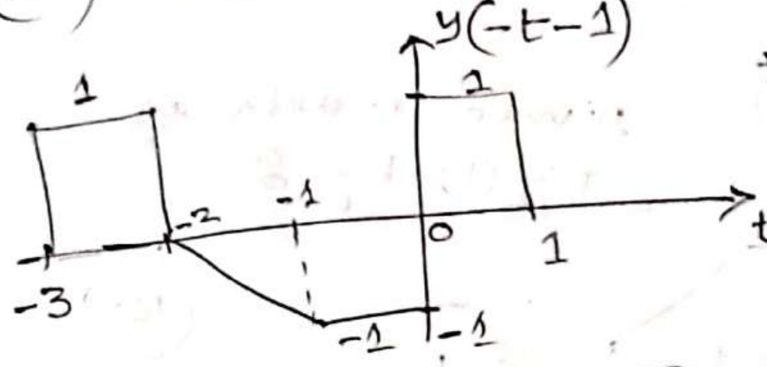


multiply (6) & (7)



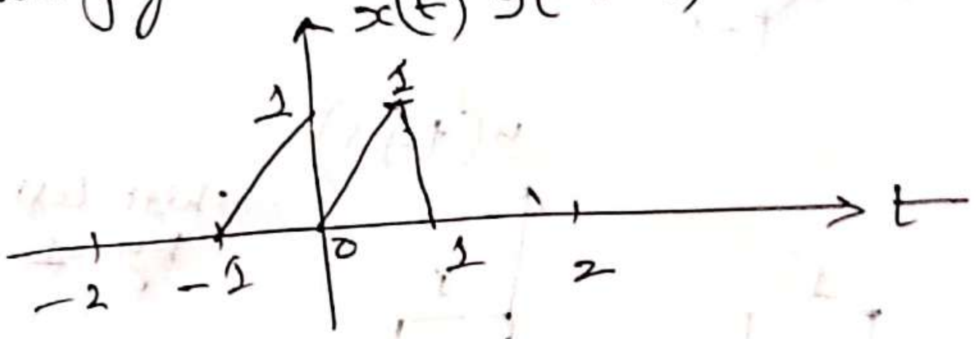
(iv) ~~x(t) · y(−t~~ $x(t) \cdot y(-1-t)$

Fold & Shift Left by 1



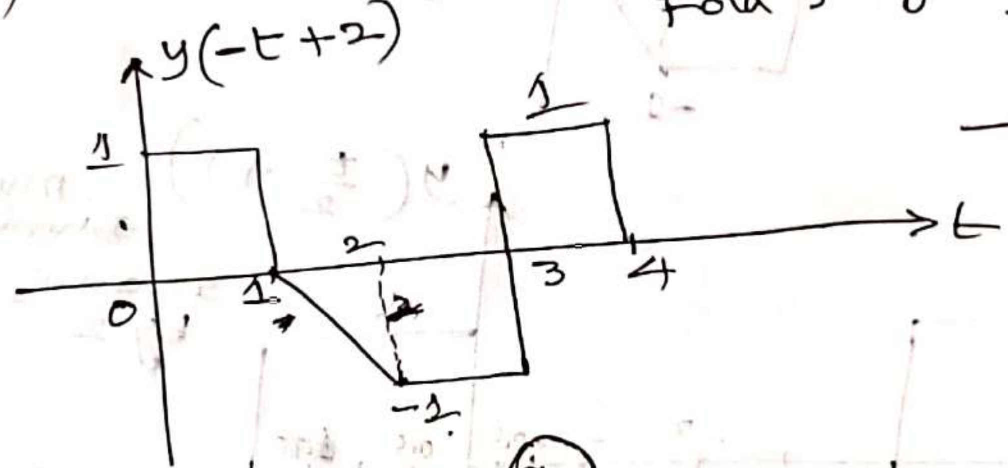
—— (8)

multiply (1) & (8)



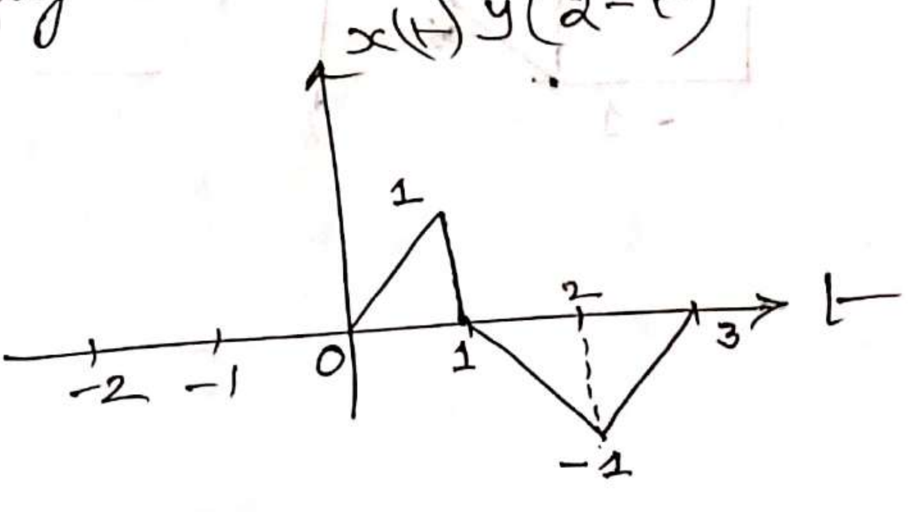
(v) $x(t)\, y(2-t)$

Fold & Right shift by 2

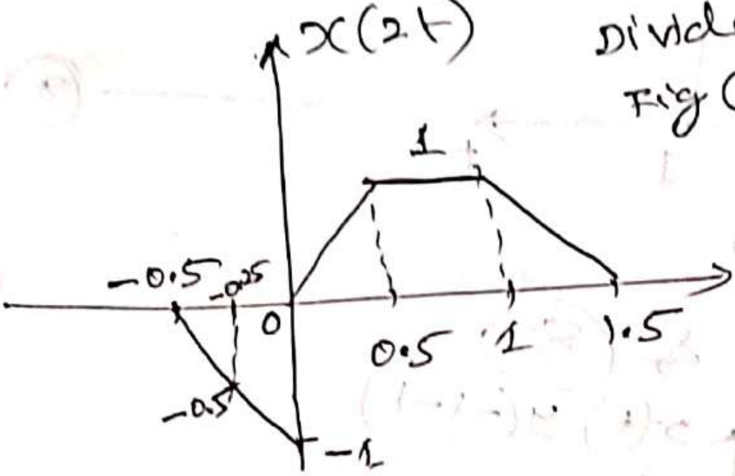


—— (9)

multiply (1) & (9)

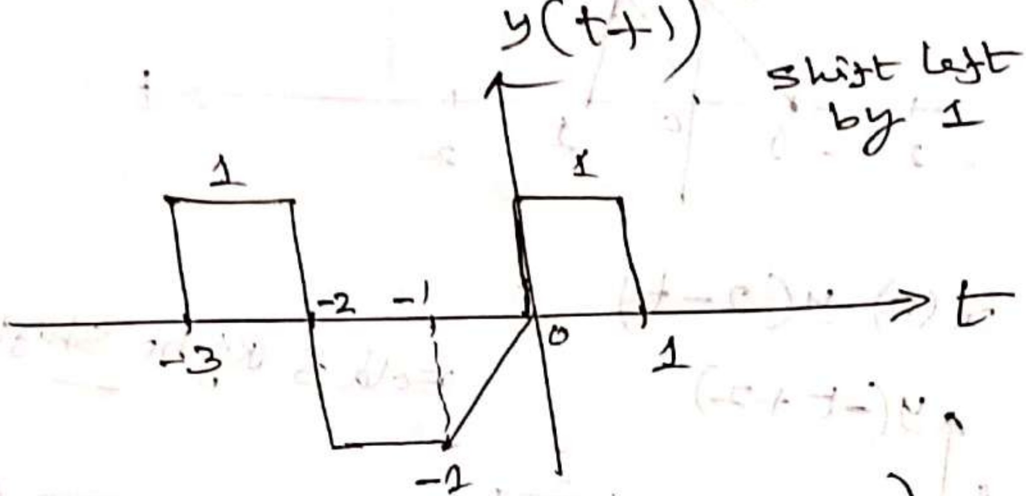


(vi) $x(2t)\, y\left(\frac{t}{2}+1\right)$

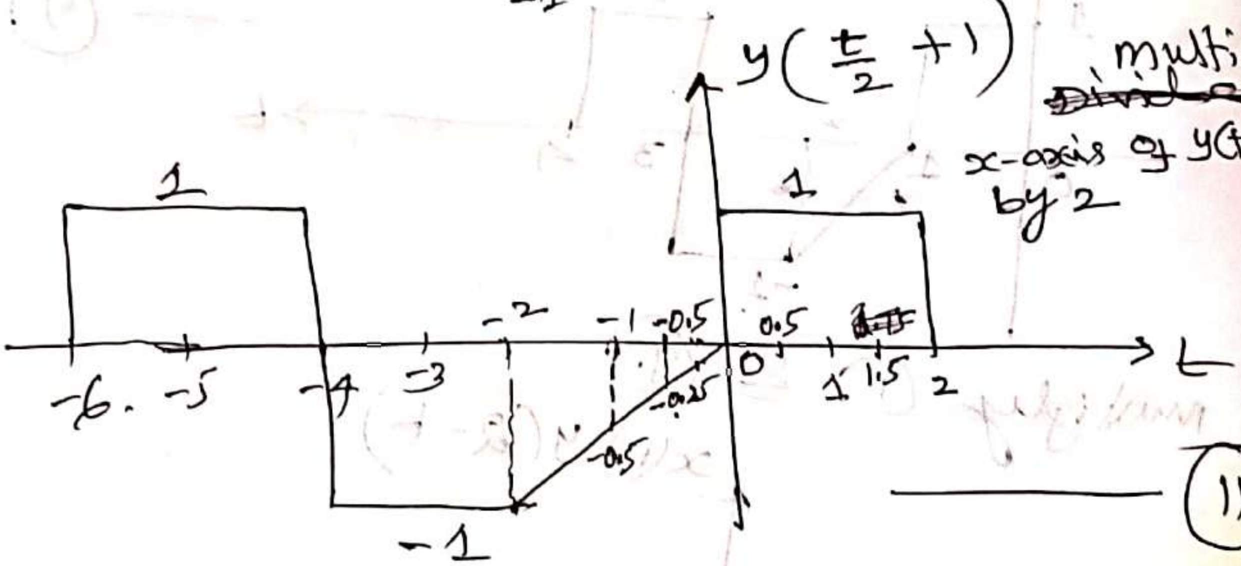


Divide x-axis of Fig (1) by 2

(10)



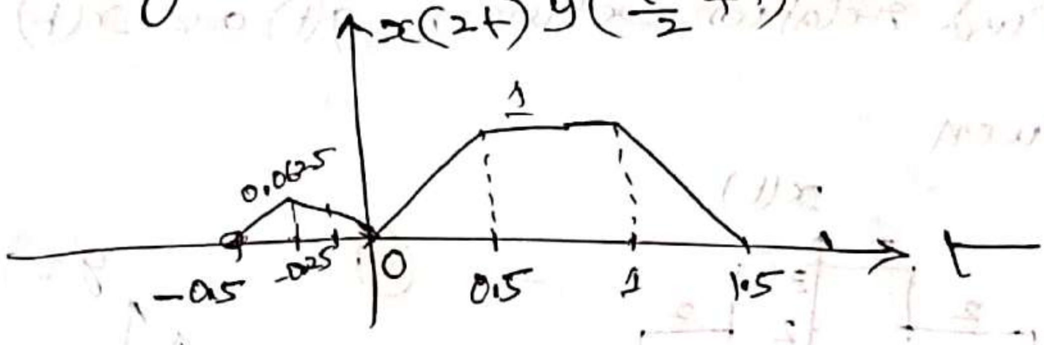
shift left y(t) by 1



multiply ~~Divide~~ x-axis of y(t+1) by 2

(11)

Multiply 10 & 11



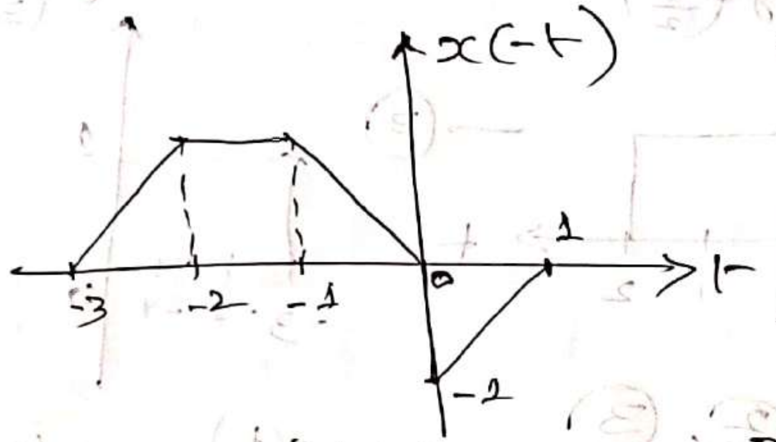
At, $t = -0.5$, magnitude $= 0 \times (-0.25) = 0$

$t = -0.25$, Magnitude $= \cancel{-0.25}\ -0.5 \times -0.125 = \cancel{+0.03125}\ 0.0625$

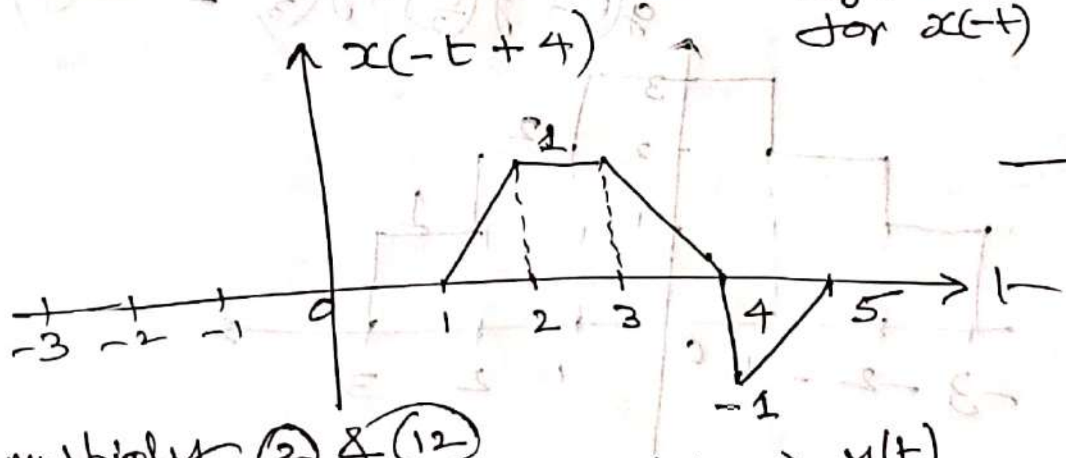
$t = -0.125$, Magnitude $= -0.75 \times -0.0625 = 0.04675$

(vii) $x(4-t)\ y(t)$

Fold $x(t)$

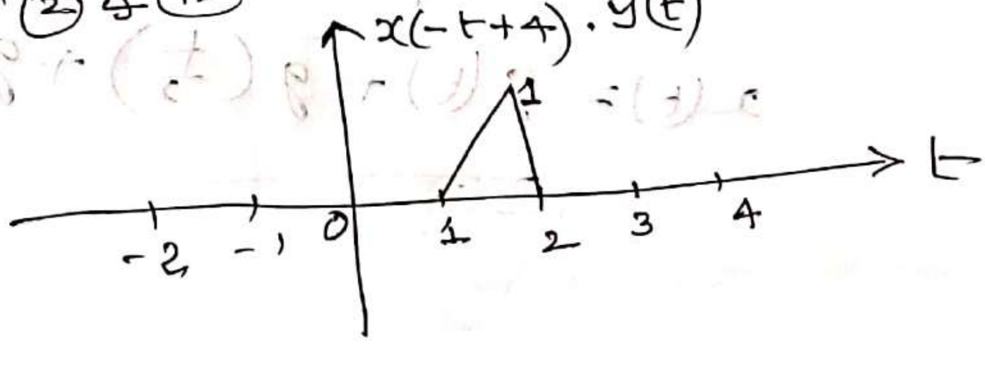


Right shift by 4 for $x(-t)$



— 12

multiply 2 & 12



Verification

For $t = 1$

$x(3)\,y(1) = 0 \times 1 = 0$

For $t = 2$

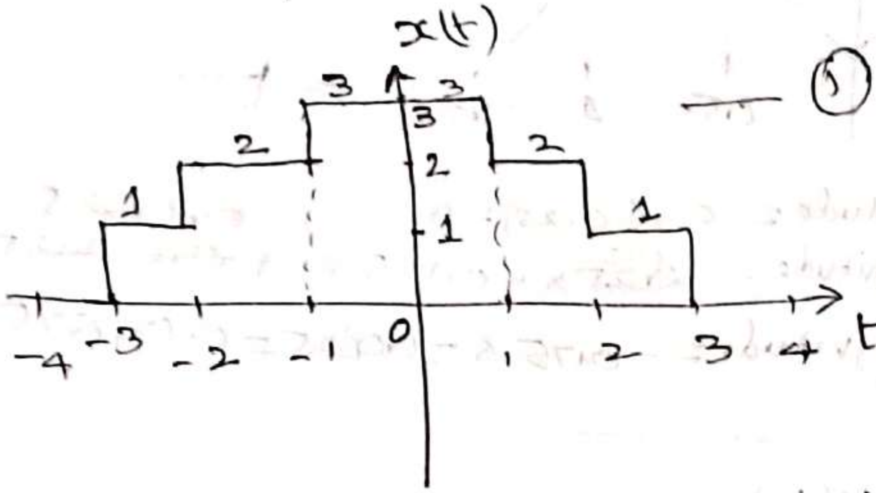
$x(2)\,y(2) = 1 \times 1 = 1$

For $t = 0$

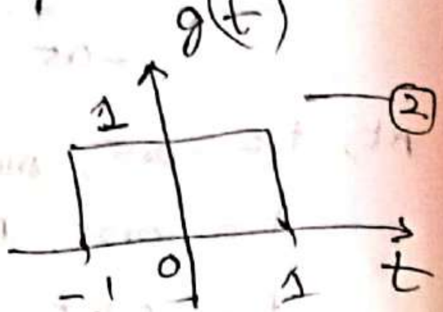
$x(4)\,y(0) = 0 \times -1 = 0$

3 Find relation between $g(t)$ and $x(t)$

Given

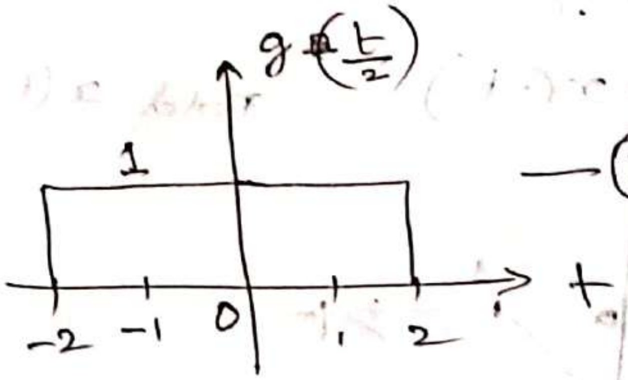


— (1)

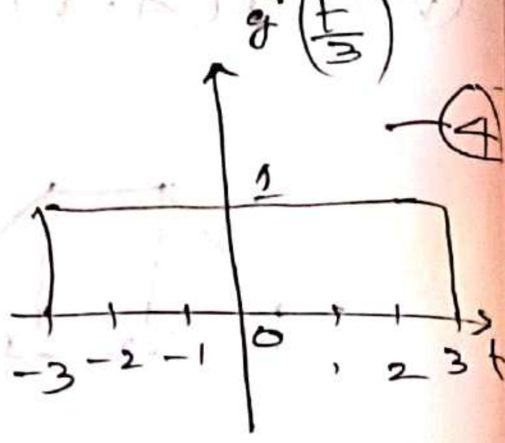


— (2)

=



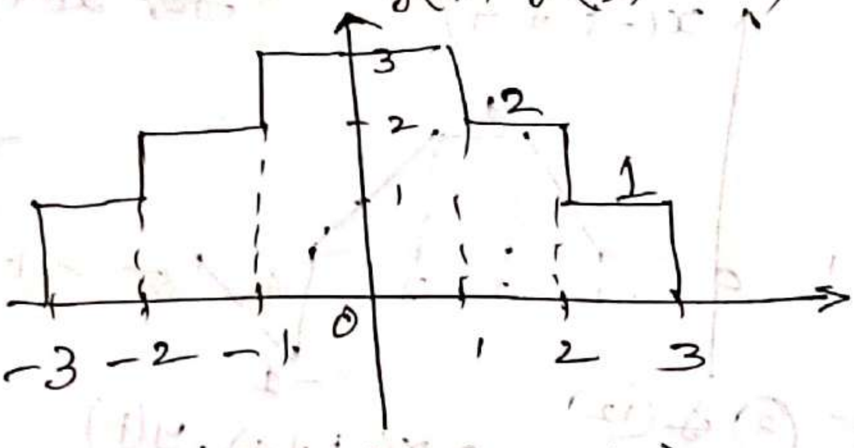
— (3)



— (4)

Add (1), (2), (3)

$g(t) + g\left(\frac{t}{2}\right) + g\left(\frac{t}{3}\right) = x(t)$

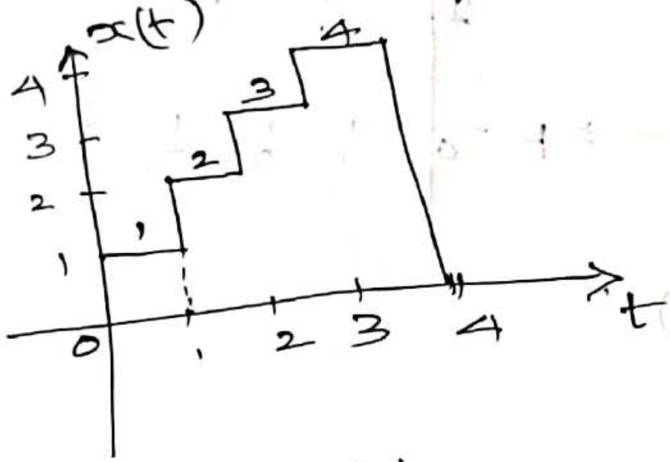


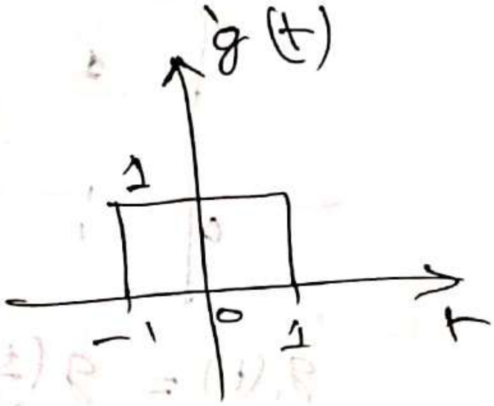
$$x(t) = g(t) + g\left(\frac{t}{2}\right) + g\left(\frac{t}{3}\right)$$

Scanned with CamScanner

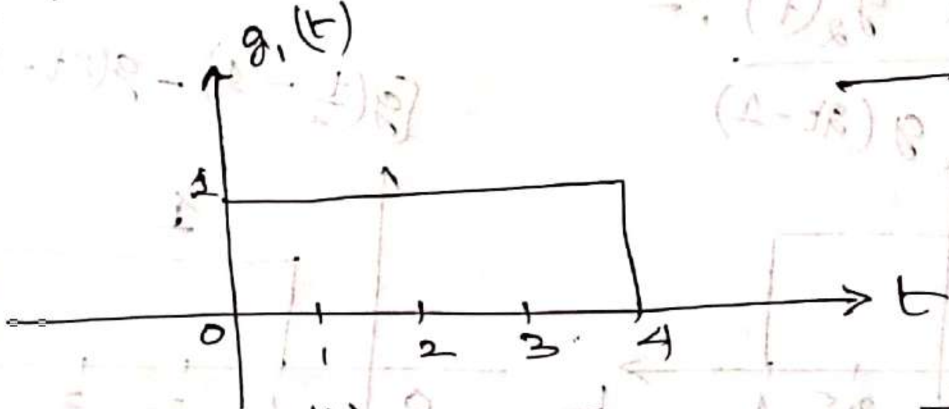
(4) Find the relationship between $x(t)$ and $g(t)$

Given

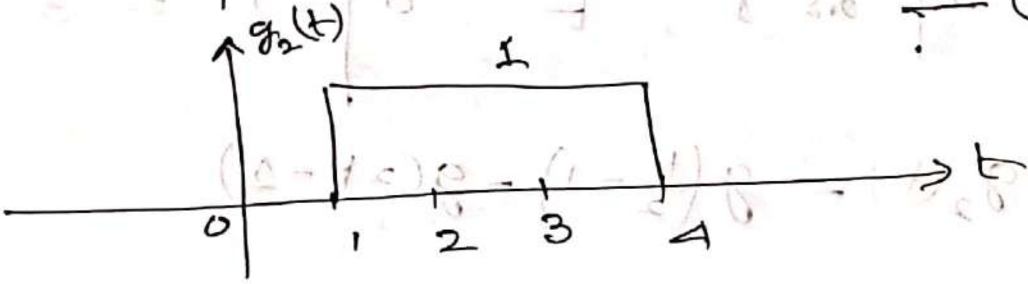




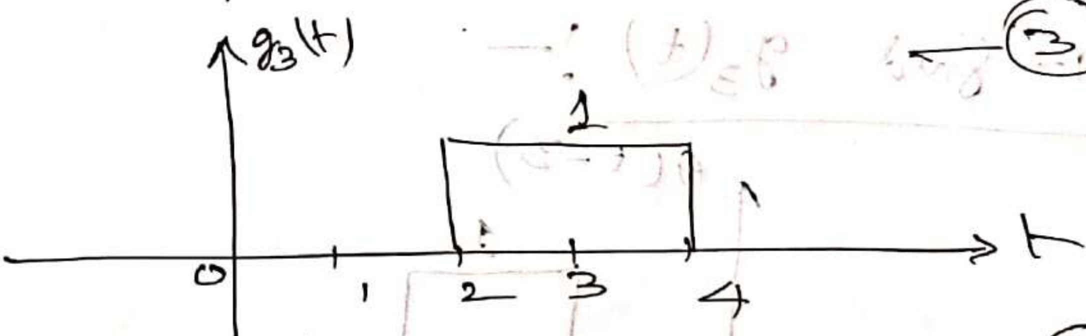
=



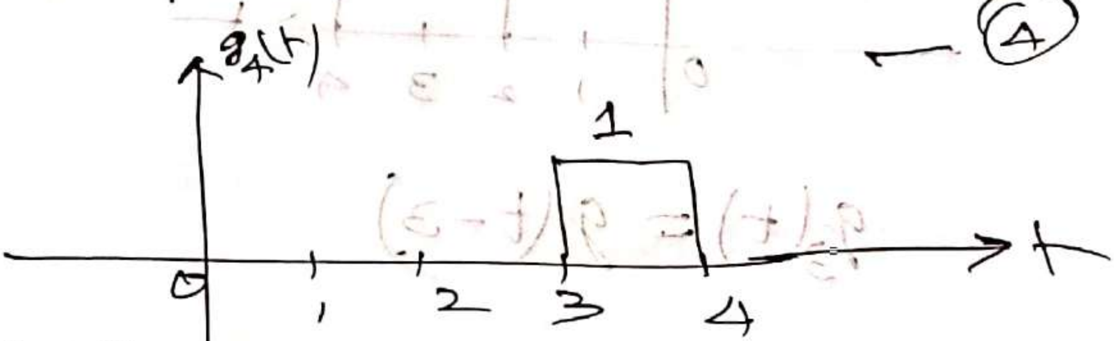
← (1)



← (2)



← (3)

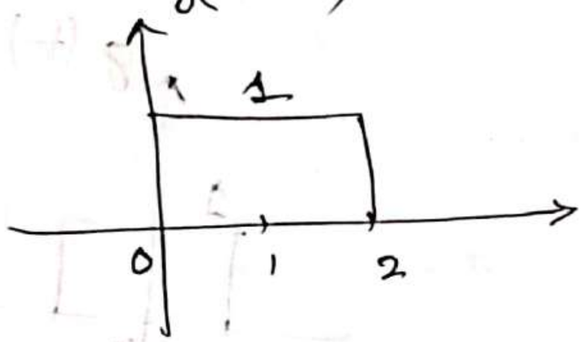


← (4)

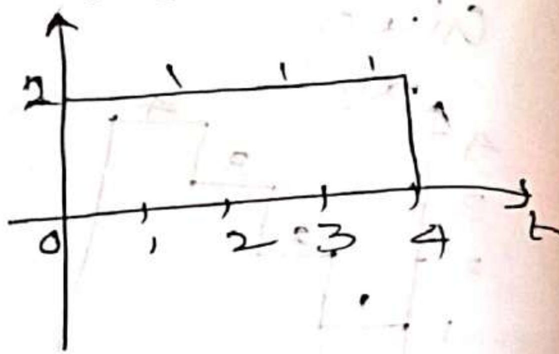
$x(t) = g_1(t) + g_2(t) + g_3(t) + g_4(t)$

To find $g_1(t)$:-

$g(t-1)$



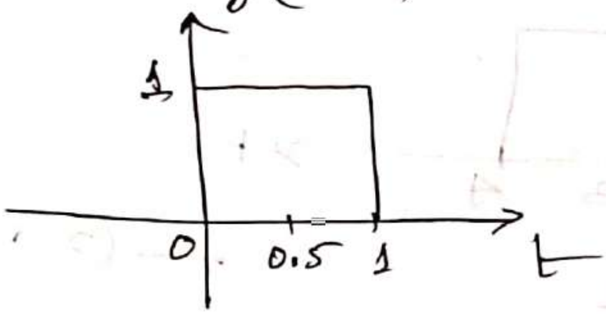
$g\left(\frac{t}{2}-1\right)$



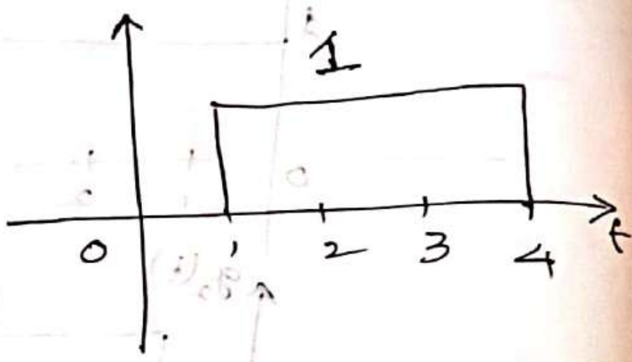
$$g_1(t) = g\left(\frac{t}{2}-1\right)$$

To find $g_2(t)$:-

$g(2t-1)$



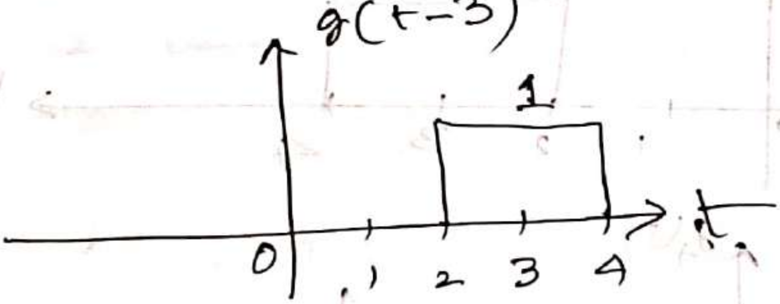
$\left[g\left(\frac{t}{2}-1\right) - g(2t-1)\right]$



$$g_2(t) = g\left(\frac{t}{2}-1\right) - g(2t-1)$$

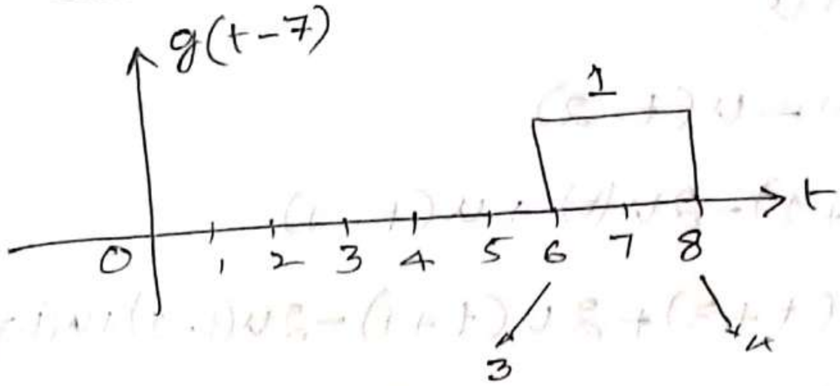
To find $g_3(t)$:-

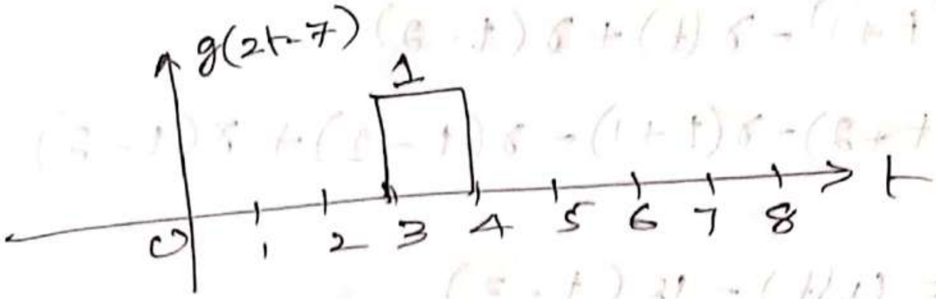
$g(t-3)$



$$g_3(t) = g(t-3)$$

To find $g_4(t)$:-





$$g_4(t) = g(2t-7)$$

$$x(t) = g_1(t) + g_2(t) + g_3(t) + g_4(t)$$

$$= g\left(\frac{t}{2}-1\right) + g\left(\frac{t}{2}-1\right) - g(2t-1)$$

$$+ g(t-3) + g(2t-7)$$

(4) Sketch the waveforms of the following signals

(i) $x(t) = u(t) - u(t-2)$

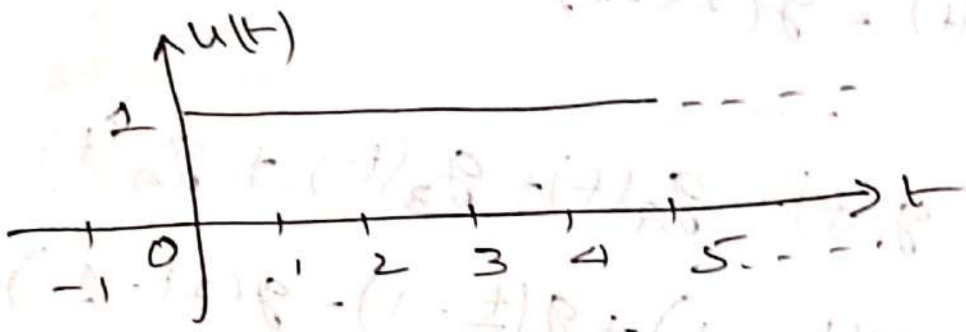
(ii) $x(t) = u(t+1) - 2u(t) + u(t-1)$

(iii) $x(t) = -u(t+3) + 2u(t+1) - 2u(t-1) + u(t-3)$

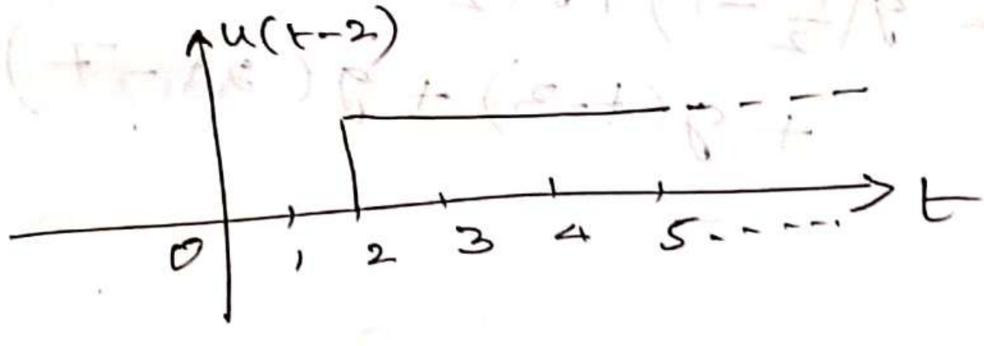
(iv) $y(t) = r(t+1) - r(t) + r(t-2)$

(v) $y(t) = r(t+2) - r(t+1) - r(t-1) + r(t-2)$

= (i) $x(t) = u(t) - u(t-2)$



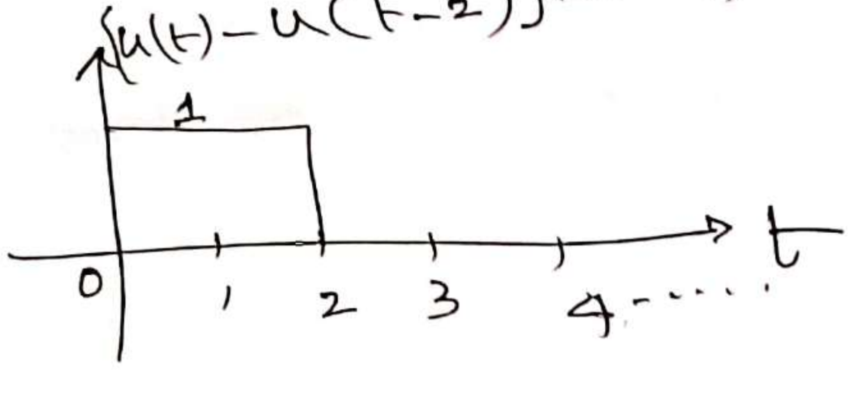
— (1)



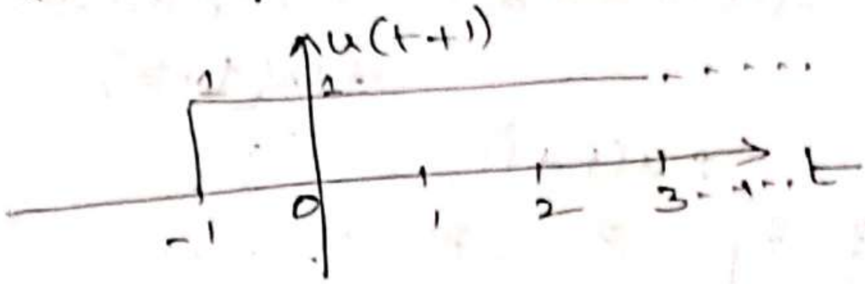
— (2)

(1) − (2)

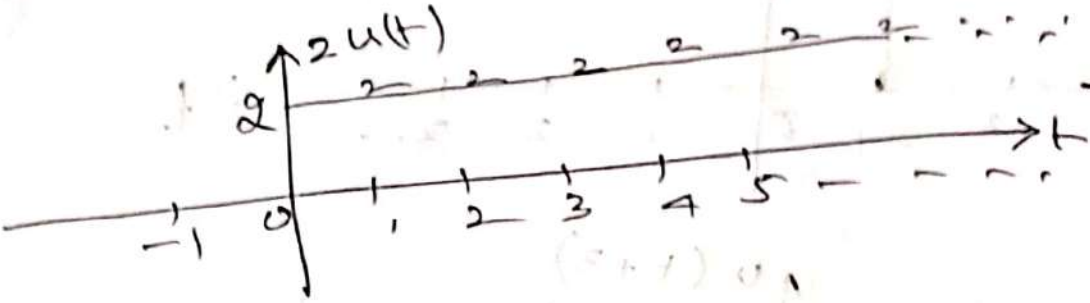
$[u(t) - u(t-2)] = x(t)$



(ii) $x(t) = u(t+1) - 2u(t) + u(t-1)$

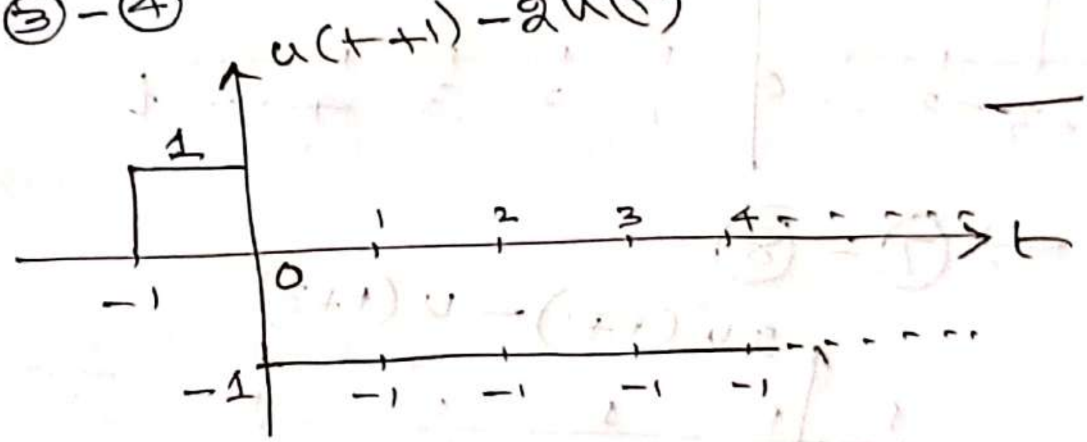


— (3)

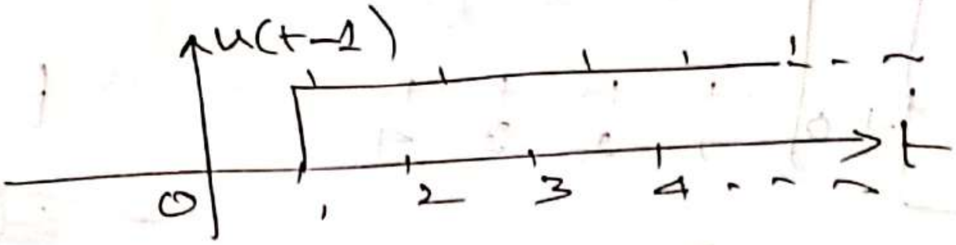


— (4)

(3) − (4)

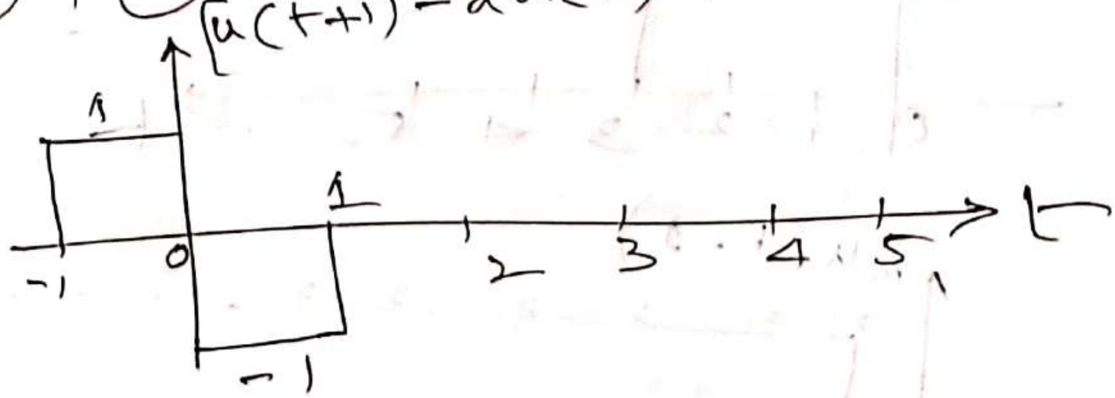


— (5)

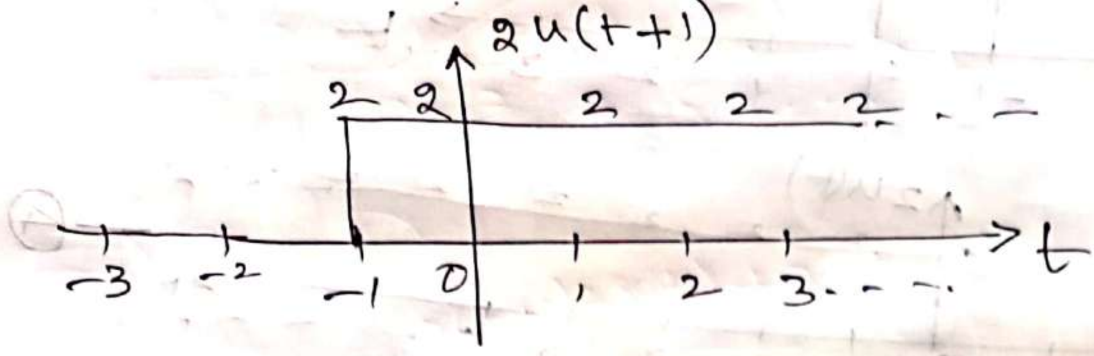


— (6)

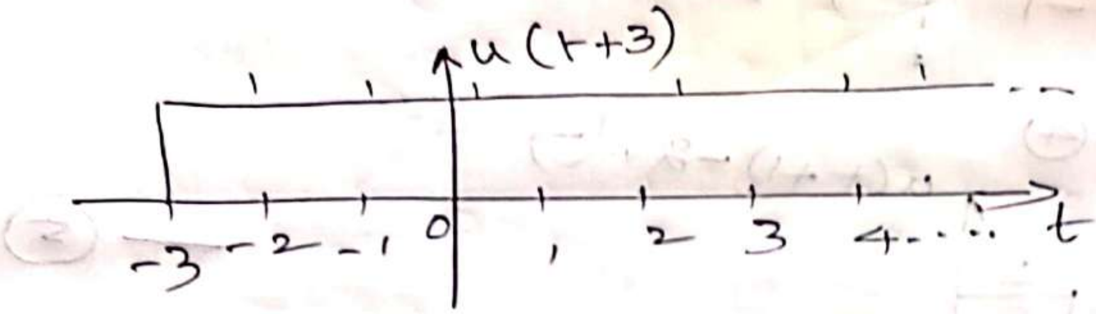
(5) + (6) $[u(t+1) - 2u(t) + u(t-1)] = x(t)$



(iii) $x(t) = -u(t+3) + 2u(t+1) - 2u(t-1) + u(t-3)$

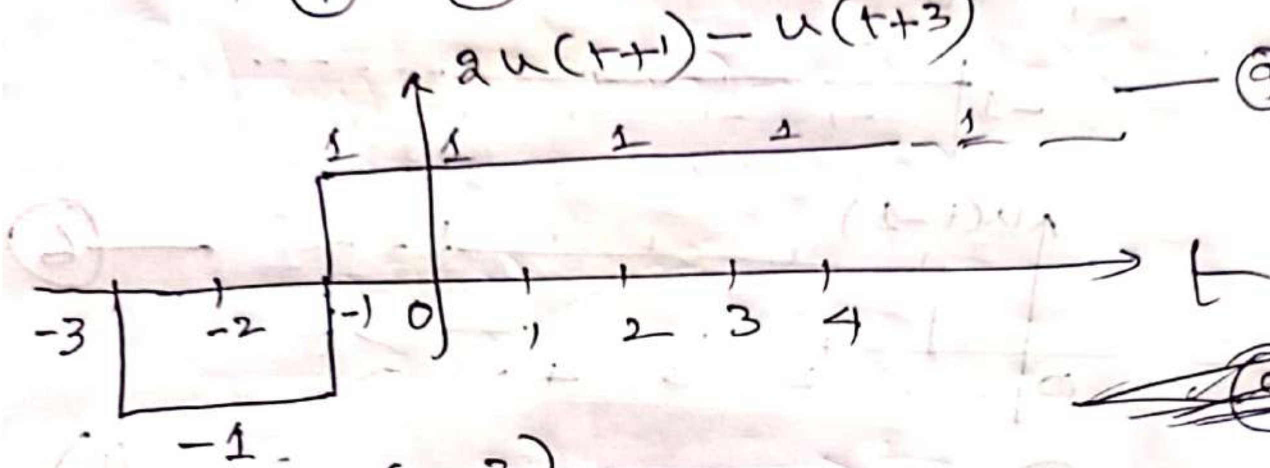


— (7)

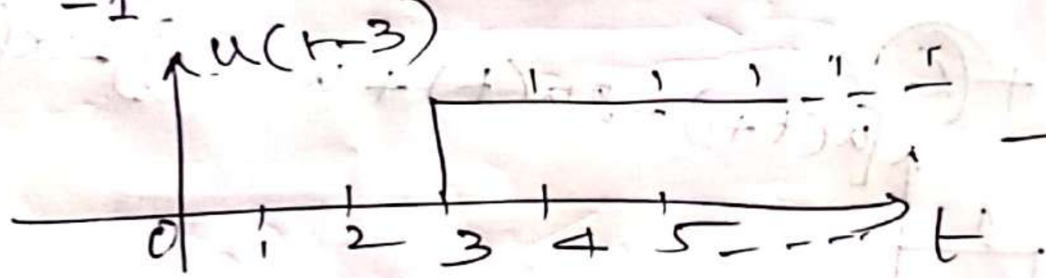


— (8)

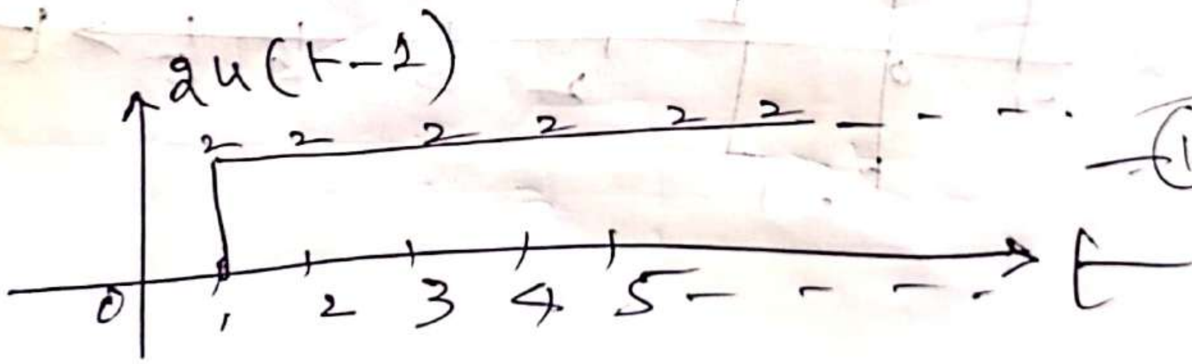
→ (7) − (8)



— (9)



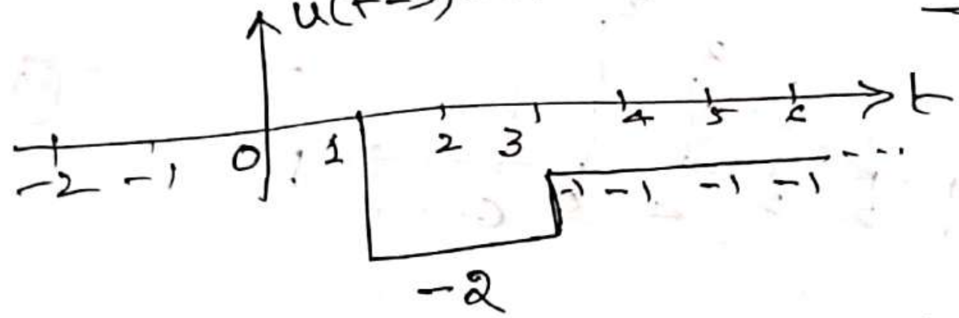
— (10)



— (11)

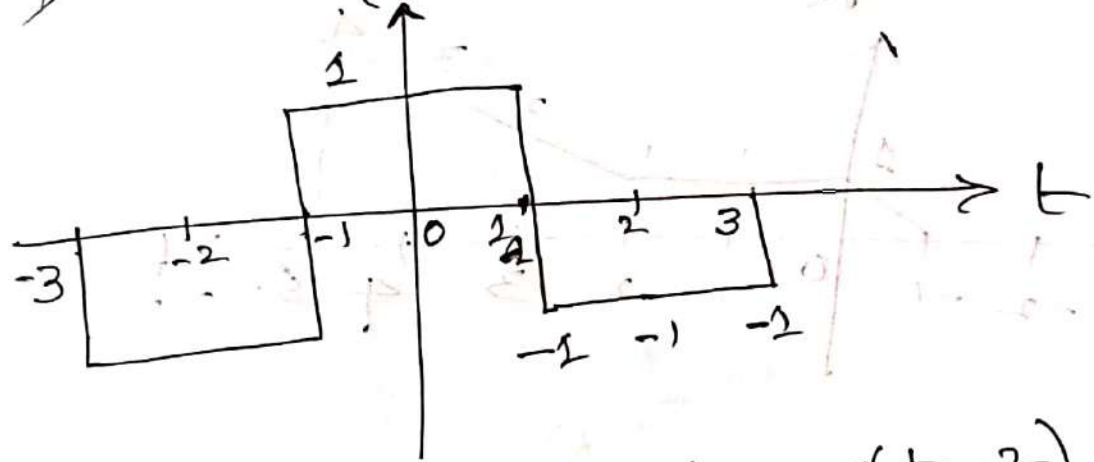
→ 10 − 11

$u(t-3) - 2u(t-1)$ ← 12



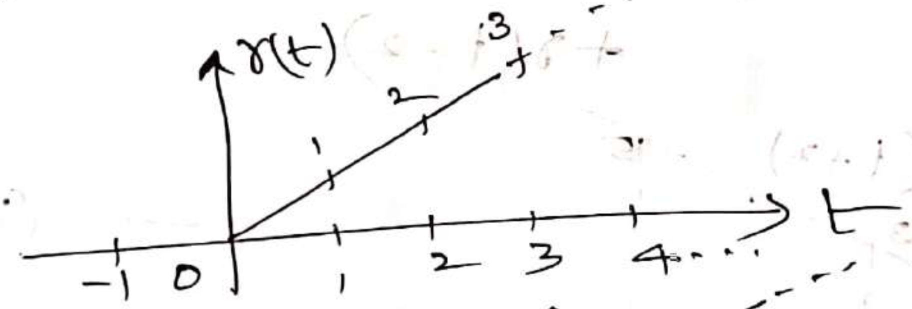
⇒ 9 + 12

$[2u(t+1) - u(t+3) + u(t-3) - 2u(t-1)] = x(t)$

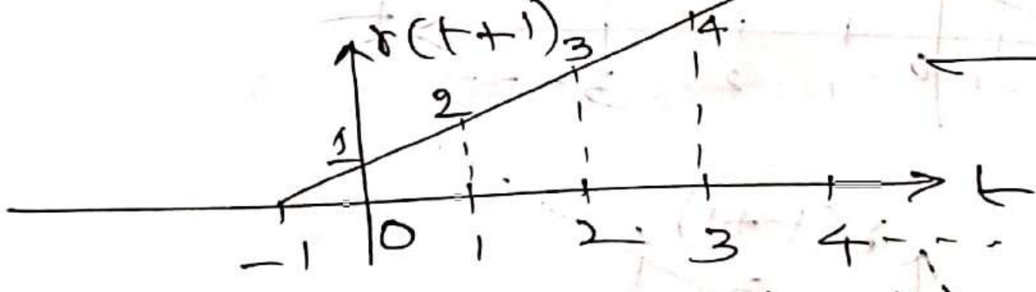


(iv) $y(t) = r(t+1) - r(t) + r(t-2)$

$r(t)$ ← 1

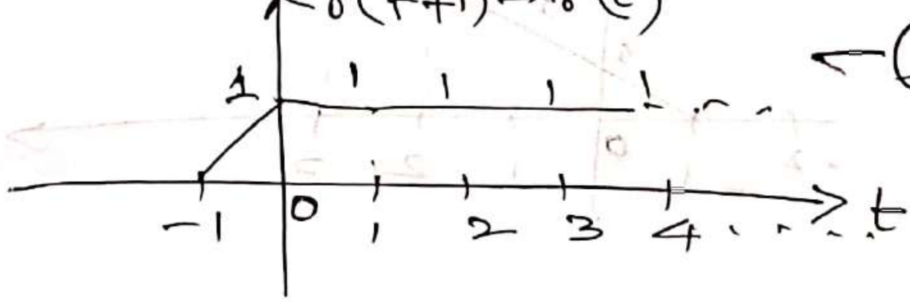


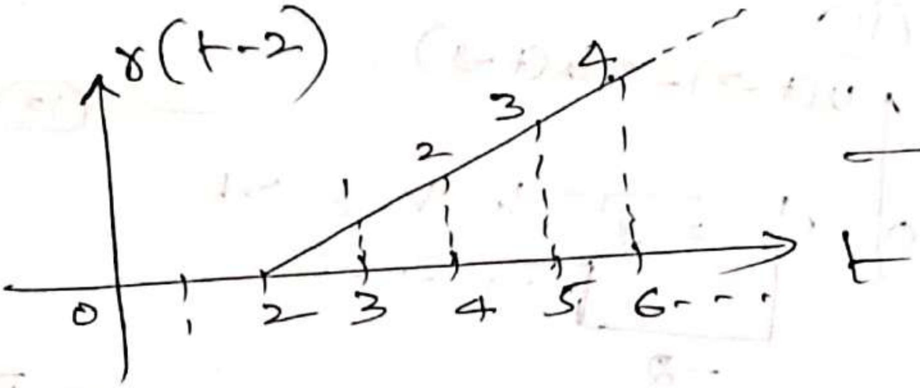
$r(t+1)$ ← 2



⇒ 2 − 1

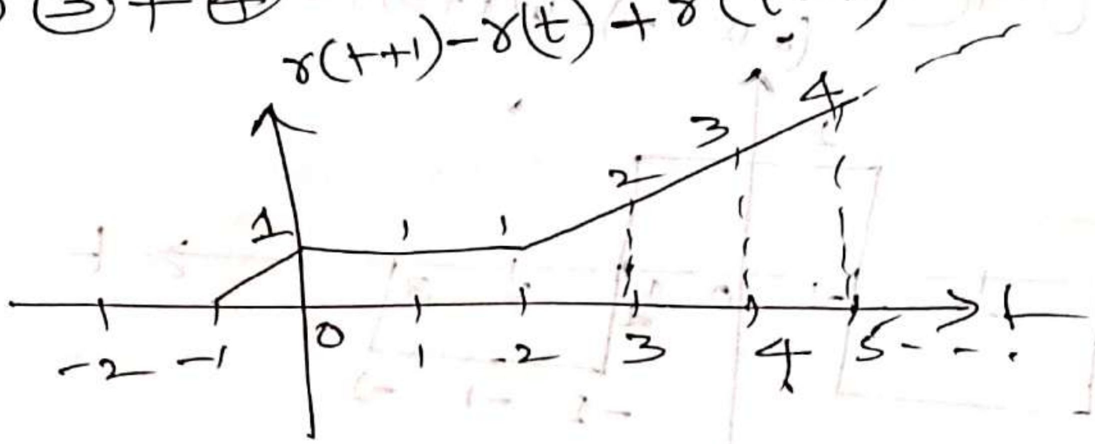
$r(t+1) - r(t)$ ← 3



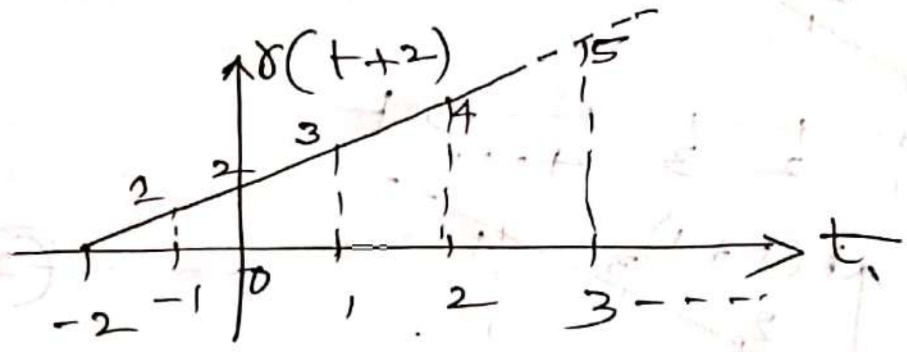


— (4)

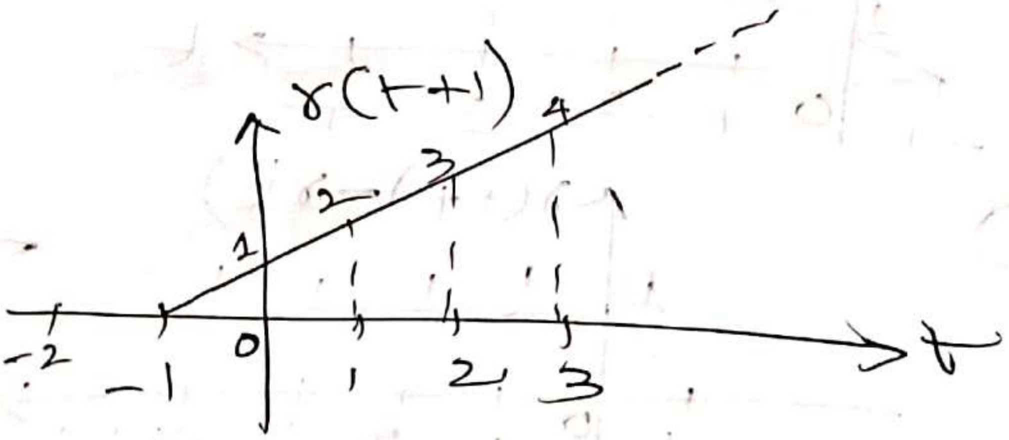
$\Rightarrow (3) + (4)$ $r(t+1) - r(t) + r(t-2)$



(v) $y(t) = r(t+2) - r(t+1) - r(t-1) + r(t-2)$



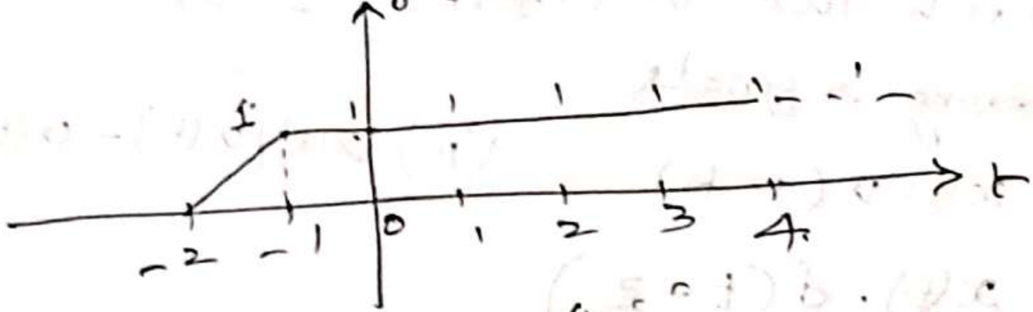
— (1)



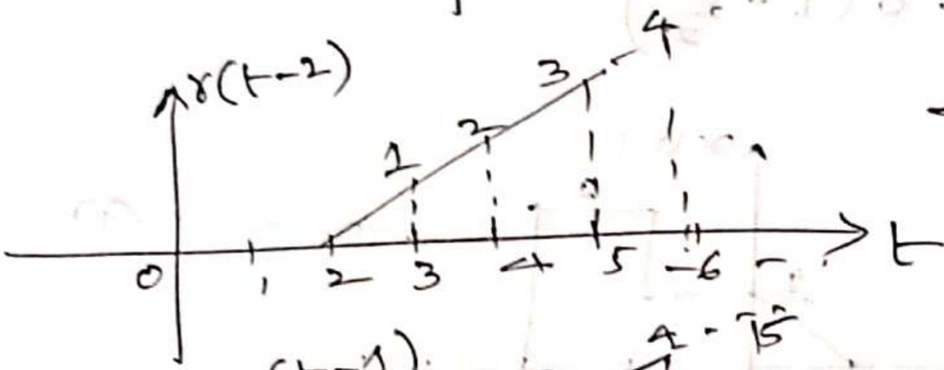
— (2)

$\Rightarrow$ ① − ②

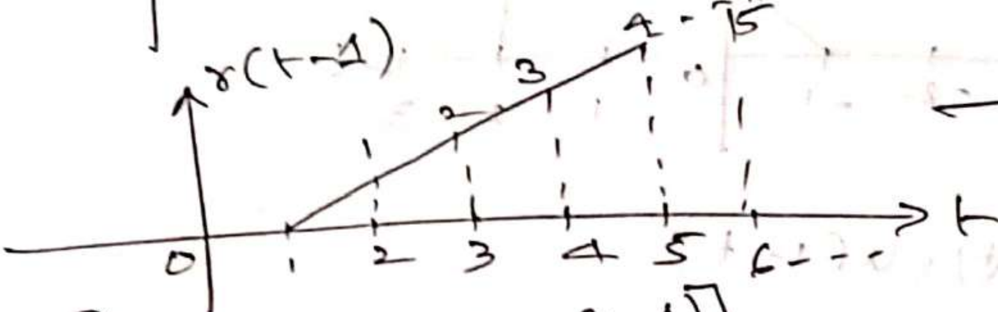
$r(t+2) - r(t+1)$



— ③



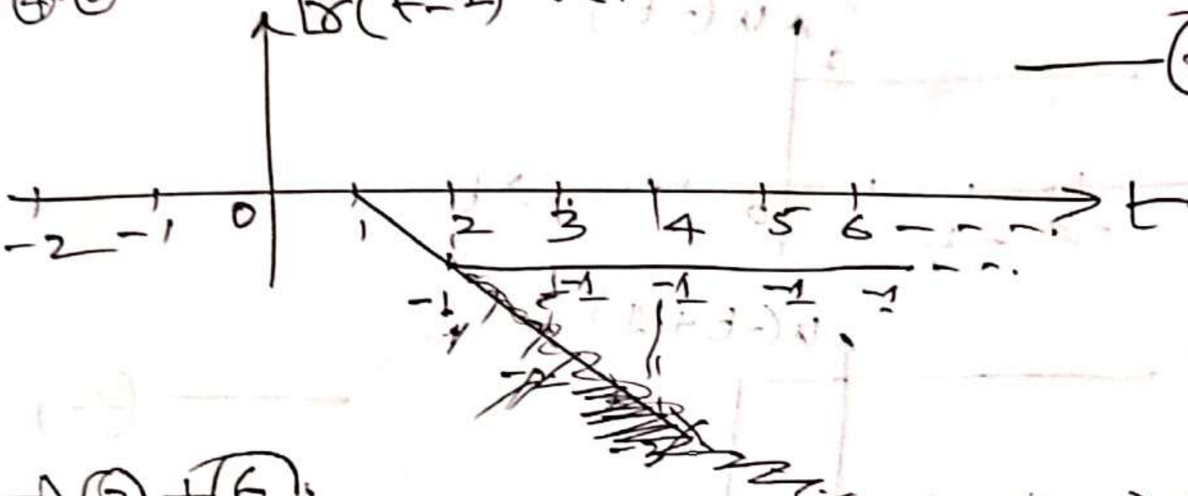
— ④



— ⑤

$\Rightarrow$ ④ − ⑤

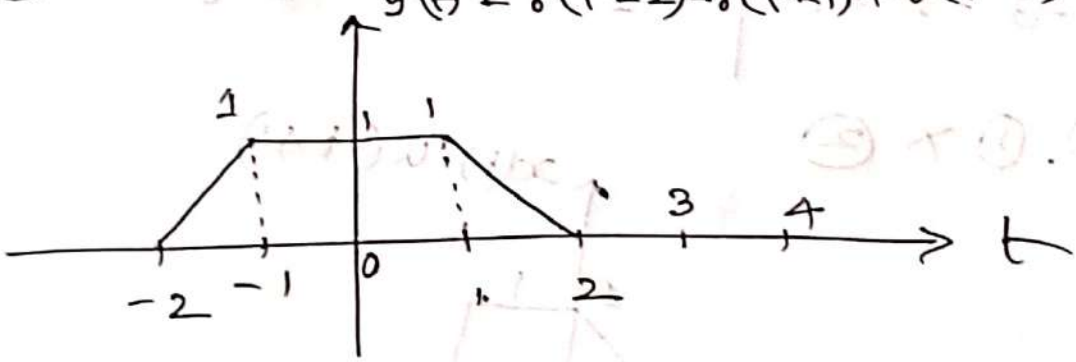
$[r(t-2) - r(t-1)]$



— ⑥

$\Rightarrow$ ③ + ⑥

$y(t) = r(t+2) - r(t+1) + r(t-2) - r(t-1)$

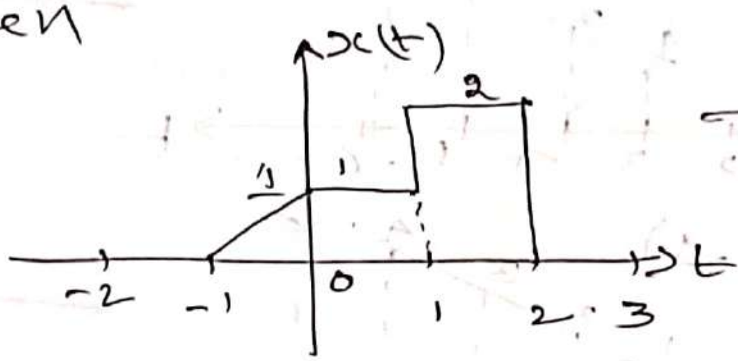


(5) Sketch the waveforms of the following signals

(i) $x(t)\,u(1-t)$ (ii) $x(t)[u(t)-u(t-1)]$

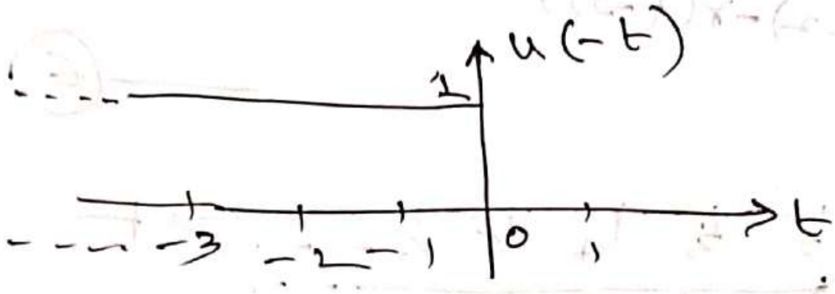
(iii) $x(t)\cdot\delta(t-\frac{3}{2})$

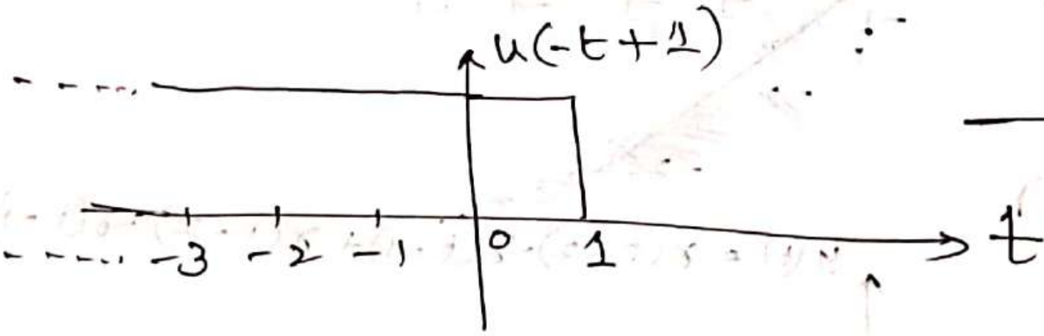
Given



———— (1)

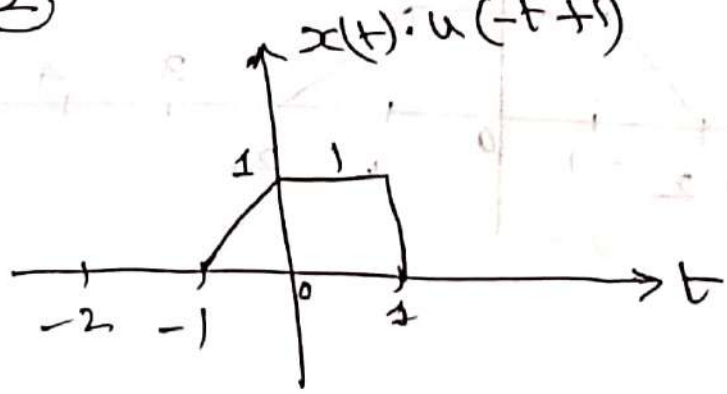
(i) $x(t)\cdot u(1-t)$



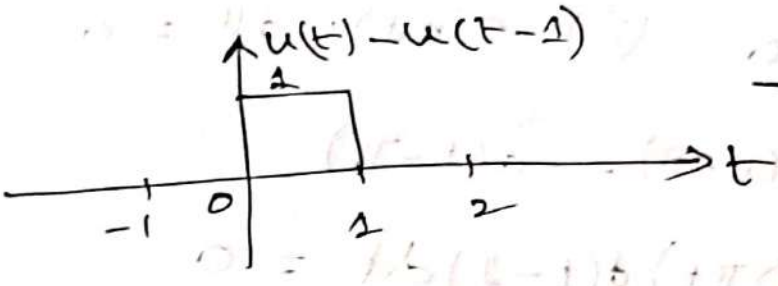


———— (2)

$\Rightarrow$ (1) × (2)

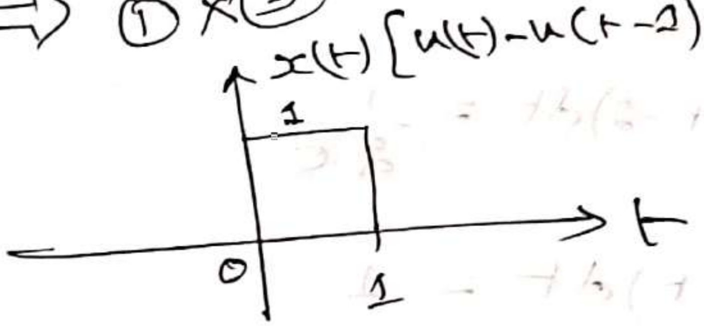


(ii) $x(t)\,[u(t)-u(t-1)]$

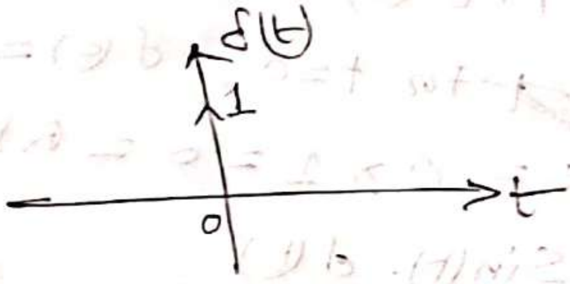


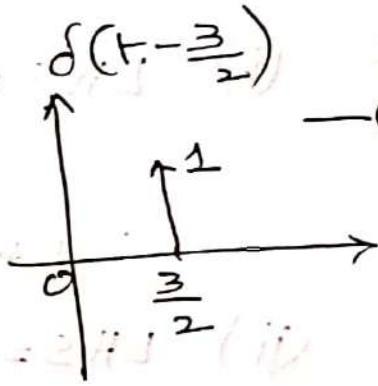
—— ③

$\Longrightarrow$ ① × ③



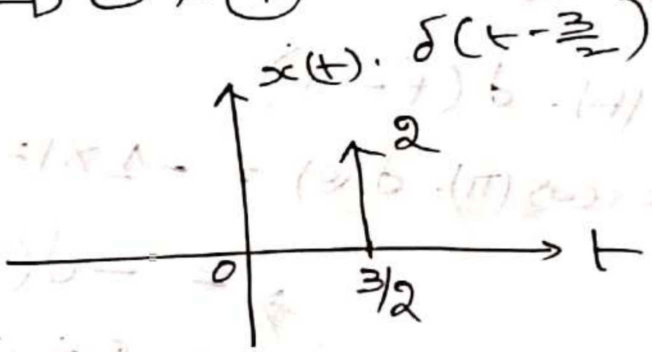
(iii) $x(t)\cdot\delta\left(t-\frac{3}{2}\right)$





—— ④

$\Longrightarrow$ ① × ④



⑥ Show that

(i) $t \cdot \delta(t) = 0$ (ii) $\sin(t) \cdot \delta(t) = 0$

(iii) $\cos(t) \cdot \delta(t-\pi) = -\delta(t-\pi)$

(iv) $\int_{-\infty}^{\infty} (t^2 + \cos \pi t)\, \delta(t-1)\, dt = 0$

(v) $\int_{-\infty}^{\infty} e^{-t} \cdot \delta(2t-2)\, dt = \frac{1}{2e}$

(vi) $\int_{-\infty}^{\infty} e^{-t} \cdot \delta(t)\, dt = 1$

= (i) $LHS = t \cdot \delta(t)$

For $t=0$, $\delta(0) = 1$

$\therefore LHS = 0 \times 1 = 0 = RHS$

(ii) $LHS = \sin(t) \cdot \delta(t)$

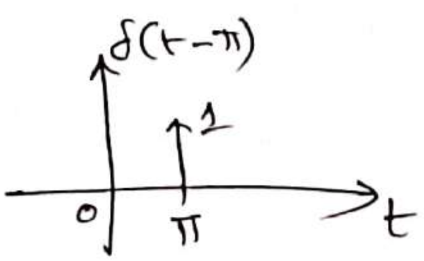
$= \sin(0) \cdot \delta(0) = 0 \times 1 = 0 = RHS$

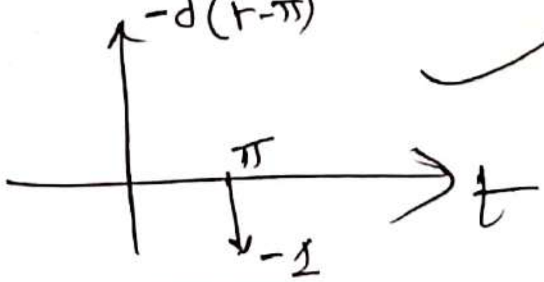
(iii) $LHS = \cos(t) \cdot \delta(t-\pi)$

For $t=\pi$, $= \cos(\pi) \cdot \delta(0) = -1 \times 1 = -1$

$= -\delta(t-\pi)$

$= RHS$





(iv) $LHS = \int_{-\infty}^{\infty} (t^2 + \cos \pi t)\, \delta(t-1)\, dt$

$= \int_{1} (t^2 + \cos \pi t) \cdot \delta(t-1)\, dt$

$= (1^2 + \cos \pi)\, \delta(0) = (1-1)\, 1 = 0 = RHS$

(v) $LHS = \int_{-\infty}^{\infty} e^{-t}\, \delta(2t-2)\, dt$

Let $2t = u$, $t = \frac{u}{2}$

$dt = \frac{du}{2}$

$LHS = \int_{-\infty}^{\infty} e^{-u/2} \cdot \delta(u-2) \cdot \frac{du}{2}$

$= \frac{1}{2} \int_{-\infty}^{\infty} e^{-u/2}\, \delta(u-2)\, du$

$= \frac{1}{2} \int_{2} e^{-u/2}\, \delta(u-2)\, du$

$= \frac{1}{2} \cdot e^{-2/2} \cdot \delta(0) = \frac{1}{2e} = RHS$

(vi) $LHS = \int_{-\infty}^{\infty} e^{-t}\, \delta(t) \cdot dt$

$= \int_{0} e^{-t}\, \delta(t)\, dt$

$= e^{-0}\, \delta(0) = 1 \times 1 = 1 = RHS$

(7) Sketch the graph of the following signals

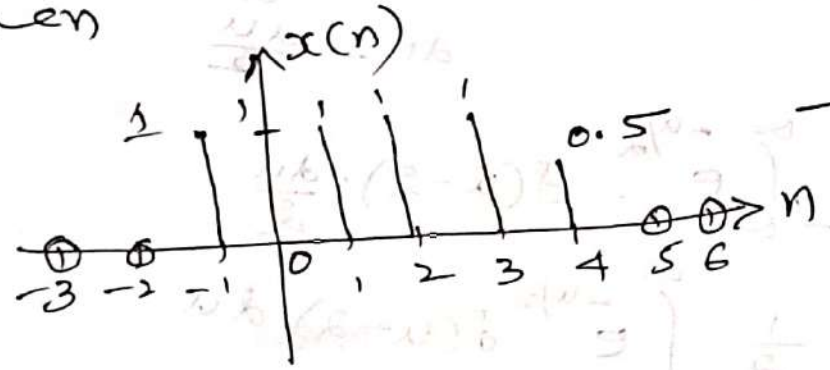
(i) $x(4-n)$ (ii) $x(2n+1)$

(iii) $x(n)\,u(2-n)$ (iv) $x(n-1)\,\delta(n-3)$

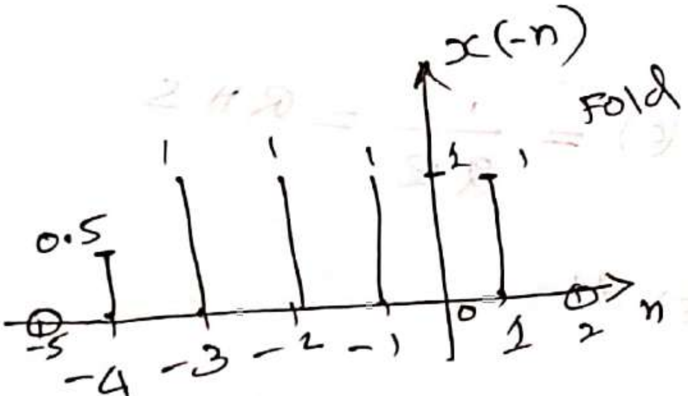
(v) $\frac{1}{2}x(n)+\frac{1}{2}(-1)^n x(n)$

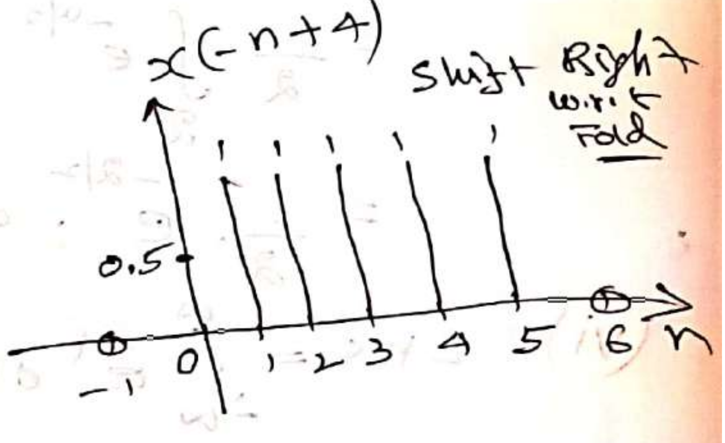
(vi) $x(3n-1)$ (vii) $x(2-2n)$

Given

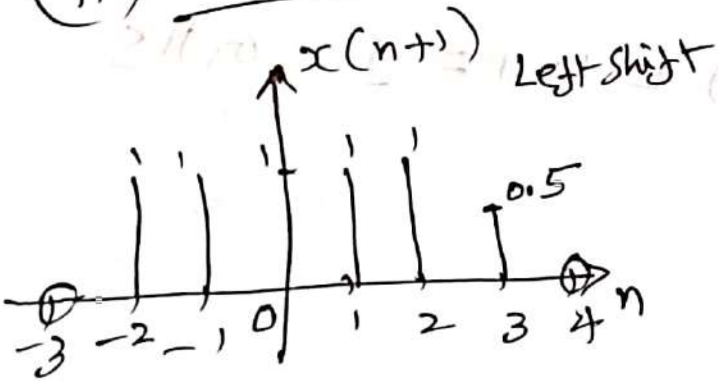


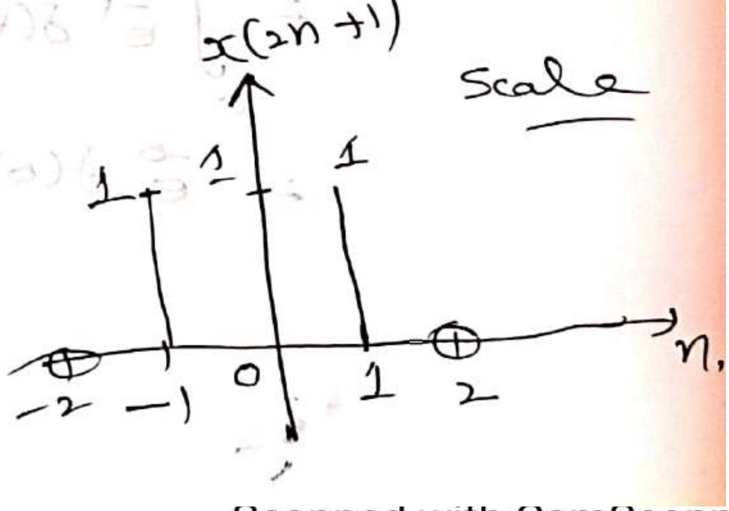
(i) $x(4-n)$:—



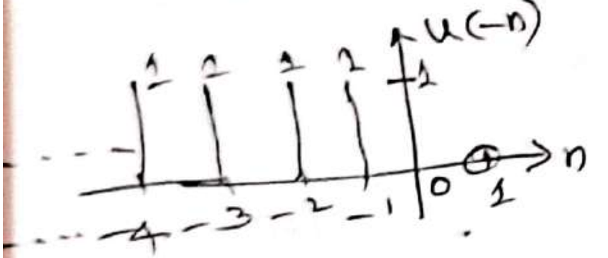


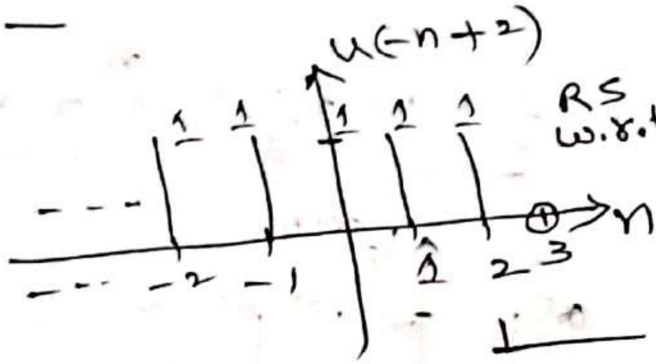
(ii) $x(2n+1)$:—





(iii) $\underline{x(n)\, u(2-n)}$:—

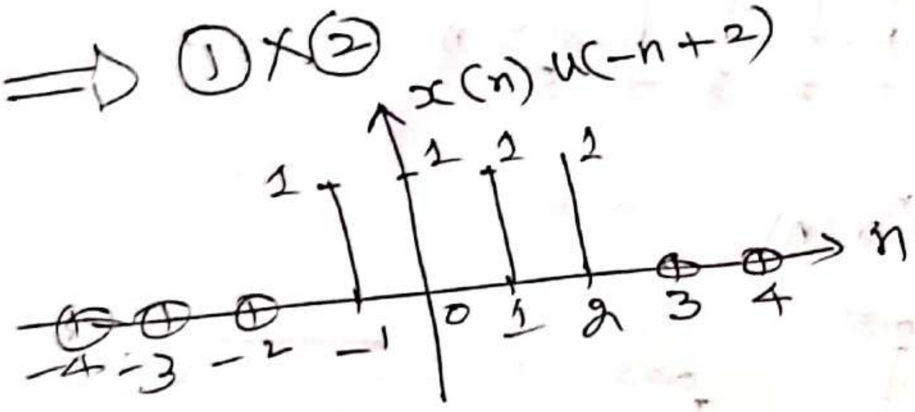




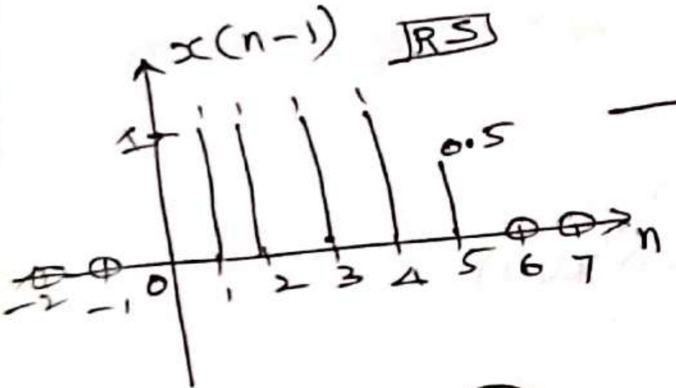
RS w.r.t. Fold

— (2)

$\Rightarrow$ (1) × (2)

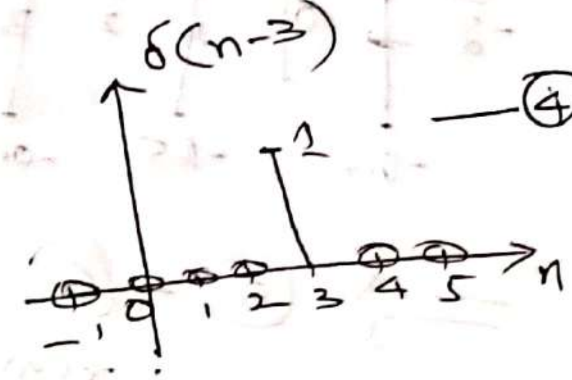


(iv) $\underline{x(n-1) \cdot \delta(n-3)}$:—



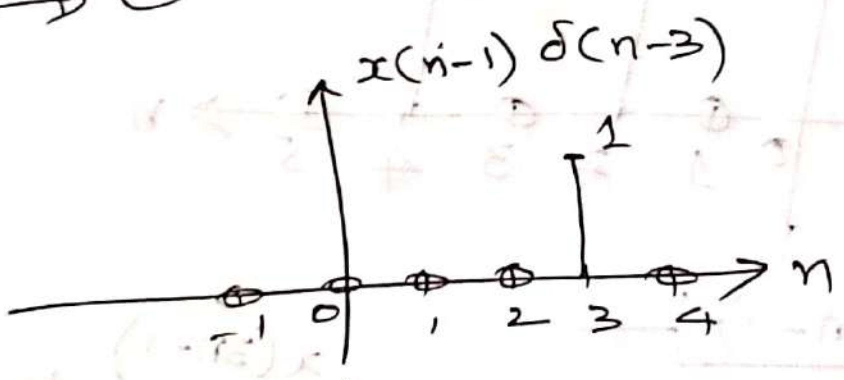
$\boxed{RS}$

— (3)

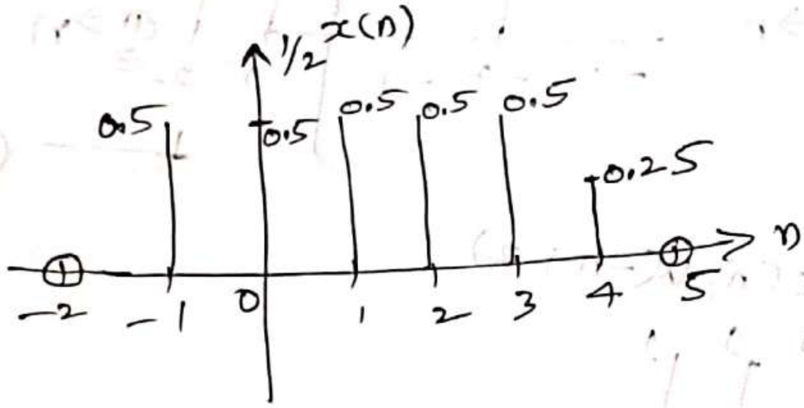


— (4)

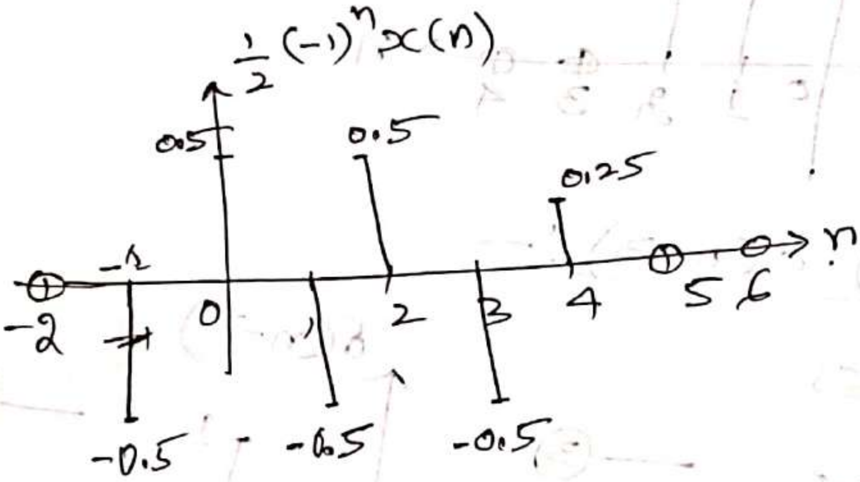
$\Rightarrow$ (3) × (4)



(v) $\underline{\frac{1}{2}x(n) + \frac{1}{2}(-1)^n x(n)}$:—

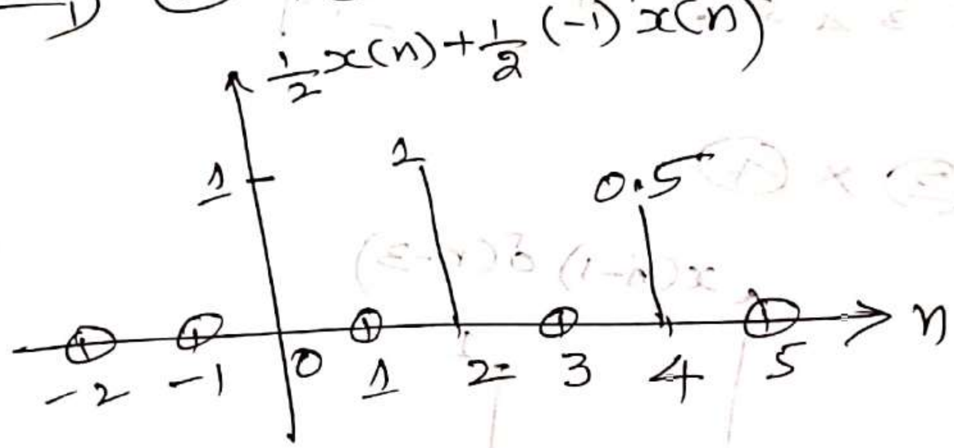


— (5)

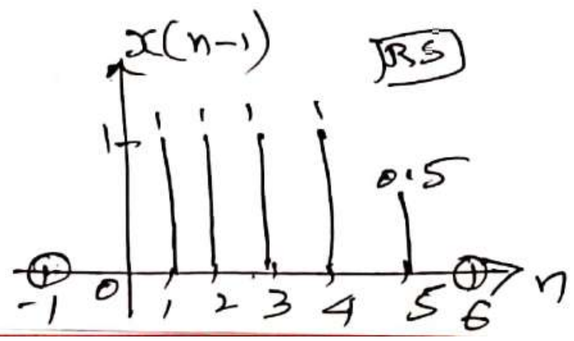


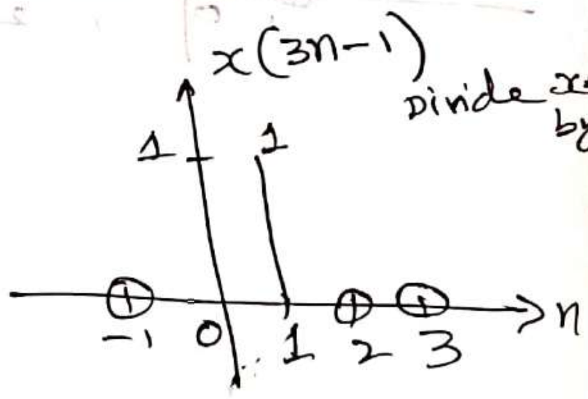
— (6)

$\Rightarrow$ (5) + (6)



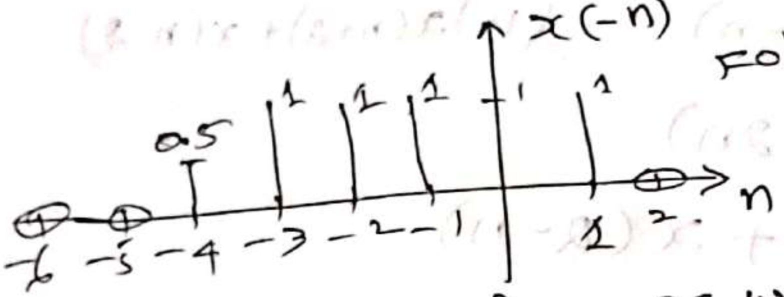
(vi) $\underline{x(3n-1)}$:—





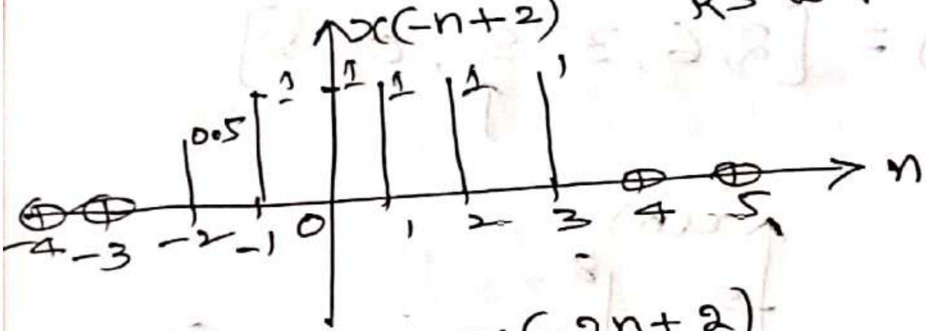
Divide x-axis by 3

(vii) $x(2-2n)$:—

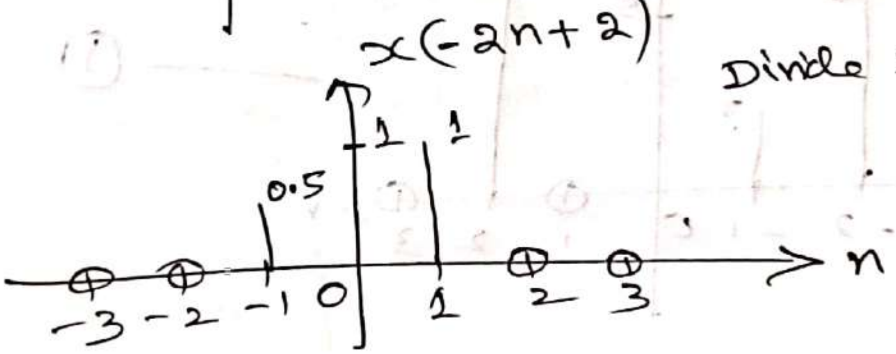


Fold.

RS w.r.t. Fold



Divide x-axis by 2



(52)

(8) Sketch the following signals

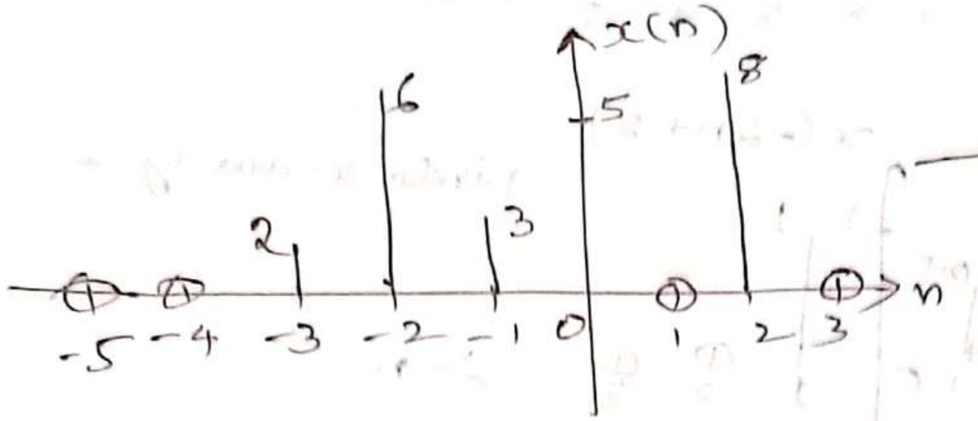
(i) $x(n) + 2x(-n)$ (ii) $x(n+2) + x(n-2)$

(iii) $x(2n) + x(-2n)$

(iv) $x(2n+1) + x(2-n)$

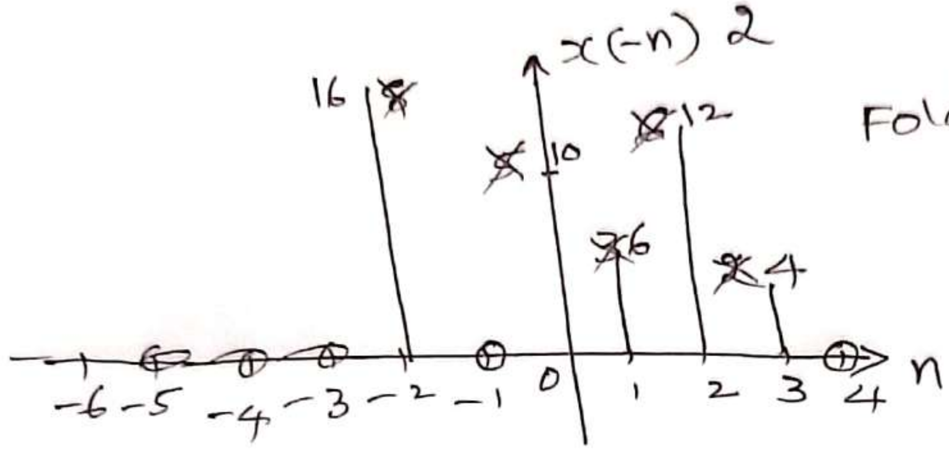
Given $x(n) = [2, 6, 3, \underset{\uparrow}{5}, 0, 8]$

=



—— (1)

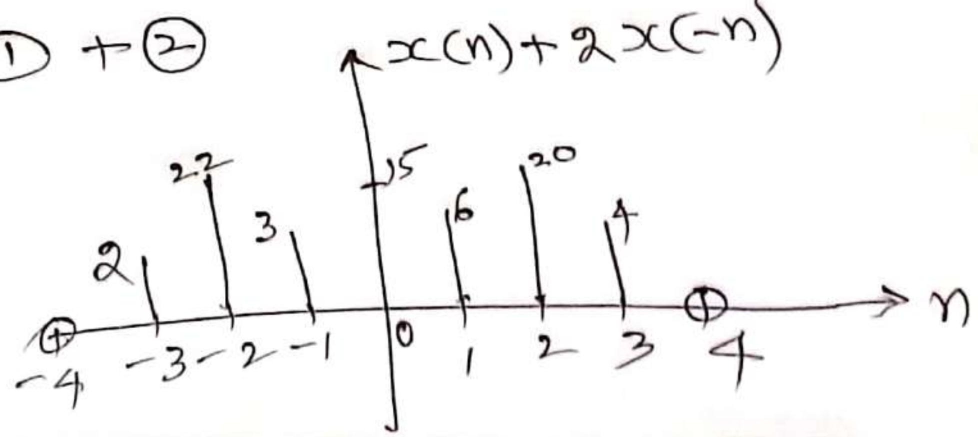
(i) $\underline{x(n) + 2x(-n)}$:—



Fold

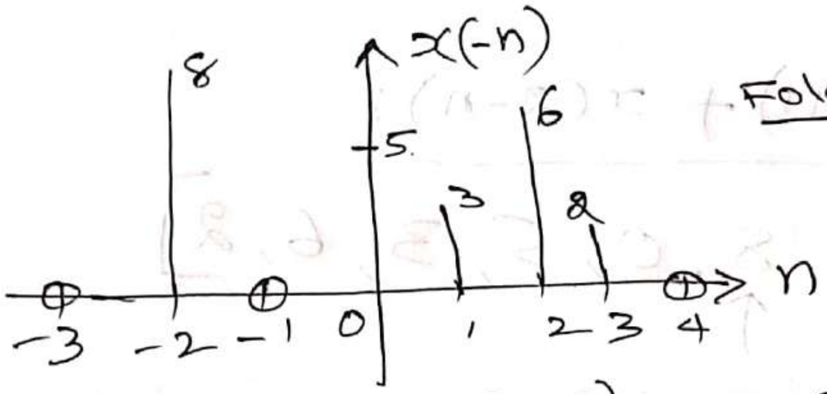
—— (2)

$\Rightarrow$ (1) + (2)

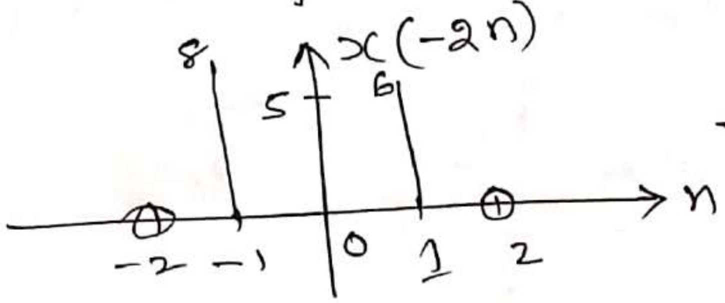


$$[x(n) + 2x(-n)] = [2, 22, 3, \underset{\uparrow}{15}, 6, 20, 4]$$

(iii) $\underline{x(2n) + x(-2n)}$:-

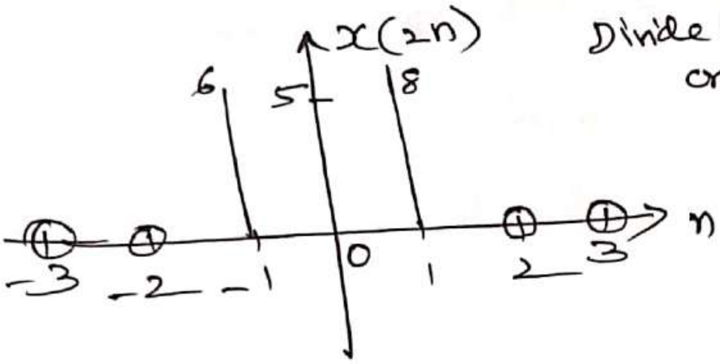


Fold



Divide x-axis by 2 on Fold

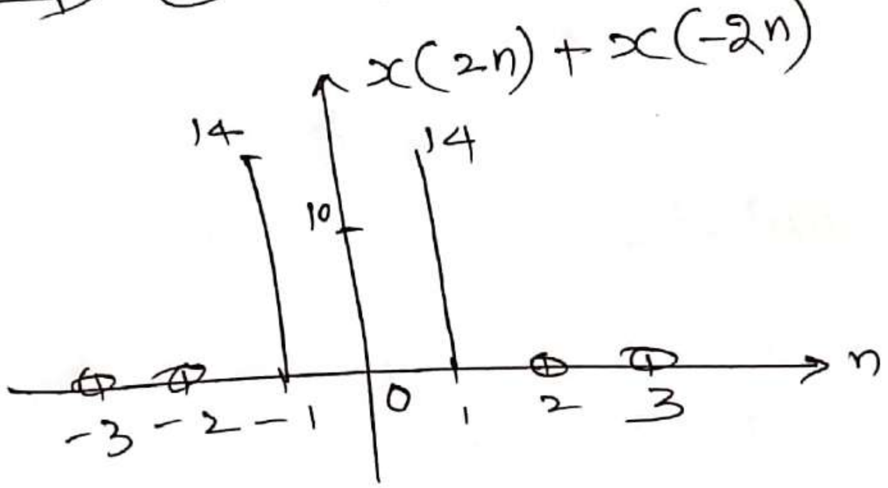
3



Divide x-axis by 2 on x(n)

4

$\Longrightarrow$ 3 + 4



$$[x(2n) + x(-2n)] = [14, \underset{\uparrow}{10}, 14]$$

(iii) $x(n+2) + x(n-2)$:-

$= [2, 6, 3, 5, 2, 14, 3, 5, 0, 8]$

(iv) $x(2n+1) + x(2-n)$:-

$= [2, 3, 8, 0, 5, 3, 6, 2]$

⑨ Find and sketch the derivatives of the following signals

(i) $x(t) = u(t) - u(t-t_0)$, $t_0 > 0$

(ii) $x(t) = t[u(t) - u(t-t_0)]$, $t_0 > 0$

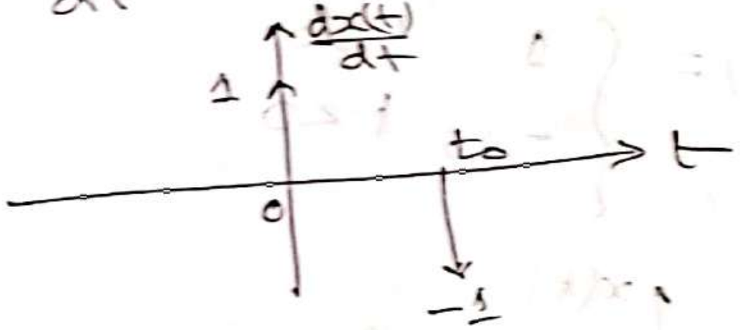
(iii) $x(t) = sgn(t) = \begin{cases} 1, & t > 0 \\ -1, & t < 0 \end{cases}$

= (i) $x(t) = u(t) - u(t-t_0)$

D. w.r. t

$$\frac{dx(t)}{dt} = \frac{du(t)}{dt} - \frac{d[u(t-t_0)]}{dt}$$

$$\frac{dx(t)}{dt} = \delta(t) - \delta(t-t_0)$$

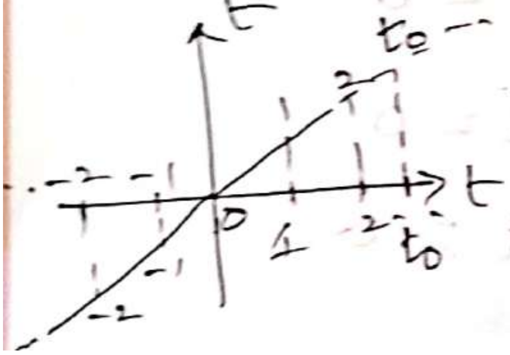


— ①

(ii) $x(t) = t[u(t) - u(t-t_0)]$

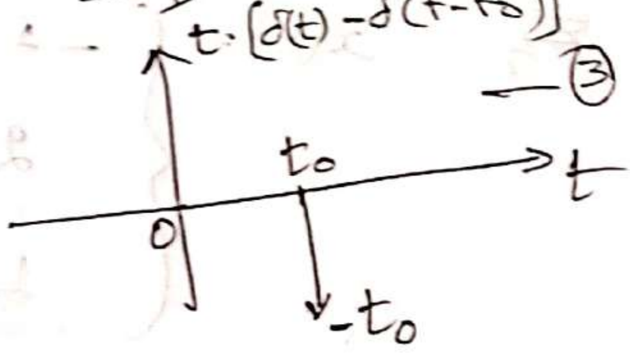
D. w.r.t.

$$\frac{dx(t)}{dt} = t\cdot[\delta(t) - \delta(t-t_0)] + [u(t) - u(t-t_0)]\times 1$$



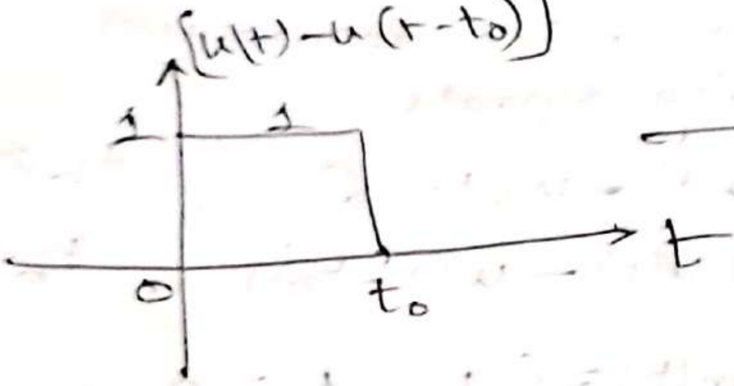
— ②

$\Longrightarrow$ ① × ②



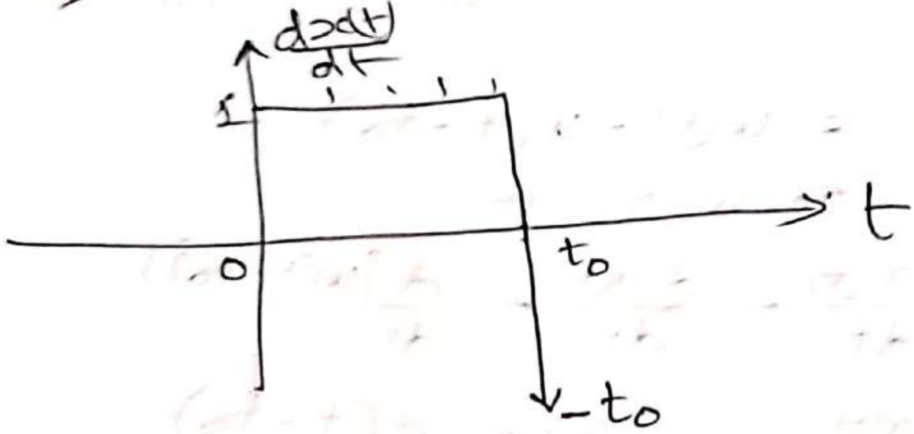
— ③

(5)

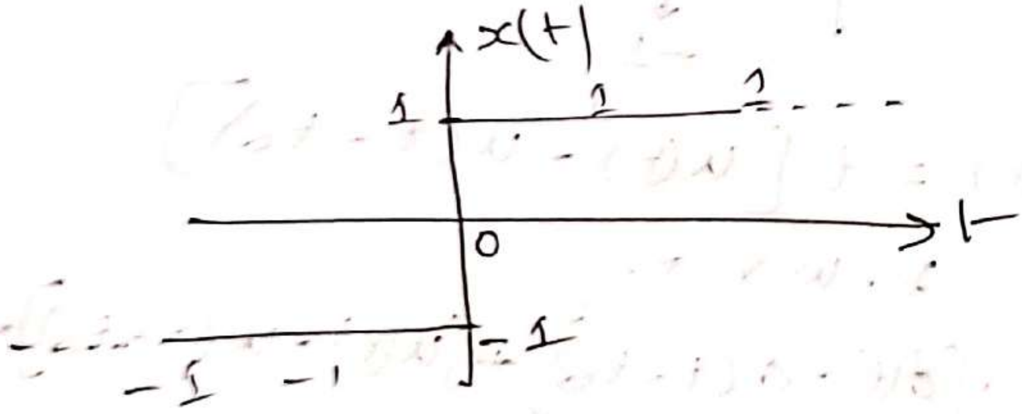


—— (4)

$\Longrightarrow$ (3) + (4)



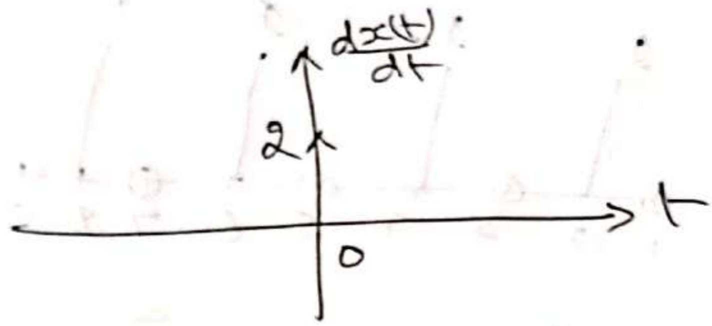
(iii) $x(t) = \begin{cases} 1, & t > 0 \\ -1, & t < 0 \end{cases}$



$x(t) = \begin{cases} -1, & t < 0 \\ 2, & t = 0 \\ 1, & t > 0 \end{cases}$

D.w.r.t t

$$\frac{dx(t)}{dt} = \begin{cases} 0, & t<0 \\ 2, & t=0 \\ 0, & t>0 \end{cases}$$



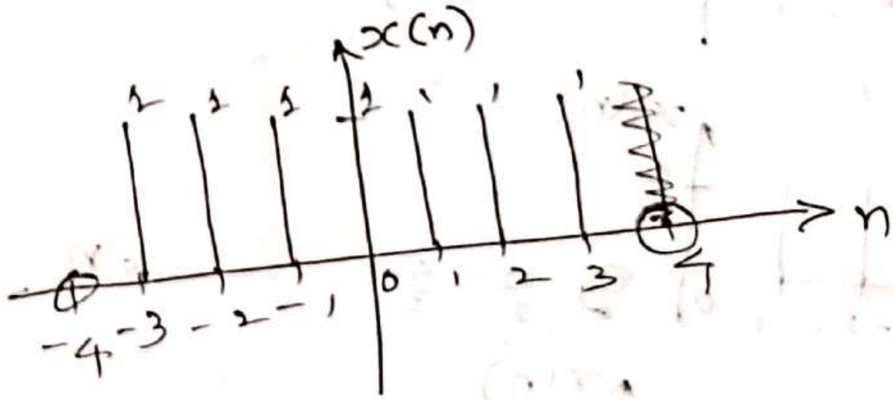
(10) Sketch the following signals

(i) $x(n) = \sum_{k=-3}^{3} \delta(n-k)$

(ii) $x(n) = \sum_{k=0}^{\infty} \delta(n-2.k)$

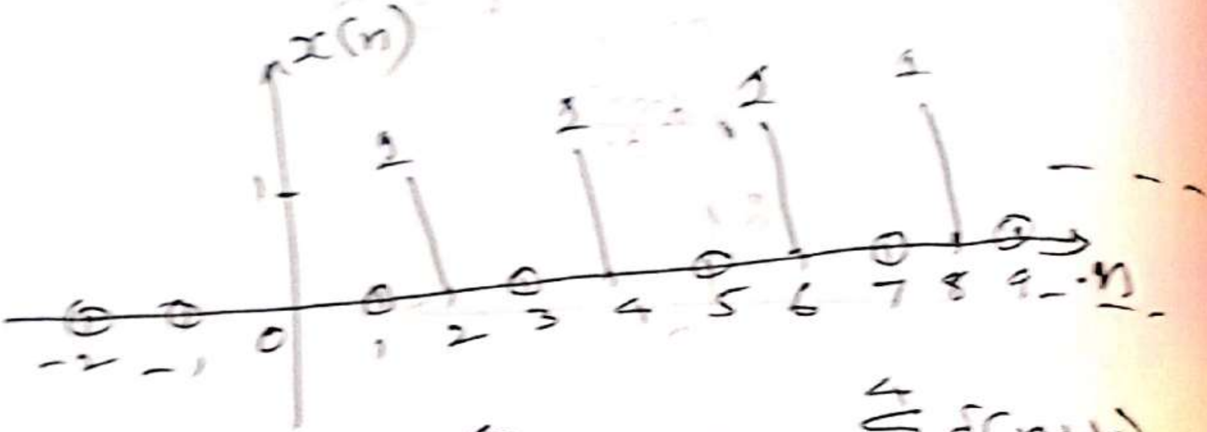
(iii) $x(n) = \sum_{k=0}^{4} [\delta(n-k) + \delta(n+k)]$

= (i) $x(n) = \delta(n+3) + \delta(n+2) + \delta(n+1) + \delta(n)$
$+ \delta(n-1) + \delta(n-2) + \delta(n-3)$



(ii) $x(n) = \sum_{k=0}^{\infty} \delta(n-2k)$

$x(n) = \delta(n) + \delta(n-2) + \delta(n-4) + \delta(n-6) + \ldots$

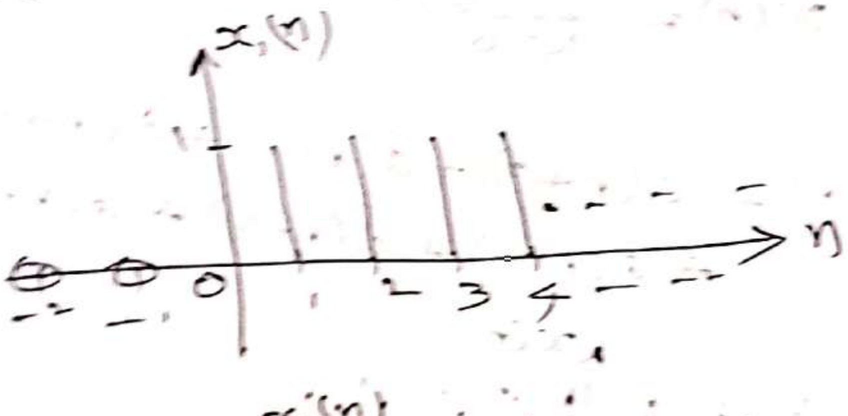


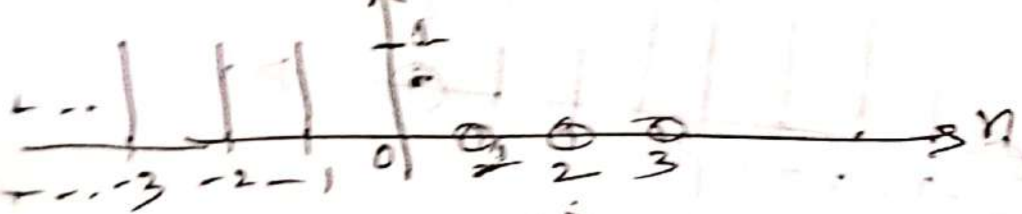
(iii) $x(n) = \sum_{k=0}^{4} \delta(n-k) + \sum_{k=0}^{4} \delta(n+k)$

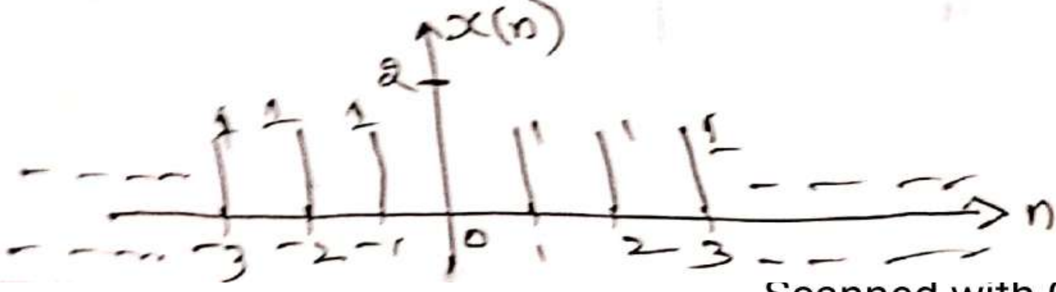
$x(n) = x_1(n) + x_2(n)$

$x_1(n) = \delta(n) + \delta(n-1) + \delta(n-2) + \ldots$

$x_2(n) = \delta(n) + \delta(n+1) + \delta(n+2) + \ldots$







(11) Sketch the following signals

(i) $y(-2n+2) + 3x(-n)$

(ii) $y(2-n) \cdot x(2+n)$

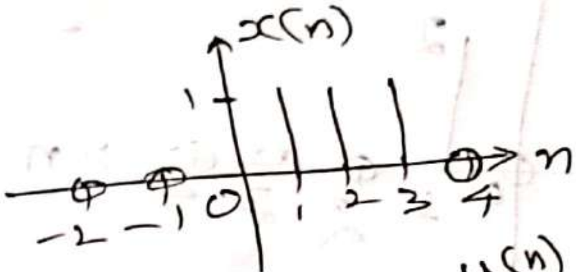
(iii) $[y(n-3) + x(n+2)]\, u(-n+2)$

Given $x(n) = u(n) - u(n-4)$

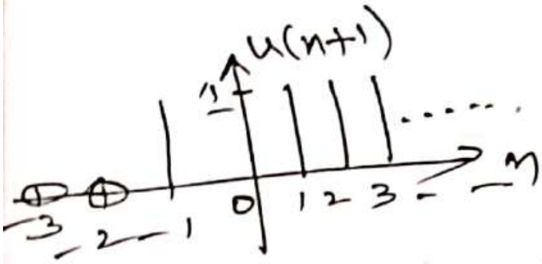
~~$y(n) = n[u(n+1) + u(n) +$~~

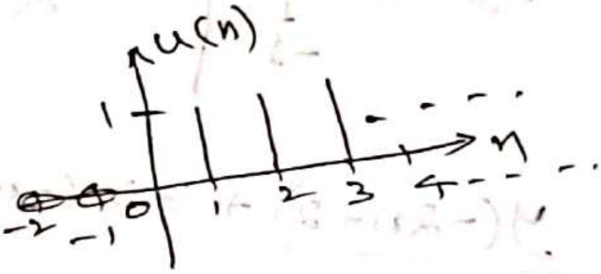
$y(n) = n[u(n+1) + u(n) - 2u(n-2)]$

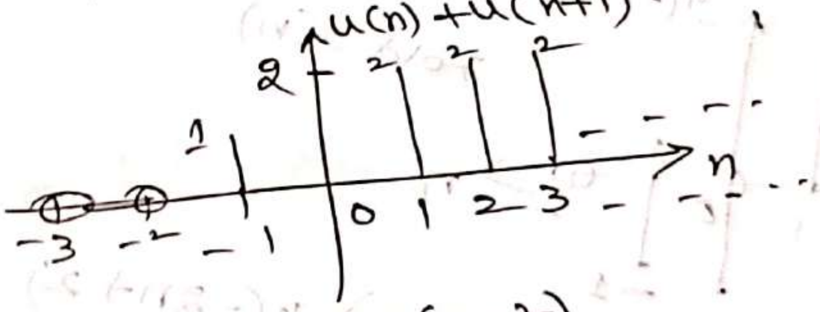
$\Rightarrow$



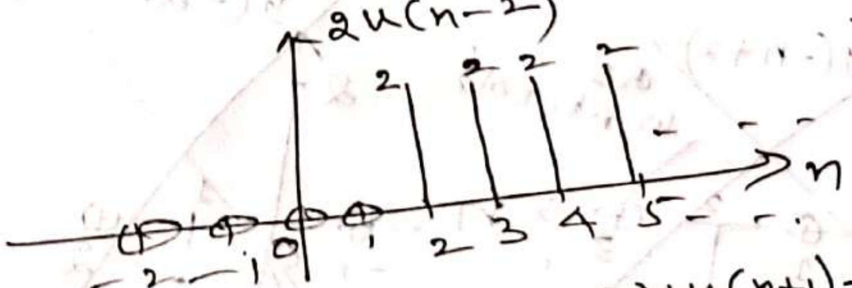
— (1)





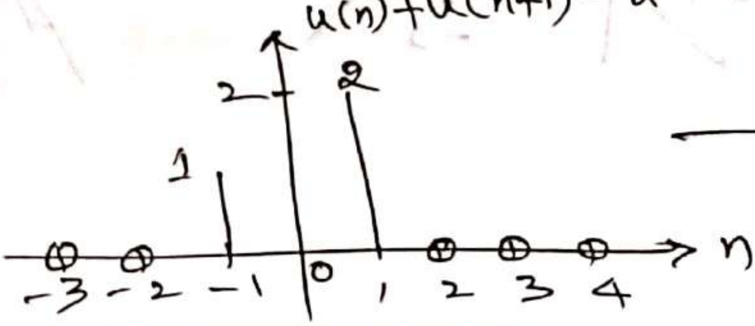


— (2)

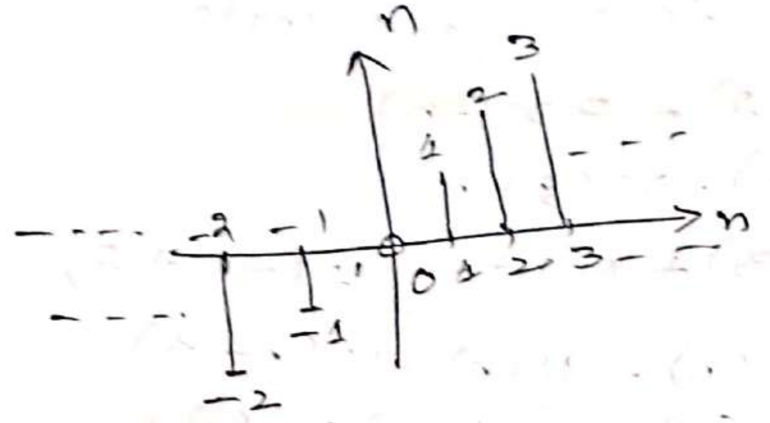


— (3)

$\Rightarrow$ (2) − (3)

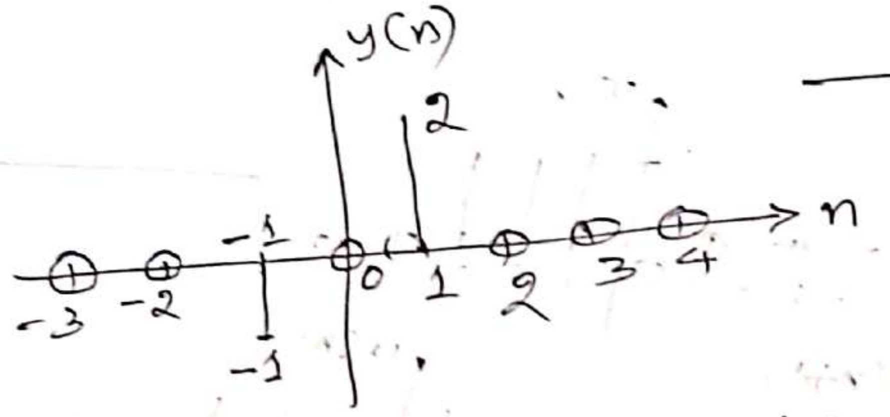


— (4)



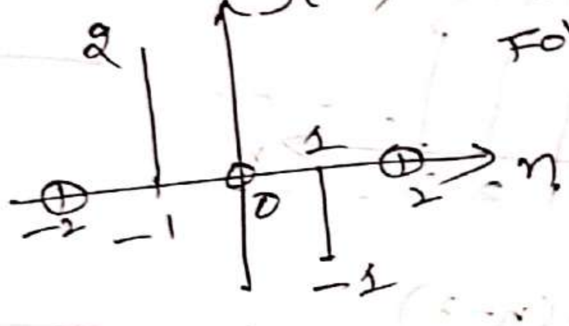
— 5

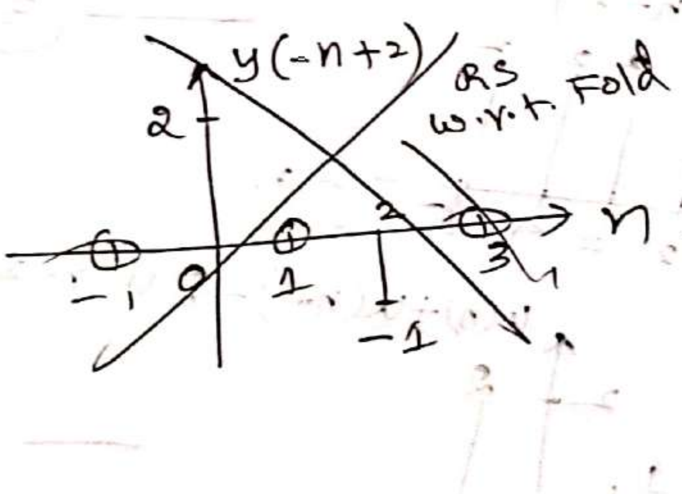
$\Longrightarrow$ 4 × 5

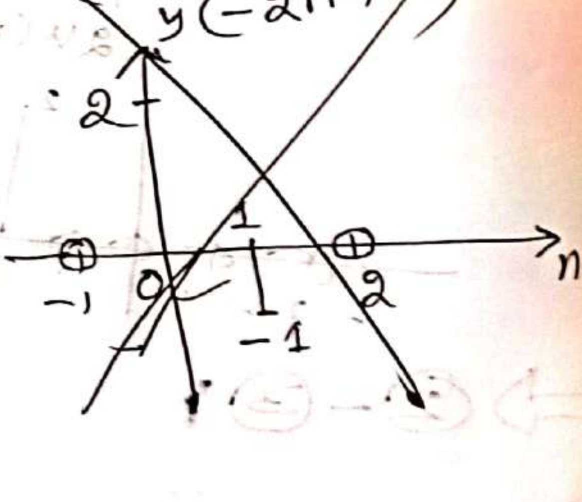


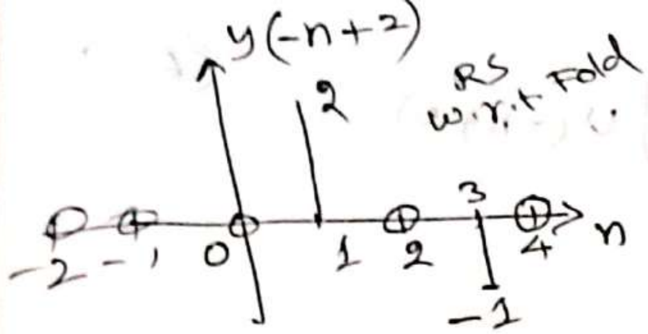
— 6

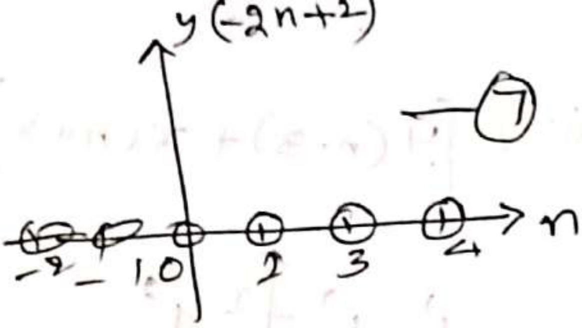
(j) $\underline{y(-2n+2) + 3x(-n)}$:—



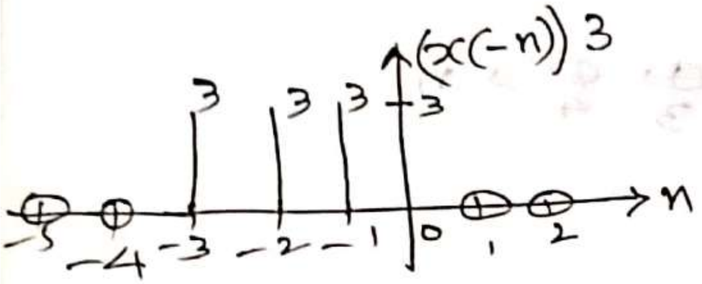






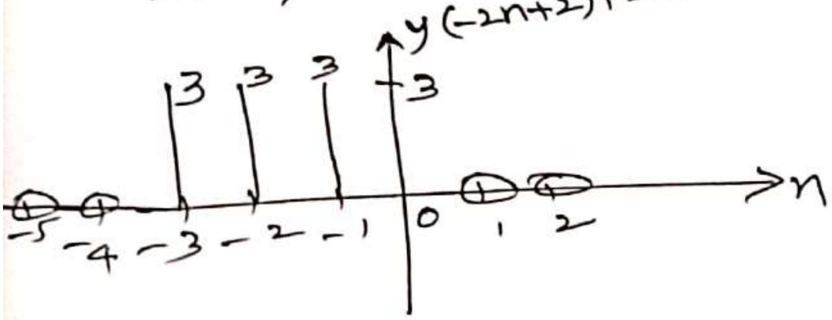


— 7



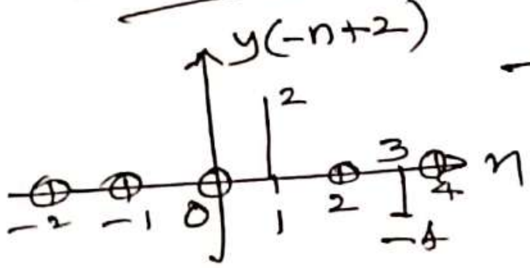
— 8

$\Longrightarrow$ 7 + 8

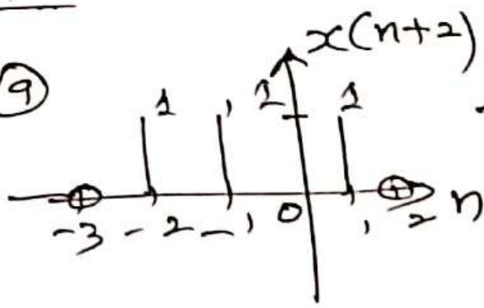


$= [3, 3, 3, \underset{\uparrow}{3}]$

(ii) $y(2-n) \cdot x(2+n)$:—

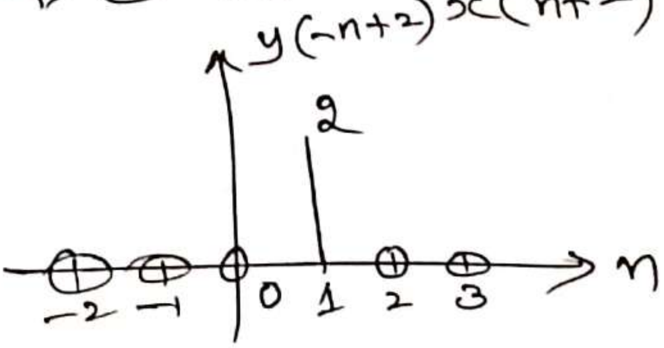


— 9

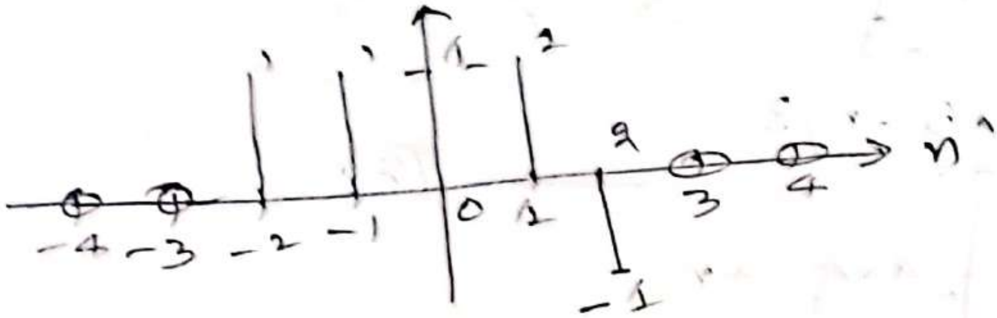


— 10

$\Longrightarrow$ 9 × 10



(iii) $[y(n-3) + x(n+2)]\,u(-n+2)$



Classification of Signals:

The signals are classified into different groups based on different features

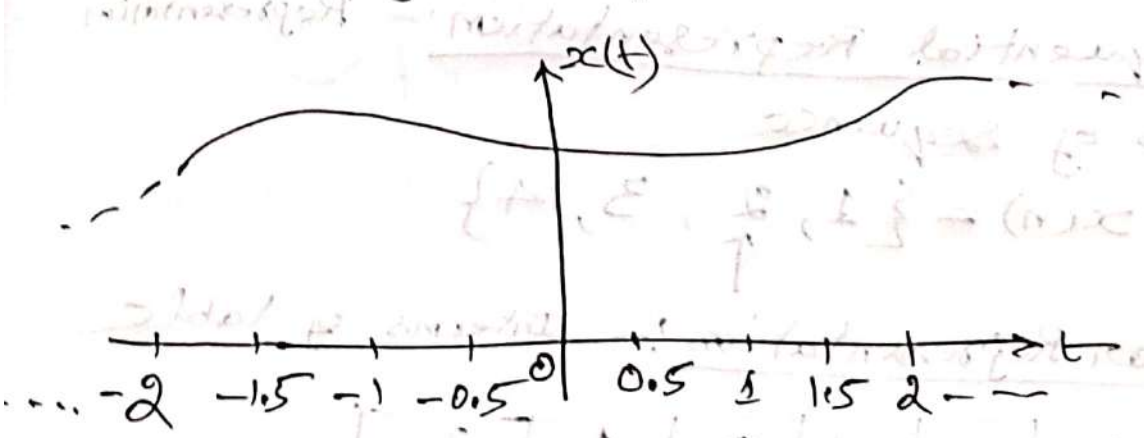
1. Continuous time and Discrete time Signals
2. Periodic and Non periodic Signals
3. Even and odd signals
4. Energy and power signals
5. Deterministic and Random signals

1. Continuous Time (CT) and Discrete Time (DT) Signals

CT Signal

A signal $x(t)$ is said to be CT signal if it has amplitude values for all values of time 't'. The signal amplitudes varies continuously with time 't' i.e., signal amplitudes exists for integer values of time and also fraction values of time.

Example:- Speech Signal, CT Sin Signal



DT Signal :-

A signal $x(n)$ is said to be DT signal if it has amplitudes only for integer values of time 'n'

Example :- DT speech signal

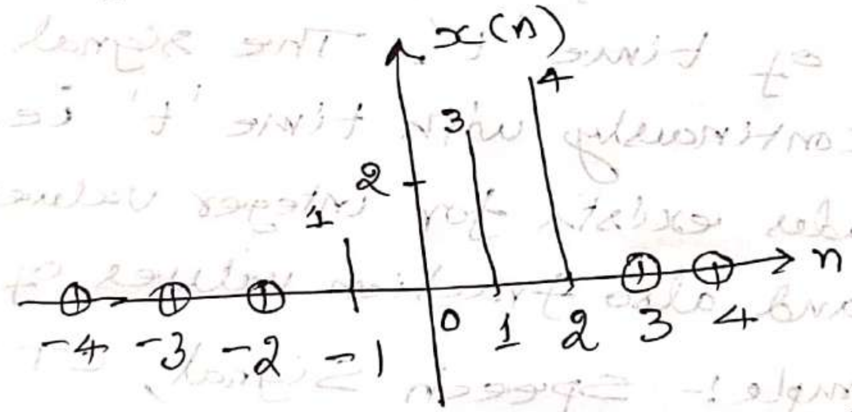
Functional Representation :-

The DT signal $x(n)$ in mathematical form

$$x(n) = \begin{cases} (n+2), & -2 \le n \le 2 \\ 0, & \text{otherwise} \end{cases}$$

Graphical Representation :-

plot of signal for various values of 'n'



Sequential Representation :- Representation

interms of sequence

$$x(n) = \{1, \underset{\uparrow}{2}, 3, 4\}$$

Tabular Representation :- Interms of table

n	-2	-1	0	1	2
$x(n)$	0	1	2	3	4

2. Periodic and Non periodic signals

(i) CT periodic and non periodic signals

A CT signal $x(t)$ is said to be periodic if it satisfies the following condition

$$\boxed{x(t) = x(t+T)}, \text{ for all values of } 't'$$

where $T \rightarrow$ Fundamental period of $x(t)$
$\rightarrow$ Time taken by the signal $x(t)$ to complete one cycle (seconds)

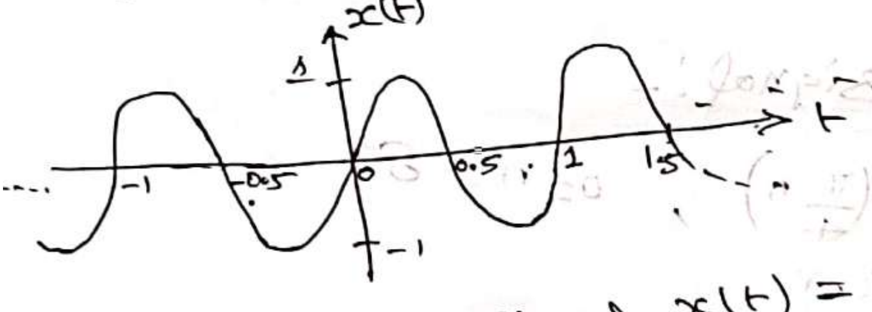
Fundamental Frequency $f = \frac{1}{T}$ Hertz (Hz)

Fundamental Angular Frequency $\omega = 2\pi f$ radians/sec

A CT signal $x(t)$ is said to be non-periodic if it satisfies the following equation

$$\boxed{x(t) \neq x(t+T)}$$

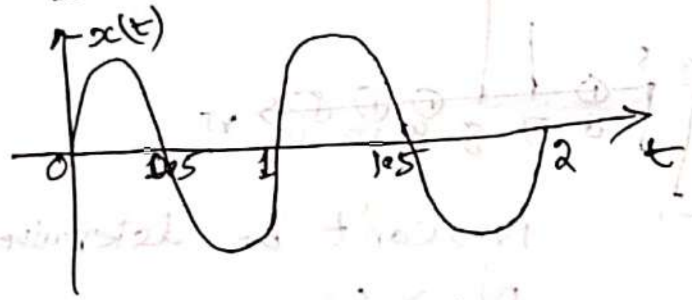
Example :- Periodic signal $x(t) = \sin(2\pi t)$



$\boxed{T = 1 \text{ Second}}$

The waveform repeats for every 1 sec.

Non Periodic Signal $x(t) = \sin(2\pi t)$, $0 \leq t \leq 2$



$T \rightarrow$ can't be determined

$\therefore T \rightarrow \infty$

(ii) DT periodic and non periodic Signals

A DT signal $x(n)$ is said to be periodic if it satisfies the following conditions

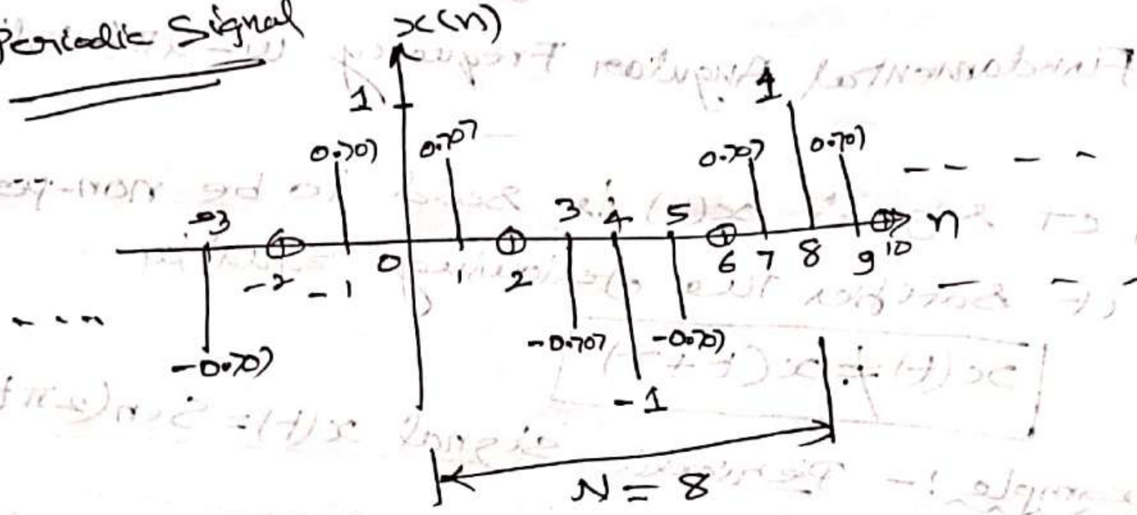
$$\boxed{x(n) = x(n+N)}, \text{ for all 'n' values}$$

where N is fundamental period

A DT signal $x(n)$ is non periodic signal if $x(n) \neq x(n+N)$

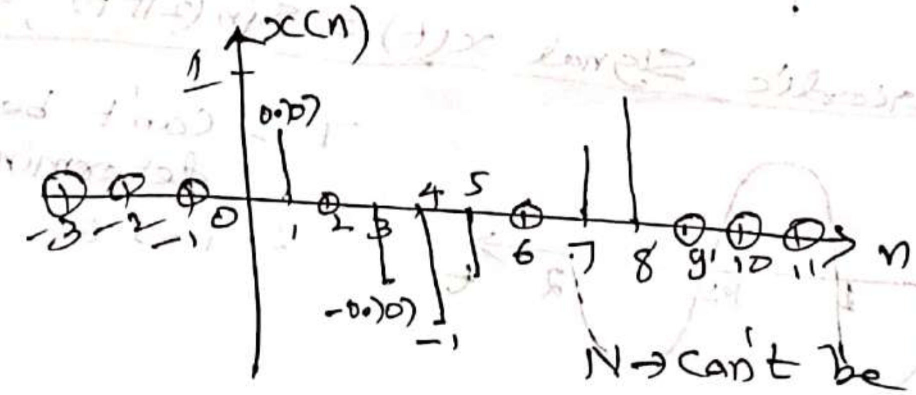
Example :- $x(n) = \cos\left(\frac{\pi}{4}n\right)$, for all 'n'

Periodic Signal



Non periodic Signal :-

$$x(n) = \cos\left(\frac{\pi}{4}n\right), \quad 0 \le n \le 8$$



N → can't be determined

N → ∞

Problems :— Determine, whether the following signals are periodic. If they are periodic, find the fundamental period.

$\cos(\omega t)$
$\sin(\omega t)$
$e^{j\omega t}$

1. $x(t) = \cos(2\pi t)$ —— (1)

ω = Analog angular frequency

$2\pi f = 2\pi$ radians/second

$f = \frac{2\pi}{2\pi} = 1$ Hz

$T = \frac{1}{f} = 1$ Second.

Replace t by t+T in Equation (1)

$x(t+T) = \cos(2\pi(t+T)) = \cos(2\pi t + 2\pi T)$

$= \cos(2\pi t + 2\pi \times 1)$

$x(t+T) = \cos(2\pi t) = x(t)$ for all t

Hence it is periodic with fundamental period $T = 1$ sec.

2. $x(t) = \cos(2\pi t) \cdot u(t)$

The signal is not repeating for time $t = -\infty$ to $+\infty$. ie., the signal appears only for $t \geq 0$.
Hence the signal is Non periodic & fundamental period can't be determined.

3. $x(t) = \cos^2(2\pi t)$

$$x(t) = \frac{1+\cos(4\pi t)}{2} \quad \text{—— (1)}$$

$\omega = 4\pi$

$2\pi f = 4\pi$

$f = 2$ Hz

$T = \frac{1}{2} = 0.5$ sec.

substitute $t = t+T$ in Equation (1)

$$x(t+T) = \frac{1+\cos(4\pi(t+T))}{2}$$

$$= \frac{1+\cos(4\pi t + 4\pi T)}{2}$$

$$= \frac{1+\cos(4\pi t + 4\pi \times 0.5)}{2}$$

$$= \frac{1+\cos(4\pi t + 2\pi)}{2}$$

$$= \frac{1+\cos(4\pi t)}{2}$$

$x(t+T) = x(t)$

Hence periodic with $T = 0.5$ sec.

3. $x(t) = \sin^2\left(t - \frac{\pi}{6}\right)$

$$x(t) = \frac{1-\cos\left(2\left(t-\frac{\pi}{6}\right)\right)}{2}$$

$$x(t) = \frac{1-\cos\left(2t - \frac{\cancel{2}\pi}{\cancel{6}\,3}\right)}{2} \quad \text{—— (1)}$$

$\omega = 2$ rad/sec

$2\pi f = 2$, $f = \frac{1}{\pi}$, $T = \pi$ sec

Substitute $t = t+T$ in Equation ①

$$x(t+T) = \frac{1-\cos\left[2(t+T)-\frac{\pi}{3}\right]}{2}$$

$$= \frac{1-\cos\left[2t+2T-\frac{\pi}{3}\right]}{2}$$

$$= \frac{1-\cos\left[2t+2\pi-\frac{\pi}{3}\right]}{2}$$

$$= \frac{1-\cos\left[\left(2t-\frac{\pi}{3}\right)+2\pi\right]}{2}$$

$$x(t+T) = \frac{1-\cos\left(2t-\frac{\pi}{3}\right)}{2}$$

$$\boxed{x(t+T) = x(t)}$$

Hence it is periodic, with $T = \pi$ sec.

④ $x(t) = 2\cos\left(3t+\frac{\pi}{4}\right)$

periodic with $T = \frac{2\pi}{3}$ seconds

⑤ $x(t) = e^{j\pi t}$ —— ①

$=$ $\omega = \pi$, $2\pi f = \pi$, $f = \frac{1}{2}$, $T = 2$ sec.

Substitute $t = t+T$ in equation ①

$$x(t+T) = e^{j\pi(t+T)} = e^{j\pi(t+2)}$$

$$= e^{j\pi t} \cdot e^{j2\pi}$$

$$= e^{j\pi t}$$

$$\boxed{x(t+T) = x(t)}$$

Hence periodic with $T = 2$ sec.

(6) $x(t) = e^{j(\pi t - 1)}$ —— (1)

= $\omega = \pi$ rad. $2\pi f = \pi$

$f = \frac{1}{2}$, $T = 2$ sec.

Substitute $t = t+T$ in equation (1)

$x(t+T) = e^{j[\pi(t+T) - 1]}$

$= e^{j[\pi t + \pi \times 2 - 1]}$

$= e^{j\pi t} \cdot e^{j2\pi} \cdot e^{-j}$

$x(t+T) = e^{j(\pi t - 1)}$

$\boxed{x(t+T) = x(t)}$ Hence periodic with $T = 2$ sec.

(7) $x(t) = \cos\frac{\pi}{3}t + \sin\frac{\pi}{4}t$

= $x(t) = x_1(t) + x_2(t)$

Consider $x_1(t)$:-

$\omega_1 = \frac{\pi}{3}$ rad/sec

$2\pi f_1 = \frac{\pi}{3}$

$f_1 = \frac{\pi}{2\pi 3} = \frac{1}{6}$ Hz

$T_1 = \frac{1}{f_1} = 6$ Sec.

Consider $x_2(t)$:-

$\omega_2 = \frac{\pi}{4}$ rad/sec

$2\pi f_2 = \frac{\pi}{4}$

$f_2 = \frac{1}{8}$ Hz

$T_2 = \frac{1}{f_2} = 8$ Sec.

Ratio: $\frac{T_1}{T_2} = \frac{\cancel{6}^{3}}{\cancel{8}_{4}} = \frac{3}{4}$ Rational

Hence $x(t)$ is periodic

Fundamental Period $T = 4T_1$ or $3T_2$

$= 4\times 6$ or 3×8

$T = 24$ sec.

(8) $x(t) = \cos(t) + \sin\sqrt{2}t$

= $x(t) = x_1(t) + x_2(t)$

Consider $x_1(t)$

$T_1 = 2\pi$ sec.

Consider $x_2(t)$

$T_2 = \frac{2\pi}{\sqrt{2}}$ sec.

Ratio: $\frac{T_1}{T_2} = \frac{2\pi}{2\pi/\sqrt{2}} = \frac{\sqrt{2}}{1}$ not rational

Hence $x(t)$ is non-periodic

(9) $x(t) = \cos 10t + 2\sin 3\pi t$

= $= x_1(t) + x_2(t)$

$T_1 = \frac{\pi}{5}$ $\quad T_2 = \frac{2}{3}$, $\frac{T_1}{T_2} = \frac{3\pi}{10}$ Not Rational

Hence $x(t)$ non-periodic.

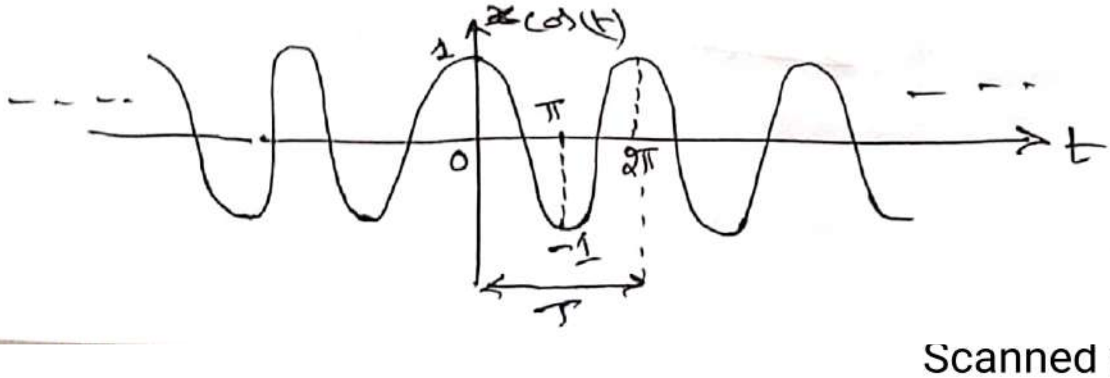
(10) $x(t) = v(t) + v(-t)$

where $v(t) = \cos(t)\cdot u(t)$

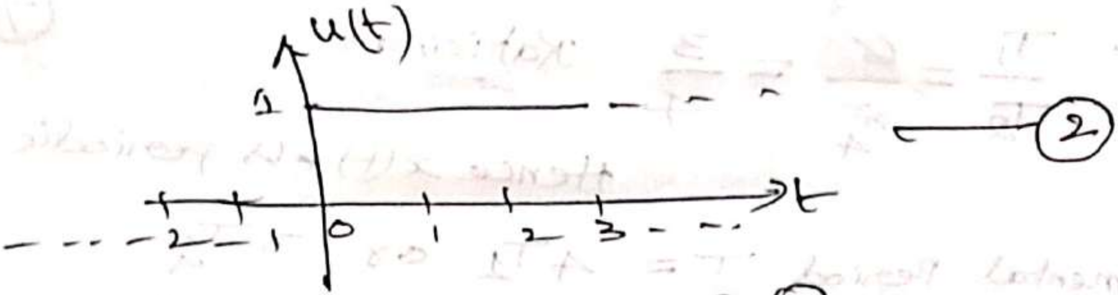
= Consider $v(t) = \cos(t)\cdot u(t)$

Consider $\cos(t)$, $\omega = 1$, $2\pi f = 1$, $f = \frac{1}{2\pi}$

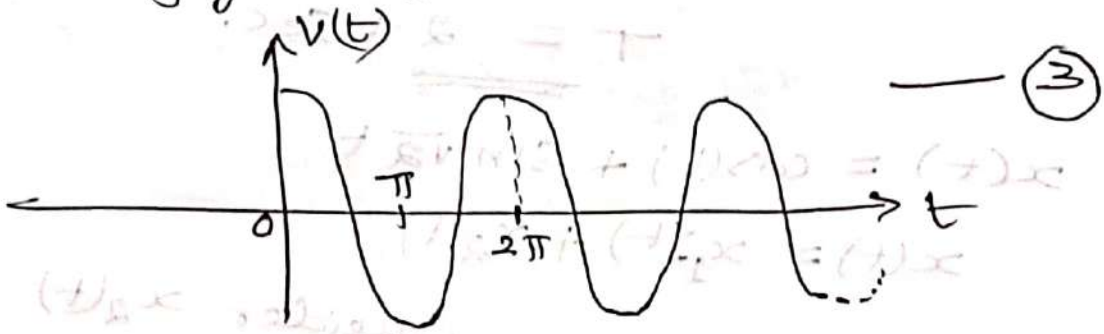
$\therefore T = 2\pi$

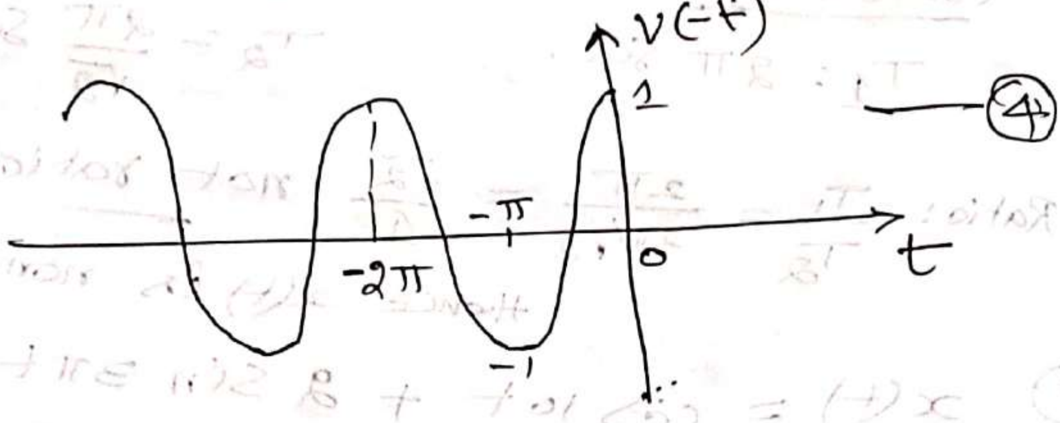


(1)

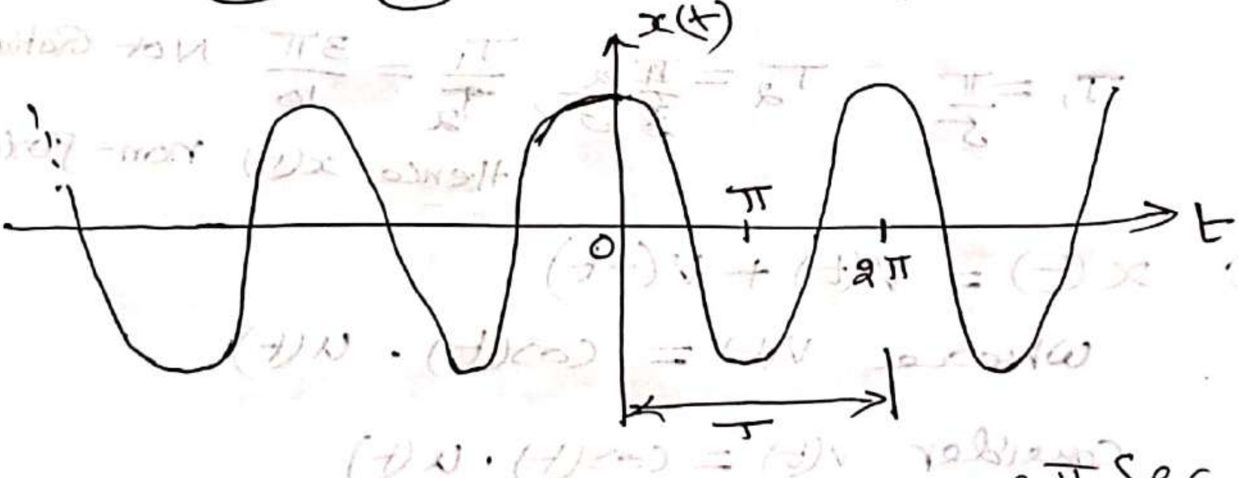


multiply Graphs 1 & 2





3 + 4



$x(t)$ is periodic with $T = 2\pi$ sec.

(11) $x(t) = v(t) + v(-t)$, ~~where $v(t) = \sin(t)$~~

where $v(t) = \sin(t) \cdot u(t)$

= Non periodic Signal

(12) $x(n) = \cancel{\cos(2\pi n)}\ \cos\left(\frac{3\pi}{2} n\right)$ — (1)

$\cos(\Omega n)$
$\sin(\Omega n)$
$e^{j\Omega n}$

= $\cancel{\Omega = 2\pi \text{ rad}}$

$\Omega = \frac{3\pi}{2}$ rad

$2\cancel{\pi} f = \frac{3\cancel{\pi}}{2}$

$f = \frac{3}{4} = \frac{K}{N}$ Rational

Hence $x(n)$ is periodic with $N = 4$ Samples

Substitute $n = n + N$ in equation (1)

$x(n+N) = \cos\left[\frac{3\pi}{2}(n+N)\right]$

$= \cos\left[\frac{3\pi}{2} n + \frac{3\pi}{\cancel{2}} \times \cancel{4}^{2}\right]$

$= \cos\left[\frac{3\pi}{2} n + 6\pi\right]$

$x(n+N) = \cos\left(\frac{3\pi}{2} n\right)$

$x(n+N) = x(n)$ Hence Periodic.

*(13) $x(n) = \cos\left(\frac{3}{2} n\right)$

= $\Omega = \frac{3}{2}$, $2\pi f = \frac{3}{2}$, $f = \frac{3}{4\pi}$

Not Rational

Hence Non-Periodic

14 $x(n) = \cos(0.125\, n\pi)$

$= \quad x(n) = \cos\left(\frac{1}{16} n\pi\right)$

Periodic, with $N = 16$ Samples

15 $x(n) = \cos\left(\frac{4\pi}{13} n + \pi\right)$

$= \quad \Omega = \frac{4\pi}{13}, \quad \cancel{2\pi} f = \frac{\cancel{4\pi}^{2}}{13}$

$f = \frac{2}{13} = \frac{K}{N}$ Rational

Periodic, with $N = 13$ Samples

16 $x(n) = \cos\left(\frac{n\pi}{12}\right) + \sin\left(\frac{n\pi}{18}\right)$

$= \quad = x_1(n) + x_2(n)$

Consider $x_1(n)$

$\Omega_1 = \frac{\pi}{12}$

$2\cancel{\pi} f_1 = \frac{\cancel{\pi}}{12}$

$f_1 = \frac{1}{24} = \frac{K_1}{N_1}$

$N_1 = 24$ Samples

Consider $x_2(n)$

$\Omega_2 = \frac{\pi}{18}$

$2\cancel{\pi} f_2 = \frac{\cancel{\pi}}{18}$

$f_2 = \frac{1}{36} = \frac{K_2}{N_2}$

$N_2 = 36$ Samples

Ratio $\frac{N_1}{N_2} = \frac{24}{36} = \frac{2}{3}$ Rational

Hence $x(n)$ is periodic with

$N = N_1 \times 3$ or $N_2 \times 2$

$= 24 \times 3$ or 36×2

$N = 72$ Samples.

17) $x(n) = Re\{e^{j\frac{n\pi}{12}}\} + Im\{e^{j\frac{n\pi}{18}}\}$

$= e^{j\frac{n\pi}{12}} = \cos\frac{n\pi}{12} + j\sin\frac{n\pi}{12}$

$Re\{e^{j\frac{n\pi}{12}}\} = \cos\left(\frac{n\pi}{12}\right)$

$e^{j\frac{n\pi}{18}} = \cos\left(\frac{n\pi}{18}\right) + j\sin\left(\frac{n\pi}{18}\right)$

$Im\{e^{j\frac{n\pi}{18}}\} = \sin\left(\frac{n\pi}{18}\right)$

$\therefore x(n) = \cos\left(\frac{n\pi}{12}\right) + \sin\left(\frac{n\pi}{18}\right)$

Periodic, with $N = 72$ Samples

18) $x(n) = e^{j\left(\frac{2\pi n}{3}\right)} + e^{j\left(\frac{3\pi n}{8}\right)}$

$= x_1(n) + x_2(n)$

$\Omega_1 = \frac{2\pi}{3}$ $\qquad$ $\Omega_2 = \frac{3\pi}{8}$

$\cancel{2\pi} f_1 = \frac{\cancel{2\pi}}{3}$ $\qquad$ $\cancel{2\pi} f_2 = \frac{3\cancel{\pi}}{8}$

$f_1 = \frac{1}{3} = \frac{K_1}{N_1}$ $\qquad$ $f_2 = \frac{3}{16} = \frac{K_2}{N_2}$

$N_1 = 3$ Samples $\qquad$ $N_2 = 16$ Samples

Ratio $\frac{N_1}{N_2} = \frac{3}{16}$ Rational, Hence periodic

$N = N_1 \times 16$ or $N_2 \times 3$

$= 3 \times 16$ or 16×3

$= 48$ Samples

Scanned with CamScanner

(19) $x(n) = \cos\left(\frac{n\pi}{8}\right) \cdot \cos\left(\frac{3n\pi}{7}\right)$

$\cos A \cos B = \frac{\cos(A+B) + \cos(A-B)}{2}$

= ~~x(n) = cos~~

$$x(n) = \frac{\cos\left(\frac{31 n\pi}{56}\right) + \cos\left(\frac{17 n\pi}{56}\right)}{2}$$

$\Omega_1 = \frac{31\pi}{56}$; $\Omega_2 = \frac{17\pi}{56}$

$2\cancel{\pi} f_1 = \frac{31\cancel{\pi}}{56}$; $2\cancel{\pi} f_2 = \frac{17\cancel{\pi}}{56}$

$f_1 = \frac{31}{112} = \frac{K_1}{N_1}$; $f_2 = \frac{17}{112} = \frac{K_2}{N_2}$

Ration $\frac{N_1}{N_2} = \frac{112}{112} = \frac{1}{1}$ Rational

Hence periodic

$N = N_1 \times 1$ or $N_2 \times 1$

$= 112$ Samples

(20) $x(n) = 2\sin\left(\frac{n\pi}{3}\right)\cos\left(\frac{n\pi}{5}\right)$

$\sin A \cos B = \frac{\sin(A+B) + \sin(A-B)}{2}$

Periodic
with $N = 15$ samples

(21) $x(n) = (-1)^n$

$= x(n) = \left[e^{j\pi}\right]^n = e^{j\pi n}$

Periodic, with $N = 2$ Samples

2. Even and odd Signals

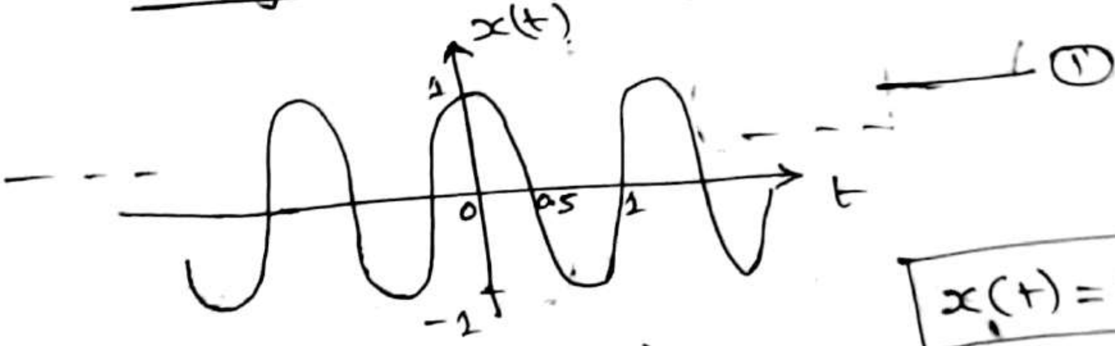
1. Even signal (Symmetrical signal):-

A signal $x(t)$ or $x(n)$ is said to be even signal if it satisfies the condition.

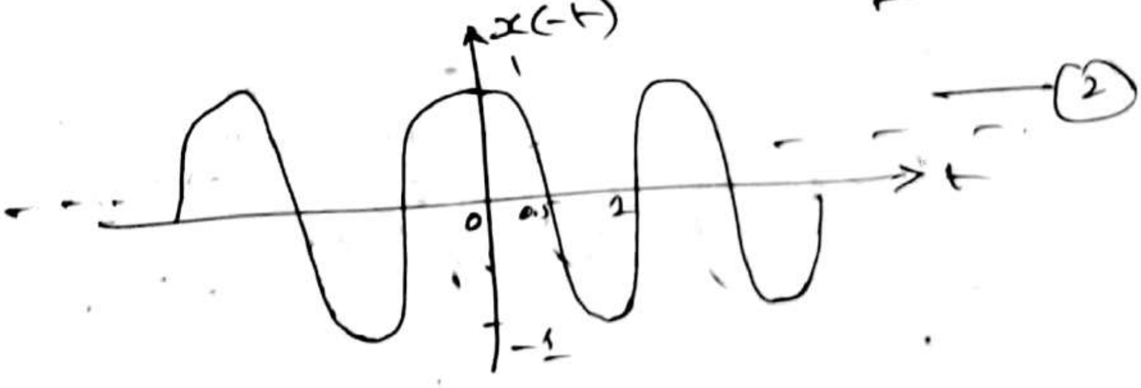
$$x(t) = x(-t) \text{ for all 't'}$$
$$x(n) = x(-n) \text{ 'for 'all' n'}$$

original signal = Folded signal

Example:- $x(t) = \cos(2\pi t) \Rightarrow \cos$ signal.



$$\boxed{x(t) = x(-t)}$$

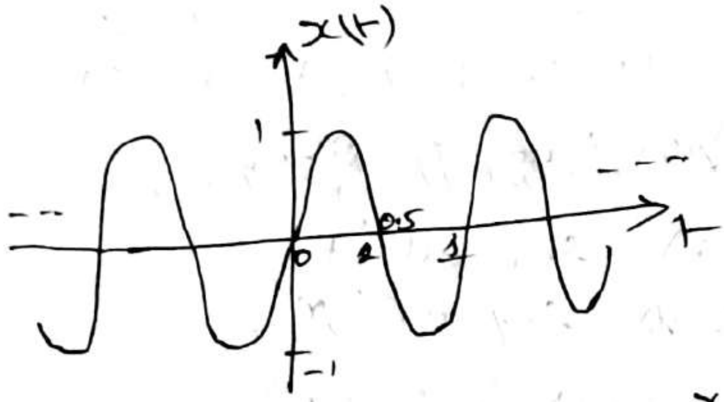


2. odd signal (Antisymmetric):-

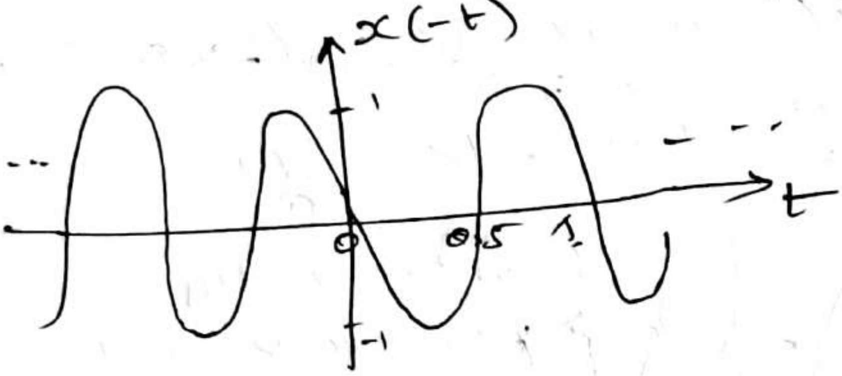
A signal $x(t)$ or $x(n)$ is said to be odd signal if it satisfies the condition

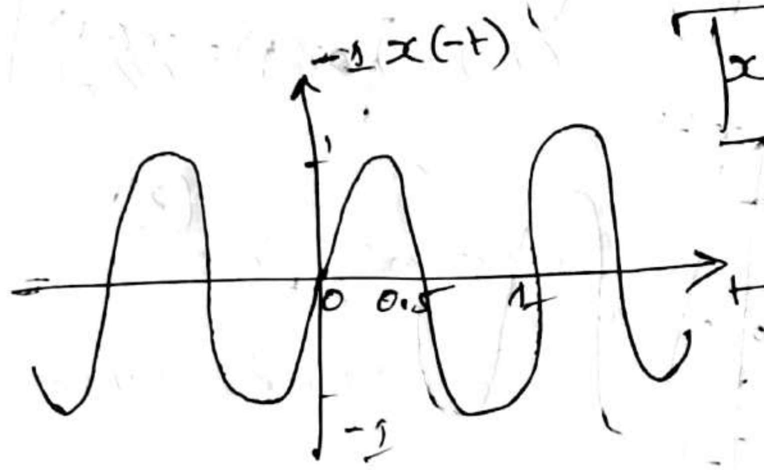
$$x(t) = -x(-t) \quad (\text{or}) \quad x(-t) = -x(t)$$
$$x(n) = -x(-n) \quad (\text{or}) \quad x(-n) = -x(n)$$

Example:- sin signal, $x(t) = \sin(2\pi t)$



$x(t) \neq x(-t)$





$\boxed{x(t) = -x(-t)}$

The amplitude at the origin for odd signal is always Zero

*****

Decomposition of Signal:-

The $x(t)$ can be decomposed into even $x_e(t)$ signal and odd $x_o(t)$ signal

$$x(t) = x_e(t) + x_o(t) \quad \text{--- (1)}$$

Replace 't' by $-t$

$$x(-t) = x_e(-t) + x_o(-t) \quad \text{--- (2)}$$

W.K.T $x_e(-t) = x_e(t)$, $x_o(-t) = -x_o(t)$ — (3)

Substitute (3) in (2)

$$x(-t) = x_e(t) - x_o(t) \quad \text{— (4)}$$

Add (1) and (4)

$$x(t) = x_e(t) + \cancel{x_o(t)} \quad \text{— (1)}$$
$$x(-t) = x_e(t) - \cancel{x_o(t)} \quad \text{— (4)}$$

$$x(t) + x(-t) = 2x_e(t)$$

$$\therefore \boxed{x_e(t) = \frac{x(t) + x(-t)}{2}}$$

Similarly for DT signals

$$\boxed{x_e(n) = \frac{x(n) + x(-n)}{2}}$$

* Substract (4) ~~from~~ (1)

$$x(t) = \cancel{x_e(t)} + x_o(t)$$
$$x(-t) = \cancel{x_e(t)} \mp x_o(t)$$

$$x(t) - x(-t) = 2x_o(t)$$

$$\therefore \boxed{x_o(t) = \frac{x(t) - x(-t)}{2}}$$

Similarly for DT signals

$$\boxed{x_o(n) = \frac{x(n) - x(-n)}{2}}$$

Problems :-

1. Find the even and odd components of the signal

$$x(t) = \cos(t) + \sin(t) + \sin(t)\cos(t) \quad \text{——} \ (1)$$

⇒ substitute $t = -t$ in equation (1)

$$x(-t) = \cos(-t) + \sin(-t) + \sin(-t)\cos(-t)$$

$$x(-t) = \cos t - \sin t - \sin t \cos t \quad \text{——} \ (2)$$

~~Add (1) and (2) to g~~

Even signal component :-

$$x_e(t) = \frac{x(t) + x(-t)}{2} = \cos(t)$$

odd signal component :-

$$x_o(t) = \frac{x(t) - x(-t)}{2} = \sin(t)\left[1 + \cos t\right]$$

(2) Find even and odd components of the signal $x(t) = (1+t^3)\cos^3(10t)$

⇒

$$x_e(t) = \cos^3(10t)$$

$$x_o(t) = t^3\cos^3(10t)$$

(3) Determine weather the following signals are even, odd or neither signals

(i) $x(t) = 3 + t^2 + 2t^4$

(ii) $x(t) = 2t + 4t^3$

(iii) $x(n) = |n|$ for all 'n'

(iv) $x(n) = n$ for all 'n'

(v) $x(t) = 1 + 4t + 3t^2$ for all 't'

(vi) $x(n) = 2n + 8$ for all 'n'

= (i) $x(t) = 3 + t^2 + 2t^4$

Substitute $t = -t$

$x(-t) = 3 + (-t)^2 + 2(-t)^4$

$x(-t) = 3 + t^2 + 2t^4$ Hence Even Signal

$\Longrightarrow x(t) = x(-t)$

(ii) $x(t) = 2t + 4t^3$

Substitute t by $-t$

$x(-t) = 2(-t) + 4(-t)^3 = -2t - 4t^3$

$x(-t) = -[2t + 4t^3] = -x(t)$

Hence signal is odd

(iii) $x(n) = |n|$ for all n

Substitute $n = -n$

$x(-n) = |-n| = |n| = x(n)$

Hence signal is even

(iv) $x(n) = n$ for all n

Substitute $n = -n$

$x(-n) = -n = -x(n)$ Hence its odd

~~(v) x(t) =~~

(v) $x(t) = 1 + 4t + 3t^2$ for all 't'

Substitute $t = -t$

$$x(-t) = 1 - 4t + 3(-t)^2 = 1 - 4t + 3t^2$$

$$x(-t) \neq x(t) \text{ (or) } x(-t) = -x(t)$$

$\therefore$ signal neither even nor odd

(vi) $x(n) = 2n + 8$ for all 'n'

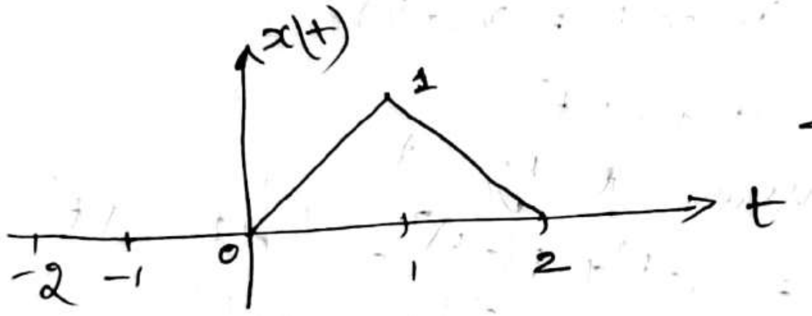
Substitute $n = -n$

$$x(-n) = -2n + 8$$

$$x(-n) \neq x(n) \text{ (or) } x(-n) \neq -x(n)$$

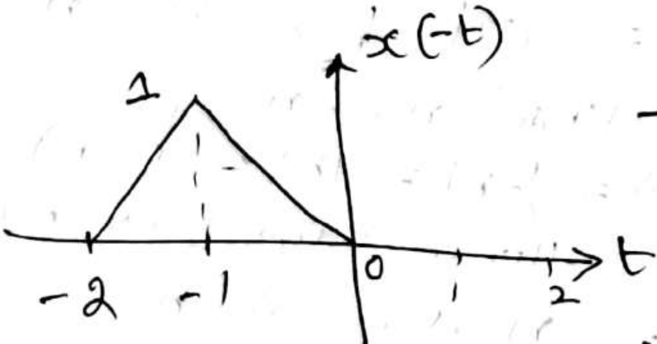
$\therefore$ signal is neither even nor odd

4) Determine and sketch the even and odd components of the signal shown



— (1)

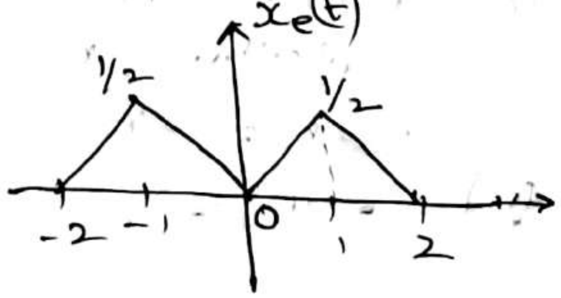
= Fold the signal



— (2)

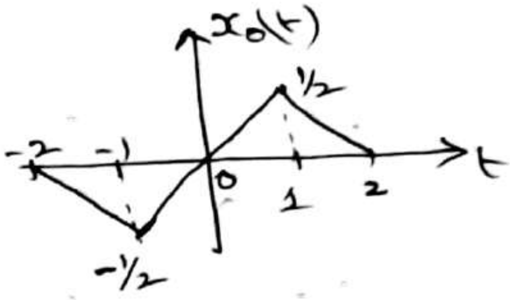
Even signal

$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

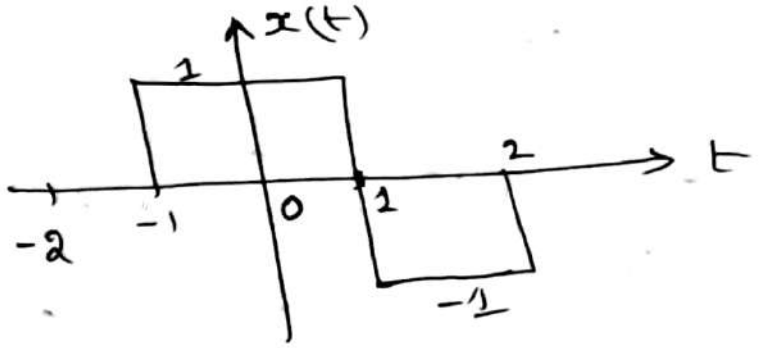


odd signal

$$x_o(t) = \frac{x(t) - x(-t)}{2}$$

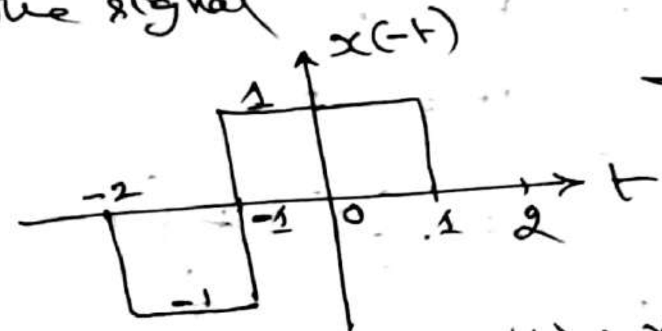


(5) Determine and sketch the even and odd parts of the signal shown



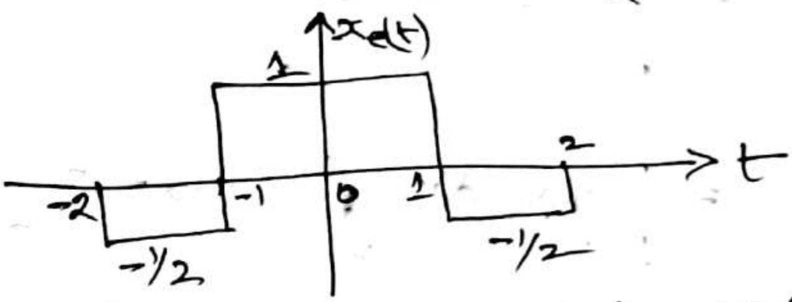
← (1)

= Fold the signal

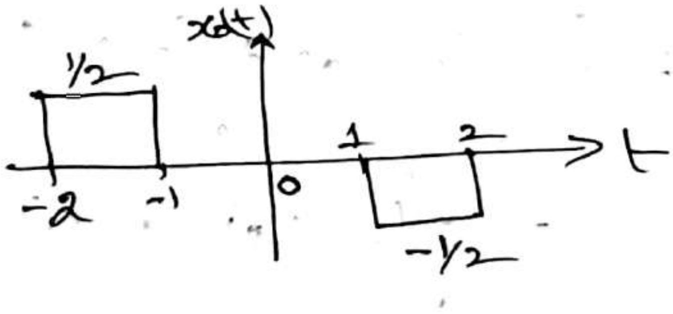


← (2)

Even signal: $x_e(t) = \dfrac{x(t) + x(-t)}{2}$



odd signal :- $x_o(t) = \dfrac{x(t) - x(-t)}{2}$



⑥ (i) S.T $\int_{-a}^{a} x(t)\,dt = 2\int_{0}^{a} x(t)\,dt$

If $x(t)$ is even

(ii) S.T $\int_{-a}^{a} x(t)\cdot dt = 0$

If $x(t)$ is odd

= (i) $LHS = \int_{-a}^{a} x(t)\,dt = \int_{-a}^{0} x(t)dt + \int_{0}^{a} x(t)dt$

Substitute $t = -t$ in I term

$dt = -dt$

$$= \int_{a}^{0} x(-t)\cdot[-dt] + \int_{0}^{a} x(t)\,dt$$

$$= -\int_{a}^{0} x(-t)dt + \int_{0}^{a} x(t)\,dt$$

$$= \cancel{\int_{0}^{a} x(t)dt} \int_{0}^{a} x(-t)dt + \int_{0}^{a} x(t)dt$$

For even $x(-t) = x(t)$

$$\therefore \; LHS = \int_{0}^{a} x(t)dt + \int_{0}^{a} x(t)dt$$

$$= 2\int_{0}^{a} x(t)dt = RHS$$

(ii) LHS $= \int_0^a x(-t)dt + \int_0^a x(t)\,dt$

For odd, $x(-t) = -x(t)$

$$= -\int_0^a x(t)\,dt + \int_0^a x(t)\,dt = 0 = RHS$$

7 (i) ST $\sum_{n=-\infty}^{\infty} x(n) = 0$ if $x(n)$ is odd

(ii). S.T $\sum_{n=-\infty}^{\infty} x(n) = x(0) + 2\sum_{n=1}^{\infty} x(n)$ if $x(n)$ is even

= (i) LHS $= \sum_{n=-\infty}^{\infty} x(n)$

$$= \sum_{n=-\infty}^{-1} x(n) + \sum_{n=0} x(n) + \sum_{n=1}^{\infty} x(n)$$

Substitate $n = -n$ in I term

$$= \sum_{n=1}^{\infty} x(-n) + x(0) + \sum_{n=1}^{\infty} x(n)$$

For odd Signal $x(-n) = -x(n)$ & $x(0) = 0$

$$= -\sum_{n=1}^{\infty} x(n) + 0 + \sum_{n=1}^{\infty} x(n) = 0 = RHS$$

(ii) For Even signal $x(-n) = x(n)$.

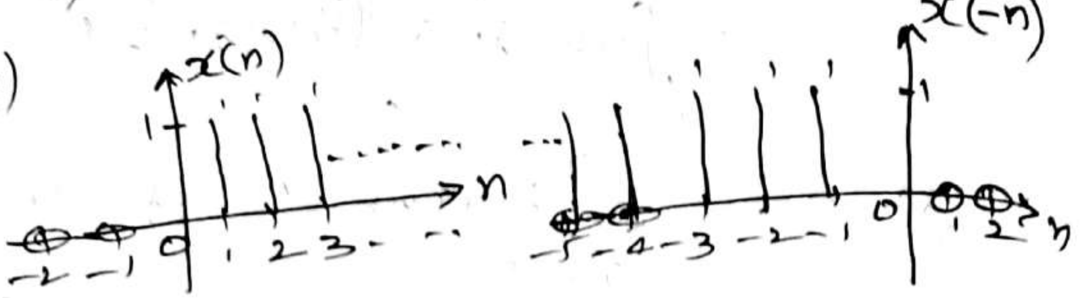
$$\therefore \text{LHS} = \sum_{n=1}^{\infty} x(n) + x(0) + \sum_{n=1}^{\infty} x(n)$$

$$= x(0) + 2\sum_{n=1}^{\infty} x(n) = RHS$$

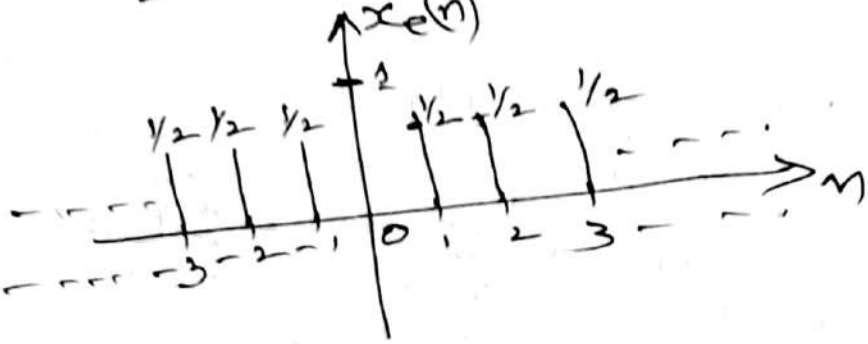
(8) Find Even and odd components of the following signals

(i) $x(n) = u(n)$, (ii) $x(n) = \alpha^n u(n)$

= (i)

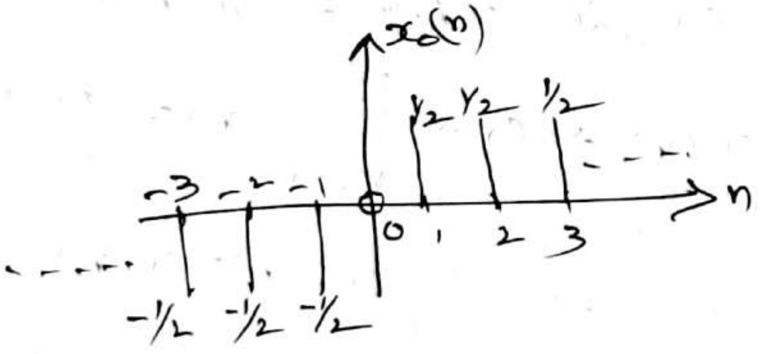


Even Signal $x_e(n) = \dfrac{x(n) + x(-n)}{2}$



$$x_e(n) = \begin{cases} 1, & n = 0 \\ 0.5, & n \neq 0 \end{cases}$$

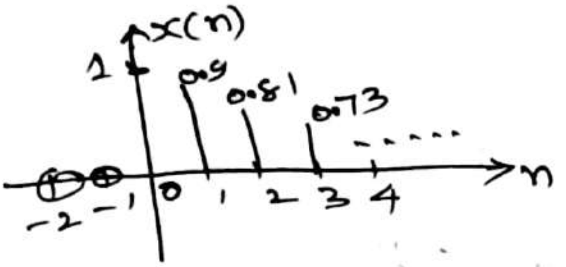
odd signal:- $x_o(n) = \dfrac{x(n) - x(-n)}{2}$

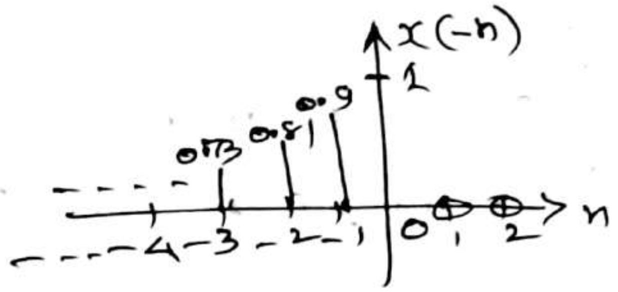


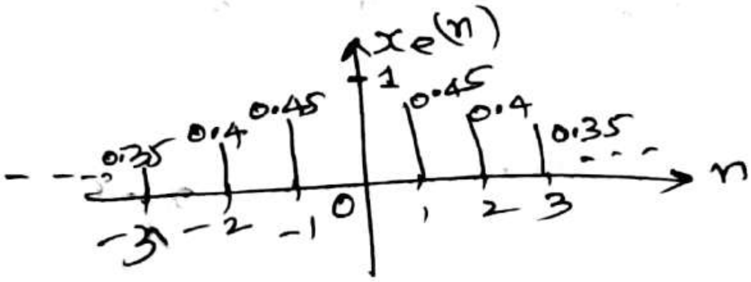
$$x_o(n) = \begin{cases} 0, & n = 0 \\ -0.5, & n < 0 \\ 0.5, & n > 0 \end{cases}$$

(ii) $x(n) = \alpha^{n} u(n)$

If $\alpha = 0.9$

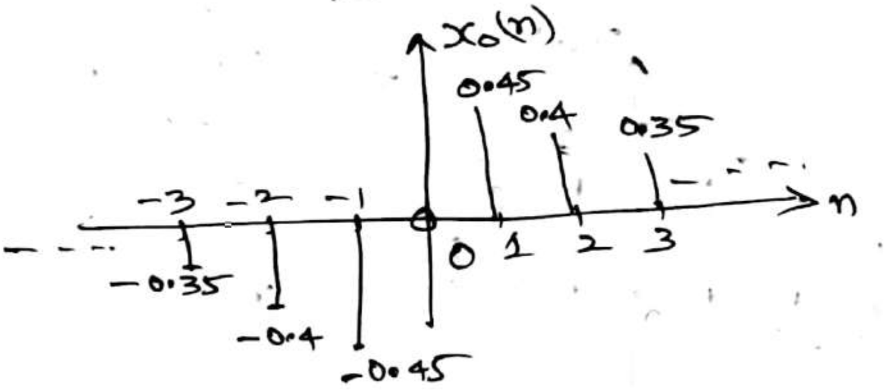






Even Signal

$$x_e(n) = \begin{cases} 1, & n = 0 \\ \dfrac{\alpha^{|n|}}{2}, & n \neq 0 \end{cases}$$



$$x_o(n) = \begin{cases} 0, & n = 0 \\ \dfrac{\alpha^{n}}{2}, & n > 0 \\ -\dfrac{\alpha^{-n}}{2}, & n < 0 \end{cases}$$

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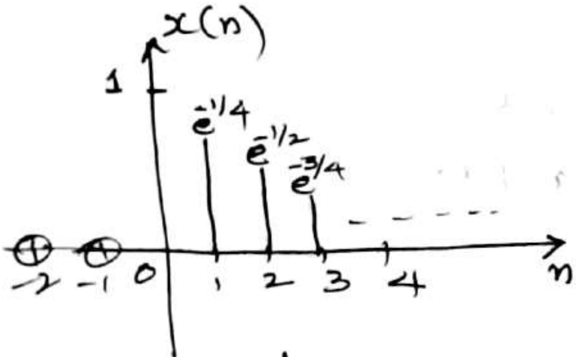
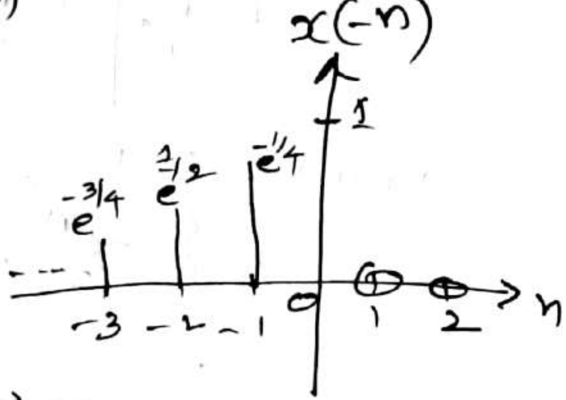
⑨ Find and sketch the even and odd components of the following signals

(i) $x(n) = e^{-n/4}\, u(n)$

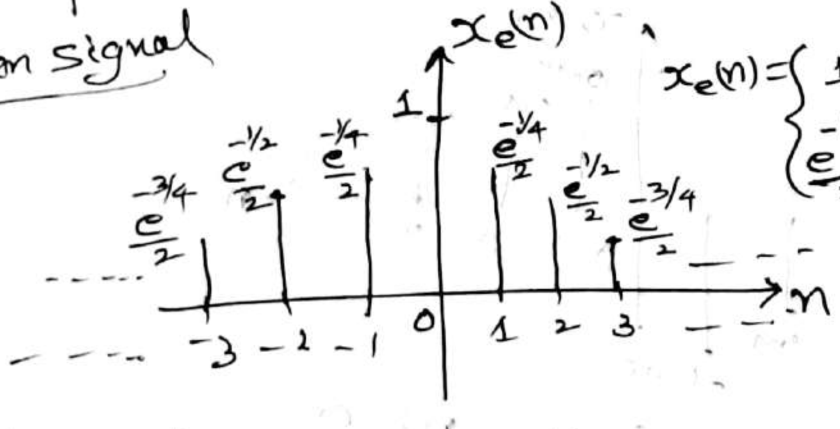
(ii) $x(t) = \begin{cases} t, & 0 \le t \le 1 \\ 2-t, & 1 \le t \le 2 \end{cases}$

= (i) $x(n) = e^{-n/4} \cdot u(n)$





Even signal



$x_e(n) = \begin{cases} 1, & n = 0 \\ \dfrac{e^{-|n|/4}}{2}, & n \ne 0 \end{cases}$

odd signal

$x_o(n) = \begin{cases} 0, & n = 0 \\ \dfrac{e^{-n/4}}{2}, & n > 0 \\ -\dfrac{e^{+n/4}}{2}, & n < 0 \end{cases}$

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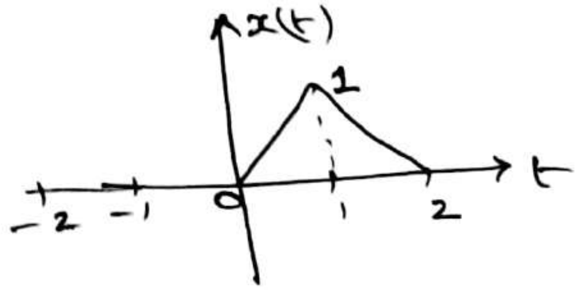
(ii) $x(t) = \begin{cases} t, & 0 \le t \le 1 \\ 2-t, & 1 \le t \le 2 \end{cases}$

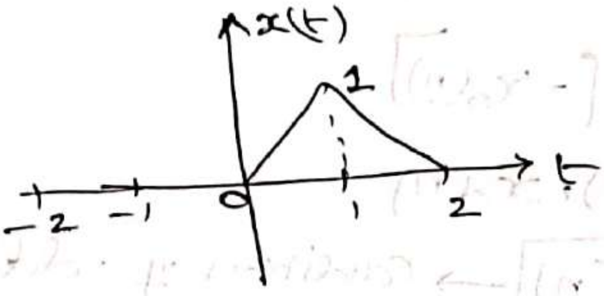
=



Problem is same as problem no: 4

(ii) $x(t) = \begin{cases} t, & 0 \leq t \leq 1 \\ 2-t, & 1 \leq t \leq 2 \end{cases}$

$=$



Problem is same as problem no: 4

(10) Find even and odd components of the signal

$x(n) = [3, 4, 6, \underset{\uparrow}{8}, 2, -6, 7, 6]$ — ①

$=$ Fold it

$x(-n) = [6, 7, -6, 2, \underset{\uparrow}{8}, 6, 4, 3]$ — ②

$\frac{① + ②}{2} = x_e(n)$

$\therefore x_e(n) = [3, 5, -1, 4, \underset{\uparrow}{8}, 4, -1, 5, 3]$

$\frac{① - ②}{2} = x_o(n)$

$\therefore x_o(n) = [-3, -2, 5, 2, \underset{\uparrow}{0}, -2, -5, 2, 3]$

(11) S.T. Product of Even and odd ~~signal~~ signals result in odd signal

$\cancel{x_o(n)}\; x_e(n) \cdot x_o(n) = x_o(n)$

$=$ Let ~~LHS~~ $= x_e(n) \cdot x_o(n) = y(n)$ (say)

Substitute $n = -n$

$y(-n) = x_e(-n) \cdot x_o(-n)$ — ①

For $\left.\begin{array}{l} x_e(-n) = x_e(n) \\ x_o(-n) = -x_o(n) \end{array}\right\} - (2)$

Substitute (2) in (1)

$$y(-n) = x_e(n)\left[-x_o(n)\right]$$

$$y(-n) = -x_e(n) \cdot x_o(n)$$

$$\boxed{y(-n) = -y(n)} \rightarrow \text{condition of odd}$$

$\therefore$ Product of even and odd is odd only

---

(12) S.T $\sum\limits_{n=-\infty}^{\infty} x^2(n) = \sum\limits_{n=-\infty}^{\infty} x_e^2(n) + \sum\limits_{n=-\infty}^{\infty} x_o^2(n)$

$$= \text{LHS} = \sum_{n=-\infty}^{\infty} x^2(n)$$

$$= \sum_{n=-\infty}^{\infty} \left[x_e(n) + x_o(n)\right]^2$$

$$= \sum_{n=-\infty}^{\infty} \left[x_e^2(n) + x_o^2(n) + 2\, x_e(n) \cdot x_o(n)\right]$$

$$= \sum_{n=-\infty}^{\infty} x_e^2(n) + \sum_{n=-\infty}^{\infty} x_o^2(n) + 2\sum_{n=-\infty}^{\infty} x_e(n)\, x_o(n)$$

4. Energy and Power Signals

In electrical systems, a signal is represented by voltage or a current through the resistor 'R'.

(i) CT Signal :- The resistor value is $R = 1\,\Omega$, while computing power and Energy.

The Energy of CT signal $x(t)$ is

$$\boxed{E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt}$$

If $0 < E < \infty$, then $x(t)$ is called Energy Signal

Ex: Non periodic signals

The power of CT signal $x(t)$ is

$$\boxed{P = \frac{1}{T} \int_{0}^{T} |x(t)|^2 \, dt}$$

, $T \rightarrow$ Fundamental Period.

If $0 < P < \infty$, then $x(t)$ is power signal

Ex :- Periodic signal

(ii) DT Signal :-

The Energy of DT signal $x(n)$ is

$$\boxed{E = \sum_{n=-\infty}^{\infty} |x(n)|^2}$$ If $0 < E < \infty$, then ES

The power of DT signal $x(n)$ is

$$\boxed{P = \frac{1}{N} \sum_{n=0}^{N-1} |x(n)|^2}$$, $N \to$ Fundamental Period

If $0 < P < \infty$, then PS

Problems :-

1. Determine, whether the following signals are energy signals, power signals or neither

(i) $x(t) = u(t)$, (ii) $x(t) = t \cdot u(t)$

(iii) $x(t) = e^{-at} \cdot u(t)$, (iv) $x(t) = \cos(5\pi t)$

(v) $x(t) = 8 \cdot e^{j4\pi t} \cdot u(t)$

= (i) $x(t) = u(t)$ $\boxed{\text{Neither}}$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |u(t)|^2 dt = \int_{0}^{\infty} 1 \cdot dt$$

$$= t\Big|_0^{\infty} = \infty \quad \therefore x(t) \text{ is not } \boxed{ES}$$

$T \to$ can't be determ

$\therefore P = 0$, not PS

$$P = \frac{1}{T}\int_0^T dt = \cancel{\frac{1}{T} t\Big|_0^T} = \cancel{\frac{1}{T}[T-0] = 1} \; \cancel{\therefore \boxed{PS}}$$

(ii) $x(t) = t \cdot u(t)$

$$= \quad E = \int_{-\infty}^{\infty} |t \cdot u(t)|^2 \cdot dt = \int_{0}^{\infty} t^2 dt$$

$$= \frac{t^3}{3}\Big]_0^{\infty} = \frac{1}{3}[\infty - 0] = \infty$$

$\boxed{\therefore \text{Not ES}}$

$$P = \frac{1}{T}\int_0^T t^2 dt = \cancel{\frac{1}{T}\frac{t^3}{3}\Big]_0^T}$$

$$\cancel{= \frac{1}{3T}[T^3 - 0] = \frac{T^2}{3}} \rightarrow \infty$$

T can't be determined

$\therefore P = 0$, $\therefore$ $\boxed{\text{Not PS}}$

Signal is neither.

(iii) $x(t) = e^{-at} \cdot u(t)$

$$= \quad E = \int_{-\infty}^{\infty} |e^{-at} u(t)|^2 dt = \int_0^{\infty} e^{-2at} \cdot dt$$

$$= \frac{e^{-2at}}{-2a}\Big|_0^{\infty} = -\frac{1}{2a}[0 - 1]$$

$E = \frac{1}{2a}$, $\therefore$ Finite value $\therefore$ $\boxed{\text{ES}}$

$$P = \frac{1}{T}\int_{\cancel{-\infty}}^{\cancel{\infty}} |x(t)|^2 dt$$

T → can't be determined

$\therefore P = 0$, $\therefore$ Not PS

(iv) $x(t) = \cos(5\pi t)$

$$= \quad P = \frac{1}{T}\int_0^T |x(t)|^2 dt = \frac{1}{T}\int_0^T \cos^2(5\pi t)\, dt$$

$\omega = 5\pi$

$2\cancel{\pi} f = 5\cancel{\pi}$

$f = \frac{5}{2}$

$\therefore T = \frac{2}{5}$ sec.

$\boxed{P = \frac{1}{2}}$ watts

$0 < P < \infty$

$\therefore$ PS

(v) $x(t) = 8 \cdot e^{j4\pi t} \cdot u(t)$

$$= \quad E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |8 \cdot e^{j4\pi t} \cdot u(t)|^2 dt$$

$e^{j4\pi t} = \cos(4\pi t) + j\sin(4\pi t)$

$|e^{j4\pi t}| = \sqrt{\cos^2(4\pi t) + \sin^2(4\pi t)}$

$|e^{j4\pi t}| = 1$

$$\therefore E = 64\int_0^{\infty} dt = 64\, t\Big]_0^{\infty} = \infty$$

$\therefore$ $\boxed{\text{NOT ES}}$

$$P = \frac{1}{T}\int_0^T |8 \cdot e^{j4\pi t} \cdot u(t)|^2 dt$$

$\omega = 4\pi$

$2\pi f = 4\pi$

$f = 2$

$T = \frac{1}{2}$ sec.

$P = 2 \times 64 \times t\Big]_0^T$

$= 128[T] = 128 \times \frac{1}{2} = 64$ watt

$\boxed{\therefore \text{PS}}$

(2) Find energy and power of the following signals

(i) $x(t) = \begin{cases} t, & 0 \le t \le 1 \\ 2-t, & 1 \le t \le 2 \\ 0, & \text{otherwise} \end{cases}$

(ii) $x(t) = A \cdot e^{j\omega_0 t}$

(iii) $x(t) = A \cdot e^{-3|t|}$

(iv) $x(t) = e^{\alpha t} \cdot u(-t)$

= (i) $x(t) = \begin{cases} t, & 0 \le t \le 1 \\ 2-t, & 1 \le t \le 2 \\ 0, & \text{otherwise} \end{cases}$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_0^1 t^2 . dt + \int_1^2 (2-t)^2 dt$$

$$= \frac{t^3}{3}\Big|_0^1 + \frac{(2-t)^3}{3}\Big|_1^2$$

$E = \frac{2}{3}$ Joules, $\underline{\underline{\text{Finite}}}$

$P = \frac{1}{T}\int_0^T |x(t)|^2 dt$, $T \rightarrow$ can't be determined

$\rightarrow \infty$

$\therefore P = 0$

(ii) $x(t) = A \cdot e^{j\omega_0 t}$

$=$ $\omega = \omega_0$

$2\pi f = \omega_0$

$f = \frac{\omega_0}{2\pi}$

$T = \frac{1}{f} = \frac{2\pi}{\omega_0}$

$\cancel{E=\int}$ $P = \frac{1}{T}\int_0^T |x(t)|^2 dt$

$\cancel{P = \frac{2\pi}{\omega_0}\int_0^T A^2 dt}$

$\cancel{= \frac{2\pi}{\omega_0} A^2 \cdot t\Big]_0^T}$

$\cancel{= \frac{2\pi}{\omega_0} \cdot A^2 [T]}$

$P = \frac{1}{T}\int_0^T A^2 \cdot dt = \frac{1}{T} \cdot A^2 \cdot t\Big]_0^T$

$$\boxed{P = \frac{A^2}{T}[T-0] = A^2 \text{ watts}}$$

$E = \int_{-\infty}^{\infty} A^2 \cdot dt = A^2 t\Big]_{-\infty}^{\infty} = \infty$

(iii) $x(t) = A \cdot e^{-3|t|}$

$= E = A^2 \int_{-\infty}^{\infty} \left[e^{-3|t|}\right]^2 dt = A^2 \left\{ \int_{-\infty}^{0} \left[e^{-3\times(-t)}\right]^2 dt + \int_0^{\infty} \left[e^{-3\times(t)}\right]^2 dt \right\}$

$= A^2 \left\{ \int_{-\infty}^{0} e^{6t} dt + \int_0^{\infty} e^{-6t} dt \right\}$

$= A^2 \left\{ \frac{e^{6t}}{6}\Big|_{-\infty}^{0} + \frac{e^{-6t}}{-6}\Big|_0^{\infty} \right\}$

$E = A^2 \left[\frac{1}{6}(1-0) + \left(\frac{-1}{6}\right)(0-1) \right\} = A^2\left[\frac{1}{6}+\frac{1}{6}\right] = \frac{A^2}{3}$

$$P = \frac{1}{T}\int_0^T |x(t)|^2 dt \quad , \quad T \to \infty \quad \because \text{Not periodic}$$

$$\therefore P = 0$$

(iv) $x(t) = e^{\alpha t} u(-t)$

$$= \quad E = \int_{-\infty}^{\infty} (e^{\alpha t} u(-t))^2 dt = \int_{-\infty}^{0} e^{2\alpha t} dt$$

$$= \left. \frac{e^{2\alpha t}}{2\alpha} \right|_{-\infty}^{0} = \frac{1}{2\alpha}[1 - 0]$$

$$\boxed{E = \frac{1}{2\alpha}} \qquad P = 0 \quad \because \text{Non Periodic}$$

(3) Find energy and power of the following signals
(i) $x(n) = u(n)$ (ii) $x(n) = r(n)$
(iii) $x(n) = A \cdot e^{j8n}$,

= (i) $x(n) = u(n)$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \sum_{n=-\infty}^{\infty} |u(n)|^2$$

$$\boxed{E = \sum_{n=0}^{\infty} 1 = \infty}$$

$$P = \frac{1}{N} \sum_{n=0}^{N-1} |x(n)|^2 \quad , \quad N = 1 \text{ Sample.}$$

$$= \frac{1}{N} (N) = 1 \text{ watt.}$$

(ii) $x(n) = r(n)$

Ramp function, Non periodic

$\therefore N \rightarrow$ can't be determ

$$\therefore P = \frac{1}{N} \sum_{n=0}^{N-1} |x(n)|^2 \rightarrow \infty$$

$$\boxed{P = 0}$$

$$E = \sum_{n=-\infty}^{\infty} |r(n)|^2 = \sum_{n=0}^{\infty} (n)^2 = \infty$$

(iii) $x(n) = A \cdot e^{j8n}$

~~$\omega = 8$~~
~~$2\pi f = 8$~~
~~$f =$~~

$$P = \frac{1}{N} \sum_{n=0}^{N-1} |(A \cdot e^{j8n})|^2$$

$$= \frac{1}{N} \sum_{n=0}^{N-1} A^2$$

$$= \frac{1}{N} A^2 \sum_{n=0}^{N-1} (1)^2$$

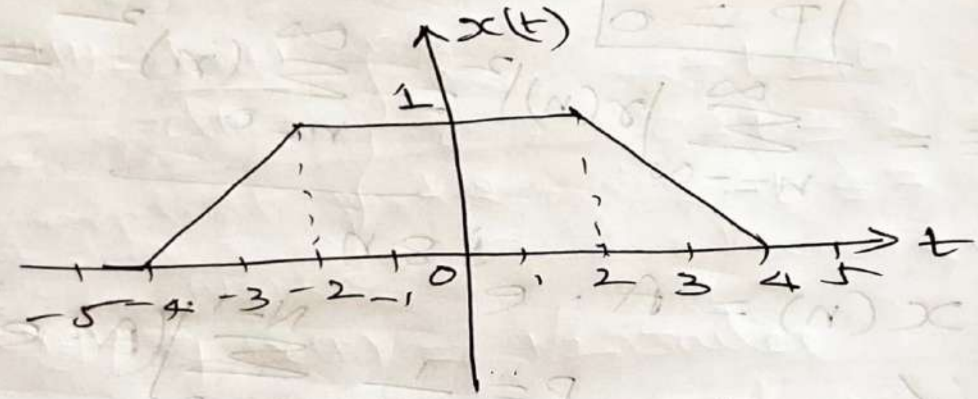
$$= \frac{1}{\cancel{N}} A^2 \times \cancel{N} = A^2$$

$$E = A^2 \sum_{n=-\infty}^{\infty} (1) = \infty$$

100

(4) Find the energy of trapezoidal signal $x(t)$ and $y(t) = 3 \cdot \frac{dx(t)}{dt}$

$$x(t) = \begin{cases} 4+t, & -4 \le t \le -2 \\ 2, & -2 \le t \le 2 \\ 4-t, & 2 \le t \le 4 \\ 0, & \text{otherwise} \end{cases} \quad \text{—(1)}$$



Energy of $x(t)$, $E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$

$$= \int_{-4}^{-2} (4+t)^2 dt + \int_{-2}^{2} (2)^2 dt + \int_{2}^{4} (4-t)^2 dt$$

$$= \left. \frac{(4+t)^3}{3} \right|_{-4}^{4} + \left. 4t \right|_{-2}^{2} + \left. \frac{(4-t)^3}{-3} \right|_{2}^{4}$$

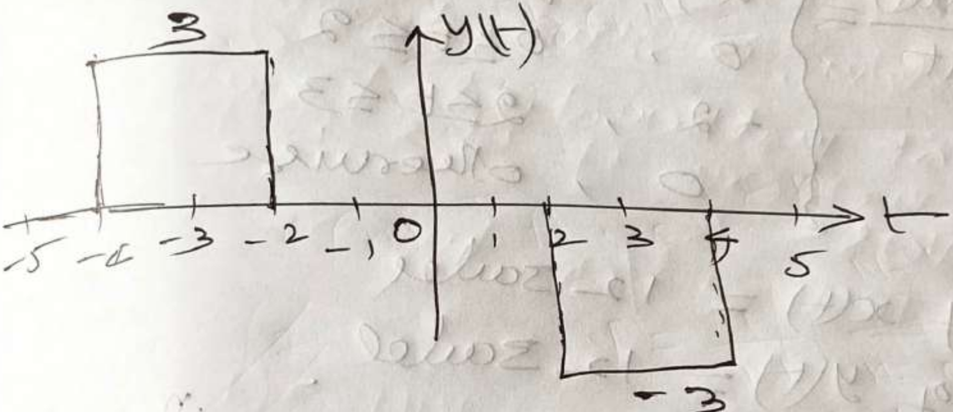
$$\boxed{E = \frac{64}{3} \text{ Joule}}$$

To find Energy of y(t):-

Differentiate Equation ①

$$\frac{dx(t)}{dt} = \begin{cases} 1, & -4 \le t \le -2 \\ 0, & -2 \le t \le 2 \\ -1, & 2 \le t \le 4 \\ 0, & \text{otherwise} \end{cases}$$

$$y(t) = 3\frac{dx(t)}{dt} = \begin{cases} 3, & -4 \le t \le -2 \\ 0, & -2 \le t \le 2 \\ -3, & 2 \le t \le 4 \\ 0, & \text{otherwise} \end{cases}$$



Energy of y(t), $E = \int_{-\infty}^{\infty} |y(t)|^2 dt$

$$= \int_{-4}^{-2} 3^2 dt + \int_{2}^{4} (-3)^2 dt$$

$$= \int_{-4}^{-2} 9\,dt + \int_{2}^{4} 9\,dt = 9t\Big|_{-4}^{-2} + 9t\Big|_{2}^{4}$$

$$= 9[-2+4] + 9[4-2] = 9(2) + 9(2)$$

$E = 18 + 18 = 36$ Joule

(5) The system input and output is related by $y(t) = \frac{dx(t)}{dt}$

Given input $x(t) = \begin{cases} 2t+2, & -2 \le t \le 0 \\ 2, & 0 \le t \le 2 \\ 6-2t, & 2 \le t \le 3 \\ 0, & \text{otherwise} \end{cases}$

Find energy of both input and outputs

=

$$y(t) = \frac{dx(t)}{dt} = \begin{cases} 2, & -2 \le t \le 0 \\ 0, & 0 \le t \le 2 \\ -2, & 2 \le t \le 3 \\ 0, & \text{otherwise} \end{cases}$$

Energy of $x(t) = 12$ Joule

Energy of $y(t) = 12$ Joule

(6) Consider the sequence $x(n) = \left(\frac{3}{2}\right)^n u(-n)$

(i) Find the numerical value of $A = \sum_{n=-\infty}^{\infty} x(n)$

(ii) Compute the power in $x(n)$

= (i) $A = \sum_{n=-\infty}^{\infty} \left(\frac{3}{2}\right)^n u(-n) = \sum_{n=-\infty}^{0} \left(\frac{3}{2}\right)^n$

Replace n by $-n$, $A = \sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n = \frac{1}{1-\frac{2}{3}} = 3$

(ii) power $P = \frac{1}{N} \sum_{n=0}^{N-1} |x(n)|^2$, Signal $x(n)$ is Non periodic
$\therefore N \to \infty$

Hence $\underline{P = 0}$

Formulae ✻✻✻✻✻

1. $$\sum_{n=0}^{\infty}(a)^n = \begin{cases} \frac{1}{1-a}, & a<1 \quad \text{Converges} \\ \infty, & a \geq 1 \quad \text{Diverges} \end{cases}$$

2. $$\sum_{n=\square}^{\infty}(a)^n = \begin{cases} \frac{a^{\square}}{1-a}, & a<1 \quad \text{Converges} \\ \infty, & a \geq 1 \quad \text{Diverges.} \end{cases}$$

3. $$\sum_{n=0}^{N-1}(a)^n = \begin{cases} \frac{1-a^{N-1+1}}{1-a} & a \neq 1 \\ N-1+1 & a = 1 \end{cases}$$

4. $$\sum_{n=\square}^{N-1}(a)^n = \begin{cases} \frac{a^{\square}-a^{N-1+1}}{1-a}, & a \neq 1 \\ N-1+1-\square, & a = 1 \end{cases}$$

5. Deterministic and Random Signals

(i) ~~Detern~~ Deterministic Signals:—

The signal behave in a fixed known way w.r.t. time 't' or 'n'
ie., no uncertainity w.r.t. time

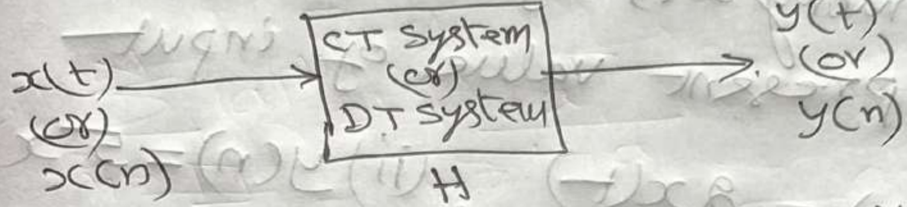
Ex:— sin, cos, ---

(ii) **Random Signal**:— The magnitudes are random, ie., uncertainity w.r.t. time
No explicit mathematical formulae

Ex:— Noise

System

A system is a device, which process the input signal $x(t)$ (or) $x(n)$ to produce another signal $y(t)$ (or) $y(n)$



$$H = \frac{(o/p)}{(i/p)} = \frac{y(t)}{x(t)} \text{ (or) } \frac{y(n)}{x(n)}$$

i.e., $y(t) = H[x(t)]$ (or) $y(n) = H[x(n)]$

Properties of System:– The properties of CT and DT system are discussed

1. → Static and Dynamic system
   → Memoryless and memory system
2. Causality
3. Linearity
4. Time invariance
5. Stability
6. Invertability.

1. Memory (or) Memory less Systems

→ Memoryless (or) Static :— A system is called memoryless if its output depends only on present value of input

Ex: (i) $y(t) = 2x(t)$, (ii) $y(n) = 5x(n)$

(iii) $y(n) = n\,x(n)$ (iv) $y(n) = x(n) + u(n-1)$

→ Memory (or) Dynamic :— A system is said to be memory if its output depends on past and/OR future values of input

Ex :- (i) $y(t) = x(t^2)$ (ii) $y(t) = x(t-2)$

(iii) $y(n) = x(n+1) + x(n) + x(n-2)$

(iv) $y(n) = x(2n)$

2. Causality :—

(i) Causal :— A system is called causal if its output depends only on ~~Pa~~ present and/OR past values of input. ie., the output not depends on future input

Ex:- (i) $y(t) = 5x(t)$ (ii) $y(t) = x(t) + x(t-2)$

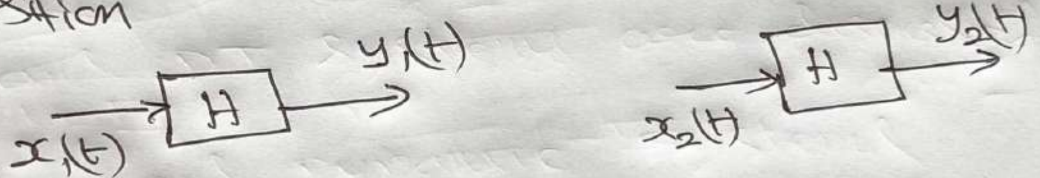
(iii) $y(n) = e^{x(n)}$

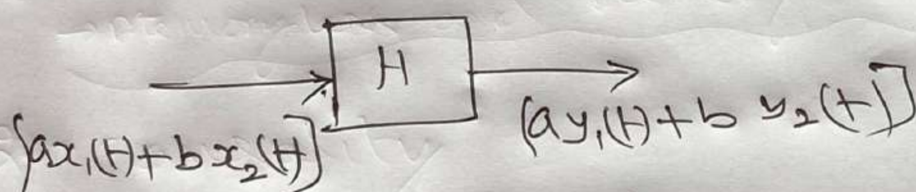
(ii) <u>non causal</u> :- A system is called non causal if its output depends on <u>future</u> values of input.

Ex:- (i) $y(t) = x(t+1)$ (ii) $y(t) = x(t) + x(t+2)$

(iii) $y(n) = 2x(n) + x(n-1) + x(n+2)$

3. <u>linearity</u> :- A system is called linear if it satisfies the principle of superposition





<u>superposition Principle</u>

$$\boxed{x_3(t) = [ax_1(t) + bx_2(t)] \longrightarrow y_3(t) = [ay_1(t) + by_2(t)]}$$

4) Time Invariance :-

→ A system is said to be time invariant, if its input and output characteristics do not change with time

→ The time shift of the input signal causes a corresponding shift in the output.

$x(t) \longrightarrow \boxed{H} \longrightarrow y(t) = H[x(t)]$

$x(n)$ $\qquad y(n) = H[x(n)]$

shift output $\quad y(n-n_0) = H[x(n-n_0)]$ — ①

varies in time i.e., shift

Shift input

$x(t-t_0) \longrightarrow \boxed{H} \longrightarrow y(t-t_0) = H[x(t-t_0)]$

$x(n-n_0)$ $\qquad y(n-n_0) = H[x(n-n_0)]$ — ②

∵ ① = ②, ∴ TIV

Example :- consider input $x(n) =$ ~~[u(n) - u(n-3)]~~

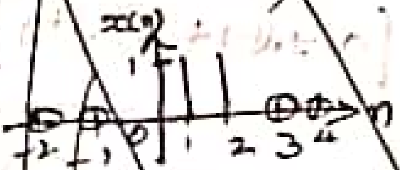
System $\boxed{y(n) = e^{-n}\, x(n)}$

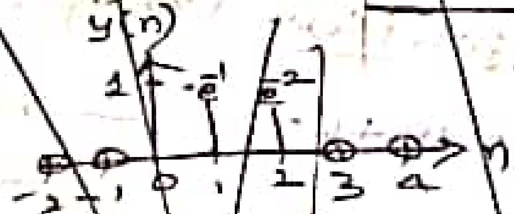
original system

$x(n) \longrightarrow \boxed{H = e^{-n}} \longrightarrow$

$y(n) = H(x(n))$

$y(n) = e^{-n} x(n)$ — ①

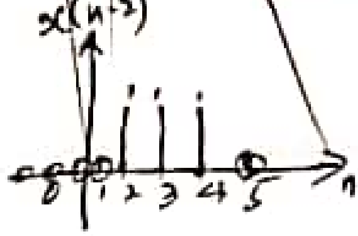


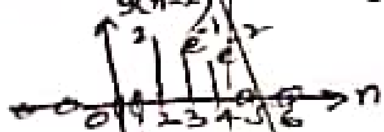


Shift it by 2

$x(n-2) \longrightarrow \boxed{H = e^{-(n-2)}} \longrightarrow y(n-2) = H[x(n-2)]$

$= e^{-(n-2)} \cdot x(n-2)$ — ②





① = ②, ∴ Time Invariant

Interms of Equation

For comparision shift Graph ① by 2

Replace n by $n-2$ in $y(n)$

$$y_1(n) = y(n-2) = e^{-(n-2)} \cdot x(n-2) \quad \text{—— ①}$$

From ② $$y_2(n) = y(n-2) = e^{-(n-2)} x(n-2) \quad \text{—— ②}$$

① = ② ∴ TIV

(or)

From Fig ②

consider i/p $x(n)$ and shift by 2 and find output

$x(n) \to x(n-2)$, $y(n) = e^{-n} \cdot x(n)$ original

~~$y_1(n) = e^{-n} x(n-2)$~~

1) Example:- $y(t) = 2x(t)$

$$x(t) \longrightarrow \boxed{\times 2} \longrightarrow y(t) = 2x(t)$$

(i) shift input $x(t)$ ~~$x(t) = x(t-2)$ find o/p~~

~~$y_1(t) = 2x(t-2)$~~

$$x_1(t) = x(t-2) \longrightarrow \boxed{\times 2} \longrightarrow y_1(t) = 2x_1(t) = 2x(t-2) \quad \text{—— ①}$$

(ii) shift output ~~$y(t)$~~

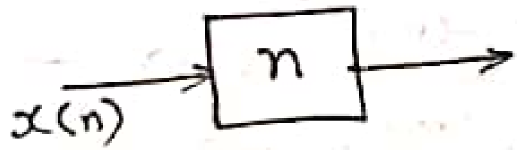
$$x_2(t) = x(t) \longrightarrow \boxed{\times 2} \longrightarrow y_2(t) = 2x_2(t) = 2x(t)$$

shift o/p

$$y_2(t-2) = 2x_2(t-2) = 2x(t-2) \quad \text{—— ②}$$

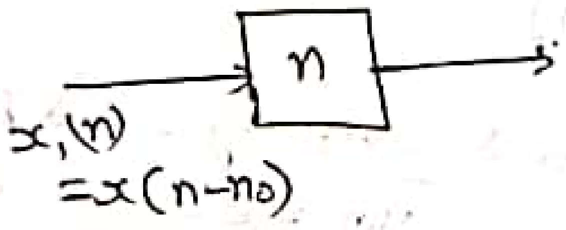
① = ② Time Invariant

Example 2 : $y(n) = n \cdot x(n)$



$y(n) = n \cdot x(n)$

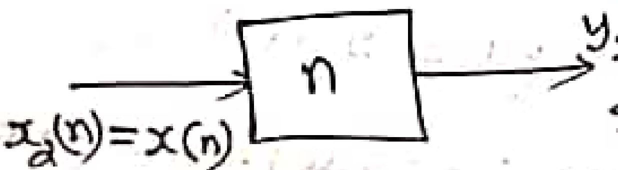
(i) shift input, find corresponding output



$y_1(n) = n \cdot x_1(n)$

$y_1(n) = n \cdot x(n-n_0)$ —— ①

(ii) shift output, find corresponding output



$y_2(n) = n \cdot x_2(n) = n \cdot x(n)$

shift o/p

$y_2(n-n_0) = (n-n_0)\, x(n-n_0)$ —— ②

① ≠ ②

Hence Time variant

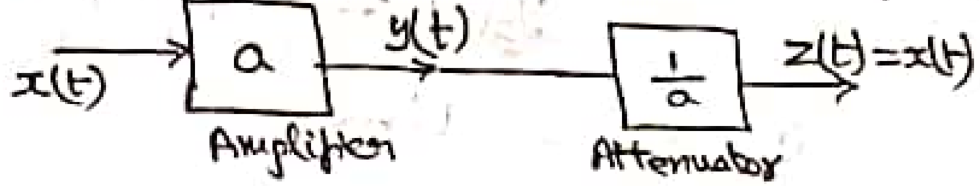
5) **Stability** :— A system is said to be stable if finite input produces finite output

→ Bounded input produces bounded output (BIBO)

6) <u>Invertibility</u> :- A system is said to be invertible, if the input of the system can be recovered from the system output.

<u>Ex</u> :-

<u>Invertible system</u>



$y(t) = a\,x(t)$, $z(t) = \frac{1}{a} y(t) = \frac{1}{a} \cdot a\,x(t)$

$z(t) = x(t)$

<u>Problems</u> :— Determine whether the following systems are (i) memory (ii) causal (iii) Linear (iv) Time Invariant (v) Stable

1) $y(t) = x\left(\frac{t}{2}\right)$

(i) <u>Memory</u> :-

if $t=0$, $y(0) = x(0)$ present input

$t=-1$, $y(-1) = x(-0.5)$ Future input

$t = 1$, $y(1) = x(0.5)$ past input

Present out depends on past and future inputs, ∴ System has $\boxed{\text{memory}}$

(ii) <u>Causality</u> :- Present output depends on future input, ∴ System is $\boxed{\text{non-causal}}$

(iii) <u>Linearity</u> :- $x_1(t) \longrightarrow y_1(t)$

$x_2(t) \longrightarrow y_2(t)$

$x_3(t) \longrightarrow y_3(t)$

Superposition Principle

$x_3(t) = [a x_1(t) + b x_2(t)] \longrightarrow y_3(t) = [a y_1(t) + b y_2(t)]$ ⟵ ①

$y_1(t) = x_1\left(\frac{t}{2}\right)$, $y_2(t) = x_2\left(\frac{t}{2}\right)$, $y_3(t) = x_3\left(\frac{t}{2}\right)$

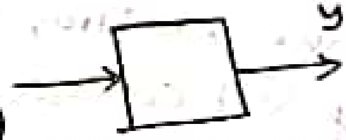
$$y_3(t) = x_3\left(\frac{t}{2}\right) = a\,x_1\left(\frac{t}{2}\right) + b\,x_2\left(\frac{t}{2}\right)$$

$$y_3(t) = a\,y_1(t) + b\,y_2(t) \quad \text{--- (2)}$$

(1) = (2); $\therefore$ Linear

(iv) <u>Time Invariant</u> :-

<u>Shift input</u> :-



$x_1(t) = x(t-t_0)$

$y_1(t) = \cancel{x(\tfrac{t}{2})}\ x_1\left(\frac{t}{2}\right)$

$$y_1(t) = x\left(\frac{t}{2} - t_0\right) \quad \text{--- (1)}$$

<u>output Shift</u> :-

$x_2(t) = x(t)$

$y_2(t) = x_2\left(\frac{t}{2}\right) = x\left(\frac{t}{2}\right)$

shift output

$$y_2(t-t_0) = x\left(\frac{t-t_0}{2}\right) \quad \text{--- (2)}$$

(1) $\neq$ (2) $\therefore$ <u>Time Variant</u>

(v) <u>Stability</u> :- If $x(t)$ is finite

then $y(t) = \underset{\text{Time Expanded}}{x\left(\frac{t}{2}\right)} \rightarrow$ Finite

$\therefore$ BIBO <u>Stable</u>

2) $y(n) = x^2(n)$

(i) Memory :- If $n=0$, $y(0) = x^2(0)$ present input

If $n=-1$, $y(-1) = x^2(-1)$ present input

If $n=1$, $y(1) = x^2(1)$ present input.

output depends on present input $\therefore$ ~~causal~~ $\boxed{\text{Memoryless}}$

(ii) causality :-

System is causal, because output is not depends on future input.

(iii) linearity :- $y_1(n) = x_1^2(n)$, $y_2(n) = x_2^2(n)$

$y_3(n) = x_3^2(n)$

SPP :- $x_3(n) = [a x_1(n) + b x_2(n)] \rightarrow y_3(n) = (a y_1(n) + b y_2(n))$ —— ①

$$y_3(n) = x_3^2(n) = (a x_1(n) + b x_2(n))^2$$

$$y_3(n) = a^2 x_1^2(n) + b^2 x_2^2(n) + 2ab\, x_1(n) \cdot x_2(n) \quad \text{—— ②}$$

① ≠ ②, Hence $\boxed{\text{non linear}}$

(iv) Time Invariant :-

Shift Input :-

$x_1(n) = x(n-n_0) \rightarrow \boxed{} \rightarrow y_1(n) = x_1^2(n) = (x(n-n_0))^2$ —— ①

Shift output :-

$x_2(n) = x(n) \rightarrow \boxed{} \rightarrow y_2(n) = x_2^2(n)$ ~~shift~~ $= x^2(n)$

Shift $y_2(n-n_0) = x_2^2(n-n_0)$ —— ②

① = ② $\therefore$ Time Invariant

(v) Stability :- If $x(n) \rightarrow$ Finite

$y(n) = x^2(n) \rightarrow$ Finite

Hence BIBO Stable.

3) $y(t) = 2\, e^{x(t)}$

= (i) memoryless (ii) causal

(iii) linearity :- $y_1(t) = 2e^{x_1(t)}$

$y_2(t) = 2e^{x_2(t)}$, $y_3(t) = 2e^{x_3(t)}$

SPR:-

$x_3(t) = [a x_1(t) + b x_2(t)] \rightarrow y_3(t) = (a y_1(t) + b y_2(t))$ —— (1)

$y_3(t) = 2e^{x_3(t)} = 2e^{[a x_1(t) + b x_2(t)]}$

$= 2 \cdot e^{a x_1(t)} \cdot e^{b x_2(t)}$

$y_3(t) = 2 (e^{x_1(t)})^a \cdot (e^{x_2(t)})^b$ —— (2)

$\therefore$ (1) $\neq$ (2) $\therefore$ Non linear.

(iv) Time Invariant :-

Input Shift

$y_1(t) = 2e^{x_1(t)} = 2 \cdot e^{x(t-t_0)}$ —— (1)

$x_1(t) = x(t-t_0)$

output shift

$x_2(t) = x(t) \rightarrow \boxed{} \rightarrow y_2(t) = 2e^{x_2(t)} = 2\cdot e^{x(t)}$

shift o/p

$~~y_2(t)~~ \; y_2(t-t_0) = 2\cdot e^{x(t-t_0)}$ —— (2)

(1) = (2)

∴ Time Invariant

(v) Stability :- If input is finite

$x(t) \rightarrow$ Finite

$y(t) = 2\cdot e^{x(t)} = 2\cdot e^{\text{Finite}} =$ Finite.

∴ BIBO stable

(4) $y(t) = \cos(x(t))$

(i) memory less (ii) causal

(iii) linearity :- $y_1(t) = \cos(x_1(t))$

$y_2(t) = \cos(x_2(t))$, $y_3(t) = \cos(x_3(t))$

SPP: $x_3(t) = [a x_1(t) + b x_2(t)] \rightarrow y_3(t) = [a y_1(t) + b y_2(t)]$ —— (1)

$y_3(t) = \cos[x_3(t)] = \cos[a x_1(t) + b x_2(t)]$ —— (2)

(1) ≠ (2) Non linear

(iv) Time Invariance :-

Shift i/p :-

$x_1(t) = x(t-t_0) \rightarrow \boxed{} \rightarrow y_1(t) = \cos[x_1(t)]$

$= \cos[x(t-t_0)]$ —— (1)

Scanned with CamScanner

Shift-o/p

$x_2(t) = x(t)$

$y_2(t) = \cos[x_2(t)] = \cos[x(t)]$

Shift-o/p

$y_2(t-t_0) = \cos[x(t-t_0)]$ —— ②

① = ②

∴ Time invariant

(v) Stability :- $x(t) \rightarrow$ Finite.

$y(t) = \cos[x(t)] = \cos(\text{Finite}) = \text{Finite}.$

BIBO Stable.

⑤ $y(t) = x(t)\cos(\omega_0 t)$

(i) memoryless (ii) causal

(iii) linearity :- $y_1(t) = x_1(t)\cos(\omega_0 t)$

$y_2(t) = x_2(t)\cos(\omega_0 t)$, $y_3(t) = x_3(t)\cos(\omega_0 t)$

SPP

$x_3(t) = [a x_1(t) + b x_2(t)] \rightarrow y_3(t) = [a y_1(t) + b y_2(t)]$ —— ①

$y_3(t) = x_3(t)\cos(\omega_0 t)$

$= [a x_1(t) + b x_2(t)]\cos(\omega_0 t)$

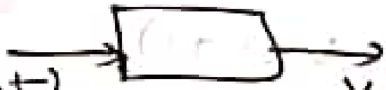
$= a x_1(t)\cos(\omega_0 t) + b x_2(t)\cos(\omega_0 t)$

$y_3(t) = a y_1(t) + b y_2(t)$ —— ②

① = ② ∴ Linear.

(iv) Time Invariance :-

Shift input



$x_1(t) = x(t-t_0)$

$y_1(t) = x_1(t)\cos(\omega_0 t)$

$y_1(t) = x(t-t_0)\cdot\cos(\omega_0 t)$ ——— (1)

output shift :-

$x_2(t) = x(t)$

$y_2(t) = x_2(t)\cos(\omega_0 t)$

$y_2(t) = x(t)\cos(\omega_0 t)$

shift o/p, Replace t by $(t-t_0)$

$y_2(t-t_0) = x(t-t_0)\cos[\omega_0(t-t_0)]$ ——— (2)

$(1) \neq (2)$

$\therefore$ Time Variant

(v) Stability:

$x(t) \rightarrow$ Finite

$y(t) = x(t)\cos(\omega_0 t)$

= Finite × Finite

= Finite

$\therefore$ BIBO stable.

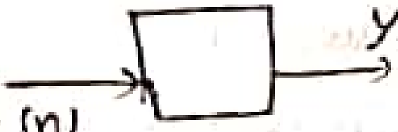
(6) $y(n) = 2x(n)\cdot u(n+1)$

= (i) memory less (ii) causal

(iii) linear (iv)

(iv) Time Invariant :-

Shift Input



$$y_1(n) = 2x_1(n) \cdot u(n+1)$$
$$= 2x(n-n_0) \cdot u(n+1) \quad \text{--- (1)}$$

$x_1(n) = x(n-n_0)$

Shift output

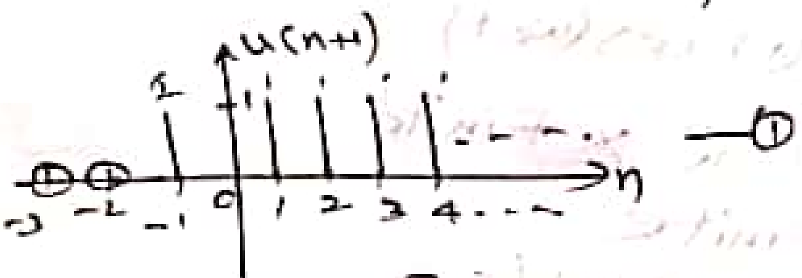
$x_2(n) = x(n)$

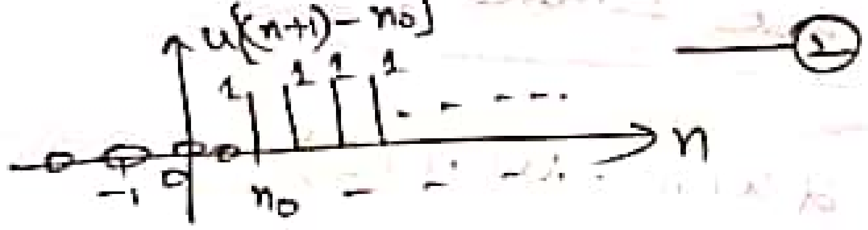
$$y_2(n) = 2x_2(n)\, u(n+1)$$
$$y_2(n) = 2x(n)\, u(n+1)$$

shift o/p

$$\cancel{y_2(n)}$$
$$y_2(n-n_0) = 2x(n-n_0)\, u(n-n_0+1)$$

$$y_2(n-n_0) = 2x(n-n_0) \cdot u[(n+1)-n_0] \quad \text{--- (2)}$$





Graphs (1) & (2) are same except 2nd graph shifted right by 'n_0' units

∴ Equations (1) & (2) are same

∴ Time invariant

(v) Stability:

$x(n) \rightarrow$ Finite

$y(n) = 2x(n)\ u(n+1)$

$= 2 \times \text{Finite} \times \text{Finite} = \text{Finite}$

$\therefore$ Stable.

(7) $y(n) = n \cdot x(n)$

= (i) Memoryless (ii) causal (iii) linear.

(iv) Time variant

(v) Stability:- $x(n) \rightarrow$ Finite

$y(n) = n \cdot x(n) = n \cdot \text{Finite}.$

$= \infty$ for $n \rightarrow \infty$

$\therefore$ Unstable.

---

(8) $y(n) = x(-n)$

= (i) Memory (ii) Non causal (v) stable.

(iii) Linear (iv) Time variant

(9) $y(n) = x(n) + 2x(n-1)$

= (i) memory (ii) causal (iii) linear.

(iv) Time invariant (v) Stable.

(10) $y(t) = \text{Log}[x(t)]$

= (i) Memoryless (ii) causal (iii) Non linear

(iv) Time invariant (v) Stable

---

(11) $y(n) = e^{x(n)}$

= Memoryless, causal, Non linear
Time invariant, stable

(12) strait line equation
$y(n) = a\,x(n) + b$

= memory less, causal, non linear,
time invariant, stable

(13) $y(n) = \sum_{k=n-n_0}^{n+n_0} x(k)$

= if $n_0 = 1$

$y(n) = \sum_{k=n-1}^{n+1} x(k)$

$y(n) = x(n-1) + x(n) + x(n+1)$

(i) Memory :- The system is ~~non~~ memory since, the present output depends on past and future inputs

(ii) Non causal, ∵ output depends on future input

(iii) Linearity :- $y_1(n) = \sum_{k=n-n_0}^{n+n_0} x_1(k)$

$y_2(n) = \sum_{k=n-n_0}^{n+n_0} x_2(k)$, $y_3(n) = \sum_{k=n-n_0}^{n+n_0} x_3(k)$

SPP

$x_3(k) = [a x_1(k) + b x_2(k)] \rightarrow y_3(n) = (a y_1(n) + b y_2(n))$ —— (1)

system output $y_3(n) = \sum_{k=n-n_0}^{n+n_0} x_3(k)$

$$= \sum_{k=n-n_0}^{n+n_0} [a x_1(k) + b x_2(k)]$$

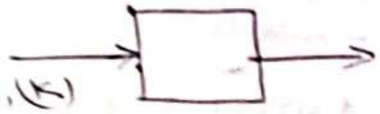
$$= a \sum_{k=n-n_0}^{n+n_0} x_1(k) + b \sum_{k=n-n_0}^{n+n_0} x_2(k)$$

$$y_3(n) = a\, y_1(n) + b\, y_2(n) \quad \text{--- (2)}$$

(1) = (2), Hence System is linear

(iv) Time invariance :-

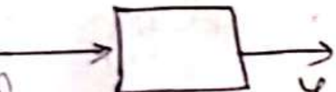
Shift input



$x_1(k) = x(k-k_0)$

$$y_1(n) = \sum_{k=n-n_0}^{n+n_0} x_1(k) = \sum_{k=n-n_0}^{n+n_0} x(k-k_0) \quad \text{--- (1)}$$

Shift output



$x_2(k) = x(k)$

$$y_2(n) = \sum_{k=n-n_0}^{n+n_0} x_2(k) = \sum_{k=n-n_0}^{n+n_0} x(k)$$

Shift output, i.e., Replace n by $(n-k_0)$

$$y_2(n-k_0) = \sum_{k=n-k_0-n_0}^{n-k_0+n_0} x(k)$$

$$y_2(n-k_0) = \sum_{k=[(n-n_0)-k_0]}^{[(n+n_0)-k_0]} x(k) \quad \text{--- (2)}$$

Equation (1) = Equation (2) Hence Time invariant.

Equation (2) is shifted Right by k_0 units compared to Equation (1)

(v) <u>Stability</u> :- Consider $x(k) \rightarrow$ Finite

$$y(n) = \sum_{k=n-n_0}^{n+n_0} x(k) = \sum_{k=(n-n_0)}^{(n+n_0)} (\text{Finite})$$

System is <u>stable</u> for <u>finite limits</u>

(14) $$y(n) = \sum_{k=n_0}^{n} x(k)$$

$=$ If $n_0 = 0$, $$y(n) = \sum_{k=0}^{n} x(k)$$

$$y(n) = \sum_{k=0}^{n} x(k) = x(0) + x(1) + \cdots + x(n)$$

if $n = -1$, $y(-1) = \sum_{k=0}^{-1} x(k) = x(0) + x(-1)$

$\cancel{y(-1) = x(0) + x(1) + \cdots x(-1)}$

$y(-1) = x(0) + x(-1)$ Future inputs

$n = 1$, $y(1) = x(0) + x(1)$ past inputs

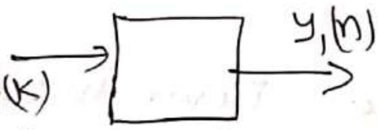
(i) memory system, $\because$ output depends on past and future inputs

(ii) non causal, $\because$ output depends on future inputs

(iii) <u>linearity</u> :- linear

(iv) <u>Time invariance</u> :-

<u>Shift input</u>

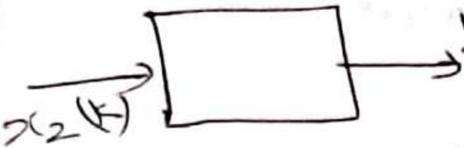


$x_1(k) = x(k-k_0)$

$$y_1(n) = \sum_{k=n_0}^{n} x_1(k)$$

$$= \sum_{k=n_0}^{n} x(k-k_0) \quad \text{——— (1)}$$

Shift output



$x_2(k) = x(k)$

$$y_2(n) = \sum_{k=n_0}^{n} x_2(k)$$

$$y_2(n) = \sum_{k=n_0}^{n} x(k)$$

Shift output by k_0 units,

$$y_2(n-k_0) = \sum_{k=n_0}^{n-k_0} x(k) \quad \text{—— (2)}$$

For comparision (1) & (2)

consider Equ (1) $y_1(n) = \sum_{k=n_0}^{n} x(k-k_0)$

let $k-k_0 = m$, $\Rightarrow k = m+k_0$

limits :- lower: $k = n_0$ upper: $k = n$

$m+k_0 = n_0$ $\qquad$ $m+k_0 = n$

$\boxed{m = n_0+k_0}$ $\qquad$ $\boxed{m = n+k_0}$

$$y_1(n) = \sum_{m=n_0+k_0}^{n+k_0} x(m) \quad \text{—— (3)}$$

consider Eqn (2) $y_2(n-k_0) = \sum_{k=n_0}^{n-k_0} x(k)$ — (2)

(2) $\neq$ (3) $\because$ Number of sample in limits are different.

$\therefore$ Time variant

(v) Stability :- if $x(k) \rightarrow$ Finite

$$y(n) = \sum_{k=n_0}^{n} x(k) = \sum_{k=n_0}^{n} (\text{Finite})$$

For $n \rightarrow \infty$, $y(n) \rightarrow \infty$ $\therefore$ Unstable.

15 $y(n) = g(n)\, x(n)$

= memoryless, causal, linear

Time variant,

Stability :- $x(n) \to$ Finite.

$y(n) = g(n)$. Finite.

The system is Stable if $g(n)$ Finite

Unstable if $g(n) \to \infty$

16 $y(n) = x(n) \sum_{k=-\infty}^{\infty} \delta(n-2k)$

= memoryless, causal

Linearity :- $y_1(n) = x_1(n) \sum_{k=-\infty}^{\infty} \delta(n-2k)$

$y_2(n) = x_2(n) \sum_{k=-\infty}^{\infty} \delta(n-2k)$

$y_3(n) = x_3(n) \sum_{k=-\infty}^{\infty} \delta(n-2k)$

SPP

$x_3(n) = [a x_1(n) + b x_2(n)] \to y_3(n) = [a y_1(n) + b y_2(n)]$ —— 1

system output

$y_3(n) = x_3(n) \sum_{k=-\infty}^{\infty} \delta(n-2k)$

$= [a x_1(n) + b x_2(n)] \sum_{k=-\infty}^{\infty} \delta(n-2k)$

$= a x_1(n) \sum_{k=-\infty}^{\infty} \delta(n-2k) + b x_2(n) \sum_{k=-\infty}^{\infty} \delta(n-2k)$

$y_3(n) = a y_1(n) + b y_2(n)$ —— 2

1 = 2 Linear

Time invariant :-

shift input

$$x_1(n) \rightarrow \boxed{} \rightarrow y_1(n) = x_1(n) \sum_{k=-\infty}^{\infty} \delta(n-2k)$$

$= x(n-n_0)$

$$y_1(n) = x(n-n_0) \sum_{k=-\infty}^{\infty} \delta(n-2k) \quad \text{--- (1)}$$

shift output

$$x_2(n) \rightarrow \boxed{} \rightarrow y_2(n) = x_2(n) \sum_{k=-\infty}^{\infty} \delta(n-2k)$$

$= x(n)$

$$y_2(n) = x(n) \sum_{k=-\infty}^{\infty} \delta(n-2k)$$

Shift output

$$y_2(n-n_0) = x(n-n_0) \sum_{k=-\infty}^{\infty} \delta(n-n_0-2k)$$

$$= x(n-n_0) \sum_{k=-\infty}^{\infty} \delta((n-2k)-n_0) \quad \text{--- (2)}$$

(1) = (2)

Hence Time invariant

Equation (2) is shifted version of Eqn (1) by n_0 units

stability :- If $x(n) \rightarrow$ Finite

$$y(n) = x(n) \sum_{k=-\infty}^{\infty} \delta(n-2k)$$

$$= [\text{Finite}]\,[\text{Finite}] = \text{Finite}$$

$\therefore$ stable

(17) $$y(n) = \sum_{k=-\infty}^{n} x(k+2)$$

= memory, non causal, Linear, Time invariant

Stable