ESS 411/511 Geophysical Continuum Mechanics Class #14

Highlights from Class #13 – John-Morgan Manos
Today's highlights on Monday – Alysa Fintel

Our text doesn't cover our next topics very thoroughly, so we will use a few other sources, which are posted on the class web site under READING & NOTES. https://courses.washington.edu/ess511/NOTES/notes.shtml

For Monday class – Please read

- Stein and Wysession 5.7.2
- Stein and Wysession 5.7.3/4
- Raymond notes on failure

Also see slides about upcoming topics

- Failure and Mohr's circles – slides

ESS 411/511 Geophysical Continuum Mechanics

Broad Outline for the Quarter

- Continuum mechanics in 1-D
- 1-D models with springs, dashpots, sliding blocks
- Attenuation
- Mathematical tools – vectors, tensors, coordinate changes
- Stress – principal values, Mohr's circles for 3-D stress
- Coulomb failure, pore pressure, crustal strength
- Measuring stress in the Earth
- Strain – Finite strain; infinitesimal strains
- Moments – lithosphere bending; Earthquake moment magnitude
- Conservation laws
- Constitutive relations for elastic and viscous materials
- Elastic waves; kinematic waves

Midterm

- Study questions will be posted this weekend.
- HW session next Thursday will be devoted to the study questions

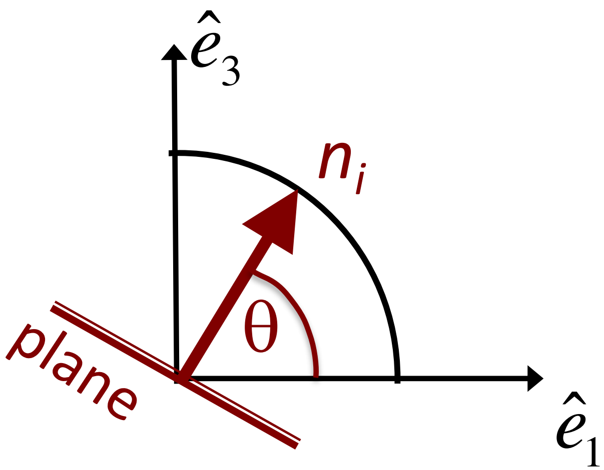
Problem Set #1

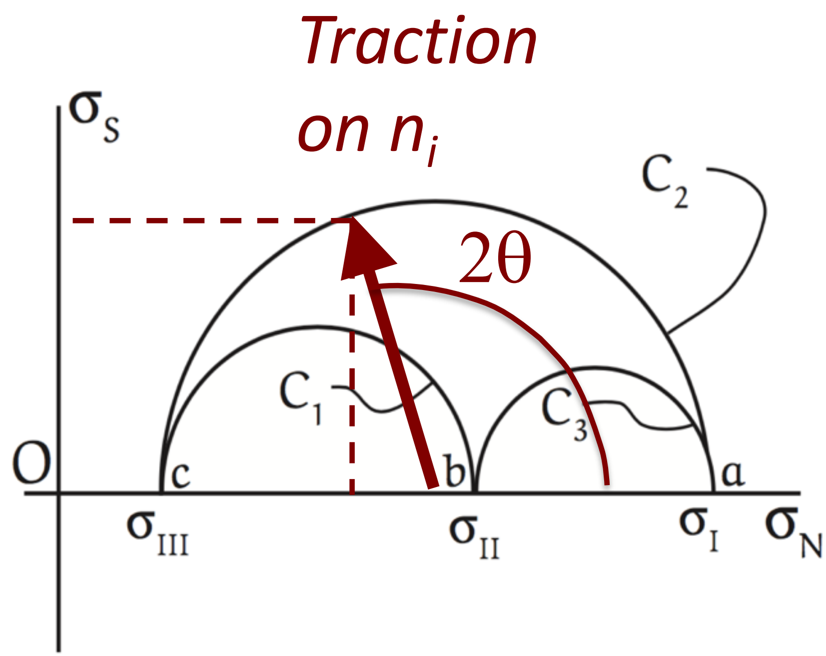
- Remember to include *units* when you evaluate expressions
- Q 1 – some of you got the correct equation for stress evolution n (a), then totally ignored it when drawing your graphs.
- Some of you interpreted 1(f) to be about shearing the lithosphere. If you used the thickness of the asthenosphere (~200 km), you would have found that the shear strain rate fell below the minimum strain rate needed for failure in (c).
- 1(f) some of you didn't use the thickness of the asthenosphere (100 – 200 km), or the typical rate of plate motion (1-10 cm per year) to estimate the strain rate.

Question 2 – viscosity of silly putty

- Some of you are clearly theoreticians rather than experimentalists. You never told me how you would set up an experiment in your Lab.
- Data from some of your experiments would require very complicated interpretation.

Mohr's circle in 2-D view





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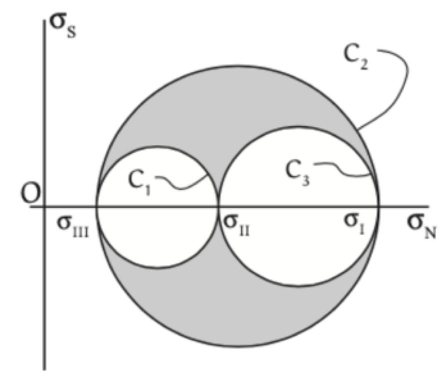
Revisiting creation of 3 Mohr's circles for stress state σ_{ij}

- σ_I, σ_{II}, σ_{III} are principal stresses
- n_i is normal vector to a plane at point P
- σ_N and σ_S are normal and shear components of traction vector $t_i^{(\boldsymbol{n})}=\sigma_{ij}\, n_j$ on that plane



Projection of $t_i^{(\boldsymbol{n})}$ onto n_i

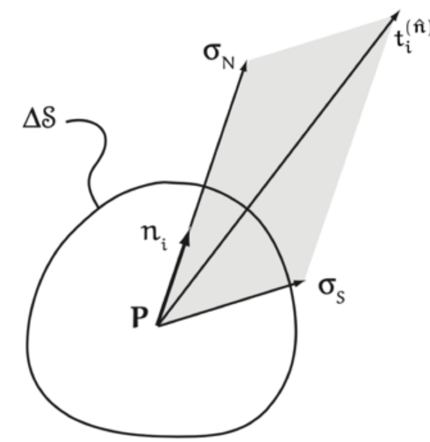
$$\sigma_N = \sigma_I n_1^2 + \sigma_{II} n_2^2 + \sigma_{III} n_3^2$$

Pythagoras gives σ_s

$$\sigma_N^2 + \sigma_S^2 = \sigma_I^2 n_1^2 + \sigma_{II}^2 n_2^2 + \sigma_{III}^2 n_3^2$$

n_i is a unit vector

$$n_1^2 + n_2^2 + n_3^2 = 1$$



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Revisiting creation of 3 Mohr's circles

We can use those 3 equations to find n_i, the plane on which traction σ_N and σ_S exists

$$n_1^2 = \frac{(\sigma_N - \sigma_{II})(\sigma_N - \sigma_{III}) + \sigma_S^2}{(\sigma_I - \sigma_{II})(\sigma_I - \sigma_{III})}$$

Denominator >0

use those 3 equations to find all the values of σ_N and σ_S that satisfy the equality in

$$n_2^2 = \frac{(\sigma_N - \sigma_{III})(\sigma_N - \sigma_I) + \sigma_S^2}{(\sigma_{II} - \sigma_{III})(\sigma_{II} - \sigma_I)}$$

Denominator <0

$$n_3^2 = \frac{(\sigma_N - \sigma_I)(\sigma_N - \sigma_{II}) + \sigma_S^2}{(\sigma_{III} - \sigma_I)(\sigma_{III} - \sigma_{II})}$$

Denominator >0

We can also multiply each equation by the denominator on the RHS term
Th LHS squares are always non-negative.
Knowing the signs of the denominators, we can create 3 inequalities involving the RHS numerators

This gives the Mohr's circle connecting σ_{II} and σ_{III}

The inequality allows all σ_N and σ_S that fall outside the circle

$$(\sigma_N - \sigma_{II})(\sigma_N - \sigma_{III}) + \sigma_S^2 \geqslant 0$$

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We can also use those 3 equations to find all the values of σ_N and σ_S that satisfy the equality in

$$(\sigma_N - \sigma_{II})(\sigma_N - \sigma_{III}) + \sigma_S^2 \geqslant 0$$

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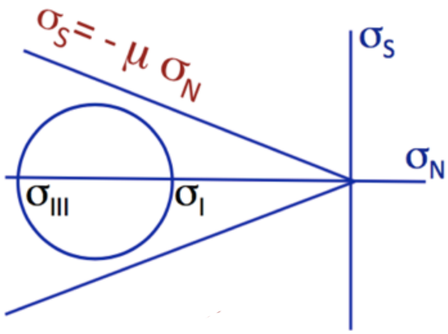
Class-prep questions for today

Failure of materials

Faults can slip when shear stress σ_S is large enough to overcome frictional resistance. Frictional resistance to failure can be modeled as increasing proportional to the normal traction σ_N.

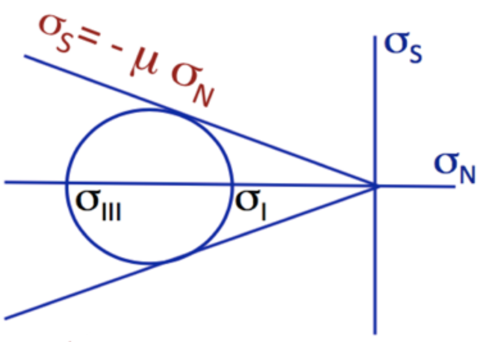
In stress space, if a stress state σ_N and σ_S exists that intersects or touches the frictional line, then the plane represented at that point can fail.



In the diagrams, all principal stresses are negative.

- Are they compressive or extensile?
- In the first diagram, do any stress states exist outside the circle shown?
- Can any faults fail in this stress field?

In the second diagram (below):



- What has changed in the stress field?
- Can any faults fail in this new stress field?
- If yes, how many different faults can fail?
- How could you identify the orientation(s) from the Mohr's circle?

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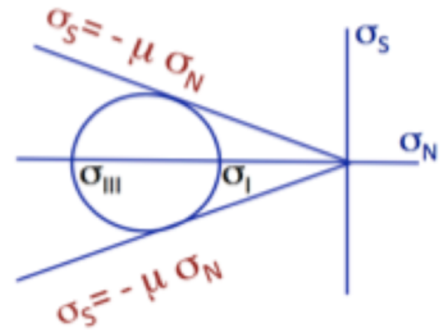
Prep for Class 15 on Monday

Failure of materials

Last class, we looked at frictional sliding on pre-existing fractures or faults with a coefficient of friction μ.

- What physical characteristics of a surface cause friction?



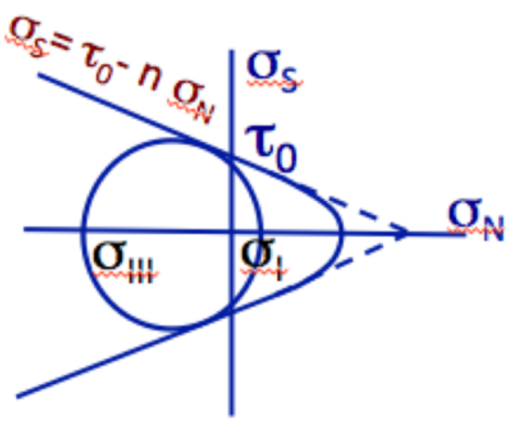
Now we are going to actually break new rocks.

Mohr–Coulomb failure

$\sigma_S = \tau_0 - n\,\sigma_N$

n = ***coefficient of internal friction*** for fracture on a new fault surface

τ_0 = cohesion of the material



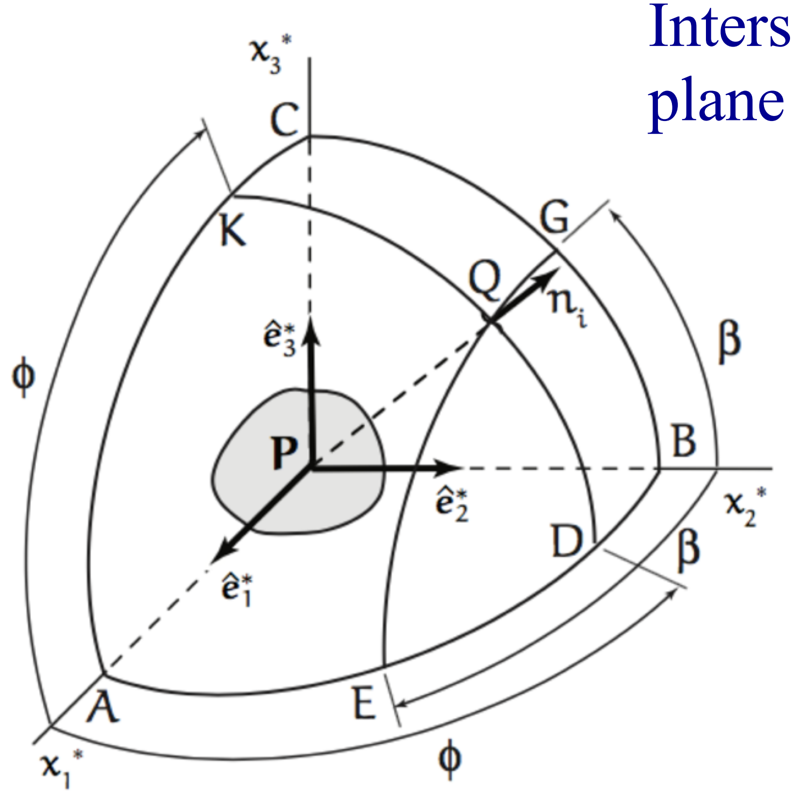
- Explain what you think n and τ_0 might mean in terms of micro-scale processes at the micro-crack, crystalline, or lattice scales.
- Why do you think the failure envelope is rounded off at the right? Think about the sign of σ_N and the processes that might contribute to internal friction.

Cartesian Space *vs* Stress Space

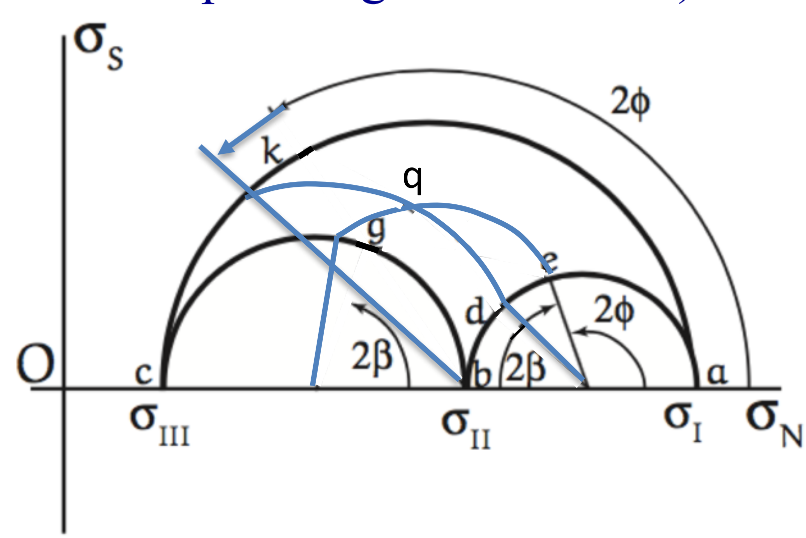
$\hat{e}_i^*$ are principal directions defining principal planes at **P**.
Small circles in Cartesian space (e.g. EQG) map onto circles (e.g. eqg) concentric with primary Mohr's circles (e.g. akc).
Similarly, KQD maps onto kqd.

Intersection at q shows σ_N and σ_s on plane defined by normal vector n_i at Q.

(I have attempted to (sort of) correct the stress-plane figure below. ☺)

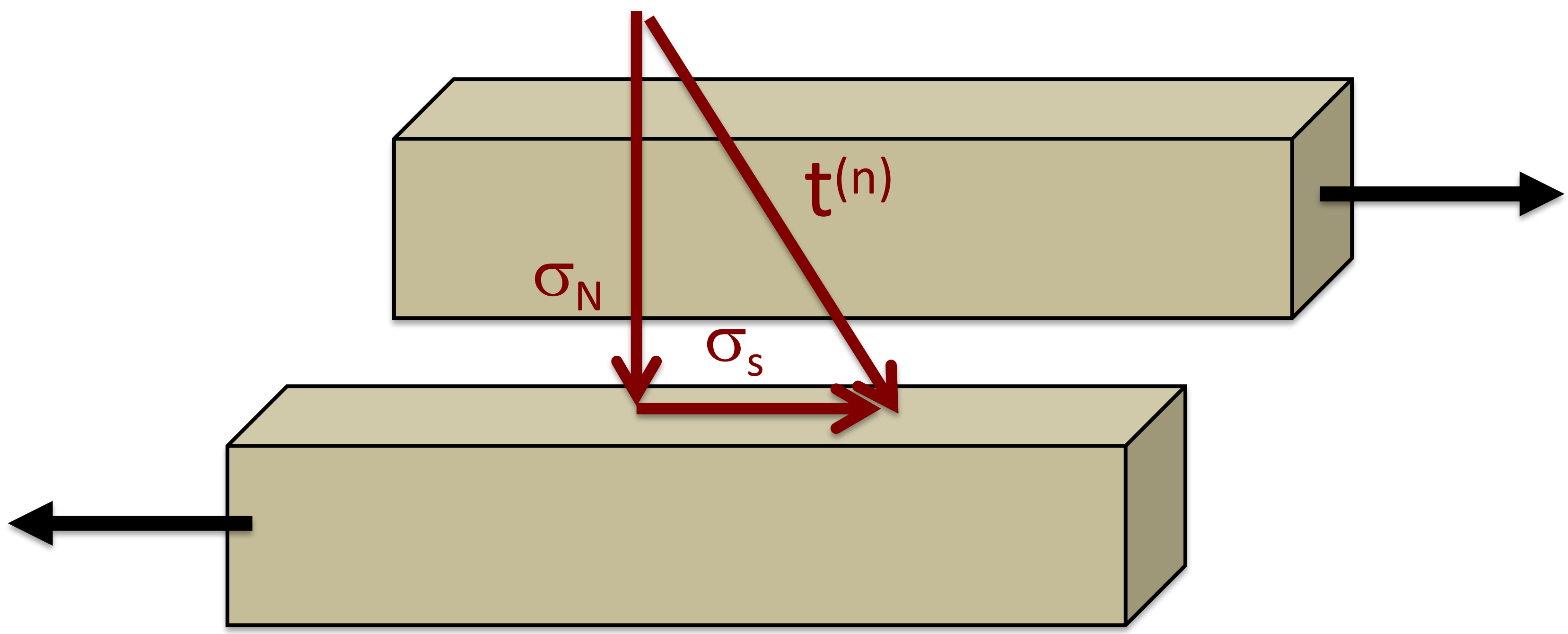


(a) Reference angles ϕ and β for intersection point Q on surface of body octant.



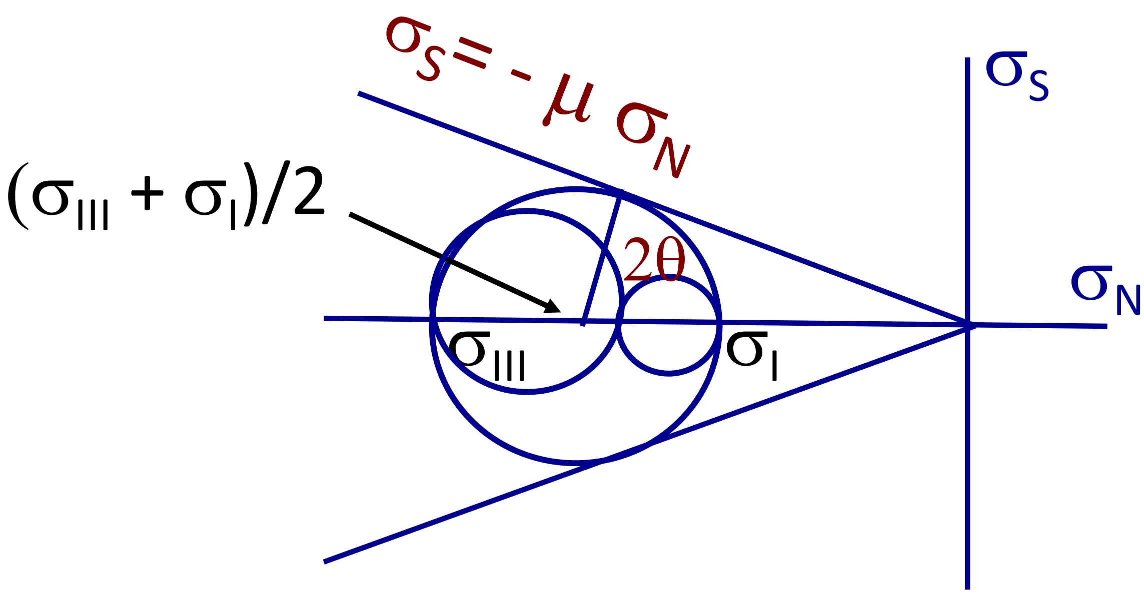
(b) Mohr's stress semicircle for octant of Fig. 3.15(a).

Sliding friction



$\sigma_S = - \mu \, \sigma_N$ μ is ***coefficient of friction*** for sliding on a pre-existing break

Frictional sliding



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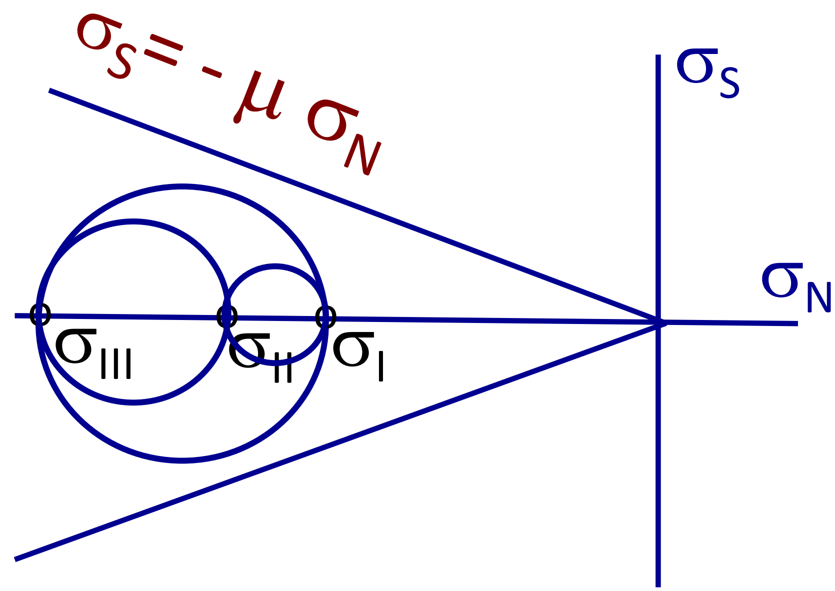
Differential stress $\sigma_{III} - \sigma_{I}$

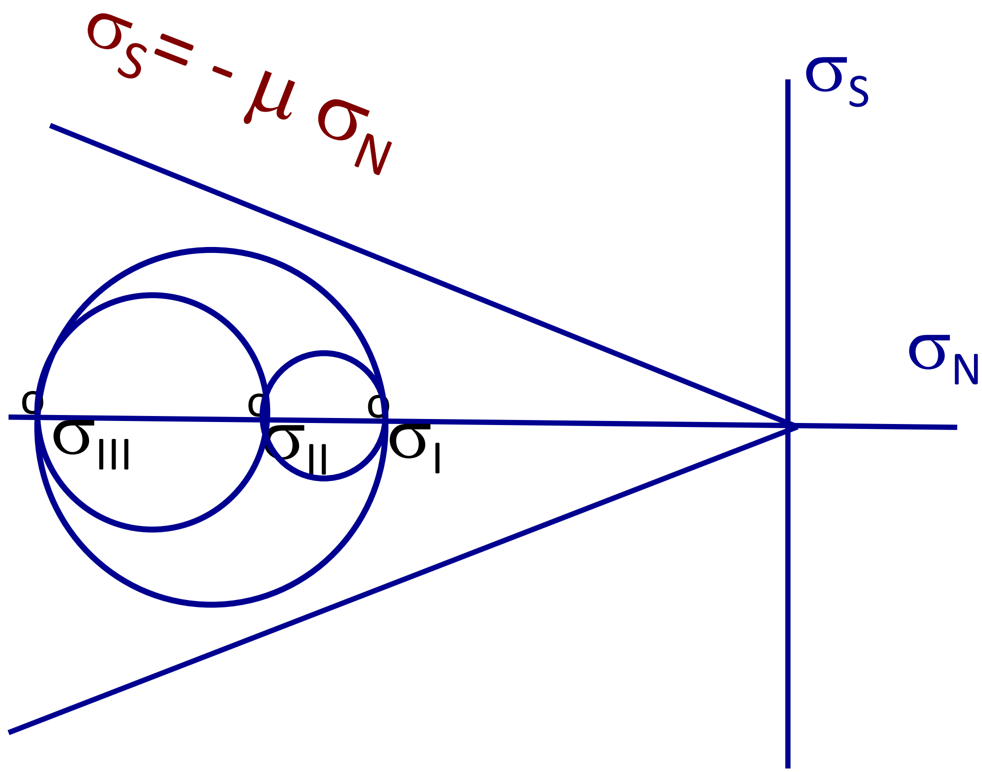


But, if $\sigma_{III} = \sigma_{I}$, all 3 principal stresses are equal

- What do the 3 Mohr's circle look like?
- Describe this state of stress inside the body.
- Is frictional failure possible, if differential stress is zero?

Differential stress

$\sigma_{III} - \sigma_{I}$



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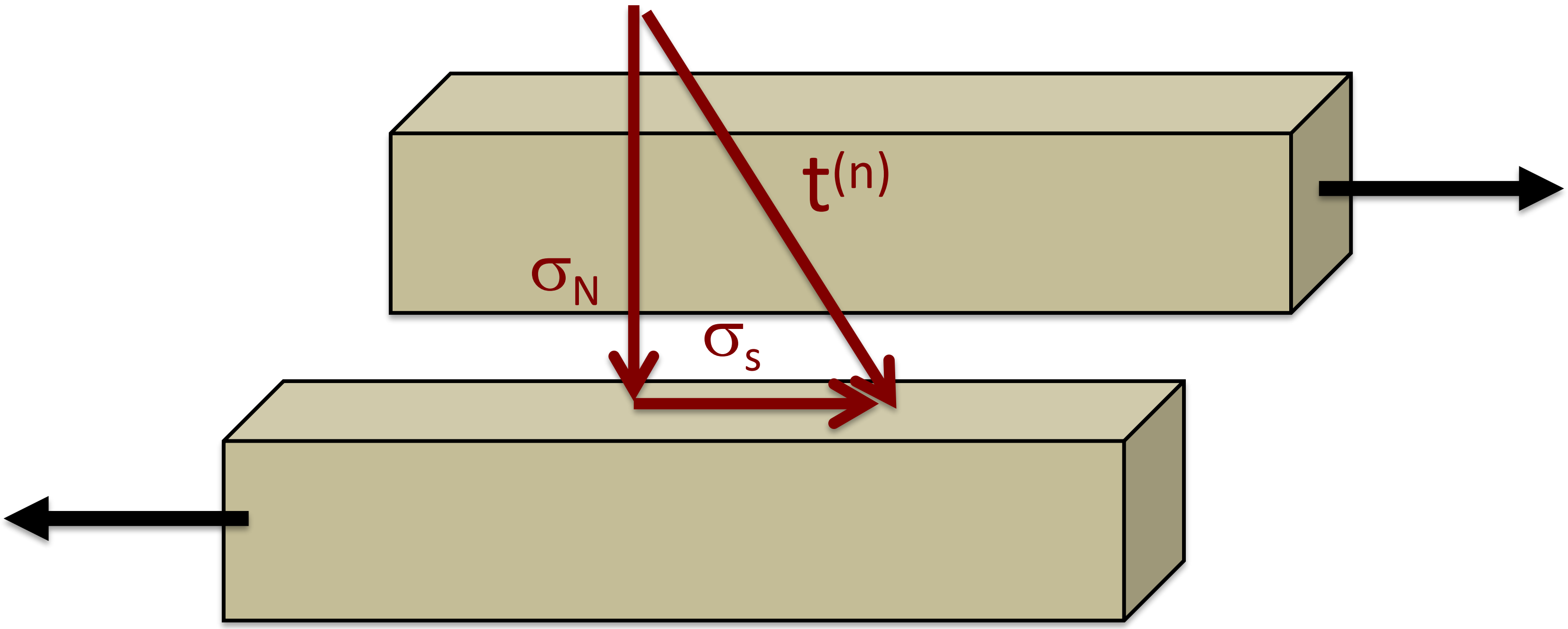
Mohr-Coulomb Fracture



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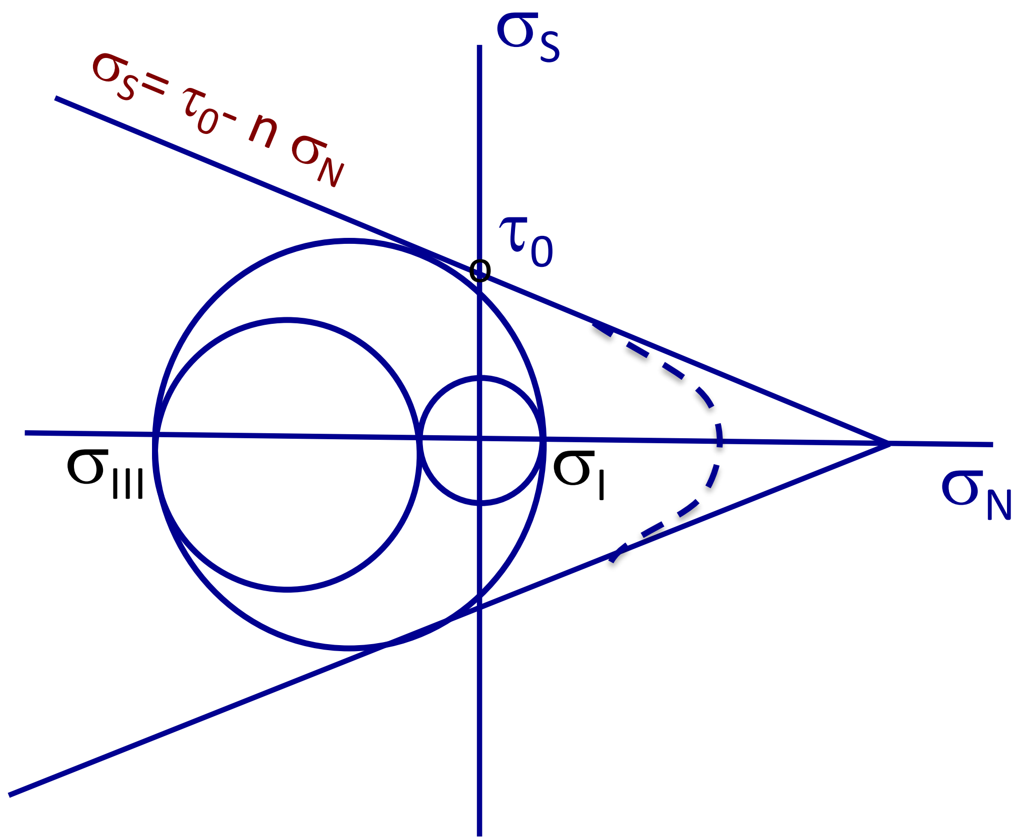

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Failure in shear

- Why is failure is not on the plane with maximum shear stress?
- Why are there 2 conjugate failure planes?