Introduction

If y be any f^n of x then its derivative $\frac{dy}{dx}$ will be a f^n of x whose diff^n can be done

The succesive derivative of y w.r.t x are denoted by $\frac{dy}{dx}, \frac{d^2y}{dx^2}, \frac{d^3y}{dx^3}, \ldots \frac{d^ny}{dx^n}$

n^{th} derivative of some standard functions

(i) $D^n\left(e^{ax+b}\right) = a^n e^{ax+b}$

(ii) $D^n\left(a^x\right) = \left(\log a\right)^n a^x$

(iii) $D^n(ax+b)^m = m(m-1)(m-2)\ldots(m-n+1)a^n(ax+b)^{m-n}$ $\{m<n\}$

$D^n(ax+b)^m = 0$ $\{n>m\}$

$D^n(ax+b)^n = n!\,a^n$ $\{n=m\}$

$D^n(ax+b)^{-1} = \dfrac{(-1)^n n!\,a^n}{(ax+b)^{n+1}}$ $\{m=-1\}$

(iv) $D^n \sin(ax+b) = a^n \sin\left(ax+b+\frac{1}{2}n\pi\right)$

(v) $D^n \cos(ax+b) = a^n \cos\left(ax+b+\frac{1}{2}n\pi\right)$

Leibnitz Theorem

If 2 functions u & v of x then

$$D^n(uv) = D^n(u)\cdot v + {}^nC_1 D^{n-1}(u)\cdot D(v) + {}^nC_2 D^{n-2}(u)\cdot D^2(v) + \cdots + {}^nC_n(u)\cdot D^n(v)$$

Angle b/w radius vector & tangent

$$\tan(\phi) = r\frac{d\theta}{dr}$$

Steps to solve:-

1) Take log
2) Differentiate w.r.t θ
3) Reciprocate $\frac{dr}{d\theta}$
4) Use formula

Angle b/w 2 polar curves:-

$$\alpha = |\phi_2 - \phi_1|$$

Steps to solve:-

1) Find θ
2) Find ϕ_1
3) Find ϕ_2
4) Apply formula

Pedal Equation:-

$$p = r\sin\phi$$

Steps to solve:-

1) Find θ

2) Find ϕ

3) Use formula

4) Substitute θ from step-1

Radius of curvature for cartesian curve:

Suppose $y = f(x)$ is a curve of Cartesian form, then amount of bending of a curve at given point is called Radius of curvature

$$\rho = \frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2}}{\frac{d^2y}{dx^2}}$$

$$\rho = \frac{\{1+(y_1)^2\}^{3/2}}{y_2}$$

If $y_1 \to \infty$ at any point then radius of curvature is

$$\rho = \frac{(1+x_1^2)^{3/2}}{x_2}$$

ROC of Polar Curves:-

Let $r=f(\theta)$ be a polar curve then ROC is given by

$$\boxed{\rho = \frac{(r^2+r_1^2)^{3/2}}{r^2+2r_1^2-rr_2}}$$ where $r_1 = \frac{dr}{d\theta}$; $r_2 = \frac{d^2r}{d\theta^2}$

ROC of parametric curves:-

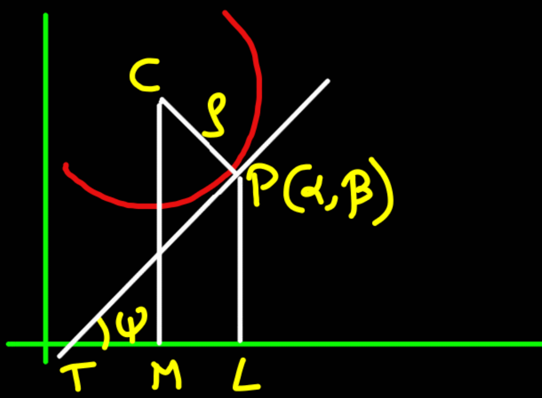
Let $f(t)=x$ & $g(t)=y$ be 2 parametric functions

$$\rho = \frac{(\dot{x}^2+\dot{y}^2)^{3/2}}{\dot{x}\ddot{y}-\dot{y}\ddot{x}}$$, where $\dot{x}=f'(t)$; $\ddot{x}=f''(t)$
$\dot{y}=g'(t)$; $\ddot{y}=g''(t)$

$t \rightarrow$ parameter

Centre of Curvature

Let $C(\bar{x},\bar{y})$ be the centre of curvature then Coordinates of C is



$$\boxed{\begin{aligned} \bar{x} &= x - \frac{y_1(1+y_1^2)}{y_2} \\ \bar{y} &= y + \frac{1+y_1^2}{y_2} \end{aligned}}$$ → Coordinates of centre of curvature

Eq^n of Circle:-

$$\boxed{(x-\bar{x})^2+(y-\bar{y})^2 = \rho^2}$$ → Circle of curvature

Evolutes:-

The locus of center of curvature is called evolute & the curve itself is called involute

Working rule for evolute:-

① Find the coordinates of the center of curvature

$$\alpha = x - \frac{y_1(1+y_1^2)}{y_2} \; ; \; \beta = y + \frac{1+y_1^2}{y_2}$$

The coordinates are (α, β)

② Eliminate the parameters x, y to find a relation b/w α & β

③ Replace α & β with x & y respectively to find locus

The locus is the evolute of the curve

Singular points:-

Let $f(x,y)$ be a curve on a plane.

The slope of the tangent is $\boxed{\frac{dy}{dx} = -\frac{\partial f/\partial x}{\partial f/\partial y}}$

If at some point $p(x_1, y_1)$ of the curve, $\frac{\partial f}{\partial x}$ & $\frac{\partial f}{\partial y}$ vanish, i.e.,

i.e., must lie on the curve

$$\left(\frac{\partial f}{\partial x}\right)_{\substack{x=x_1\\y=y_1}} = 0 \quad \text{and} \quad \left(\frac{\partial f}{\partial y}\right)_{\substack{x=x_1\\y=y_1}} = 0$$

then the point $p(x_1, y_1)$ is called singular point of the curve

Double points:-

Double points of a curve are obtained by solving the eq^{n}s $\frac{\partial f}{\partial x} = 0$ & $\frac{\partial f}{\partial y} = 0$

Nature of double points:-

$$\Delta = \left(\frac{\partial^2 f}{\partial x \partial y}\right)^2 - \left(\frac{\partial^2 f}{\partial x^2}\right)\left(\frac{\partial^2 f}{\partial y^2}\right)$$

i) $\Delta > 0$ then point is Node

ii) $\Delta = 0$ then point is Cusp

iii) $\Delta > 0$ then point is Isolated

Asymptotes:-

Def^n:- Tangent at ∞

Parallel Asymptotes:- Working Rule

① To find the asymptote ||el to x-axis, equate real coefficient of highest power of x to zero

② To find the asymptote ||el to y-axis, equate real coefficient of highest power of y to zero

Oblique Asymptotes: Working Rule

① Write given eq^n in the form

$$H_n + H_{n-1} + H_{n-2} + \cdots + \text{Constant} = 0$$

where H_i is the homogeneous exp^r in x & y of degree i

② Put $x=1$, $y=m$, we get

$$\phi_n(m) + \phi_{n-1}(m) + \cdots + \text{Constant} = 0$$

③ Solve $\phi_n(m)=0$ to get slopes of asymptotes

④ For non-repeated value of m, find c by using

$$c = -\frac{\phi_n'(m)}{\phi_{n-1}(m)}$$

⑤ For repeated value of m (say twice repeated)

$$\frac{c^2}{2!}\phi_n''(m) + \frac{c}{1!}\phi_{n-1}'(m) + \phi_{n-2}(m) = 0$$

In this case, we get 2 ||el asymptotes

Note:- If highest degree of curve is n then there exists n no. of asymptotes