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$\int \log x \, dx = x\log x - x + C$ $\int \log x \, dx = x\log x - x + C$

By definition of SHM;

$F_{restoring} \propto -x$ ; $x \rightarrow$ displacement.

$F_{rest} = -kx \Rightarrow (1)$ ; $F_{restoring} = F_{rest} =$ restoring force acting on oscilator

By Newton's $2^{nd}$ law

$F = ma \Rightarrow (2)$

from (1) & (2);

$$ma = -kx$$

$$m\frac{d^2x}{dt^2} + kx = 0 \quad \left(\because a = \frac{d^2x}{dt^2}\right) \Rightarrow (3)$$

Eq$^n$ (3) $\div$ m;

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \Rightarrow (4)$$

Assume $\frac{k}{m} = \omega^2 \Rightarrow (5)$

$$\Rightarrow \omega = \sqrt{\frac{k}{m}} \quad ; (\omega = \text{Angular freq.})$$

Eq$^n$ (5) in (4);

(*) $$\frac{d^2x}{dt^2} + \omega^2 x = 0 \Rightarrow (6)$$

This is the diff. Eq$^n$ for simple harmonic vibration.

$\therefore$ Soln. of this Eqⁿ will be of the form.

$$x = e^{\alpha t} \quad \Rightarrow (7)$$

where $\alpha \to$ some constant
$x \to$ displacement / Amplitude
$t \to$ time.

differentiating above Eqⁿ we get

$$\frac{dx}{dt} = \alpha e^{\alpha t} \Rightarrow (7)$$

differentiating again:

$$\frac{d^2x}{dt^2} = \alpha^2 e^{\alpha t} \Rightarrow (8)$$

Eqⁿ (7) & (8) in (6);

$$\alpha^2 e^{\alpha t} + \omega^2 e^{\alpha t} = 0$$

$$e^{\alpha t}(\alpha^2 + \omega^2) = 0$$

$\because$ displacement / amplitude $\neq 0$, $e^{\alpha t} \neq 0$

$$\therefore \alpha^2 + \omega^2 = 0$$

$$\alpha^2 = -\omega^2$$

$$\alpha = i\omega \quad \Rightarrow (9)$$

Gen Soln. of Eqⁿ (6);

$$x = A \cdot e^{i\omega t} + Be^{-i\omega t} \quad \Rightarrow (10)$$

where A & B are constants.

Also,

$$e^{i\omega t} = \cos\omega t + i\sin\omega t \quad \Rightarrow (11)$$

$$e^{-i\omega t} = \cos\omega t - i\sin\omega t \quad \Rightarrow (12)$$

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Eq$^n$ (11) & (12) in (10);

$$x = A(\cos\omega t + i\sin\omega t) + B(\cos\omega t - i\sin\omega t)$$

$$x = A\cos\omega t + Ai\sin\omega t + B\cos\omega t - Bi\sin\omega t$$

$$x = \cos\omega t(A+B) + i\sin\omega t(A-B)$$

Replacing $A+B$ with $R\sin\phi$
$i(A-B)$ with $R\cos\phi$

$$x = R\sin\phi\cos\omega t + \sin\omega t\, R\cos\phi$$

(*) $$x = R\sin(\omega t + \phi)$$

This is the final Eq$^n$ of SHM. where,
$R$ = Amplitude of Oscillatory System
$\phi + \omega t$ = Phase of vibrat$^n$
$x = R\sin(\omega t + \phi)$, represents displacement of SHM

Calculation of time ~~perea~~ period;

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{k}{m}}}$$

Calculat$^n$ of frequency.

$$f = \frac{1}{T} = \frac{\sqrt{k/m}}{2\pi}$$

If $t$ is increased by $\frac{2\pi}{\omega}$

$$t = t + \frac{2\pi}{\omega}$$

$$x = R\sin\left(\omega\left(t+\frac{2\pi}{\omega}\right)+\phi\right)$$

$$x = R\sin(\omega t + 2\pi + \phi)$$

$$x = R\sin(2\pi + (\omega t + \phi))$$

$$\Rightarrow x = R\sin(\omega t + \phi)$$

$\Rightarrow$ Displacement remains same even after $t$ is increased by $\frac{2\pi}{\omega}$.

ii) <u>Damped Oscillation</u> / <u>vibration</u> — For a free oscillation the energy remains constant and hence oscillator continues indefinitely.

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February 2005

Notes

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But in reality, the amplitude of the oscillatory system gradually decreases due to the damping forces like friction (air) and resistance of the media.

"The oscillator whose amplitude, in successive oscillations goes on decreasing due to the presence of resistive forces are called

25 tue

as damped oscillators and oscillations are called damped oscillations (DHO)."

The damping force always act in a opposite directions to the motion of oscillatory body and velocity dependent.

# Damped Oscillation

$F_{damped} \propto -v$

$F_{damp} = -\gamma v$ $\Rightarrow$ ① ; $\gamma \to$ damping constant

; $v \Rightarrow$ damping force velocity

$F_{damp} \Rightarrow$ damping force

$F_{restoring} = -kx$ $\Rightarrow$ ②

from Newton's 2nd law,

$F_{net} = ma$ $\Rightarrow$ ③

from ①, ② & ③;

$F_{net} = F_{restoring} + F_{damping}$

$ma = -\gamma v - kx$

$$m\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + kx = 0$$

÷ by $m$;

$$\frac{d^2x}{dt^2} + \frac{\gamma}{m}\frac{dx}{dt} + \frac{k}{m}x = 0 \Rightarrow ④$$

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Assume, $\frac{k}{m} = \omega^2$ ; $\omega \Rightarrow$ Angular freq. $\Rightarrow$ (5)

$\frac{\nu}{m} = 2\beta$ ; $2\beta \rightarrow$ damping coefficient $\Rightarrow$ (6)

Eq (5) & (6) in (4)

(*) $$\Rightarrow \frac{d^2x}{dt^2} + 2\beta \frac{dx}{dt} + \omega^2 x = 0 \Rightarrow (7)$$

This is the diff. Eqn for damped oscillator

The Gen soln. of the above Eqn;

$x = Ae^{\alpha t}$ $\Rightarrow$ (8) ; $x \Rightarrow$ disp. / Amplitude.

differentiating above Eqn;

$$\frac{dx}{dt} = A\alpha e^{\alpha t} \Rightarrow (9)$$

differentiate again;

$$\frac{d^2x}{dt^2} = A\alpha^2 e^{\alpha t} \Rightarrow (10)$$

from 7, 9, 10 ;

$$A\alpha^2 e^{\alpha t} + 2\beta A\alpha e^{\alpha t} + \omega^2 A e^{\alpha t} = 0$$

$$Ae^{\alpha t}(\alpha^2 + 2\beta\alpha + \omega^2) = 0$$

$\because Ae^{\alpha t}$ = displacemnt & $\because$ disp. $\neq 0$

$\Rightarrow Ae^{\alpha t} \neq 0$

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$\Rightarrow \alpha^2 + 2\beta\alpha + \omega^2 = 0$

Two roots of the Eq (iii);

$$\alpha = \frac{-2\beta \pm \sqrt{4\beta^2 - 4(1)(\omega^2)}}{2(1)}$$

$$\alpha = \frac{-2\beta \pm \sqrt{4(\beta^2 - \omega^2)}}{2}$$

$$\alpha_1 = +\frac{2}{2}\left(-\beta + \sqrt{\beta^2 - \omega^2}\right) = \underline{\underline{-\beta + \sqrt{\beta^2 - \omega^2}}}$$

$$\alpha_2 = \frac{2}{2}\left(-\beta - \sqrt{\beta^2 - \omega^2}\right) = \underline{\underline{-\beta - \sqrt{\beta^2 - \omega^2}}}$$

$\therefore$ Gen Soln. of Eq (iii) f (i);

$$x = Ae^{\alpha_1 t} + Be^{\alpha_2 t}$$

$$* \quad x = Ae^{(-\beta + \sqrt{\beta^2 - \omega^2})t} + Be^{(-\beta - \sqrt{\beta^2 - \omega^2})t} \Rightarrow \text{(ii)}$$

$$\cancel{x = Ae^{-\beta t} + Ae^{(\sqrt{\beta^2 - \omega^2})t} + Be^{\beta t} + Be^{-\sqrt{\beta^2 - \omega^2}t}}$$

Actual form of Eq (ii) depends on wheather

$k^2 > \omega^2 \rightarrow$ heavy damping

$k^2 < \omega^2 \rightarrow$ Low "

$k^2 = \omega^2 \rightarrow$ Critical "

Case : 1; $\beta^2 > \omega^2$ (heavy damping)

If the damping is high, then $\sqrt{\beta^2 - \omega^2} = c$ is a real quantity of +ve.

$$\Rightarrow x = e^{-\beta t}\left(Ae^{(\sqrt{\beta^2-\omega^2})t} + Be^{(-\sqrt{\beta^2-\omega^2})t}\right)$$

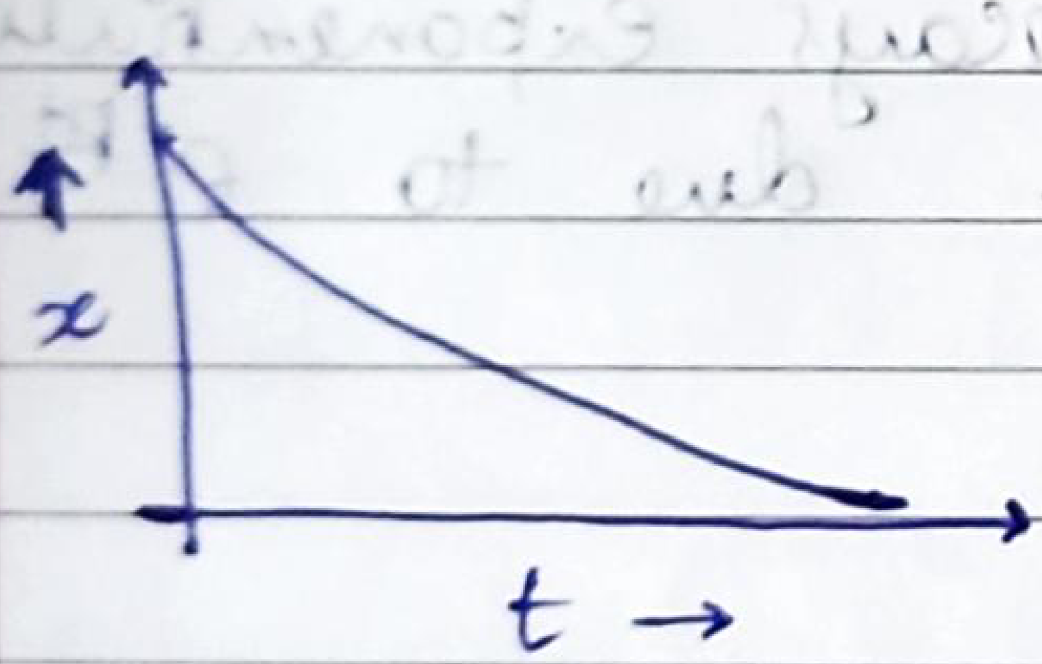
$$\Rightarrow x = e^{-\beta t}\left(Ae^{ct} + Be^{-ct}\right); \quad c = \sqrt{b^2 - \omega^2}$$

$$\Rightarrow x = Ae^{-(\beta - c)t} + Be^{-(c+\beta)t}$$

$\Rightarrow \because b > c$, both the powers are -ve, $\therefore$ disp. $x \downarrow$ continuously with respect to time. So, if a particle is displaced from the mean posi$^n$ it reaches the equilibrium posi$^n$ slowly.

$\Rightarrow c^2 = b^2 - \omega^2$

$\Rightarrow b > c$




This is Dead Beat

Non Oscillatory / Periodic motion.

Ex: Pendulum in viscious medium

Case : II : $\beta^2 < \omega^2$ (Under damping)

$\Rightarrow \sqrt{\beta^2 - \omega^2}$ is imaginary.

So, $\sqrt{\beta^2 - \omega^2} = \sqrt{-(\omega^2 - \beta^2)} = i\sqrt{\omega^2 - \beta^2}$

Let $\sqrt{\omega^2 - \beta^2} = \omega_1$

$\Rightarrow \sqrt{\beta^2 \cdot \omega^2} = i\omega_1 \quad \Rightarrow (12)$

$Eq^n$ (12) in (11)

$$x = e^{-\beta t}\left(Ae^{i\omega_1 t} + Be^{-i\omega_1 t}\right)$$

$$x = e^{-\beta t}\left(A(\cos\omega_1 t + i\sin\omega_1 t) + B(\cos\omega_1 t - i\sin\omega_1 t)\right)$$

$$x = e^{-\beta t}\left((A+B)\cos\omega t + i(A-B)\sin\omega t\right)$$

Assume;

$\Rightarrow A+B = a_0 \sin\phi$

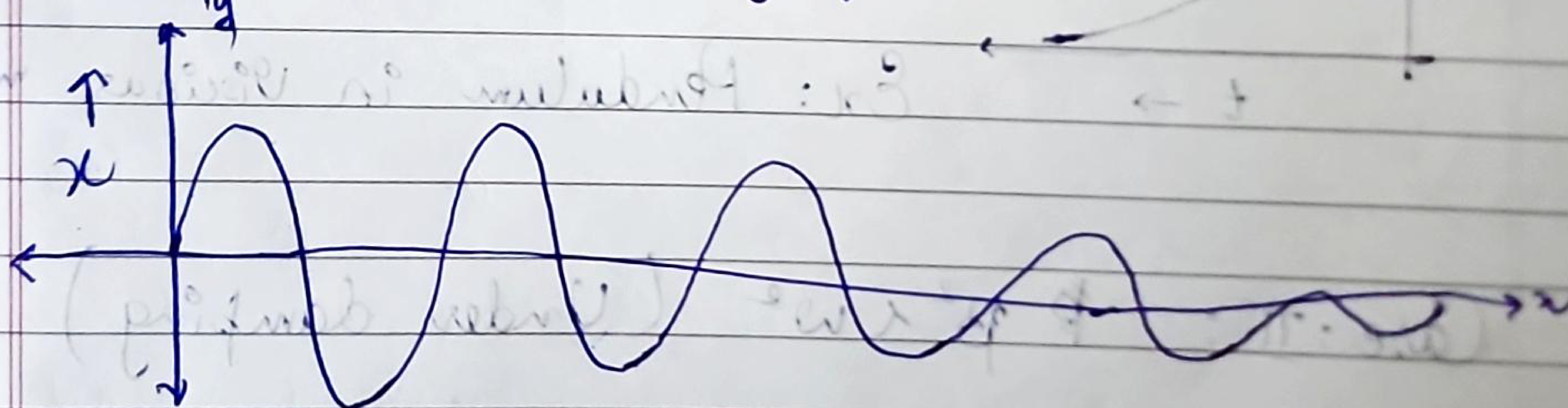
$i(A-B) = a_0 \cos\phi$

$$x = e^{-\beta t}\left(a_0\sin\phi\cos\omega t + a_0\cos\phi\sin\omega t\right)$$

$$x = a_0 e^{-\beta t}\left(\sin(\omega t+\phi)\right) \rightarrow (13)$$

$\Rightarrow$ $\cancel{a_0}$ $a_0 e^{-\beta t}$ $\rightarrow$ Amplitude of oscillation it decays exponentially with time due to $e^{-\beta t}$!

$e^{-\beta t}$ = Damping force

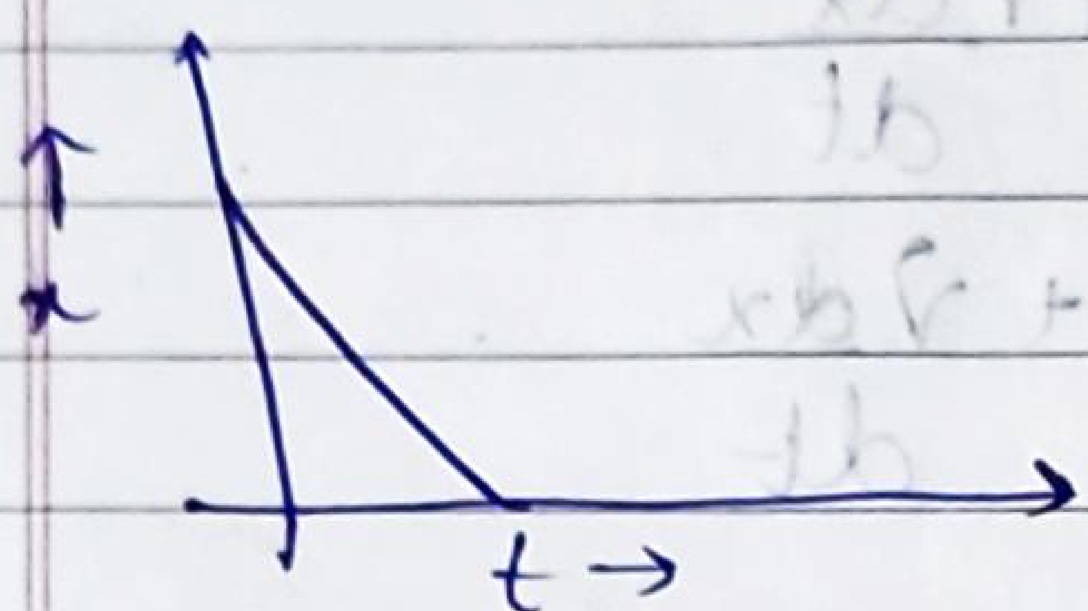



Case: III $\beta^2 = \omega^2$ (critical damping)

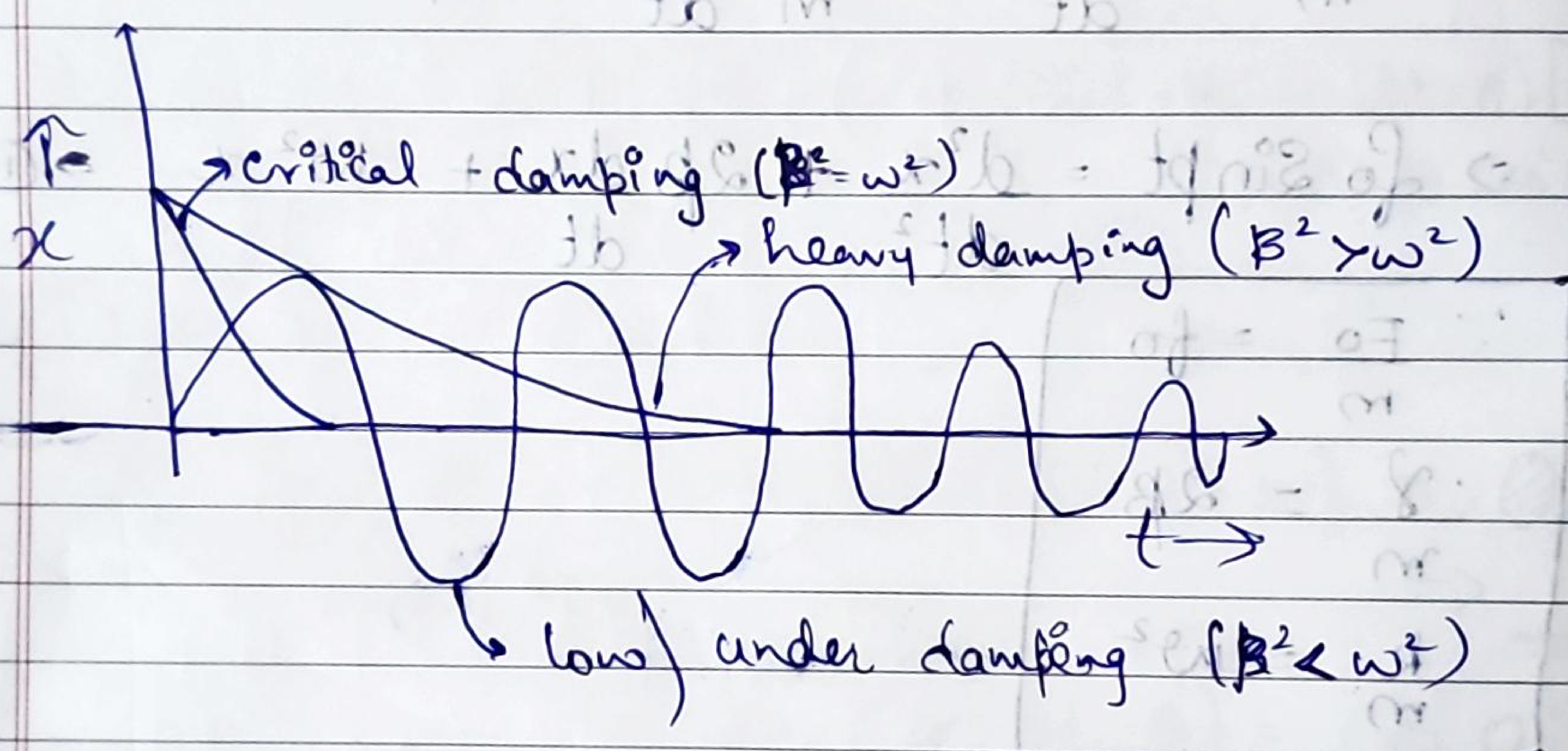
$Eq^n$ (11) will be, $x = (A+B)e^{-\beta t}$

$\therefore$ Disp. approaches zero asymptomatically for given value of initial $pos^n$.

But here the critically damped oscillator approaches to the equilibrium posi$^n$ more rapidly than compared to heavy damping.




Curves of all 3 cases




③ Forced harmonic Oscillator

When oscillator is subjected to external periodic force and it oscillates with the frequency of external periodic force, it is called as a forced harmonic oscillator.

$F_{net} = F_{restoring} + F_{damping} + F_{ext\ periodic\ force}$

$= -kx - \gamma v + F_0 \sin pt$

$$ma = -Kx - \gamma \frac{dx}{dt} + F_0 \sin pt$$

$$\Rightarrow F_0 \sin pt = ma + Kx + \gamma \frac{dx}{dt}$$

$$\Rightarrow F_0 \sin pt = m\frac{d^2x}{dt^2} + Kx + \gamma \frac{dx}{dt}$$

$\div m$

$$\Rightarrow \frac{F_0}{m} \sin pt = \frac{d^2x}{dt^2} + \frac{\gamma}{m}\frac{dx}{dt} + \frac{k}{m}x$$

$$\Rightarrow f_0 \sin pt = \frac{d^2x}{dt^2} + 2\beta \frac{dx}{dt} + \omega^2 x \Rightarrow (1)$$

$$\boxed{\because \frac{F_0}{m} = f_0,\quad \frac{\gamma}{m} = 2\beta,\quad \frac{k}{m} = \omega^2}$$

Let Soln. of Eq. (1) be

$$x = A \sin(pt - \theta) \Rightarrow (2)$$

where p is the driving frequency.

Diff Eq. (2),

$$\frac{dx}{dt} = Ap \cos(pt - \theta) \Rightarrow (3)$$

diff again,

$$\frac{d^2x}{dt^2} = -Ap^2 \sin(pt - \theta) \Rightarrow (4)$$

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Eq ③ & ④ in ①,

$$f_0 \sin pt = \left(-Ap^2 \sin(pt-\theta)\right) + 2\beta\left(Ap\cos(pt-\theta)\right) + \omega^2 A \sin(pt-\theta)$$

$$f_0\left(\sin\left[(pt-\theta)+\theta\right]\right) = -Ap^2\sin(pt-\theta) + 2Ap\beta\cos(pt-\theta) + \omega^2 A \sin(pt-\theta)$$

$\sin(A+B) =$

$$f_0 \sin(pt-\theta)\cos\theta + f_0\cos(pt-\theta)\sin\theta = A\sin(pt-\theta)\left[\omega^2 - p^2\right] + 2Ap\beta\cos(pt-\theta) \Rightarrow ⑤$$

⇒ Case: 1

If $pt - \theta = 0 \Rightarrow \therefore \sin(pt-\theta) = \sin(0) \Rightarrow 0$

& $\cos(pt-\theta) = \cos 0 \Rightarrow 1$.

Eq ⑤ with above assumption.

$$f_0 \cancel{\sin(pt-\theta)}^{0}\cos\theta + f_0\cancel{\cos(pt-\theta)}^{1}\sin\theta = A\cancel{\sin(pt-\theta)}^{0}(\omega^2-p^2) + 2Ap\beta\cancel{\cos(pt-\theta)}^{1}$$

⇒ $f_0 \sin\theta = 2\beta Ap$ ⇒ ⑥ ; when $pt - \theta = 0$

Case: 2: If $pt - \theta = \pi/2$,

$\sin(pt-\theta) = \sin(\pi/2) \Rightarrow 1$

$\cos(pt-\theta) = \cos(\pi/2) = 0$

Eq (5) with above assumption.

$$f_0 \cancel{\sin(pt-\theta)}^{1} \cos\theta + f_0 \sin\theta (\cancel{\cos(pt-\theta)}^{0}) = A(\cancel{\sin(pt-\theta)}^{1})(\omega^2-p^2) + 2Ap\beta(\cancel{\cos(pt-\theta)}^{0})$$

$$f_0 \cos\theta = A(\omega^2-p^2) + 2\beta A p(0)$$

$$\Rightarrow f_0 \cos\theta = A(\omega^2-p^2) \; ; \text{ when } pt-\theta = \pi/2 \Rightarrow (7)$$

Squaring & adding Eq 6 & 7;

$$f_0^2(\sin^2\theta + \cos^2\theta) = A^2(\omega^2-p^2)^2 + 4\beta^2 A^2 p^2$$

$$f_0^2 = A^2((\omega^2-p^2)^2 + 4\beta^2 p^2)$$

$$A^2 = \frac{f_0^2}{(\omega^2-p^2)^2 + 4\beta^2 p^2}$$

$$A = \frac{f_0}{\sqrt{(\omega^2-p^2)^2 + 4\beta^2 p^2}} \Rightarrow (8)$$

Eq (8) in (2);

$$x = \frac{f_0}{\sqrt{(\omega^2-p^2)^2 + 4\beta^2 p^2}} \sin(pt-\theta) \Rightarrow (9)$$

$\therefore$ This is the Gen $Eq^n$ of forced harmonic oscillation

finding the phase of forced vibrat$^n$; ( $\theta$ )

$Eq^n$ (6) $\div$ (7);

$$\frac{f_0 \sin\theta}{f_0 \cos\theta} = \frac{2\beta A p}{A(\omega^2 - p^2)}$$

$$\tan\theta = \frac{2\beta p}{\omega^2 - p^2}$$

$$\theta = \tan^{-1}\left(\frac{2\beta p}{\omega^2 - p^2}\right)$$

$\therefore$ final Gen. $Eq^n$ of FHO.

$$x = \frac{f_0}{\sqrt{(\omega^2 - p^2)^2 + 4\beta^2 p^2}} \sin\left(pt - \left(\tan^{-1}\left(\frac{2\beta p}{\omega^2 - p^2}\right)\right)\right)$$

Optional to write

a) When $\omega >> p$ (Damping is low)
i.e. Driving freq. is very less than natural freq.

$Eq^n$ (8);

$$A = \frac{f_0}{\sqrt{(\omega^2 - p^2)^2 + 4\beta^2 p^2}}$$

$$A = \frac{f_0}{\omega^2}$$

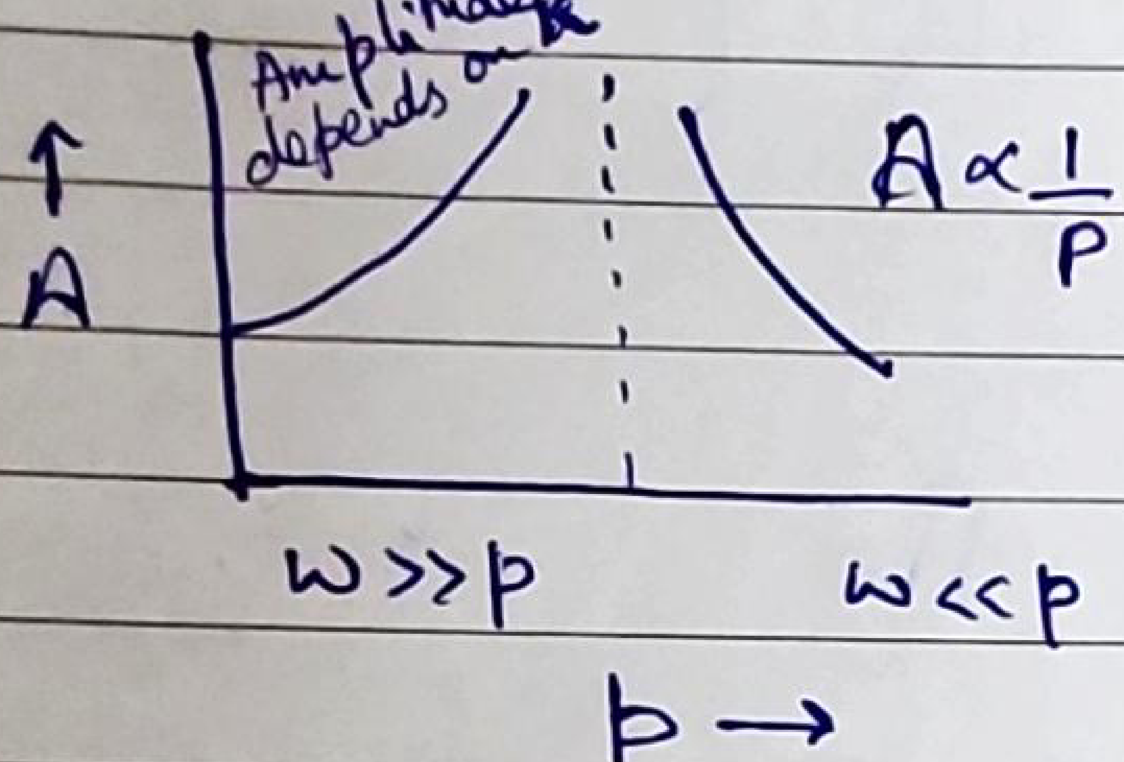
Couldn't understand how!

Case: II ; If $\omega << p$ ; Damping is low

$\Rightarrow$ Eq$^n$ (8)

$$A = \frac{f_0}{p^2}$$

$$A \propto \frac{1}{p^2}$$




Case: III when $\omega = p$ (Resonance Case) :

For resonance - Amplitude is max.

Eq$^n$ (8) = $A = \frac{f_0}{\sqrt{4\beta^2 p^2}} = \frac{f_0}{2\beta p}$ or

$= \frac{f_0}{2\beta\omega}$

## Resonance

$$a = \frac{f_0}{\sqrt{(\omega^2 - p^2) + 4\beta^2 p^2}} \Rightarrow (1)$$

For $a$ to be $a_{max}$

(i) Dr of (1) $\rightarrow$ min

(ii) $\omega = p$

(iii) $\therefore$ finding minima of $D^r$ of Eq$^n$ (1)

$$\frac{d}{dp}\left(4\beta^2p^2 + (\omega^2-p^2)^2\right) = 0$$

$$8\beta^2 p + 2(\omega^2-p^2)(-2p) = 0$$

$$8\beta^2 p - 4(p)(\omega^2-p^2) = 0$$

$\div 4p$

$$2\beta^2 - (\omega^2-p^2) = 0$$

$$2\beta^2 - \omega^2 + p^2 = 0$$

$$p^2 = \omega^2 - 2\beta^2$$

$\because$ we also know $a_{max}$ for $p=\omega$, from above we have $p^2=\omega^2$, iff $\beta$ is verry-verry small.

$$\therefore a_{max} = \frac{f_0}{\sqrt{(\omega^2-\omega^2)^2 + 4\beta^2p^2}}$$

$$\Rightarrow \frac{f_0}{2\beta\omega} \text{ or } \frac{f_0}{2\beta p}$$

## Velocity Resonance.

The displacement of a driven Oscillation is given by

$$x = A\sin(pt-\theta) \Rightarrow ①$$

$$\because A = \frac{f_0}{\sqrt{(\omega^2-p^2)^2 + 4\beta^2p^2}}$$

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$$x = \frac{f_0}{\sqrt{(\omega^2-p^2)^2 4\beta^2 p^2}} \sin(pt-\theta)$$

The velocity of the driven oscillator

$\Rightarrow \frac{dx}{dt} = PA\cos(pt-\theta) \rightarrow$ derivative of Eqⁿ ①

$$v = P \times \frac{f_0}{\sqrt{(\omega^2-p^2)^2 4\beta^2 p^2}} \cos(pt-\theta)$$

$$\Rightarrow v = v_0 \sin\left(\frac{\pi}{2} + (pt-\theta)\right) \Rightarrow ②$$

$$v_0 = \frac{f_0 p}{\sqrt{(\omega^2-p^2)+4\beta^2 p^2}}$$

We know that $\theta = \tan^{-1}\left(\frac{2\beta p}{\omega^2-p^2}\right)$

$v_0$ is the velocity amplitude & it varies with $p$

The velocity attains its max value if driving freq. is equal to natural freq. $(p=\omega)$, we call it as velocity resonance

Light damping
Medium damping
heavy damping

## Sharpness of Resonance

At resonance, the amplitude of Oscillating System becomes max, after that it ↓ from max value

Sharpness of resonance means rate of fall of amplitude with change in forced frequency on either side of resonant freq.

$\therefore$ Change in amplitude $= A_{max} - A$

$$\Rightarrow \frac{f_0}{2\beta \omega} - \frac{f_0}{\sqrt{(\omega^2-p^2)+4\beta^2p^2}}$$

(Assume $\omega^2 - p^2 = 0$)

$$\Rightarrow \frac{f_0}{2\beta\omega} - \frac{f_0}{2\beta p}$$

$$\Rightarrow \frac{f_0}{2\beta}\left(\frac{1}{\omega} - \frac{1}{p}\right)$$

$$\Rightarrow \frac{f_0(p-\omega)}{2\beta\omega p}$$

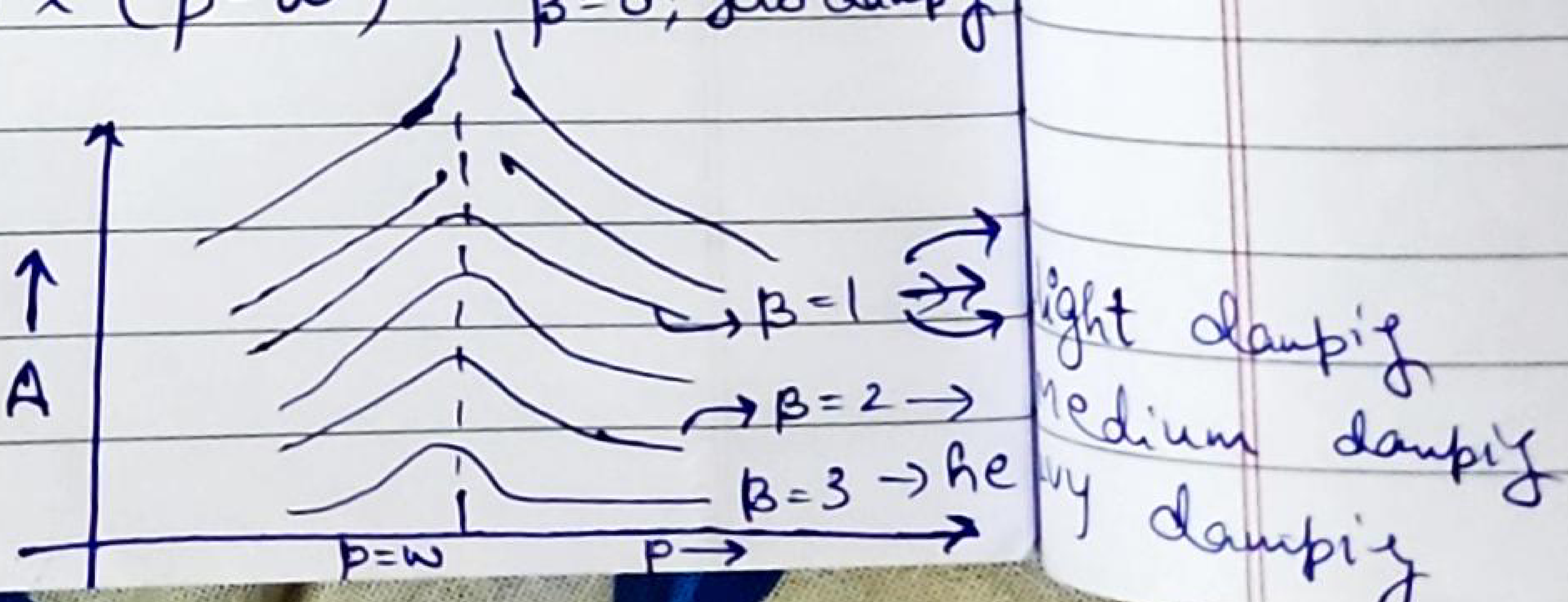
Change in frequency $= p - \omega$

Sharpness of resonance $= \dfrac{\text{Change in amplitude}}{\text{Change in freq.}}$

$$= \frac{f_0(p-\omega)}{2\beta\omega p \times (p-\omega)}$$

Sharpness $\Rightarrow \dfrac{f_0}{2\beta\omega p}$

$\Rightarrow$ Sharpness $\propto \dfrac{1}{\beta}$



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Note: If $\beta \uparrow$, curves are flat
$\therefore$ resonance flat.

If $\beta \downarrow$: curves are sharp.

If $\beta = 0$ A is Infinity

When the damping is low, the amplitude falls very rapidly on either side of Resonant frequency for which the resonance is sharp.

On the other hand, for high damping the amplitude falls of very slowly on either side of resonance.

Resonance is either flat or sharp depending on the damping for the oscillating system is large or small.

Example for flat resonance is observed in the resonance of an air coloumn of a aspirator bottle with tuning fork. Due to its large damping the air column respond with tuning fork over a wide range in the neighbourhood of resonance. Thus in this case, it is actually dificult to predict the exact point of resonance. and hence the resonance is said to flat.

While in the case of sonometer wire, the damping is small and responds only with one particular frequency i.e its own natural frequency, hence the resonance is sharp.

## Quality Factor

The amount of damping is described by the quantity called quality factor.

$$Q = \frac{\omega}{2b}$$

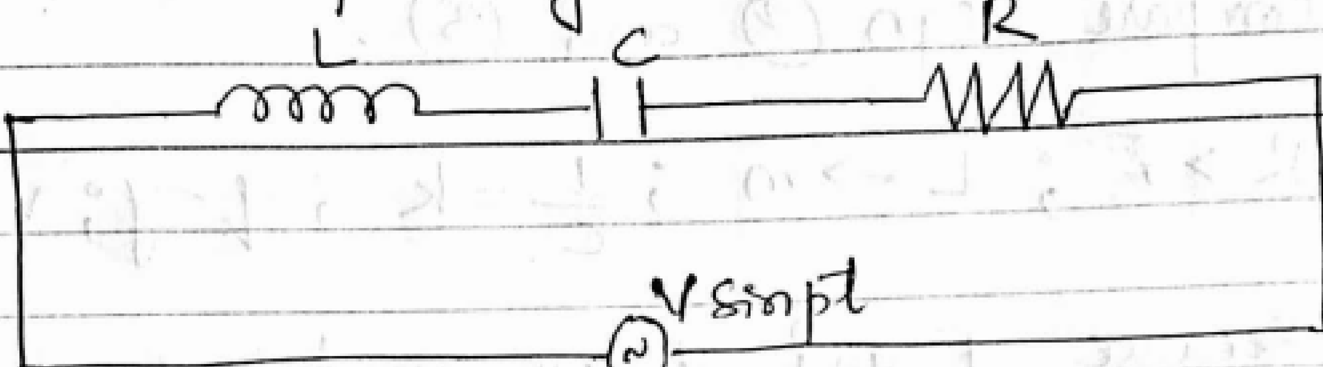
Quality factor is the number of cycles required for the energy to fall off by a factor $e^{2\pi}$ (≈ 535)

If Q value is more indicated the sustained oscillations overcoming the resistive forces.

Quality factor describes how much under damped is the oscillatory system.

# LCR Resonance

Consider a series LCR circuit in which $V \sin pt$ is the applied AC voltage. The voltage drop across the inductor $L$, Resistor $R$ and capacitor $C$ respectively.

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$$V_L = L\frac{di}{dt} \qquad V_R = iR \qquad V_C = \frac{q}{C}.$$

Since the applied voltage must be equal to the sum of the voltage drop across each of the elements.

9 sat

$$V_L + V_R + V_C = V \sin pt \longrightarrow (1).$$

$$L\frac{di}{dt} + iR + \frac{1}{C}q = V \sin pt.$$

But $i = \frac{dq}{dt}$

$$\therefore L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = V \sin pt.$$

Divide above eqn by $L$.

$$\frac{d^2q}{dt^2} + \left(\frac{R}{L}\frac{dq}{dt}\right) + \left(\frac{1}{LC}\right)q = \frac{V}{L}\sin pt$$

$$\longrightarrow (2)$$

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The above eqn resembles the eqn of motion of the vibrating system given by.

$$\frac{d^2y}{dt^2} + \left(\frac{r}{m}\right)\frac{dy}{dt} + \left(\frac{k}{m}\right)y = \frac{F}{m}\sin pt \rightarrow (3).$$

Compare eqn (2) and (3).

12 tue

$R \rightarrow r$ ; $L \rightarrow m$ ; $\frac{1}{C} = k$ ; $q - y$ ; $V \rightarrow F$ ;

Because q and y are analogous quantity.

Thus $\frac{1}{LC} = \frac{K}{m} = \omega^2 \Rightarrow \omega = \sqrt{\frac{K}{m}}$.

13 wed Resonance condition, $p = \omega = \sqrt{\frac{K}{m}}$.

$$\boxed{\frac{1}{LC} = \omega^2}$$

$\therefore$ In LCR circuit at Resonance ; $\boxed{\omega^2 = \frac{1}{LC}}$

★ Numericals

Q A free particle is executing SHM in a straight line period of 25 sec. After it has cross equilibrium point. The velocity is found out to be 0.7 $ms^{-1}$ find the displacement at the end of 10 sec. & also the amplitude of oscillation

$y = ?$ $A = ?$

Data given

period of oscillation, $T = 25$ sec

Velocity, $v = 0.7 ms^{-1}$ at time $t_1 = 5$ sec after crossing the equilibrium

To find:-

Amplitude of oscillation

Displacement ~~of~~ ~~after~~ at time $t_2 = 10s$

$y = \sin \omega t$

Angular freq, $\omega = \frac{2\pi}{25}$

$V = \frac{dy}{dt} = a\omega\cos\omega t$

$0.7 = a\,\frac{2\pi}{25}\cos\left(\frac{2\pi\times 8}{25}\right)$

$\cancel{\frac{0.7\times 25}{2\pi}} = a$

$\frac{0.7\times 25}{2\pi\times 0.309} = a$

$\boxed{a = 9.06\,m}$

Displacement at the end of 10 sec is

$y = 9.06 \times \sin\left(\frac{2\pi}{25}\times 10\right)$

$\cancel{\boxed{y = 0.39\,m}}$ $\boxed{y = 5.33\,m}$

Q Evaluate the resonance frequency of ~~force~~ a spring of force constant of $\boxed{1974\ N/m}$ Carrying a mass of 2 kg.

$k = 1974\ N/m$ ~~2 kg~~ $\therefore m = 2\,kg$

~~f = k~~ $\boxed{\omega = \sqrt{\frac{k}{m}}}$

$2\pi f = \sqrt{\frac{k}{m}}$

$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$\boxed{f = 5\,Hz}$

Q A body of mass 500g is attached to a spring & System is driven by an external ~~period~~ for force of ~~amplitude~~ 15 N & frequency 0.796 Hz. The spring extends by an length of 88mm under the given load. Calculate the amplitude of the oscillation. If the resistance co-efficient of the medium is 5.05 kg/s (Ignore the mass of spring)

Solⁿ:-

$m = 500 \times 10^{-3}$ kg $\quad l = 0.88 \times 10^{-3}$ m

$F = 15$ N $\quad r = 5.05$ kg/s

$f = 0.796$ Hz

$p = 2\pi f$

$$A = \frac{f_0}{\sqrt{(\omega^2 - p^2)^2 + 4\beta^2 p^2}}$$

$f_0 = \frac{f}{m} = \frac{0.796}{0.5}$

$\beta = \frac{r}{2m}$ $r \rightarrow$ damping constan

$2m \rightarrow$ mass

$$f_0 = \frac{f}{m}$$

$p = 2 \times \pi \times 0.796$

$\boxed{p = 5}$

$\omega^2 = \frac{k}{m}$

$k = \frac{F}{x} = \frac{0.5 \times 9.81}{0.088}$

$\boxed{k = 55.74}$

$\omega^2 = \frac{55.74}{0.5}$

$\boxed{\omega^2 = 111.48}$

$\beta = \frac{5.05}{0.5}$

$\beta = 5.05$

$f_0 = \frac{F}{m} = \frac{0.796}{0.5}$

$\boxed{f_0 = 1.592}$

$f_0 = \frac{15}{0.5}$

$f_0 = 30$

$a = \frac{1.592}{\sqrt{(111.48-25) + 4(0.1)^2(25)}}$

$= \frac{1.592}{\sqrt{(111.48-25) + 4(5.05)^2(25)}}$

$= \frac{1.592}{151.349}$

$\boxed{a = 0.03\,m}$

$\boxed{a = 3\text{ cm}}$

$= \frac{30}{51.349}$

$a = \frac{30}{\sqrt{(111.48-25)^2 + 4(5.05)(25)}}$

$= \frac{30}{100}$

$\boxed{a = 0.3}$

Q A Body of mass 1000g is attach to a spring. If the System is driven an external for amplitude 20N & frequency 1 Hz - The spring extends by a length of 100mm. Under the given load. Calculate the amplitude of oscillation If the resistance co-efficient of medium 10kg/s.

Q Calculate the peak amplitude of vibration of body ~~ab~~ whose natural freq is 1000Hz when it oscillates in a resistive medium for which value of damping / unit mass ~~of~~ is 0.008 rad/s. Under the action of external peridic force ~~of~~ per unit mass of amplitude 5N/kg with tunable freq

$\beta = 0.008$ rad/s

$$\boxed{\frac{f}{m} = 5 = f_0}$$

~~p = ω~~ = f = 1000 Hz

$P = 2\pi \, 1000$

$P = 2000\pi$

$P = 6283.18$

$$a = \frac{5}{\sqrt{4(0.008)^2(6283.18)}}$$

$\boxed{a = 1.97m}$

Q. An Automobile of mass 2000kg is supported by 4 equal shock absorbing spring. It executes under damped ossilaton of (which is a bad design). The under damped oscillation

Evaluate force constant of shock absorber & period of oscillation, when 4 persons each of weight 60 kg board the automobile

# Elasticity

Elasticity is one of the general properties of matter. Anybody which is not free to move when acted upon by a suitable force, undergoes a change in form. This change in the form is called deformation. The change could be in shape, size or both of them. If the changes are within limit, the body regains its original shape or size when the system of deforming forces is withdrawn. This property of a substance is called elasticity.

Thus elasticity can be defined as, that property of matter by virtue of which a deformed body returns to its original state after the removal of the deforming force.

## Stress

Force applied per unit area is known as stress.

$$\boxed{\text{Stress} = \frac{\text{Force}}{\text{Area}}}$$

## Strain

The ratio of change in dimension to original dimension is known as strain.

$$\boxed{\text{Strain} = \frac{\text{Change in Dimension}}{\text{Original Dimension}}}$$

## Hooke's Law

Stress is proportional to strain for any material within the elastic limit.

$$\text{Stress} \propto \text{Strain}$$

$$\boxed{\frac{\text{Stress}}{\text{Strain}} = \text{Constant} / \text{Modulus}}$$

## Elastic Moduli

### Young's Modulus (Y)

The ratio of Longitudinal stress to linear strain within the elastic limit is known as Young's Modulus.

$$\text{Young's Modulus} = \frac{\text{Longitudinal Stress}}{\text{Linear Strain}}$$

$$Y = \frac{F/a}{x/L}$$

$$\boxed{Y = \frac{FL}{xa} \ N/m^2}$$

### Bulk Modulus (K)

The ratio of Compressive stress to the volume strain without change in shape of the body within the elastic limits is called the bulk modulus.

$$\text{Bulk Modulus} = \frac{\text{Compressive Stress}}{\text{Volume Strain}}$$

$$K = \frac{F/a}{v/V}$$

$$\boxed{K = \frac{FV}{va} \ N/m^2}$$

### Rigidity Modulus

The ratio of tangential stress to shear strain is known as rigidity modulus.

$$\text{Rigidity Modulus} = \frac{\text{Tangential Stress}}{\text{Shear Strain}}$$

$$\eta = \frac{F/a}{x/L}$$

$$\boxed{\eta = \frac{FL}{xa} \ N/m^2}$$

## poisson's Ratio

within the elastic limits of a body, the ratio of lateral strain to longitudinal strain is a constant called as poisson's ratio.

$$\text{poisson's Ratio} = \frac{\text{Lateral Strain}}{\text{Longitudinal strain}}$$

$$\sigma = \frac{d/D}{x/L}$$

$$\boxed{\sigma = \frac{dL}{xD}}$$

There are no units for poisson's ratio. It is a number & hence a dimensionless quantity.

## Longitudinal Strain Coefficient

The longitudinal strain produced per unit stress is known as longitudinal strain coefficient.

$$\alpha = \frac{\text{Longitudinal Strain}}{\text{unit Stress}}$$

$$\alpha = \frac{x/L}{T}$$

$$\boxed{\alpha = \frac{x}{LT}}$$ where $T \rightarrow$ unit Stress

## Lateral Strain Coefficient

The lateral strain produced per unit stress is called lateral strain coefficient.

$$\beta = \frac{d/D}{T}$$

$$\boxed{\beta = \frac{d}{DT}}$$

$d \rightarrow$ Change in dimension

$D \rightarrow$ original dimension

$T \rightarrow$ unit stress

## Relation b/n $\alpha$, $\beta$ and $\sigma$

Consider the ratio,

$$\beta/\alpha = \frac{d/D\cancel{T}}{x/L\cancel{T}}$$

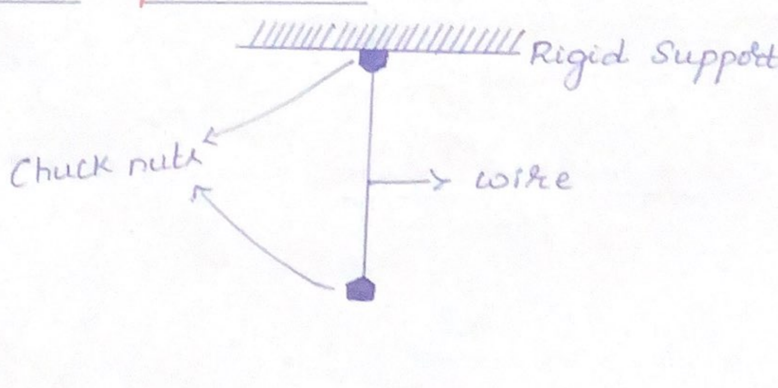
$$= \frac{d/D}{x/L}$$

$$\beta/\alpha = \frac{dL}{xD}$$

WKT $\sigma = \frac{dL}{xD}$

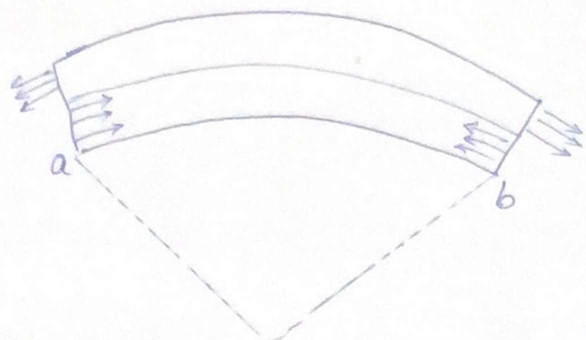
$$\therefore \boxed{\beta/\alpha = \sigma}$$

## Torsional pendulum



## Neutral Surface

when a straight bar is bent as shown in figure, the outer layers get elongated while the inner layers get contracted. However in between there will be a layer which is neither elongated nor contracted. This layer is known as neutral surface.




## Neutral axis

The line of intersection of the neutral surface and the plane of bending is known as neutral axis.

## Types of Beams

There are four types of beams.

a) Simply supported beam: Ends of the beam are made to rest freely.

Ex Girders of railway over bridges

b) Fixed beam: Beam is fixed at both ends.

Ex: Bridges & other structures

c) cantilever beam: Beam is fixed at one end and free at another end.

Ex: Found in big buildings & appartments as balcony.

d) Continiously supported beam: More than two supports are provided to the beam.

Ex: Metro bridges

consider a wire which is clamped vertically with the help of a chuck nut at the top and carrying a uniform body like disc, bar or cylinder at the other end. when the body is given a slight twist and let go, the system executes oscillation in the horizontal plane about the wire as axis. These oscillations are called torsional oscillations and the arrangement is called torsional pendulum.

The expression for time period of torsional oscillations is given by,

$$\boxed{T = 2\pi\sqrt{I/C}}$$

C → Couple per unit twist

I → Moment of Inertia

## Applications

a) The working of torsion pendulum clocks is based on torsional oscillations.

b) To determine the frictional forces between solid surfaces & flowing liquid using forced torsion pendulums

c) Torsion springs are used in torsion pendulum clocks.

d) To determine the rigidity modulus of the material given.

## Bending of Beams

### Beam

A beam is defined as a rod or bar of uniform cross section [circular or cylindrical] whose length is very much larger than its thickness so that the shearing stresses over any cross section are small and may be neglected.

## Bending Moment

The sum of the moments of all internal elastic forces acting over the whole cross section of the beam is known as bending moment.

The expression for bending moment is given by,

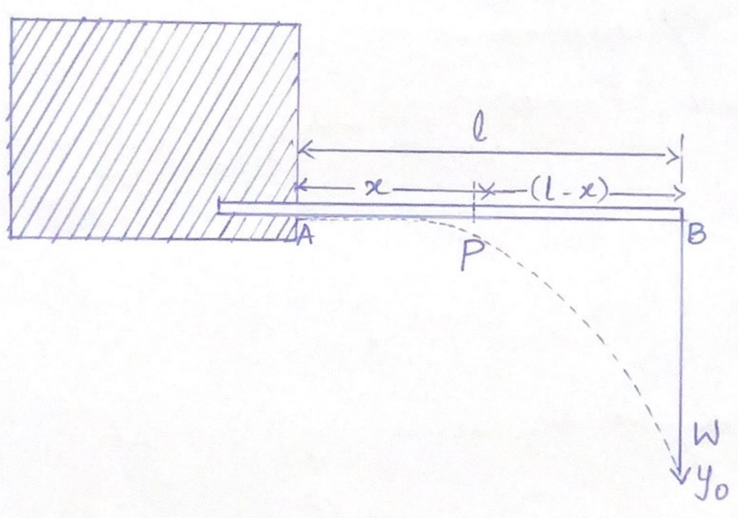
$$\boxed{BM = \frac{qI}{R}}$$

where $q$ -> young's modulus of the material

$I$ -> Moment of Inertia

$R$ -> Radius of curvature

## Cantilever Experiment to determine young's Modulus




Let AB be a beam of length 'l' fixed at the end 'A' and loaded by a weight 'W' at B. The bent position is shown by the dotted line. Consider a section of the beam at 'p', distant 'x' from the fixed end 'A'.

Since the beam is in equilibrium in the bent position, the external bending couple at 'p' is balanced by the internal bending moment.

ie $w(l-x) = \frac{EI}{R} \rightarrow ①$

But according to differential calculus, 'R' is given by,

$\frac{1}{R} = \frac{d^2y}{dx} \rightarrow ②$ where $\frac{dy}{dx} \rightarrow$ Slope of the beam

$\therefore$ eqn ① can be written as,

$$W(l-x) = EI \frac{d^2y}{dx^2}$$

$$\frac{d^2y}{dx^2} = \frac{W}{EI}(l-x) \rightarrow ③$$

Integrating above eqn, we get

$$\frac{dy}{dx} = \frac{W}{EI}\left[lx - \frac{x^2}{2}\right] + C_1 \rightarrow ④$$

where $C_1 \rightarrow$ Constant of integration

At the end A, $x=0$, $\frac{dy}{dx}=0$

$\Rightarrow C_1 = 0$

$$\therefore \frac{dy}{dx} = \frac{W}{EI}\left[lx - \frac{x^2}{2}\right] \rightarrow ⑤$$

Integrating again the above eqn, we get

$$y = \frac{W}{EI}\left[\frac{lx^2}{2} - \frac{x^3}{6}\right] + C_2 \rightarrow ⑥$$

where, $C_2 \rightarrow$ Constant of integration

Again at the end A, $x=0$ & $y=0$

$\Rightarrow C_2 = 0$

$$\therefore y = \frac{W}{EI}\left[\frac{lx^2}{2} - \frac{x^3}{6}\right] \rightarrow ⑦$$

If $y_0$ is the minimum depression at the end 'B' of the beam, ie at $x = l$,

$$y_0 = \frac{W}{qI}\left[\frac{l^3}{2} - \frac{l^3}{6}\right]$$

$$y_0 = \frac{Wl^3}{3qI} \longrightarrow (8)$$

For a beam of rectangular cross section,

$$I = \frac{bd^3}{12}$$

$$\therefore\ y_0 = \frac{Wl^3}{\cancel{3}q}\left(\frac{\cancel{12}^{4}}{bd^3}\right)$$

$$y_0 = \frac{4Wl^3}{qbd^3}$$

$$W = mg$$

$$\therefore\ y_0 = \frac{4mgl^3}{qbd^3}$$

$$\boxed{q = \frac{4mgl^3}{bd^3 y_0}}$$

Using the above expression, the value of young's Modulus can be determined.

Numericals on Elasticity 1/12/2023

Q Calculate the force req. to produce an extension of 1mm in steel wire of length 2m & diameter 1mm. (Young's modulus

$Y = 2 \times 10^{11}\ N/m^2$)

Data Given: Extension to be produced $\ell$ = 1 mm = $10^{-3}$ m

Length of the wire, L = 2 m

Diameter d = 1 mm = $10^{-3}$ m

Young's modulus of the steel, $Y = 2 \times 10^{11}\ N/m^2$

To find: The force

$$Y = \frac{F/A}{\Delta L/L}$$

$$Y \times \frac{\Delta L}{L} \times A = F$$

$$F = \frac{Y \times \Delta L \times A}{L}$$

$$= \frac{\cancel{2} \times 10^{11} \times 1 \times 10^{-3} \times \pi \left(\frac{10^{-3}}{2}\right)^2}{\cancel{2}}$$

$$= \frac{10^{11} \times 10^{-3} \times 10^{-6} \times \pi}{4}$$

$$= 0.7853 \times 10^{2}$$

$$F = 78.53\ N$$

Q A rod of crossection of $A = 1cm \times 1cm$ is rigidly planted into the earth vertically. A string which can withstand a max. T of 2kg is tied to the upper end of the rod and pulled horizontally. If the length of the rod from the ground level is 2m. Calculate the distance through which is upper end is displaced just before the string snaps.

Data given:

Breadth $b = 1cm = 10^{-2}m$

Thickness $d = 1cm = 10^{-2}m$

Mass $m = 2kg$

Length $= 2m$

$Y = 2 \times 10^{11} N/m^2$

To find:

Distance through which the upper end of the rod is displaced before the string.

$$\cancel{\frac{Wl^3}{Y}} \quad y_0 = \frac{4mgl^3}{bd^3 Y}$$

$$= \frac{4 \times \cancel{2} \times 9.8 \times 2^3}{10^{-2} \times 10^{-6} \times \cancel{2} \times 10^{11}} \quad = \cancel{\frac{4 \times 2 \times 2^{82}}{10^{-2} \times 10^{-2} \quad 2 \times 10^{11}}}$$

$$= 313.6 \times 10^{-3} \quad = \cancel{\frac{16 \times 9.8}{10^{4}}}$$

$$y_0 = 0.3131 \quad = \cancel{16 \times 10^{-7} N/m^2} \quad =$$

~~W~~ The string snaps when the following force becomes equal to max. the string can with stand i.e when ~~W = mg~~ ~~tension~~ tension is 2kg, there tension is as

~~W~~ = as same as load

$$W = mg$$

$$= 2 \times 9.8$$

$$W = 19.6N$$

Q Calculate the torque req. to twist a wire of length 1.5 m, radius $0.0425 \times 10^{-2}$ m through an angle $\left(\frac{\pi}{45}\right)$ rad, if the value of rigidity modulus of its material is $8.3 \times 10^{10}$ N/m$^2$.

Data Given.

Length of wire = 1.5 m

Radius = $0.0425 \times 10^{-2}$ m

Angle, $\theta = \frac{\pi}{45}$ rad = $4°$

Rigidity Modulus $\eta$ = $8.3 \times 10^{10}$ N/m$^2$

$\frac{N}{m^2 \times m} \times m^4$

Couple $C = \frac{\eta \pi R^4}{2L}$

$= \frac{8.3 \times 10^{10} \times \pi \times (0.0425 \times 10^{-2})^4}{2 \times 1.5}$

$C = \cancel{15699.45}$, $2.83 \times 10^{-3}$ N/m

Torque is require, $T = C\theta$

$= \cancel{15699.45 \times \frac{\pi}{45}}$

$= 2.83 \times 10^{-3} \times \frac{\pi}{45}$

$= 1.97 \times 10^{-4}$ N/m

Q Calculate the torque req. to twist a wire of length 5m radius 2mm through an angle $\frac{\pi}{90}$ rad, if the value of rigidity modulus of its material is given by $10\times10^{10}\ N/m^2$

$$\text{Couple, } C = \frac{\eta \pi R^4}{2L}$$

$$= \frac{10\times10^{10}\times\pi\,(2\times10^{-3})^4}{2\times5}$$

$$C = 0.5$$

$$\text{torque} = 0.5\times\frac{\pi}{90}$$

$$= 0.017\ N/m$$

$$= 1.7\times10^{-2}\ N/m$$

---

Q An increment in length by 1mm was observed in a gold wire of diameter 0.3mm, when it was subjected to longitudinal force of 2N, a twist of 0.1rad was observed in same wire, when its one end of subjected to a torque of $7.9\times10^{-7}$ Nm while its other end was fixed. Calculate the value of Poisson's ratio for gold.

Data given:

$\Delta L = 1\,mm = 1\times10^{-3}\,m$

$D = 0.3\times10^{-3}\,m$

$F = 2N$

~~$\theta = 0.1\,rad$~~

$\tau = 7.9\times10^{-7}\,NM$

$$\tau = C\theta$$

$$\frac{7.97\times10^{-7}}{0.1} = C$$

$$\boxed{7.97\times10^{-6} = C}$$

$$\boxed{7.9\times10^{-6} = \frac{\eta\pi R^4}{2L}}$$

$$Y = \frac{F/A}{\Delta L/L}$$

$$\boxed{Y = \frac{FL}{\pi R^2 x}} \rightarrow (1)$$

$$Y = \frac{2L}{\pi\left(0.15\times10^{-3}\right)^2 \cdot 10^{-3}}$$

$$C = \frac{\eta\pi R^4}{2L}$$

$$\eta = \frac{C\,2L}{\pi R^4} \rightarrow (2)$$

$$\frac{(1)}{(2)} \Rightarrow \frac{F\cancel{L}}{\cancel{\pi}\cancel{R^2}x} \times \frac{\cancel{\pi}R^{\cancel{4}\,2}}{2C\cancel{L}}$$

$$= \frac{\cancel{2}\times\left(0.15\times10^{-3}\right)^2}{10^{-3}\times\cancel{2}\times7.9\times10^{-6}}$$

$\frac{y'}{n} = 2.84\,m$

We have relation

$\frac{y}{n} = 2(1+\sigma)$

$\frac{\cancel{2.84}^{1.42}}{\cancel{2}} - 1 = \sigma$

$\boxed{0.424 = \sigma}$

---

Q The free end of single cantilever depresses 10mm under a certain load of 120g. Calculate the Young's modulus of single cantilever for which the length of lever is given 0.5m., width 1m & thickness $0.3\times10^{-3}$

Find Young's modulus

$Y = \frac{4mgl^3}{bd^3}$

$$Y = \frac{4 \times 120 \times 10^{-3} \times (0.5)^3 \times 9.8}{1 \times (0.3\times10^{-3})^3}$$

$$\boxed{Y = 2.17 \times 10^{10}\ N/m^2}$$

Q. A solid lead sphere of radius of 10.3m is subjected to a normal pressure 10 N/m² acting all over the surface. Determine the change in its volume given the bulk modulus for lead $4.58 \times 10^{10}$ N/m².

To find change in vol. of sphere

i.e. $r = 10.3m$

$P = 10 N/m^2$

$K = 4.58 \times 10^{10} N/m^2$

Sol: To find change in volume

$V = \frac{4}{3}\pi r^3$ $K = \frac{pV}{v}$

$$v = \frac{pV}{K}$$

$$= \frac{10 \times \frac{4}{3}\pi (10.3)^3}{4.58 \times 10^{10}}$$

$v = 9.99 \times 10^{-7} m^3$

$\boxed{v = 10^{-6} m^3}$

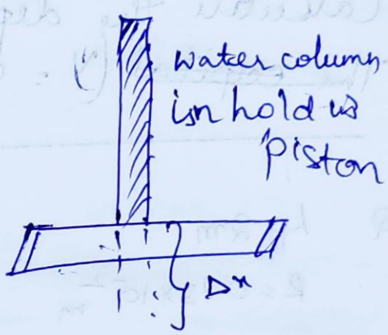
Q. A ~~bottle~~ water of column length 1m is held in a cylinder b/w its base and tightly fitted piston. Calculate the distance through which the piston would move, if the water column is subjected to a pressure of $205 \times 10^4\ N/m^2$

Given bulk modulus for water $K = 2.05 \times 10^9\ N/m^2$

To find: Distance of movement of the piston, $\Delta x = ?$

Let $r$ be the radius of the cylinder,

$l_2$ be the length after completion




Distance through which the piston moves

$\Delta x = l_1 - l_2$

Initial volume of water column

$V = \pi r^2 . l_1 \rightarrow ①$

$l_2 = 1 - 0.001$

$\boxed{l_2 = 0.99m}$

Final volume of water column

$V' = \pi r^2 l_2 \rightarrow ②$

Chang in volum $= V - V'$

$\Delta V = \pi r^2 (l_1 - l_2)$

$\Delta V = \Delta x\ \pi r^2$

$\cancel{\Delta x = \frac{10 \times 1}{2.05 \times 10^9}}$

$\cancel{\Delta x = 0.48 \times 10^{-8} m}$

$K = \frac{pV}{\Delta V}$

$\Delta x = \frac{205 \times 10^4}{2.05 \times 10^9}$

$\Delta V = \frac{P(\pi r^2 l_1)}{K}$

$= 10^2 \times 10^{-5}$

$\cancel{\pi r^2} \Delta x = \frac{P(\cancel{\pi r^2} l_1)}{K}$

$\Delta x = 10^{-3}$

$\Delta x = \frac{P l_1}{K}$

$\boxed{\Delta x = 1mm}$

uw

Q Calculate the extension produced in a wire of length 2m & radius $0.13\times10^{-2}$ m due to a force of 14.7N applied along its length. Given $Y = 2.1\times10^{11}$ N/m²

Q A brass bar of length 1m, 0.1m² in section is clamped firmly in a horizontal position at 1end. A weight of one kg is applied at the other end. Calculate the depression that would be produced. ~~The capital~~ ($Y = 9.78\times10^{10}$ N/m²). $\boxed{y_0 = 0.0134\text{ m}}$

---

Q $L = 2m$

$R = 0.13\times10^{-2}$ m

$F = 14.7$ N

$Y = 2.1\times10^{11}$ N/m²

$$Y = \frac{F/A}{\Delta L / L}$$

$$\Delta L = \frac{FL}{AY}$$

$$= \frac{14.7\times2}{(0.13\times10^{-2})^2\times2.1\times10^{11}}$$

$$\boxed{\Delta L = 8.28\times10^{-5}\text{ m}}$$

$L = 1m$

$A = 0.1 m^2$ $\quad$ $W = 1 kg$

$Y = 9.78 \times 10^{10} N/m^2$

$y_0 = 0.0134 m$

$$Y = \frac{4 mg l^3}{b d^3 y_0}$$

$l \times b = 0.1$

$1 \times b = 0.1$

$b = 0.1$

$$9.78 \times 10^{10} = \frac{4 \times 1 \times 1^3}{\cancel{b}0.1 \times d^3 \times 0.0134}$$

$$d^3 = \frac{4}{0.1 \times 9.78 \times 10^{10} \times 0.0134}$$

$\cancel{d} = 3.125 \times 10^{-3} m$