

Day 2, Session 2: Logs/Exponentiation

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Exponentiation

- A mathematical operation corresponding to repeated multiplication
- The second in the order of operations! (PEMDAS)
- Composed of two numbers: a base, b , and an exponent, n
- $b^n = \underbrace{b \times b \times \cdots \times b}_{n \text{ times}}$

Positive vs negative exponents

- Exponents correspond to multiplication
- Positive exponent: multiplication, e.g. $2^2 = 2 \times 2$
- Negative exponent: multiplication of reciprocals, i.e.
 $2^{-2} = \frac{1}{2} \times \frac{1}{2}$

Properties of exponents

- For any base b and any n an integer:
 - $b^0 = 1$
 - $b^1 = b$
 - $b^{n+1} = b^n \times b$
- For $b \neq 0$ and any n an integer:
 - $b^n = b^{n+1}/b$
 - $b^{-n} = 1/b^n$

Exponent identities

- For all $b, c \neq 0$:
 - $b^{m+n} = b^m \times b^n$
 - $b^{m \times n} = (b^m)^n$
 - $(b \times c)^n = b^n \times c^n$

Example: integer exponent properties and identities

- Take $b = 2$
- $2^0 = 1$
- $2^1 = 2$
- $2^2 = 2 \times 2$
- $2^3 = 2^2 \times 2 = 8$
- $(2 \times 3)^2 = 2^2 \times 3^2 = 4 \times 9 = 36$ (check:
 $2 \times 3 = 6, 6^2 = 36$)

Rational exponents (roots)

- n th root of b : the number x such that $x^n = b$
- Written as $b^{1/n}$ or $\sqrt[n]{b}$
- Some identities (for b positive):
 - $b = (b^n)^{1/n}$
 - $b^{m/n} = (b^m)^{1/n} = \sqrt[n]{b^m}$

Exponential function

- An important constant: e , approximately 2.718
- Useful as a base for powers
- Define $\exp(x) = e^x$
- Useful identity: $\exp(x + y) = e^{x+y} = \exp(x) \times \exp(y)$

Exercise: exponents and the exponential function

Solutions: exponents and the exponential function

Logarithms

- Exponents correspond to multiplication