Appendices

Appendix-I

Butterworth polynomials in standard and factored forms

Standard form

$$B_N(s) = a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0$$

a_7	a_6	a_5	a_4	a_3	a_2	a_1	a_0	n
						1	1	1
					1	$\sqrt{2}$	1	2
				1	2	2	1	3
			1	2.613	3.414	2.613	1	4
		1	3.236	5.236	5.236	3.236	1	5
	1	3.864	7.464	9.141	7.464	3.864	1	6
1	4.494	10.103	14.606	14.606	10.103	4.494	1	7

Factored form

$B_N(s)$	n
$s + 1$	1
$s^2 + \sqrt{2}s + 1$	2
$(s^2 + s + 1)(s + 1)$	3
$(s^2 + 0.76536s + 1)(s^2 + 1.84776s + 1)$	4
$(s + 1)(s^2 + 0.6180s + 1)(s^2 + 1.6180s + 1)$	5
$(s^2 + 0.5176s + 1)(s^2 + \sqrt{2}s + 1)(s^2 + 1.9318s + 1)$	6
$(s + 1)(s^2 + 0.4450s + 1)(s^2 + 1.2456s + 1)(s^2 + 1.8022s + 1)$	7

Butterworth filter

$$H_N(s) = \frac{1}{a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + 1} = \frac{1}{B_n(s)}$$

Appendix-II

Polynomials $V_N(s)$ used in Chebyshev I filter design for $\frac{1}{2}$, 1, 2, and 3 dB ripples

Chebyshev filter $H_N(s) = \frac{K_N}{V_N(s)}$, where $K_N = \begin{cases} \frac{b_0}{\sqrt{1+\epsilon^2}} & \text{for } N \text{ even} \\ b_0 & \text{for } N \text{ odd} \end{cases}$

$$V_N(s) = s^N + b_{N-1}s^{N-1} + \cdots + b_1 s + b_0$$

N	b_0	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9
				a. $\frac{1}{2}$ dB Ripple ($\epsilon = 0.3493114$, $\epsilon^2 = 0.1220184$)						
1	2.8627752									
2	1.5162026	1.4256245								
3	0.7156938	1.5348954	1.2529130							
4	0.3790506	1.0254553	1.7168662	1.1973856						
5	0.1789234	0.7525181	1.3095747	1.9373675	1.1724909					
6	0.0947626	0.4323669	1.1718613	1.5897635	2.1718446	1.1591761				
7	0.0447309	0.2820722	0.7556511	1.6479029	1.8694079	2.4126510	1.1512176			
8	0.0236907	0.1525444	0.5735604	1.1485894	2.1840154	2.1492173	2.6567498	1.1460801		
9	0.0111827	0.0941198	0.3408193	0.9836199	1.6113880	2.7814990	2.4293297	2.9027337	1.1425705	
10	0.0059227	0.0492855	0.2372688	0.6269689	1.5274307	2.1442372	3.4409268	2.7097415	3.1498757	1.1400664
				b. 1 dB Ripple ($\epsilon = 0.5088471$, $\epsilon^2 = 0.2589254$)						
1	1.9652267									
2	1.1025103	1.0977343								
3	0.4913067	1.2384092	0.9883412							
4	0.2756276	0.7426194	1.4539248	0.9528114						
5	0.1228267	0.5805342	0.9743961	1.6888160	0.9368201					
6	0.0689069	0.3070808	0.9393461	1.2021409	1.9308256	0.9282510				
7	0.0307066	0.2136712	0.5486192	1.3575440	1.4287930	2.1760778	0.9231228			
8	0.0172267	0.1073447	0.4478257	0.8468243	1.8369024	1.6551557	2.4230264	0.9198113		
9	0.0076767	0.0706048	0.2441864	0.7863109	1.2016071	2.3781188	1.8814798	2.6709468	0.9175476	
10	0.0043067	0.0344971	0.1824512	0.4553892	1.2444914	1.6129856	2.9815094	2.1078524	2.9194657	0.9159320

N	b_0	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9
c. 2 dB Ripple ($\epsilon = 0.7647831$, $\epsilon^2 = 0.5848932$)										
1	1.3075603									
2	0.6367681	0.8038164								
3	0.3268901	1.0221903	0.7378216							
4	0.2057651	0.5167981	1.2564819	0.7162150						
5	0.0817225	0.4593491	0.6934770	1.4995433	0.7064606					
6	0.0514413	0.2102706	0.7714618	0.8670149	1.7458587	0.7012257				
7	0.0204228	0.1660920	0.3825056	1.1444390	1.0392203	1.9935272	0.6978929			
8	0.0128603	0.0729373	0.3587043	0.5982214	1.5795807	1.2117121	2.2422529	0.6960646		
9	0.0051076	0.0543756	0.1684473	0.6444677	0.8568648	2.0767479	1.3837464	2.4912897	0.6946793	
10	0.0032151	0.0233347	0.1440057	0.3177560	1.0389104	1.1585287	2.6362507	1.5557424	2.7406032	0.6936904
d. 3 dB Ripple ($\epsilon = 0.9976283$, $\epsilon^2 = 0.9952623$)										
1	1.0023773									
2	0.7079478	0.6448996								
3	0.2505943	0.9283480	0.5972404							
4	0.1769869	0.4047679	1.1691176	0.5815799						
5	0.0626391	0.4079421	0.5488626	1.4149847	0.5744296					
6	0.0442467	0.1634299	0.6990977	0.6906098	1.6628481	0.5706979				
7	0.0156621	0.1461530	0.3000167	1.0518448	0.8314411	1.9115507	0.5684201			
8	0.0110617	0.0564813	0.3207646	0.4718990	1.4666990	0.9719473	2.1607148	0.5669476		
9	0.0039154	0.0475900	0.1313851	0.5834984	0.6789075	1.9438443	1.1122863	2.4101346	0.5659234	
10	0.0027654	0.0180313	0.1277560	0.2492043	0.9499208	0.9210659	2.4834205	1.2526467	2.6597378	0.5652218

poles

Zeros of polynomial $V_N(s)$ derived from the Chebyshev I approximation for $\frac{1}{2}$, 1, 2, and 3 dB ripples

Chebyshev filter $H_N(s) = \frac{K_N}{V_N(s)}$, where $K_N = \begin{cases} b_0 & \text{for } N \text{ odd} \\ \frac{b_0}{\sqrt{1+\epsilon^2}} & \text{for } N \text{ even} \end{cases}$

$N=1$	$N=2$	$N=3$	$N=4$	$N=5$	$N=6$	$N=7$	$N=8$	$N=9$	$N=10$
a. $\frac{1}{2}$ dB Ripple ($\epsilon = 0.3493114$, $\epsilon^2 = 0.1220184$)									
−2.8627752	−0.7128122 ±j1.0040425	−0.6264565	−0.1753531 ±j1.0162529	−0.3623196	−0.0776501 ±j1.0084608	−0.2561700	−0.0436201 ±j1.0050021	−0.1984053	−0.0278994 ±j1.0032732
		−0.3132282 ±j1.0219275	−0.4233398 ±j0.4209457	−0.1119629 ±j1.0115574	−0.2121440 ±j0.7382446	−0.0570032 ±j1.0064085	−0.1242195 ±j0.8519996	−0.0344527 ±j1.0040040	−0.0809672 ±j0.9050658
				−0.2931227 ±j0.6251768	−0.2897940 ±j0.2702162	−0.1597194 ±j0.8070770	−0.1859076 ±j0.5692879	−0.0992026 ±j0.8829063	−0.1261094 ±j0.7182643
						−0.2308012 ±j0.4478939	−0.2192929 ±j0.1999073	−0.1519873 ±j0.6553170	−0.1589072 ±j0.4611541
								−0.1864400 ±j0.3486869	−0.1761499 ±j0.1589029
b. 1 dB Ripple ($\epsilon = 0.5088471$, $\epsilon^2 = 0.2589254$)									
−1.9652267	−0.5488672 ±j0.8951286	−0.4941706	−0.1395360 ±j0.98333792	−0.2894933	−0.0621810 ±j0.9934115	−0.2054141	−0.0350082 ±j0.9964513	−0.1593305	−0.0224144 ±j0.9977755
		−0.2470853 ±j0.9659987	−0.3368697 ±j0.4073290	−0.0894584 ±j0.9901071	−0.1698817 ±j0.7272275	−0.0457089 ±j0.9952839	−0.0996950 ±j0.8447506	−0.0276674 ±j0.9972297	−0.1013166 ±j0.7143234
				−0.2342050 ±j0.6119198	−0.2320627 ±j0.2661837	−0.1280736 ±j0.7981557	−0.1492041 ±j0.5644443	−0.0796652 ±j0.8769490	−0.0650493 ±j0.9001063
						−0.1850717 ±j0.4429430	−0.1759983 ±j0.1982065	−0.1220542 ±j0.6508954	−0.1276664 ±j0.4586271
								−0.1497217 ±j0.3463342	−0.1415193 ±j0.1580321

$N = 1$	$N = 2$	$N = 3$	$N = 4$	$N = 5$	$N = 6$	$N = 7$	$N = 8$	$N = 9$	$N = 10$
				c. 2 dB Ripple ($\epsilon = 0.7647831$, $\epsilon^2 = 0.5848932$)					
−1.3075603	−0.4019082 $\pm j$ 0.6893750	−0.3689108	−0.1048872 $\pm j$ 0.9579530	−0.2183083	−0.0469732 $\pm j$0.9817052	−0.1552958	−0.0264924 $\pm j$0.9897870	−0.1206298	−0.0169758 $\pm j$0.9934868
		−0.1844554 $\pm j$0.9230771	−0.2532202 $\pm j$0.3967971	−0.0674610 $\pm j$0.9734557	−0.1283332 $\pm j$0.7186581	−0.0345566 $\pm j$0.9866139	−0.0754439 $\pm j$0.8391009	−0.0209471 $\pm j$0.9919471	−0.0767332 $\pm j$0.7112580
				−0.1766151 $\pm j$0.6016287	−0.1753064 $\pm j$0.2630471	−0.0968253 $\pm j$0.7912029	−0.1129098 $\pm j$0.5606693	−0.0603149 $\pm j$0.8723036	−0.0492657 $\pm j$0.8962374
						−0.1399167 $\pm j$0.4390845	−0.1331862 $\pm j$0.1968809	−0.0924078 $\pm j$0.6474475	−0.0966894 $\pm j$0.4566558
								−0.1133549 $\pm j$0.3444996	−0.1071810 $\pm j$0.1573528
				d. 3 dB Ripple ($\epsilon = 0.9976283$, $\epsilon^2 = 0.9952623$)					
1.0023773	−0.3224498 $\pm j$0.7771576	−0.2986202	−0.0851704 $\pm j$0.9464844	−0.1775085	−0.0382295 $\pm j$0.9764060	−0.1264854	−0.0215782 $\pm j$0.9867664	−0.0982716	−0.0138320 $\pm j$0.9915418
		−0.1493101 $\pm j$0.9038144	−0.2056195 $\pm j$0.3920467	−0.0548531 $\pm j$0.9659238	−0.1044450 $\pm j$0.7147788	−0.0281456 $\pm j$0.9826957	−0.0614494 $\pm j$0.8365401	−0.0170647 $\pm j$0.9895516	−0.0401419 $\pm j$0.8944827
				−0.1436074 $\pm j$0.5969738	−0.1426745 $\pm j$0.2616272	−0.0788623 $\pm j$0.7880608	−0.0919655 $\pm j$0.5589582	−0.0491358 $\pm j$0.8701971	−0.0625225 $\pm j$0.7098655
						−0.1139594 $\pm j$0.4373407	−0.1084807 $\pm j$0.1962800	−0.0752804 $\pm j$0.6458839	−0.0787829 $\pm j$0.4557617
								−0.0923451 $\pm j$0.3436677	−0.0873316 $\pm j$0.1570448