Regression Fit

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Ways to understand model fit and advice on what to do

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6. More . . .

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Several definitions of R^2

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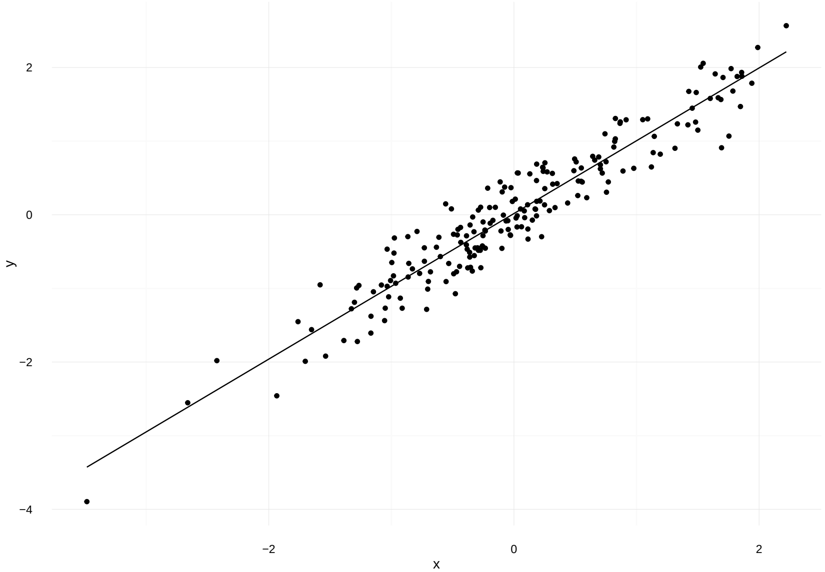
- $R^2 \in [0, 1]$ where 1 is all points are on a line/plane

R-squared is dependent on scale of X



$\hat{\sigma}^2 = 0.3, \; R^2 = 0.91$

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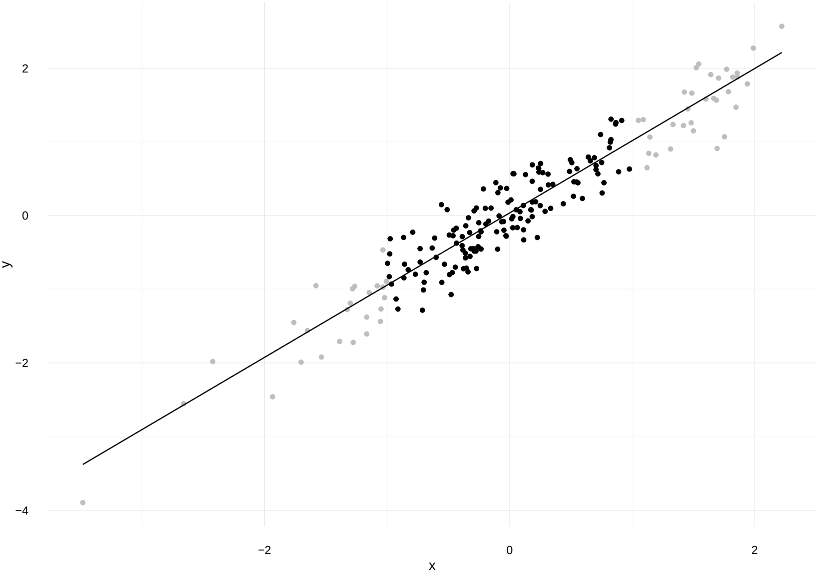
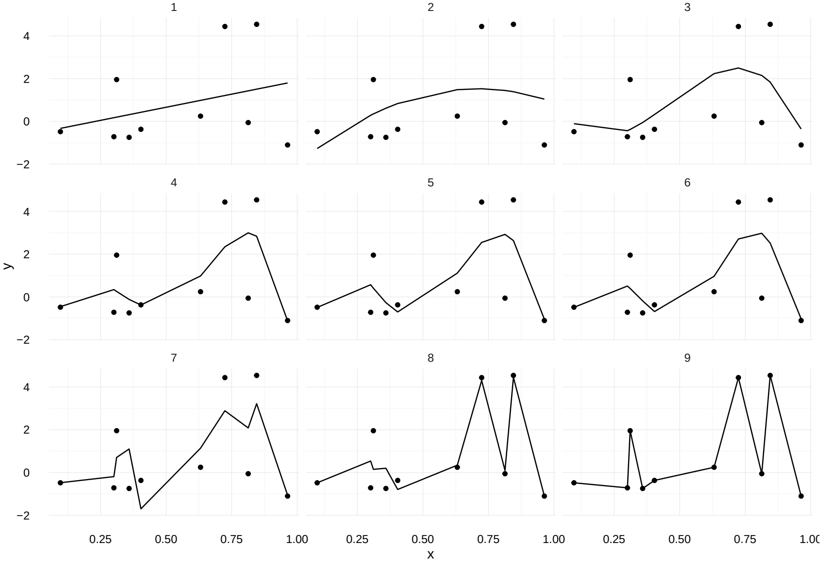
Same data, regression on subset



$\hat{\sigma}^2 = 0.29, \ R^2 = 0.75$

In-sample fit always increases as variables are added

$y = x + \epsilon,\ \epsilon_i \sim N(0, 2)$



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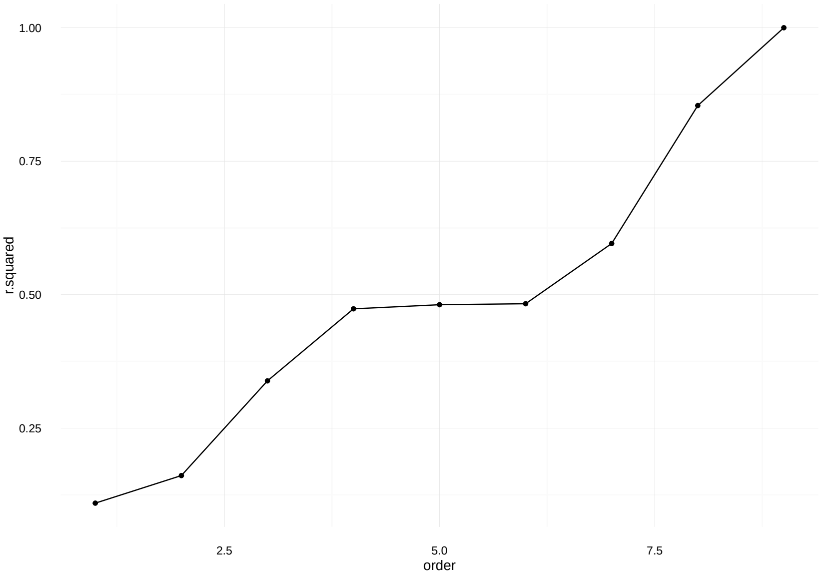

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3. Cannot compare different datasets (including transformed Y)
4. Not variance “explained” in causal sense

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- On the same scale as $\boldsymbol{y}$
- Often suggested as alternative to R^2

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- Doesn't fix any important problem with R^2. Pointless for comparing models

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3. To interpret $\hat{\sigma}$ need to compare to scale (variance) of $\boldsymbol{y}$, but then almost the same as R^2.

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- Assumes that linear model is correct, not whether it is a good model

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   1.1 If prediction: out of sample performance
   1.2 If causation:
       - identification of β (omitted variable bias or design)
       - assumptions of model (other diagnostics)
2. Focus on results/average of many models - not the "best" model

Next time

Comparing predictive performance of models using cross-validation

References

- Gary King "How Not to Lie With Statistics: Avoiding Common Mistakes in Quantitative Political Science."

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- R-squared: useful or evil?