AA/EE/ME 548: Linear Multivariable Control

Lecture 03

4/7/2025

Announcements/reminders

- Sign-up sheet for homework/worked example
- Office hours
  - Karen: Wednesday 1PM (after lecture)
  - Oliver: Tuesday 4PM
  - May add more as the quarter goes on
  - Encourage using EdDiscussion otherwise (benefits the whole class)

- (Small) Typo in homework
  - Should be argmin instead of argmax

Last week

- State-space representation
  - States, controls, dynamics, continuous time vs discrete time, linearization
  - $x_{a:b} = x_a, x_{a+1}, \ldots, x_b, u_{a:b} = u_a, u_{a+1}, \ldots, u_b$
  - $\xi_{x_0,t_0}^{f,u(\cdot)}(t) = x(t)$, where $x(t_0) = x_0$
- Connection with Homework 1
  - Use numerical integration to get discrete-time dynamics (using Jax)
  - Leverage automatic differentiation to linearize DT dynamics

This week

- Intro to optimization
  - https://uw-ctrl.github.io/lmc-book/lectures/optimization.html

- Control Barrier Functions and Control Lyapunov Functions (connection with Lyapunov Theory from 547)

Combining CLFs and CBFs to safe trajectory planning

Xiao, W. and Belta, C., *High-Order Control Barrier Functions,* IEEE Transactions on Automatic Control, Vol. 67, No. 7, 2022

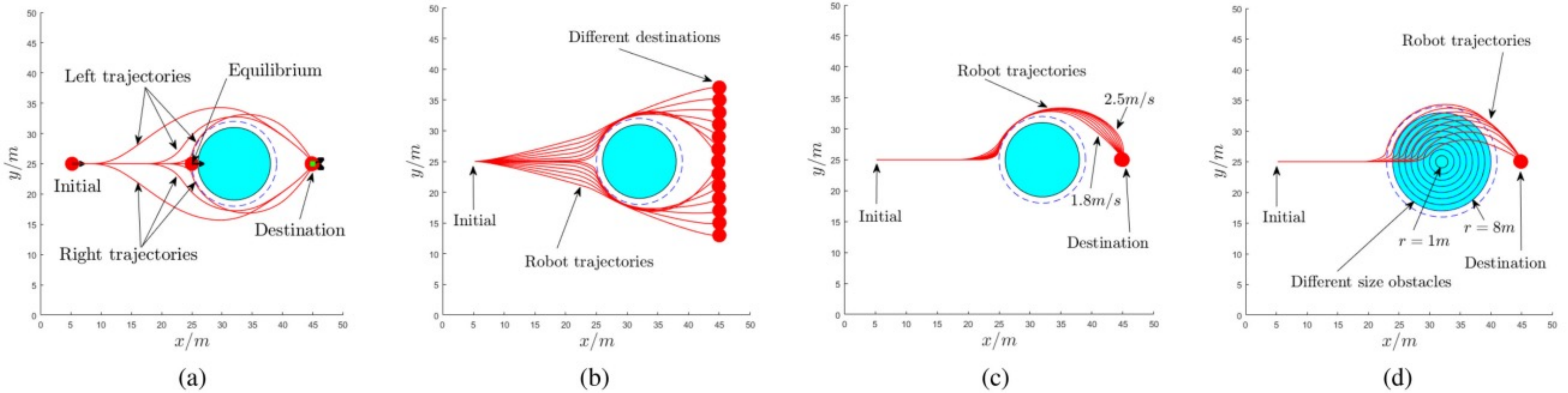


Fig. 3. Robot control problem using HOCBFs and the parameterization method. (a) Trajectories under different parameters. (b) Trajectories under different obstacle-approaching angles. (c) Trajectories under different obstacle-approaching speeds. (d) Trajectories for obstacles of different sizes.

https://ieeexplore-ieee-org.offcampus.lib.washington.edu/document/9516971

Combining CLF with nonlinear MPC to improve stability

Grandia, R., Taylor, A. J., Singletary, A., Hutter, M. and Ames, A. D, Nonlinear Model Predictive Control of Robotic Systems with Control Lyapunov Functions, Robotics: Science and Systems, 2020

NMPC-β:

$$\begin{aligned}
\min_{X,U}\ & \beta V(x_N, t_N) + \sum_{k=0}^{N-1} \eta(x_k, t_k)^\top Q \eta(x_k, t_k) + \frac{1}{2} u_k^\top u_k && \text{(25a)}\\
\text{s.t}\quad & x_0 - \hat{x} = 0, && \text{(25b)}\\
& x_{k+1} - f_k^d(x_k, u_k) = 0, \quad k = 0, \dots, N-1, && \text{(25c)}\\
& \underline{u} \le u_k \le \bar{u}, \quad k = 0, \dots, N-1, && \text{(25d)}
\end{aligned}$$

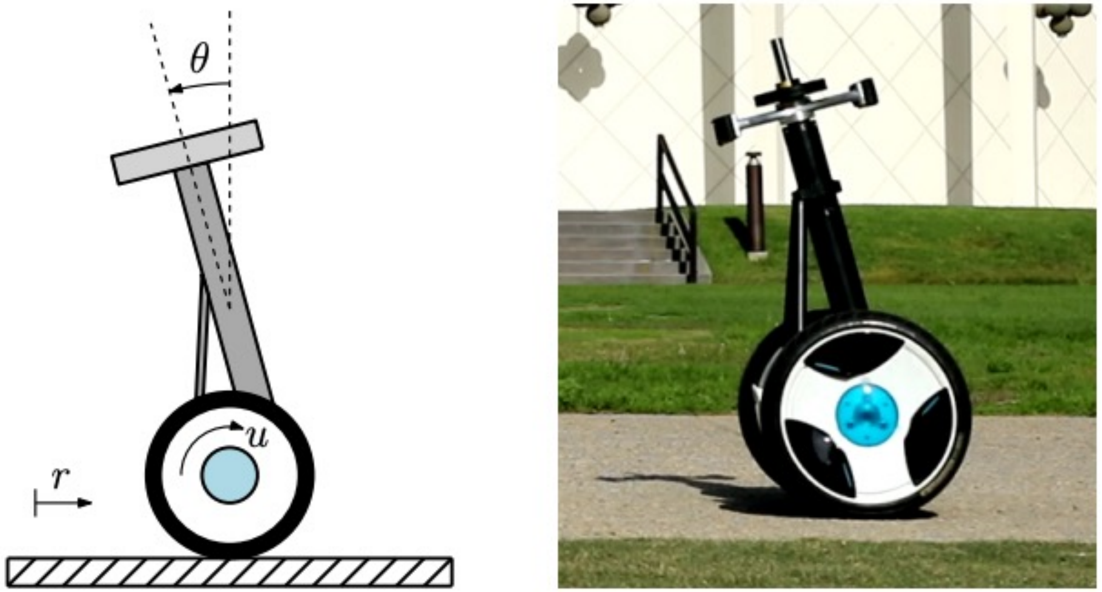
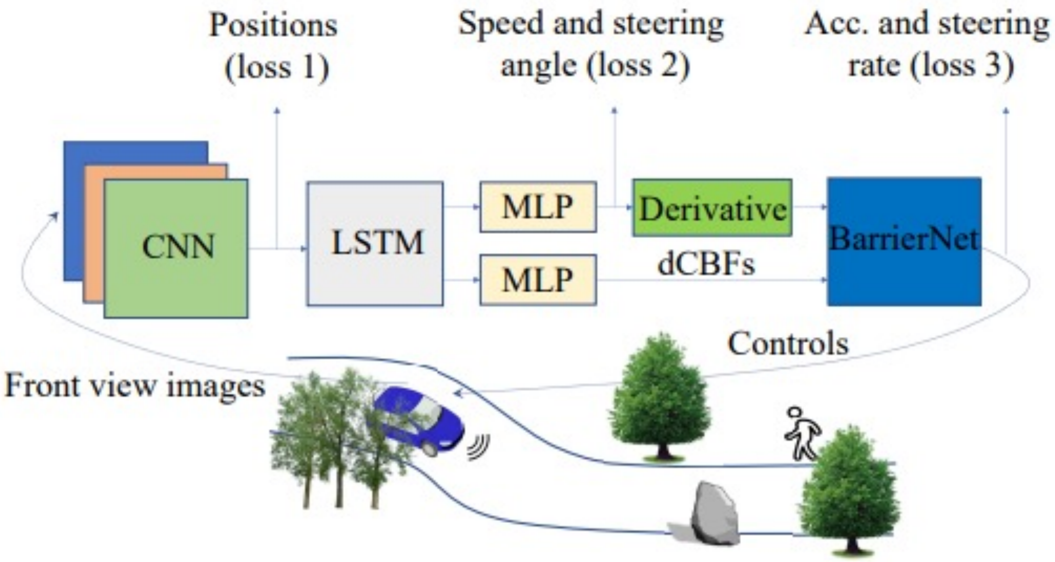


Fig. 1. Left: Segway model for simulation and control design. Right: Physical Segway system in outdoor experiment environment.

https://arxiv.org/abs/2006.01229

Combining CBFs with a vision-based neural controller

Xiao, W., Wang, T-H., Chahine, M., Amini, A., Hasani, R. and Rus, D., *Differential Control Barrier Function for Vision-based End-to-End Autonomous Driving, arXiv, 2022*



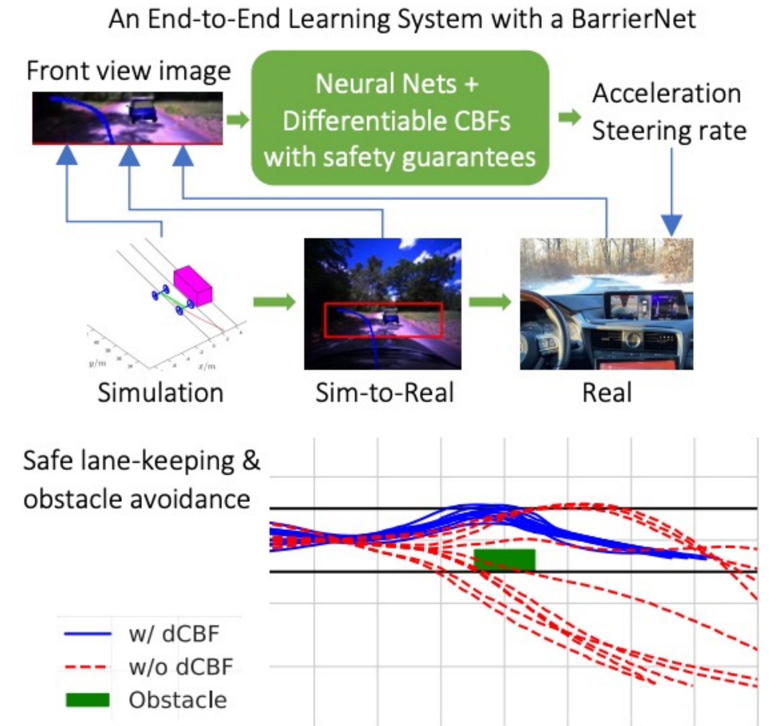
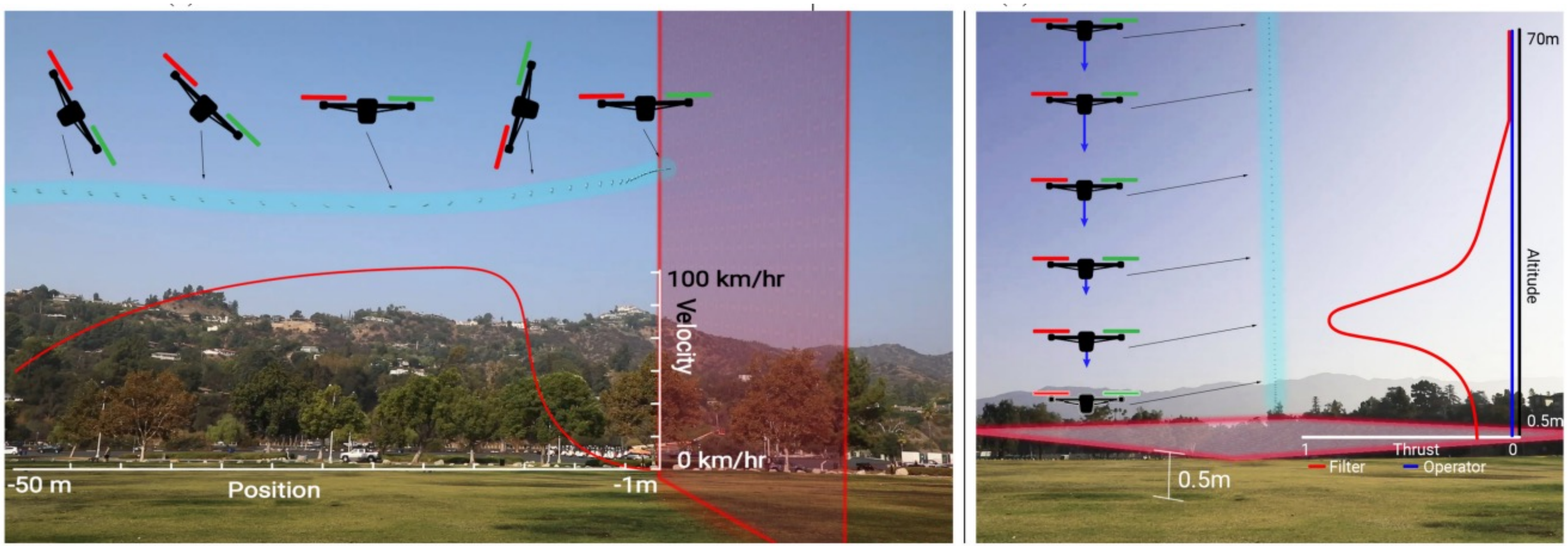


Fig. 1: Vision-based end-to-end autonomous driving with differentiable CBFs in a BarrierNet. Lane keeping and collision avoidance are guaranteed.

https://arxiv.org/abs/2203.02401

CBF-based geofencing for high-speed drones

Singletary, A., Swann, A., Chen, Y. and Ames, A. D., *Onboard Safety Guarantees for Racing Drones: High-Speed Geofencing With Control Barrier Functions*, IEEE Robotics and Automation Letters, Vol. 7, No. 2, 2022

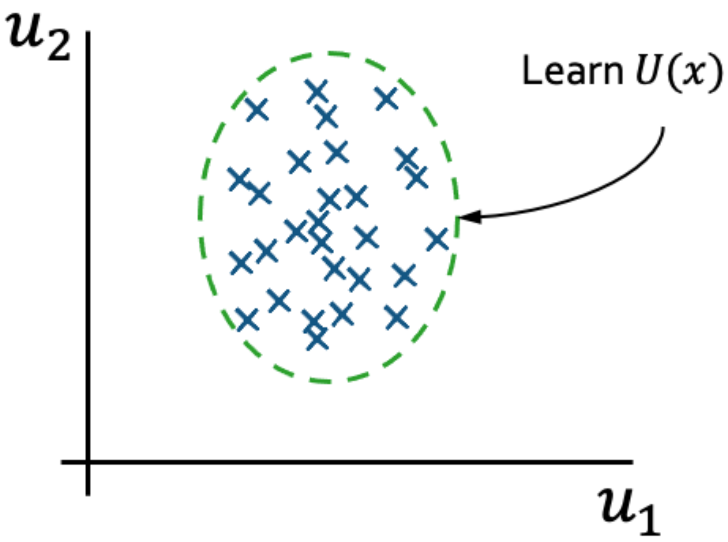


(c) Actual drone flight during the two showcased example is highlighted in blue.

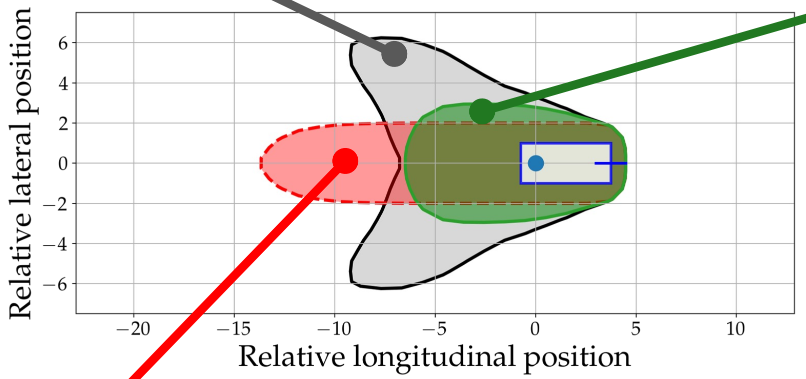
http://ames.caltech.edu/singletary2022onboard.pdf

Learning reasonable human behaviors using CBFs

Leung, K, Veer, S. Schmerling, E. and Pavone, M., *Learning Autonomous Vehicle Safety Concept from Demonstrations*, American Control Conference, 2023



Worst case analysis: can be over-conservative



Data-informed: generated by propagating learned control bounds through dynamics

Fixed policy (braking): assumes too much; over-optimistic

https://arxiv.org/abs/2210.02761

Combining preference learning with safety-critical control

Cosner, R. K., Tucker, M., Taylor, A. J., Li, K. Molnar, T. G., Ubellacker, W., Alan, A., Orosz, G., Yue, Y., and Ames, A. D., *Safety-aware Preference-based Learning for Safety-critical Control, Learning for Dynamics and Control, 2022.*

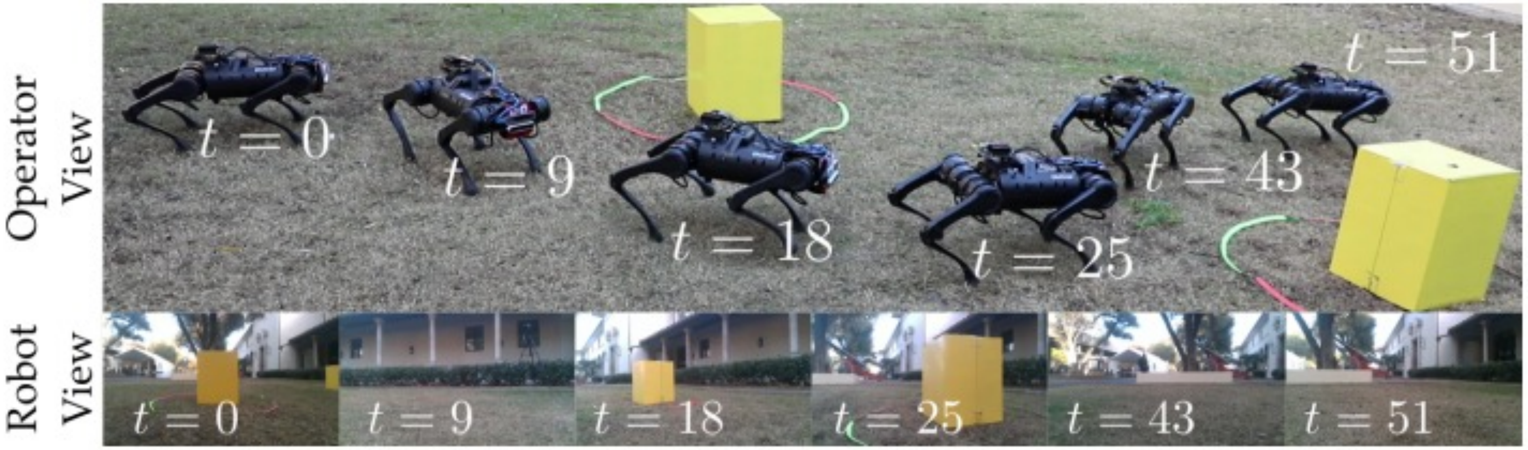


Figure 4. The preferred action, $\hat{\mathbf{a}}^*_{40} = (5, 0.1, 0.4, 0.02)$, after simulation, indoor experiments, and 3 additional iterations of 3 actions in an outdoor environment is shown alongside views from the onboard camera.

https://arxiv.org/abs/2112.08516

Using CBFs to learn different driving responsibility allocations from data

Cosner, R. K., Chen, Y., Leung, K. and Pavone, M., *Learning Responsibility Allocations for Safe Human-Robot Interaction with Applications to Autonomous Driving,* IEEE International Conference on Robotics and Automation, 2023

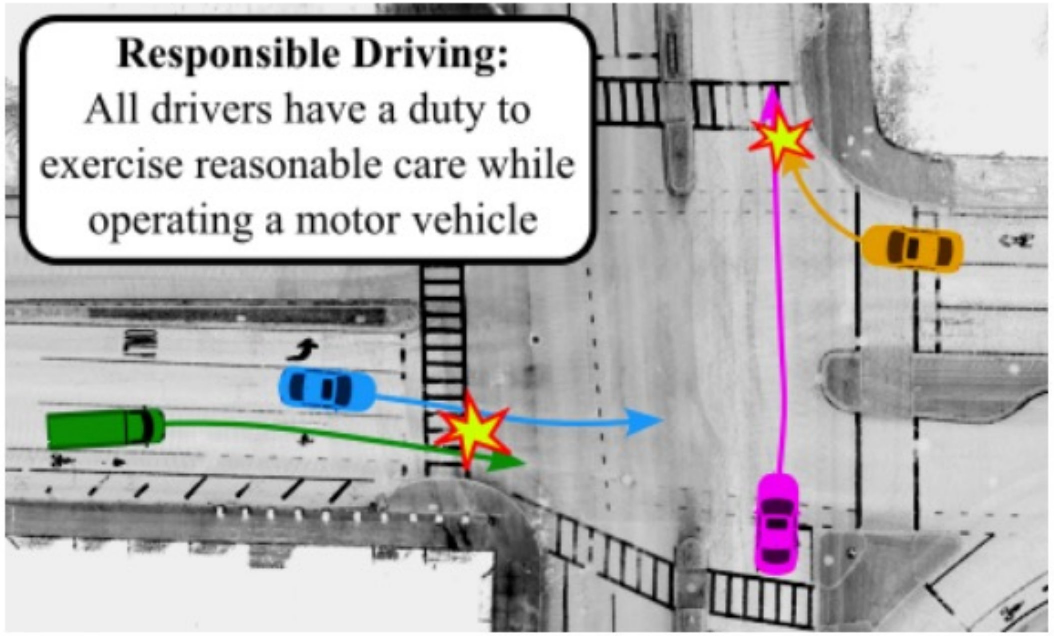
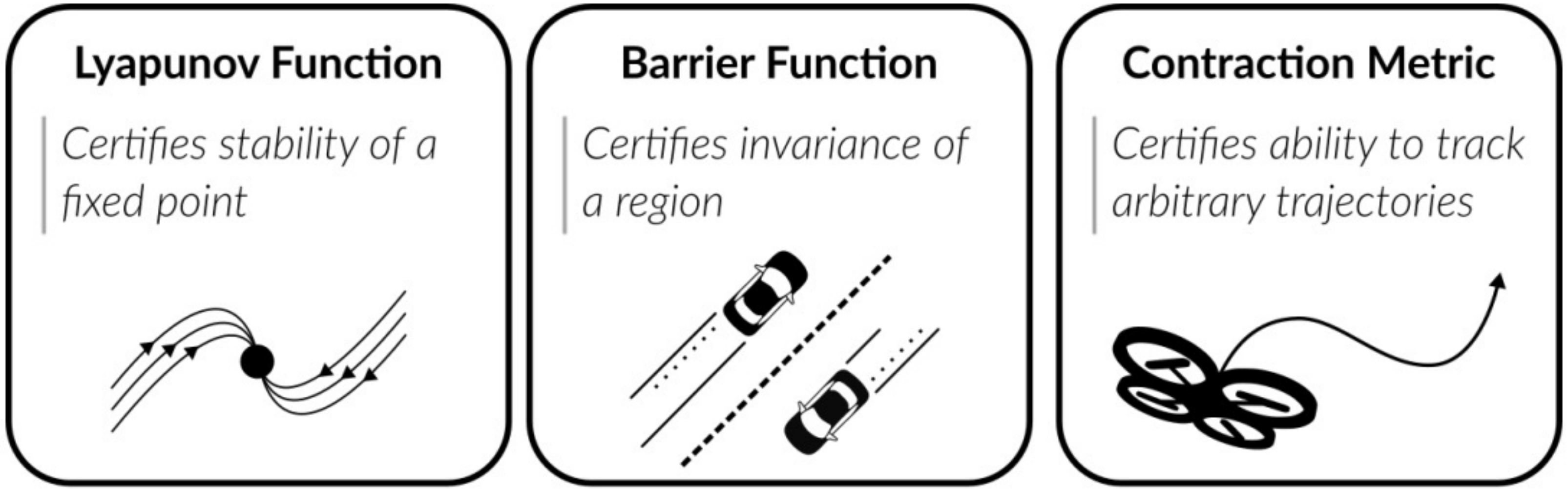


Fig. 1. In human driving, vehicles can be expected to demonstrate a reasonable duty of care. For example, a trailing vehicle (green) takes responsibility for not colliding with the car in front of it (blue) and a merging vehicle (orange) follows formal and informal rules to avoid a collision with the vehicles in the lane (pink). How can we ensure that autonomous vehicles act according to such informal driving etiquette?

https://arxiv.org/abs/2303.03504

Safe Control With Learned Certificates: A Survey of Neural Lyapunov, Barrier, and Contraction Methods for Robotics and Control



https://arxiv.org/abs/2202.11762