ESS 411/511 Geophysical Continuum Mechanics

Broad Outline for the Quarter

- Continuum mechanics in 1-D
- 1-D models with springs, dashpots, sliding blocks
- Attenuation
- Mathematical tools – vectors, tensors, coordinate changes
- Stress – principal values, Mohr's circles for 3-D stress
- Coulomb failure, pore pressure, crustal strength
- Measuring stress in the Earth
- Strain – Finite strain; infinitesimal strains
- Moments – lithosphere bending; Earthquake moment magnitude
- Conservation laws
- Constitutive relations for elastic and viscous materials
- Elastic waves; kinematic waves

ESS 511 Term projects

This Friday Nov 5:

- 60 second updates

Next Friday Nov 12:

- 1-page reports (outline, refs, …)

The following Friday Nov19:

- 60 second updates

Class-prep questions

Check out these websites about measuring stress in the Earth:

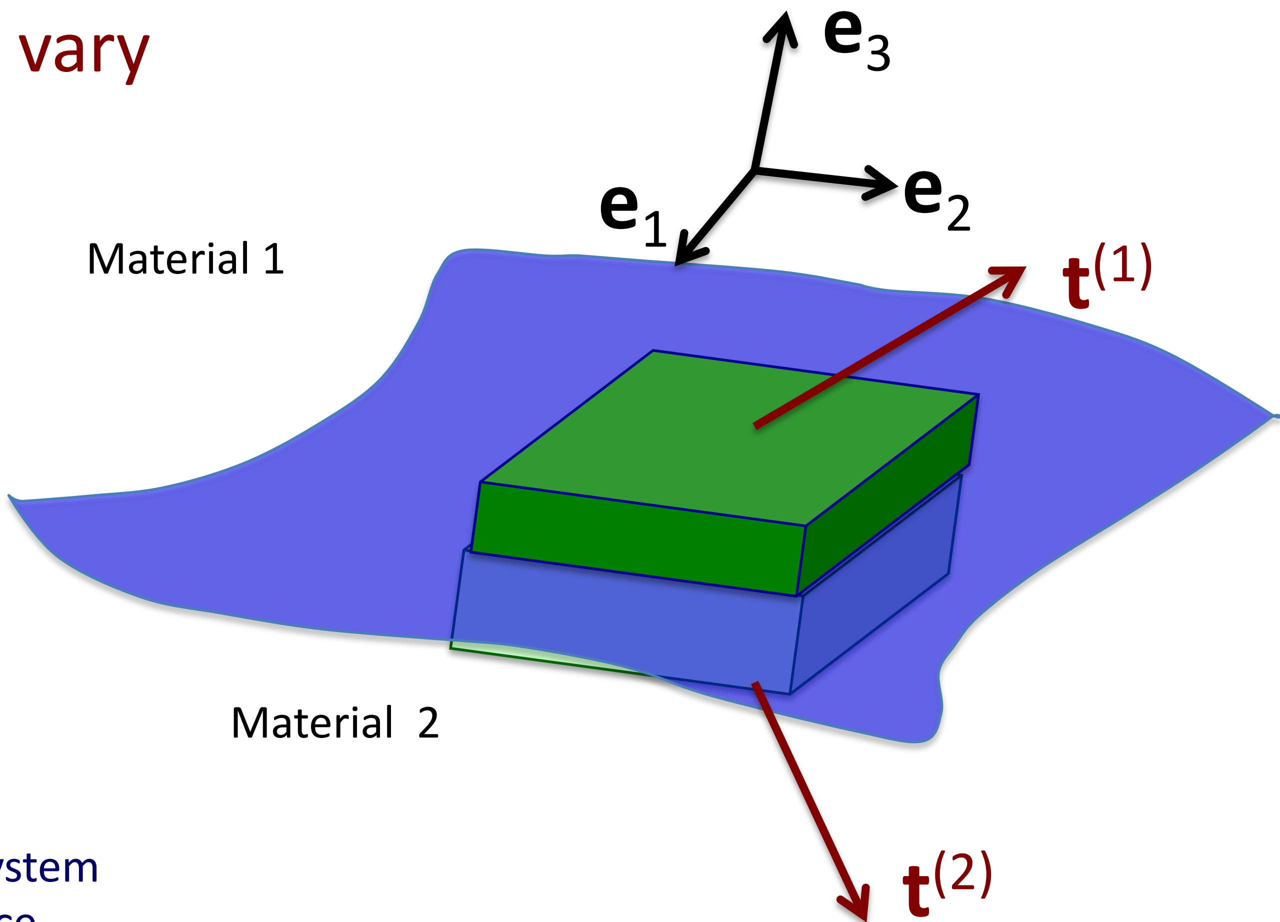
https://www.eoas.ubc.ca/courses/eosc433/lecture-material/L7-InSituStress.pdf

http://www.hydrofrac.com/hfo_home.html

https://courses.washington.edu/ess511/NOTES/SLIDE_SHOWS/PDF/stress_class_show_2017_all.pdf

Assignment:
What is a flatjack?
How can a flatjack be used to measure stress?

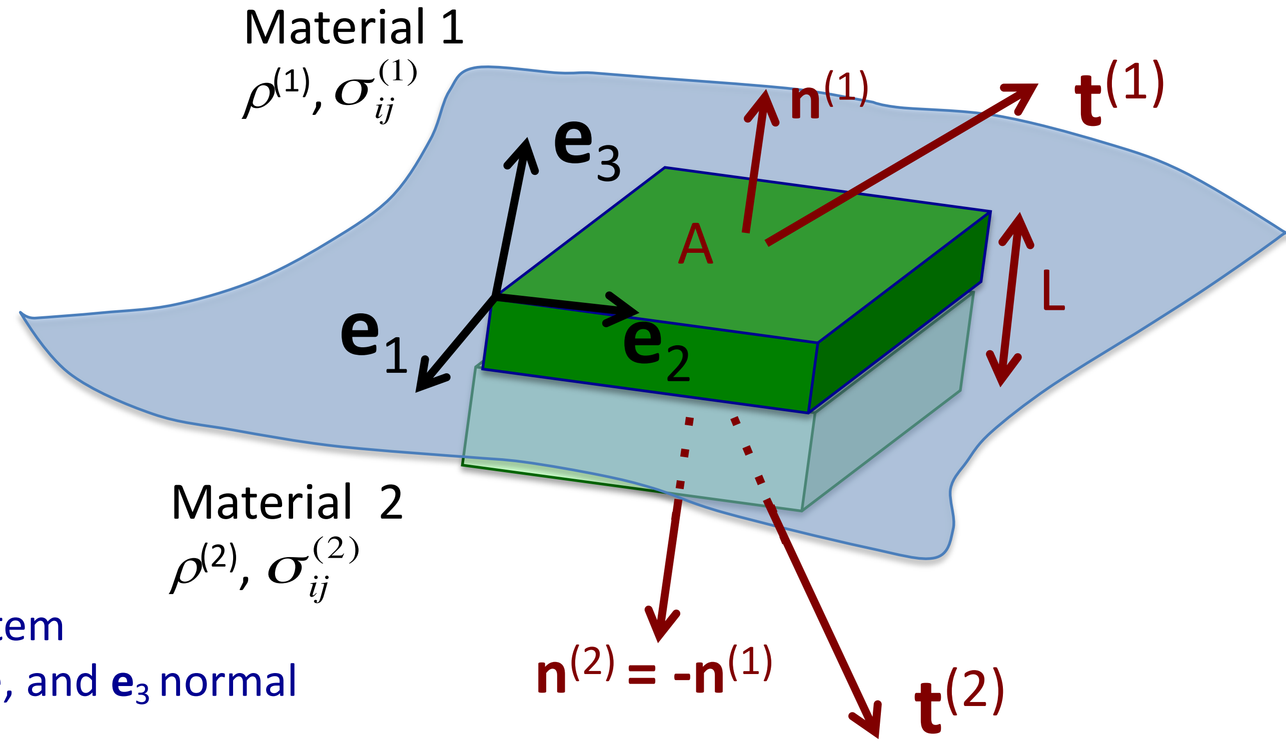
How do stresses vary across material boundaries?



Let's choose a coordinate system with $\mathbf{e}_1$ and $\mathbf{e}_2$ in the interface

- now calculate all the forces on the little box
- Let the thickness of the box go to zero

How do stresses vary across material boundaries?



Let's choose a coordinate system with $\mathbf{e}_1$ and $\mathbf{e}_2$ in the interface, and $\mathbf{e}_3$ normal to the interface

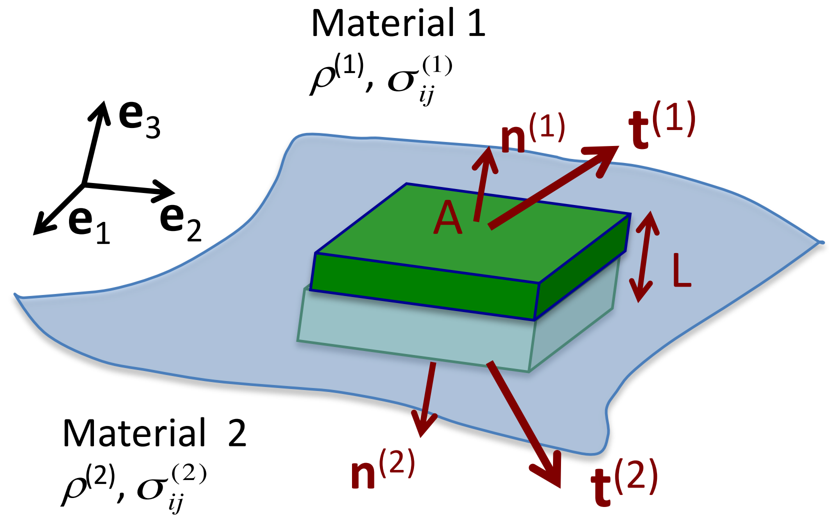
- Now add up all the forces on the surface of the little box
- For equilibrium, $\Sigma f_j = 0$
- Let the thickness L of the box go to zero, so the areas of the sides go to zero

The only remaining nonzero forces are:

$$f_i^{(top)} = A\ t_i^{(n^{(1)})} = A\ \sigma_{ij}^{(1)} n_j^{(1)}$$

$$f_i^{(bot)} = A\ t_i^{(n^{(2)})} = A\ \sigma_{ij}^{(2)} n_j^{(2)} = -A\ \sigma_{ij}^{(2)} n_j^{(1)}$$

How do stresses vary across material boundaries?



$$f_i^{(top)} = A\ \sigma_{ij}^{(1)} n_j^{(1)}$$

$$= A \begin{bmatrix} \sigma_{11}^{(1)} & \sigma_{12}^{(1)} & \sigma_{13}^{(1)} \\ \sigma_{12}^{(1)} & \sigma_{22}^{(1)} & \sigma_{23}^{(1)} \\ \sigma_{31}^{(1)} & \sigma_{32}^{(1)} & \sigma_{33}^{(1)} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = A \begin{bmatrix} \sigma_{13}^{(1)} \\ \sigma_{23}^{(1)} \\ \sigma_{33}^{(1)} \end{bmatrix}$$

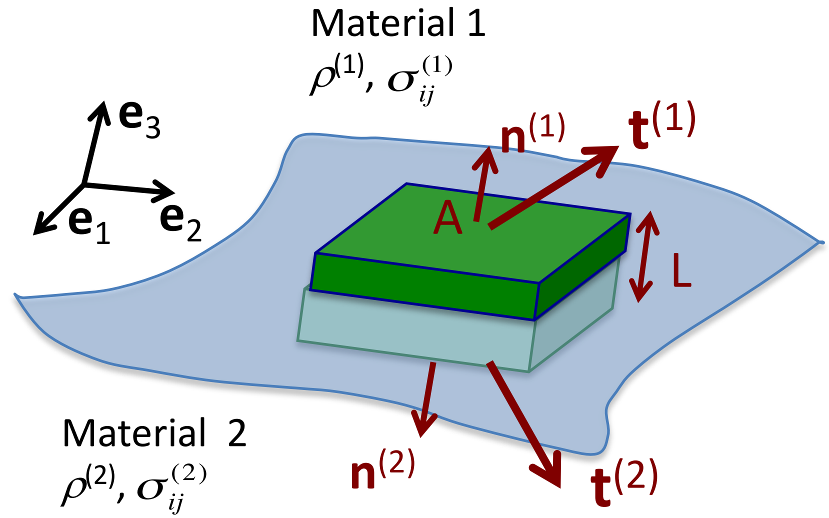
$$f_i^{(top)} + f_i^{(bot)} = 0$$

$$f_i^{(bot)} = -A\ \sigma_{ij}^{(2)} n_j^{(1)}$$

$$= A \begin{bmatrix} \sigma_{11}^{(2)} & \sigma_{12}^{(2)} & \sigma_{13}^{(2)} \\ \sigma_{12}^{(2)} & \sigma_{22}^{(2)} & \sigma_{23}^{(2)} \\ \sigma_{31}^{(2)} & \sigma_{32}^{(2)} & \sigma_{33}^{(2)} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} = A \begin{bmatrix} -\sigma_{13}^{(2)} \\ -\sigma_{23}^{(2)} \\ -\sigma_{33}^{(2)} \end{bmatrix}$$

$$\Rightarrow \left\{ \begin{array}{c} \sigma_{13}^{(1)} = \sigma_{13}^{(2)} \\ \sigma_{23}^{(1)} = \sigma_{23}^{(2)} \\ \sigma_{33}^{(1)} = \sigma_{33}^{(2)} \end{array} \right\}$$

How do stresses vary across material boundaries?



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So σ_{13}, σ_{23}, and σ_{33} must be continuous across the interface.

- σ_{31} and σ_{32} are also continuous, since σ_{ij} is symmetric

There are no restrictions on σ_{11}, σ_{12}, σ_{21}, or σ_{22}.

- They act only within their own material.
- This is the principle of stress guides

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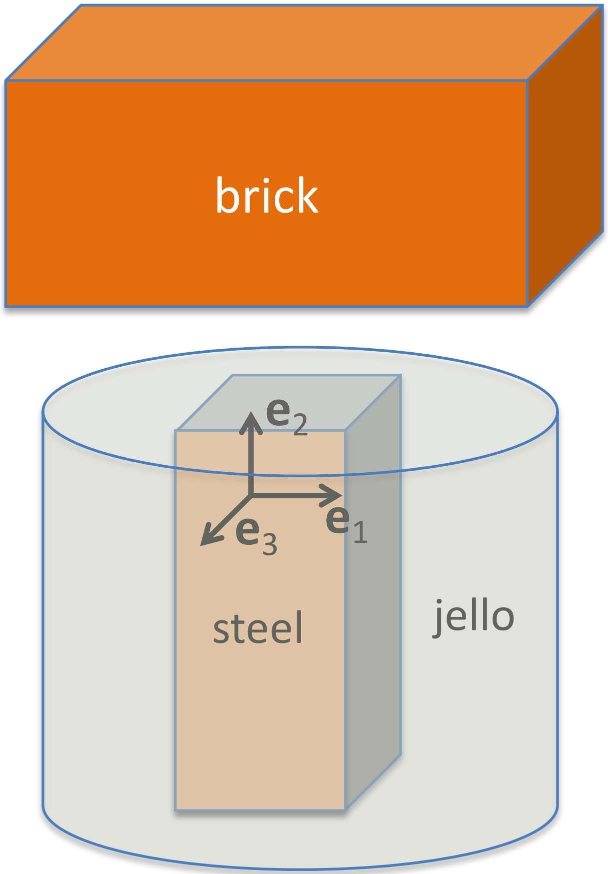


Is σ_{22} continuous across the jello/steel interface?
What about σ_{33} ?

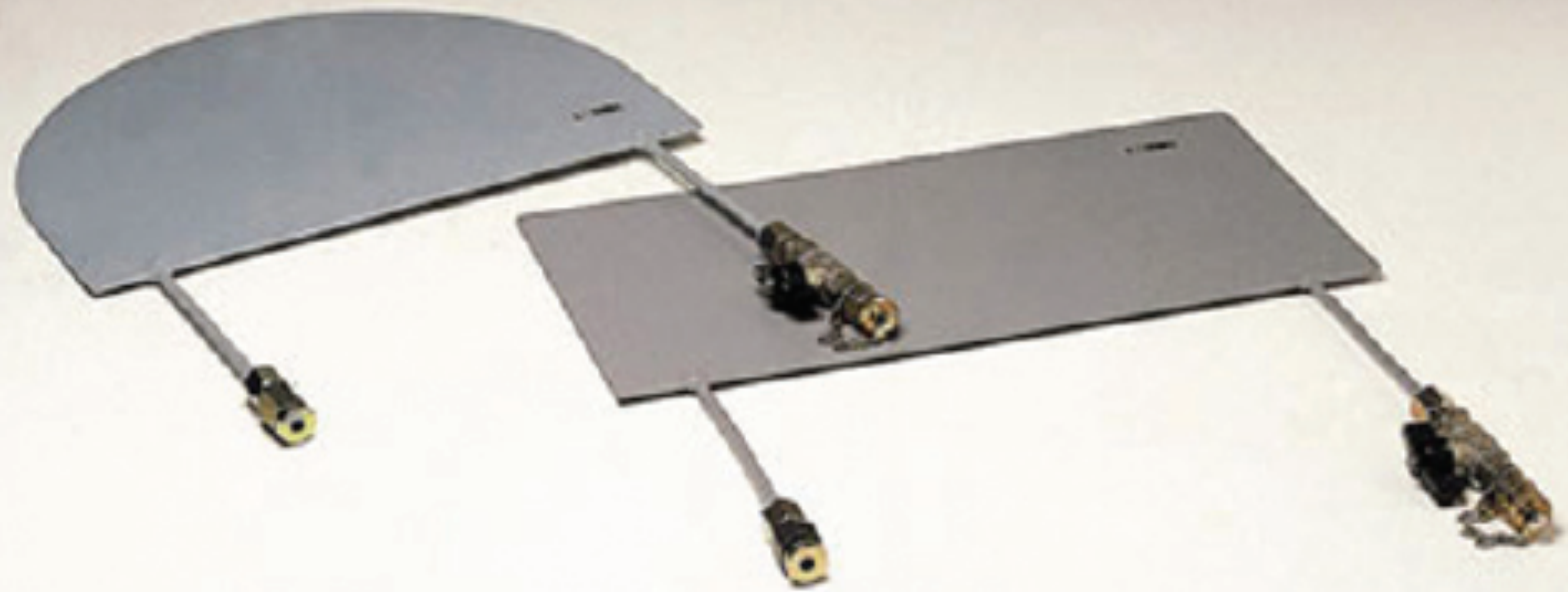
Why do we care about stresses in the Earth?

Measuring stress around an excavation

- Mine shaft
- Tunnel
- Well
- Others?

Let’s take a break for some flatjacks with syrup ...

Flatjack Tests

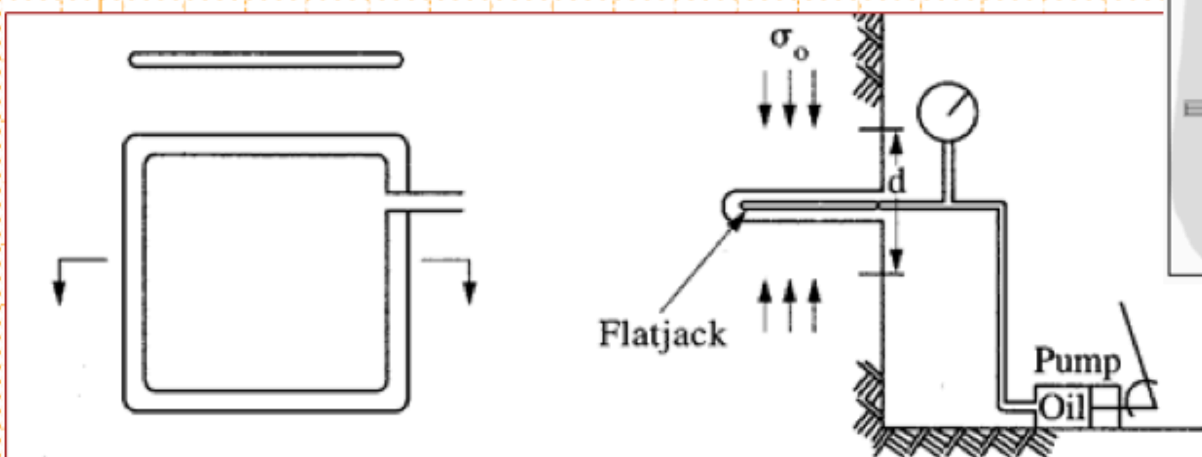


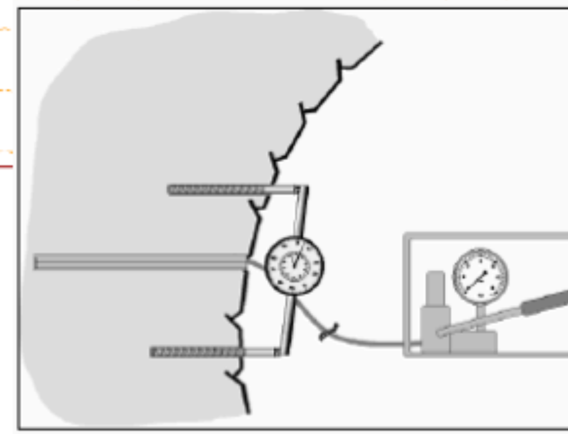
- What do you think the test might measure?

http://www.walterpmoore.com/

Flatjack Method

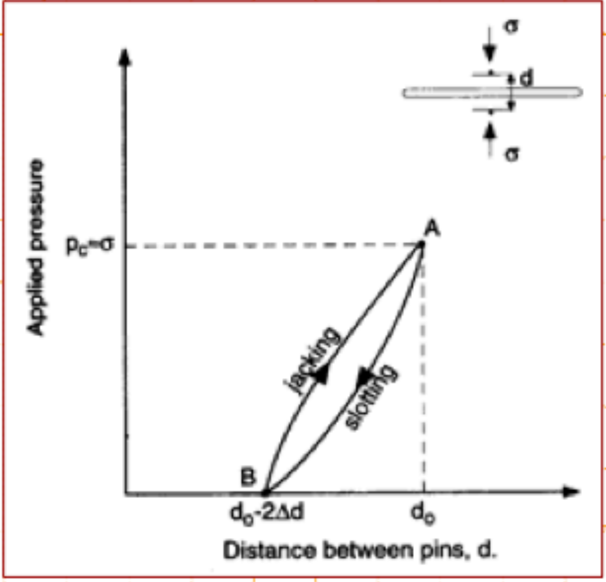
The flatjack method involves the placement of two pins fixed into the wall of an excavation. The distance, *d*, is then measured accurately. A slot is cut into the rock between the pins. If the normal stress is compressive, the pins will move together as the slot is cut. The flatjack is then placed and grouted into the slot.

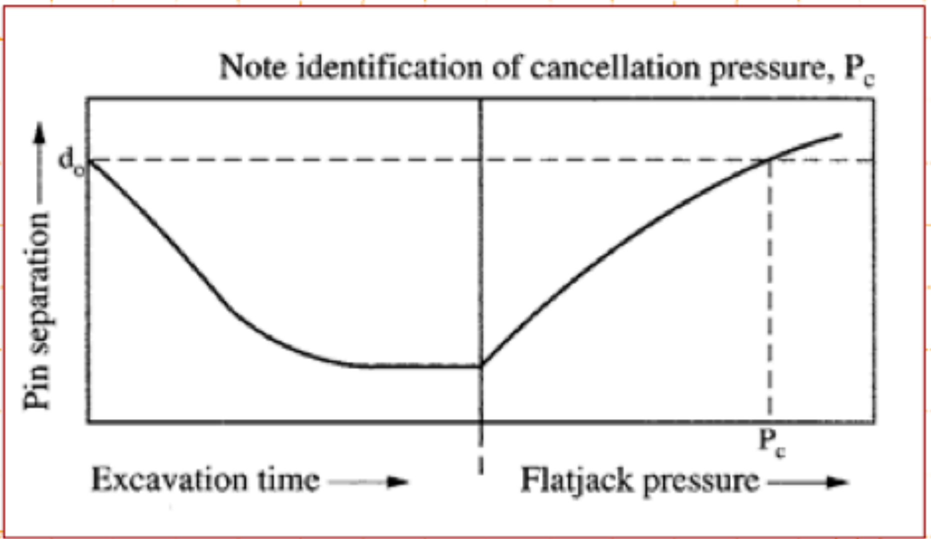




Flatjack Method

On pressurizing the flatjack, the pins will move apart. It is assumed that, when the pin separation distance reaches the value it had before the slot was cut, the force exerted by the flatjack on the walls of the slot is the same as that exerted by the pre-existing normal stress.

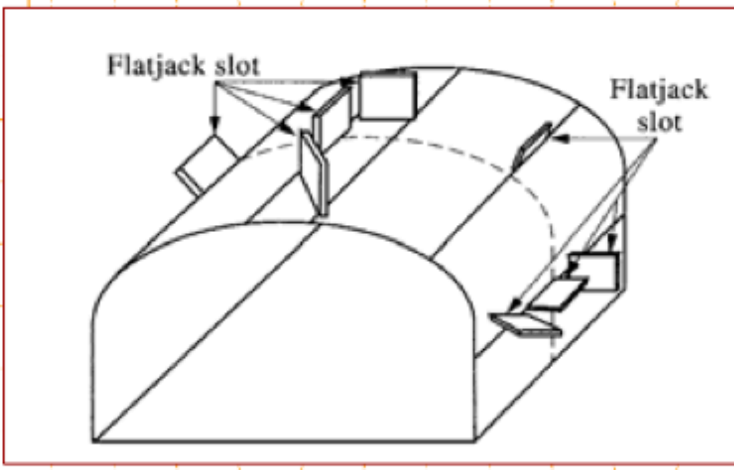




Hudson & Harrison (1997)

Flatjack Method

The major disadvantage with the system is that the necessary **minimum number of 6 tests**, at different orientations, have to be conducted **at 6 different locations** and it is therefore necessary to distribute these around the boundary walls of an excavation.



1. **Flatjack**

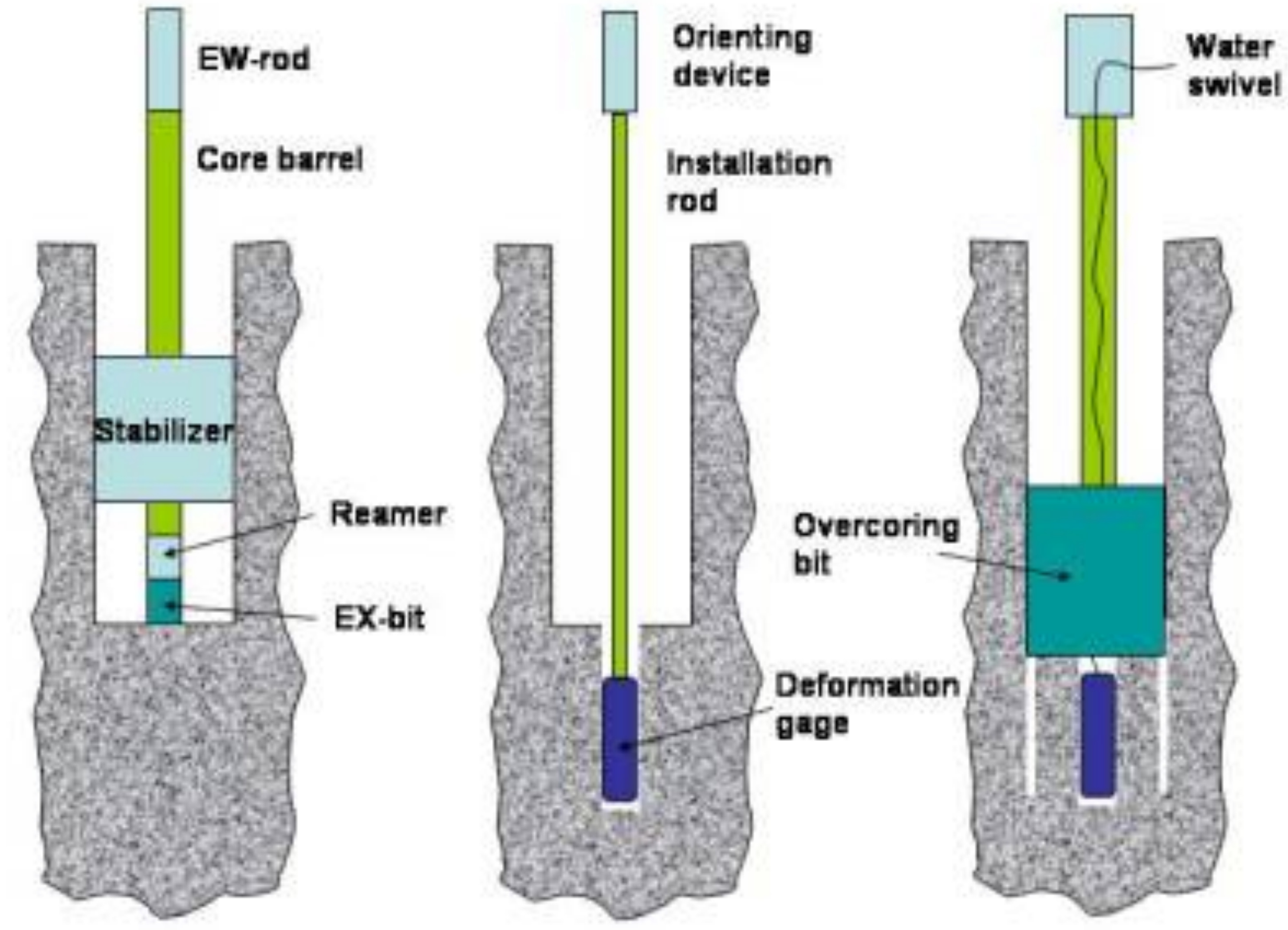
$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ & \sigma_{yy} & \tau_{yz} \\ \text{Symm.} & & \sigma_{zz} \end{bmatrix}$$

One normal stress component determined, say parallel to x-axis.

Hudson & Harrison (1997)

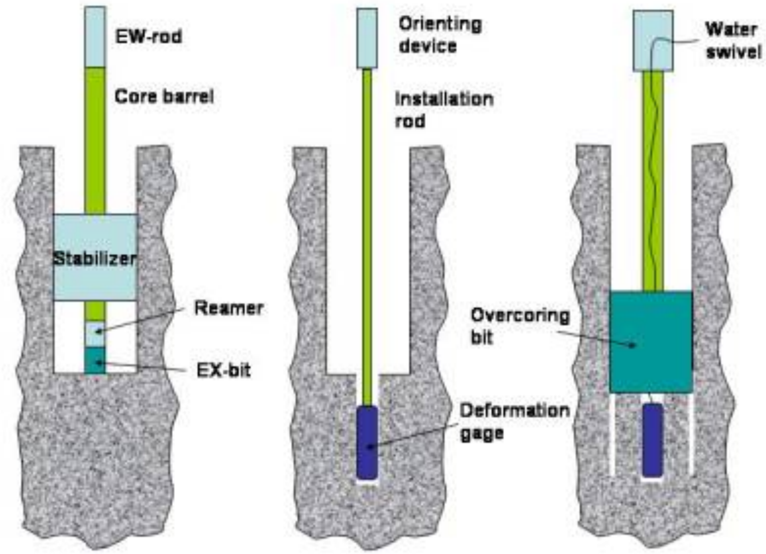
It is also important to note that the excavation from which the tests are made will disturb the pre-existing stress state, and so the new redistribution of stresses should be accounted for.

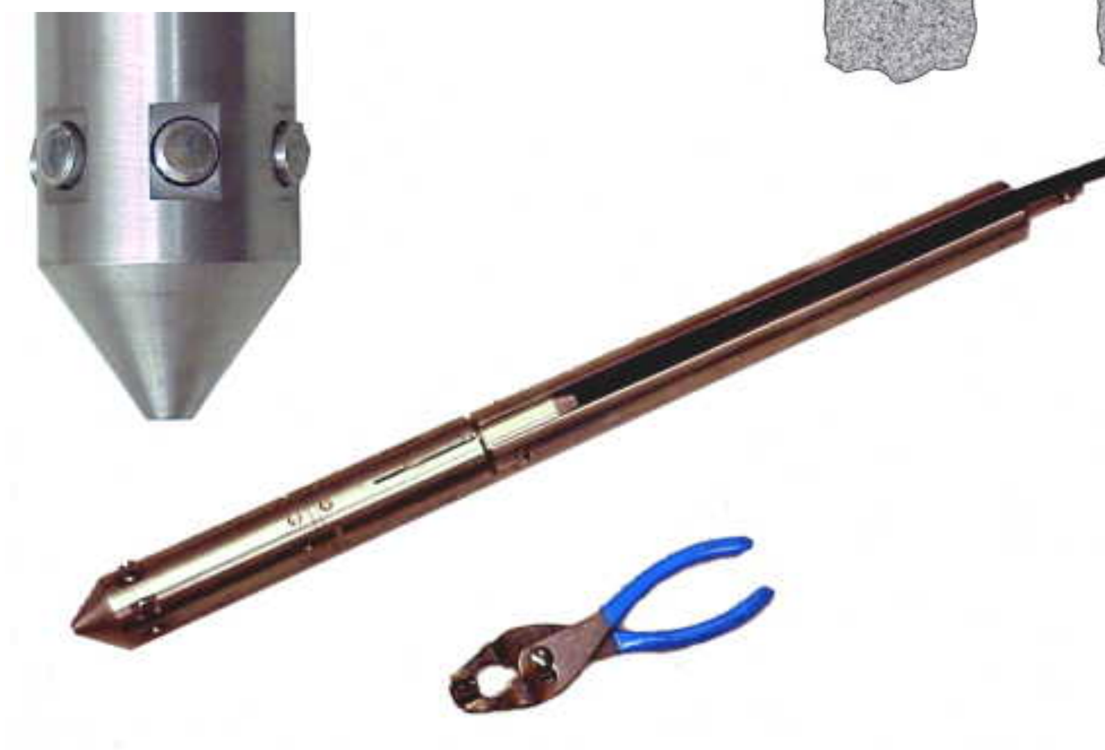
Overcoring – the operation



http://www.hydrofrac.com/hfo_home.html

Overcoring – the tool





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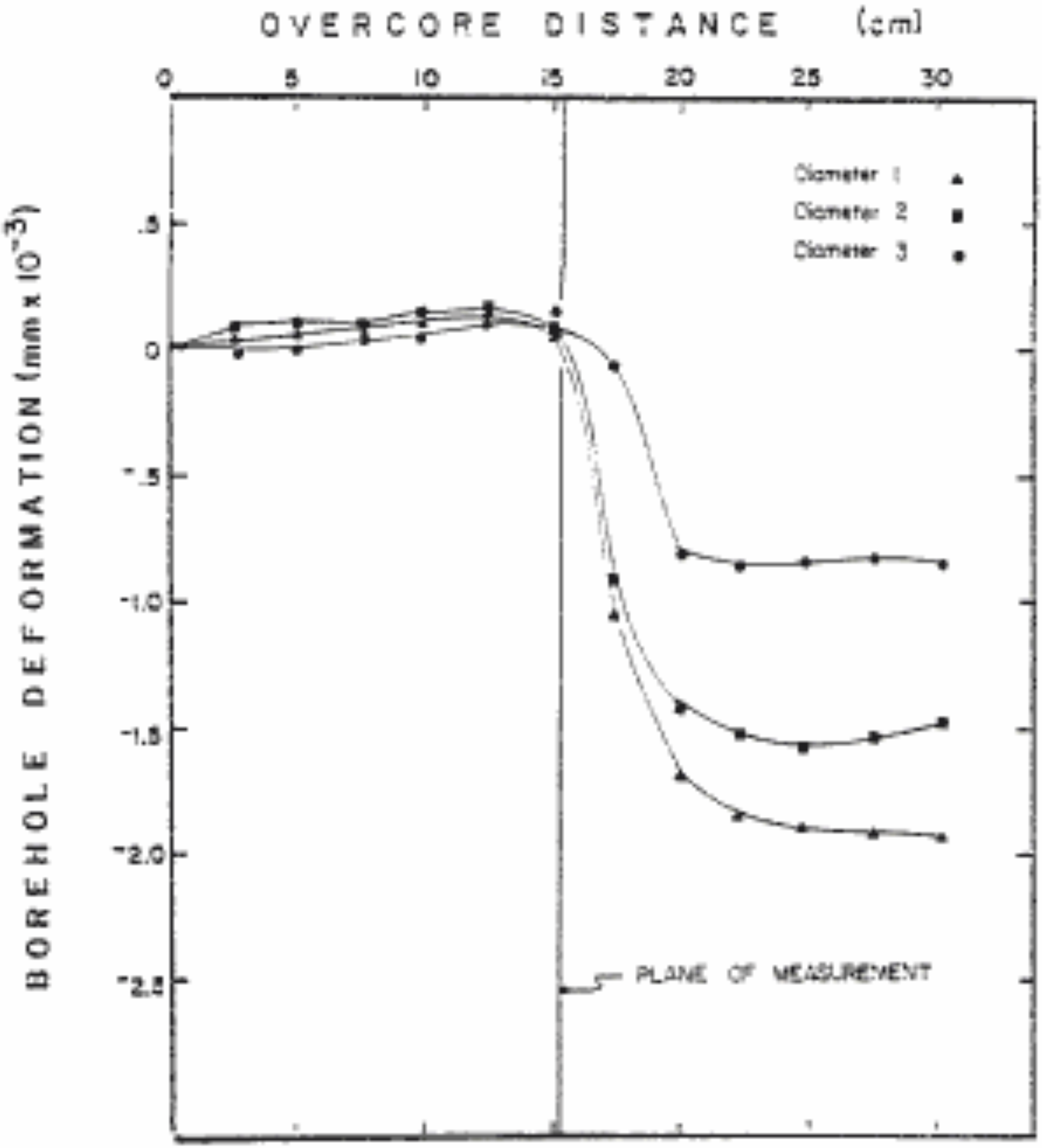
Overcoring – the data

Three channels of diametral deformations logged continuously as the deformation gage is overcored. As the overcoring bit passes through the plane of measurements, the stresses are relieved and the results are shown as diametral deformations.



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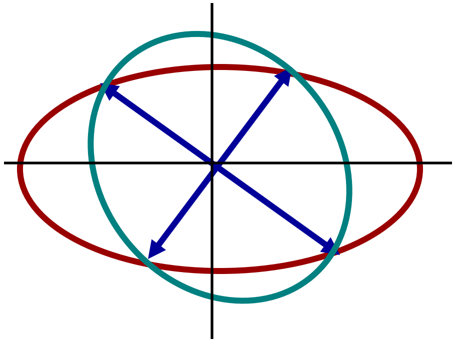
Why 3 sensors at 120°?

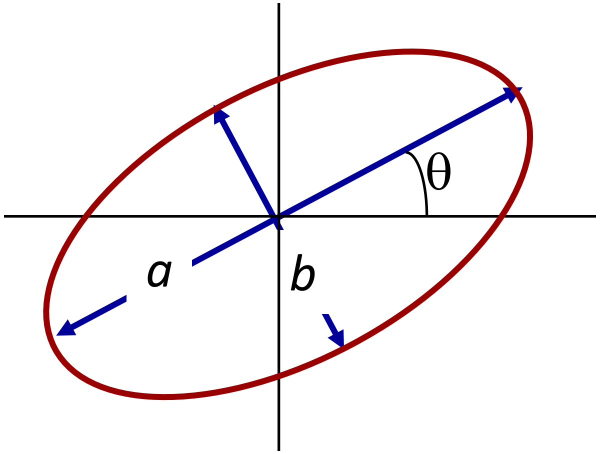
Why not 2 sensors at 90°?

- The hole will deform into an ellipse.
- Many ellipses can fit 2 diameters.

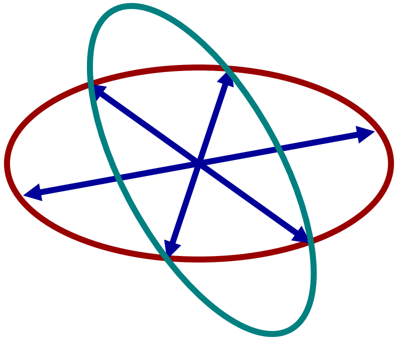


How many parameters are needed to describe an ellipse?

- Major axis a
- minor axis b
- orientation angle θ



With 3 diameters, there is a unique ellipse solution



Overcoring - calibration

Now we know the deformation of the hole, but not the stress in the surrounding rock.

What's next?

Rheological properties

- Pressure tests and tri-axial tests on the recovered cores to relate measured strain to relieved stress.

http://www.hydrofrac.com/hfo_home.html

Modes of Cracking

Mode I:	Mode II:	Mode III:
Opening	In-plane shear	Out-of-plane shear