ESS 411/511 Geophysical Continuum Mechanics Class #13

Highlights from Class #12 – Xinyu Wan
Today's highlights on Friday – Maleen Kidiwela

For Friday class

- Please read Mase, Smelser, and Mase, Ch 3 through Section 3.8

Your short CR/NC Pre-class prep writing assignment (1 point) in Canvas

- It will be due in Canvas at the start of class.
- I will send another message when it is posted in Canvas.

ESS 411/511 Geophysical Continuum Mechanics

Broad Outline for the Quarter

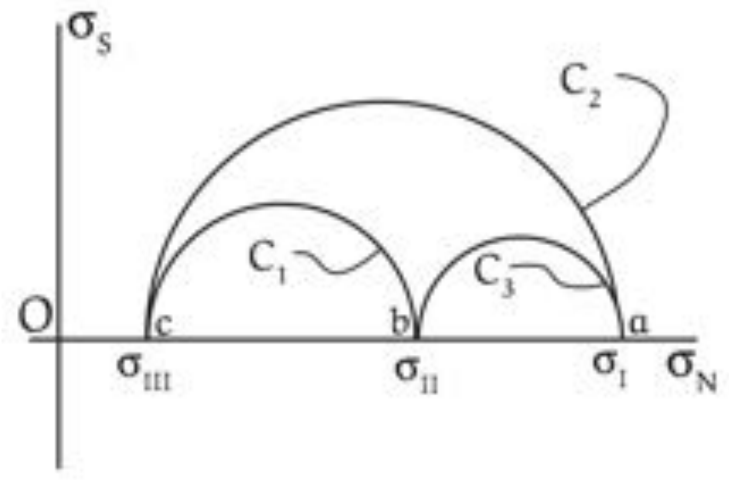
- Continuum mechanics in 1-D
- 1-D models with springs, dashpots, sliding blocks
- Attenuation
- Mathematical tools – vectors, tensors, coordinate changes
- Stress – principal values, Mohr's circles for 3-D stress
- Coulomb failure, pore pressure, crustal strength
- Measuring stress in the Earth
- Strain – Finite strain; infinitesimal strains
- Moments – lithosphere bending; Earthquake moment magnitude
- Conservation laws
- Constitutive relations for elastic and viscous materials
- Elastic waves; kinematic waves

Class-prep questions for today (break-out rooms)

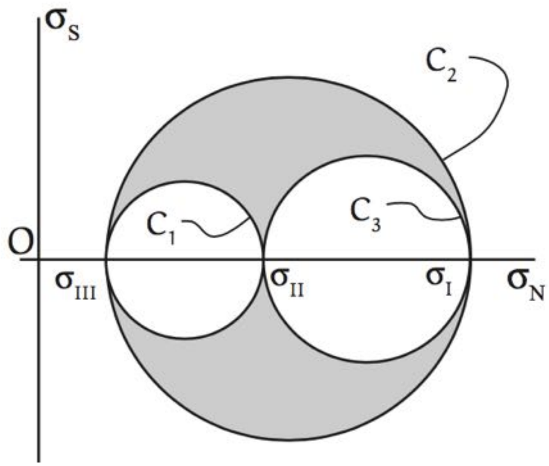
Mohr's circles

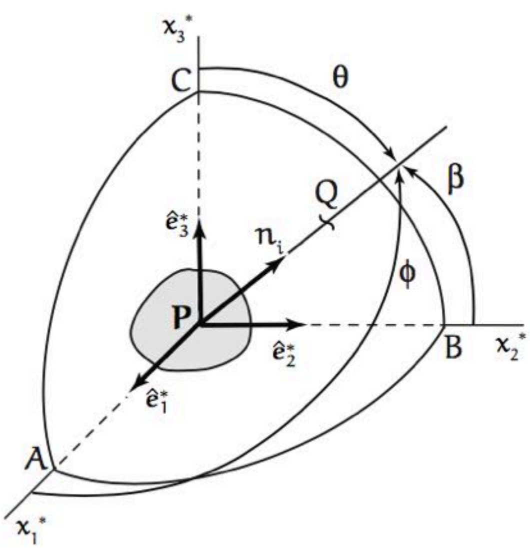
Why are Mohr's circles actually *circles* in stress space, and not some other shape, such as general ellipses for example?

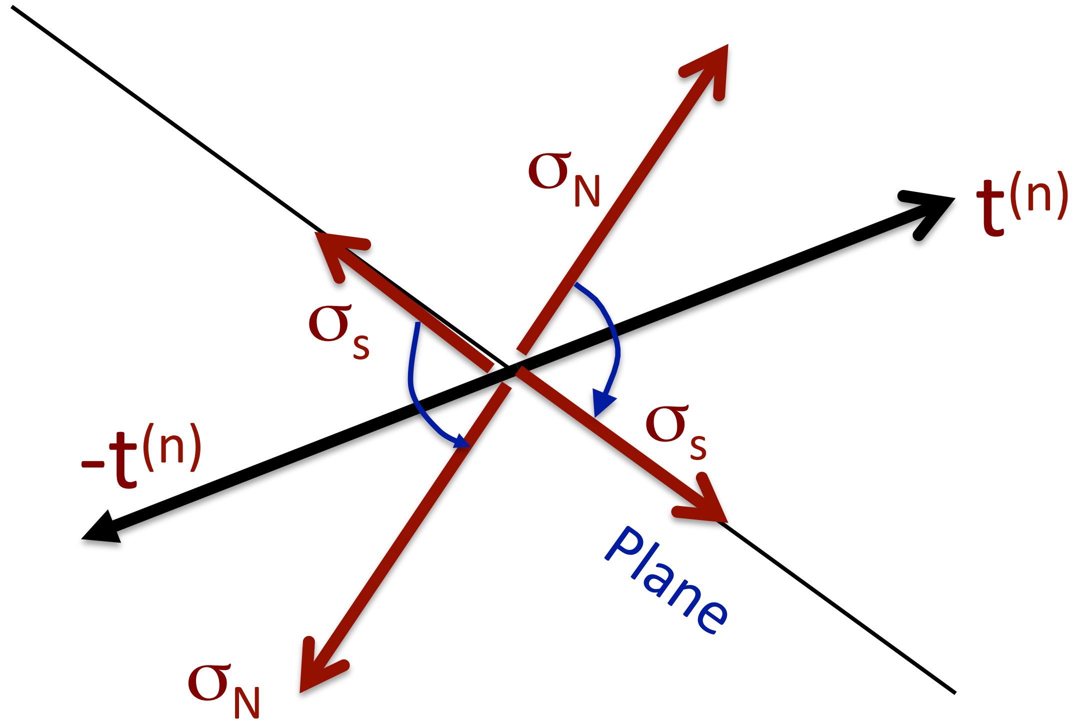
For a given stress tensor t_{ij} in stress space, with principal stresses
$\sigma_I > \sigma_{II} > \sigma_{III}$
the normal and shear components of t_{ij}, σ_N and σ_S for all possible planes with normal vectors n_i plot in a restricted areas in stress space, and all other areas are "out of bounds".
In which area do *all* the possible stress states (σ_N, σ_S) plot in the stress-space diagram?
Why do Mohr's circles always seem to stop at $\sigma_S = 0$? Is a negative shear component impossible?



Mohr's circles in 4th quadrant







Normal vectors n_j defining a plane can point in either direction, e.g. into opposite octant in Cartesian space

Problem Set #1

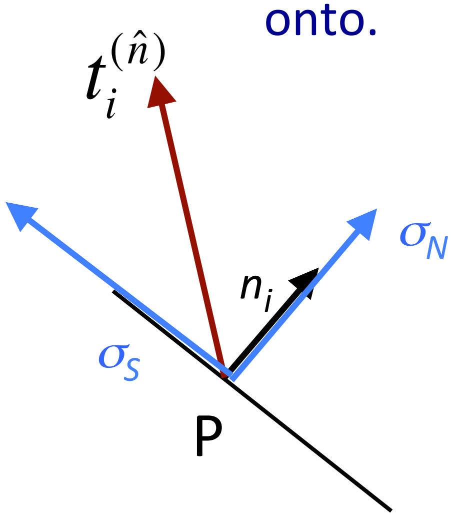
- Remember to include *units* when you evaluate expressions
- Some of you interpreted 1(f) to be about shearing the lithosphere.
  If you used the thickness of the asthenosphere (~200 km), you would have found that the strain rate fell below the minimum strain rate needed for failure.

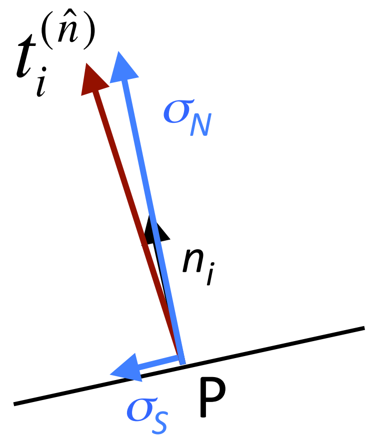
Question 2 – viscosity of silly putty

- Some of you are clearly theoreticians rather than experimentalists. You never told me how you would set up an experiment in your Lab.

Section 3.7 – Minimum and maximum stress values

σ_N and σ_S can vary dramatically depending on which plane the traction vector $t_i^{(\hat{n})}$ is resolved onto.



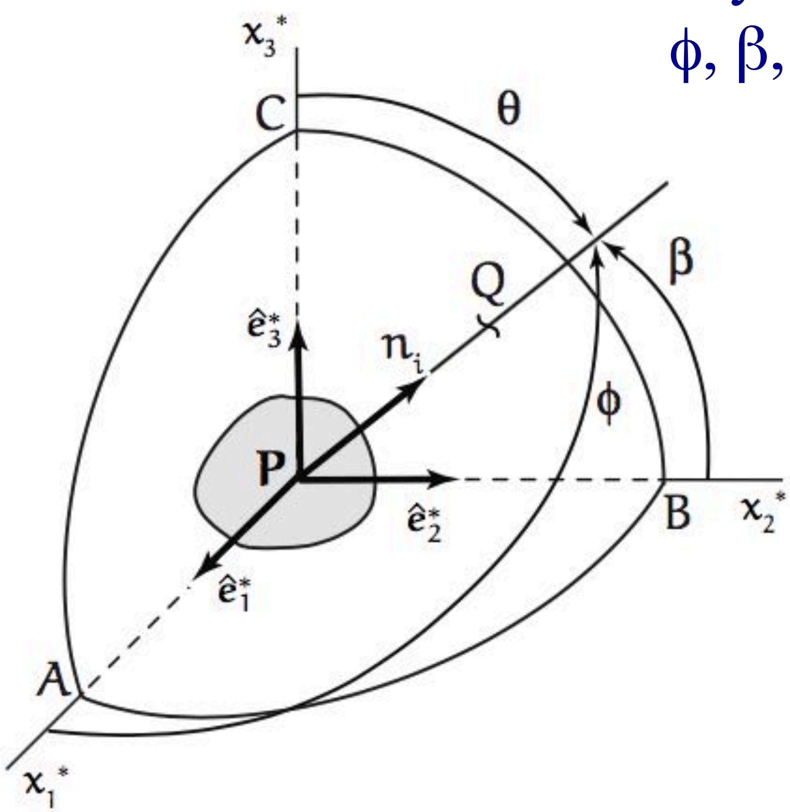


Cartesian Space vs Stress Space

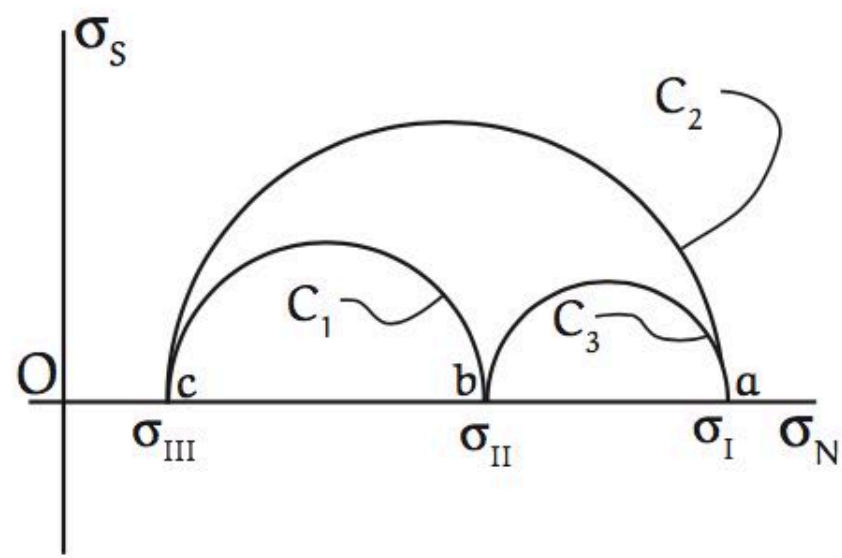
$\hat{e}_i^*$ are principal directions defining principal planes at **P**.
Lower-case letters in stress space correspond to upper-case letters in Cartesian space.

End point **Q** of unit vector n_i can fall anywhere on unit sphere centered at **P**.
ϕ, β, θ relate Q to coordinate axes.

The 3 circles correspond to Q lying in a principal plane.



(a) Octant of small spherical portion of body together with plane at **P** with normal n_i referred to principal axes $Ox_1^*x_2^*x_3^*$.



(b) Mohr's stress semicircle for octant of Fig. 3.14(a).

Cartesian Space *vs* Stress Space

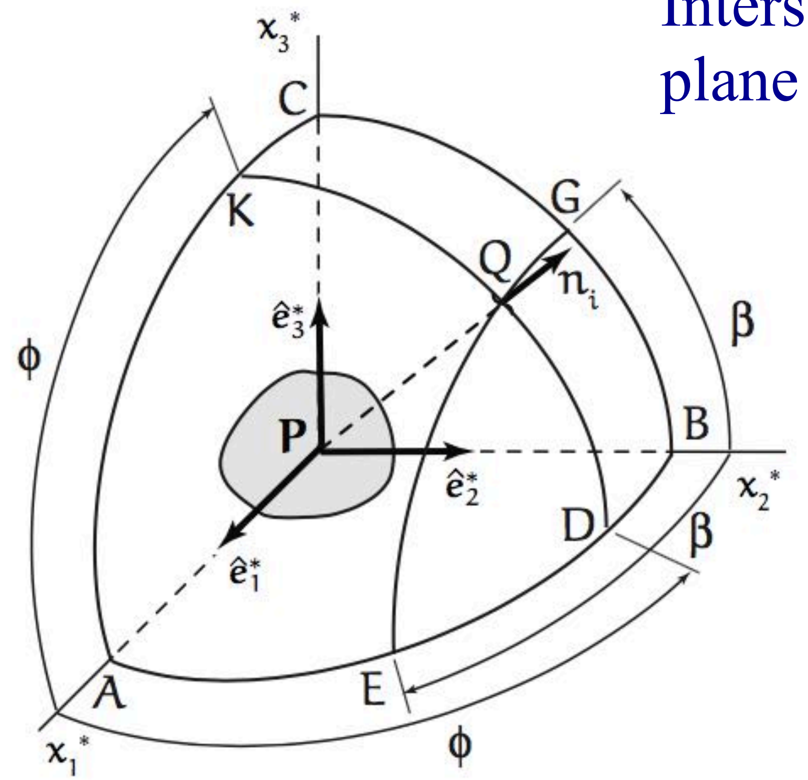
$\hat{e}_i^*$ are principal directions defining principal planes at **P**.

Small circles in Cartesian space (e.g. EQG) map onto circles (e.g. eqg) concentric with primary Mohr's circles (e.g. akc).

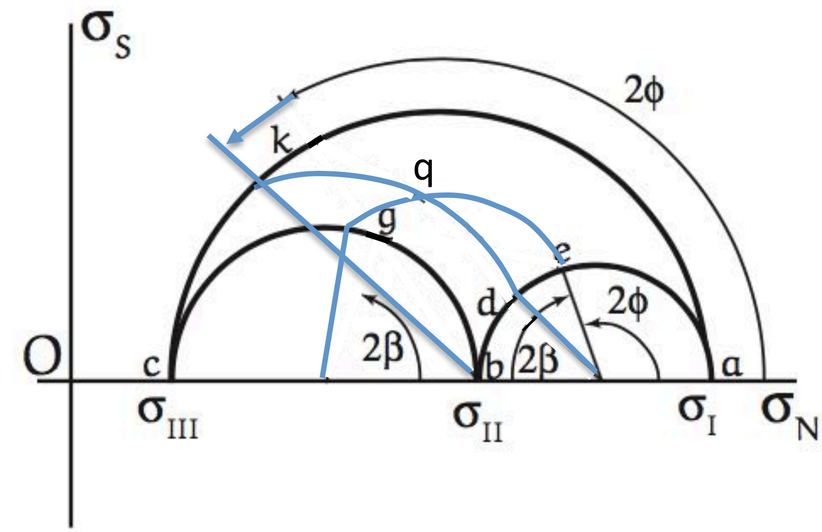
Similarly, KQD maps onto kqd.

Intersection at q shows σ_N and σ_s on plane defined by normal vector n_i at Q.

(I have attempted to (sort of) correct the stress-plane figure below. ☺)

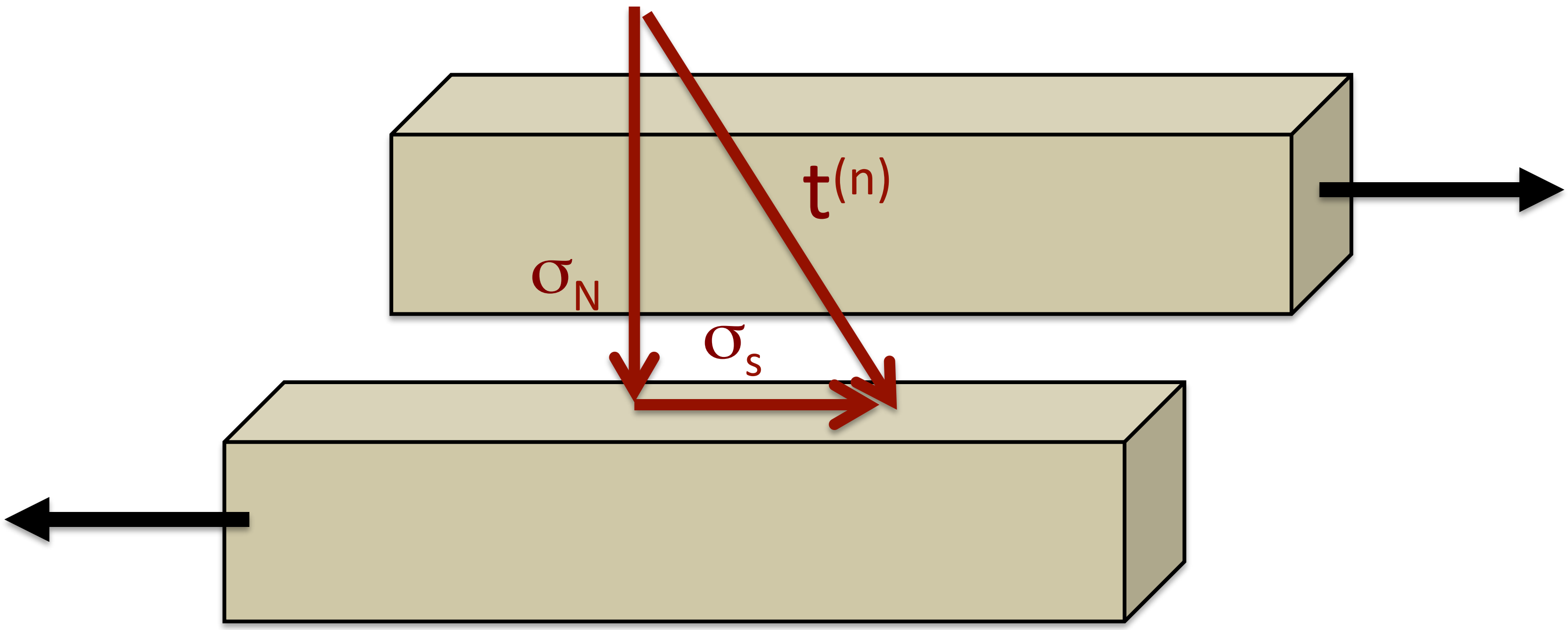


(a) Reference angles ϕ and β for intersection point Q on surface of body octant.



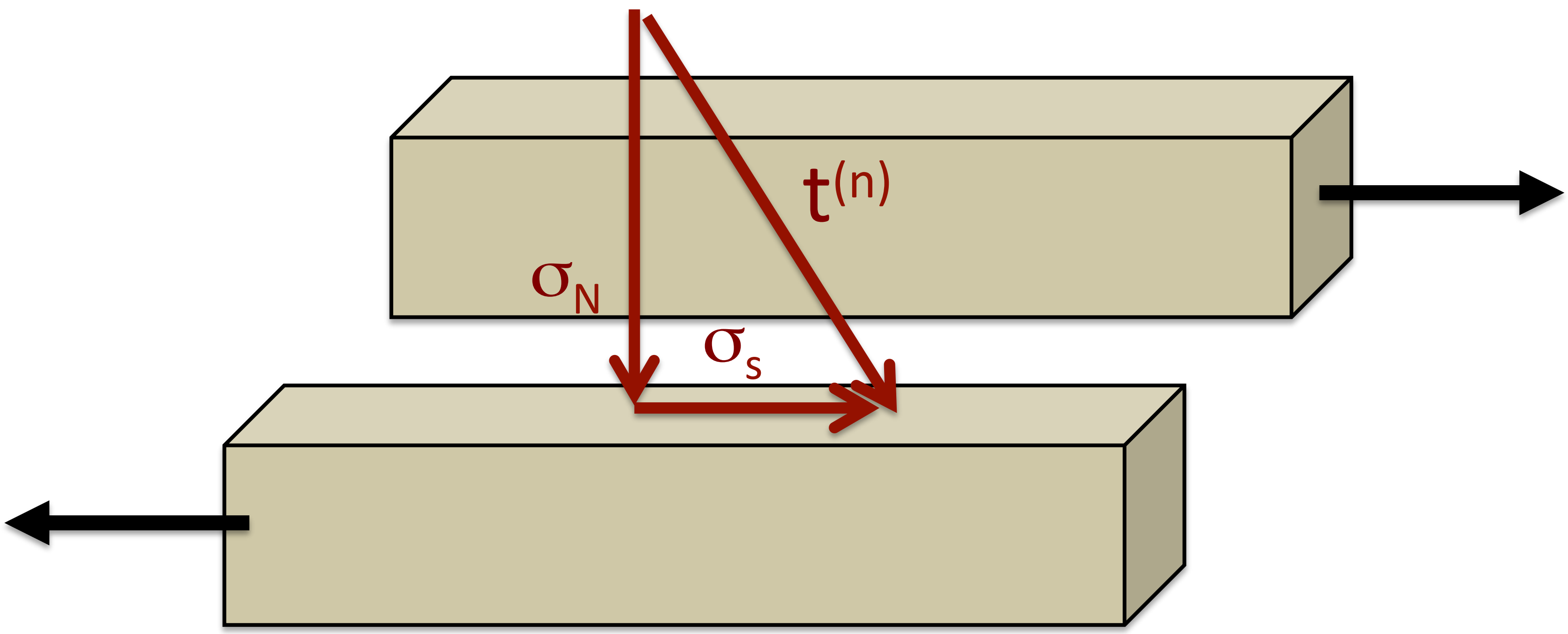
(b) Mohr's stress semicircle for octant of Fig. 3.15(a).

Sliding friction



$\sigma_S = -\mu\,\sigma_N$ μ is ***coefficient of friction*** for sliding on a pre-existing break

Mohr-Coulomb Fracture



$\sigma_S = -\, n\, \sigma_N$ n is ***cohesion***, or internal friction opposing rupture in unbroken material