Sequence:-

$$\langle a_n \rangle = \frac{1}{x^n} \Rightarrow \langle a_n \rangle = \left\{ \frac{1}{x}, \frac{1}{x^2}, \frac{1}{x^3}, \dots, \frac{1}{x^n} \right\}$$

Bounded Sequences:-

Bounded Sequence

- Bounded Above → If for a real no. m, $a_n \geq m$
- Bounded Below → If for a real no. M, $a_n \leq M$

↓

If $m \leq a_n \leq M$, then it is called Bounded Sequence

Limit of a Sequence:-

$$\lim_{n \to \infty} u_n = l$$

, $u_n \to n^{th}$ term of sequence

Convergence of Sequence:-

$$\lim_{n \to \infty} u_n = l \text{ (some finite value)}$$

↳ Convergent

$$\lim_{n \to \infty} u_n = \pm\infty \to \text{Divergent}$$

neither converge or diverge

↳ Oscillatory

If $\lim\limits_{n \to \infty} u_n = 0$ → Null sequence

Monotonic Sequences :-

1) If $a_{n+1} \geq a_n$ → Monotonically $\uparrow$ing sequence
↳ $a_{n+1} > a_n$ → Strictly $\uparrow$ing

2) If $a_{n+1} \leq a_n$ → Monotonically $\downarrow$ing sequence
↳ $a_{n+1} < a_n$ → Strictly $\downarrow$ing

Properties of Monotonic seq^n :-

1. Monotonic $\uparrow$ing seq^n & Bounded above → Convergent
2. Monotonic $\downarrow$ing seq^n & Bounded below → Divergent
3. Monotonic $\uparrow$ing seq^n → Not bounded above → Diverges to $+\infty$
4. Montonic $\downarrow$ing seq^n → Not bounded below → Diverges to $-\infty$

To Remember :-

1. $\lim\limits_{n \to \infty} (n)^{1/n} = 1$

2. $\lim\limits_{n \to \infty} \left(1 + \frac{1}{n}\right)^n = e$

Series of Real Numbers:-

Sum of terms of Sequences, ie, $u_1+u_2+u_3+\dots\infty$

Denoted by $\sum\limits_{n=1}^{\infty} u_n$

Partial Sum:-

If $\sum\limits_{n=1}^{\infty} u_n = u_1+u_2+u_3+\dots+u_n+\dots\infty$

$$\left.\begin{aligned} S_1 &= u_1 \\ S_2 &= u_1+u_2 \\ &\vdots \\ S_n &= u_1+u_2+u_3+\dots+u_n \end{aligned}\right\} \text{Partial Sum}$$

Cond^n for Convergence of Series:-

① $\lim\limits_{n\to\infty} u_n = l$ (finite & unique value) → Convergent

② $\lim\limits_{n\to\infty} u_n = \pm\infty$ → Divergent

Types of Series:-

Series

- +ve term
- Alternating Series

Tests for +ve term Series

P-Series Test :-

If $\sum_{n=1}^{\infty} u_n = \frac{1}{n^p}$ be a +ve term series, then :-

① If $p > 1 \rightarrow$ Convergent

② If $p \leq 1 \rightarrow$ Divergent

Comparision Test :-

Let $\sum u_n$ & $\sum v_n$ be 2 positive terms series &

$\exists\, m \in \mathbb{N} \ni |u_n| \leq |v_n| \ \forall n \in \mathbb{N}$

There exists

Smaller Bigger

1) $\sum v_n$ converges $\Rightarrow \sum u_n$ converges

2) $\sum u_n$ diverges $\Rightarrow \sum v_n$ diverges

Limit Comparision Test :-

Let $\sum u_n$ and $\sum v_n$ be two +ve term series such that

$\lim_{n \to \infty} \frac{u_n}{v_n} = l$, where l is non-zero finite number

then $\sum u_n$ & $\sum v_n$ are convergent or divergent together

Method of finding V_n

Let $u_n = \dfrac{C_1 n^q + C_2 n^{q-1} + C_3 n^{q-2} + \dots}{b_1 n^p + b_2 n^{p-1} + b_3 n^{p-2} + \dots}$

then we take $\boxed{V_n = \dfrac{n^q}{n^p} = \dfrac{1}{n^{p-q}}}$ i.e $V_n = \dfrac{\text{Highest power of } N^r}{\text{Highest power of } D^r}$

Not Considering constant of both N^r & D^r

d'Alembert Ratio Test

Let $\sum u_n$ be a +ve term series

1) If $\lim\limits_{n\to\infty} \dfrac{u_{n+1}}{u_n} < 1 \rightarrow$ Series is Convergent

2) If $\lim\limits_{n\to\infty} \dfrac{u_{n+1}}{u_n} > 1 \rightarrow$ Series is divergent

3) If $\lim\limits_{n\to\infty} \dfrac{u_{n+1}}{u_n} = 1 \rightarrow$ Test fails

Apply this when series has $n!$ or $n(n+1)$ type

Raabe's Test:-

Let u_n be a +ve term series then

1) If $\lim_{n\to\infty} n\left(\frac{u_n}{u_{n+1}}-1\right)>1 \rightarrow$ Convergent

2) If $\lim_{n\to\infty} n\left(\frac{u_n}{u_{n+1}}-1\right)<1 \rightarrow$ Divergent

3) If $\lim_{n\to\infty} n\left(\frac{u_n}{u_{n+1}}-1\right)=1 \rightarrow$ Test fails

Apply when D'Alembert Ratio test fails & power of u_n is 1

Cauchy's Root Test:- Apply when power has n in it

The series $\sum u_n$ of +ve terms is

1) Convergent if $\lim_{n\to\infty} (u_n)^{1/n} < 1$

2) Divergent if $\lim_{n\to\infty} (u_n)^{1/n} > 1$

3) If $\lim_{n\to\infty} (u_n)^{1/n} = 1 \rightarrow$ Test fails

Cauchy's Integral Test :-

If for $x \geq 1$, f be a non-negative monotonic decreasing integrable f^n $\ni$ $f(n) = u_n$ $\forall n \in \mathbb{N}$ then $\sum_{n=1}^{\infty} u_n$ and $\int_1^{\infty} f(x)\,dx$ either both converge or diverge

1) $\sum_{n=1}^{\infty} u_n$ is convergent iff $\lim_{x \to \infty} \int_1^{x} f(x)\,dx$ finitely exists

2) $\sum_{n=1}^{\infty} u_n$ is divergent iff $\lim_{x \to \infty} \int_1^{\infty} f(x)\,dx$ does not finitely exist

Tests for Alternating Series

Leibnitz Test :-

Let $\langle u_n \rangle$ be a sequence such that

a) $u_n \geq 0$; b) $u_{n+1} \leq u_n$; c) $\lim_{n \to \infty} u_n = 0$

Then the alternating series $\sum_{n=1}^{\infty} (-1)^{n-1} u_n$ is convergent

Absolute Convergence:-

Alternating Series is said to be Absolutely Converging

iff $\sum |(-1)^{n-1} u_n|$ is convergent

Conditional Convergence:-

An alternating series is said to be Conditionally converging

if $\sum u_n$ is convergent but not Absolutely Convergent