ASTR 501 — **Homework Assignment 1a**

September 18, 2024 — (due September 24, 2024)

Question 1

The burning of ^{3}He in the vicinity of the H-burning shell may drive thermohaline mixing (salt-fingering convection) in the radiative zone of a low-mass upper RGB star? Estimate the maximum relative decrease of the mean molecular weight ($\Delta\mu/\mu$) resulting from the reaction ^{3}He(^{3}He,2p)^{4}He in the H-rich layer adjacent to the H-burning shell, where $X = 0.70$, $Y = 0.28$, and $Z = 0.014$, for the mass fraction of ^{3}He in the radiative zone equal to 0.006, assuming that all that mass fraction of ^{3}He is consumed there.

Question 2

Write a system of differential equations that describe the evolution (changes with time) of the ^{12}C, ^{13}C, ^{14}N, and ^{15}N isotopic abundances, $y_i = X_i/A_i$ (mole g^{-1}), occurring in the first CNO cycle, ^{12}C(p,γ)^{13}N(e$^+\nu$)^{13}C(p,γ)^{14}N(p,γ)^{15}O(e$^+\nu$)^{15}N(p,α)^{12}C (also known as the CN branch). Assume that the positron decays of ^{13}N and ^{15}O are instantaneous. Use these equations to show that the total mass fraction of the CN isotopes is conserved and to find the equilibrium (when all the time derivatives are zero, and the abundances do not change) isotopic ratio $y(^{12}\mathrm{C})/y(^{13}\mathrm{C})$ at $T = 25 \times 10^6$ K. To answer the last question, use the NACRE adopted rates $\lambda_1 = \langle\sigma v\rangle_1 N_\mathrm{A} = 1.28 \times 10^{-12}$ (cm^3 s^{-1} mole^{-1}) and $\lambda_2 = \langle\sigma v\rangle_2 N_\mathrm{A} = 5.18 \times 10^{-12}$ (cm^3 s^{-1} mole^{-1}) for the reactions ^{12}C(p,γ)^{13}N and ^{13}C(p,γ)^{14}N, respectively, at this temperature.