Unit : 1 : Intro to Signals

In the below expression you can replace t within

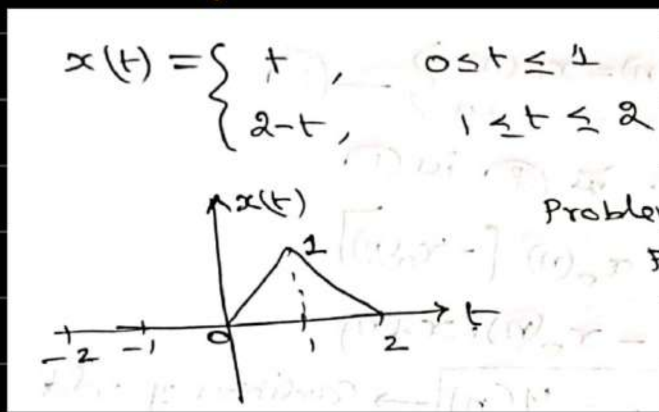
$x(t-2) \rightarrow$ Right Shift.
$x(t+2) \rightarrow$ Left Shift
$x(-t-2) \rightarrow$ Fold & left Shift
$x(-t+2) \rightarrow$ Fold & Right shift

* For doing $x(At \pm B)$
$\rightsquigarrow x(3t+2)$
$\Rightarrow$ Do 'A' part first i.e divide 't' axis by 3
$\Rightarrow$ Then right Shift or left shift B units, 2 in this case

$x(2t) \rightarrow \frac{t}{2}$ $\Rightarrow$ x-axis or time domain Compressed by 2

$x(t/2) \Rightarrow 2t$ $\Rightarrow$ Expanded by 2 times

$2\,x(t)$ $\Rightarrow$ Amplitude increased by 2



Some important formulas :-

1) $$\sum_{n=0}^{\infty} (a)^n = \begin{cases} \frac{1}{1-a}, & a<1 \\ \infty, & a \geq 1 \end{cases}$$

2) $$\sum_{n=t}^{\infty} (a)^n = \begin{cases} \frac{a^t}{1-a}, & a<1 \\ \infty, & a \geq 1 \end{cases}$$

3) $$\sum_{n=0}^{N-1} (a)^n = \begin{cases} \frac{1-(a)^{\overbrace{(N-1)+1}^{\text{(Upper limit+1)}}}}{1-a}, & a \neq 1 \\ \underbrace{(N-1)+1}_{\text{(Upper limit+1)}}, & a=1 \end{cases}$$

Remember in words

Remember in words

4) $$\sum_{n=t}^{N-1} (a)^n = \begin{cases} \dfrac{a^t - (a)^{\overbrace{(N-1)+1}^{(\text{Upper limit}+1)}}}{1-a}, & a \neq 1 \\ \underbrace{(N-1)+1}_{(\text{Upper limit}-1)}, & a = 1 \end{cases}$$

Remember in words

Remember in words

1. Unit Impulse Function

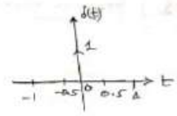
(i) CT- Unit Impulse Function $\delta(t)$:-

It is defined as

$$\delta(t) = \begin{cases} 1, & t = 0 \\ 0, & t \neq 0 \end{cases}$$

and the area is unity ie., $\int_{-\infty}^{\infty} \delta(t)\,dt = 1$

$\delta(t)$ is called as "Dirac-delta"



(ii) DT - Unit Impulse Function $\delta(n)$

It is defined as

$$\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

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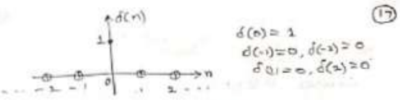


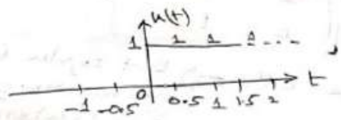
$\delta(0) = 1$
$\delta(-1) = 0,\ \delta(-2) = 0$
$\delta(1) = 0,\ \delta(2) = 0$

2. Unit Step Function

(i) CT - unit Step Function $u(t)$

It is defined as

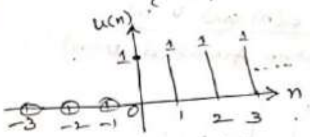
$$u(t) = \begin{cases} 1, & 0 \leq t \\ 0, & t < 0 \end{cases}$$



(ii) DT - unit Step Function $u(n)$

It is defined as

$$u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

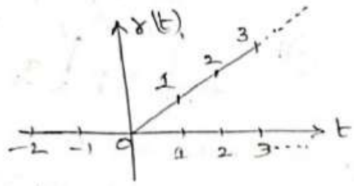


$u(0) = 1,\ u(1) = 1$
$u(2) = 1,\ u(3) = 1$
$u(-1) = 0,\ u(-2) = 0$

③ Ramp Function

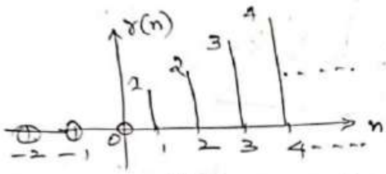
(i) CT - Ramp Function $r(t)$

It is defined as $r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$



(ii) DT - Ramp Function $r(n)$

It is defined as $r(n) = \begin{cases} n, & n \geq 0 \\ 0, & n < 0 \end{cases}$



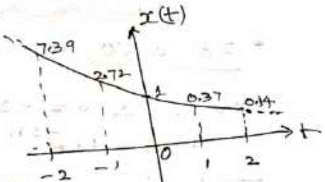
Types of Signals.

• If $a < 0$, ie, $a \to -ve \Rightarrow$ Decaying exponential

If $A = 1$, & $a = -1$

$x(t) = 1 \cdot e^{-1t}$

t	~~$x(t)$~~	$x(t)$
-2	~~0.14~~	7.39
-1	~~0.37~~	2.72
0	~~1~~	1
1	~~2.72~~	0.37
2	~~7.39~~	0.14



Decaying exponential

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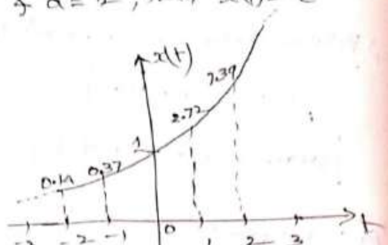
• If $a > 0$, $a \to +ve \Rightarrow$ Growing Exponential

If $A = 1$, & $a = 1$, then $x(t) = e^t$

t	$x(t)$
-2	0.14
-1	0.37
0	1
1	2.72
2	7.39



Growing Exponential

Relation between Signals:

i) $\delta(t) = \frac{d\, u(t)}{d}$ or $u(t) = \int \delta(t)\, dt$

ii) $u(n) = \sum_{R=0}^{\infty} \delta(n-R)$

iii) $\frac{d\, r(t)}{dt} = u(t)$ or $r(t) = \int u(t)\, dt$

Periodicity:

→ A signal is periodic if $x(n) = x(N+n)$
$x(t) = x(T+t)$

N → Period

T → Time period

For CT Signal:

In Qns, → first find the time period T, by sub. generally ω part inside part of funct^n to $2\pi f$, get f, then $\frac{1}{f}$ i.e T

$\cos(\omega t)$

$\sin(\omega t)$

$e^{j\omega t}$

Coefficient of t

→ Check if $\underbrace{x(t+T) = x(t)}$

If both equal, the given function is periodic with period T

Note: If Qn is in $\sin^2(\sim)$ or $\cos^2(\sim)$ Convert it to Sin & Cos form first.

If 2 functions in Summation, then find the time period of each one seperately say T_1 & T_2;

If Periodic $\frac{T_1}{T_2} \Rightarrow$ Rational

Eg: If $T_1 = 6$
$T_2 = 8$; $\frac{T_1}{T_2} = \frac{\cancel{6}\,3}{\cancel{8}\,4}$

Fundamental period $= T_1 \times 4$ Or $T_2 \times 3$

$\Rightarrow 6 \times 4$ 8×3

$\underline{\underline{24}}$ $\underline{\underline{24}}$

For DT Signals

Same procedure as previously till substituting $2\pi f$ finding f; timeperiod $\frac{1}{f}$ etc.

Eg: If $f = \frac{3}{4} = \frac{k}{N}$

Period $\underline{\underline{= N}} = 4$

But both Nr & Dr of f should be rational, if $f = \frac{3}{4\pi}$; Signal is Non periodic

And method of finding fundamental period incase of 2 periods is same as before (")

Even & Odd Signals:

$x(t) = x(-t) \Rightarrow$ Even

$x(t) = -x(-t) \Rightarrow$ Odd

The amplitude of origin of odd signal $= 0$

A signal can be decomposed into odd & even signals

$$x_e(t) = \frac{x(t) + x(-t)}{2} \qquad x_o(t) = \frac{x(t) - x(-t)}{2}$$

Same applies even for $x(n)$ DT signal

Energy & Power Signal

Juoles (J) Watts (W)

CT Sig →

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$; If $0 < E < \infty$ the $x(t)$ is Energy signal

Fundamental Period

$$P = \frac{1}{T} \int_0^{T} |x(t)|^2 \, dt$$; If $0 < P < \infty$, then $x(t)$ is power signal

DT Sig →

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$; If $0 < E < \infty$ the $x(n)$ is Energy signal

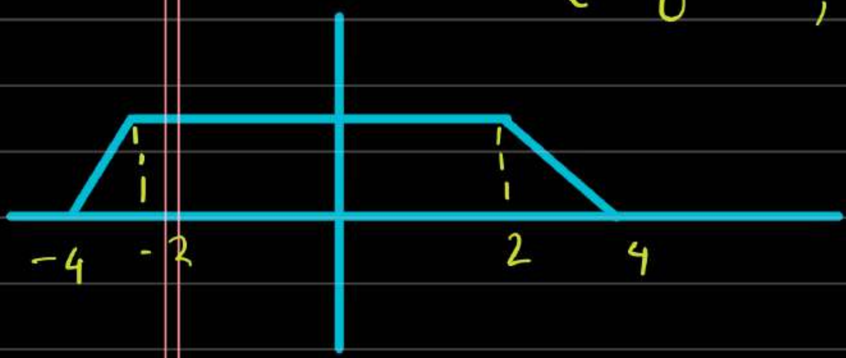
$$P = \frac{1}{N} \sum_{n=0}^{N-1} |x(n)|^2 \, dt \quad ; \quad \text{If } 0 < P < \infty \text{, then } x(n) \text{ is power signal}$$

N → Fundamental Period

For $u(n)$, $r(n)$, $\delta(n)$ etc where period can't be determined, so write period $N = \infty$ & $T = \infty$ and solve for power signal & tell, not power signal

If a signal ;

$$x(t) = \begin{cases} 4+t & ; \quad -4 \le t \le -2 \\ 2 & ; \quad -2 \le t \le 2 \\ 4-t & ; \quad 2 \le t \le 4 \\ 0 & ; \quad \text{otherwise} \end{cases}$$



Now $3 \frac{d\,x(t)}{dt} = ?$

use normal diff. & diff. the signal in given time range

$$\frac{d(x(t))}{dt} = \begin{cases} 1 & -4 \le t \le -2 \\ 0 & -2 \le t \le 2 \\ -1 & 2 \le t \le 4 \\ 0 & \text{otherwise} \end{cases}$$

Now for $3 \frac{d(x(t))}{dt}$;

$$3 \frac{d(x(t))}{dt} \begin{cases} 3 & -4 \le t \le 2 \\ 0 & -2 \le t \le 2 \\ -3 & 2 \le t \le 4 \\ 0 & \rightarrow \text{otherwise} \end{cases}$$

Properties of a System:

① ⇝ Static (memoryless) & Dynamic (Memory) system:

Present only
$$\begin{cases} y(t) = 2x(t) \\ y(n) = 5x(n) \\ y(n) = n\,x(n) \\ y(n) = x(n) + u(n-1) \end{cases}$$

$y(t) = x(t^2)$

$y(t) = x(t-2) \rightarrow$ Past

$y(n) = \underbrace{x(n+1)}_{\text{future}} + \underbrace{x(n)}_{\text{present}} + \underbrace{x(n-2)}_{\text{past}}$

$y(n) = x(2n)$

In Problems
↪ * Check the i/p & o/p @ $n = -1, 1, 0$

② ⇝ Causality: It is Causal if o/p depends on present & past value & Not dependent on future. It is non Causal if o/p depends on the future i/p

Causal	Non Causal
$y(t) = 5x(t)$, $y(n) = e^{x(n)}$	$y(t) = x(t+1)$
$y(t) = x(t) + x(t-2)$	$y(t) = x(t) + x(t+2)$
	$y(n) = \underbrace{2x(n)}_{\text{Present}} + \underbrace{x(n-1)}_{\text{Past}} + \underbrace{x(n+2)}_{\text{Future}}$

③ ⇝ Linear: A system is linear if it satisfies Super position principle.

④ ⇝ Time invariance: I/P & output doesnt Change with time then time invarient System

Method to Check:

⇝ Shift i/p by '$-n_0$' units obtain final o/p → $y_1(t)$

⇝ Shift o/p by '$-n_0$' units Obtain final o/p → $y_2(t-t_0)$

If both above expressions equal $y_1(t) = y_2(t-t_0)$ then System is Time invarient

(5) ⇝ Stability : If bounded i/p produces bounded o/p system is Stable.

$y(n) = n\,x(n) \rightarrow$ unstable system, if $n \rightarrow \infty$

(6) ⇝ Invertiblity : A System is invertible if i/p can be recovered from the o/p.

* Linear Time Invarient System :

Note :

If $x(t) = \delta(t) \Rightarrow$ o/p is Impulse Response

$x(t) = u(t) \Rightarrow$ " " step "

$x(t) = r(t) \Rightarrow$ " " ramp "

Convolution → used to find o/p of :

* $$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k) \longrightarrow \text{DT LTI signal}$$

$$y(t) = \int_{\tau=-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau \longrightarrow \text{CT LTI Signal}$$

Property of LTI ⇒

4) Memory : $y(n) = h(n) * x(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$

$$\underset{\text{Present output}}{y(n)} = \underbrace{\sum_{k=-\infty}^{-1} h(k)x(n-k)}_{\text{Left side portion}} + \underbrace{\sum_{k=0} h(k)x(n)}_{\text{origin}} + \underbrace{\sum_{k=1}^{\infty} h(k)x(k-n)}_{\text{Right side portion}}$$

Left side portion ← Future Inputs

origin ← Present Input

Right side portion ← Past Inputs

∴ LTI system is memory system

★ Condition for memory less system :-

Remove all the future & past inputs

$$\left.\begin{array}{l} h(1); h(2); \dots h(\infty) \Rightarrow 0 \longrightarrow h(k)=0,\ k>0 \\ h(-1); h(-2); \dots h(-\infty) \Rightarrow 0 \longrightarrow h(k)=0,\ k<0 \end{array}\right\} h(k)=0,\ k\neq 0$$

Unit : 2
Z - Transform

Partial fraction Formulas ⟹

S.No.	Form of the rational function	Form of the partial fraction
1.	$\frac{px+q}{(x-a)(x-b)}, a \neq b$	$\frac{A}{x-a}+\frac{B}{x-b}$
2.	$\frac{px+q}{(x-a)^2}$	$\frac{A}{x-a}+\frac{B}{(x-a)^2}$
3.	$\frac{px^2+qx+r}{(x-a)(x-b)(x-c)}$	$\frac{A}{x-a}+\frac{B}{x-b}+\frac{C}{x-c}$
4.	$\frac{px^2+qx+r}{(x-a)^2(x-b)}$ go till 2nd power	$\frac{A}{(x-a)^1}+\frac{B}{(x-a)^2}+\frac{C}{x-b}$
5.	$\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$	$\frac{A}{x-a}+\frac{Bx+C}{x^2+bx+c}$
	• where x^2+bx+c cannot be factorised further	

(11) Find IZT of $X(z) = \frac{z^2+z}{(z-\frac{1}{4})(z-\frac{1}{2})^3}$ ROC: $|z| > \frac{1}{2}$ (32)

$$= \frac{X(z)}{z} = \frac{z+1}{(z-\frac{1}{4})(z-\frac{1}{2})^3} = \frac{A}{z-\frac{1}{4}} + \frac{B}{(z-\frac{1}{2})^1} + \frac{C}{(z-\frac{1}{2})^2} + \frac{D}{(z-\frac{1}{2})^3}$$

go till 3rd power

① $$Z[x(n)] = X(Z) = \sum_{n=-\infty}^{\infty} x(n)\, Z^{-n}$$

$$Z = re^{j\omega}$$

⇝ ROC outside circle for Causal seq.

⇝ " inside " " Non " ".

⇝ ROC b/w 2 circles for 2 sided seq.

When we Equate D^r of final answer to 0 to find the points of ROC, Dr should of form $1 \pm (m)$

$$X(z) = \frac{1}{1-az^{-1}}$$

$$= \frac{z}{z-a} \text{ ; then for ROC take ( )}$$

$\Rightarrow az^{-1} < 1$

$\Rightarrow a < z \Rightarrow \underbrace{z > a}_{\text{Outside circle}}$

⇝ For Stability purpose ROC does not contain any poles

Properties {Only most used ones}

→ Time shift : $x(n+n_0) \longleftrightarrow Z^{+n_0} X(Z)$ → z transform without n_0 term ; i.e Z transform of n

→ $Z(n^m u_n) = \left(-Z\frac{d}{dz}\right)^m U(z)$ → z transform without n term ; i.e Z transform of u(n)

m^{th} derivative

SL NO	Signal $x(n)$	ZT	ROC
1	$n \cdot u(n)$	$\frac{z}{(z-1)^2}$	$\lvert z\rvert > 1$
2	$n \cdot a^n u(n)$	$\frac{az}{(z-a)^2}$	$\lvert z\rvert > \lvert a\rvert$
3	$n^2 u(n)$	$\frac{z(z+1)}{(z-1)^3}$	$\lvert z\rvert > 1$
4	$n^2 a^n u(n)$	$\frac{az(z+a)}{(z-a)^3}$	$\lvert z\rvert > \lvert a\rvert$
5	$\delta(n)$	1	Entire z-p

} First 2 imp

⇝ In Inverse ZT if they ask you to find Causality of a System by giving points.

→ Apply partial fraction to $\frac{x(z)}{z}$.

→ In $\frac{x(z)}{z}$ multiply the Dr z back to RHS

→ Draw circles & Check for causality

→ If that particular term or circle is Causal multiply with $u(n)$ & non Causal multiply with $-u(-n-1)$

⇒ For Easy method of partial fraction refer Pg: 26, Q(3) of unit: 2 notes

Inverse Z transform { For $\frac{\text{funct}^n \text{ of } z}{\text{funct}^n \text{ of } z}$ }

(11) Find IZT of $X(z) = \dfrac{z^2+z}{\left(z-\frac{1}{4}\right)\left(z-\frac{1}{2}\right)^3}$ (32)

ROC: $|z| > \frac{1}{2}$

$$= \frac{X(z)}{z} = \frac{z+1}{\left(z-\frac{1}{4}\right)\left(z-\frac{1}{2}\right)^3} = \frac{A}{z-\frac{1}{4}} + \frac{B}{z-\frac{1}{2}} + \frac{C}{\left(z-\frac{1}{2}\right)^2} + \frac{D}{\left(z-\frac{1}{2}\right)^3}$$

Method of finding A, B, C, D

$$A = \frac{(z+1)\cancel{\left(z-\frac{1}{4}\right)}}{\cancel{\left(z-\frac{1}{4}\right)}\left(z-\frac{1}{2}\right)^3}\Bigg|_{z=\frac{1}{4}} = -80$$

$$D = \frac{(z+1)\cancel{\left(z-\frac{1}{2}\right)^3}}{\left(z-\frac{1}{4}\right)\cancel{\left(z-\frac{1}{2}\right)^3}}\Bigg|_{z=\frac{1}{2}} = 6$$

$$C = \frac{1}{1!}\frac{d}{dz}\{\text{Equation of } D\} = \frac{d}{dz}\left\{\frac{(z+1)}{\left(z-\frac{1}{4}\right)}\right\}\Bigg|_{z=\frac{1}{2}}$$

$$C = \frac{\left(z-\frac{1}{4}\right)\cdot 1 - (z+1)\cdot 1}{\left(z-\frac{1}{4}\right)^2}\Bigg|_{z=\frac{1}{2}} = -20$$

$$B = \frac{1}{2!}\frac{d}{dz}\{\text{Equation of } C\}\Bigg|_{z=\frac{1}{2}}$$

$$= \frac{1}{2!}\frac{d}{dz}\left\{\frac{\left(z-\frac{1}{4}\right)-(z+1)}{\left(z-\frac{1}{4}\right)^2}\right\}\Bigg|_{z=\frac{1}{2}}$$

$$= \frac{1}{2\times1} = \frac{d}{dz}\left[\left(-\frac{5}{4}\right)\left(z-\frac{1}{4}\right)^{-2}\right]\Bigg|_{z=\frac{1}{2}}$$

$$B = \frac{1}{2}\left[\left(-\frac{5}{4}\right)(-2)\left(z-\frac{1}{4}\right)^{-3}\right]\Bigg|_{z=\frac{1}{2}} = 80$$

$$n\cdot a^n u(n) \longleftrightarrow a\frac{z}{(z-a)^2}$$

$$[n(n-1)a^n u(n)] \longleftrightarrow a^2\frac{2z}{(z-a)^3}$$

$$X(z) = -80\left[\frac{z}{z-\frac{1}{4}}\right] + 80\left(\frac{z}{z-\frac{1}{2}}\right) - 20\left(\frac{z}{\left(z-\frac{1}{2}\right)^2}\right) + 6\left(\frac{z}{\left(z-\frac{1}{2}\right)^3}\right), \quad \text{ROC } |z| > \tfrac{1}{2} \text{ causal}$$

$$= -80\left[\frac{z}{z-\frac{1}{4}}\right] + 80\left[\frac{z}{z-\frac{1}{2}}\right] - 20\left[\frac{\left(\frac{1}{2}\right)z}{\left(\frac{1}{2}\right)\left(z-\frac{1}{2}\right)^2}\right] + 6\left[\frac{\left(\frac{1}{2}\right)^2\times 2z}{\left(\frac{1}{2}\right)^2\times2\times\left(z-\frac{1}{2}\right)^3}\right]$$

$$X(z) = -80\left[\frac{z}{z-\frac{1}{4}}\right] + 80\left(\frac{z}{z-\frac{1}{2}}\right) - 40\left[\frac{\frac{1}{2}z}{\left(z-\frac{1}{2}\right)^2}\right] + 12\left[\frac{\left(\frac{1}{2}\right)^2 2z}{\left(z-\frac{1}{2}\right)^3}\right]$$

Take IZT

$$x(n) = \left[-80\left(\tfrac{1}{4}\right)^n + 80\left(\tfrac{1}{2}\right)^n - 40\cdot n\left(\tfrac{1}{2}\right)^n + 12\cdot n(n-1)\left(\tfrac{1}{2}\right)^n\right]u(n)$$

⇝ Apply partial fraction to $\frac{X(z)}{z}$

⇝ Take IZT of ans. above

{V.V. Imp}

Eg: Ans of P. Fracⁿ ⇒ $7 + \frac{1}{2}\left[\frac{z}{z-(-1)}\right] + \frac{1}{2}\left(\frac{z}{z-1}\right) + \frac{z}{z-2}$

Req. IZT ⇒ $7\delta(n) + \frac{1}{2}(-1)^n u(n) + \frac{1}{2}1^n u(n) + 2^n u(n)$

Inverse Z.T of $z^k \rightarrow \delta(n+k)$

Note: If given functⁿ has N^r & D^r power, then partial fractⁿ can't be applied for IZT.

∴ Do long division;

Quotient + $\frac{\text{Remainder}}{\text{Divisor}}$

Solution To diffrence Eqⁿ

⇝ $Z(y(n-1)) = z^{-1}(y(z))$ → No initial Cond.

⇝ $Z(y(n-1)) = z^{-1}\left[y(z) + y(-1)z^{1}\right]$ → with inital Cond

⇝ $Z(y(n-2)) = z^{-2}(y(z))$ → without Inital Cond

⇝ $Z(y(n-2)) = z^{-2}\left(y(z) + y(-1)z^{1} + y(-2)z^{2}\right)$

V.V.V.V Imp terms

① System Function $H(z) = \frac{Y(z)}{X(z)}$

↳ Also the Z transform of impulse response

(2) The impulse/unit sample response $h(n)$ = IZT of $H(z)$

Partial fractⁿ & Take inverse

(3) Step response $S(n)$ = ZT of the given Qn & then take its IZT

⇝ For Converting $H(z)$ to diffrence Eqⁿ,

→ write $H(z)$ as $\frac{Y(z)}{X(z)}$

Take highest power of z out from Nr & Dr

&

And then Cross multiply

i.e $Y(z)(\ldots) = X(z)(\ldots)$

Then take IZT

⇝ Output of the system for a particular input → $x(n)$ is

Convolution of $x(n)$ & $h(n)$

$x(n)$ ↓ given input

$h(n)$ ↓ impulse resp. ↓ IZT of $H(z)$ ↓ $\frac{Y(z)}{X(z)}$

For Convolution use the basic Conv. formula

Compulsorily refer notes for UZT they are different type

⇝ Unilateral Z transform (UZT):

$$X(z) = \sum_{n=0}^{\infty} x(n) \cdot z^{-n}$$

→ Normal ZT but limits from $0 \to \infty$

DTFT

I/P ← Non periodic DT signal
DTFT
O/P ↳ Continuous Periodic Signal

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\Omega n}$$

DT Signal

Periodic CT

IDTFT : $x(n) = \frac{1}{2\pi}\int_0^{2\pi} x(e^{j\Omega}) \times e^{j\Omega n}\, d\Omega$

DT Signal

For frequency response ; Convert the answer of DTFT i.e. $X(e^{j\omega})$ to Sin & cos terms, for magnitude spectrum c/o $\sqrt{a^2+b^2}$ {If $a+ib$} & phase by $\tan^{-1}\left(\frac{b}{a}\right)$

Both are Sin & Cos terms

V.V. Imp

While finding IDTFT;

$$X(e^{j\Omega}) \begin{cases} 0 , & |\Omega| \le \Omega_c \\ 1 & \Omega_c \le |\Omega| \le \pi \end{cases}$$

Notice limits

$$x(n) = \frac{1}{2\pi}\left[\int_{-\pi}^{-\Omega_c} 1 \times e^{nj\Omega}\, d\Omega + \int_{\Omega_c}^{\pi} 1 \times e^{j\Omega n}\, d\Omega\right]$$

If magnitude & phase (argument : arg $X(e^{j\Omega})$) is given seperately then,

$$X(e^{j\omega}) = \text{Magnitude} \times e^{j\,\text{phase}}$$

Then apply IDTFT to the above Signal

Diffrence Equation:

Eg: $3y(n) - 4y(n-1) + y(n-2) = 3x(n)$

$3Y(e^{j\Omega}) - 4e^{-j\Omega}Y(e^{j\Omega}) + e^{-2j\Omega}Y(e^{j\Omega}) = 3X(e^{j\Omega})$

Note:

Freq. Resp $= \dfrac{Y(e^{jw})}{X(e^{jw})}$

Inverse of Freq. Response

Impulse Response $= \left.\dfrac{H(e^{jw})}{e^{jw}}\right\}$ → Write in this format, apply partial fraction & take IDTFT

$\sin(x) = \dfrac{e^{jx} - e^{-jx}}{2j}$

$\cos(x) = \dfrac{e^{jx} + e^{-jx}}{2j}$ → Imp Formulas

$e^{j\theta} = \cos\theta + j\sin\theta$

$e^{-j\theta} = \cos\theta - j\sin\theta$

$\boxed{1 - e^{-2jw} = 2j\sin(w)}$ → Used in one of the problem

Properties of DTFT;

i) Time shift = $x(n-n_0) \xleftrightarrow{DTFT} e^{j\Omega(-n_0)} \cdot X(e^{j\Omega})$

ii) Freq. " = $e^{j\Omega_0 n} \cdot x(n) \longleftrightarrow X(e^{j(\Omega-\Omega_0)})$

iii) $n\,x(n) \xleftrightarrow{DTFT} j\dfrac{d}{d\Omega}[X(e^{j\Omega})]$

Unit: 3 DFT

I/P ↙ DT signal with cont. freq. domain
DTFT
O/P ↳ DT signal with freq. sampling

$$X(K) = \sum_{n=0}^{N-1} x(n)\, W_N^{kn} \qquad W_N = e^{-j\frac{2\pi}{N}}$$

Both always periodic in DFT

$$\text{IDFT:} \quad x(n) = \frac{1}{N}\sum_{K=0}^{N-1} X(K) \times W_N^{-kn}$$

DT Signal ↵

While computing DFT i.e. $x(n) \to$ given, $X(k) = ?$

$$X(k) = \sum_{0}^{N-1} \left(W_N^{(k-1)}\right)^n$$

$$X(k) = \underbrace{N}_{\text{upperlimit } + 1}\, \delta(k-1)$$

V.V.V.V Imp **Properties of DFT**

Circular Time Shift

$$x((n-n_0))_N \xleftrightarrow{DFT} W_N^{n_0 k}\, X(k)$$

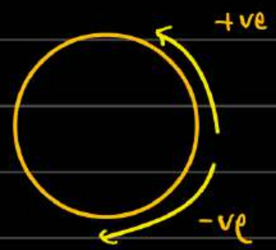
↳ time → freq.

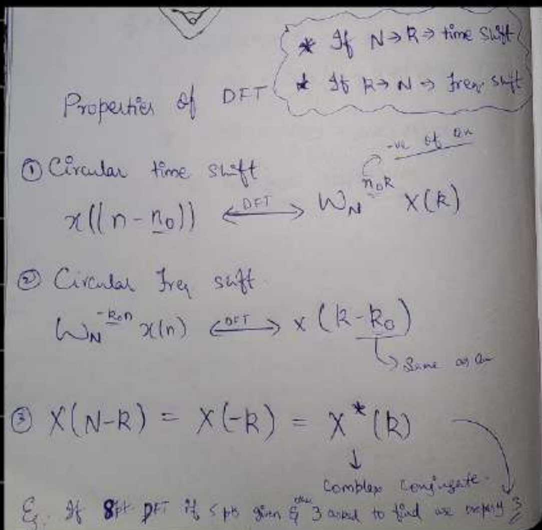
Circular Freq. Shift

$$W_N^{-k_0 n}\, x(n) \xleftrightarrow{DFT} X(k-k_0)$$

* **Convolution:**

1) **Circular Convolution:** $x(n) = \sum_{m=0}^{N-1} x(m) \times h(n-m)$





Properties of DFT
* If $N \to k \Rightarrow$ time shift
* If $k \to N \Rightarrow$ freq. shift

① Circular time shift

$$x((n-n_0)) \xleftrightarrow{DFT} W_N^{n_0 k}\, X(k)$$

-ve of ex

② Circular freq shift

$$W_N^{-k_0 n}\, x(n) \xleftrightarrow{DFT} X(k-k_0)$$

↳ Same as ex

③ $X(N-k) = X(-k) = X^*(k)$

↓ complex conjugate

Eg. If 8pt DFT it's 5 pts given & the 3 asked to find use property ③

(ii) **Matrix method:**

$x_1(n) = [1, 2, 0, 1]$

$x_2(n) = [2, 2, 1, 1]$

$$X_1 \cdot X_2 = \begin{bmatrix} 1 & 1 & 0 & 2 \\ 2 & 1 & 1 & 0 \\ 0 & 2 & 1 & 1 \\ 1 & 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \\ 6 \\ 5 \end{bmatrix}$$

Methods under Circular Convolution

Be thorough with all 3 parts

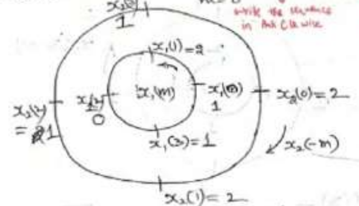
1. Find circular convolution of two sequences $x_1(n) = [\underset{\uparrow}{1}, 2, 0, 1]$ and $x_2(n) = [\underset{\uparrow}{2}, 2, 1, 1]$ using (i) circle method (ii) Matrix method (iii) DFT and IDFT → Study properly till end

= (i) Circle Method :–

$$x(n) = x_1(n) \, (N) \, x_2(n) = \sum_{m=0}^{N-1} x_1(m) x_2(n-m)$$

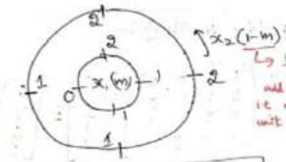
For n=0 $\quad x(0) = \sum_{m=0}^{3} x_1(m) \cdot x_2(0-m)$

write the sequence in Anti Clk wise $\quad$ $-m$ → write seq. in Clk wise



$$\boxed{x(0) = 2+2+0+2 = 6}$$

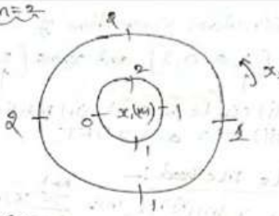
For n=1 :– $\quad x(1) = \sum_{m=0}^{3} x_1(m) x_2(1-m)$



$1-m$ → add 1 to $-m$ i.e. move $-m$ by 1 unit in −ve dir → as the 1

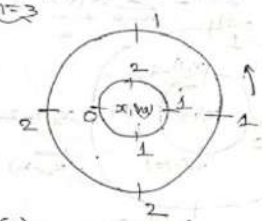
$$\boxed{x(1) = 2+4+0+1 = 7}$$

For n=2



$$x(2) = 1+4+0+1 = 6$$

For n=3



$$x(3) = 1+2+0+2 = 5$$

$$x(n) = [6, 7, 6, 5]$$

(ii) Matrix method :–

$$x(n) = \begin{bmatrix} 1 & 1 & 0 & 2 \\ 2 & 1 & 1 & 0 \\ 0 & 2 & 1 & 1 \\ 1 & 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \\ 6 \\ 5 \end{bmatrix}$$

$$x(n) = [6, 7, 6, 5]$$

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(23)

(iii) DFT and IDFT method → If the x_n is func^n form only then use this. Else, if the x_n in seq. form use method on Pg. 34 Q(4)

$$x(n) = [x_1(n) \, (N) \, x_2(n)] \longleftrightarrow X(k) = X_1(k) \cdot X_2(k)$$

$$X_1(k) = \sum_{n=0}^{N-1} x_1(n) W_N^{kn} = x_1(0) + x_1(1) W_N^{k} + x_1(2) W_N^{k2} + x_1(3) W_N^{k3}$$

$$\boxed{X_1(k) = 1 + 2W_N^{k} + 0 + 1 W_N^{k3}}$$

$$X_2(k) = \sum_{n=0}^{N-1} x_2(n) W_N^{kn} = x_2(0) + x_2(1) W_N^{k} + x_2(2) W_N^{k2} + x_2(3) W_N^{k3}$$

$$\boxed{X_2(k) = 2 + 2W_N^{k} + W_N^{k2} + W_N^{k3}}$$

$$X(k) = X_1(k) \cdot X_2(k) = (1 + 2W_N^{k} + W_N^{k3})(2 + 2W_N^{k} + W_N^{k2} + W_N^{k3})$$

$$= 2 + 2W_N^{k} + W_N^{k2} + W_N^{k3} + 4W_N^{k} + 4W_N^{2k} + 2W_N^{k3} + 2W_N^{4k} + 2W_N^{k3} + 2W_N^{k4} + W_N^{k5} + W_N^{k6}$$

$$= 2 + 6W_N^{k} + 5W_N^{k2} + 5W_N^{k3} + 4W_N^{k4} + W_N^{k5} + W_N^{k6}$$

$$= 2 + 6W_N^{k} + 5W_N^{k2} + 5W_N^{k3} + 4 + W_N^{k1} + W_N^{k2}$$

$$X(k) = 6 + 7W_N^{k} + 6W_N^{k2} + 5W_N^{k3}$$

Take IDFT

$$x(n) = 6\delta(n) + 7\delta(n-1) + 6\delta(n-2) + 5\delta(n-3)$$

$$x(n) = [6, 7, 6, 5]$$

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(9) Compute circular convolution using DFT & IDFT for

$$x_1(n) = [\underset{\uparrow}{2}, 3, 1, 1], \quad x_2(n) = [\underset{\uparrow}{1}, 3, 5, 3]$$

Verify the result

= $\quad x(n) = x_1(n) \, (N) \, x_2(n) \longleftrightarrow X(k) = X_1(k) \cdot X_2(k)$

$$X_1(k) = [7, (1-2j), 1, (1+2j)]$$

$$X_2(k) = [12, -4, 6, -4]$$

$$X(k) = X_1(k) \cdot X_2(k) = [84, (-4+j8), 6, (-4-j8)]$$

Refer 4f Doc

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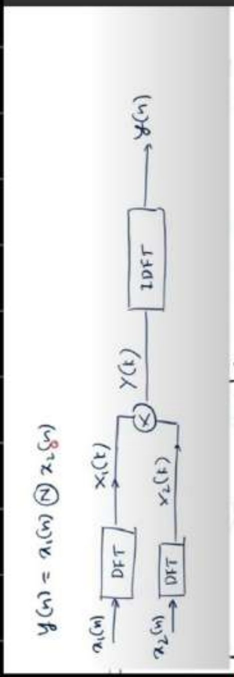


(34)

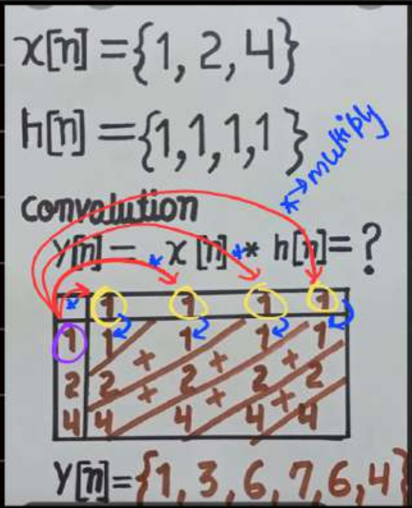
Take IDFT of $X(k)$

$$x(n) = [\cancel{20.5}\ 19, \cancel{15.5}\ 17, \cancel{24.5}\ 23, \cancel{23.5}\ 25]$$

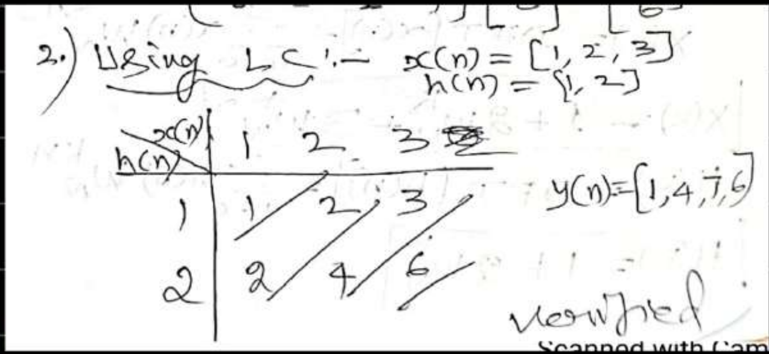
Verification :–

$$x(n) = \begin{bmatrix} 2 & 1 & 1 & 3 \\ 3 & 2 & 1 & 1 \\ 1 & 3 & 2 & 1 \\ 1 & 1 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 5 \\ 3 \end{bmatrix} = \begin{bmatrix} 19 \\ 17 \\ 23 \\ 25 \end{bmatrix}$$

(iii) Linear Convolution.



New length = $L_1 + L_2 - 1$



(iv) Direct multiplication method :-

P.S You already got $X_1(k)$ & $X_2(k)$ from $x_1(n)$ & $x_2(n)$

Eg : $X_1(k) = 1 + 2W_N^{k} + W_N^{3k}$
$X_2(k) = 2 + 2W_N^{k} + W_N^{2k} + W_N^{3k}$

$X_1 \times X_2 = \left(1 + 2W_N^{k} + W_N^{3k}\right)\left(2 + 2W_N^{k} + W_N^{2k} + W_N^{3k}\right)$

→ Even if given in Seq. Form only X(k) can be directly multiplied by other X(k) & obtain Conv. directly. If Seq. of x(n) seq given use matrix method or Convert to X(k)

$x_1(k) = (1, 2, 3, 4)$
$x_2(k) = (2, 3, 4, 5)$
↓ Conv.
$(2, 6, 12, 20)$
Only if initial sequence in terms of k, i.e x(k)

FFT

Radix 2 → No. of Samples = 2^n
⇒ 2, 4, 8, 16, 32, ...

Radix 3 → No. of Samples = 3^n
⇒ 3, 9, 27, ...

Complex multiplications & Complex additions:

① Normal FFT : For N point FFT

Complex Mul = $\underline{\log_2 N} \times \underline{N}$

No. of stage ↩ ↳ CM per stage

Complex addition : Same formula as above

② Butterfly FFT :

If 5 pt FFT & only 5 pts given, assume 3 other pts as 0 to make it standard 8 pt FFT

Complex multiplication = $\log_2 N \times \frac{N}{2}$

" addition = $\log_2 N \times N$

Gen. Exp. of FFT

$$X(k) = \underbrace{G(k)}_{\text{Even}} + \overset{\text{multiplier}}{W_N^k} \underbrace{H(k)}_{\text{odd}}$$

↳ k ⇒ 0-7
↳ 8 pt FFT

For N point DFT :

Complex multiplication = N^2

Complex addition = $(N-1)N$

Note: * In FFT & IFFT, Both I/P & O/P are Bit reversed

→ In FFT, $x(n)$ is always given & $x(k)$ is found
" IFFT, $x(k)$ " " " " $x(n)$ " "

In FFT

Left side	Right side
always $x(n)$ ↳ time domain	always $x(k)$ ↳ freq. domain
If DIT; I/P decimated & O/P retained	If DIF; O/P decimated & I/P retained

Unit : 4 : IIR Filters

→ Analog Butterworth Filter:

① System Funct^n / Gain : $|H(j\Omega)|^2 = \dfrac{1}{1+\left(\dfrac{\Omega}{\Omega_c}\right)^{2n}}$; n = order of Filter

$\Omega_c \to$ Cutoff freq.

⇝ Gain is 3dB / 0.707 at cutoff freq. Ω_c

⇝ Max gain of a filter is 1 at $\Omega = 0$
↳ 0dB

② Order of Filter ⇒ $n = \dfrac{\log\left(\frac{1}{d}\right)}{\log\left(\frac{1}{k}\right)}$

If $\Omega \to$ Hz ;
do $2\pi \underset{\hookrightarrow Hz}{\Omega} \to \underset{\hookrightarrow rad/s}{Ans}$

$d = \sqrt{\dfrac{(1-\delta_p)^{-2}-1}{\delta_s^{-2}-1}}$; $1-\delta_p \to$ Gain in normal scale

; $\delta_s \to$ Stop band attenu^n " "

$k = \dfrac{\Omega_p}{\Omega_s}$ rad/s

Normal scale = $10^{\frac{\text{dB scale}}{20}}$

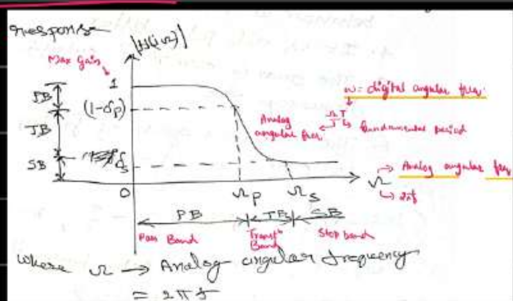
③ Cutoff frequency = $\Omega_{c1} \Rightarrow \dfrac{\Omega_p}{\left((1-\delta_p)^{-2}-1\right)^{1/2n}}$

$\Omega_{c2} = \dfrac{\Omega_c}{\left(\delta_s^{-2}-1\right)^{1/2n}}$

$\Omega_c = \dfrac{\Omega_{c1}+\Omega_{c2}}{2}$

$\Omega_c = \Omega_p$
Iff $\underset{\hookrightarrow gain}{1-\delta_p} = 3dB$ or $\overset{1/\sqrt{2}}{0.707}$

4) Normalised Filter ⇒ $H_n(s) \Rightarrow \dfrac{1}{(\sim)}$ → Table



⑤ Desired filter: $H_n(s)\Big|_{s \to \frac{s}{\Omega_c}}$

→ Analog Chebyshev Filter:

① System Funct^n / Gain: $|H(j\Omega)|^2 = \dfrac{1}{1+\varepsilon^2[T_n(\Omega)]^2}$; ε = Ripple related factor

$T_n(\Omega)$ = Cheby poly.

⇝ Gain is 3dB | 0.707 at cutoff freq. Ω_c

⇝ Max gain of a filter is 1 at $\Omega = 0$
↳ 0dB

② Order of Filter ⇒ $n = \dfrac{\cosh^{-1}\left(\frac{1}{d}\right)}{\cosh^{-1}\left(\frac{1}{k}\right)}$

If $\Omega \to$ Hz ;
do $2\pi \underbrace{\Omega}_{\text{Hz}} \to \underbrace{\text{Ans}}_{\text{rad/s}}$

$d = \sqrt{\dfrac{(1-\delta_p)^{-2}-1}{\delta_s^{-2}-1}}$; $1-\delta_p$ → Gain in normal scale

; δ_s → Stop band attenu^n " "

$k = \dfrac{\Omega_p}{\Omega_s}$ rad/s

Normal scale = $10^{\frac{\text{dB scale}}{20}}$

③ Cutoff frequency = $\Omega_c = \Omega_p$
↳ Always

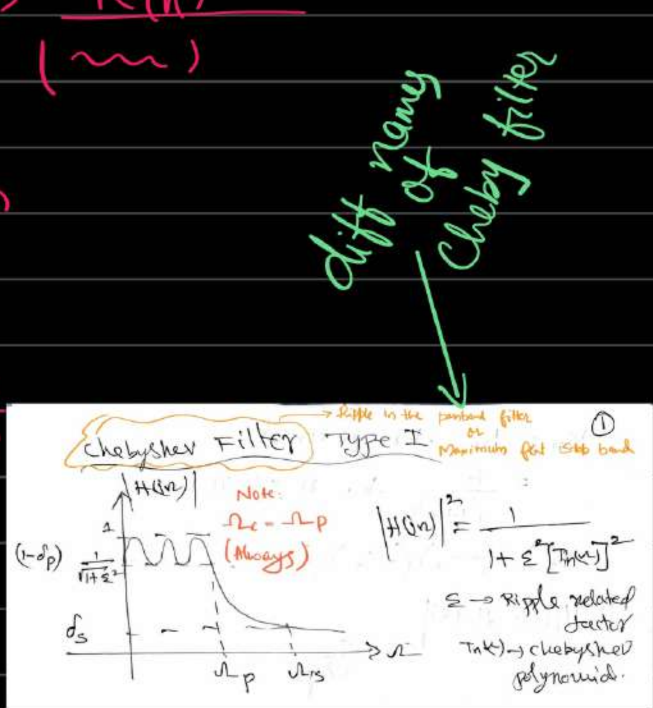
4) Normalised Filter ⇒ $H_n(s) \Rightarrow \dfrac{K(n)}{(\sim\sim)}$ → Table

$K_n = b_0$ for n = odd

$= \dfrac{b_0}{\sqrt{1+\varepsilon^2}}$ for n = even

diff names of Cheby filter

⑤ Desired filter: $H_n(s)\Big|_{s \to \frac{s}{\Omega_p}}$
↳ $H_d(s)$



Analog → Digital filter {IIT}

$$\frac{1}{S - S_k} = \frac{1}{1 - e^{S_k T} z^{-1}}$$

$\frac{1}{S-S_k} \downarrow H_d(s)$ $\qquad$ $\frac{1}{1 - e^{S_k T} z^{-1}} \downarrow H(z)$

Analog → Digital filter {BLT}

While designing analog filter, replace all the Ω, i.e Ω_p & Ω_s by Ω^*_p & Ω^*_s & use Ω^*_p & Ω^*_s ONLY for for all values of Ω;

Step 1

So,

$$\Omega^* = \frac{2}{T} \tan\left(\frac{\Omega T}{2}\right)$$

Ω ⇒ given Ω in rad/sec

Replace T by the time period given in Quest. if freq. given then $T = \frac{1}{f}$

** If T not given in Quest, take T = 1 sec.

Step 2

After finding $H_d(s)$, replace all the values of s by;

$$\frac{2}{T}\left[\frac{1 - z^{-1}}{1 + z^{-1}}\right]$$

If T value not given assume it to be 1 sec

→ Sub. T value from Quest.

$$H(z) = \frac{1 + 2z^{-1} + z^{-2}}{3.41 + 0.59 z^{-2}}$$

↳ Divide Nr. & Dr by 3.41 for realization of filter structure

$$H(z) = \frac{0.29 + 0.58 z^{-1} + 0.29 z^{-2}}{1 + 0.17 z^{-2}}$$

Final ans should be in the above format where Dr → 1 + ...

For verification, in the final Step i.e $H(z) = H_d(s)$,

This is exactly how you find DC gain in digital filters;

The very final digital one

*** Substitution is of z not z^{-1}

In that Substitute z with e^{jw}

So, it becomes $H(e^{jw})$ & then

Equate $\omega = 0$; If the ans Comes to be ≈ 1,

The ans is correct as the max gain Came to be $\underline{\underline{1}}$

Ans got after $w=0$ is DC gain in digital filters

⇝ If they ask for pass band verification then Substitute Ω_p & see if you get corresponding gain i.e $1-\delta p$

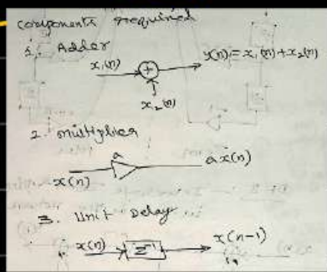
Not Ω^*_p

Not $z^{-1}, z^{-2}, \ldots$

First Sub z with e^{jw}, Convert to Sin & Cos form & take mag. of Nr. & Dr $|a+ib| = \sqrt{a^2+b^2}$

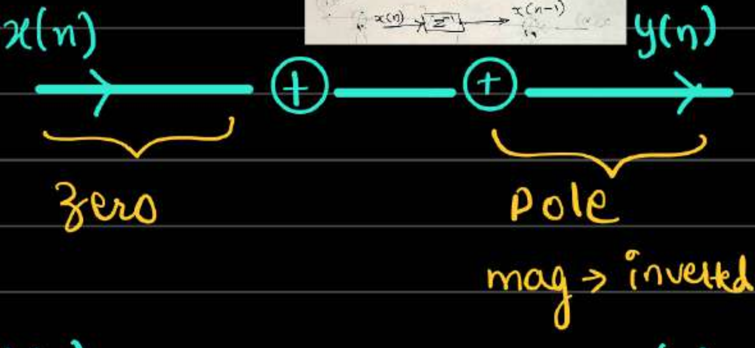
III^{ly} when you Sub. Ω_s value in $H(e^{j\Omega})$ you should get δ_s

If you get both the values i.e $1-\delta p$ & δs for corresponding values of freq. Ans is correct

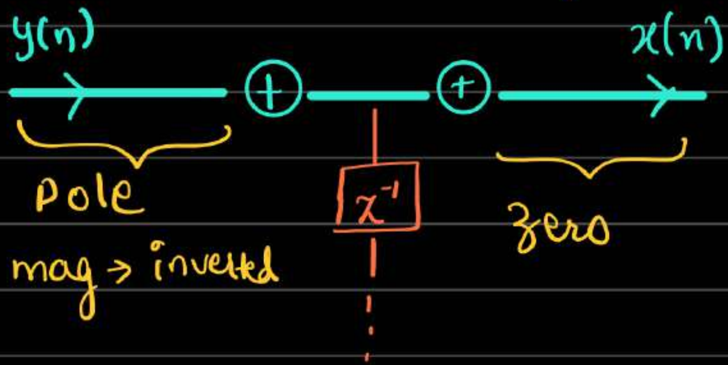


Cascading of IIR filters:

⇝ Direct form 1 :



⇝ Direct form 2 :



→ Cascade form ; Convert $\underbrace{H(z) \rightarrow H_1(z) \times H_2(z)}$

By finding roots of Nr & Dr

$$\frac{(x-a)(x-b)}{(y-a)(y-b)}$$

but variable in Nr & Dr $\Rightarrow z^{-1}$.

Draw DF 2 Structures of $H_1(z)$ & $H_2(z)$ & Connect using adder.

→ Parallel form : $\underbrace{H(z) = H_1(z) + H_2(z)}_{\text{Partial fractions}} + \cdots$

Draw the diagrams individually one below other & put them in parallel.

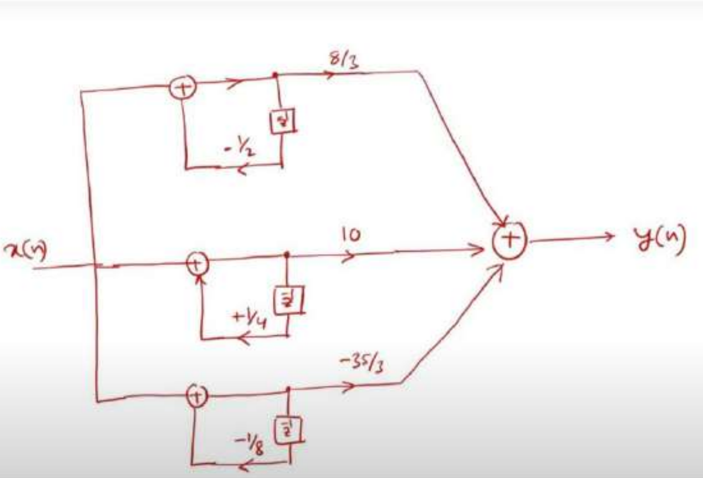
Obtain a parallel realization for the system described by

$$H(z) = \frac{(1+z^{-1})(1+2z^{-1})}{\left(1+\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)\left(1+\frac{1}{8}z^{-1}\right)} \quad \text{— (1)}$$

$\Rightarrow \quad H(z) = H_1(z) + H_2(z) + H_3(z) + \cdots + H_m(z)$

Eqn (1) can be simplified using partial fraction expansion

$$H(z) = \underbrace{\frac{8/3}{1+\frac{1}{2}z^{-1}}}_{H_1(z)} + \underbrace{\frac{10}{1-\frac{1}{4}z^{-1}}}_{H_2(z)} + \underbrace{\frac{-35/3}{1+\frac{1}{8}z^{-1}}}_{H_3(z)}$$



Unit : 5 : FIR Filters

Question Format :

$$H_d(e^{j\omega}) = \begin{cases} e^{-j2\omega} , & |\omega| \leq \frac{\pi}{4} \\ 0 , & \frac{\pi}{4} \leq |\omega| \leq \pi \end{cases}$$

$2 \rightarrow \alpha$

here cutoff freq. is $|\omega| \leq \frac{\pi}{4}$, as signal exists only for that range, &

$\because |\omega|$ it is from $-\frac{\pi}{4}$ to $\frac{\pi}{4}$

Cutoff freq. $\omega_c \Rightarrow$ $-\frac{\pi}{4} \rightarrow a$, $\frac{\pi}{4} \rightarrow b$

from Qn, 2 in this case

① $\alpha = \frac{N-1}{2}$; $N \Rightarrow$ found

α also called as delay

N → Order of filter

② $$H_d(n) = \frac{1}{2\pi} \int_a^b H_d(e^{j\omega}) \cdot e^{j\omega n} \, d\omega$$

Shortcut; Ans $\Rightarrow$ $$\pm \frac{\sin[\omega_c(n-\alpha)]}{\pi(n-\alpha)} \Rightarrow ①$$

-ve, if Ω_c given between 2 values or given between 2 range (Case 1)

Case : 2 { + If Ω_c given just $\geq$ or $\leq$ w.r.t one value

*** But ω_c should be in terms of π for this to work

③ Refer the below table & obtain corresponding value of $w(n)$ for the given winow in Qn

window	Equation, $w(n)$, $0 \leq n \leq N-1$
Rectangular $w_R(n)$	$w_R(n) = 1$
Bartlet	$w_{Bar}(n) = 1 - \frac{2\left\lvert n - \left(\frac{N-1}{2}\right)\right\rvert}{N-1}$
Hanning	$w_{Han}(n) = 0.5 - 0.5\cos\left(\frac{2\pi n}{N-1}\right)$
Hamming	$w_{Ham}(n) = 0.54 - 0.46\cos\left(\frac{2\pi n}{N-1}\right)$
Blackman	$w_{Blk}(n) = 0.42 - 0.5\cos\left(\frac{2\pi n}{N-1}\right) + 0.08\cos\left(\frac{4\pi n}{N-1}\right)$

In Case 1 ONLY

Use the below Eq① to find the invalid h(n)

$$\frac{\sin(\pi(n-\alpha)) - \sin(\omega_c(n-\alpha))}{\pi(n-\alpha)}$$

*** But ω_c should be in terms of π for this to work

Now inorder to find the invalid h(n), eliminate / hide all the problem making terms including sin

$W(n) = \sim\sim \Rightarrow ②$

Refer Above Pg

(4) Final FIR filter expression => (1) × (2)

i.e -ve, if Ω_c given between 2 values or given between 2 range

$$h(n) \Rightarrow \frac{\mp \sin[\omega_c(n-\alpha)]}{\pi(n-\alpha)} \times W(n)$$

+ If Ω_c given just $\geq$ or $\leq$ w.r.t one value

(5) Substitute Corresponding values of n & obtain the Seq.

If $N \Rightarrow 4$

$n = 0 \rightarrow N-1 \Rightarrow 0, 1, 2, 3$

$h(n) = [\; \ldots, \;\ldots, \;\ldots, \;\ldots \;]$

$h(0) \quad h(1) \quad h(2) \;\ldots$

Obtained

~> Freq. Response;

$$H(e^{j\omega}) \Rightarrow \sum_{0}^{N-1} h(n)e^{-j\omega n}$$

use Seq. obtained above, $h(0) + h(1) + \ldots$

Convert exp terms to trigo.; take Common out

$(\sim + \sim + \sim + \sim)\, e^{-j\omega\alpha}$

Const + Cos term + Cos term + ...

Take the Quest. Common Out

Const + $(\sim)e^{-j\omega\alpha} + (\sim)e^{-j\omega\alpha} + \ldots$

Convert from

~> Mag. Response : Ans = $\underbrace{(\sim\sim\sim)}$ $\cancel{\boxed{e^{-jA\Omega}}}$

Only the bracket of freq. response

~> Phase Response : Ans = $-j\underline{\underline{A}}$

$$(\sim\sim\sim)\, e^{-jA\Omega}$$

$$H_d(e^{j\omega}) = j\omega$$
$$-\pi \le \omega \le \pi$$

Note :

~> If they give $\underline{\text{cutoff freq.}}$, Sampling freq. etc.

Used in Analog filters

Calculate $\underbrace{\text{Time period } T \text{ from Sampling freq.}}$ i.e $\frac{1}{f_s}$

This is used in digital filters

Ω analog = Ω digital

$$\Omega_c T = \omega_c$$

$$2\pi f_c T = \omega_c$$

Cutoff freq. $\underbrace{\downarrow}$; $T \to \frac{1}{f_s}$, Calculated above

Substitute this in Qn now!

~> If funct. itself is a constant, like 1, 2, etc.

New funct is $\underline{1} \times e^{-j\omega\alpha}$
↳ given constant.

Find α by $N-1/2$; $N \to$ given in Qn

Symmetric & Antisymmetric FIR filter

$\rightarrow h(n) = h(M-1-n) \quad \ldots \quad n = 0, 1 \ldots M-1$

M = number of samples

h(n) is symmetric then filter is symmetric

$h(n) = -h(M-1-n) \quad n = 0, 1 \ldots M-1$

Filter is antisymmetric

DC gain in dB $\Rightarrow 20 \log \sum_{n=0}^{N-1} w(n)$

↳ Eq of Window

derived by taking FT of W(n) & equating ω in $e^{-j\omega n}$ as zero.

$$\boxed{\text{Note: } FT(w_n) = \sum_{n=0}^{N-1} w_n e^{-j\omega n}}$$