

# Day 2, Session 1: Graphs

Brian Williamson

EPI/BIOST Bootcamp 2016

26 September 2016

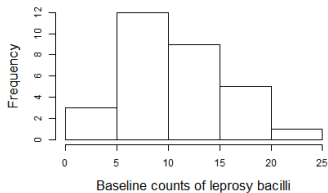
# Graphs

- Why do we use graphs?
  - Describe relationships in the data
  - Visualize functions
- Graphs are very useful in exploratory analyses, or for description

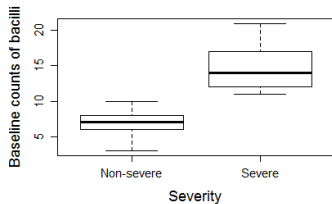
# Example data analysis

- Data on 30 patients with leprosy
- Counts of leprosy bacilli measured at baseline and at a further time point
- Three treatments and an indicator of severity of the leprosy

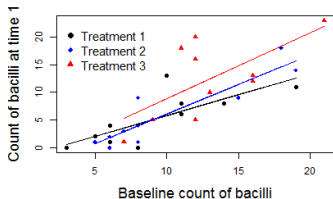
# Common types of graphs in data analysis



(a) Histogram



(b) Boxplot



(c) Scatterplot

# What do graphs tell us?

- Histograms: summaries of one-dimensional distributions
  - Counts or frequencies of each occurrence
- Boxplots: summaries of two-dimensional distributions
  - measures of center (typically median)
  - measures of spread (typically inter-quartile range)
- Scatterplots: summaries of two-dimensional distributions
  - Can visualize the whole data
  - Trends in two or more dimensions by using different colors/shapes for strata

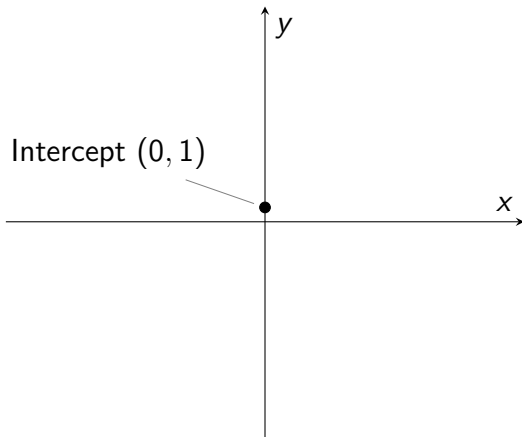
# Linear trends

- A common way to describe data (think linear regression!)
- Lines are easy to compute
  - Only need a point and a slope
  - Two common forms of linear equations

# Slope-intercept form

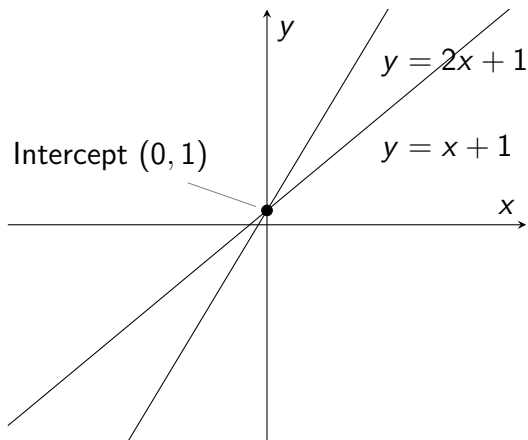
- $y = mx + b$
- Slope:  $m$ 
  - Rate of change, i.e. how does  $y$  change with each one unit change in  $x$ ?
  - Example: speed, the distance traveled with each unit change in time
- Intercept:  $b$ 
  - The point where the line crosses the  $y$ -axis

## Slope-intercept form: determining a line





## Slope-intercept form: determining a line



# Point-slope form

- $y - y_1 = m(x - x_1)$
- Point:  $(x_1, y_1)$ 
  - A point on the line (can be any point! Even the intercept!)
- Slope:  $m$ 
  - Same as in slope-intercept form!
- Example:  $y - 1 = 2(x - 0)$  is the same as  $y = 2x + 1$  in slope-intercept form!

## Exercise: slopes and intercepts

1. What is the slope of the line  $y = 2x - 3$ ?
2. What is the  $y$ -intercept of the line  $y = 2x - 3$ ?
3. What is the slope of the line  $y + 1 = 2(x - 1)$ ?
4. What point did we use to create the line  $y + 1 = 2(x - 1)$ ?

## Solution: slopes and intercepts

1. The equation is in slope-intercept form, so the slope is 2
2. The equation is in slope-intercept form, so the intercept is  $-3$
3. The equation is in point-slope form, so the slope is 2
4. The equation is in point-slope form, so the point is  $(1, -1)$

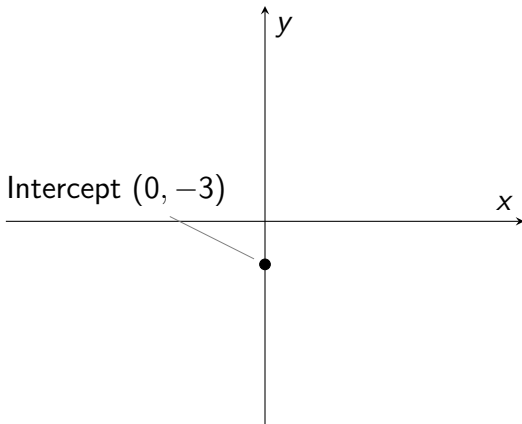
# Creating a graph using an equation

- Steps:
  1. Draw axes
  2. Place a point at the  $y$ -intercept (slope-intercept form) or at the starting point (point-slope form)
  3. Increase  $x$  by one unit, increase  $y$  by  $m$  units, place a new point
  4. Draw a line between the old point and the new point!

## Example: creating a graph using an equation

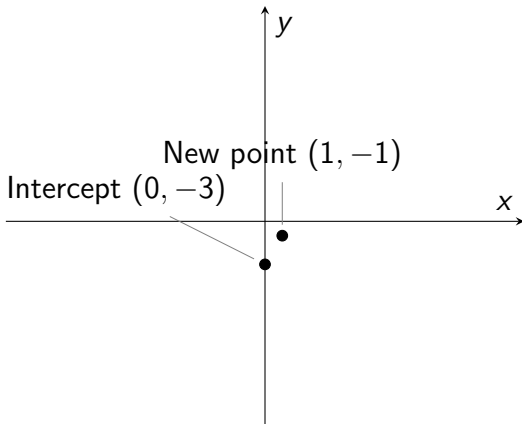
- Equation  $y = 2x - 3$
- Slope: 2, Intercept:  $-3$

1. Draw a point at  $(0, -3)$



## Example: creating a graph using an equation

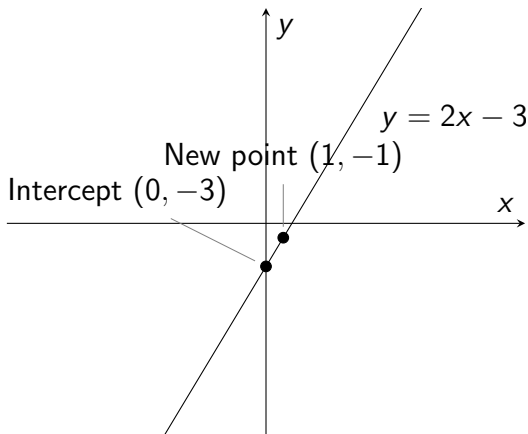
- Equation  $y = 2x - 3$
  - Slope: 2, Intercept:  $-3$
2. Increase  $x$  to 1, increase  $y$  by 2. New point at  $(1, -1)$



## Example: creating a graph using an equation

- Equation  $y = 2x - 3$
- Slope: 2, Intercept:  $-3$

3. Draw a line!

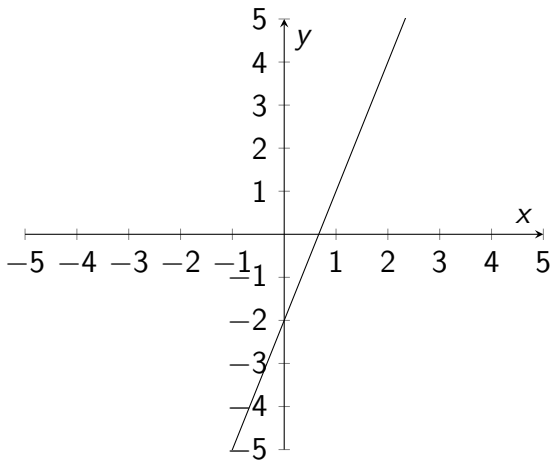




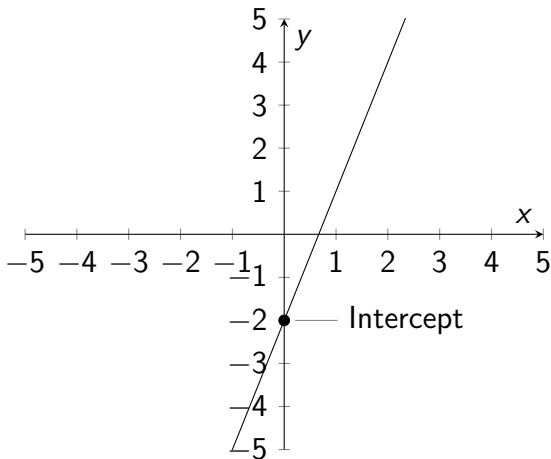
# Reading an equation from a graph

- Two options:
  1. Slope-intercept form
    - 1.1 Find the  $y$ -intercept
    - 1.2 Find the slope: how much does  $y$  change with each 1 unit difference in  $x$ ?
  2. Point-slope form
    - 2.1 Choose any point on the line
    - 2.2 Find the slope: how much does  $y$  change with each 1 unit difference in  $x$ ?

## Example: reading an equation from a graph

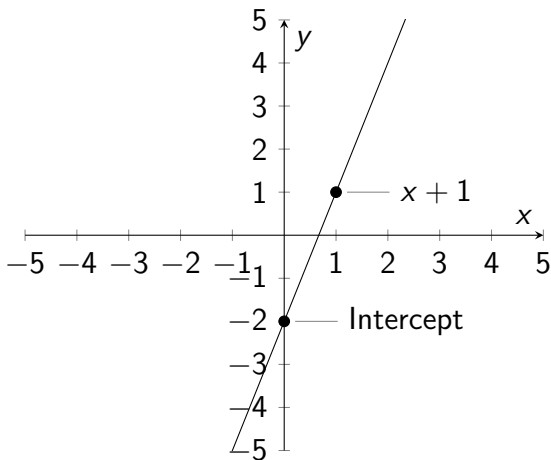


## Example: reading an equation from a graph



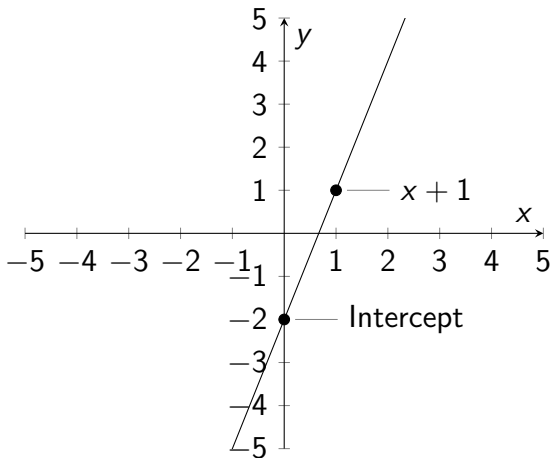
## Example: reading an equation from a graph

1. Find the intercept. Here it is  $-2$



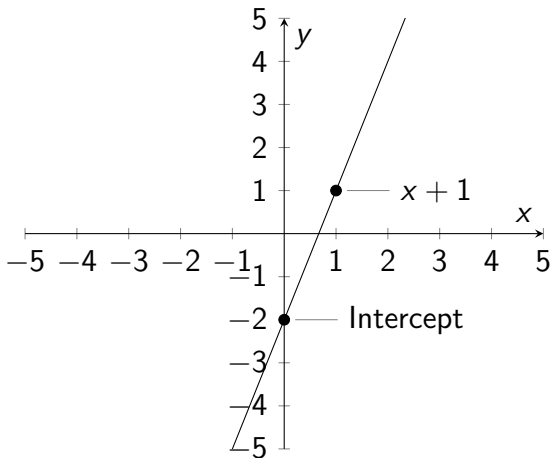
## Example: reading an equation from a graph

2. Increase  $x$  by one to find the slope. The  $y$  value at  $x = 1$  is 1, so  $y$  changed by 3



## Example: reading an equation from a graph

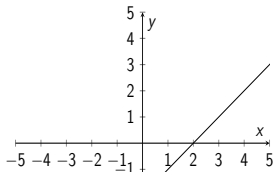
3. Slope-intercept:  $y = 3x - 2$ , point-slope:  
 $y + 2 = 3(x - 0)$



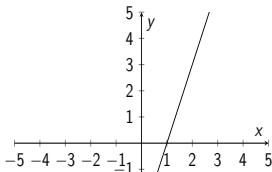
# Exercise: matching graphs to equations

1. Which is the graph of  $y = 2x - 3$ ?

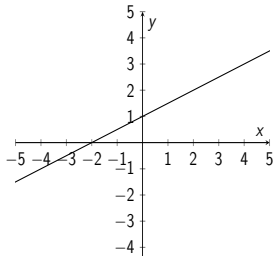
(a)



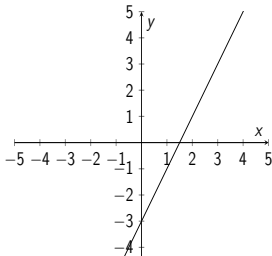
(b)



(c)



(d)



## Solution: matching graphs to equations

(a) Intercept:  $-2$ , slope:  $1$

(b) Intercept:  $-3$ , slope:  $3$

(c) Intercept:  $1$ , slope:  $1$

(d) Intercept:  $-3$ , slope:  $2$  ✓



# Quadratics

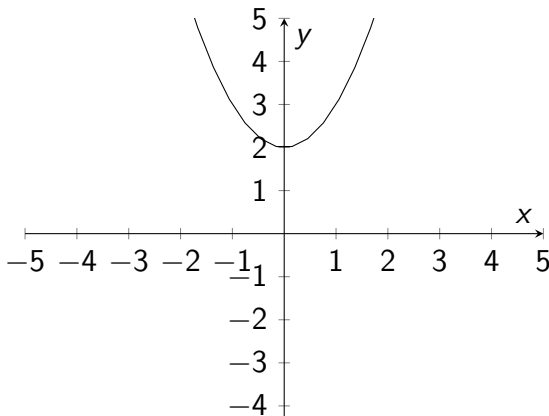
- Sometimes we believe a trend is higher order than linear
- Higher order terms allow more flexibility in modeling
- Quadratics are the natural next step from linear terms, and are shaped like parabolas

# Defining a quadratic

- Standard form:  $y = ax^2 + bx + c$
- $a$  determines the direction of the tails and the degree of curvature
  - $a > 0$  means the curve faces up (convex)
  - $a < 0$  means the curve faces down (concave)
  - large  $|a| > 1$  means steep slope
  - $0 < |a| < 1$  means shallow slope
- $b$  and  $a$  together determine the  $x$ -coordinate of the vertex:  $x = -\frac{b}{2a}$
- $c$  controls the height

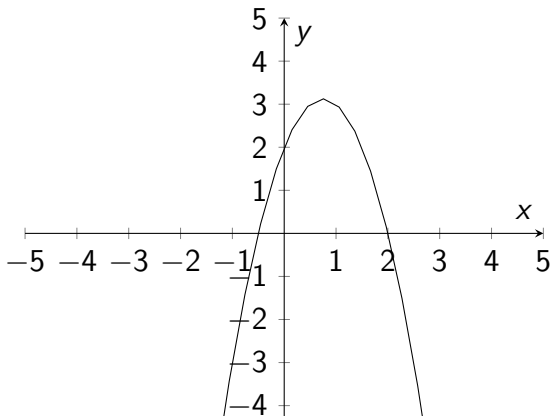
## Example: quadratics

- $y = x^2 + 2$
- $a = 1$ ,  $b = 0$ ,  $c = 2$
- Not too steep, convex, and vertex is at  $(0, 2)$



## Example: quadratics

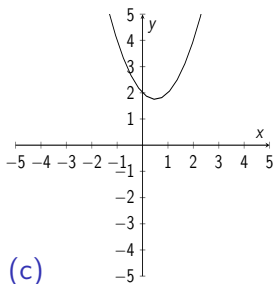
- $y = -2x^2 + 3x + 2$
- $a = -2$ ,  $b = 3$ ,  $c = 2$
- Steeper than before, concave, and vertex is at  $(3/4, 2)$



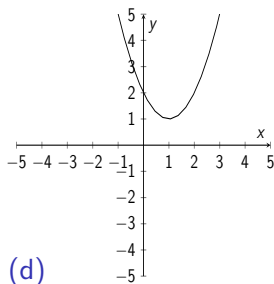
## Exercise: quadratics

1. Which is a plausible plot of  $x^2 - x + 2$ ?

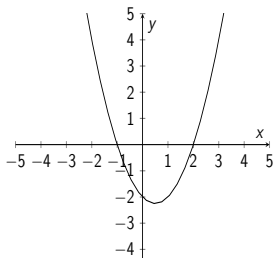
(a)



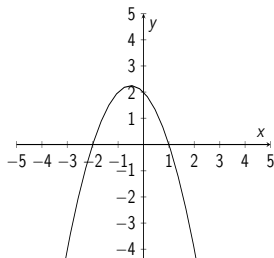
(b)



(c)



(d)



## Solution: quadratics

1.  $a = 1, b = -1, c = 2$

- We are looking for a plot where the vertex has  $y$ -coordinate 2
- This rules out (b) and (c)
- Of the two remaining, (d) is concave, so it has  $a < 0$
- (a) is the solution!

# Transforming graphs

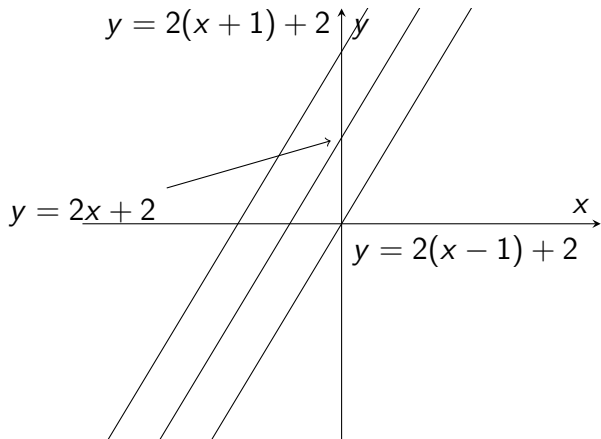
- Once we have one graph, how do we get another?
- Two main types of transformations: shifting and stretching

# Shifting graphs

- Sometimes we want to change the interpretation of the intercept
- Once we know the properties of a graph, shifting doesn't change much!
- Shift left: add to  $x$
- Shift right: subtract from  $x$
- Shift up: add to intercept
- Shift down: subtract from intercept
- Why? Adding to  $x$ : smaller  $x$ 's now have the same  $y$ .  
Subtracting from  $x$ : larger  $x$ 's now have the same  $y$ .



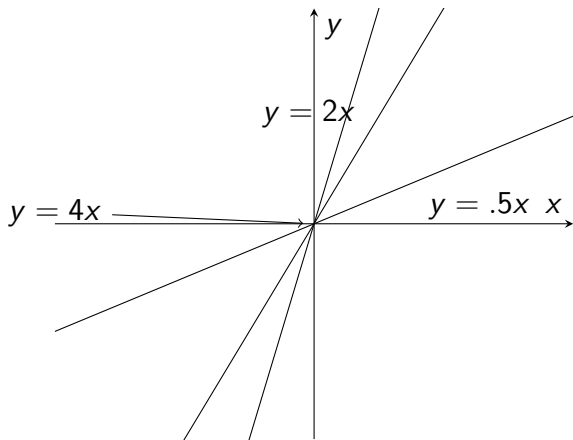
## Example: shifting graphs



# Stretching graphs

- Make the graph steeper, or shrink it: make  $|m|$  larger in a linear equation, and make  $|a|$  larger in a quadratic equation
- Make the graph shallower, or stretch it: make  $|m|$  smaller in a linear equation, and make  $|a|$  smaller in a quadratic equation

## Example: stretching graphs



## Exercise: transforming graphs

1. How do we shift the graph of  $y = 2x + 3$  one unit right?
2. How do we transform the graph of  $y = 2x + 3$  to have a shallower slope?
3. How do we transform the graph of  $y = 2x + 3$  to have a slope of 1 and a  $y$ -intercept of 4?

## Solution: shifting graphs

1. Subtract 1 from  $x$ ! New equation:  $y = 2(x - 1) + 3$
2. Multiply by a number less than 1; for example, take  $x/2$ .  
This gives new equation  $y = 2(x/2) + 3$ , or  $y = x + 3$
3. To get a slope of 1, divide  $x$  by 2. To make the  $y$ -intercept 4, shift left by adding  $1/2$  to  $x$ . New equation:  $y = 2 * (x/2 + 1/2) + 3$ , or  $y = x + 4$