R-squared

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R-squared, similar model fit stats, and advice on what to do

1. R-squared
2. Adjusted R-squared
3. Standard error of the regression
4. F-test
5. Advice

Several definitions of R^2

- Ratio of variance of fitted values to sample y

$$R^2 = \frac{\text{Var}(\hat{\boldsymbol{y}})}{\text{Var}\,\boldsymbol{y}}$$

- Ratio of variance "explained" by the regression

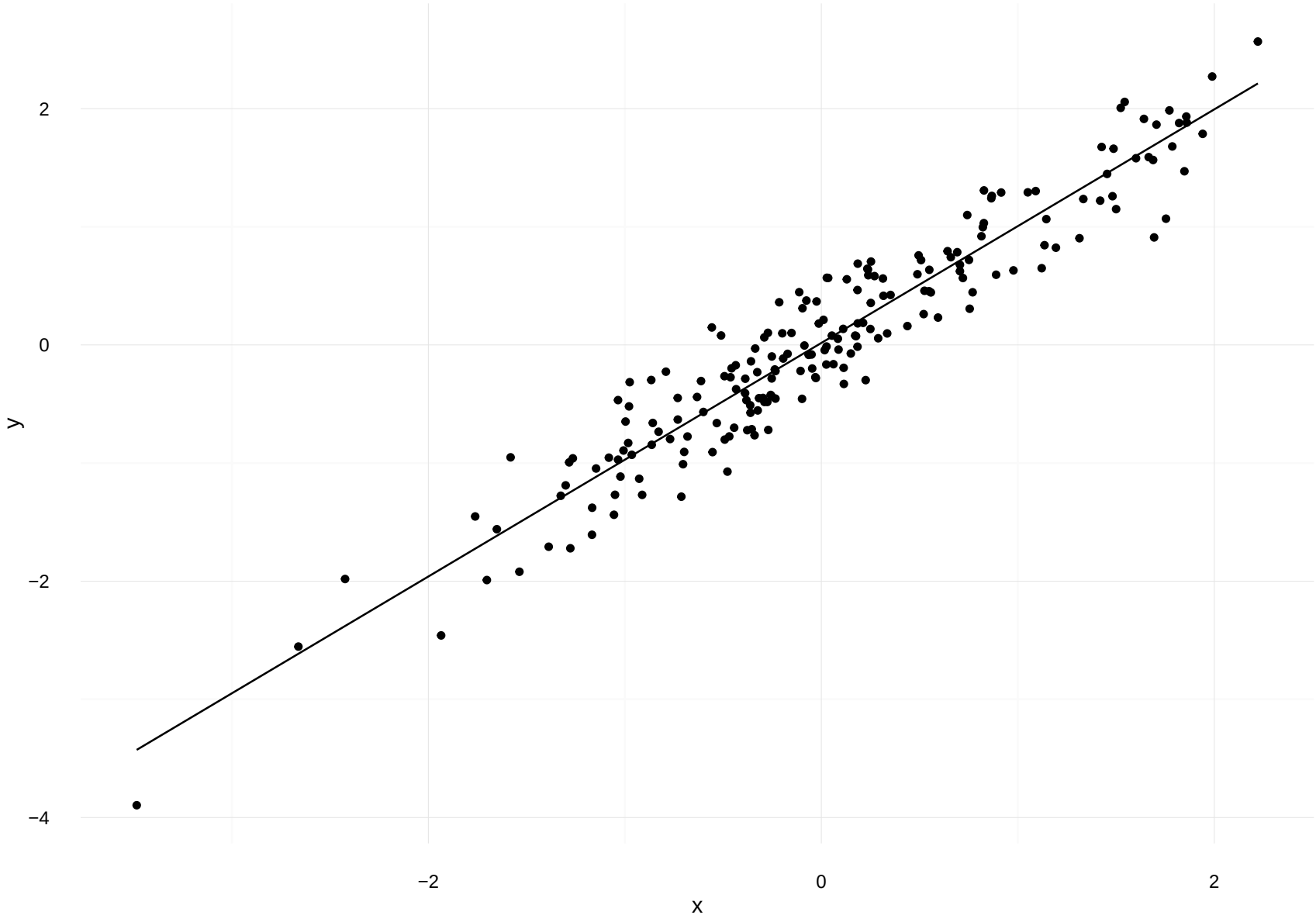
$$R^2 = 1 - SSE/SST = 1 - \frac{\sum(y_i - \hat{y}_i)^2}{\sum(y_i - \bar{\boldsymbol{y}})^2}$$

- For bivariate regression, correlation of Y and X squared,

$$R^2 = \text{Cor}(\boldsymbol{x}, \boldsymbol{y})^2 = \hat{\beta}_1 \frac{\text{sd}\,\boldsymbol{y}}{\text{sd}\,\boldsymbol{x}}$$

- $R^2 \in [0, 1]$ where 1 is all points are on a line/plane

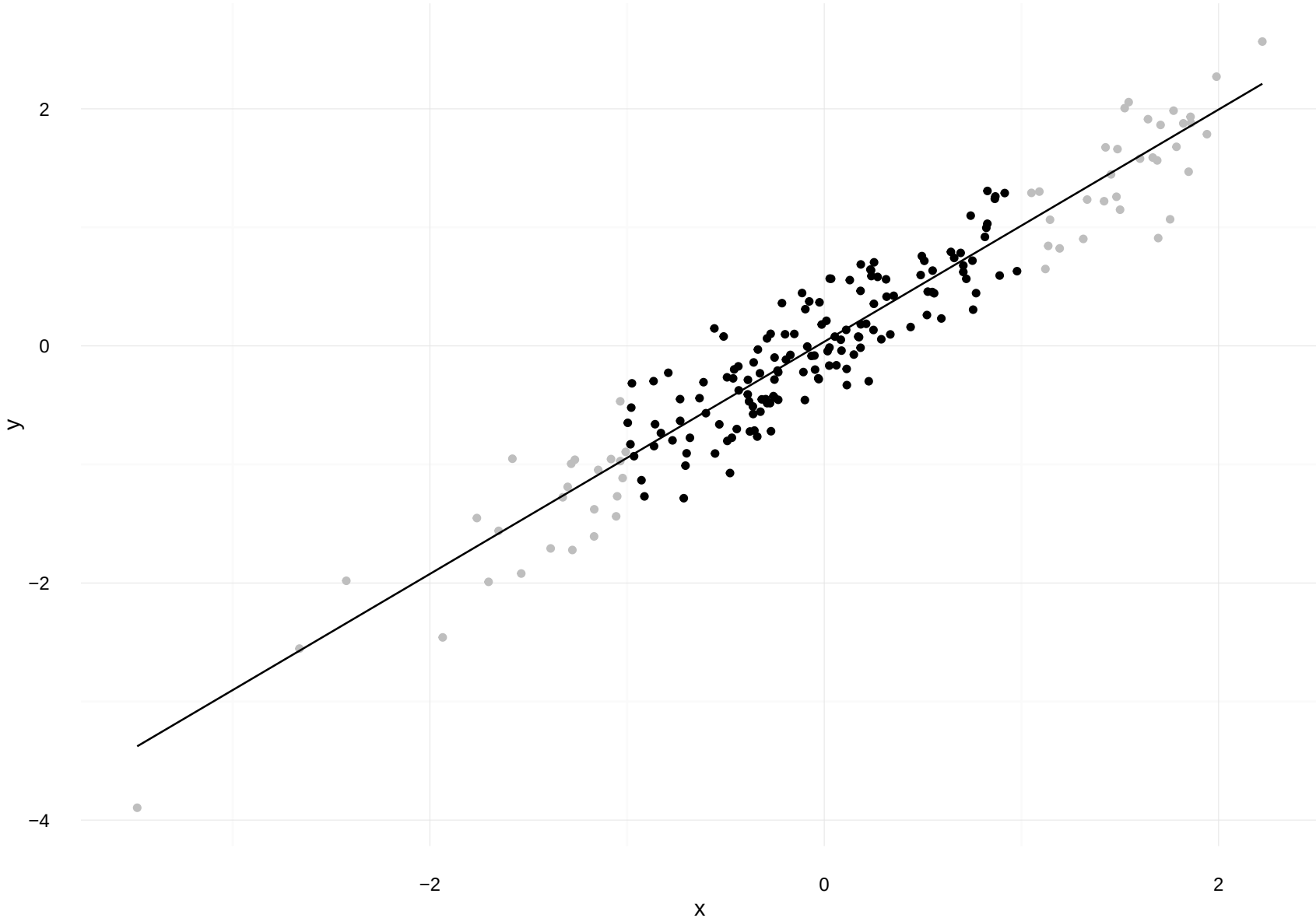
R-squared is dependent on scale of X



$\hat{\sigma}^2 = 0.3,\ R^2 = 0.91$

R-squared is dependent on scale of X

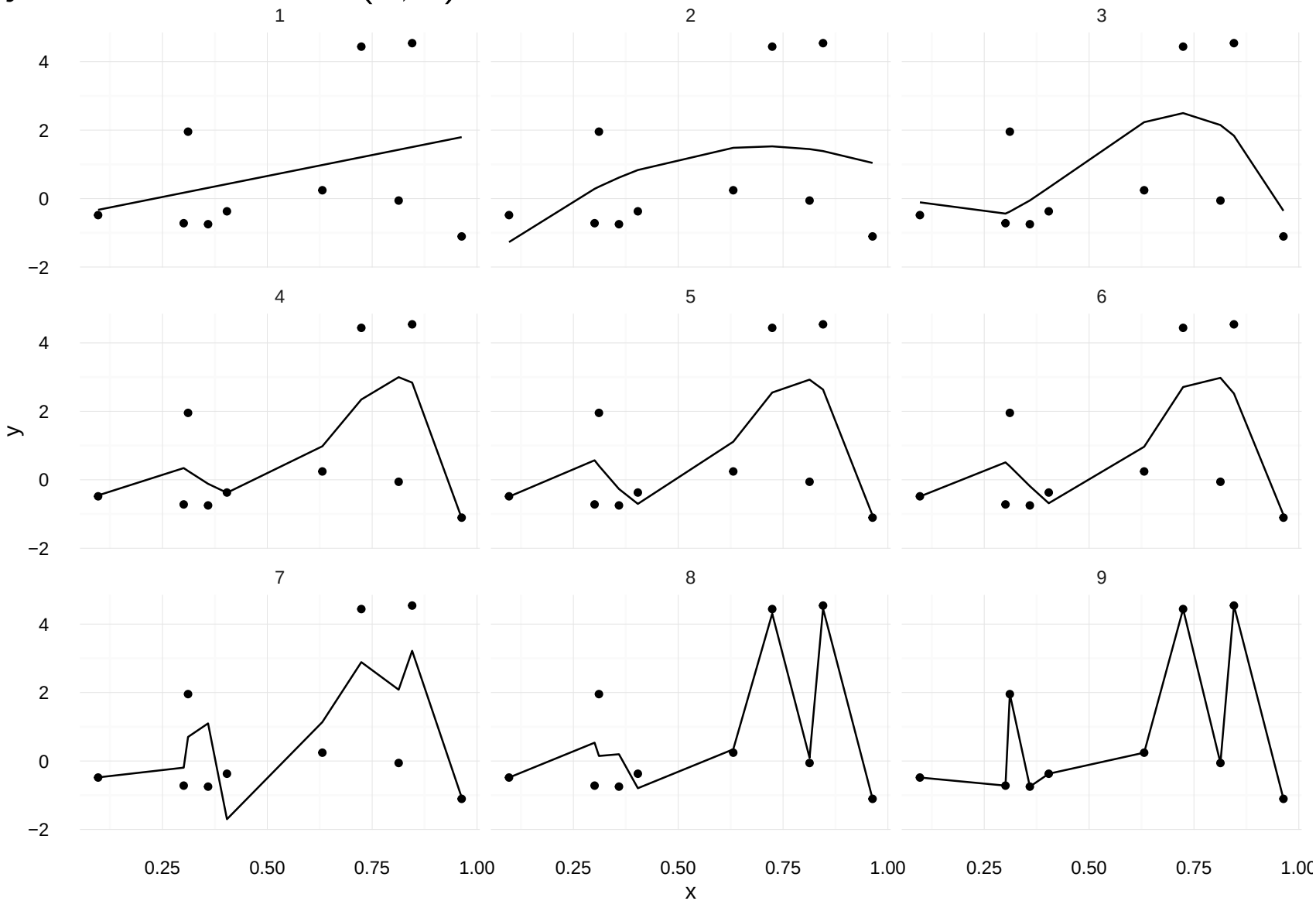
Same data, regression on subset



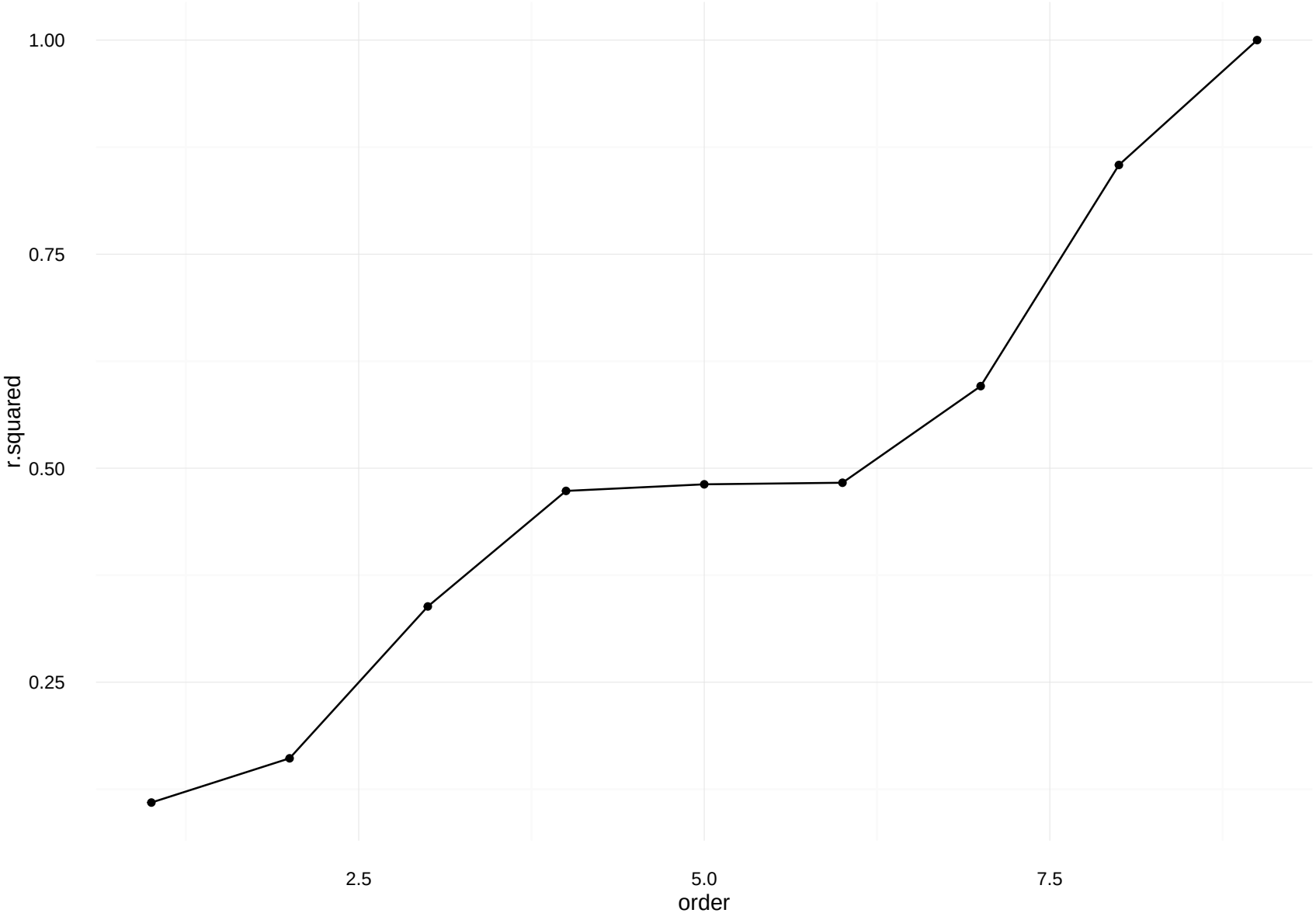
$\hat{\sigma}^2 = 0.29,\ R^2 = 0.75$

In-sample fit always increases as variables are added

$y = x + \epsilon,\ \epsilon_i \sim N(0, 2)$



R^2 always increases as variables are added



Other problems with R^2

1. Does not measure goodness of fit
   1.1 To get R^2 large, make X spread out
   1.2 To get R^2 small, make X not spread out
2. Does not measure prediction
3. Cannot compare different datasets (including transformed Y)
4. Not variance "explained" in causal sense

Adjusted R^2

Adjust R^2 for sample size and variables,

$$R^2 = 1 - \frac{SSE/(N-K-1)}{SST/(N-1)}$$

- Slightly penalizes R^2 for more variables
- Adjustment only relevant for cases where $N \approx K$
- Atheoretical
- Doesn't fix any important problem with R^2.
- Pointless for comparing models

Standard error of the regression ($\hat{sigma}$)

$$\hat{\sigma} = \sqrt{\frac{1}{N-K-1}\sum \varepsilon_i^2}$$

- "Average" error
- RMSE is similar, with denominator N instead of $N-K-1$.
- On the same scale as $\boldsymbol{y}$ - substantive interpretation
- Often suggested as alternative to R^2

Problems with $\hat{\sigma}$

2. All insample problems with R^2 apply to $\hat{\sigma}$
3. To interpret $\hat{\sigma}$ need to compare to scale (variance) of $\boldsymbol{y}$, but then almost the same as R^2.

F-test

- R^2 and $\hat{\sigma}$ are statistics, but generally not used in tests
- F-test with $H_0 : \beta_1 = \cdots = \beta_K = 0$
- F-statistic is a function of the SSE of models
- Inherits most of the same problems as R^2
- Assumes that linear model is correct, not whether it is a good model

What to do about it?

1. Focus on what's important:
   - 1.1 If prediction: out of sample performance
   - 1.2 If causation:
     - identification of β (omitted variable bias or design)
     - assumptions of model (other diagnostics)
2. Focus on results/average of many models - not the "best" model

Next time

Comparing predictive performance of models using cross-validation

References

- Gary King "How Not to Lie With Statistics: Avoiding Common Mistakes in Quantitative Political Science."
- Cosmo Shalizi, F-Tests, R2, and Other Distractions.
- Gelman and King. R-squared: useful or evil?