

Day 2, Session 1: Order of operations and negative numbers

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EPI/BIOST Bootcamp 2016

26 September 2016

Outline for Session 1

- Order of operations
- Negative numbers
- Fractions
- Algebra
- Graphs

Evaluating expressions

- Example expression: $3(1 + 2) + 5$
- How do we evaluate the above expression? In other words:
 - Which terms do we compute first?
 - Are there rules for evaluating expressions?

Order of operations

- Rules for evaluating expressions:
 1. Parentheses
 2. Exponents
 3. Multiplication and division
 4. Addition and subtraction
- A handy memory device: PEMDAS — Please Excuse My Dear Aunt Sally

Example: order of operations in action!

- Example from slide 2: $3(1 + 2) + 5$
- This notation is equivalent to $3 \times (1 + 2) + 5$

- Apply PEMDAS:

1. Parentheses: add 1 and 2

2. Exponents: none

3. Multiplication: multiply 3 and 3

4. Division: none

5. Addition: add 9 and 5

6. Subtraction: none

Current Expression

$3(3) + 5$

$3(3) + 5$

$9 + 5$

$9 + 5$

14

- The final answer is 14!

Example: order of operations with exponents!

- Expression: $\frac{(2^2 + 5)^2}{3 \times 3} + 5$

- Apply PEMDAS:

1. Parentheses: $2^2 + 5$.

Need to apply PEMDAS again!

- 1.1 Parentheses: none

- 1.2 Exponents: $2^2 = 4$

- 1.3 Multiplication/division: none

- 1.4 Addition/subtraction: $4 + 5 = 9$

2. Exponents: $9^2 = 81$

3. Multiplication: $3 \times 3 = 9$

4. Division: $81/9 = 9$

5. Addition/subtraction: $9 + 5 = 14!$

Current Expression

$$\frac{(2^2+5)^2}{3 \times 3} + 5$$

$$\frac{(2^2+5)^2}{3 \times 3} + 5$$

$$\frac{(4+5)^2}{3 \times 3} + 5$$

$$\frac{(4+5)^2}{3 \times 3} + 5$$

$$\frac{(9)^2}{3 \times 3} + 5$$

$$\frac{81}{3 \times 3} + 5$$

$$\frac{81}{9} + 5$$

$$9 + 5$$

Order of operations: nesting

- Earlier, we needed to apply PEMDAS a second time, within the evaluation of the parentheses
- This is common!
- Apply PEMDAS as many times as necessary within each sub-expression, like $(2^2 + 5)$ in the previous example

Exercise: order of operations

- Try to work out the following examples by yourself or in pairs:

1. $(5 \times 6) + 4$

2. $5(4 - 2)^2$

3. $[(2 + 1)^2 + 1]^2$

Note on exercises

- Solutions for the exercises are usually in the slides
- However, please attempt them first without looking at the solution!

Solution: order of operations

1. $(5 \times 6) + 4 = 34$. PEMDAS:
 - Parentheses: 5×6 . Nested PEMDAS:
 - Multiplication: $5 \times 6 = 30$
 - Addition: $30 + 4 = 34$
2. $5(4 - 2)^2 = 20$. PEMDAS:
 - Parentheses: $4 - 2$. Nested PEMDAS:
 - Subtraction: $4 - 2 = 2$
 - Exponents: $2^2 = 4$
 - Multiplication: $5 \times 4 = 20$
3. $[(2 + 1)^2 + 1]^2 = 100$. PEMDAS:
 - Parentheses: $(2 + 1)^2 + 1$. Nested PEMDAS:
 - Parentheses: $2 + 1$. Nested PEMDAS — Addition:
 $2 + 1 = 3$
 - Exponent: $3^2 = 9$
 - Addition: $9 + 1 = 10$
 - Exponent: $10^2 = 100$

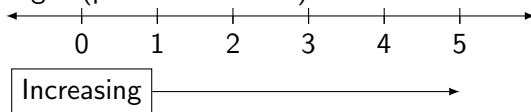
Negative numbers: what are they?

- Ways to think about negative numbers:
 - A positive number subtracted from zero
 - Opposites of positive numbers: $-4 + 4 = 0$
 - Movement left on the number line

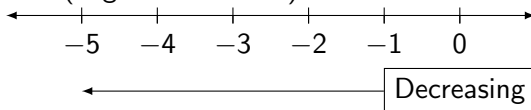
The number line

- Movement on the number line:

- Right (positive numbers):



- Left (negative numbers):



- Example: move left 3, starting at 100?
 - Subtract 3
 - Add negative 3

Properties of negative numbers

- Represent opposites of positive numbers, or movement left on the number line
- Subtraction = adding a negative number
- The product of two negatives is a positive
- Movement left on the numberline \rightarrow smaller numbers

Negative fractions

- First, positive fractions: for the same numerator, a larger denominator makes a smaller number, e.g. $1/4 < 1/2$
- Negatives are opposites: think of zero as a mirror
- So for the same numerator (a negative number), a larger denominator makes a more negative number, e.g. $-1/2 < -1/4$

Example: two negatives make a positive

- Expression: -1×-1
- Answer: 1!
- Why?
 - -1 is a negative number
 - Negative numbers mean opposites; the opposite of -1 is 1

Example: ordering negative numbers

- Expression: $-3 \text{ ___ } -2$
- Answer: $-3 < -2$
- Why?
 - Negative numbers are left motion on number line! -3 is further left than -2

Exercise: negative numbers

- Try to work out the following examples by yourself or in pairs:

1. $-5.2 - (-11.3)$

2. $-5 - 6$

3. $(-1) \times (-5) + (-3)$

4. $-\frac{3}{7} - \left(-\frac{1}{4}\right)$

Solution: negative numbers

1. $-5.2 - (-11.3) = 6.1$

- $-(-11.3) = -1 \times (-11.3) = 11.3$
- $-5.2 + 11.3 = 11.3 - 5.2 = 6.1$

2. $-5 - 6 = -11$

3. $(-1) \times (-5) + (-3) = 2$

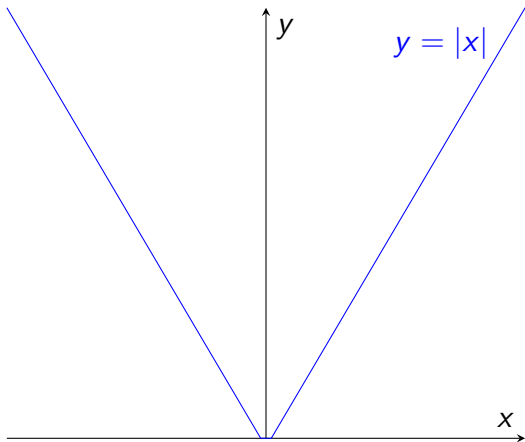
- $-1 \times (-5) = 5$
- $5 + (-3) = 5 - 3 = 2$

4. $-\frac{3}{7} - \left(-\frac{1}{4}\right) = -5/28 \approx -0.17$

- $-\frac{3}{7} - \left(-\frac{1}{4}\right) = \frac{3}{7} + \frac{1}{4}$
- $-\frac{3}{7} + \frac{1}{4} = -12/28 + 7/28 = -5/28$

Related concepts: absolute value

- Magnitudes: how “large” is a number, with no direction
 - Examples: speed (how fast an object is moving), length
- Symbol for absolute value is $|\cdot|$



Example: absolute value of a positive number

- Expression: $|4|$
- Answer is 4! Positive numbers already measure size, with no direction

Example: absolute value of a negative number

- Expression: $|-4|$
- Answer is 4!
- Why?
 - Negatives are opposites of positives
 - Absolute value has no direction
 - 4 and -4 are equally far away from zero

Related concepts: negative numbers and inequalities

- Expression from before: $-3 < -2$
- What happens if we multiply both sides by -1 ?
- Negatives are opposites: signs change and inequality flips, yielding $2 < 3$

Back to negative fractions

- Recall that for a fixed negative numerator, a larger denominator means a more negative number
- Absolute value makes this more clear!
- Example: $-1/2$, $-1/4$
 1. $|-1/2| = 1/2$, $|-1/4| = 1/4$
 2. We already know that $1/4 < 1/2$
 3. Multiply both sides by -1 , yielding (with inequality rules) $-1/2 < -1/4$

Exercise: absolute value, negative numbers

- Try to work out the following examples by yourself or in pairs:
 1. $|-5|$ and $|5|$
 2. Is $|-5| < 4$?
 3. Is $-15 > -14$?
 4. Is $-(3 + 1) \times 5 < -(4 + 1) \times 3$?

Solution: absolute value, negative numbers

1. $|-5| = |5| = 5$
2. $|-5| = 5$, and $5 > 4$; answer is no
3. -15 is further from 0 than -14 ; also, $14 \nmid 15$. Hence $-15 < -14$
4. Two ways to solve this:
 - $-(3+1) \times 5 = -1 \times (4) \times 5 = -20$, and $-(4+1) \times 3 = -1 \times (5) \times 3 = -15$. So $-20 < -15$
 - If $-(3+1) \times 5 < -(4+1) \times 3$, then $(3+1) \times 5 > (4+1) \times 3$. But this means $20 > 15$, which is true!

Summary

- The order of operations is a recipe for solving expressions
- A handy memory tool is PEMDAS: Parentheses, Exponents, Multiplication/Division, Addition/Subtraction
- Negative numbers decrease as we move away from zero (left on the number line)
- Absolute value measures the magnitude of a number — how far away from zero is it?