Taylor & Maclaurin Series for 1 variable f^n:-

$$f(x)=f(a)+(x-a)f'(a)+\frac{(x-a)^2}{2!}f''(a)+\frac{(x-a)^3}{3!}f'''(a)+\ldots$$

Expanding $f(x)$ in terms of $(x-a)$ or about $x=a$

Taylor Series Expansion

$$f(x)=f(0)+xf'(0)+\frac{x^2}{2!}f''(0)+\frac{x^3}{3!}f'''(0)$$

Maclaurin Series Expansion

Taylor & Maclaurin Series for 2-variable f^n:-

$$f(x,y)=f(a,b)+\frac{1}{1!}\left[(x-a)f_x+(y-b)f_y\right]+\frac{1}{2!}\left[(x-a)^2f_{xx}+(y-b)^2f_{yy}+2(x-a)(y-b)f_{xy}\right]$$
$$+\frac{1}{3!}\left[(x-a)^3f_{xxx}+(y-b)^3f_{yyy}+3(x-a)^2(y-b)f_{xxy}+3(x-a)(y-b)^2f_{xyy}\right]$$

Taylor Series

where, $f_x=\frac{\partial f}{\partial x}$; $f_y=\frac{\partial f}{\partial x}$; $f_{xx}=\frac{\partial^2 f}{\partial x^2}$; $f_{yy}=\frac{\partial^2 y}{\partial y^2}$

$f_{xy}=\frac{\partial^2 f}{\partial x\partial y}$; $f_{xxx}=\frac{\partial^3 f}{\partial x^3}$; $f_{yyy}=\frac{\partial^3 f}{\partial y^3}$; $f_{xxy}=\frac{\partial^3 f}{\partial x^2\partial y}$

$f_{xyy}=\frac{\partial^3 f}{\partial x\partial y^2}$

Put $a=0$ & $b=0$, we get Maclaurin Series

Indeterminate forms:-

$F(x)$ at $x=a \rightarrow \underbrace{\frac{0}{0}, \frac{\infty}{\infty}, 0\times\infty, \infty-\infty, 0^0, \infty^0, 1^\infty}_{\text{7 Indeterminate forms}}$

All values are approaching values, NOT exact value!

DHOKA!! :- $\infty+\infty$, $\infty-0$, $(\infty)^\infty$, 0^∞, $\infty\times\infty$, $\frac{0}{\infty}$

$$\lim_{x\to\infty} \frac{\ln x}{2^x} \quad \ln x \to \infty, \quad 2^x \to \infty$$

$\frac{0}{\square}$ → Starts app. 0 → Takes the entire term to 0

$\frac{\square}{0}$ → Starts app 0 → Whole term → $\pm\infty$

Tug of War

⇓

Indeterminate

No Tug of War, only pyaar → then not indeterminate

$(0)^\infty = 0$

$(\;)^\infty$: <1 → $(0)^\infty = 0$; >1 → $(2)^\infty = \infty$

No tug of war

$\frac{0}{0}$ form → $\frac{\text{Zero Approaching}}{\text{Zero Approaching}} = \frac{0\uparrow}{0\uparrow}$

$\frac{\text{EXACT Zero}}{\text{ZERO approaching}} = \frac{0}{0\uparrow} = 0$ | $\underbrace{\frac{0}{0} \;\&\; \frac{0\uparrow}{0}}_{\text{NOT DEFINED}}$

L'Hôpital Rule:-

For $\frac{0}{0}$ & $\frac{\infty}{\infty}$ forms ONLY

$\lim_{x \to a} f(x) = 0$; $\lim_{x \to a} g(x) = 0$

$$\boxed{\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)} = \lim_{x \to a} \frac{f''(x)}{g''(x)}}$$

Points to remember:-

① $\lim_{x \to 0} \frac{\sin x}{x} = 1$

② $\lim_{x \to 0} \frac{\tan x}{x} = 1$

③ $\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$

④ Questions in the form of

$[f(x)]^{g(x)}$ → Eg:- $(\cos x)^{1/x^2}$

$\downarrow$

Let $k = \lim_{x \to a} [f(x)]^{g(x)}$

Taking $\log_e$ on both sides

$\log_e k = \lim_{x \to a} g(x) \log_e (f(x))$

↳ Solve further

Maxima & Minima for one variable f^n:

Method of Solving:-

① Find $f'(x)$ & equate to 0

one or more than one

② Solve $f'(x) = 0$ to get value of a

③ Find $f''(x)$ & evaluate $f''(a)$

If $f''(a) < 0 \rightarrow x = a$ is point of local maximum

If $f''(a) > 0 \rightarrow x = a$ is point of local minimum

Maxima & Minima of two variable f^n:-

If the f^n is $f(x,y)$; Steps to solve:-

① Find stationary points $f_x = 0$ & $f_y = 0$

② Find $\Rightarrow$ $\left.\begin{aligned} A &= f_{xx} \\ B &= f_{xy} \\ C &= f_{yy} \end{aligned}\right\}$ Find $AC - B^2$

If $AC - B^2 > 0$ & $A < 0 \rightarrow$ Points are maxima

If $AC - B^2 > 0$ & $A > 0 \rightarrow$ Points are minima

If $AC - B^2 \leq 0$ & $A = 0 \rightarrow$ Saddle points

Curve Tracing

Eg:- $x^2=4ay$

① **Symmetry:-** If power of x is even $\Rightarrow$ Symmetric about y-axis

Eg:- $y^2=4ax$ $\longleftarrow$ If power of y is even $\Rightarrow$ Symmetric about x-axis

Eg:- $x^2+y^2=a^2$ $\longleftarrow$ If power of x & y is even $\Rightarrow$ Symmetric about x & y- axes

② **Origin:-** If we replace $f(x,y)$ with $f(0,0)$ & get $f(0,0)=0$ then it passes through origin $\Rightarrow$ Eg:- $x^2=4ay$, $y^2=4ax$ $\rightarrow$ Passes

$x^2+y^2=a^2$ $\rightarrow$ Does not pass

③ **Tangent at Origin:-** Tangent at origin exists iff curve passes through origin

Finding TAO:- By putting least degree term as zero

Ex:- $y^2=4ax$	$x^2=4ay$
$x=0$ is TAO	$y=0$ is TAO

④ **Intersection with Coordinate axis:-**

Cutting x-axis $\Rightarrow$ Put $y=0$

Cutting y-axis $\Rightarrow$ Put $x=0$

⑤ Asymptotes

⑥ Region of Existence