Analog Filters :- The analog input signal with all frequencies is connected to the filter and an analog output is generated with restricted frequency band based on filter design such as LPF, HPF, BPF, BSF.

Analog Filter

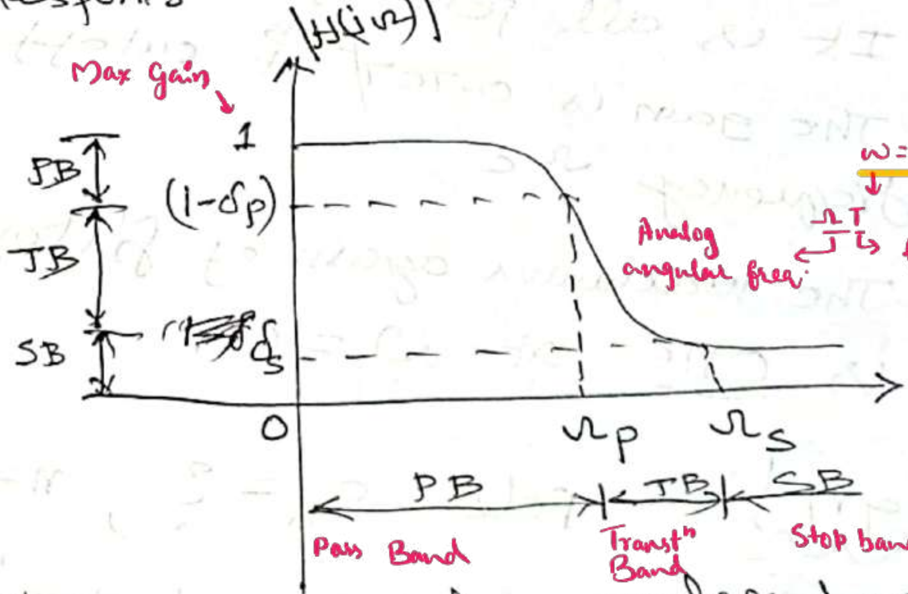
$x(t) \rightarrow$ [$h(t)$, $H(e^{j\Omega})$] $\rightarrow y(t)$

$X(e^{j\Omega})$ $\qquad$ $Y(e^{j\Omega})$

There are basically 2 types of analog filters such as (i) Butterworth filter and (ii) chebyshev filters

Design of Analog $\boxed{\text{Butterworth Filter}}$ → Monotonic / flat PB/TB/SB filter

Consider low pass filter (LPF) frequency response



ω = digital angular freq.

ΩT → fundamental period

→ Analog angular freq

Ω → $2\pi f$

Where $\Omega \rightarrow$ Analog angular frequency

$= 2\pi f$

The analog Butterworth filter represented as magnitude squared frequency response as given in equation 1.

$$|H(j\Omega)|^2 = \frac{1}{1+\left(\frac{\Omega}{\Omega_c}\right)^{2n}} \quad \text{--- (1)}$$

System function / gain

If 2.1 or 2.2... take 3
(Always take higher value)

Where $n \rightarrow$ the order of filter

$\Omega_c \rightarrow$ cutoff frequency
$\hookrightarrow$ 3dB frequency, rad/sec

Properties:—

1. It is a maximum ~~maximally~~ flat frequency response filter
2. The magnitude $|H(j\Omega)|$ decreases monotonically as Ω increases
3. Frequency response is monotonic behaviour in PB, TB & SB
4. It is all pole filter
5. The gain is 0.707/3dB at cutoff frequency Ω_c
6. The maximum again of filter is one at $\Omega = 0$, & in decibels it is 0dB

Max gain

\#\#*** Design:— poles $S_k = ?$, $n = ?$

$\Omega_c = ?$, Normalized system function $H_n(S) = ?$

Desired system function $H_d(S) = ?$

$$S_k = \underbrace{e^{j\left(\frac{\pi}{2} + \frac{(2k+1)\pi}{2n}\right)}}_{\text{Not Req. to rem.}} \; ; \; k = 0 \rightarrow n-1$$

Mag. e^{phase} ; $\therefore$ Magnitude = 1 poles S_k :— $s = j\Omega \Rightarrow \Omega = \frac{s}{j}$ —②

Substitute ② in ①

$$|H(s)|^2 = \frac{1}{1 + \left(\frac{s}{j\Omega_c}\right)^{2n}}$$

$\boxed{\text{For Normalized filter } \Omega_c = 1 \text{ rad/sec}}$

$$\therefore \underset{\text{LHP}}{\underset{\uparrow}{H(s)}} \cdot \underset{\text{RHP}}{\underset{\uparrow}{H(-s)}} = \frac{1}{1 + \left(\frac{s}{j}\right)^{2n}}$$

To find poles :— Substitute $Dr = 0$

$$1 + \left(\frac{s}{j}\right)^{2n} = 0$$

$$\left[\left(\frac{s}{j}\right)^2\right]^n = -1$$

$$\left[\frac{s^2}{-1}\right]^n = -1$$

$$(-s^2)^n = -1$$

$$-s^2 = (-1)^{1/n}$$

$$s^2 = (-1)(-1)^{1/n} = (-1)\left[-1 \times 1\right]^{1/n}$$

$$s^2 = \left(e^{j\pi}\right)\left[e^{j\pi} \cdot e^{j \cdot 2\pi k}\right]^{1/n}$$

$$s = e^{j\frac{\pi}{2}} \cdot e^{j\frac{\pi}{2n}} \cdot e^{j \cdot \frac{2\pi k}{2n}} = e^{j\left[\frac{\pi}{2} + \frac{(2k+1)\pi}{2n}\right]}$$

$$\boxed{S_k = e^{j\left[\frac{\pi}{2} + \frac{(2k+1)\pi}{2n}\right]}} \Rightarrow \text{Normalized poles}$$

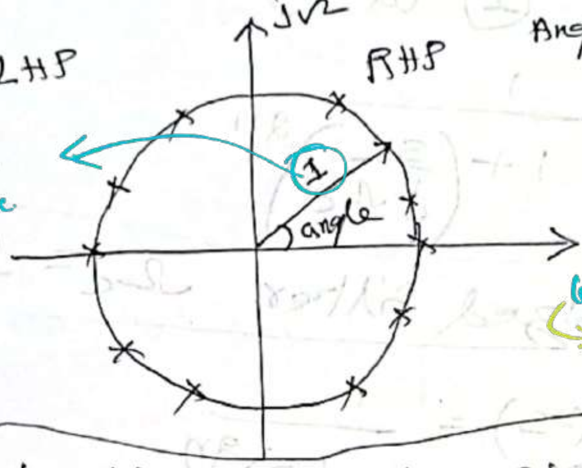
$k = 0$ to $(n-1)$

Magnitude of poles $|s_k| = 1$

Angle of poles $\angle s_k = \left[\frac{\pi}{2} + \frac{(2k+1)\pi}{2n}\right]$

S-plane (Analog filter)

$s = \sigma + j\Omega$



Phase = Angle $= \frac{\pi}{2} + \frac{(2k+1)\pi}{2n}$ → Calculated from formula below Qn

If $n = 5$

left half plane ← LHP → stable

If RHP is there system becomes unstable.
Right half plane

$\boxed{\text{poles lie on unit circle}}$

2. order of filter (n):—

consider LPF Frequency Response.

& $$|H(j\Omega)|^2 = \frac{1}{1+\left(\frac{\Omega}{\Omega_c}\right)^{2n}} \quad \text{—(1)}$$

From frequency Response

At $\Omega = \Omega_p$:- $|H(j\Omega)| = (1-\delta_p)$ —(2)

Substitute (2) in (1)

$$(1-\delta_p)^2 = \frac{1}{1+\left(\frac{\Omega_p}{\Omega_c}\right)^{2n}}$$

$$1+\left(\frac{\Omega_p}{\Omega_c}\right)^{2n} = (1-\delta_p)^{-2}$$

$$\left(\frac{\Omega_p}{\Omega_c}\right)^{2n} = \left[(1-\delta_p)^{-2} - 1\right] \quad \text{—(3)}$$

At $\Omega = \Omega_s$:- $|H(j\Omega)| = \delta_s$ —(4)

Substitute (4) in (1)

$$\delta_s^2 = \frac{1}{1+\left(\frac{\Omega_s}{\Omega_c}\right)^{2n}}$$

$$1+\left(\frac{\Omega_s}{\Omega_c}\right)^{2n} = \delta_s^{-2}$$

$$\left(\frac{\Omega_s}{\Omega_c}\right)^{2n} = \left[\delta_s^{-2} - 1\right] \quad \text{—— (5)}$$

DO → (5)/(3)

$$\left(\frac{\Omega_s}{\Omega_p}\right)^{n} = \left[\frac{(\delta_s^{-2} - 1)}{[(1-\delta_s)^{-2} - 1]}\right]^{1/2}$$

Apply Log on both sides

$$n \cdot \log\left(\frac{\Omega_s}{\Omega_p}\right) = \log\left(\sqrt{\frac{\delta_s^{-2} - 1}{(1-\delta_p)^{-2} - 1}}\right)$$

$$n = \frac{\log\sqrt{\frac{\delta_s^{-2} - 1}{(1-\delta_p)^{-2} - 1}}}{\log\left(\frac{\Omega_s}{\Omega_p}\right)} \quad ****** $$

$$\boxed{n = \frac{\log\left(\frac{1}{d}\right)}{\log\left(\frac{1}{k}\right)}}$$

order of filter

V. Imp

Where d = discremination factor $= \sqrt{\frac{(1-\delta_p)^{-2} - 1}{\delta_s^{-2} - 1}}$

K = Selectivity factor $= \frac{\Omega_p}{\Omega_s}$

3. Cutoff frequency (Ω_c)

Ω_c from Ω_p :-

From eqn 3

$$\left(\frac{\Omega_p}{\Omega_c}\right)^{2N} = (1-\delta_p)^{-2} - 1$$

$$\frac{\Omega_p}{\Omega_c} = \left[(1-\delta_p)^{-2} - 1\right]^{1/2N}$$

$$\therefore \boxed{\Omega_{c_1} = \frac{\Omega_p}{\left[(1-\delta_p)^{-2} - 1\right]^{1/2N}}}$$

Ω_c from Ω_s :-

From eqn 5

$$\left(\frac{\Omega_s}{\Omega_c}\right)^{2N} = \delta_s^{-2} - 1$$

$$\left(\frac{\Omega_s}{\Omega_c}\right) = \left[\delta_s^{-2} - 1\right]^{1/2N}$$

$$\boxed{\Omega_{c_2} = \frac{\Omega_s}{\left[\delta_s^{-2} - 1\right]^{1/2N}}}$$

$$\therefore \boxed{\Omega_c = \frac{\Omega_{c_1} + \Omega_{c_2}}{2}}$$

4. Normalized Filter :-

$$H_n(s) = \frac{1}{(s-s_0)(s-s_1)(s-s_2)\cdots}$$

5. Desired Filter with $\Omega_c \neq 1$

$$H_d(s) = H_n(s)\Big|_{s \to \frac{s}{\Omega_c}}$$

1. Given $|H_a(j\Omega)|^2 = \frac{1}{1+64\Omega^6}$, Determine the analog filter system function $H_a(s)$

= W.K.T $|H(j\Omega)|^2 = \frac{1}{1+\left(\frac{\Omega}{\Omega_c}\right)^{2n}}$ —— (1)

$|H_a(j\Omega)|^2 = \frac{1}{1+\left[\frac{\Omega}{(1/2)}\right]^{2\times3}}$ —— (2)

compare (1) & (2) $\Longrightarrow$ $n=3$, $\Omega_c = \frac{1}{2}$ rad/sec.

$n=3 \Longrightarrow$ No. of poles 3 $\longrightarrow k=0,1,2$

$s_k = e^{j\left[\frac{\pi}{2}+\frac{(2k+1)\pi}{2n}\right]}$

$k=0$, $s_0 = e^{j\left[\frac{\pi}{2}+\frac{\pi}{2\times3}\right]} = e^{j\frac{2\pi}{3}} = \cos\left(\frac{2\pi}{3}\right)+j\sin\left(\frac{2\pi}{3}\right)$

$= (-0.5+j0.87)$

$k=1$, $s_1 = e^{j\left[\frac{\pi}{2}+\left(\frac{3\pi}{6}\right)\right]} = e^{j\left[\frac{6\pi}{6}\right]} = -1$

$k=2$, $s_2 = e^{j\left[\frac{\pi}{2}+\frac{5\pi}{6}\right]} = e^{j\left(\frac{8\pi}{6}\right)} = -0.5-j0.87$

$\therefore H_n(s) = \frac{1}{(s-s_0)(s-s_1)(s-s_2)}$

$= \frac{1}{(s+0.5-j0.87)(s+1)(s+0.5+j0.87)}$

$$\boxed{H_n(s) = \frac{1}{s^3+2s^2+2s+1}}$$ ← Normalized Filter with $\Omega_c = 1$ rad/sec

For $\Omega_c = 0.5$ rad/sec

$H_a(s) = H_n(s)|_{s\to\frac{s}{0.5}} = \frac{0.125}{s^3+s^2+0.5s+0.125}$

2) Design an analog BWF that has a gain of 0.8 at 0.2π rad/sec and attenuation of 0.2 at 0.6π rad/sec.

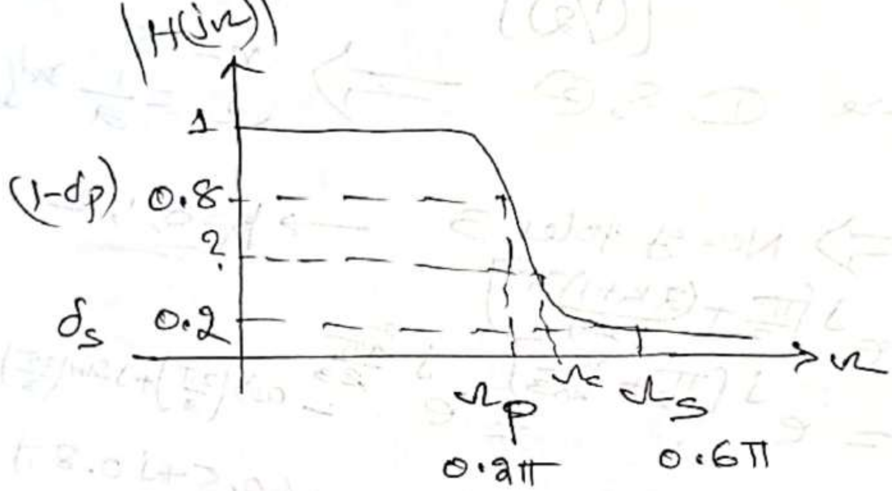
=

$$(1-\delta_p) = 0.8, \quad \delta_s = 0.2$$

$$\Omega_p = 0.2\pi \text{ rad/sec}, \quad \Omega_s = 0.6\pi \text{ r/s}$$

$n = ?, \quad \Omega_c = ?, \quad S_k = ?, \quad H_n(S) = ?$

$H_a(s) = ?$



$$n = \frac{\log\left(\frac{1}{d}\right)}{\log\left(\frac{1}{k}\right)} = 3.37 \approx 4$$

$$d = \sqrt{\frac{(1-\delta_p)^{-2}-1}{\delta_s^{-2}-1}}, \quad k = \frac{\Omega_p}{\Omega_s}$$

<u>To find Ω_c :-</u>

<u>Ω_c from Ω_p :-</u> $\Omega_{c_1} = \dfrac{\Omega_p}{\left[(1-\delta_p)^{-2}-1\right]^{1/2n}}$

$= 21.39$ r/s

<u>Ω_c from Ω_s :-</u> $\Omega_{c_2} = \dfrac{\Omega_s}{\left[\delta_s^{-2}-1\right]^{1/2n}}$

$= 22.795$ r/s

$$\Omega_c = \frac{\Omega_{c_1} + \Omega_{c_2}}{2} \approx 22 \text{ rad/sec.}$$

To find $H_n(s)$:- Normalized Filter

$$H_n(s) = \frac{1}{(s-s_0)(s-s_1)(s-s_2)(s-s_3)}$$

{Tables}

$$H_n(s) = \frac{1}{s^4 + 2.613 s^3 + 3.41 s^2 + 2.613 s + 1}$$

For $\Omega_c = 22$ rad/sec

$$H_a(s) = H_n(s)\Big|_{s \to \frac{s}{22}}$$

$$\therefore H_a(s) = \frac{(22)^4}{s^4 + 2.613 \times 22\, s^3 + 3.41 \times (22)^2 s^2 + 2.613 \times (22)^3 s + (22)^4}$$

$$n = \frac{\log\left(\frac{1}{d}\right)}{\log\left(\frac{1}{k}\right)} = 1.71 \simeq 2$$

cutoff frequency Ω_c

$$\Omega_{c_1} = \frac{\Omega_p}{\left[(1-\delta_p)^{-2} - 1\right]^{1/2n}} = 0.726 \text{ rad/sec}$$

$$\Omega_{c_2} = \frac{\Omega_s}{\left[\delta_s^{-2} - 1\right]^{1/2n}} = 0.85 \text{ rad/sec}$$

$$\Omega_c = \frac{\Omega_{c_1} + \Omega_{c_2}}{2} = 0.78 \text{ rad/sec.}$$

Poles :- $S_k = e^{j\left[\frac{\pi}{2} + \frac{(2k+1)\pi}{2n}\right]}$

$s_0 = -0.707 + j0.707$

$s_1 = -0.707 - j0.707$

$$H_n(s) = \frac{1}{(s-s_0)(s-s_1)} = \frac{1}{s^2+\sqrt{2}s+1}$$

For $\Omega_c = 1$ rad/sec

For $\Omega_c = 0.78$ rad/sec

$$H_a(s) = H_n(s)\Big|_{s \to \frac{s}{0.78}}$$

$$H_a(s) = \frac{0.62}{s^2 + 1.12s + 0.62}$$

(3) Design an analog BWF that has a gain of 2dB at 20 rad/sec and attenuation of 10dB at 30 rad/sec

= $n = ?$, $\Omega_c = ?$, $S_k = ?$, $H_n(s) = ?$, $H_a(s) = ?$

Pass band gain = 2dB at 20 r/s

Convert into normal scale $(1-\delta_p)_{dB} = -2\,dB$

$\because$ Max gain of a BWF is 1 or 0 in dB scale, here gain can't be 2, $\therefore$ it should be $\underline{-2}$.

$20\log(1-\delta_p) = -2$

$\log(1-\delta_p) = -\frac{2}{20}$

$\boxed{(1-\delta_p) = 10^{-2/20}} = 0.79$

** Remember. V. Imp Point

stop band gain = 10dB at 30 r/s

δ_s dB = −10 dB

$20 \log \delta_s = -10$

$\log \delta_s = \frac{-10}{20}$

$\boxed{\delta_s = 10^{-10/20}} = 0.31$

$\Omega_p = 20$ rad/sec. $\Omega_s = 30$ rad/sec.

$n = \frac{\log\left(\frac{1}{d}\right)}{\log\left(\frac{1}{k}\right)} = 3.37 \simeq \boxed{4}$ Higher

$d = \sqrt{\frac{(1-\delta_p)^{-2}-1}{\delta_s^{-2}-1}} =$

$k = \frac{\Omega p}{\Omega s} =$

cutoff frequency (Ωc):-

$\Omega_{c_1} = \frac{\Omega p}{\left[(1-\delta_p)^{-2}-1\right]^{1/2n}} = 21.39$ r/s

$\Omega_{c_2} = \frac{\Omega s}{\left[\delta_s^{-2}-1\right]^{1/2n}} = 22.795$ r/s

$\Omega_c = \frac{\Omega_{c_1}+\Omega_{c_2}}{2} = 22$ r/s

$Hn(s) = \frac{1}{s^4+2.6s^3+3.4s^2+2.6s+1}$

For $\Omega_c = 22$ r/s

$Ha(s) = Hn(s)\Big|_{s \to \frac{s}{22}}$

$$H_a(s) = \frac{(22)^4}{s^4 + 2.6\times 22 s^3 + 3.4\times 22^2 s^2 + 2.6\times 22^3 s + 22^4}$$

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(4) Design maximally flat filter for the specifications given below

$-1 \le |H(j\Omega)| \le 0$, for $0 \le \Omega \le 10$ r/s

$|H(j\Omega)|_{dB} \le 60$, for $\Omega \ge 50$ r/s

= Given, $(1-\delta_p)_{dB} = -1\,dB$

$20\log(1-\delta_p) = -1$

$\log(1-\delta_p) = -\frac{1}{20}$

$(1-\delta_p) = 10^{-1/20}$ =

$\delta_{s\,dB} = -60\,dB$

$20\log\delta_s = -60$

$\log\delta_s = -\frac{60}{20}$

$\delta_s = 10^{-60/20}$

$\Omega_p = 10$ r/s, $\Omega_s = 50$ r/s

$$n = \frac{\log\left(\frac{1}{d}\right)}{\log\left(\frac{1}{k}\right)} = 4.71 \simeq 5$$

$\Omega_{c_1} = 11.45$ r/s

$\Omega_{c_2} = 12.56$ r/s

$\Omega_c = 12$ r/s

$H_n(s) = ?$ $\quad H_a(s) = ?$

5) Design an analog BWF with $-3\,dB$ at 500 HZ and an attenuation of 15 dB at 750 HZ

$(1-\delta_p)\,dB = -3\,dB$ $\qquad \delta_s\,dB = -15\,dB$

$\Rightarrow \boxed{(1-\delta_p) = 0.707}$ $\qquad \delta_s = ?$

$f_s = 750\,HZ$

Given $f_p = 500\,HZ$,

$\Omega_p = ? = 2\pi f_p = 2\pi 500\ r/s = 1000\pi\ r/s$

$\Omega_s = ? = 2\pi f_s = 2\pi \times 750\ r/s = 1500\pi\ r/s$

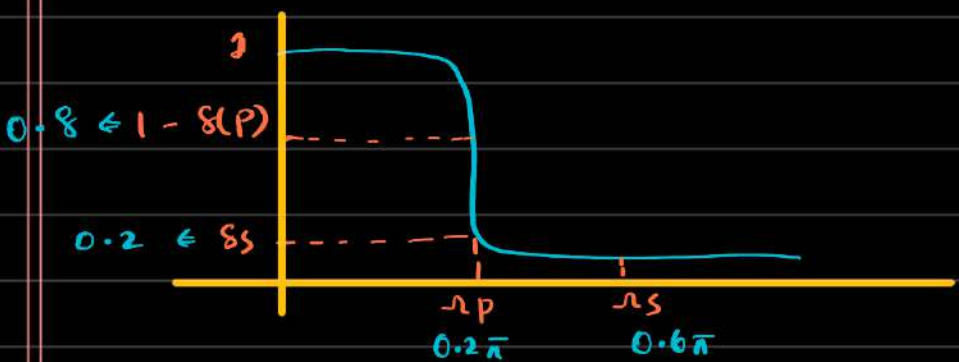
$$n = \frac{\log\left(\frac{1}{d}\right)}{\log\left(\frac{1}{k}\right)} = 4.225 \simeq 5$$

Design analog Butterworth filter for the foll spec.

① $0.8 \le |H(j\Omega)| \le 1 \quad 0 \le \Omega \le 0.2\pi$

③ $|H(j\Omega)| \le 0.2 \quad \Omega \geq 0.6$

draw the graph from these values.



$$d = \sqrt{\frac{(1-\delta p)^{-2}-1}{\delta s^{-2}-1}} \Rightarrow \sqrt{\frac{(0.8)^{-2}-1}{(0.2)^{-2}-1}}$$

$$\Rightarrow \underline{\underline{0.15}}.$$

$$k = \frac{\Omega p}{\Omega s} = \frac{0.2\pi}{0.6\pi} \Rightarrow \underline{\underline{\frac{1}{3}}}. \Rightarrow \underline{\underline{0.33}}.$$

$$n = \frac{\log\left(\frac{1}{d}\right)}{\log\left(\frac{1}{k}\right)} \Rightarrow \frac{\log(1/0.15)}{\log(3)} \Rightarrow \underline{\underline{1.7}}. \approx \underline{\underline{2}}.$$

Order always higest value.

Now we need Ω_c ;

$\because \Omega p \neq 0.7$, we need to calc Ωc

Seperately. If $\Omega p = 0.707$. ; then $\Omega p = \Omega c$

$$\Omega c = \frac{\Omega c_1 + \Omega c_2}{2}$$

$$\Omega_{c_1} = \frac{\Omega_p}{\left((1-\delta_p)^{-2}-1\right)^{1/2n}} \Rightarrow \frac{0.2\pi}{\left((0.8)^{-2}-1\right)^{\frac{1}{2(2)}}} \Rightarrow \frac{0.2\pi}{\left((0.8)^{-2}-1\right)^{1/4}}$$

$$\Rightarrow \underline{\underline{0.73}}\ r/s$$

$$\Omega_{c_2} = \frac{\Omega_s}{\left((\delta_s)^{-2}-1\right)^{\frac{1}{2n}}} \Rightarrow \underline{\underline{0.85\ r/s}}$$

$$\Omega_c = \frac{0.73+0.85}{2} \Rightarrow \underline{\underline{0.78}}\ r/s$$

From table:

$$H_n(s) = \frac{1}{B_n(s)} \Rightarrow \frac{1}{s^2+\sqrt{2}s+1} \quad \left\{ \text{for } n=2 \right\}$$

$$Hd(s) = H_n(s)\Big|_{s\to\frac{s}{\Omega_c}} \Rightarrow H_n(s)\Big|_{s\to\frac{s}{0.78}}$$

$$\Rightarrow \frac{1}{\left(\frac{s}{0.78}\right)^2 + \sqrt{2}\times\frac{s}{0.78} + 1}$$

$$\Rightarrow \frac{(0.78)^2}{s^2+\sqrt{2}\times 0.78s+(0.78)^2}$$

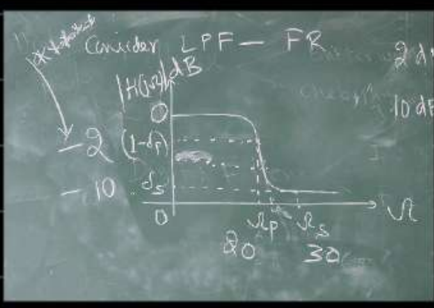
$$\Rightarrow \frac{0.616}{s^2+1.11s+0.616}$$

② Design analog BW filter that has a gain of 2dB @ 20 r/s & stop band attenuation of 10 dB at 30 rad/s.

$n = ?$ $\quad H_n(s) = ?$ $\quad H_d(s) = ?$

$\omega_p = 20$ r/s ;

$\omega_s = 30$ r/s.



$1-\delta p = ?$

$\because$ Gain in decibel : 2 dB

& $1-\delta p$ is below max Gain (0 dB), $\therefore$ Should be -ve.

hence it is $-2 \rightarrow 1-\delta p$

& $\delta s \rightarrow -10$

All our formulas in normal scale, whereas given values in dB, So, Convert dB to normal scale

(i) $(1-\delta p)\,dB \Rightarrow -2$

$20\log_{10}(1-\delta p) \Rightarrow -2$

$\log_{10}(1-\delta p) = \frac{-2}{20}$

$(1-\delta p) = 10^{-2/20} \Rightarrow \underline{\underline{0.79}}$.

(ii) δs dB $= -10$

$20\log(\delta s) = -10$

$\log(\delta s) = \frac{-10}{20}$

$\delta s = 10^{-10/20} \Rightarrow \underline{\underline{0.31}}$

Here $\because$ $1-\delta p = 0.7 \neq 0.707$

So, find Ωc_1, Ωc_2, Ωc.

$$n \Rightarrow \frac{\log\left(\frac{1}{d}\right)}{\log\left(\frac{1}{k}\right)}$$

$$d = \sqrt{\frac{(0.79)^{-2}-1}{(0.31)^{-2}-1}} \Rightarrow \underline{\underline{0.25}}.$$

$$k = \frac{\Omega p}{\Omega s} = \frac{20}{30} = 0.66$$

$$n = \frac{\log\left(\frac{1}{0.25}\right)}{\log\left(\frac{1}{2/3}\right)} \Rightarrow 3.41$$

$$\approx \underline{\underline{4}}$$

By previous method.

$\Omega_{c_1} = 21.308$ r/s
$\Omega_{c_2} = 22.669$ r/s

Ω_c = Avg. of Both = 22 rads/s.

So, $H_n(s) = \dfrac{1}{B_n(s)} = \dfrac{1}{(s^2+0.765s+1)(s^2+1.845s+1)}$

$$H_n(d) = \frac{1}{\left(\left(\frac{s}{22}\right)^2 + 0.76\left(\frac{s}{22}\right)+1\right)\left(\left(\frac{s}{22}\right)^2 + 1.84\left(\frac{s}{22}\right)+1\right)}$$

(3) Design analog BW filter with 3dB passband at 500Hz & stop band attenuation of 15dB with 750 Hz.

(ans) $\because$ $1-\delta_p$ = 3dB, now we can take $\Omega_p = \Omega_c$

Converting dB to normal values.

$(1-\delta_p) = -3$

$20\log_{10}(1-\delta_p) = -3$

$1-\delta_p = 10^{-3/20} \Rightarrow \underline{\underline{0.707}}$

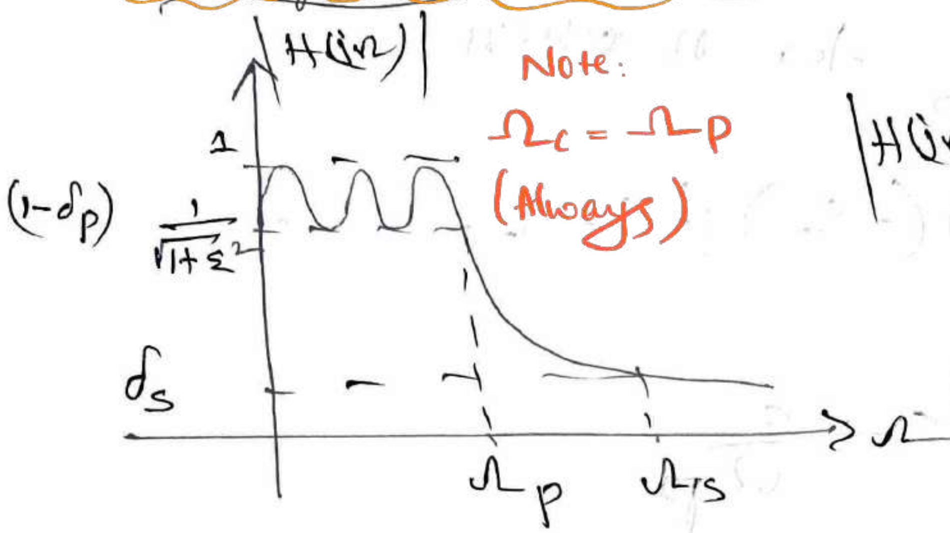
Converting Hz → r/s

$\Omega p = 2\pi fp = 2 \times \pi \times 500 \Rightarrow 1000\pi$ r/s

$\Omega s = 2\pi fs = 2 \times \pi \times 750 \Rightarrow 1500\pi$ r/s

$$d = \sqrt{\frac{(1-\delta p)^{-2} - 1}{\delta s^{-2} - 1}} =$$

Chebyshev Filter Type I → Ripple in the passband filter or Maximum flat stop band ①



Note: $\Omega_c = \Omega_p$ (Always)

$$|H(j\Omega)|^2 = \frac{1}{1+\varepsilon^2\left[T_n(\Omega)\right]^2}$$

$\varepsilon \to$ Ripple related factor

$T_n(\Omega) \to$ chebyshev polynomial.

Design $\quad n = ?\,,\ S_k = ?\,,\ H_n(S) = ?\ \ H_a(S) = ?$

$$n = \frac{\cosh^{-1}\left(\frac{1}{d}\right)}{\cosh^{-1}\left(\frac{1}{k}\right)} \qquad d = \sqrt{\frac{(1-\delta_p)^{-2}-1}{\delta_s^{-2}-1}}\ , \quad k = \frac{\Omega_p}{\Omega_s}$$

$$S_k = \sigma_k + j\Omega_k\ , \quad k = 1 \text{ to } n$$

$$\sigma_k = -a \sin\left[\frac{(2k-1)\pi}{2n}\right]$$

$$\Omega_k = b \cos\left[\frac{(2k-1)\pi}{2n}\right]$$

$$a = \frac{1}{2}\left\{\left(\frac{1+\sqrt{1+\varepsilon^2}}{\varepsilon}\right)^{1/n} - \left(\frac{1+\sqrt{1+\varepsilon^2}}{\varepsilon}\right)^{-1/n}\right\}$$

$$b = \frac{1}{2}\left\{ -//- + -//- \right\}$$

$$H_n(S) = \frac{k_n}{(S-S_1)(S-S_2)\ldots\ldots}$$

Normalized Filter

$\Omega_p = \Omega_c = 1$ r/s

$K_n = b_0$ for n odd

$= \frac{b_0}{\sqrt{1+\varepsilon^2}}$ for n even

$b_0 = (-s_1)(-s_2)(-s_3) \ldots$

$H_d(s) = H_n(s)\Big|_{s \to \frac{s}{\Omega_p}}$

$H_n(s) = \frac{K_n}{V_n(s)}$ $K_n = b_0$ for n odd

$= \frac{b_0}{\sqrt{1+\varepsilon^2}}$ for n even

$H_d(s) = H_n(s)\Big|_{s \to \frac{s}{\Omega_p}}$

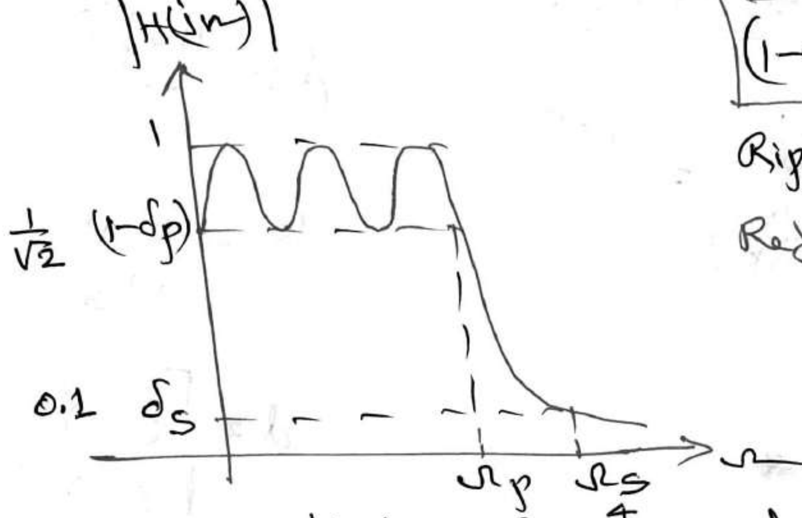
(3)

1. Design an analog chebyshev filter that satisfy the following constraints.

$$\frac{1}{\sqrt{2}} \le |H(j\Omega)| \le 1 \quad , \quad 0 \le \Omega \le 2$$

$$|H(j\Omega)| < 0.1 \qquad \Omega \ge 4$$

$\Rightarrow$



$$\boxed{(1-\delta_p)\,dB = 20\log\frac{1}{\sqrt{2}} = -3dB}$$

Ripple $\underline{3dB}$

Refere $\underline{3dB}$ Table

$$n = \frac{\cosh^{-1}\left(\frac{1}{d}\right)}{\cosh^{-1}\left(\frac{1}{k}\right)} = 3$$

$$d = \sqrt{\frac{(1-\delta_p)^{-2}-1}{\delta_s^{-2}-1}} =$$

$$k = \frac{\Omega_p}{\Omega_s} =$$

From 3dB Table

$$H_n(s) = \frac{K_n}{V_n(s)} = \qquad K_n = \cancel{b_0}\, b_0 \text{ for } n \text{ odd}$$

$$= 0.25$$

$$= \frac{0.25}{s^3+0.59s^2+0.93s+0.25} \qquad V_n(s) = s^3+b_2 s^2+b_1 s+b_0$$

for $\Omega_c = \Omega_p = 1\,r/s$

$$= s^3 + \cancel{0.25}\,0.59 s^2 + 0.93s + 0.25$$

For $\Omega_p = 2\ r/s$

$$H_d(s) = H_n(s)\Big|_{s \to \frac{s}{\Omega_p} = \frac{s}{2}}$$

$$H_d(s) = \frac{0.25 \times 2^3}{s^3 + 0.59 \times 2\, s^2 + 0.93 \times 2^2\, s + 2^3 \times 0.25}$$

(2) Design analog chebyshev LPF to satisfy the following specification (4)

(i) acceptable PB ripple of 2dB

(ii) cutoff frequency is 40 r/s

(iii) SB attenuation of 20dB at 52 r/s

= ~~Refer 2dB Table~~

$$n = \frac{\cosh^{-1}\left(\frac{1}{d}\right)}{\cancel{\log}\cosh^{-1}\left(\frac{1}{k}\right)} = 5$$

$$d = \sqrt{\frac{(1-\delta_p)^{-2}-1}{\delta_s^{-2}-1}}$$

$$k = \frac{\Omega_p}{\Omega_s}$$

$\Omega_p = 40$ r/s

$\Omega_s = 52$ r/s

$(1-\delta_p) = -2$ dB

$$d = \sqrt{\frac{(\ )^{-2}-1}{(\ )^{-2}-1}}$$

$20\log(1-\delta_p) = -2$

$$H_n(s) = \frac{k_n}{V_n(s)}$$

$\log(1-\delta_p) = \frac{-2}{20}$

$(1-\delta_p) = 10^{-2/20}$

=

δ_s dB $= -20$

$20\log\delta_s = -20$

$\log\delta_s = -1$

$\delta_s = 10^{-1}$

=

(3) Give Ripple 3dB, SB attenuation 16dB (5)
with $f_p = 1KHz$ & $f_s = 2KHz$. Design
analog chebyshev filter.

$$= \quad \Omega_p = 2\pi f_p = 2000\pi \ r/s$$

$$\Omega_s = 2\pi f_s = 4000\pi \ r/s$$

$$(1-\delta_p) = -3dB \qquad \delta_s = -16dB$$

$$20\log(1-\delta_p) = -3 \qquad 20\log\delta_s = -16$$

$$\log(1-\delta_p) = \frac{-3}{20} \qquad \log\delta_s = \frac{-16}{20}$$

$$(1-\delta_p) = 10^{-3/20} \qquad \delta_s = 10^{-16/20}$$

$$n = \frac{\cancel{\log}\ \cosh^{-1}\left(\frac{1}{d}\right)}{\cosh^{-1}\left(\frac{1}{k}\right)} = 2 \qquad d = \sqrt{\frac{(1-\delta_p)^{-2}-1}{\delta_s^{-2}-1}}$$

$$k = \frac{\Omega_p}{\Omega_s}$$

Refer 3dB Table

$$K_n = \frac{b_0}{\sqrt{1+\varepsilon^2}} \quad \text{for even}$$

$$H_n(s) = \frac{K_n}{V_n(s)} =$$

Design analog Cheby filter with pass band gain of 0.707 at 2 r/s with stop band attenuation of 0.1 @ 4 r/s

$1-\delta_p = 0.707$; $\Omega_p = 2$ r/s
$\delta_s = 0.1$; $\Omega_s = 4$ r/s

$$d = \sqrt{\frac{(0.707)^{-2}-1}{(0.1)^{-2}-1}} = \underline{\underline{0.1005}}.$$

$$k = \frac{\Omega_p}{\Omega_s} = \frac{2}{4} = \frac{1}{2}$$

$$n = \frac{\cos^{-1}\left(\frac{1}{0.1005}\right)}{\cos^{-1}(2)} = \frac{2.98}{1.31} \Rightarrow 2.29$$

$$\approx \underline{\underline{3}}.$$

$$H_n(s) = \frac{k_n}{V_n(s)}$$

$\Rightarrow k_n = b_0$ if n is odd
$\therefore$ here $\because n = 3$; odd

$\because$ 0.707 is 3dB ; Refer 3 dB table under which $n = 3$

$$H_n(s) = \frac{b_0}{V_n(s)} = \frac{0.25}{s^3 + 0.59s^2 + 0.92s + 0.25}$$

$H_n(d) \Rightarrow$ Replace $s \rightarrow \frac{s}{2}$ $\rightarrow$ $\omega_c = \omega_p$

② Design Cheby filter with pass band gain of 3dB at 1 KHz & 16 dB at 2 KHz

(ans)

$3\,dB \Rightarrow 0.707$

$16\,dB \Rightarrow 10^{-16/20} \Rightarrow 0.158$

$$d = \sqrt{\frac{(0.707)^{-2}-1}{(0.158)^{-2}-1}} \Rightarrow 0.16$$

$$k = \frac{\Omega p}{\Omega s} = \frac{2\pi \times 10^3}{2\pi \times 2\times10^3} \Rightarrow \underline{\underline{0.5}}$$

$$n = \frac{\cosh^{-1}(1/d)}{\cosh^{-1}(1/k)} = \frac{\cosh^{-1}(1/0.16)}{\cosh^{-1}(1/0.5)} \Rightarrow 1.91 \simeq \underline{\underline{2}}$$

$\Omega c = \Omega p = 2\pi \times 10^3$

$\Rightarrow \underline{\underline{6283.18}}\ r/s$

here $\because n =$ even

$$k(n) = \frac{b_0}{\sqrt{1+\varepsilon^2}} = \frac{0.7}{\sqrt{1+0.99}} \Rightarrow \underline{\underline{0.49}}$$

$$h_n(s) = \frac{k(n)}{V_n(s)} = \frac{0.49}{s^2+0.64s+0.70}$$

$$h_a(s) = \text{Replace } s \rightarrow \frac{s}{\underline{\underline{6283}}}$$

Digital Filters :—

1. Infinite Impulse Response (IIR) Filters
2. Finite Impulse Response (FIR) Filters

1. IIR Filters :—

→ Impulse Response $h(n)$ is infinite
→ Filter is unstable

The IIR filter is designed with the following assumptions i.e.,

(i) The filter is causal

realizable → $h(n) = 0, \quad n < 0$

(ii) The filter is stable

$$\sum_{n=0}^{\infty} |h(n)| < \infty$$

The IIR filters are designed using analog filters for the following reasons

(i) Analog filters are well developed

(ii) Design procedure is readily available through Tables.

Expression for IIR filters

Consider LCCDE

$$\sum_{k=0}^{N-1} a_k y(n-k) = \sum_{k=0}^{M-1} b_k x(n-k)$$

$a_0 = 1, \quad a_k \neq 0, \quad k = 1 \text{ to } N-1$

$$y(n) + \sum_{k=1}^{N-1} a_k y(n-k) = \sum_{k=0}^{M-1} b_k x(n-k)$$

$$\boxed{y(n) = \sum_{k=0}^{M-1} b_k x(n-k) - \sum_{k=1}^{N-1} a_k y(n-k)}$$

→ Time domain Equation

→ Recursive system because present output depends on its past ~~o/p~~ outputs

Take Z T

$$Y(z) = \sum_{k=0}^{M-1} b_k z^{-k} X(z) - \sum_{k=1}^{N-1} a_k z^{-k} Y(z)$$

$$Y(z)\left[1 + \sum_{k=1}^{N-1} a_k z^{-k}\right] = \sum_{k=0}^{M-1} b_k z^{-k} X(z)$$

$$\boxed{H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M-1} b_k z^{-k}}{1 + \sum_{k=1}^{N-1} a_k z^{-k}}}$$

Z-Domain Equation

→ Consists of both zeros & poles

→ ∵ of poles, the filter may be stable or Unstable

(3)

Mapping of Analog to Digital filters

→ Analog Filter

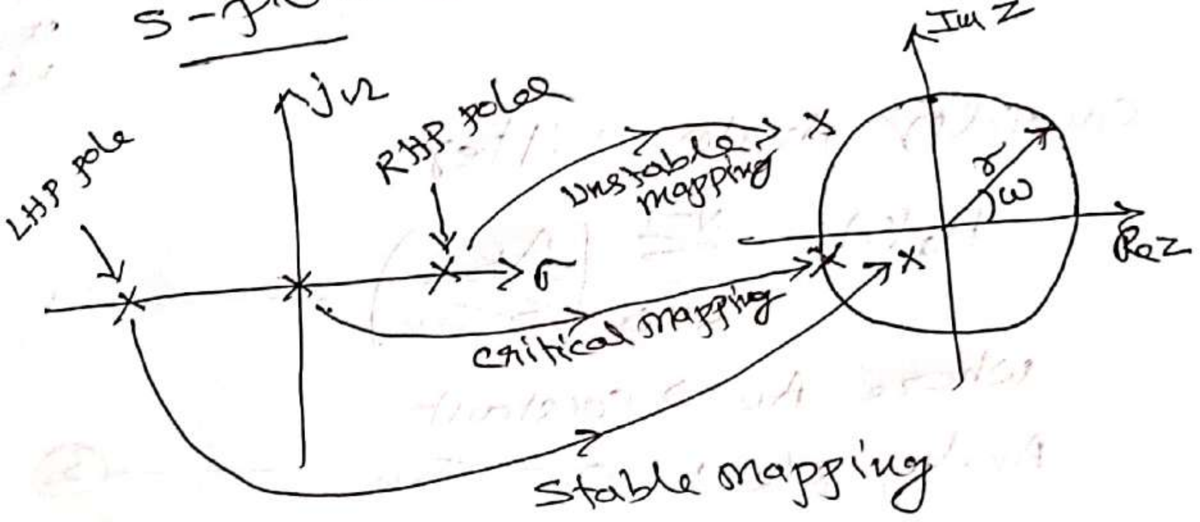
→ $s = \sigma + j\Omega$

s-plane

→ Digital Filter

→ $z = r \cdot e^{j\omega}$

z-plane



IIR Filter design by Impulse Invariance Transformation / Method

Design Process

Analog filter $H_a(s) \xrightarrow{ILT} h_a(t) \xrightarrow{Sample} h(n) \xrightarrow{ZT} H(z)$ Digital Filter

Consider Analog Filter

$$H_a(s) = \sum_{k=0}^{N-1} \left(\frac{A_k}{s-s_k}\right) \quad \text{——— (1)}$$

where $A_k \rightarrow$ constant

Analog pole : $s = s_k$ ——— (2)

Use ILT on Eqn (1)

$$h_a(t) = \sum_{k=0}^{N-1} A_k e^{s_k t}$$

Sample by substituting $t = nT$

$$h_a(nT) = \sum_{k=0}^{N-1} A_k e^{s_k nT}$$

Apply ZT

$$Z[h_a(nT)] = H(z) = \sum_{n=0}^{\infty} h_a(nT) \cdot z^{n}$$

$$= \sum_{n=0}^{\infty} \sum_{k=0}^{N-1} A_k e^{s_k nT} \cdot z^{-n}$$

$$= \sum_{k=0}^{N-1} A_k \sum_{n=0}^{\infty} e^{s_k nT} \cdot z^{-n}$$

$$= \sum_{k=0}^{N-1} A_k \cdot \sum_{n=0}^{\infty} \left(e^{s_k T} \cdot z^{-1}\right)^{n}$$

$$= \sum_{k=0}^{N-1} A_k \left[\frac{1}{1-e^{S_k T} z^{-1}} \right]$$

$\sum_{n=0}^{\infty} (a)^n = \frac{1}{1-a}, a<1$
$= \infty, a \geq 1$

$$H(z) = \sum_{k=0}^{N-1} \left[\frac{A_k}{1-e^{S_k T} z^{-1}} \right] \quad \text{— (3)}$$

Digital pole :- $z = e^{S_k T}$ — (4)

compare Equations (1) & (3)

$$\boxed{\frac{1}{S-S_k} = \frac{1}{1-e^{S_k T} z^{-1}}}$$ *****

Analog Filter → $H_a(s)$ Digital Filter → $H(z)$

Properties / characteristics of Mapping

consider analog & Digital poles

Analog pole : $S = S_k$

Digital pole : $z = e^{S_k T}$

Relationship between Analog & Digital poles

$$\boxed{z = e^{ST}}$$

Substitute $s = \sigma + j\Omega$ & $z = r.e^{j\omega}$

$$r.e^{j\omega} = e^{(\sigma + j\Omega)T} = e^{\sigma T} . e^{j\Omega T}$$

Compare real & Imaginary part

Real part :- $r = e^{\sigma T}$

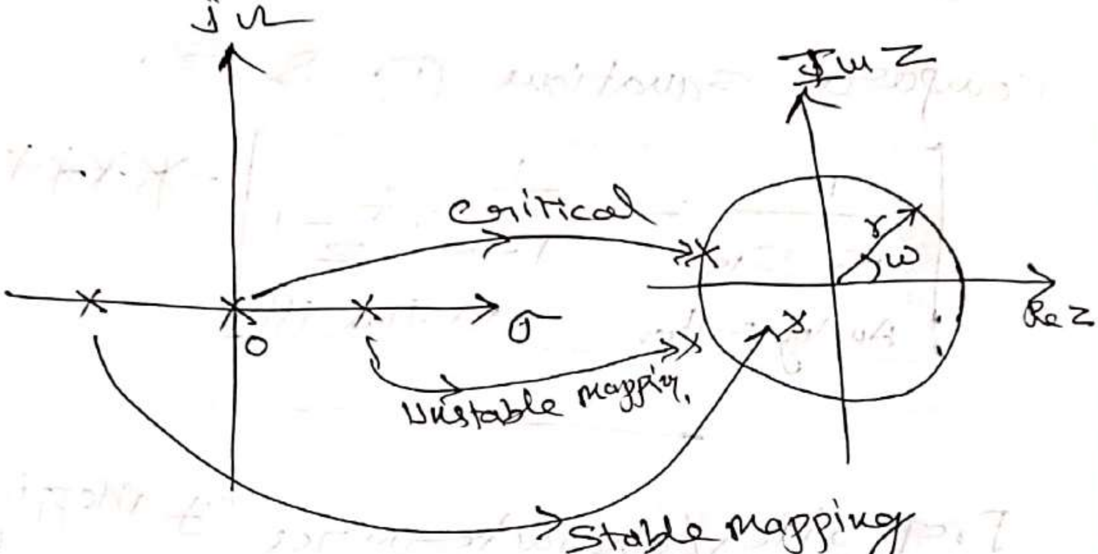
Imaginary part :- $\omega = \Omega T$

where $\omega \rightarrow$ Digital angular Frequency

$\Omega \rightarrow$ Analog angular Frequency

Consider Real part :- $r = e^{\sigma T}$

Analog Filter	Digital Filter
If $\sigma = 0$,	$r = 1$
If $\sigma < 0$,	$r < 1$
If $\sigma > 0$,	$r > 1$



$\Rightarrow$ Stable Analog Filter always give Stable digital filter

Consider imaginary part :-

$\omega = \Omega T$

$\rightarrow$ The relationship between digital & analog filter is linear (Advantage)

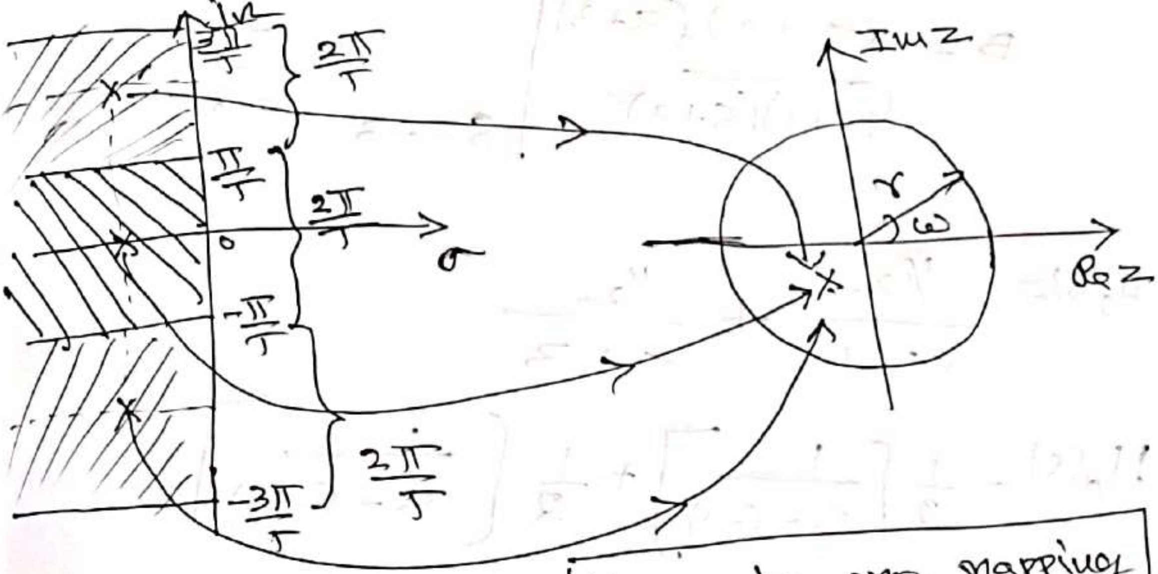
Consider Digital :- $-\pi \le \omega \le \pi$

$-\pi \le \Omega T \le \pi$

Analog :- $\frac{-\pi}{T} \le \Omega \le \frac{\pi}{T}$

Analog Filter — Digital Filter

s-plane — z-plane



→ $\boxed{\text{Many to one mapping}}$

→ Aliasing

→ Exact Reconstruction of Analog Filter from digital filter is not possible.

~~A/D — H/~~

$\boxed{\cancel{A/D - H(z) - D/A}}$ ×

---

① Find the IIT of the system

$$H_a(s) = \frac{(s+2)}{(s+1)(s+3)}$$

$= \quad H(z) = ?$

IIT

$$\boxed{\frac{1}{s-s_k} \Rightarrow \frac{1}{1-e^{s_k T} z^{-1}}}$$

$$H_a(s) = \frac{(s+2)}{(s+1)(s+3)} = \frac{A}{s+1} + \frac{B}{s+3}$$

$$A = \frac{(s+2)\cancel{(s+1)}}{\cancel{(s+1)}(s+3)}\Bigg|_{s=-1} = \frac{1}{2}$$

$$B = \frac{(s+2)\cancel{(s+3)}}{(s+1)\cancel{(s+3)}}\Bigg|_{s=-3} = \frac{-1}{-2} = \frac{1}{2}$$

$$H_a(s) = \frac{1/2}{s+1} + \frac{1/2}{s+3}$$

$$H_a(s) = \frac{1}{2}\left[\frac{1}{s-(-1)}\right] + \frac{1}{2}\left[\frac{1}{s-(-3)}\right]$$

Apply IIT

$$H(z) = \frac{1}{2}\left[\frac{1}{1-e^{-1T}z^{-1}}\right] + \frac{1}{2}\left[\frac{1}{1-e^{-3T}z^{-1}}\right]$$

$$= \frac{1}{2}\left[\frac{1-e^{-3T}z^{-1}+1-e^{-T}z^{-1}}{1-e^{-3T}z^{-1}-e^{-T}z^{-1}+e^{-4T}z^{-2}}\right]$$

$$= \frac{1}{2}\left[\frac{2-z^{-1}e^{-2T}(e^{-T}+e^{T})}{1-z^{-1}e^{-2T}(e^{-T}+e^{+T})+e^{-4T}z^{-2}}\right]$$

$$= \frac{1}{2}\left[\frac{2-z^{-1}e^{-2T}\times 2\cosh(T)}{1-z^{-1}e^{-2T}\times 2\cosh(T)+e^{-4T}z^{-2}}\right]$$

$$H(z) = \frac{1-z^{-1}e^{-2T}\cosh(T)}{1-2z^{-1}e^{-2T}\cosh(T)+e^{-4T}z^{-2}}$$

2. Determine digital filter transfer function H(z) using IIT for

$$H(s) = \frac{s+0.1}{(s+0.1)^2+9}$$

= $H(z) = ?$

$$H(s) = \frac{s+0.1}{(s+0.1)^2+3^2}$$

$x^2 + a^2 = 0$
$x^2 = -a^2$
$\underbrace{x = \pm\sqrt{a}\, i}_{\text{Roots}}$

$$= \frac{s+0.1}{(s+0.1+3j)(s+0.1-3j)}$$

$$H(s) = \frac{A}{[s-(-0.1-3j)]} + \frac{B}{[s-(-0.1+3j)]}$$

$$A = \frac{1}{2}, \quad B = \frac{1}{2}$$

$$H(s) = \frac{1}{2}\left[\frac{1}{s-(-0.1-3j)}\right] + \frac{1}{2}\left[\frac{1}{s-(-0.1+3j)}\right]$$

Take ZT

$$H(z) = \frac{1}{2}\left[\frac{1}{1-e^{(-0.1-3j)T} \cdot z^{-1}}\right] + \frac{1}{2}\left[\frac{1}{1-e^{(-0.1+3j)T} z^{-1}}\right]$$

$$H(z) = \frac{1-e^{-0.1T} z^{-1}\cos(3T)}{1-2e^{-0.1T} z^{-1}\cos(3T)+e^{-0.2T} z^{-2}}$$

Advantages of Impulse Invariance Technique/Method (IIT or IIM)

→ Analog & digital are linear ; $\omega = \Omega T$

$\omega \Rightarrow$ Digital angular freq.

$\Omega \Rightarrow$ Analog " "

Disadvantage:

Exact reconstruction of analog filter is not possible

A/D — $H(z)$ $\cancel{\rightarrow}$ D/A

① Design the IIR filter using IIT for $H(s) = \dfrac{s+1}{(s+2)(s+3)}$

$H(z) = ?$

using partial fraction

$$\frac{s+1}{(s+2)(s+3)} = \frac{A}{(s+2)} + \frac{B}{(s+3)}$$

$$s+1 = A(s+3) + B(s+2)$$

$\therefore A = -1$

$B = 2$

$$\therefore \frac{s+1}{(s+2)(s+3)} = \frac{-1}{s+2} + \frac{2}{s+3}$$

$$\Rightarrow \frac{-1}{s-(-2)} + \frac{2}{s-(-3)}$$

$$H(S) = -2\left(\frac{1}{S-(-2)}\right) + 2\left(\frac{1}{S-(-3)}\right)$$

Converting using A → D formula

$$H(z) \Rightarrow \frac{-1}{1-e^{-2T}z^{-1}} + \frac{2}{1-e^{-3T}z^{-1}}$$

$$\Rightarrow \frac{-1+e^{-3T}z^{-1}+2-2e^{-2T}z^{-1}}{1-e^{-3T}z^{-1}-e^{-2T}z^{-1}+e^{-5T}z^{-2}}$$

(Q2) Design an IIR filter using IIM for foll. constrains.

(i) Monotonic Pass band gain = 0.8 at 0.2π r/s

(ii) Stopband attn ⇒ 0.2 at 0.6π r/s.

First design Analog BW filter

$$H_d(s) = \frac{0.616}{s^2 + 1.11s + 0.616}$$

$$\frac{0.616}{s^2+1.11s+0.616} = \frac{0.616}{\{s-(-0.55+0.55j)\}\{s-(0.55-0.55j)\}}$$

Partial fractions $\Rightarrow \dfrac{0.616}{A[s-(0.55-0.55j)] + B[s-(-0.55+0.55j)]}$

$$B = \frac{-0.616}{-1.1j} \times \frac{j}{j} \Rightarrow \frac{0.616j}{1.1} \Rightarrow \underline{-0.56j}$$

$$A = \underline{0.56j}$$

$$H_d(s) = \frac{-0.56j}{\{s-(-0.5+j0.5)\}} + \frac{0.56j}{\{s-(-0.5-j0.5)\}}$$

$$H(z) = \frac{-0.56j}{1-e^{(-0.5+0.5j)T}z^{-1}} + \frac{0.56j}{1-e^{(-0.5-0.5j)T}z^{-1}}$$

Go (10) Through the Qn.

(3) Find Discrete time transfer function of analog filter $H(s) = \frac{(s+3)}{(s+1)(s+2)}$

Use IIT with a sampling frequency of 5KHZ. Also find DC gain of the filter. Plot poles and zeros on s and z planes.

=

$$H(s) = \frac{(s+3)}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

$A = 2, \ B = -1$

$$H(s) = 2\left[\frac{1}{s-(-1)}\right] - 1\left[\frac{1}{s-(-2)}\right]$$

Apply IIT

$$H(z) = 2\left[\frac{1}{1-e^{-T}z^{-1}}\right] - \left[\frac{1}{1-e^{-2T}z^{-1}}\right]$$

Given $F = 5000$ HZ

$$\therefore T = \frac{1}{F} = \frac{1}{5000} \text{ sec.} = 0.0002 \text{ sec}$$

$$\therefore e^{-T} = e^{-0.0002} = 1$$

$$e^{-2T} = e^{-0.0004} = 1$$

$$\therefore H(z) = 2\left[\frac{1}{1-z^{-1}}\right] - \left[\frac{1}{1-z^{-1}}\right]$$

$$\boxed{H(z) = \frac{1}{1-z^{-1}} = \frac{z}{z-1}}$$

DC Gain :— convert Z-T into DTFT to draw frequency response

Substitute $z = e^{j\omega}$ in $H(z)$

$$H(e^{j\omega}) = \frac{e^{j\omega}}{e^{j\omega} - 1}$$

DC gain $\rightarrow$ $\omega = 0$

$$H(e^{j\omega}) = \frac{1}{0} = \infty$$

$\Longrightarrow$ DC filter is Unstable

poles & Zeros

Analog Filter :—

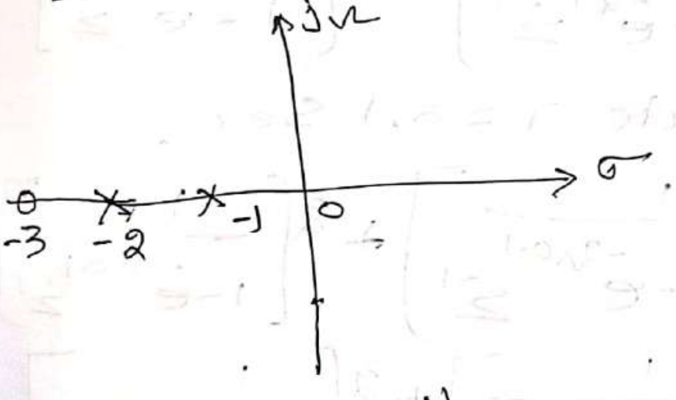
$$H(s) = \frac{(s+3)}{(s+1)(s+2)}$$

Zeros :- $s+3=0$, $\boxed{s=-3}$

poles :- $(s+1)=0$, $(s+2)=0$

$\boxed{s=-1}$, $\boxed{s=-2}$

s-plane

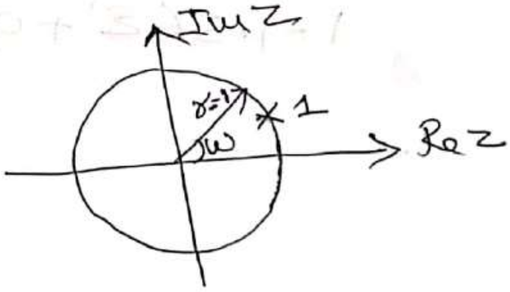


Digital Filter :- $H(z) = \frac{z}{z-1}$

Zeros: $\boxed{z=0}$

poles: $z-1=0$, $z=1$

z-plane



(4) Design digital IIR filter using IIT with $T = 0.1$ Sec. for

$$H_a(s) = \frac{s+1}{s^2+5s+6}$$

=

$H(z) = ?$ $\quad H_a(s) = \dfrac{s+1}{(s+2)(s+3)}$

$$H_a(s) = \frac{(s+1)}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$$

$A = -1$, $B = 2$

$$H_a(s) = -1\left[\frac{1}{s-(-2)}\right] + 2\left[\frac{1}{s-(-3)}\right]$$

Apply IIT

$$H(z) = -\left[\frac{1}{1-e^{-2T}z^{-1}}\right] + 2\left[\frac{1}{1-e^{-3T}z^{-1}}\right]$$

Substitute $T = 0.1$ Sec.

$$\therefore H(z) = -\left[\frac{1}{1-e^{-2\times 0.1}z^{-1}}\right] + 2\left[\frac{1}{1-e^{-3\times 0.1}z^{-1}}\right]$$

$$= -\left[\frac{1}{1-0.82z^{-1}}\right] + 2\left[\frac{1}{1-0.74z^{-1}}\right]$$

$$H(z) = \frac{1-0.896z^{-1}}{1-1.56z^{-1}+0.61z^{-2}}$$

5. The poles of 3rd order Butterworth filter are $s_1 = -0.5$, $s_{2,3} = 0.5 \pm j\,1.0$ Determine the IIR digital filter transfer function using IIM.

$$= \quad H_a(s) = \frac{1}{(s-s_1)(s-s_2)(s-s_3)}$$

$$= \frac{1}{[s-(-0.5)][s-(+0.5+j1)][s-(0.5-j1.0)]}$$

$$H_a(s) = \frac{A}{s-(-0.5)} + \frac{B}{s-(0.5+j)} + \frac{C}{s-(0.5-j)}$$

$A =$ $\qquad$ $B =$ $\qquad$ $C =$

6) Design IIR filter using IIT for the following constraints. Use BWF

$$0.8 \le |H(\Omega)| \le 1 \quad , \quad 0 \le \Omega \le 0.2\pi$$

$$|H(\Omega)| \le 0.2 \quad , \quad 0.6\pi \le \Omega$$

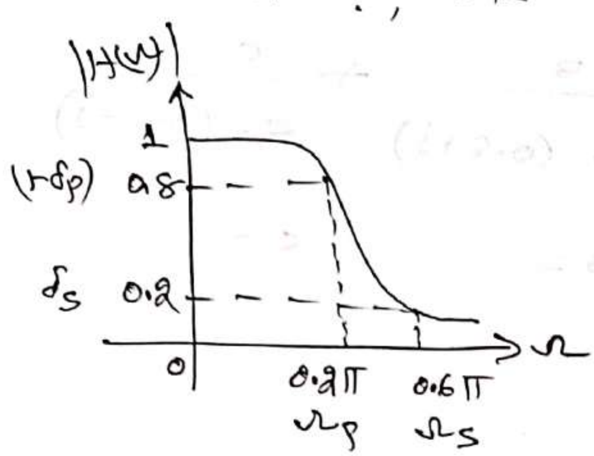
=

$$H(z) = ?$$

(i) Design of Analog BWF :-

$n = ?$, $s_k = ?$ $\Omega_c = ?$ $H_n(s) = ?$

$H_a(s) = ?$



$$n = \frac{\log\left(\frac{1}{d}\right)}{\log\left(\frac{1}{K}\right)} =$$

$$d = \sqrt{\frac{(1-\delta_p)^{-2} - 1}{\delta_s^{-2} - 1}}$$

$$d = 0.15$$

$$K = \frac{\Omega_p}{\Omega_s} = \frac{0.2\pi}{0.6\pi} = \frac{1}{3}$$

$$\therefore n = \frac{\log\left(\frac{1}{0.15}\right)}{\log\left(\frac{1}{1/3}\right)}$$

$$\boxed{n \simeq 2}$$

Poles s_k :- $s_k = e^{j\left(\frac{\pi}{2} + \frac{(2k+1)\pi}{2n}\right)}$

$k=0, \; s_0 = -0.707 + j\,0.707$

$k=1, \; s_1 = -0.707 - j\,0.707$

(or) From Tables :- $B_n(s) = s^2 + \sqrt{2}\,s + 1$

$$= [s - (-0.707 + j0.707)][s - (-0.707 - j0.707)]$$

cutoff frequencies:-

$$\Omega_{c1} = \frac{\Omega p}{\left[(1-\delta_p)^{-2} - 1\right]^{1/2n}} = 0.73 \text{ rad/sec}$$

$$\Omega_{c2} = \frac{\Omega s}{\left[\delta_s^{-2} - 1\right]^{1/2n}} = 0.85 \text{ rad/sec}$$

$$\Omega_c = \frac{\Omega_{c1} + \Omega_{c2}}{2} = 0.79 \text{ rad/sec.}$$

$$H_n(s) = \frac{1}{(s-s_0)(s-s_1)}$$

$$= \frac{1}{\left[s-(-0.707+j0.707)\right]\left[s-(-0.707-j0.707)\right]}$$

$\longrightarrow \Omega_c = 1 \text{ r/s}$

For $\Omega_c = 0.79$ rad/sec:-

$$H_d(s) = H_n(s)\Big|_{s \to \frac{s}{0.79}}$$

$$H_d(s) = \frac{1}{\left[\frac{s}{0.79} - (-0.707+j0.707)\right]\left[\frac{s}{0.79} - (-0.707-j0.707)\right]}$$

$$= \frac{(0.79)^2}{\left[s - 0.79(-0.707+j0.707)\right]\left[s - 0.79(-0.707-j0.707)\right]}$$

$$H_d(s) = \frac{0.62}{\left[s-(-0.557+j0.557)\right]\left[s-(-0.557-j0.557)\right]}$$

(ii) Transformation of Analog to Digital Filter

$$\boxed{\frac{1}{S-S_K} \Rightarrow \frac{1}{1-e^{S_K T} z^{-1}}} \quad \text{IIT}$$

Consider analog Filter

$$H_a(s) = \frac{0.62}{\left[S-(-0.557+j0.557)\right]\left[S-(-0.557-j0.557)\right]}$$

$$= \frac{A}{S-(-0.557+j0.557)} + \frac{B}{S-(-0.557-j0.557)}$$

$$A = -0.56j \qquad B = 0.56j$$

$$H_a(s) = -0.56j\left[\frac{1}{S-(-0.557+j0.557)}\right] + 0.56j\left[\frac{1}{S-(-0.557-j0.557)}\right]$$

Apply IIT

$$H(z) = \frac{-0.56j}{1-e^{(-0.557+j0.557)T} z^{-1}} + \frac{0.56j}{1-e^{(-0.557-j0.557)T} z^{-1}}$$

$$H(z) = \frac{0.56j\, z^{-1} e^{-0.55T} \cos(0.55T)}{1-z^{-1} e^{-0.55T} \cos(0.55T) + e^{-1.11T} z^{-2}}$$

7 Design IIR digital filter using IIT for the following maximally flat frequency response

$$20\log|H(\Omega)|_{\Omega=0.2\pi} \geq -1.9328\,dB$$

$$20\log|H(\Omega)|_{\Omega=0.6\pi} \leq -13.9794\,dB$$

$= \quad (1-\delta_p) = -1.9328\ dB$

$20\log(1-\delta_p) = -1.9328$

$\log(1-\delta_p) = -\frac{1.9328}{20} = -0.09664$

$(1-\delta_p) = 10^{-0.09664} = 0.8$

$\delta_s = -13.9794\,dB$

$20\log(\delta_s) = -13.9794$

$\log(\delta_s) = -\frac{13.9794}{20} = -0.69897$

$\delta_s = 10^{-0.69897} = 0.2$

$\Omega_p = 0.2\pi$ rad/sec

$\Omega_s = 0.6\pi$ rad/sec.

(i) <u>Design of Analog Butterworth Filter:-</u>

$n = ?\quad \Omega_c = ?,\quad S_k = ?,\quad H_n(s) = ?,\quad H_d(s) = ?$

$n = 1.7 \simeq 2$

$S_0 = -0.707 + j\,0.707$

$S_1 = -0.707 - j\,0.707$

$\Omega_{c_1} = \frac{\Omega_p}{\left[(1-\delta_p)^{-2}-1\right]^{1/2n}} = \cancel{0.7255}$ r/s

$$\Omega_{c2} = \frac{\Omega_s}{(\delta_s^{-2} - 1)^{1/2n}}$$

$$\Omega_c = \frac{\Omega_{c1} + \Omega_{c2}}{2}$$

$$H_a(s) = H_n(s)\Big|_{s \to \frac{s}{\Omega_c}}$$

(ii) Transformation of Analog to Digital Filter

(8) Design a digital chebyshev filter using IIT for the following constraints

$0.8 \le |H(\Omega)| \le 1$, $\quad 0 \le \Omega \le 0.2\pi$

$|H(\Omega)| \le 0.2$, $\quad 0.6\pi \le \Omega$

$\Rightarrow (1-\delta_p) = 0.8$, $\quad \delta_s = 0.2$

$\Omega_p = 0.2\pi$ rad/sec ; $\Omega_s = 0.6\pi$ rad/sec.

(i) Design of Analog chebyshev Filter:-

$n = ?$ $\quad S_k = ?$ $\quad H_n(S) = ?$ $\quad H_a(S) = ?$

$$n = \frac{\cosh^{-1}\left(\frac{1}{d}\right)}{\cosh^{-1}\left(\frac{1}{K}\right)} = 1.45 \simeq 2$$

$$d = \sqrt{\frac{(1-\delta_p)^{-2} - 1}{\delta_s^{-2} - 1}} = 0.153$$

$$K = \frac{\Omega_p}{\Omega_s} = 0.33$$

~~Using Tables:-~~

$\cancel{H_n(S) = \frac{K_n}{(S-S_1)(S-S_2)}}$

PB ripple $= 0.8$

PB ripple in dB $= 20\log 0.8$

$= -1.94$ dB

The Normalized filter Tables are not available

$\therefore$ $\boxed{\text{Find poles mannually}}$

$(1-\delta_p) = \frac{1}{\sqrt{1+\varepsilon^2}} \Rightarrow \therefore \varepsilon = 0.75$

$S_k = \sigma_k + j\Omega_k$, $\quad k = 1, 2$

$$\sigma_k = -a \sin\left[\frac{(2k-1)\pi}{2n}\right]$$

$$\Omega_k = b \cos\left[\frac{(2k-1)\pi}{2n}\right]$$

$$a = \frac{1}{2}\left[\left(\frac{1+\sqrt{1+\varepsilon^2}}{\varepsilon}\right)^{1/n} - \left(\frac{1+\sqrt{1+\varepsilon^2}}{\varepsilon}\right)^{-1/n}\right]$$

$$b = \frac{1}{2}\left[\left(\frac{1+\sqrt{1+\varepsilon^2}}{\varepsilon}\right)^{1/n} + \left(\frac{1+\sqrt{1+\varepsilon^2}}{\varepsilon}\right)^{-1/n}\right]$$

$\underline{k=1}$, $S_1 = \sigma_1 + j\Omega_1$

$S_1 = -0.408 + j0.82$

$\underline{k=2}$, $S_2 = -0.408 - j0.82$

$$H_n(s) = \frac{k_n}{(S-S_1)(S-S_2)}$$

$$k_n = \frac{b_0}{\sqrt{1+\varepsilon^2}} \quad \text{for 'n' even}$$

$$b_0 = (-S_1)(-S_2) = 0.83$$

$$k_n = \frac{b_0}{\sqrt{1+\varepsilon^2}} = 0.67$$

$$\therefore H_n(s) = \frac{0.67}{[S-(-0.408+j0.82)][S-(-0.408-j0.82)]}$$

$$H_a(s) = H_n(s)\Big|_{s \to \frac{s}{\Omega_p} = \frac{s}{0.2\pi}}$$

$$H_a(s) = \frac{0.67 \times (0.2\pi)^2}{\left[s - 0.2\pi(-0.408 + j0.82)\right]\left[s - 0.2\pi(-0.408 - j0.82)\right]}$$

(ii) Transformation of Analog to Digital filter

$$H_a(s) = \frac{A}{s - 0.2\pi(-0.408 + j0.82)} + \frac{B}{s - 0.2\pi(-0.408 - j0.82)}$$

$A =$ $\qquad\qquad$ $B =$

Apply IIT

$H(z) =$

(9) Design IIR digital chebyshev LPF using IIT for the following specifications.

(i) Passband ripple of 1dB at 0.6283 r/s

(ii) Stopband attenuation of 15dB at 0.94 r/s

(iii) $T = 1$ sec.

= (i) Design of Analog chebyshev Filter

$n = 4$

27

Advantage

A/D → H(s) → D/A

Satisfies

→ Exact reconstruction of Analog filters is possible.

IIR filter design by Bilinear Transformation (BLT):—

It is used to convert analog filters into digital filters.

analog ang. freq. → digi ang. freq.

$\Omega \rightarrow \omega$

Non linear rel^n.

Disadvantage:

Conversion is Non linear.

Design Process:—

Analog Filter → Digital Filter

$$H(s) \rightarrow Y(s) \,\&\, X(s) \xrightarrow{ILT} y(t) \,\&\, x(t) \xrightarrow{\text{Sampling}} y(n) \,\&\, x(n) \xrightarrow{ZT} Y(z) \,\&\, X(z)$$

$$\downarrow$$

$$H(z)$$

Design:—

consider Analog Filter $H(s) = \dfrac{b}{s+a}$ —— ①

where a & b are constants

$$\frac{Y(s)}{X(s)} = \frac{b}{s+a}$$

cross multiply

$$sY(s) + aY(s) = bX(s)$$

Take ILT to convert s-domain to time domain

$$\frac{dy(t)}{dt} + ay(t) = bx(t) \quad \text{—— ②}$$

Sampling Technique:— Use Trapezoidal rule for numerical integration technique

$$\int_{(n-1)T}^{nT} y(t)\,dt = \frac{T}{2}\left\{y(nT) + y[(n-1)T]\right\}$$

—— ③

Equation ② is continous time, which is converted into discrete by sampling using equation ③.

Hence equation ② is modified using $\int_{(n-1)T}^{nT}$ on both sides

$$\int_{(n-1)T}^{nT} \frac{dy(t)}{dt}\cdot dt + a\int_{(n-1)T}^{nT} y(t)\,dt = b\int_{(n-1)T}^{nT} x(t)\,dt$$

Apply Sampling Technique using Equation (3)

$$y(t)\Big|_{(n-1)T}^{nT} + a\frac{T}{2}\left[y(nT) + y[(n-1)T]\right] = b\frac{T}{2}\left[x(nT) + x[(n-1)T]\right]$$

$$y(nT) - y[(n-1)T] + a\frac{T}{2}\left[y(nT) + y[(n-1)T]\right] = \frac{bT}{2}\left[x(nT) + x[(n-1)T]\right]$$

Substitute $y(nT) = y(n)$

$y[(n-1)T] = y(n-1)$

$x(nT) = x(n)$

$x[(n-1)T] = x(n-1)$

$$y(n) - y(n-1) + a\frac{T}{2}\left[y(n) + y(n-1)\right] = \frac{bT}{2}\left[x(n) + x(n-1)\right]$$

Take ZT

$$Y(z) - z^{-1}Y(z) + \frac{aT}{2}\left[Y(z) + z^{-1}Y(z)\right] = \frac{bT}{2}\left[X(z) + z^{-1}X(z)\right]$$

$$Y(z)\left[1 - z^{-1} + \frac{aT}{2}(1 + z^{-1})\right] = X(z)\,\frac{bT}{2}\left[1 + z^{-1}\right]$$

$$\frac{Y(z)}{X(z)} = \frac{\frac{bT}{2}(1+z^{-1})}{1-z^{-1}+\frac{aT}{2}(1+z^{-1})}$$

Divide Nr & Dr of RHS by $\frac{T}{2}(1+z^{-1})$ for comparision with analog filter equation (1)

$$H(z) = \frac{b}{\frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]+a} \qquad \text{——— (4)}$$

Digital Filter

Compare equation (1) and (4) to obtain Transformation expression ****

$$\boxed{S = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right) = \frac{2}{T}\left[\frac{z-1}{z+1}\right]} \qquad \text{—— (5)}$$

BLT

Characteristics of BLT

consider equation (5) $\quad S = \frac{2}{T}\left[\frac{z-1}{z+1}\right]$

substitute $s = \sigma + j\Omega$ & $z = re^{j\omega}$

$$\sigma + j\Omega = \frac{2}{T}\left[\frac{re^{j\omega}-1}{re^{j\omega}+1}\right]$$

$$= \frac{2}{T}\left[\frac{r(\cos\omega + j\sin\omega) - 1}{r(\cos\omega + j\sin\omega) + 1}\right]$$

$$= \frac{2}{T}\left[\frac{(r\cos\omega - 1) + jr\sin\omega}{(r\cos\omega + 1) + jr\sin\omega}\right]$$

Rationalize RHS

$$= \frac{2}{T}\left[\frac{(r\cos\omega - 1) + jr\sin\omega}{(r\cos\omega + 1) + jr\sin\omega}\right] \times \left[\frac{(r\cos\omega + 1) - jr\sin\omega}{(r\cos\omega + 1) - jr\sin\omega}\right]$$

$$\sigma + j\Omega = \frac{2}{T}\left[\left(\frac{r^2-1}{r^2+1+2r\cos\omega}\right) + j\left(\frac{2r\sin\omega}{r^2+1+2r\cos\omega}\right)\right]$$

Equate real & imaginary parts

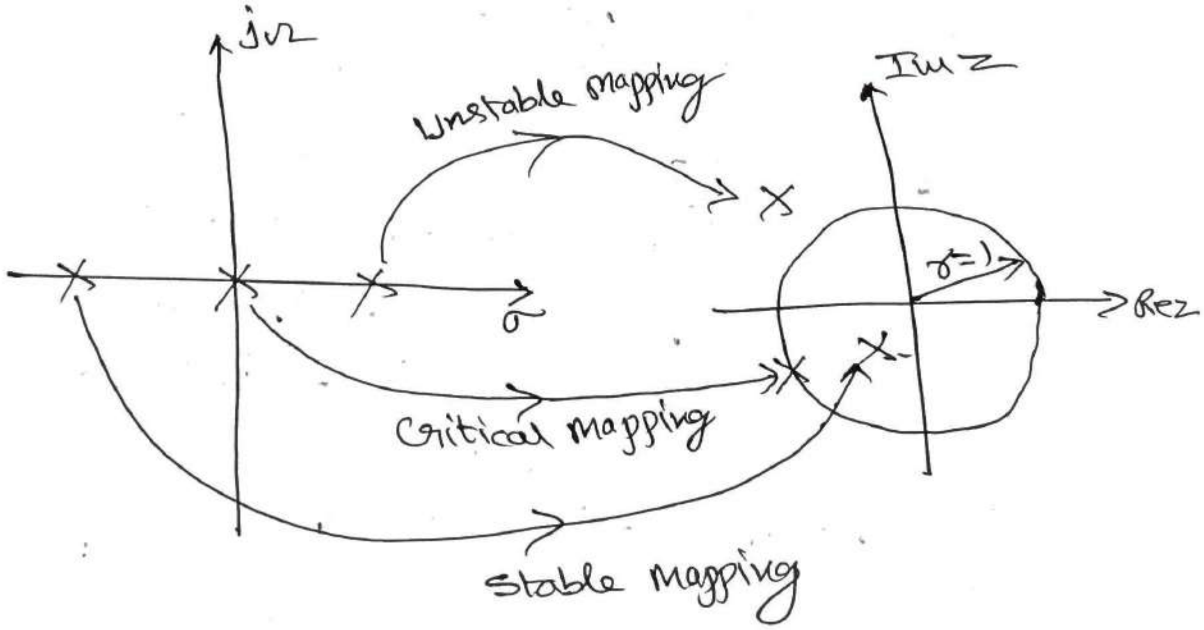
Real :- $\sigma = \frac{2}{T}\left[\frac{r^2-1}{r^2+1+2r\cos\omega}\right]$

Imaginary :- $\Omega = \frac{2}{T}\left[\frac{2r\sin\omega}{r^2+1+2r\cos\omega}\right]$

(i) Consider Real part :—

$$\sigma = \frac{2}{T}\left(\frac{r^2-1}{r^2+1+2r\cos\omega}\right)$$

Analog Filter S-Domain	Digital Filter Z-Domain
$\sigma = 0$	$r = 1$ If
$\sigma < 0$	$r < 1$ If
$\sigma > 0$	$r > 1$ If



The BLT preserves the stability of the transformed filter, ie., stable analog filter gives stable digital filter whereas unstable analog filter gives unstable digital filter.

(ii) Consider imaginary part :-

$$\Omega = \frac{2}{T}\left[\frac{2r\sin\omega}{r^2+1+2r\cos\omega}\right]$$

Substitute $r = 1$

$$\Omega = \frac{2}{T}\left[\frac{2\sin\omega}{2+2r\cos\omega}\right]$$

$$= \frac{2}{T}\left[\frac{\sin\omega}{1+\cos\omega}\right]$$

$$\boxed{\Omega = \frac{2}{T}\tan\left(\frac{\omega}{2}\right)} \quad \text{—— (6)}$$

$$\frac{\Omega T}{2} = \tan\frac{\omega}{2}$$

$$\therefore \boxed{\omega = 2\tan^{-1}\left(\frac{\Omega T}{2}\right)} \quad \text{—— (7)}$$

The relationship between digital angular frequency ω and analog angular frequency Ω is ~~non linear~~ non linear, hence disadvantage.

~~Mapping from s to z domain~~

~~Consider Digital:-~~

$$-\pi \le \omega \le \pi$$

$$-\pi \le 2\tan^{-1}\left(\frac{\Omega T}{2}\right) \le \pi$$

$$-\frac{\pi}{2} \le \tan^{-1}\left(\frac{\Omega T}{2}\right) \le \pi$$

Mapping from s to z-Domain.

Consider Digital: $\boxed{-\pi \le \omega \le \pi}$

$$-\pi \le 2\tan^{-1}\left(\frac{\Omega T}{2}\right) \le \pi$$

$$\frac{-\pi}{2} \le \tan^{-1}\left(\frac{\Omega T}{2}\right) \le \frac{\pi}{2}$$

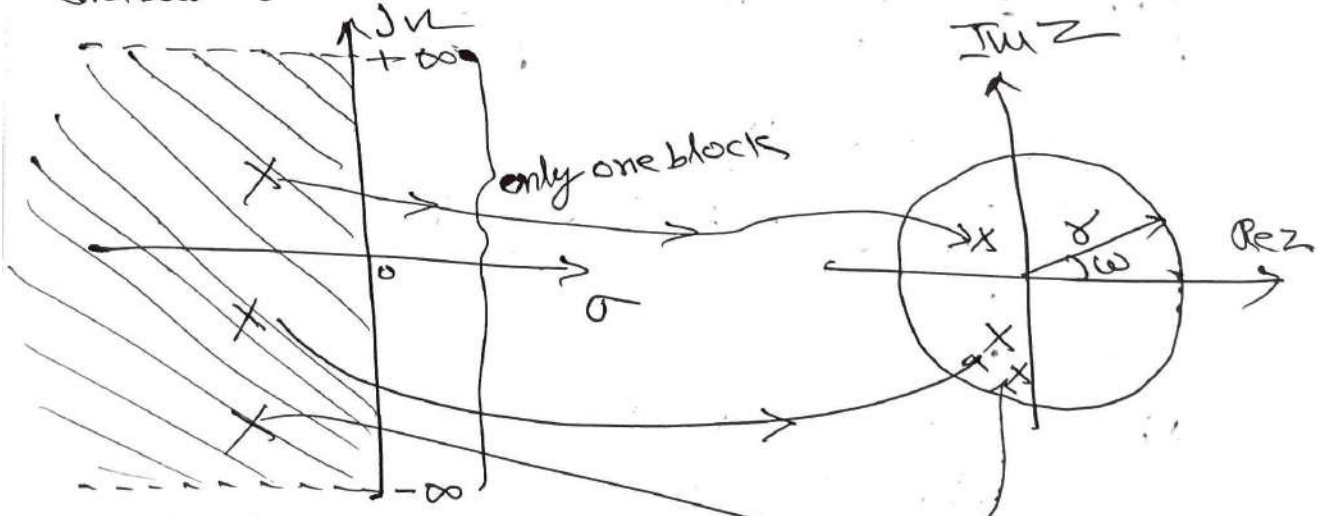
$$\tan\left(-\frac{\pi}{2}\right) \le \frac{\Omega T}{2} \le \tan\left(\frac{\pi}{2}\right)$$

$$\frac{2}{T}\tan\left(-\frac{\pi}{2}\right) \le \Omega \le \frac{2}{T}\tan\left(\frac{\pi}{2}\right)$$

Analog $\boxed{-\infty \le \Omega \le \infty}$

s-plane　　　　z-plain

Consider Left side poles



$\boxed{\text{One to one mapping}}$ Advantage

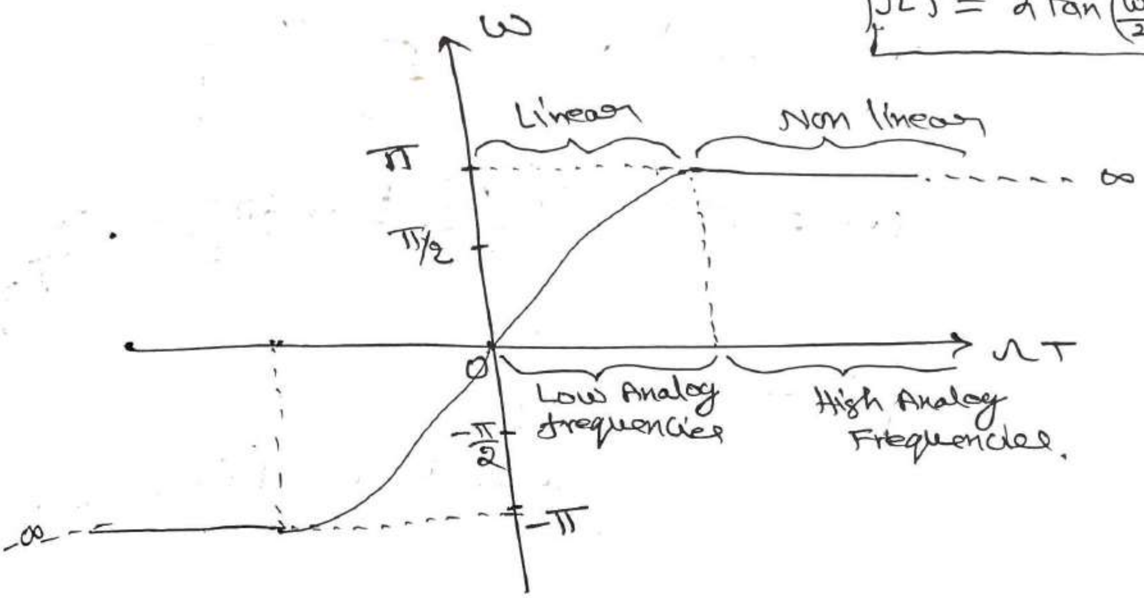
∵ only one block in s-plain

∴ No aliasing, hence exact reconstruction of analog filter from digital filter is possible. $\boxed{A/D - H(z) - D/A}$

Disadvantage:-

plot of $\underline{\omega}$ Vs $\underline{\Omega T}$

$$\boxed{\Omega T = 2\tan\left(\frac{\omega}{2}\right)}$$



For lower analog angular frequencies, the digital angular frequencies varies linearly, whereas, for higher analog frequencies, the digital angular frequencies varies non linearly.

The disadvantage of BLT, ~~for~~ is non linear for higher values of analog angular frequencies.

** Frequency prewarping:-

It is used to overcome the non linearity. All analog angular frequencies of analog filter are Prewarped.

Consider (Equation 7) $\omega = \frac{2}{\cancel{T}} \tan^{-1}\left(\frac{\Omega T}{2}\right)$ — (7)

Substitute $\Omega = \Omega^*$ in Equation (7)

Where Ω^* is prewarped frequency

$$\boxed{\Omega^* = \frac{2}{T}\tan\left(\frac{\Omega T}{2}\right)}$$ *****

Then $\omega = 2\tan^{-1}\left[\frac{\Omega^* T}{2}\right]$

$$\omega = 2\tan^{-1}\left[\cancel{\frac{2}{T}} \cdot \cancel{\tan}\left(\frac{\Omega T}{2}\right) \cdot \cancel{\frac{T}{2}}\right]$$

$$\boxed{\omega = \Omega T}$$

which is linear

Design an IIR filter that satisfies :-

$A/D \rightarrow H(z) \rightarrow A/D$

(i) Monotonic PB & SB

(ii) PB gain of 3dB @ 500Hz ($\rightarrow 0.707$)

(iii) SB attenuation 15 dB @ 750 Hz ($\rightarrow 0.177$)

(iv) Sampling 2000 sample/sec.

$\because$ its sample/s its freq.

So, $F = 2000$; $T = 1/2000$

ans) Prewrapping of Analog filter as its BLT

$$\Omega^*_p = \frac{2}{T} \tan\left(\frac{\Omega_p T}{2}\right)$$

$$\Rightarrow \frac{2}{\frac{1}{2000}} \tan\left(\frac{1000\pi \times \frac{1}{2000}}{2}\right) \Rightarrow \underline{\underline{4000 \text{ r/s}}}$$

$$\Omega^*_s = \frac{2}{\frac{1}{2000}}\left[\tan\left(\frac{1500\pi \times \frac{1}{2000}}{2}\right)\right] = \underline{\underline{9656.85 \text{ r/s}}}$$

$$d = \sqrt{\frac{(1-\delta_p)^{-2}-1}{\delta_s^{-2}-1}} = \sqrt{\frac{(0.707)^{-2}-1}{(0.177)^{-2}-1}}$$

$$\Rightarrow \underline{\underline{0.179}}$$

$$k = \frac{\Omega^*p}{\Omega^*s} = \underline{\underline{0.414}}$$

$$n = \frac{\log\left(1/d\right)}{\log\left(1/k\right)} \Rightarrow \underline{\underline{2}}$$

$$H_n(s) = \frac{1}{B_n(s)} = \frac{1}{s^2+\sqrt{2}\,s+1}$$

$\because$ 3dB; $\Omega^*_p = \Omega^*_c$

$$H_a(s) = \frac{1}{\left(\frac{s}{4000}\right)^2 + \sqrt{2}\left(\frac{s}{4000}\right)+1}$$

$$\Rightarrow \frac{16\times10^6}{s^2+5656.85s+16\times10^6}$$

$$\Rightarrow \frac{16\times10^6}{s^2+5656.85\,s+16\times10^6}$$

$\therefore$ for BLT;

Replace $s \to \frac{2}{T}\left(\frac{z-1}{z+1}\right)$

$$\Rightarrow \frac{16\times10^6}{\left(\frac{2}{T}\left(\frac{z-1}{z+1}\right)\right)^2 + 5656.56\left(\frac{2}{T}\left(\frac{z-a}{z+a}\right)\right)+16\times10^6}$$

$$T = \frac{1}{2000}$$

$$H(z) = \frac{(4000)^2}{(4000)^2\left(\frac{z-1}{z+1}\right) + \sqrt{2}\times(4000)^2\left(\frac{z-1}{z+1}\right)+4000}$$

$$\Rightarrow \frac{(1+z^{-1})^2}{(1+z^{-1})^2+\sqrt{2}\,(1-z^{-1})(1+z^{-1})+(1+z^{-1})^2}$$

1) Design IIR digital filter to satisfy the following specifications using BLT
(i) Monotonic pass band and stop band
(ii) 3 dB cutoff frequency at 500 Hz
(iii) 15 dB stop band attenuation at 750 Hz
(iv) Sampling rate of 2000 samples/sec.

$= (1-\delta_p)\,dB = -3$, $\quad \delta_s\,dB = -15$

$20\log(1-\delta_p) = -3$ $\qquad 20\log\delta_s = -15$

$\log(1-\delta_p) = \frac{-3}{20}$ $\qquad \log\delta_s = -\frac{15}{20}$

$(1-\delta_p) = 10^{-3/20}$ $\qquad \delta_s = 10^{-15/20}$

$(1-\delta_p) = 0.707$ $\qquad \delta_s = 0.17$

$f_p = 500\,Hz \rightarrow \Omega_p = 2\pi f_p = 1000\pi$ rad/sec

$f_s = 750\,Hz \rightarrow \Omega_s = 2\pi f_s = 1500\pi$ rad/sec.

$F = 2000$ samples/sec $\longrightarrow T = \frac{1}{F} = \frac{1}{2000}$ sec.

(i) Prewarping:

$$\Omega_p^* = \frac{2}{T}\tan\left(\frac{\Omega_p T}{2}\right) = 4000\tan\left(\frac{1000\pi}{4000}\right)$$

$$\Omega_p^* = 4000 \text{ rad/sec}$$

$$\Omega_s^* = \frac{2}{T}\tan\left(\frac{\Omega_s T}{2}\right) = 4000\tan\left(\frac{1500\pi}{4000}\right)$$

$$\Omega_s^* = 9658.8 \text{ rad/sec}$$

(ii) Design of Analog Butterworth Filter:-

$n = ?$, $S_K = ?$, $\Omega_c = ?$, $\Omega_c^* = ?$

$H_n(s) = ?$, $H_a(s) = ?$

$$n = \frac{\log\left(\frac{1}{d}\right)}{\log\left(\frac{1}{k}\right)}, \quad d = \sqrt{\frac{(1-\delta_p)^{-2}-1}{\delta_s^{-2}-1}}$$

$$k = \frac{\Omega_p^*}{\Omega_s^*} = \frac{4000}{9658.8} = 0.4141 \qquad = \sqrt{\frac{(0.707)^{-2}-1}{(0.17)^{-2}-1}} = \sqrt{\frac{1}{33.6}} = 0.1725$$

$$n = \frac{\log(5.797)}{\log(2.4148)}$$

$$n = \frac{0.7632}{0.38288} = 1.99$$

$$\boxed{n \simeq 2}$$

$$\Omega_c^* = \Omega_p^* = 4000 \; \cancel{\text{samples}} \text{ rad/sec.}$$

$$H_n(s) = \frac{1}{s^2+\sqrt{2}s+1} \quad \text{(From Tables)}$$

$$H_a(s) = H_n(s)\Big|_{s \to \frac{s}{4000}}$$

$$H_a(s) = \frac{(4000)^2}{s^2+\sqrt{2}\times 4000\, s+(4000)^2}$$

(iii) <u>Transformation of Analog filter into Digital filter using BLT</u>

$$H(z) = H_a(s)\Big|_{s \to \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right] = 4000\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$$

$$H(z) = \frac{(4000)^2}{\left[4000\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right]^2 + \sqrt{2}\times 4000 \times 4000\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + (4000)^2}$$

$$= \frac{1 \times (1+z^{-1})^2}{(1-z^{-1})^2 + \sqrt{2}\,(1-z^{-1})(1+z^{-1}) + (1+z^{-1})^2}$$

$$H(z) = \frac{1 + 2z^{-1} + z^{-2}}{3.41 + 0.59\,z^{-2}}$$

→ Divide Nr. & Dr by 3.41 for realization of filter structure

$$H(z) = \frac{0.29 + 0.58\,z^{-1} + 0.29\,z^{-2}}{1 + 0.17\,z^{-2}}$$

2) Design IIR digital Butterworth filter to satisfy the following specifications using BLT with sampling frequency 10 KHZ

$$-3 \le |H(\Omega)|\, dB \le 0 \quad , \quad 0 \le f \le 1\text{ KHZ}$$

$$|H(\Omega)| \le 10 \quad , \quad f > 2\text{ KHZ}$$

$= \quad (1-\delta_p)\,dB = -3 \quad , \quad \delta_s\, dB = -10$

$(1-\delta_p) = 0.707 \qquad 20\log\delta_s = -10$

$\log\delta_s = \frac{-10}{20}$

$\delta_s = 10^{-10/20} = 0.316$

$f_p = 1000\text{ HZ} \rightarrow \Omega_p = 2\pi f_p = 2000\pi$ rad/sec

$f_s = 2000\text{ HZ} \rightarrow \Omega_s = 2\pi f_s = 4000\pi$ rad/sec.

$F = 10\,000$ HZ

$T = \frac{1}{F} = \frac{1}{10\,000}$ sec.

(i) Prewarping Analog frequencies:-

$$\Omega_p^* = \frac{2}{T}\tan\left(\frac{\Omega_p T}{2}\right) = 6498 \text{ rad/sec}$$

$$\Omega_s^* = \frac{2}{T}\tan\left(\frac{\Omega_s T}{2}\right) = 14531 \text{ rad/sec}$$

(ii) Design of Analog Butterworth Filter

$n = ?\ ,\ \Omega_c^* = ?\ ,\ H_n(s) = ?\ \ H_a(s) = ?$

$$n = \frac{\log\left(\frac{1}{d}\right)}{\log\left(\frac{1}{k}\right)} = 1.3 \qquad d = \sqrt{\frac{(1-\delta_p)^{-2}-1}{\delta_s^{-2}-1}} = 0.33$$

$$\boxed{n = 2} \qquad k = \frac{\Omega_p^*}{\Omega_s^*} = 0.45$$

$$\Omega_c^* = \Omega_p^* = 6498 \text{ rad/sec}$$

$$H_n(s) = \frac{1}{s^2+\sqrt{2}s+1} \quad \text{(from Table)}$$

$$H_d(s) = H_n(s)\Big|_{s \to \frac{s}{6498}}$$

$$H_d(s) = \frac{(6498)^2}{s^2+\sqrt{2}\times 6498\, s + (6498)^2}$$

(iii) Transfor of $H_d(s)$ to $H(z)$:-

$$H(z) = H_d(s)\Big|_{s \to \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]} = 20000\left[\frac{1-z^{-1}}{1+z^{-1}}\right]$$

$$H(z) = \frac{(6498)^2(1+z^{-1})^2}{(20000)^2(1-z^{-1})^2+\sqrt{2}\times 6498\times 20000\,(1-z^{-1})(1+z^{-1}) + (1+z^{-1})^2(6498)^2}$$

$$= \frac{(1+z^{-1})^2}{9.47(1-z^{-1})^2 + 4.35(1-z^{-1})(1+z^{-1}) + (1-z^{-1})^2}$$

$$= \frac{1+2z^{-1}+z^{-2}}{6.13z^{-2} - 16.44z^{-1} + 14.81} \longleftarrow * (=1)$$

$$H(z) = \frac{0.0676 + 0.1352 z^{-1} + 0.0676 z^{-2}}{1 + 1.44 z^{-1} + 0.414 z^{-2}}$$

3. Design IIR Butterworth filter using Bilinear Transform to meet the following specifications.

$$0.8 \le |H(\Omega)| \le 1, \quad 0 \le \Omega \le 0.2\pi$$

$$|H(\Omega)| \le 0.2, \quad 0.6\pi \le \Omega$$

$\Rightarrow$ $(1-\delta_p) = 0.8$ $\qquad \Omega_p = 0.2\pi$ rad/sec.

$\delta_s = 0.2$ $\qquad \Omega_s = 0.6\pi$ rad/sec

Take $T = 1$ Sec.

(i) Prewarping Analog Frequencies :-

$$\Omega_p^* = \frac{2}{T}\tan\left(\frac{\Omega_p T}{2}\right) = 0.65 \text{ rad/sec}$$

$$\Omega_s^* = \frac{2}{T}\tan\left(\frac{\Omega_s T}{2}\right) = 2.75 \text{ rad/sec}$$

(ii) Design of analog Butterworth Filter :-

$$n = \frac{\log\left(\frac{1}{d}\right)}{\log\left(\frac{1}{K}\right)} = 1.3 \simeq 2$$

$$d = \sqrt{\frac{(1-\delta_p)^{-2} - 1}{\delta_s^{-2} - 1}}$$

$$K = \frac{\Omega_p^*}{\Omega_s^*}$$

$$\Omega_{c_1}^* = \frac{\Omega_p^*}{\left[(1-\delta_p)^{-2} - 1\right]^{1/2n}} = 0.75 \text{ rad/sec}$$

$$\Omega^*_{c_2} = \frac{\Omega^*_s}{\left[\delta_s^{-2} - 1\right]^{1/2n}} = 1.24 \text{ rad/sec.}$$

$$\Omega^*_c = \frac{\Omega^*_{c_1} + \Omega^*_{c_2}}{2} = \frac{0.75 + 1.24}{2} \simeq 1 \text{ rad/sec.}$$

$$H_n(S) = \frac{1}{S^2 + \sqrt{2}\,S + 1} \quad \text{(From Tables)}$$

$$H_d(S) = H_n(S)\Big|_{S \to \frac{S}{\Omega^*_c} = \frac{S}{1}}$$

$$H_d(S) = \frac{1}{S^2 + \sqrt{2}\,S + 1}$$

(iii) Convert $H_d(S)$ to $H(z)$:—

$$H(z) = H_d(S)\Big|_{S = \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right] = 2\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$$

$$H(z) = \frac{1}{\left(2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right)^2 + \sqrt{2} \times 2\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 1}$$

$$= \frac{(1+z^{-1})^2}{4(1-z^{-1})^2 + 2\sqrt{2}(1-z^{-1})(1+z^{-1}) + (1+z^{-1})^2}$$

$$= \frac{1 + 2z^{-1} + z^{-2}}{7.82 - 6z^{-1} + 2.18z^{-2}}$$

* Convert into 1 for realization

$$H(z) = \frac{0.128 + 0.256z^{-1} + 0.128z^{-2}}{1 - 0.767z^{-1} + 0.279z^{-2}}$$

4 ~~Design IIR filter that used in using Bilinear A/D – H(z) – D/A structure in Transformation for the following specification~~

(4) (a) Design an IIR digital filter that used in the pre-filter A/D – H(z) – A/D structure will satisfy the following analog specifications

(i) Low pass filter with 1 dB cutoff at 100 π rad/sec

(ii) stop band attenuation of 35 dB or greater at 1000 π rad/sec

(iii) Monotonic pass band and stop band

(iv) Sampling rate of 2000 Samples/sec.

(b) Verify the design.

Study this Qn

(c) Give the difference equation of the digital filter

= (a) $(1-\delta_p) = -1\,dB$ $\Omega_p = 100\pi$ rad/sec

$20\log(1-\delta_p) = -1$

$(1-\delta_p) = 0.892$

$\delta_s = -35\,dB$ $\Omega_s = 1000\pi$ rad/sec

$20\log(\delta_s) = -35$

$\delta_s = 0.0177$

$F = 2000$ Samples/sec

$\therefore T = \frac{1}{F} = \frac{1}{2000}$ sec.

(i) Prewarping Analog Frequencies :-

$$\Omega_p^* = \frac{2}{T} \tan\left(\frac{\Omega_p T}{2}\right) = 314.65 \text{ rad/sec}$$

$$\Omega_s^* = \frac{2}{T} \tan\left(\frac{\Omega_s T}{2}\right) = 3996.8 \text{ rad/sec.}$$

(ii) Design of Analog Butterworth filter

$$n = \frac{\log\left(\frac{1}{d}\right)}{\log\left(\frac{1}{K}\right)} = 1.85 \simeq 2$$

$$d = \sqrt{\frac{(1-\delta_p)^{-2} - 1}{\delta_s^{-2} - 1}}$$

$$K = \frac{\Omega_p^*}{\Omega_s^*} =$$

$$\Omega_{c_1}^* = \frac{\Omega_p^*}{\left[(1-\delta_p)^{-2} - 1\right]^{1/2n}} = 441.03 \text{ rad/sec}$$

$$\Omega_{c_2}^* = \frac{\Omega_s^*}{\left[\delta_s^{-2} - 1\right]^{1/2n}} = 531.78 \text{ rad/sec}$$

$$\Omega_c^* = \frac{\Omega_{c_1}^* + \Omega_{c_2}^*}{2} = 486.41 \text{ rad/sec}$$

$$H_n(s) = \frac{1}{s^2 + \sqrt{2}s + 1}$$ for $n = 2$ (From Table)

$$H_d(s) = H_n(s)\Big|_{s \to \frac{s}{\Omega_c^*} = \frac{s}{486.41}}$$

$$H_d(s) = \frac{1}{\left(\frac{s}{486.41}\right)^2 + \sqrt{2}\left(\frac{s}{486.41}\right) + 1}$$

$$H_d(s) = \frac{(486.41)^2}{s^2 + \sqrt{2} \times 486.41\, s + (486.41)^2}$$

(iii) Analog to Digital filter conversion :-

$$H(z) = H_d(s)\Big|_{s \to \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right] = 4000\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$$

$$H(z) = \frac{(486.41)^2}{\left[4000\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right]^2 + \sqrt{2} \times 486.41 \times 4000\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + (486.4)^2}$$

$$H(z) = \frac{0.015 + 0.029 z^{-1} + 0.015 z^{-2}}{1.187 - 1.97 z^{-1} + 0.84 z^{-2}} \quad \text{--- (1)}$$

(b) Verification :—

convert $H(z)$ into $H(e^{j\omega})$ and draw the frequency response. At $\omega = 0$, the maximum gain of the filter is 1.

Substitute $Z = e^{jw}$ in equation (1)

$$H(e^{jw}) = \frac{0.015 + 0.029\, e^{-jw} + 0.015\, e^{-j2w}}{1.187 - 1.97\, e^{-jw} + 0.84\, e^{-j2w}}$$

Substitute $w = 0$

$H(e^{jw}) = 1.04 \simeq 1$ Hence verified

(c) Difference equation realization :-

Substitute $H(z) = \frac{Y(z)}{X(z)}$ in equation (1)

$$\frac{Y(z)}{X(z)} = \frac{0.015 + 0.029\, z^{-1} + 0.015\, z^{-2}}{1.187 - 1.97\, z^{-1} + 0.84\, z^{-2}}$$

Cross multiply

$$1.187\, Y(z) - 1.97\, z^{-1} Y(z) + 0.84\, z^{-2} Y(z)$$
$$= 0.015\, X(z) + 0.029\, z^{-1} X(z) + 0.015\, z^{-2} X(z)$$

Take IZT

$$1.187\, y(n) - 1.97\, y(n-1) + 0.84\, y(n-2)$$
$$= 0.015\, x(n) + 0.029\, x(n-1) + 0.015\, x(n-2)$$

5 (a) Design IIR filter using Billinear transform to meet the following digital Low pass filter specifications

(i) Monotonic pass band and stop band

(ii) $-3.01\,dB$ cut-off at 0.5π rad

(iii) stopband attenuation of atleast $15\,dB$ at 0.75π rad

(b) Verify the design using passband and stopband specifications

(c) Give the difference equation

= (a) $(1-\delta_p) = -3.01\,dB$, $\omega_p = 0.5\pi$ rad

$20\log(1-\delta_p) = -3.01$

$(1-\delta_p) = 0.707$

$\delta_s = -15\,dB$ $\omega_s = 0.75\pi$ rad

$20\log(\delta_s) = -15$

$\delta_s = 0.178$

~~(i)~~ Let $T = 1$ sec, $\omega_p = \Omega_p T$

$\omega_s = \Omega_s T$

~~$\Omega_p = \omega_p$~~

~~$\Omega_s = \omega_s$~~

(i) Prewarp analog frequencies

$$\Omega_p^* = \frac{2}{T}\tan\left(\frac{\Omega_p T}{2}\right) = 2 \text{ rad/sec}$$

$$\Omega_s^* = \frac{2}{T}\tan\left(\frac{\Omega_s T}{2}\right) = 4.83 \text{ rad/sec.}$$

(ii) Design of Analog Butterworth Filter

$$n = \frac{\log\left(\frac{1}{d}\right)}{\log\left(\frac{1}{k}\right)} = 1.94 \approx 2$$

$$d = \sqrt{\frac{(1-\delta_p)^{-2}-1}{\delta_s^{-2}-1}} =$$

$$k = \frac{\Omega_p^*}{\Omega_s^*} =$$

$$H_n(s) = \frac{1}{s^2+\sqrt{2}\,s+1}$$

$\Omega_c^* = \Omega_p^*$ ∴ the corresponding gain is -3.01 dB

$= 2$ rad/sec

$$H_a(s) = H_n(s)\Big|_{s \to \frac{s}{\Omega_c^*} = \frac{s}{2}}$$

$$= \frac{1}{\left(\frac{s}{2}\right)^2+\sqrt{2}\times\frac{s}{2}+1}$$

$$= \frac{4}{s^2+2\sqrt{2}\,s+4}$$

(iii) Analog to Digital Filter conversion :-

$$H(z) = H_a(s)\Big|_{s \to \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right] = 2\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$$

$$H(z) = \frac{4}{\left[2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right]^2 + 2\sqrt{2} \times 2\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 4}$$

$$= \frac{(1+z^{-1})^2}{(1-z^{-1})^2 + \sqrt{2}(1-z^{-1})(1+z^{-1}) + 4(1+z^{-1})^2} \quad \text{—①}$$

$$H(z) = \frac{1 + 2z^{-1} + z^{-2}}{3.41 + 0.58 z^{-2}} \quad \text{—②}$$

(b) Verification :—

For Frequency response Substitute

~~w = e^{jw}~~ $z = e^{jw}$ in equation ①

$$H(e^{jw}) = \frac{\cancel{1 + 2e^{-jw} + e^{-j2w}}\ (1 - e^{-jw})^2}{3.41 + 0.58 e^{-j2w}}$$

Convert into magnitude response

$$H(e^{jw}) = \frac{\cancel{1 - e^{jw}}\ [1 + (\cos w - j\sin w)]^2\ \cancel{1 + 2(\cos w - j\sin w) + (\cos 2w - j\sin 2w)}}{3.41 + 0.58(\cos 2w - j\sin 2w)}$$

$$= \frac{[(1+\cos w) - j\sin w]^2\ \cancel{1 + 2\cos w - j\sin w + \cos 2w - j\sin 2w}}{3.41 + 0.58\cos 2w - j\,0.58\sin 2w}$$

$$H(e^{jw}) = \frac{\cancel{1 + 2\cos w}\ [(1+\cos w) - j\sin w]^2}{(3.41 + 0.58\cos 2w) - j\,0.58\sin 2w}$$

$|a+jb| = \sqrt{a^2+b^2}$

$$|H(e^{j\omega})| = \frac{\left[\sqrt{(1+\cos\omega)^2+(\sin\omega)^2}\right]^2}{\sqrt{(3.41+0.58\cos 2\omega)^2+(0.58\sin 2\omega)^2}}$$

$$|H(e^{j\omega})| = \frac{(1+\cos\omega)^2+(\sin\omega)^2}{\sqrt{(3.41+0.58\cos 2\omega)^2+(0.58\sin 2\omega)^2}} \quad \text{--- (3)}$$

Passband verification

at $\omega = 0.5\pi$ gain is -3.01 dB (Given data)

Substitute $\omega = 0.5\pi$ in Equation (3)

$|H(e^{j\omega})| \simeq -3.01$ dB

Stopband verification

at $\omega = 0.75\pi$, gain is -15 dB

∴ Substitute $\omega = 0.75\pi$ in Equation (3)

$|H(e^{j\omega})| = -15.4$ dB

Hence verified

6. Desig IIR filter using Bilinear Transform to meet the following specification

(i) Monotonic PB and SB

(ii) PB gain of 3 dB at 0.5 π rad

(iii) SB gain of 10 dB at 0.70483 π rad

$$(1-\delta_p) = -3\,dB \qquad \omega_p = 0.5\pi \text{ rad}$$

$$20\log(1-\delta_p) = -3$$

$$(1-\delta_p) = 0.707$$

$$\delta_s = -10\,dB \qquad \omega_s = 0.70483\pi \text{ rad}$$

$$20\log\delta_s = -10$$

$$\delta_s = 0.316 \qquad \text{let } T = 1 \text{ Sec}$$

$$\boxed{\Omega_p T = \omega}$$

(i) Prewarping Analog Frequencies

$$\Omega_p^* = \frac{2}{T}\tan\left(\frac{\Omega_p T}{2}\right)$$

$$\Omega_p^* = 2\tan\left(\frac{\omega_p}{2}\right) = 2 \text{ rad/sec}$$

$$\Omega_s^* = \frac{2}{T}\tan\left(\frac{\Omega_s T}{2}\right)$$

$$= \frac{2}{T}\tan\left(\frac{\omega_s}{2}\right) = 4 \text{ rad/sec}$$

(ii) Design of Analog Butterworth Filter

$$n = \frac{\log\left(\frac{1}{d}\right)}{\log\left(\frac{1}{k}\right)} = 2$$

$$d = \sqrt{\frac{(1-\delta_p)^{-2}-1}{\delta_s^{-2}-1}} = \qquad k = \frac{\Omega_p^*}{\Omega_s^*} =$$

~~(iii) Analog to Digital Filter conversion~~

$$H_n(s) = \frac{1}{s^2+\sqrt{2}s+1}, \quad \Omega_c^* = \Omega_p^* = 2 \text{ rad/sec.}$$

$$H_d(s) = H_n(s)\Big|_{s \to \frac{s}{\Omega_c^*} = \frac{s}{2}}$$

$$= \frac{1}{\left(\frac{s}{2}\right)^2 + \sqrt{2}\left(\frac{s}{2}\right) + 1}$$

$$H_d(s) = \frac{4}{s^2+2\sqrt{2}s+4}$$

(iii) Analog to Digital Filter conversion :-

$$H(z) = H_d(s)\Big|_{s \to \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right] = 2\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$$

$$H(z) = \frac{4}{\left[2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right]^2 + 2\sqrt{2}\times 2\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 4}$$

$$H(z) = \frac{1+2z^{-1}+z^{-2}}{3.41+0.58z^{-2}}$$

(*) convert into 1 for realization

$$H(z) = \frac{0.29+0.58z^{-1}+0.29z^{-2}}{1+0.17z^{-2}}$$

7) Design IIR Butterworth filter using BLT to meet the following specifications

(i) PB gain of 2dB at 0.033π rad

(ii) SB attenuation of 20dB at 0.2π rad

= $(1-\delta_p) = -2dB$

$20 \log(1-\delta_p) = -2$

$\log(1-\delta_p) = \frac{-2}{20}$

$(1-\delta_p) = 10^{-2/20}$

$(1-\delta_p) = 0.79$

$\omega_p = 0.33\pi$ rad

$\delta_s = -20 dB$

$20 \log \delta_s = -20$

$\log \delta_s = \frac{-20}{20} = -1$

$\delta_s = 10^{-1} = 0.1$

$\omega_s = 0.2\pi$ rad.

let $T = 1$ sec.

$\omega = \Omega T$

(i) Prewarping Analog frequency:-

$$\Omega_p^* = \frac{2}{T} \tan\left(\frac{\Omega_p T}{2}\right) = 2 \tan\left(\frac{\omega_p}{2}\right)$$

$$= 0.10 \text{ rad/sec}$$

$$\Omega_s^* = \frac{2}{T} \tan\left(\frac{\Omega_s T}{2}\right) = 2 \tan\left(\frac{\omega_s}{2}\right)$$

$$= 0.65 \text{ rad/sec}.$$

(ii) Design of Analog Butterworth Filter:-

$$n = \frac{\log\left(\frac{1}{d}\right)}{\log\left(\frac{1}{k}\right)} = 1.37 \simeq 2$$

$$H_n(s) = \frac{1}{s^2+\sqrt{2}s+1}$$

$$\Omega^*_{c_1} = \frac{\Omega^*_p}{\left[(1-\delta_p)^{-2}-1\right]^{1/2n}} = 0.1136 \text{ rad/sec.}$$

$$\Omega^*_{c_2} = \frac{\Omega^*_s}{\left(\delta_s^{-2}-1\right)^{1/2n}} = \cancel{0.16}\ 0.206 \text{ rad/sec}$$

$$\therefore \Omega^*_c = \frac{\Omega^*_{c_1}+\Omega^*_{c_2}}{2} = 0.16 \text{ rad/sec.}$$

$$H_a(s) = H_n(s)\Big|_{s \to \frac{s}{\Omega^*_c} = \frac{s}{0.16}}$$

$$= \frac{1}{\left(\frac{s}{0.16}\right)^2+\sqrt{2}\left(\frac{s}{0.16}\right)+1}$$

$$= \frac{(0.16)^2}{s^2+\sqrt{2}\times 0.16\, s+(0.16)^2}$$

$$H_a(s) = \frac{0.026}{s^2+0.23s+0.026}$$

(iii) Conversion of Analog to Digital Filter

$$H(z) = H_a(s)\Big|_{s \to \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right] = 2\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$$

$$= \frac{0.026}{\left[2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right]^2+0.23\times 2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+0.026}$$

$$H(z) = \frac{0.026\,(1+z^{-1})^2}{4(1-z^{-1})^2 + 0.46\,(1-z^{-1})(1+z^{-1}) + 0.026(1+z^{-1})^2}$$

$$= \frac{0.026(1+2z^{-1}+z^{-2})}{4(1+z^{-2}-2z^{-1}) + 0.46\,(1-z^{-2}) + 0.026(1+2z^{-1}+z^{-2})}$$

$$= \frac{0.026+0.052z^{-1}+0.026z^{-2}}{4+4z^{-2}-8z^{-1}+0.46-0.46z^{-2}+0.026 + 0.052z^{-1}+0.026z^{-2}}$$

$$H(z) = \frac{0.026+0.052z^{-1}+0.026z^{-2}}{4.49-7.95z^{-1}+3.57z^{-2}}$$

(8) Determine H(z) using BLT ~~foor~~ for the following digital specifications

(i) Monotonic PB & SB

(ii) 3 dB PB gain at 0.2π rad

(iii) 25 dB SB attenuation at 0.45π rad

$= (1-\delta_p) = -3\,dB$ $\qquad \omega_p = 0.2\pi$ rad

$20\log(1-\delta_p) = -3$

$(1-\delta_p) = 0.707$

$\delta_s = -25\,dB$ $\qquad \omega_s = 0.45\pi$ rad

$20\log\delta_s = -25$

$\delta_s = 0.056$ $\qquad * \; \omega = \Omega T$

let T = 1 sec.

~~(i) Design of Analog Butterworth Filter~~

~~$n = \frac{\log(\frac{1}{d})}{\log(\frac{1}{k})} = 3.55 \simeq 4$~~

(i). Prewarping Analog Frequencies

$$\Omega_p^* = \frac{2}{T}\tan\left(\frac{\Omega_p T}{2}\right) = 2\tan\left(\frac{\omega_p}{2}\right) = 0.65 \text{ rad/sec}$$

$$\Omega_s^* = \frac{2}{T}\tan\left(\frac{\Omega_s T}{2}\right) = 2\tan\left(\frac{\omega_s}{2}\right) = 1.71 \text{ rad/sec}$$

(ii) Design of Analog Butterworth Filter :-

$$n = \frac{\log(\frac{1}{d})}{\log(\frac{1}{k})} = 3.55 \simeq 4$$

$$\Omega_{c1}^* = \frac{\Omega_p^*}{\left[(1-\delta_p)^{-2} - 1\right]^{1/2n}}$$

$$= \frac{0.65}{\left[(0.707)^{-2} - 1\right]^{1/8}} = 0.65 \text{ rad/sec}$$

$$\Omega_{c2}^* = \frac{\Omega_s^*}{\left(\delta_s^{-2} - 1\right)^{1/2n}}$$

$$= \frac{1.71}{\left[0.056^{-2} - 1\right]^{1/8}} = 0.83 \text{ rad/sec}$$

$$\Omega_c^* = \frac{\Omega_{c1}^* + \Omega_{c2}^*}{2} = 0.74 \text{ rad/sec}$$

$$H_d(s) = H_n(s)\Big|_{s \to \frac{s}{\Omega_c^*} = \frac{s}{0.74}}$$

$$H_n(s) = \frac{1}{(s^2 + 0.77s + 1)(s^2 + 1.85s + 1)}$$

$$H_d(s) = H_n(s)\Big|_{s \to \frac{s}{0.74}}$$

$$H_d(s) = \frac{1}{\left[\left(\frac{s}{0.74}\right)^2 + 0.77 \times \frac{s}{0.74} + 1\right]\left[\left(\frac{s}{0.74}\right)^2 + 1.85 \times \frac{s}{0.74} + 1\right]}$$

(10) Design digital LPF using BLT. The filter is required to have a PB ripple satisfying the following analog specifications

$-1 \leq |H(\Omega)| dB \leq 0 \quad , \quad 0 \leq \Omega \leq 1404\pi$ r/s

$|H(\Omega)| \leq 60 dB \quad , \quad \Omega \geq 8268\pi$ r/s

sampling interval is 10^{-4} sec

$=$ $(1-\delta_p) = -1 dB$ $\qquad \Omega_p = 1404\pi$ r/s

$20 \log(1-\delta_p) = -1$

$(1-\delta_p) = 0.8912$

$\delta_s \, dB = -60$ $\qquad \Omega_s = 8268\pi$ r/s

$20 \log \delta_s = -60$

$\delta_s = 10^{-3}$

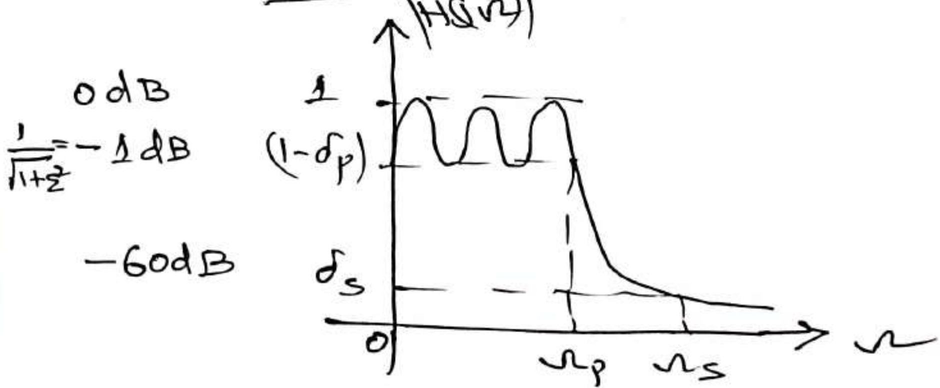
$T = 10^{-4}$ sec.

(i) Prewarp Analog Frequencies :-

$$\Omega_p^* = \frac{2}{T} \tan\left(\frac{\Omega_p T}{2}\right) = 4484 \text{ rad/sec.}$$

$$\Omega_s^* = \frac{2}{T} \tan\left(\frac{\Omega_s T}{2}\right) = 71690 \text{ rad/sec}$$

(ii) Design of Analog Chebyshev Filter :-



$$n = \frac{\cosh^{-1}\left(\frac{1}{d}\right)}{\cosh^{-1}\left(\frac{1}{k}\right)} = 2.39 \simeq 3$$

$$d = \sqrt{\frac{(1-\delta_p)^{-2}-1}{\delta_s^{-2}-1}} =$$

$$k = \frac{\Omega_p^*}{\Omega_s^*} =$$

Refer 1 dB chebyshev Filter Table

$$\varepsilon = 0.5088$$

$$H_n(s) = \frac{K_n}{(s+0.49)(s^2+0.49s+0.994)}$$

$K_n = b_0$ $\because$ n is odd

$= 0.49$ from Table.

$$\therefore H_n(s) = \frac{0.49}{(s+0.49)(s^2+0.49s+1)}$$

For $\Omega_p^* = 4484$ rad/se

$$H_d(s) = H_n(s)\Big|_{s \to \frac{s}{4484}}$$

$$= \frac{0.49}{\left(\frac{s}{4484}+0.49\right)\left(\left(\frac{s}{4484}\right)^2+0.49\left(\frac{s}{4484}\right)+1\right)}$$

$$= \frac{4.4\times10^{10}}{(s+2216)(s^2+2217s+19985562)}$$

(iii) Analog to Digital filter conversion :-

$$H(z) = H_a(s)\Big|_{s \to \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$$

$$= \frac{4.29 \times 10^{-3}(1+z^{-1})(1+2z^{-1}+z^{-2})}{(1-0.8z^{-1})(1-1.64z^{-1}+0.81z^{-2})}$$

(11) Design an IIR filter H(z) that used as prefilter in a A/D – H(z) – D/A structure will satisfy the following analog chebyshev prototype design

(i) LPF with 2 dB cutoff at 100Hz
(ii) SB attenuation of 20dB at 500Hz
(iii) sampling rate 4000 samples/sec
(iv) obtain difference equation realization.

= $(1-\delta_p) = -2\,dB$ $\qquad f_p = 100\,Hz$

$20\log(1-\delta_p) = -2$ $\qquad \Omega_p = 2\pi f_p$

$\log(1-\delta_p) = -\frac{2}{20}$ $\qquad = 2\pi \times 100$

$\qquad = 200\pi$ rad/sec.

$(1-\delta_p) = 10^{-2/20}$

$(1-\delta_p) =$

$\delta_s = -20\,dB$ $\qquad f_s = 500\,Hz$

$20\log\delta_s = -20$ $\qquad \Omega_s = 2\pi f_s$

$\qquad = 2\pi \times 500$

$\log\delta_s = -1$ $\qquad = 1000\pi$ rad/sec.

$\delta_s = 10^{-1}$ $\qquad F = 4000$ samples/sec

$\delta = 0.1$ $\qquad T = \frac{1}{F} = \frac{1}{4000}$ sec.

(i) Prewarp Analog Frequencies :-

$\Omega_p^* = \frac{2}{T}\tan\left(\frac{\Omega_p T}{2}\right) = 630$ rad/sec

$\Omega_s^* = \frac{2}{T}\tan\left(\frac{\Omega_s T}{2}\right) = 3314$ rad/sec.

(ii) Design of Analog chebyshev filter

$$n = \frac{\cosh^{-1}\left(\frac{1}{d}\right)}{\cosh^{-1}\left(\frac{1}{K}\right)} = 1.38 \simeq 2$$

$$d = \sqrt{\frac{(1-\delta_p)^{-2}-1}{\delta_s^{-2}-1}}$$

$$K = \frac{\Omega_p^*}{\Omega_s^*}$$

Refer 2dB ripple chebyshev Filter Table

For $n = 2 \longrightarrow \varepsilon = 0.76$

$$H_n(s) = \frac{K_n}{s^2 + 0.8s + 0.64}$$

*

∴ of ~~Ksn~~ $K_n = \frac{b_0}{\sqrt{1+\varepsilon^2}} = 0.51$

n is even

$b_0 = 0.64$

$$\therefore H_n(s) = \frac{0.51}{s^2 + 0.8s + 0.64}$$

for $\Omega_p^* = 1$ rad/sec.

For $\Omega_p^* = 630$ rad/sec

$$H_a(s) = H_n(s)\Big|_{s \to \frac{s}{630}}$$

$$H_d(s) = \frac{0.51}{\left[\left(\frac{s}{630}\right)^2 + 0.8\left(\frac{s}{630}\right) + 0.64\right]}$$

$$H_d(s) = \frac{202419}{s^2 + 504s + 254016}$$

(iii) Analog to Digital Filter Conversion :-

$$H(z) = H_d(s)\Big|_{s \to \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right] = 4000\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$$

$$= \frac{202419}{\left(4000\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right)^2 + 504 \times 4000\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 254016}$$

$$= \frac{202419(1+z^{-1})^2}{(4000)^2(1-z^{-1})^2 + 504 \times 4000(1-z^{-1})(1+z^{-1}) + 254016(1+z^{-1})^2}$$

$$= \frac{202419(1+2z^{-1}+z^{-2})}{(4000)^2(1-2z^{-1}+z^{-2}) + 504 \times 4000(1+\cancel{z^{-1}}-\cancel{z^{-1}}-z^{-2}) + 254016(1+2z^{-1}+z^{-2})}$$

$$= \frac{202419(1+2z^{-1}+z^{-2})}{(4000)^2(1-2z^{-1}+z^{-2}) + 504 \times 4000(1-z^{-2}) + 254016(1+2z^{-1}+z^{-2})}$$

$$H(z) = \frac{1+2z^{-1}+z^{-2}}{90.26 - 155.58z^{-1} + 70.34z^{-2}}$$

* Divide by 90.26

Difference Equation realization

$$H(z) = \frac{0.01 + 0.02 z^{-1} + 0.01 z^{-2}}{1 - 1.72 z^{-1} + 0.78 z^{-2}}$$

$$\frac{Y(z)}{X(z)} = \frac{0.01 + 0.02 z^{-1} + 0.01 z^{-2}}{1 - 1.72 z^{-1} + 0.78 z^{-2}}$$

Cross multiply

$$Y(z) - 1.72 z^{-1} Y(z) + 0.78 z^{-2} Y(z)$$
$$= 0.01 X(z) + 0.02 z^{-1} X(z) + 0.01 z^{-2} X(z)$$

Take IZT

$$y(n) - 1.72 y(n-1) + 0.78 y(n-2)$$
$$= 0.01 \overset{x(n)}{\cancel{X(z)}} + 0.02 x(n-1) + 0.01 x(n-2)$$

(12) (a) Design Low pass digital filter that will operate on sample analog data such that analog cut-off frequency is 200Hz at 1dB acceptable ripple and stop band attenuation of 20dB at 400Hz with monotonic shape. The sampling rate is 2000 samples/sec Use Bilinear Transformation method

(b) For the designed filter in part (a) plot $20\log_{10}(H(e^{j\omega}))$

= (a) $(1-\delta p) = -1\,dB$

$20\log(1-\delta p) = -1$

$\log(1-\delta p) = -\frac{1}{20}$

$(1-\delta p) = 10^{-1/20} =$

$f_p = 200\ Hz$

$\Omega_p = 2\pi f_p$

$= 2\pi \times 200$

$= 400\pi$ rad/sec.

$\delta_s\,dB = -20$

$20\log\delta_s = -20$

$\log\delta_s = -1$

$\delta_s = 10^{-1} = 0.1$

$f_s = 400\ Hz$

$\Omega_s = 2\pi f_s$

$\Omega_s = 800\pi$ rad/sec.

$F = 2000$ samples/sec, $T = \frac{1}{F} = \frac{1}{2000}$ sec

(i) Prewarp analog frequencies

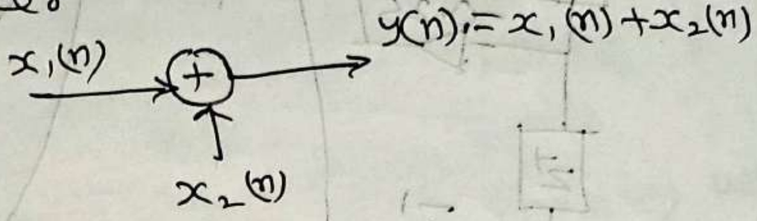
$\Omega_p^* = \frac{2}{T}\tan\left(\frac{\Omega_p T}{2}\right) =$

$\Omega_s^* = \frac{2}{T}\tan\left(\frac{\Omega_s T}{2}\right) =$

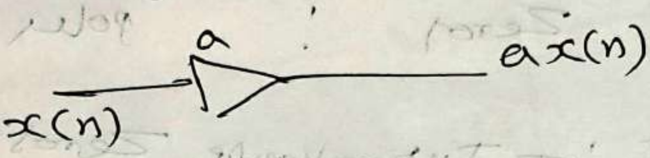
IIR Filter Structures

Components required

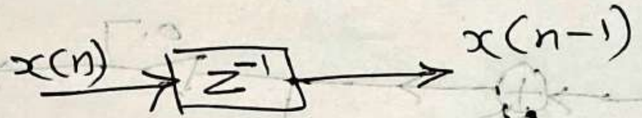
1. Adder



2. Multiplier



3. Unit Delay



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1. Realize the function

$$H(z) = \frac{0.7 - 0.25z^{-1} - z^{-2}}{1 + 0.1z^{-1} - 0.72z^{-2}}$$

Using Direct Form I, Direct Form II, Cascade form and parallel form

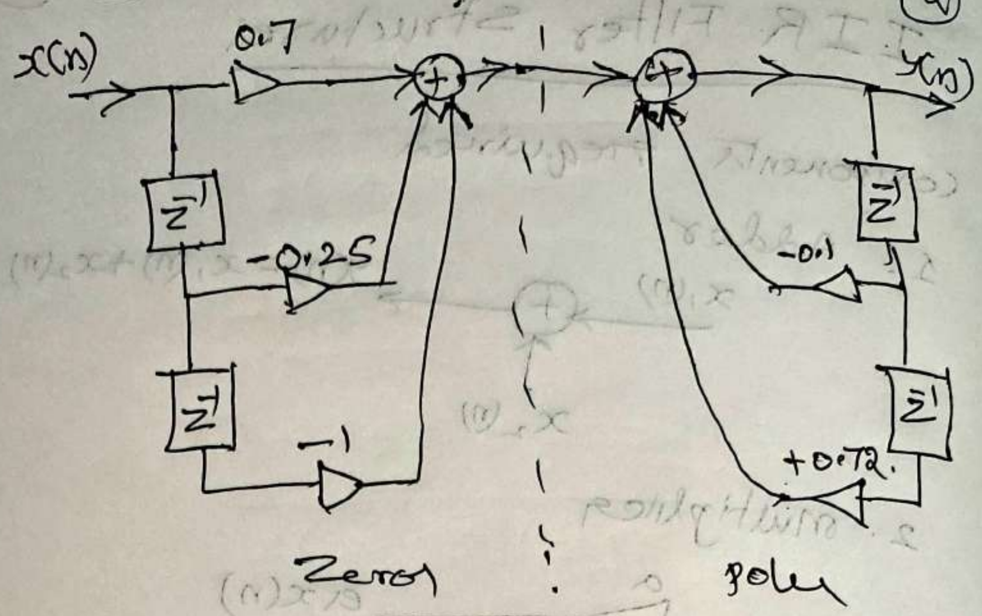
$$= \frac{Y(z)}{X(z)} = \frac{0.7 - 0.25z^{-1} - z^{-2}}{1 + 0.1z^{-1} - 0.72z^{-2}}$$

$$Y(z) + 0.1z^{-1}Y(z) - 0.72z^{-2}Y(z)$$
$$= 0.7X(z) - 0.25z^{-1}X(z) - z^{-2}X(z)$$

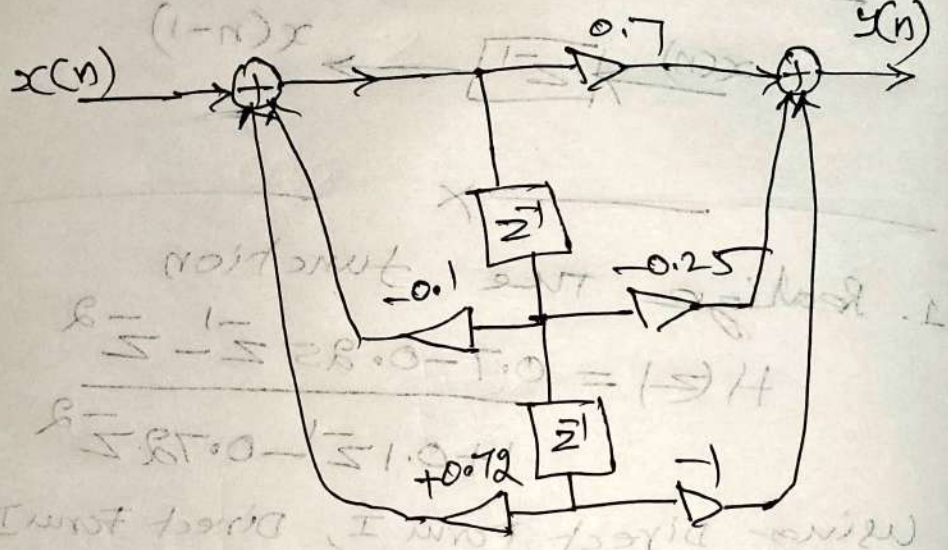
Take IZT

$$y(n) = 0.7x(n) - 0.25x(n-1) - x(n-2)$$
$$- 0.1y(n-1) + 0.72y(n-2)$$

(1) DF I

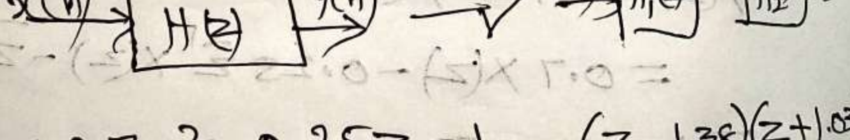


DF II :- Interchange Zeros & poles in the network



Cascade Form :-

$$H(z) = H_1(z) \cdot H_2(z) \cdot \ldots$$



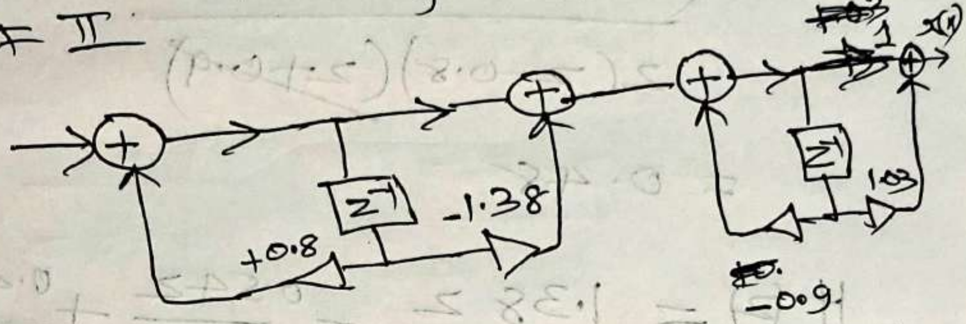
$$H(z) = \frac{0.7z^2 - 0.25z - 1}{z^2 + 0.1z - 0.72} = \frac{(z-1.38)(z+1.03)}{(z-0.8)(z+0.9)}$$

$$H(z) = \left[\frac{z-1.38}{z-0.8}\right]\left[\frac{z+1.03}{z+0.9}\right]$$

$$H(z) = \left[\frac{1-1.38z^{-1}}{1-0.8z^{-1}}\right]\left[\frac{1+1.03z^{-1}}{1+0.9z^{-1}}\right]$$

$\qquad\qquad H_1(z) \qquad\qquad\qquad H_2(z)$

write structures of $H_1(z)$ & $H_2(z)$ using DF II



$H_1(z)$ $\qquad\qquad\qquad\qquad$ $H_2(z)$

Parallel

$$H(z) = H_1(z) + H_2(z) + \cdots$$

$$H(z) = \frac{0.7z^2 - 0.25z - 1}{(z-0.8)(z+0.9)}$$

Do PFE

$$\frac{H(z)}{z} = \frac{0.7z^2 - 0.25z - 1}{z(z-0.8)(z+0.9)}$$

$$= \frac{A}{z} + \frac{B}{z-0.8} + \frac{C}{z+0.9}.$$

$$A = \left.\frac{(0.7z^2 - 0.25z - 1)\cancel{z}}{\cancel{z}(z-0.8)(z+0.9)}\right|_{z=0}$$

$$= \frac{-1}{(-0.8)(0.9)} = \frac{1}{0.72} = 1.38$$

4

$$B = \frac{(0.72z^2 - 0.25z - 1)\cancel{(z-0.8)}}{z\cancel{(z-0.8)}(z+0.9)}\Bigg|_{z=0.8}$$

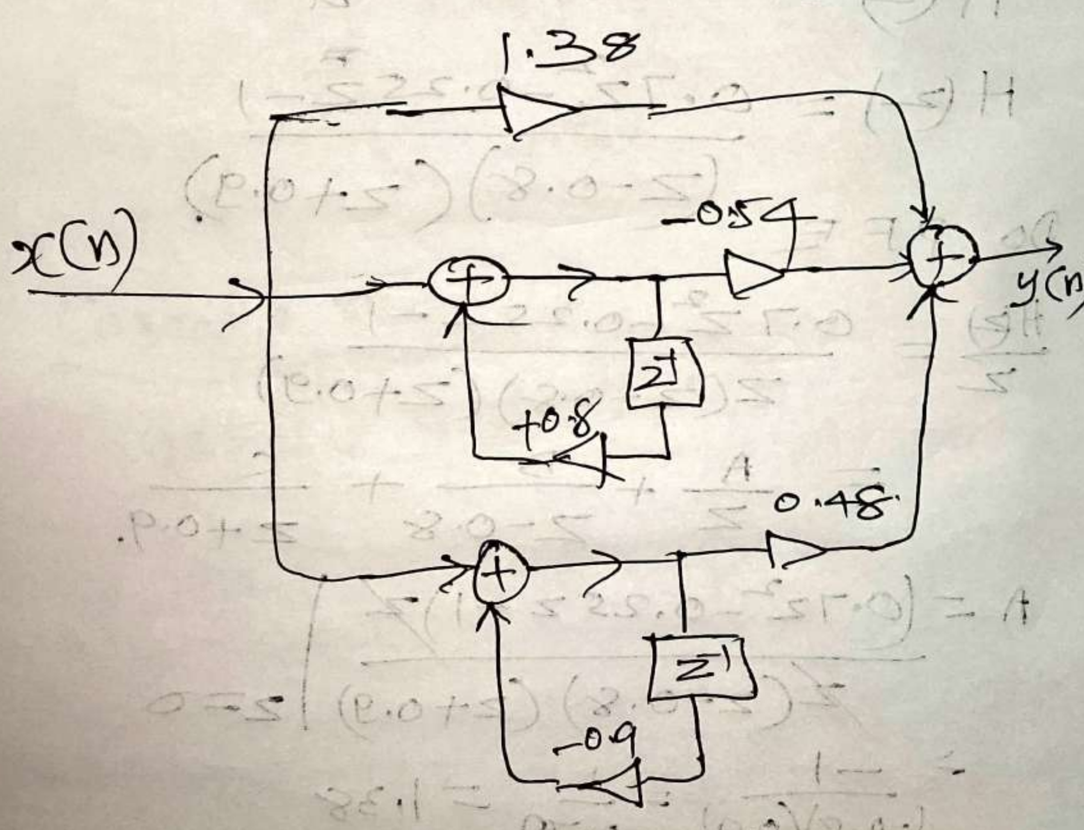
$$= -0.54$$

$$C = \frac{(0.72z^2 - 0.25z - 1)\cancel{(z+0.9)}}{z(z-0.8)\cancel{(z+0.9)}}\Bigg|_{z=-0.9}$$

$$= 0.48$$

$$H(z) = 1.38\frac{z}{z} - \frac{0.54z}{z-0.8} + \frac{0.48z}{z+0.9}$$

$$= 1.38 - \frac{0.54}{1-0.8z^{-1}} + \frac{0.48}{1+0.9z^{-1}}$$



(5)

(2) Realize $H(z) = \dfrac{3z^2 + 3.6z + 0.6}{z^2 + 0.1z - 0.2}$

using DF I, DF II, cascade & parallel form

$$= H(z) = \frac{3 + 3.6z^{-1} + 0.6z^{-2}}{1 + 0.1z^{-1} - 0.2z^{-2}}$$