STAT/CSSS 564: Assignment 3

May 2nd, 2017

1. Fork this repository to your account
2. Edit the file `solutions.Rmd` with your solutions to the problems.
3. Submit a pull request to have it graded. Include either or both a HTML and PDF file.

For updates and questions follow the Slack channel: #assignment3.

This assignment will require the following R packages:

```
library("rstan")
```

1 Statistical Simulation

Read @KingTomzWittenberg2000a. They propose a statistical simulation approach for interpreting statistical analysis; see section "Simulation-Based Approaches to Interpretation". Compare and contrast this to a full Bayesian approach.

2 Student-t Prior

The robust regression with Student-t error example uses the following prior on the degrees of freedom parameter.

$$\nu \sim Gamma(2, 0.01)$$

The Student-t distribution is used because it has wider tails and thus is less sensitive to outliers than a normal distribution. However, the researcher generally has no information about the value of the degrees of freedom.

1. Plot this prior distribution, and the values of the 5th and 95th quantiles. You can use `dgamma(x, 2, scale = 0.01)` and `qgamma(x, 2, scale = 0.01)`. What is
2. Additionally, the prior is truncated at 2. Why? Hint: What moments of the Student-t distribution are not-defined for values between 2.

3 Student-t as a Mixture of Normals

The Student-t distribution is a scale mixture of normals.[1] This means that a Student-t distribution can be represented as normal distributions in which the variances are drawn from different distributions. Suppose X is distributed Student-t with degrees of freedom ν, location μ, and scale σ,

$$X \sim t_\nu(\mu, \sigma).$$

Samples from Y can be drawn by

$$x_i \sim N(\mu, \lambda_i^2 \gamma^2)$$

If the local variance parameters are distributed inverse-gamma

$$1/\lambda^2 \sim \text{Gamma}(\nu/2, \nu/2).$$

Many distributions used in regression shrinkage: Double Exponential (Laplace), and Hierarchical Shrinkage (Horseshoe), have this representation.

[1] mix

You can draw a sample from this:

```
df <- 10
n <- 1000
sigma <- rgamma(n, 0.5 * df, 0.5 * df)
x <- rnorm(n, sd = sqrt(1 / sigma ^ 2))
```

Plot samples drawn in this way against either samples or theoretical values of the Student-t distribution. Try a few values of the degrees of freedom. Try something small (3) and large (100).

You can draw samples directly from a Student-t with `rt`. A quantile-quantile plot (geom_qq) or a density plot with the function (geom_density and stat_function).

Note: there isn't a right answer to this. Well, actually, there is, and you know it. They are equivalent, a proof is in the link. So for credit, do a little work, and show it. This pattern appears often, so wrap your head around it.

4 Separation

Continue what was covered in class.

5 Transformations of Coefficients

@Rainey2016b notes that unbiased estimators of parameters does not imply that transformations of those parameters are unbiased estimators.