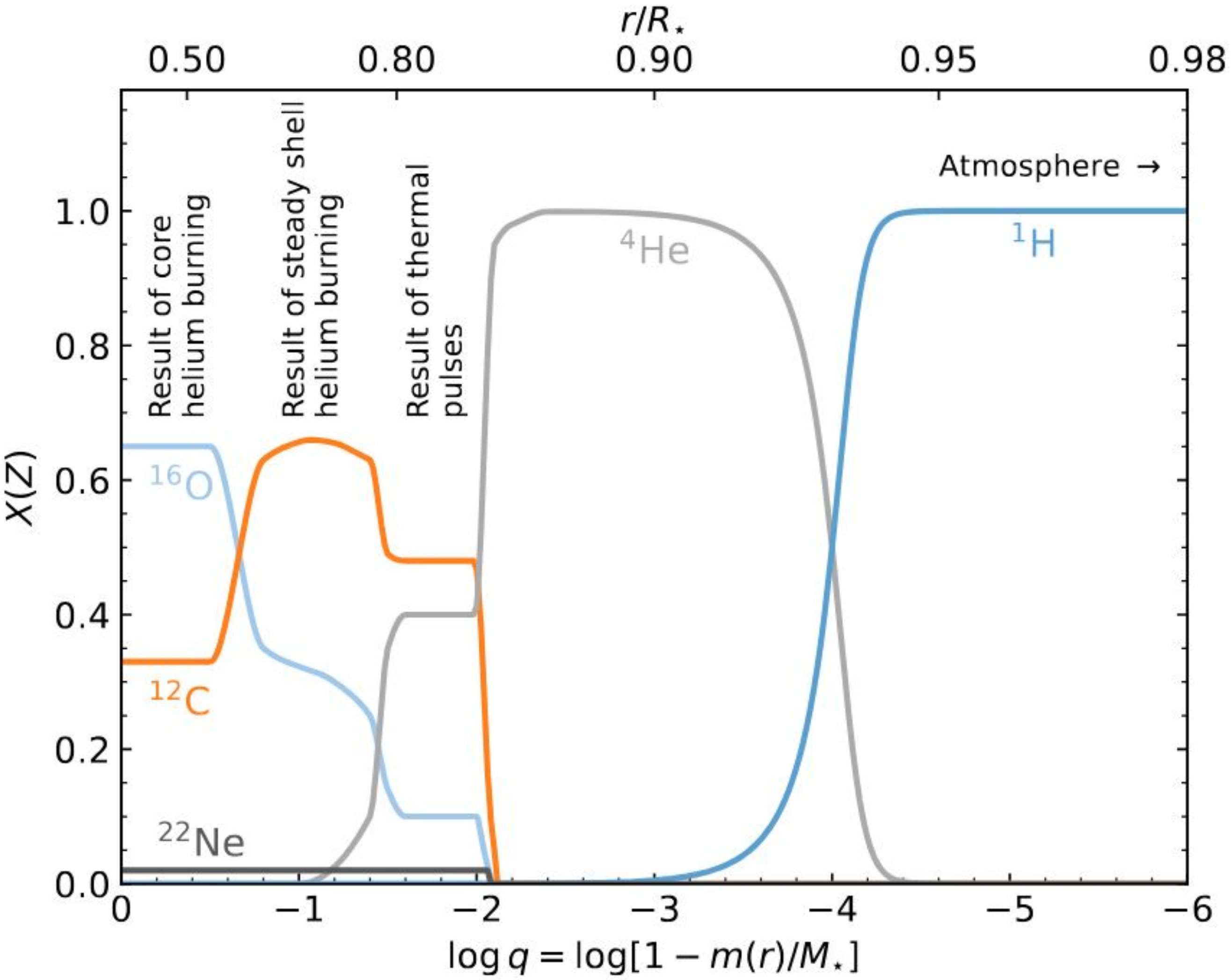
White dwarf stars

Simon Blouin - 2024-09-25

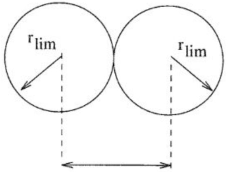


r/R⋆
0.50
0.80
0.90
0.95
0.98
Atmosphere →
Result of core helium burning
Result of steady shell helium burning
Result of thermal pulses
^{4}He
^{1}H
^{16}O
^{12}C
^{22}Ne
X(Z)
1.0
0.8
0.6
0.4
0.2
0.0
0
−1
−2
−3
−4
−5
−6
log q = log[1 − m(r)/M⋆]

WDs and equations of state

Saha ionization/dissociation

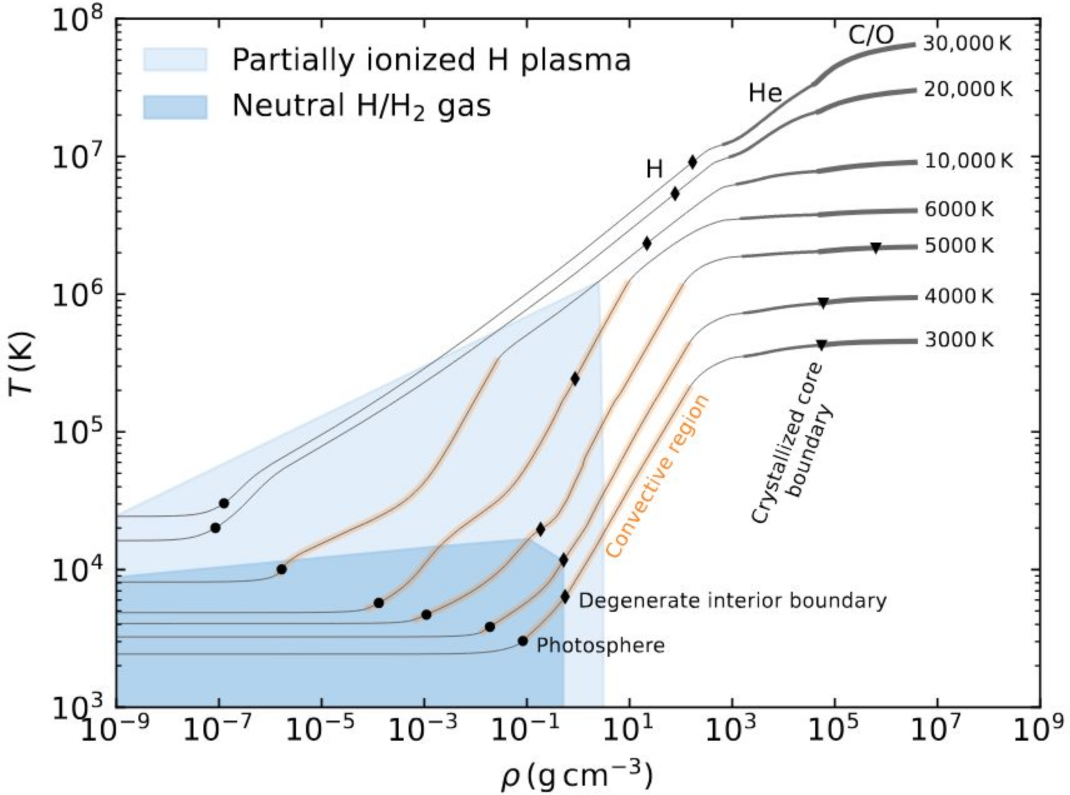
Pressure ionization



Onset of convection

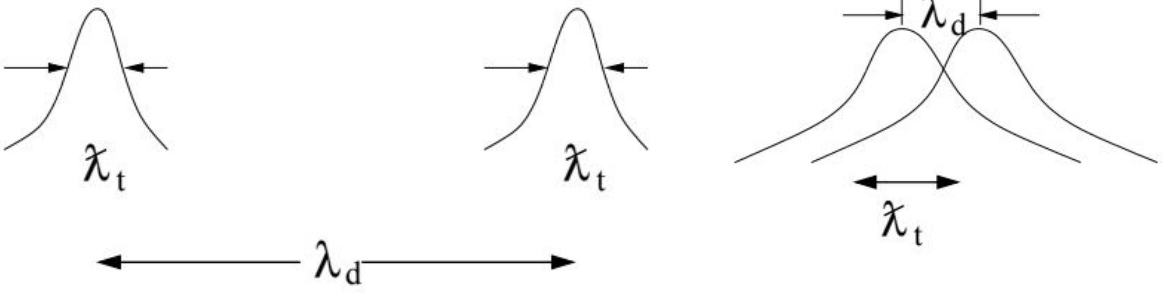
Degeneracy

Crystallization



Characteristic distances for plasma EOS

$$\lambda_d = \left(\frac{3}{4\pi n}\right)^{1/3}$$



$$\lambda_t \simeq \frac{h}{(mkT)^{1/2}}$$

$$\lambda_a \simeq \frac{Z^2 e^2}{kT}$$

$$\Gamma \equiv \frac{\lambda_a}{\lambda_d} = \frac{Z^2 e^2}{kT}\left(\frac{4\pi n}{3}\right)^{1/3}$$

Mass-radius relation

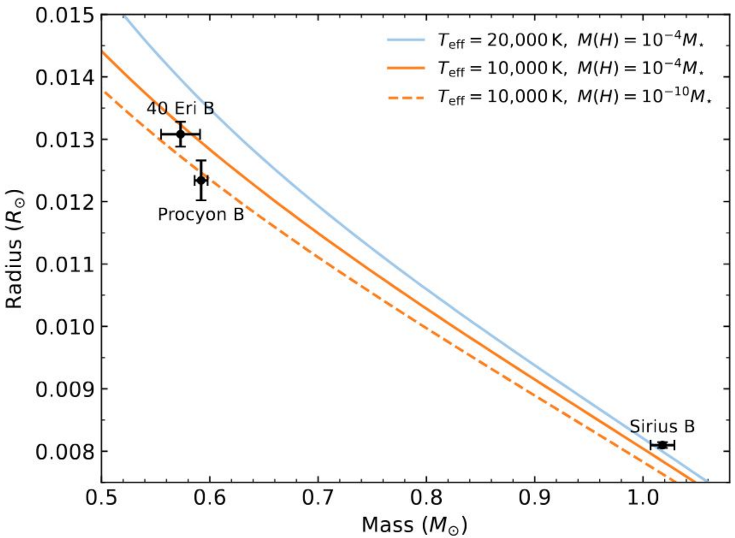
$$\frac{dP}{dr} = -\frac{Gm\rho}{r^2} \qquad \frac{dm}{dr} = 4\pi r^2 \rho$$

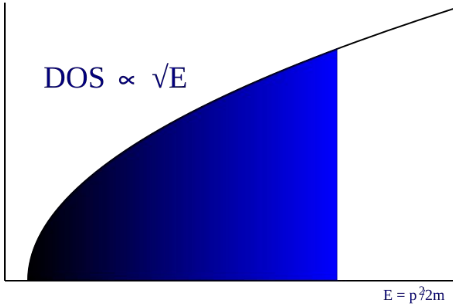
$$\frac{dP}{dr} \approx \frac{P_c}{R} \qquad \rho g \approx \frac{M}{R^3}\frac{GM}{R^2}$$

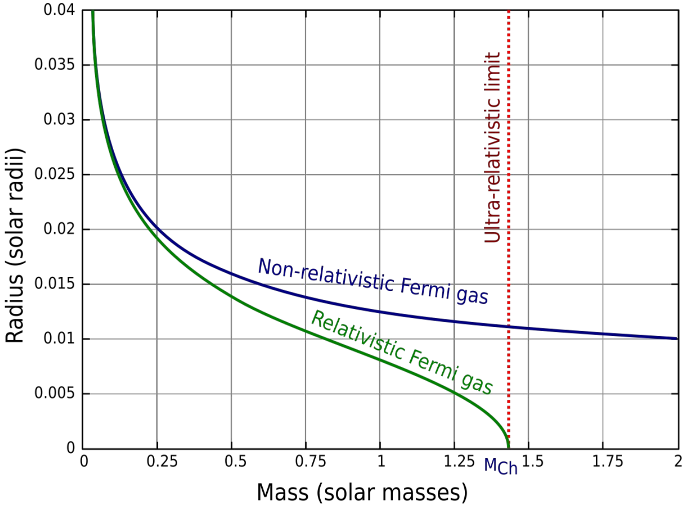
$$P_c \approx \frac{GM^2}{R^4} \propto \rho^\gamma \propto \left(\frac{M}{R^3}\right)^\gamma \Rightarrow M^{\gamma-2} \propto R^{3\gamma-4}$$

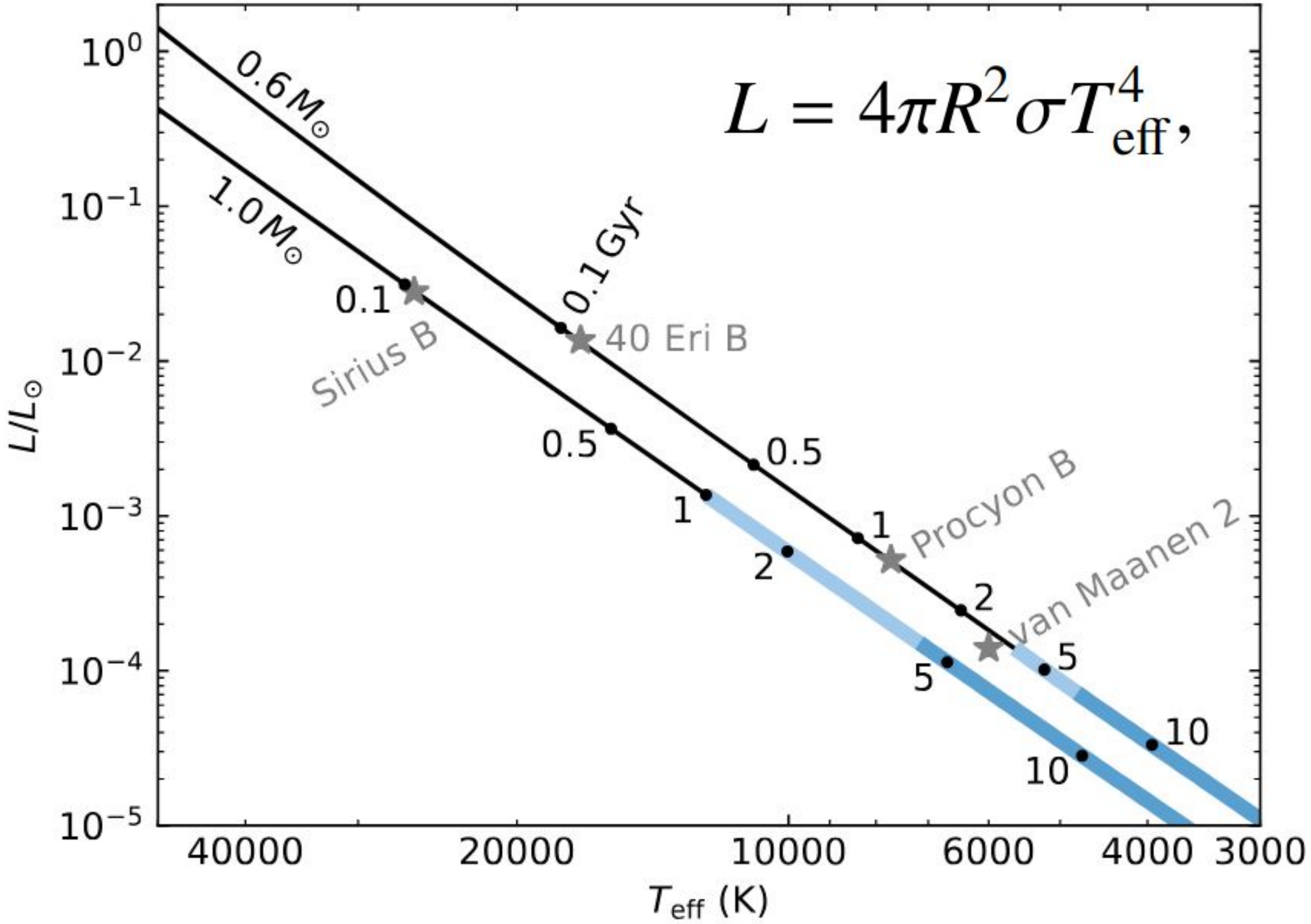
$$P = \frac{\left(3\pi^2\right)^{2/3}}{5}\frac{\hbar^2}{m_e}n_e^{5/3} \Rightarrow R \propto M^{-1/3}$$

$$P = \frac{\left(3\pi^2\right)^{1/3}}{4}\hbar c n_e^{4/3}$$



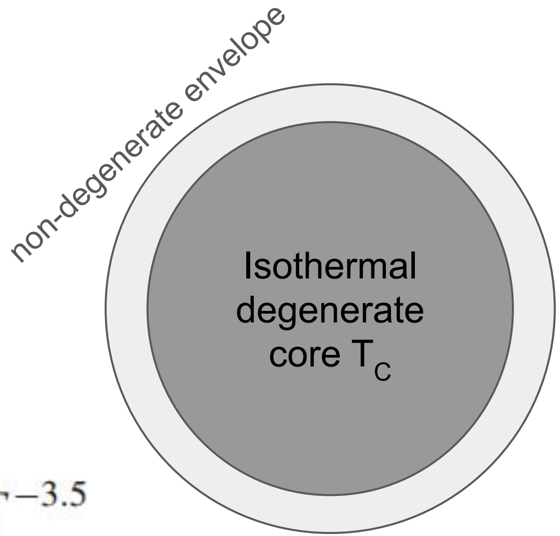






$L = 4\pi R^2 \sigma T_{\mathrm{eff}}^4,$
$0.6\,M_\odot$
$1.0\,M_\odot$
0.1 Gyr
0.1
0.5
1
2
5
10
Sirius B
40 Eri B
Procyon B
van Maanen 2
$L/L_\odot$
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
T_{eff} (K)
40000
20000
10000
6000
4000
3000

The Mestel (1952) cooling model



We start from HSE and photon diffusion equation

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{Gm\rho}{r^2}, \qquad \frac{\mathrm{d}T}{\mathrm{d}r} = -\frac{3}{4ac}\frac{\kappa\rho}{T^3}\frac{L_r}{4\pi r^2},$$

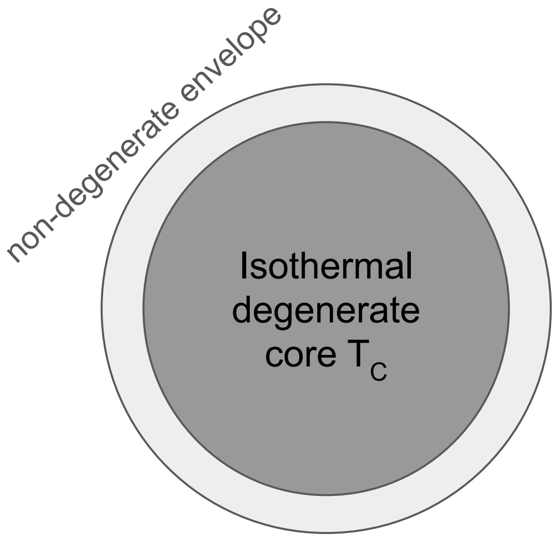
We divide these two equations and approximate $\kappa = \kappa_0 \rho\, T^{-3.5}$

$$\frac{\mathrm{d}P}{\mathrm{d}T} = \frac{16\pi ac}{3\kappa_0}\frac{GMT^{6.5}}{\rho L}$$

We can now integrate assuming an ideal gas EOS $P = \frac{\Re}{\mu}\rho T$ with $\mu^{-1} = \sum_i (1+Z_i)\frac{X_i}{A_i}$

$$P\,\mathrm{d}P = \frac{16\pi ac}{3\kappa_0}\frac{\Re GM}{L\mu}T^{7.5}\,\mathrm{d}T$$

The Mestel (1952) cooling model (cont'd)



$$P\,\mathrm{d}P = \frac{16\pi ac}{3\kappa_0}\frac{\Re GM}{L\mu}T^{7.5}\,\mathrm{d}T$$

Integrating from the surface (P=0, T=0) to the transition layer, we get the relation

$$\rho_{\mathrm{tr}} = \left(\frac{32\pi ac}{8.53\kappa_0}\frac{\mu}{\Re}\frac{GM}{L}\right)^{1/2} T_{\mathrm{tr}}^{3.25}$$

But this is also where T=Tc and $P = \frac{\left(3\pi^2\right)^{2/3}}{5}\frac{\hbar^2}{m_e}n_e^{5/3} = \frac{\Re}{\mu_e}\rho_{\mathrm{tr}}T_{\mathrm{tr}}$

$$L = 5.7\times10^5\,\frac{\mu}{\mu_{\mathrm{e}}^2}\,\frac{1}{Z(1+X)}\,\frac{M}{M_\odot}\,T_{\mathrm{c}}^{3.5}\ \mathrm{erg/s}$$

The Mestel (1952) cooling model (cont'd)

$$L = 5.7 \times 10^5 \, \frac{\mu}{\mu_e^2} \, \frac{1}{Z(1+X)} \, \frac{M}{M_\odot} \, T_c^{3.5} \ \mathrm{erg/s}$$

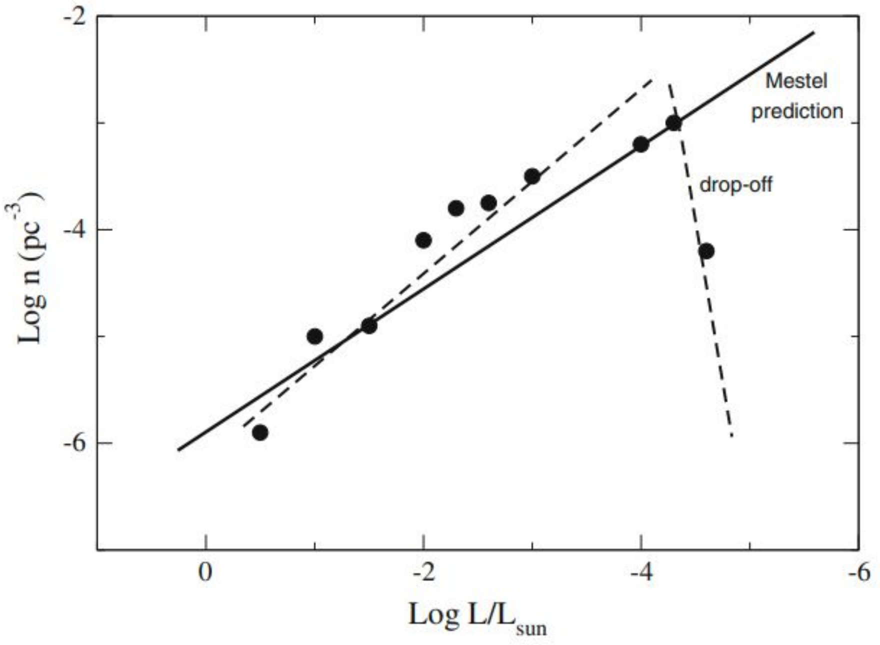
$$L = -\frac{\mathrm{d}E}{\mathrm{d}t} = -\frac{\mathrm{d}}{\mathrm{d}t}(E_{\mathrm{ion}} + E_{\mathrm{elec}} + E_{\mathrm{grav}}).$$

Gravitational contraction and the heat capacity of the electrons are small. We can also neglect the contribution of the envelope

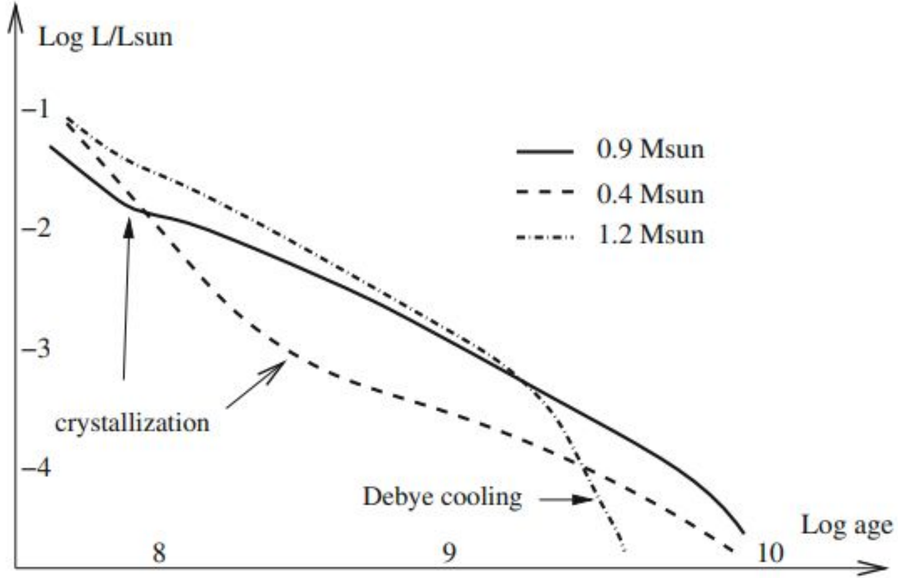
$$CMT_c^{3.5} = -\langle C_V^{\mathrm{ion}} \rangle M \frac{\mathrm{d}T_c}{\mathrm{d}t}$$

Assuming that the heat capacity is 1.5 kB per ion $C_V^{\mathrm{ion}} = 3\Re/2A$

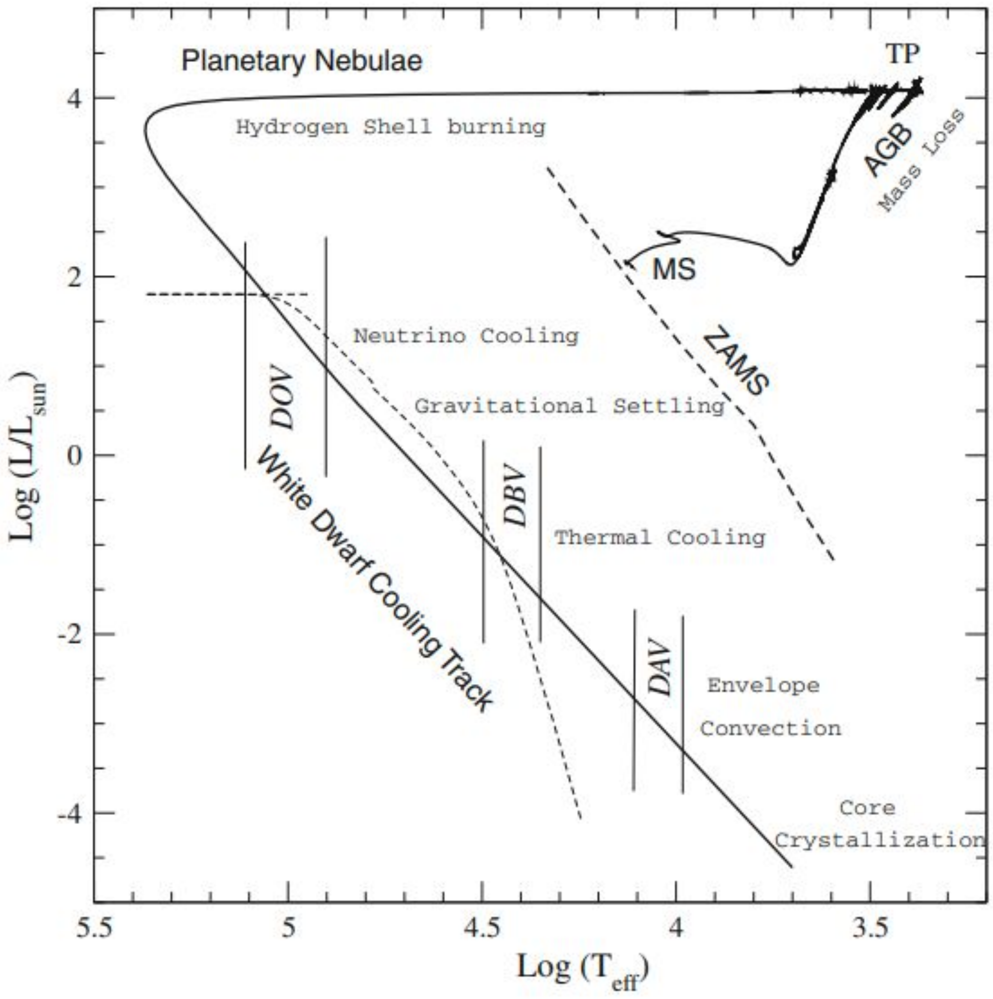
$$t_{\mathrm{cool}} \approx \frac{10^8}{A} \left(\frac{M/M_\odot}{L/L_\odot} \right)^{5/7} \ \mathrm{years}$$



-2
-4
-6
Log n (pc^{-3})
Mestel
prediction
drop-off
0
-2
-4
-6
Log L/L$_{sun}$



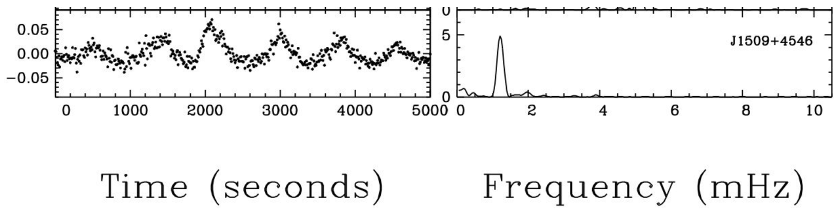
Log L/Lsun
−1
−2
−3
−4
0.9 Msun
0.4 Msun
1.2 Msun
crystallization
Debye cooling
Log age
8
9
10



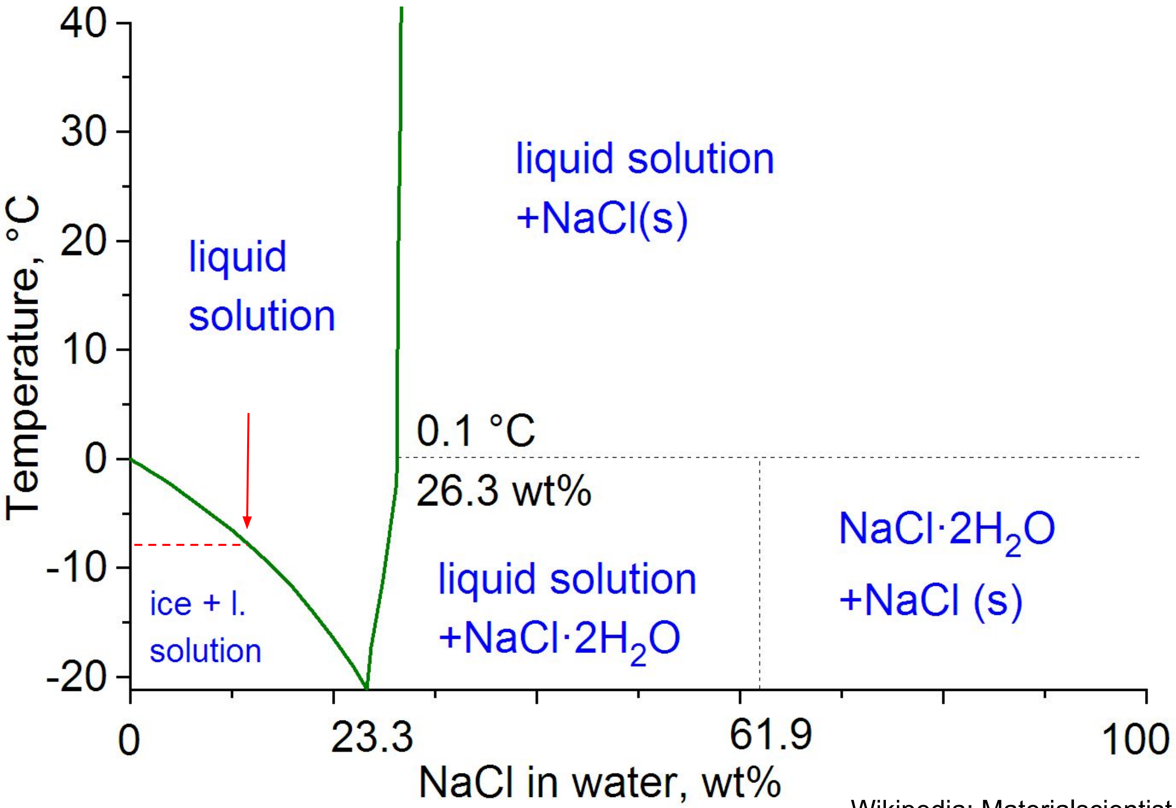
In partial ionization zones, the opacity *increases* during compression. This drives pulsations.

$$\bar{\kappa} \propto \rho T^{-7/2}$$

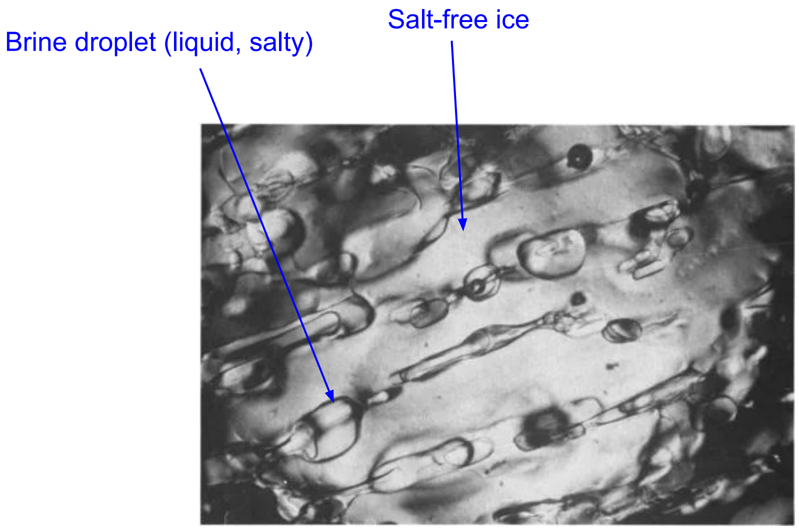
Radiation is trapped, further heating the layer. It eventually expands and cools down. The cycle repeats -> pulsations.



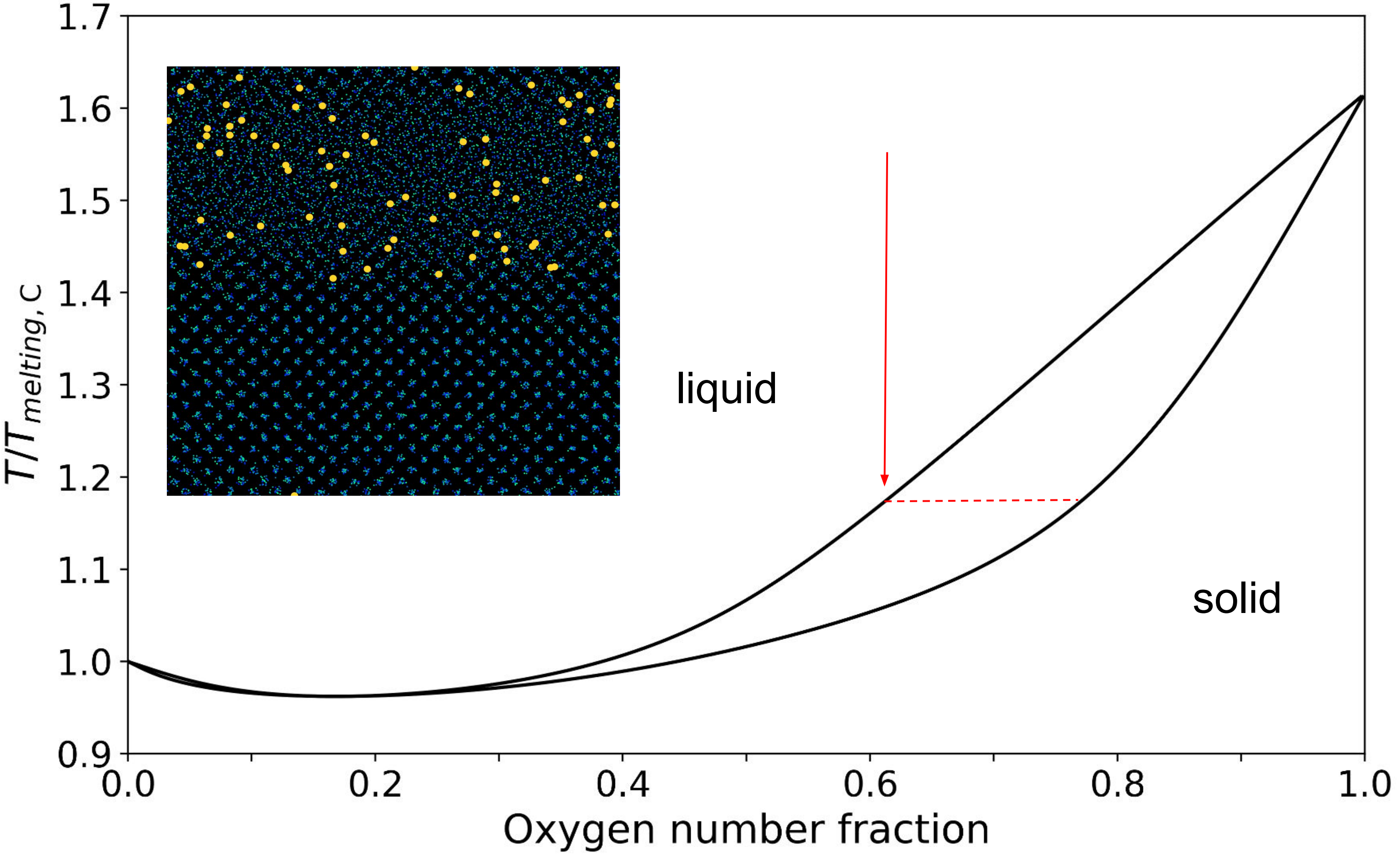
Phase diagram of NaCl + H_2O



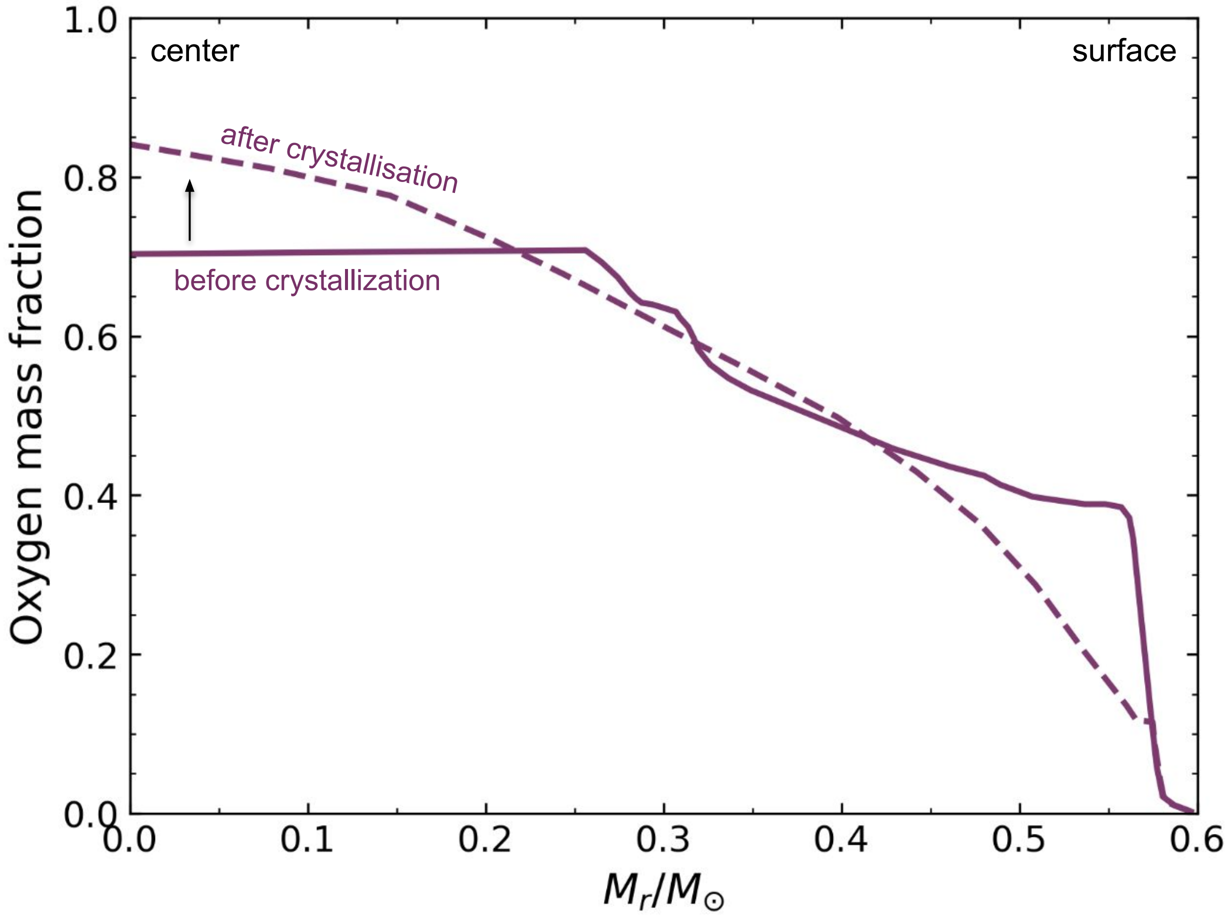
Wikipedia: Materialscientist

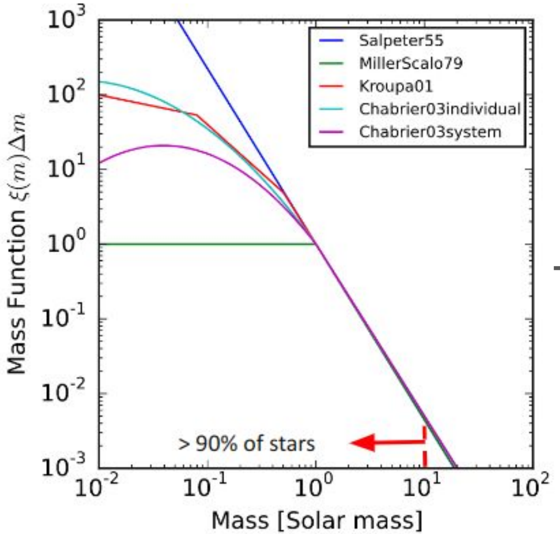


Phase diagram of C-O (in an electron degenerate plasma)

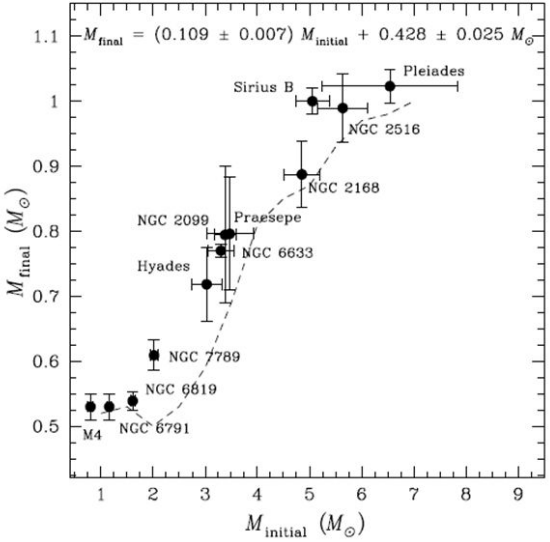


Change in composition profile due to C-O fractionation

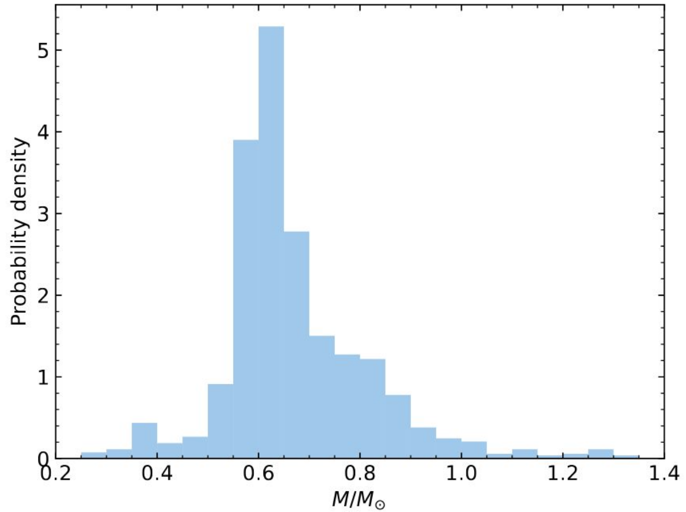


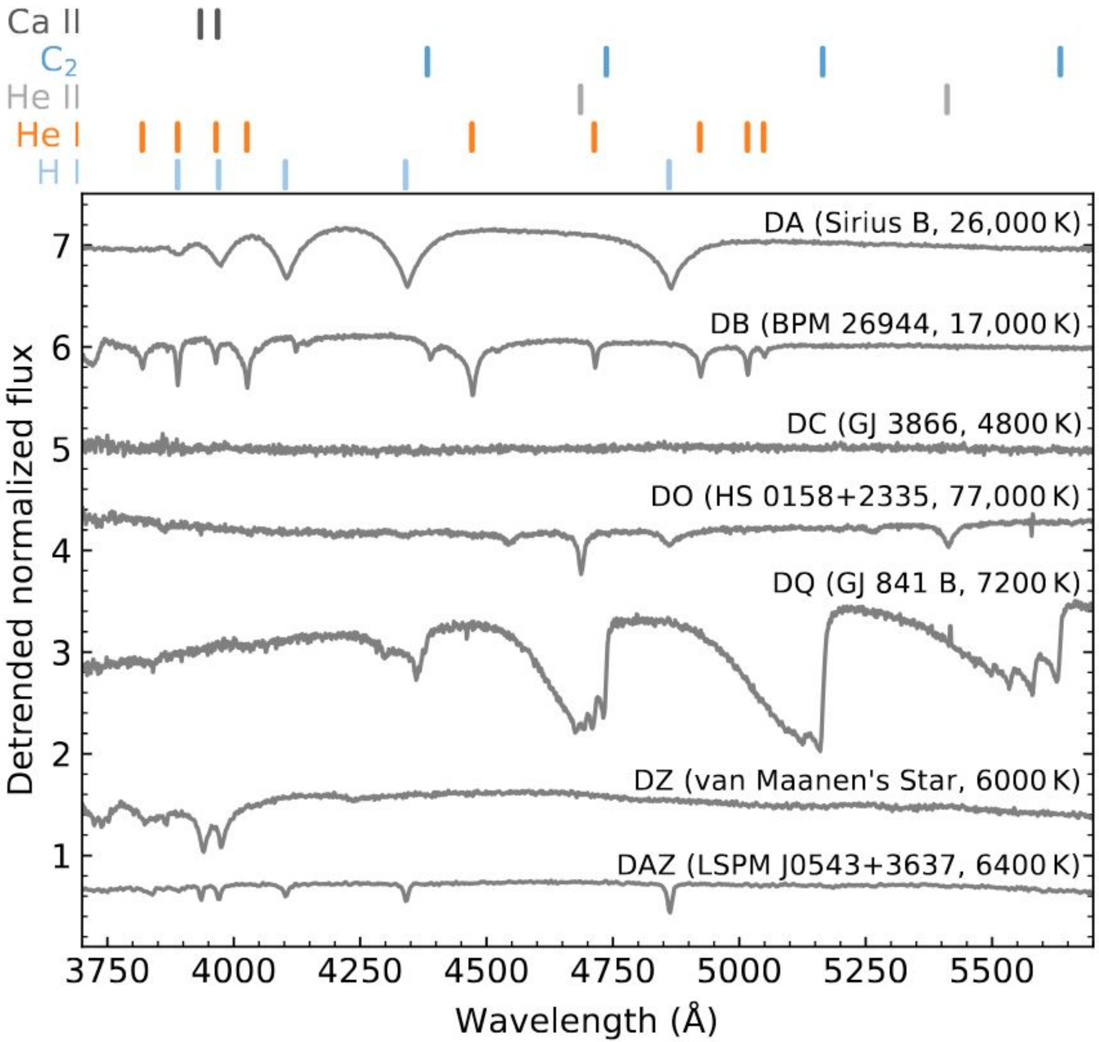

Salpeter55
MillerScalo79
Kroupa01
Chabrier03individual
Chabrier03system
> 90% of stars
Mass Function $\xi(m)\Delta m$
Mass [Solar mass]

+

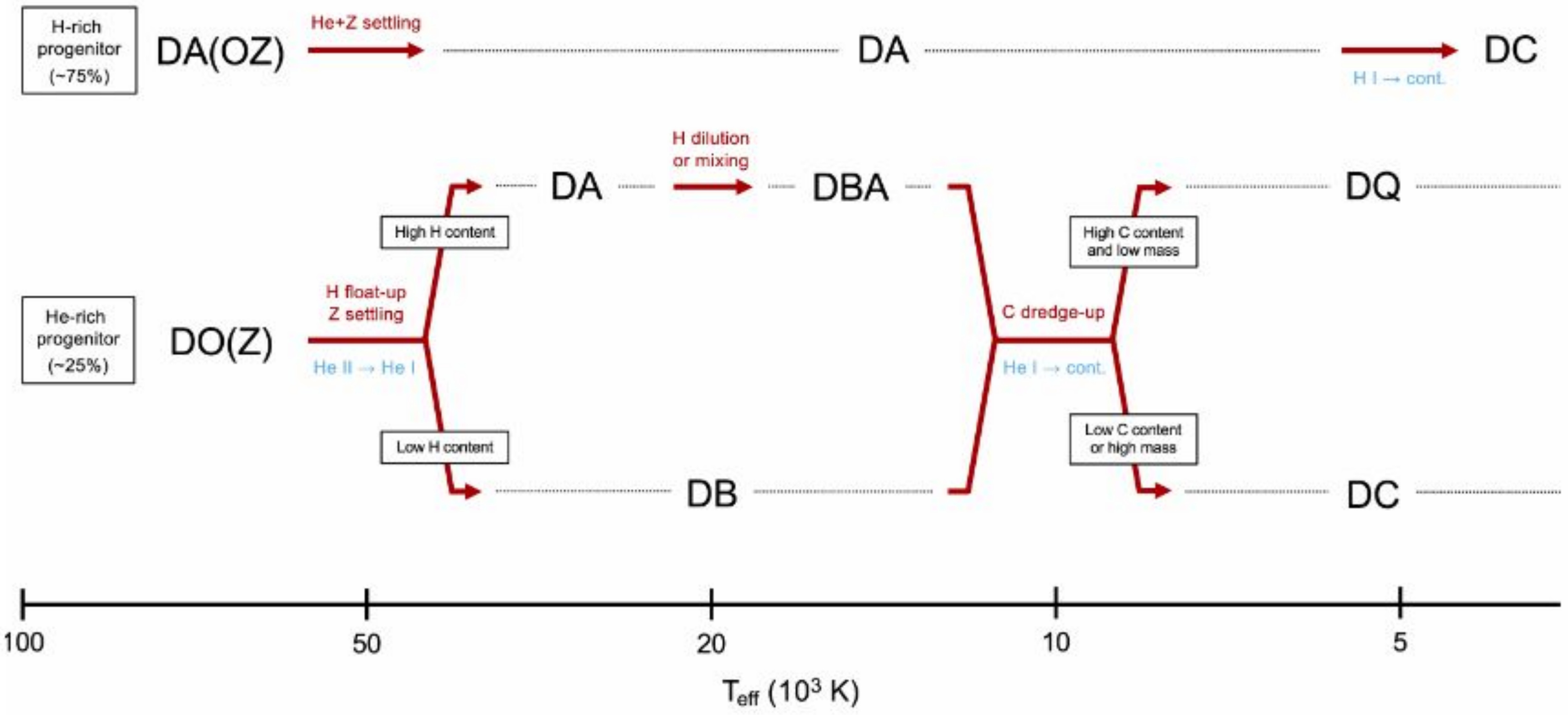

$M_{\rm final} = (0.109 \pm 0.007)\, M_{\rm initial} + 0.428 \pm 0.025\, M_\odot$
Pleiades
Sirius B
NGC 2516
NGC 2168
NGC 2099
Praesepe
NGC 6633
Hyades
NGC 7789
NGC 6819
NGC 6791
M4
$M_{\rm final}$ $(M_\odot)$
$M_{\rm initial}$ $(M_\odot)$

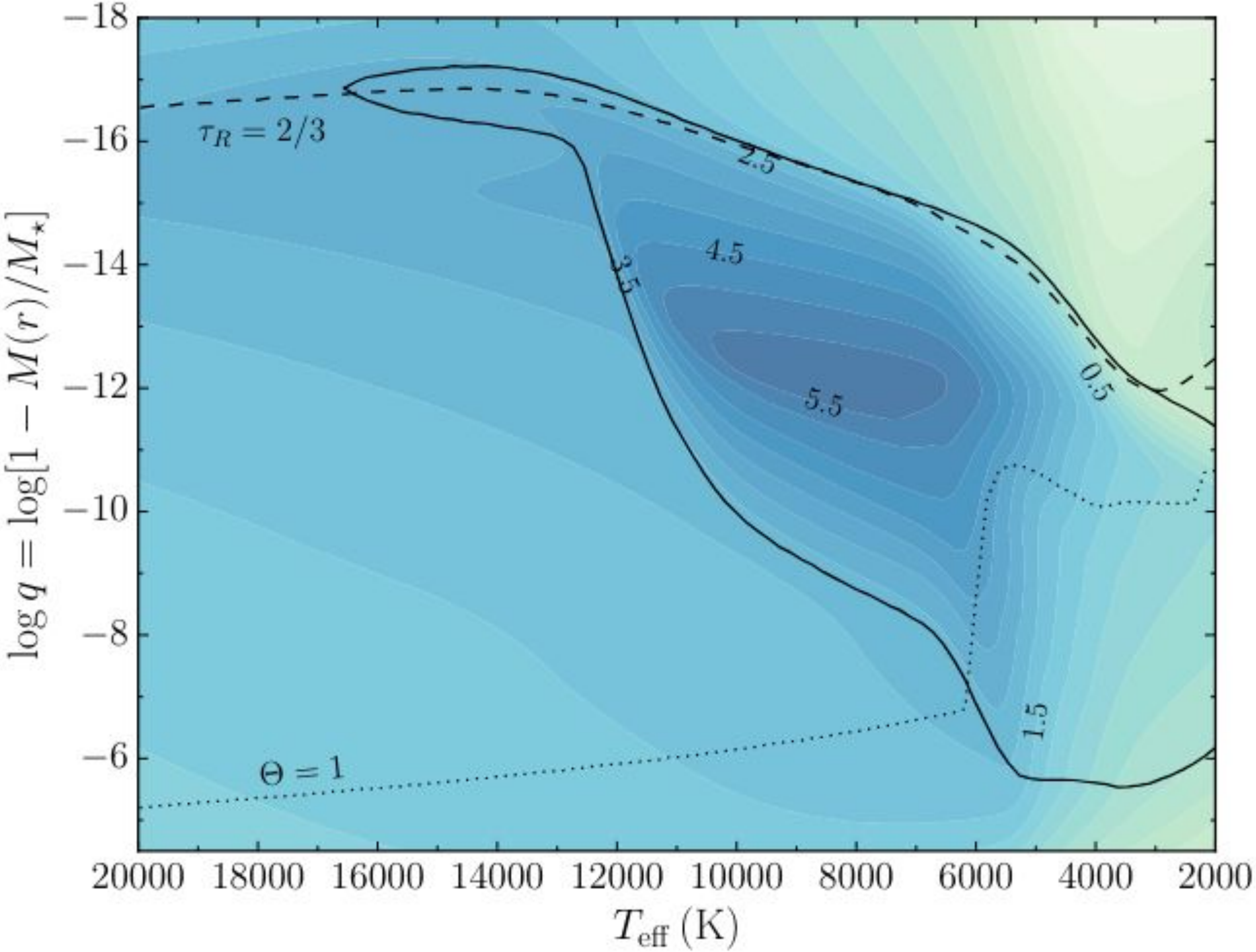
=


Probability density
$M/M_\odot$



Ca II
C2
He II
He I
H I
DA (Sirius B, 26,000 K)
DB (BPM 26944, 17,000 K)
DC (GJ 3866, 4800 K)
DO (HS 0158+2335, 77,000 K)
DQ (GJ 841 B, 7200 K)
DZ (van Maanen's Star, 6000 K)
DAZ (LSPM J0543+3637, 6400 K)
Detrended normalized flux
1
2
3
4
5
6
7
3750
4000
4250
4500
4750
5000
5250
5500
Wavelength (Å)


H-rich progenitor (~75%)
DA(OZ)
He+Z settling
DA
DC
H I → cont.
He-rich progenitor (~25%)
DO(Z)
H float-up
Z settling
He II → He I
High H content
Low H content
DA
H dilution or mixing
DBA
DB
C dredge-up
He I → cont.
High C content and low mass
Low C content or high mass
DQ
DC
100
50
20
10
5
Teff (10³ K)

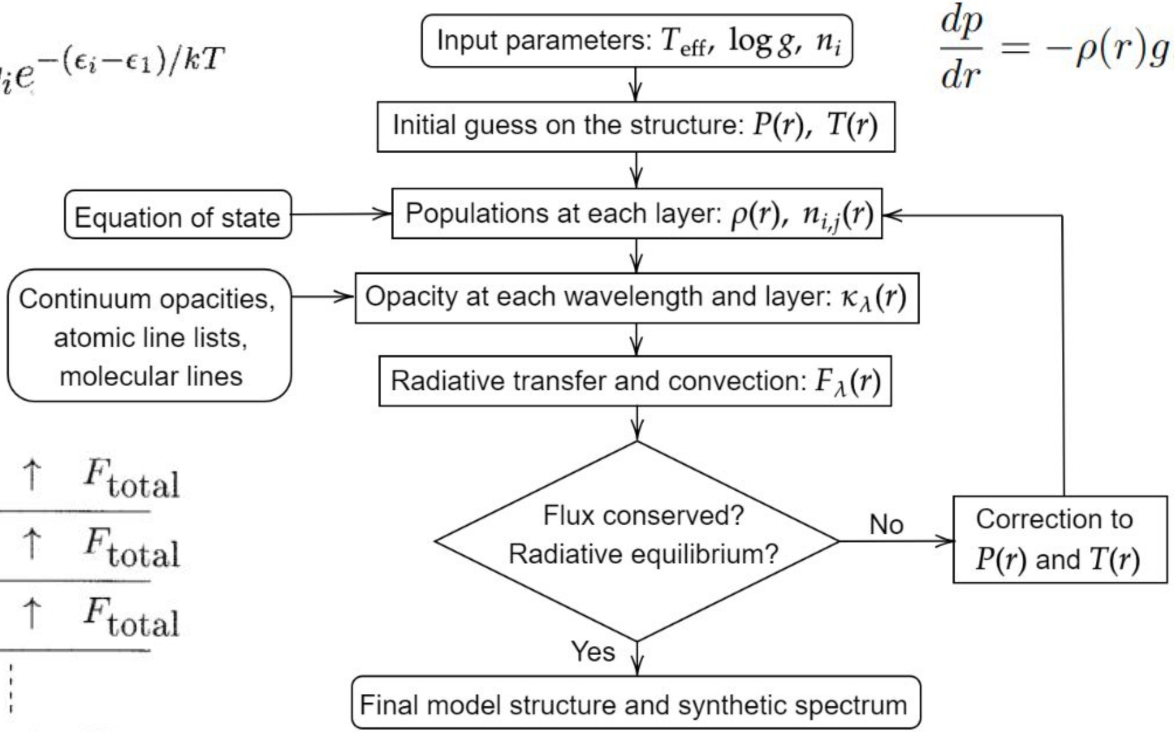


$\tau_R = 2/3$
2.5
3.5
4.5
5.5
0.5
1.5
$\Theta = 1$
$\log q = \log[1 - M(r)/M_\star]$
$T_{\rm eff}$ (K)
−18
−16
−14
−12
−10
−8
−6
20000
18000
16000
14000
12000
10000
8000
6000
4000
2000

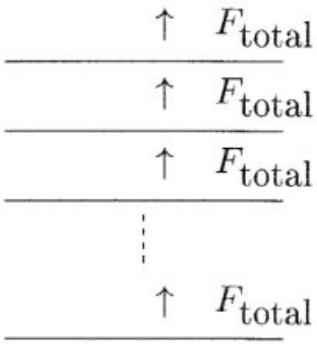
$$\frac{n_{Z+1} n_e}{n_Z} = \frac{z_{Z+1} 2}{z_Z} \left(\frac{2\pi m_e k T}{h^2} \right)^{3/2} e^{-\chi_Z / kT}$$

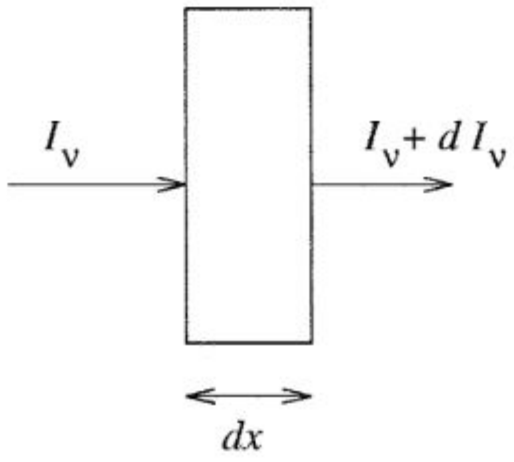
$$\frac{n_k}{n} = \frac{g_k e^{-(\epsilon_k - \epsilon_1)/kT}}{z} \qquad z \equiv \sum_i g_i e^{-(\epsilon_i - \epsilon_1)/kT}$$

$$\frac{dp}{dr} = -\rho(r) g$$



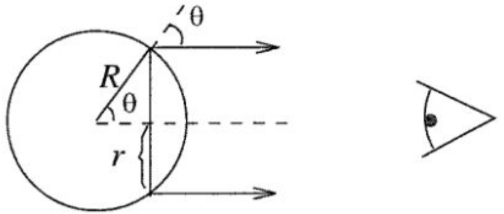
$$\frac{F_{\rm rad} + F_{\rm conv} - \sigma_S T_{\rm eff}^4}{\sigma_S T_{\rm eff}^4} \approx 0$$



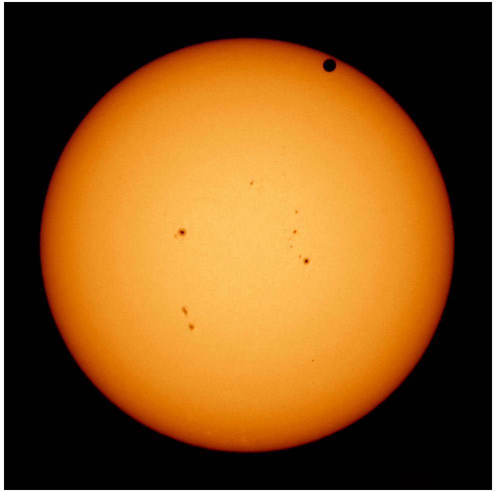


$$dI_\nu = -\underbrace{\kappa_\nu I_\nu \rho dx}_{\text{absorption}} + \underbrace{j_\nu \rho dx}_{\text{émission}} .$$

$$d\tau_\nu \equiv \kappa_\nu \rho dx \qquad S_\nu \equiv \frac{j_\nu}{\kappa_\nu}$$



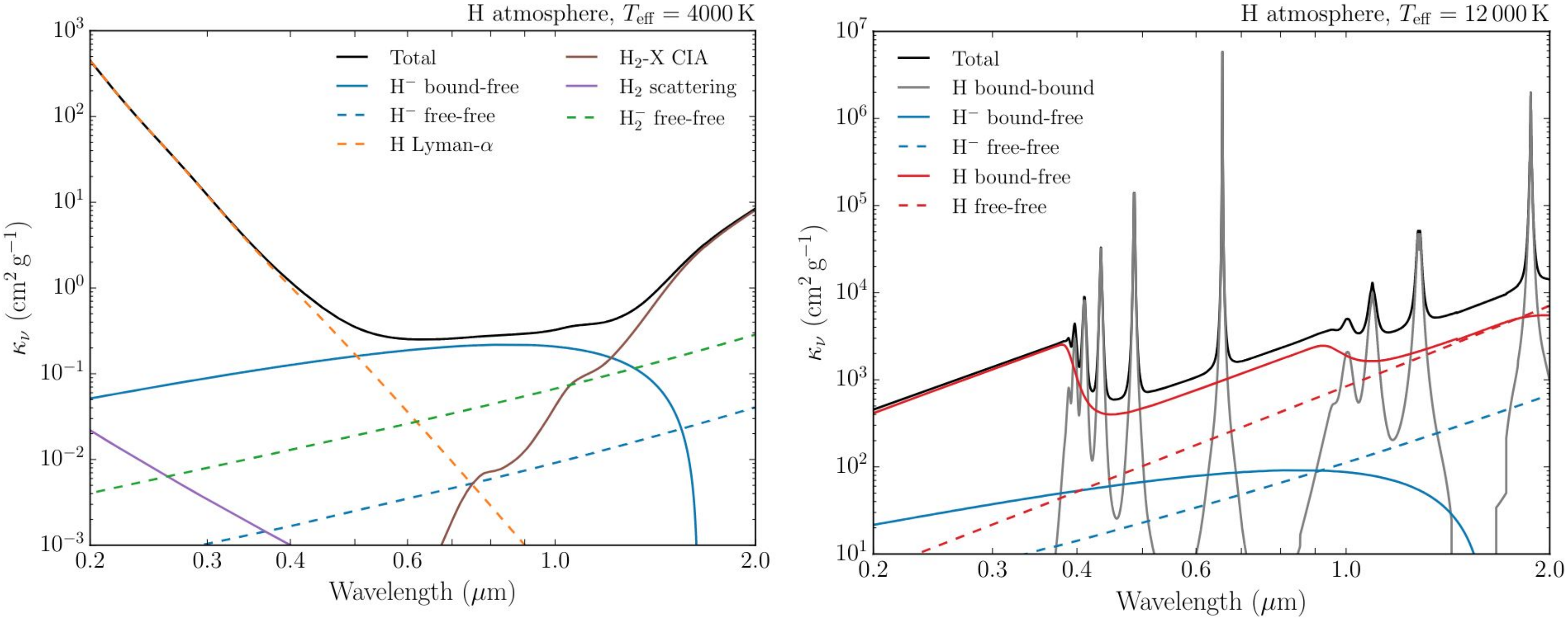
$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$$



$$dI_\nu = -\kappa_\nu I_\nu \rho dx$$

$$dI_\nu = j_\nu \rho dx$$

$$I_\nu(\tau_\nu) = \int_0^{\tau_\nu} S_\nu(t_\nu) e^{-(\tau_\nu - t_\nu)} dt_\nu + I_\nu(0) e^{-\tau_\nu}$$



H atmosphere, $T_{\rm eff} = 4000\,$K
Total
H^- bound-free
H^- free-free
H Lyman-α
H_2-X CIA
H_2 scattering
H_2^- free-free
κ_ν ($cm^2\,g^{-1}$)
Wavelength (μm)
H atmosphere, $T_{\rm eff} = 12\,000\,$K
Total
H bound-bound
H^- bound-free
H^- free-free
H bound-free
H free-free
κ_ν ($cm^2\,g^{-1}$)
Wavelength (μm)

Bound-bound transitions

$$\kappa_\nu(n', n) = \frac{N(n')}{\rho} \frac{\pi e^2}{mc} f(n', n) \varphi_\nu(n', n)$$

$$\int_0^\infty \varphi_\nu(n', n) d\nu = 1.$$

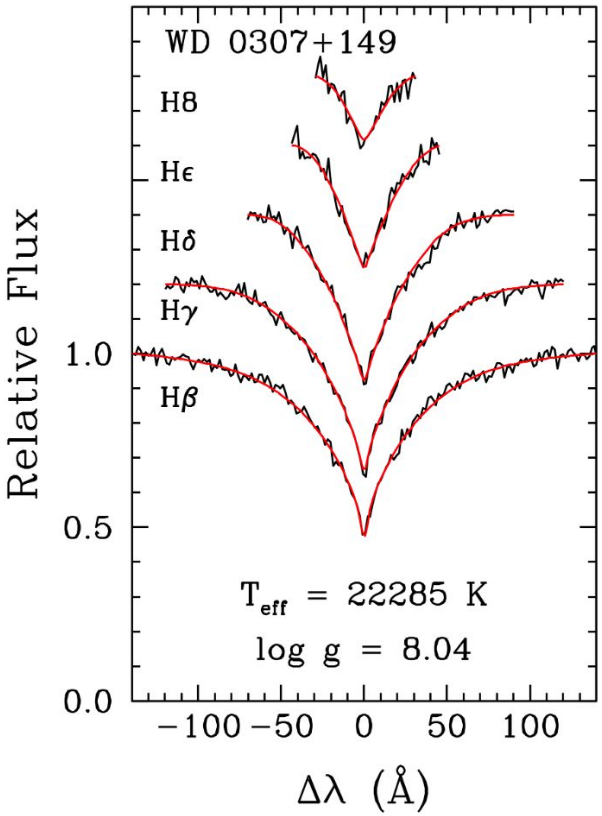
- “Natural” broadening (uncertainty principle)
- Doppler broadening
- Collisional broadening

$$\varphi_\nu(n', n) = \frac{\gamma(n', n)/(2\pi)}{(\Delta\nu)^2 + (\gamma(n', n)/2)^2}$$

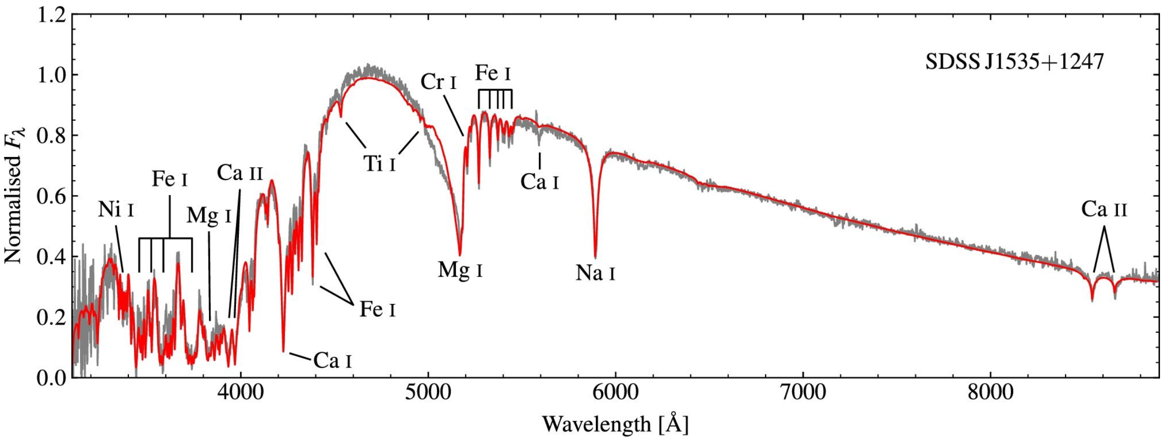
NIST Atomic Spectra Database Lines Data

Fe I: 10031 Lines of Data Found

Observed Wavelength Air (nm)	Unc. (nm)	Ritz Wavelength Air (nm)	Unc. (nm)	Rel. Int. (?)	A_{ki} (s^{-1})	f_{ik}	Acc.	E_i (cm^{-1})		E_k (cm^{-1})	Lower Level Conf., Term, J		
400.0252	0.0003	400.025248	0.000023	457bl	2.2e+06	5.2e-03	E	26 339.696	-	51 331.052	$3d^6(^5D)4s4p(^3P°)$	z $^5D°$	2
400.04570	0.00005	400.045722	0.000023	1480	1.01e+06	2.43e-03	C+	24 118.819	-	49 108.896	$3d^64s^2$	b ^3G	4
400.16615	0.00005	400.166094	0.000023	4470	7.47e+05	1.79e-03	C+	17 550.181	-	42 532.741	$3d^7(^4P)4s$	a ^5P	3



WD 0307+149
H8
Hϵ
Hδ
Hγ
Hβ
Relative Flux
1.0
0.5
0.0
−100 −50 0 50 100
Δλ (Å)
T_eff = 22285 K
log g = 8.04



SDSS J1535+1247
Normalised F_λ
Wavelength [Å]
1.2
1.0
0.8
0.6
0.4
0.2
0.0
4000
5000
6000
7000
8000
Ni I
Fe I
Mg I
Ca II
Fe I
Ca I
Ti I
Cr I
Fe I
Mg I
Ca I
Na I
Ca II