

Taylor & Maclaurin Series for 1 variable f^n :

$$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!}f''(a) + \frac{(x-a)^3}{3!}f'''(a) + \dots$$

Taylor Series Expansion

Expanding $f(x)$
in terms of $(x-a)$
or about $x=a$

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \dots$$

Maclaurin Series
Expansion

Taylor & Maclaurin Series for 2-variable f^n :

$$f(x,y) = f(a,b) + \frac{1}{1!}[(x-a)f_x + (y-b)f_y] + \frac{1}{2!}[(x-a)^2f_{xx} + (y-b)^2f_{yy} + 2(x-a)(y-b)f_{xy}] + \frac{1}{3!}[(x-a)^3f_{xxx} + (y-b)^3f_{yyy} + 3(x-a)^2(y-b)f_{xxy} + 3(x-a)(y-b)^2f_{xyy}] + \dots$$

Taylor Series where, $f_x = \frac{\partial f}{\partial x}$; $f_y = \frac{\partial f}{\partial y}$; $f_{xx} = \frac{\partial^2 f}{\partial x^2}$; $f_{yy} = \frac{\partial^2 f}{\partial y^2}$

$$f_{xy} = \frac{\partial^2 f}{\partial x \partial y}; f_{xxx} = \frac{\partial^3 f}{\partial x^3}; f_{yyy} = \frac{\partial^3 f}{\partial y^3}; f_{xxy} = \frac{\partial^3 f}{\partial x^2 \partial y}$$

$$f_{xyy} = \frac{\partial^3 f}{\partial x \partial y^2}$$

Put $a=0$ & $b=0$, we get Maclaurin Series

Indeterminate forms:-

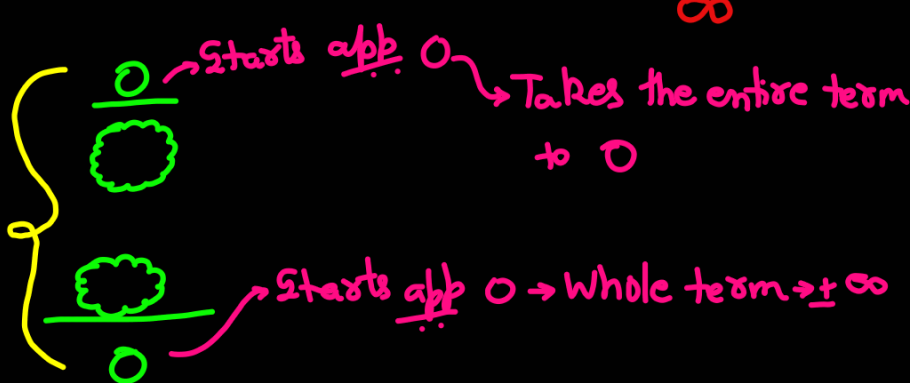
$$F(x) \text{ at } x=a \rightarrow \frac{0}{0}, \frac{\infty}{\infty}, 0 \times \infty, \infty - \infty, 0^0, \infty^0, 1^\infty$$

All values are approaching values, NOT exact value!

7 Indeterminate forms

DHOKA!! :- $\infty + \infty, \infty - 0, (\infty)^\infty, 0^\infty, \infty \times \infty, \frac{0}{\infty}$

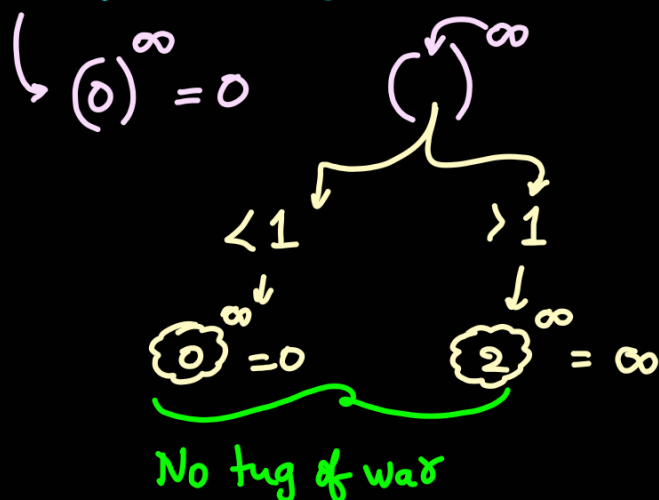
$$\lim_{x \rightarrow \infty} \frac{\ln x}{2^x}$$



Tug of war

Indeterminate

No Tug of war, only by $a^x \rightarrow$ then not indeterminate



$$\frac{0}{0} \text{ form} \rightarrow \frac{\text{Zero Approaching}}{\text{Zero Approaching}} = \frac{0^\uparrow}{0^\uparrow}$$

$$\frac{\text{EXACT Zero}}{\text{ZERO approaching}} = \frac{0}{0^\uparrow} = 0 \quad \left| \quad \frac{0}{0} \neq \frac{0^\uparrow}{0} \right.$$

NOT DEFINED

L'Hôpital Rule:-

For $\frac{0}{0}$ & $\frac{\infty}{\infty}$ forms ONLY

$$\lim_{x \rightarrow a} f(x) = 0 ; \lim_{x \rightarrow a} g(x) = 0$$

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)} = \lim_{x \rightarrow a} \frac{f''(x)}{g''(x)}$$

Points to remember:-

$$\textcircled{1} \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\textcircled{2} \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$$

$$\textcircled{3} \lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$$

④ Questions in the form of

$$[f(x)]^{g(x)} \rightarrow \text{Eg: } (\cos x)^{1/x^2}$$

$$\text{Let } k = \lim_{x \rightarrow a} [f(x)]^{g(x)}$$

Taking \log_e on both sides

$$\log_e k = \lim_{x \rightarrow a} g(x) \log_e(f(x))$$

↳ Solve further

Maxima & Minima for one variable f^n :

Method of Solving:-

① Find $f'(x)$ & equate to 0 one or more than one

② Solve $f'(x) = 0$ to get value of a

③ Find $f''(x)$ & evaluate $f''(a)$

If $f''(a) < 0 \rightarrow x=a$ is point of local maximum

If $f''(a) > 0 \rightarrow x=a$ is point of local minimum

Maxima & Minima of two variable f^n :-

If the f^n is $f(x, y)$; steps to solve:-

① Find stationary points $f_x = 0$ & $f_y = 0$

② Find $\Rightarrow \begin{matrix} A = f_{xx} \\ B = f_{xy} \\ C = f_{yy} \end{matrix} \left. \vphantom{\begin{matrix} A = f_{xx} \\ B = f_{xy} \\ C = f_{yy} \end{matrix}} \right\} \text{Find } AC - B^2$

If $AC - B^2 > 0$ & $A < 0 \rightarrow$ Points are maxima

If $AC - B^2 > 0$ & $A > 0 \rightarrow$ Points are minima

If $AC - B^2 \leq 0$ & $A = 0 \rightarrow$ Saddle points

Curve Tracing

Eg:- $x^2 = 4ay$

① Symmetry:- If power of x is even \Rightarrow Symmetric about y -axis

Eg:- $y^2 = 4ax$ \leftarrow If power of y is even \Rightarrow Symmetric about x -axis

Eg:- $x^2 + y^2 = a^2$ \leftarrow If power of x & y is even \Rightarrow Symmetric about x & y -axes

② Origin:- If we replace $f(x, y)$ with $f(0, 0)$ & get $f(0, 0) = 0$ then it passes through origin \Rightarrow Eg:- $x^2 = 4ay, y^2 = 4ax \rightarrow$ Passes
 $x^2 + y^2 = a^2 \rightarrow$ Does not pass

③ Tangent at Origin:- Tangent at origin exists iff curve passes through origin

Finding TAO:- By putting least degree term as zero

Ex:- $y^2 = 4ax$	$x^2 = 4ay$
$x = 0$ is TAO	$y = 0$ is TAO

④ Intersection with coordinate axis:-

Cutting x -axis \Rightarrow Put $y = 0$

Cutting y -axis \Rightarrow Put $x = 0$

⑤ Asymptotes

⑥ Region of Existence