ESS 411/511 Geophysical Continuum Mechanics Class #24

Highlights from Class #23 – Madeleine Lucas
Today's highlights on Monday – Maleen Kidiwela

For Monday November 30

Rock Failures

Please review the slides at:

http://courses.washington.edu/ess511/CLASS_MATERIALS/LECTURES/CLASS_25/Class_25_prep_slides.pdf

and watch these two short videos

Rockbursts in Olmos tunnel, Peru

http://www.youtube.com/watch?feature=player_detailpage&v=RtzNhss2h4w

Exfoliating granite dome, Sierra Nevada CA

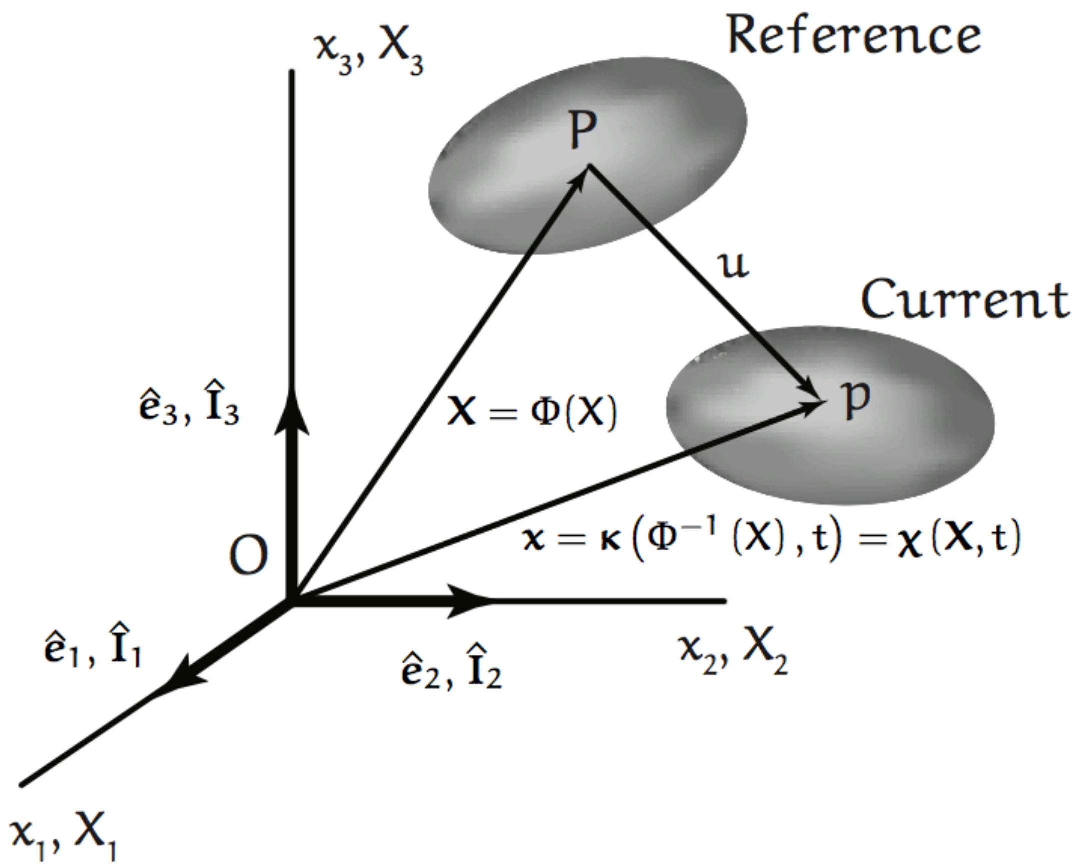
https://www.youtube.com/watch?v=yAZ1V_DJKV8

ESS 411/511 Geophysical Continuum Mechanics

Broad Outline for the Quarter

- Continuum mechanics in 1-D
- 1-D models with springs, dashpots, sliding blocks
- Attenuation
- Mathematical tools – vectors, tensors, coordinate changes
- Stress – principal values, Mohr's circles for 3-D stress
- Coulomb failure, pore pressure, crustal strength
- Measuring stress in the Earth
- Strain – Finite strain; infinitesimal strains
- Moments – lithosphere bending; Earthquake moment magnitude
- Conservation laws
- Constitutive relations for elastic and viscous materials
- Elastic waves; kinematic waves

Initial and Final Configurations



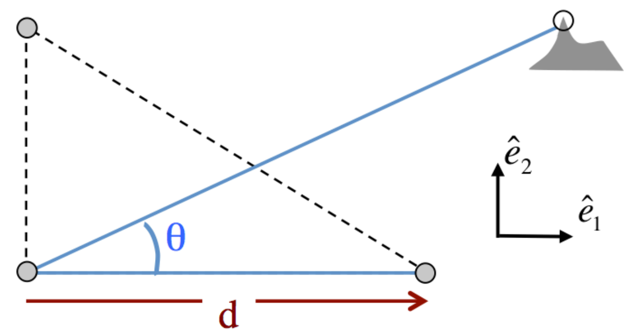
Class-prep: Moving Magma (Break-out rooms)

For small displacements and strains, the strain tensor can be written as (Eq. 4.72)

$$[\epsilon_{ij}] = \begin{bmatrix} \epsilon_{11} & \frac{1}{2}\gamma_{12} & \frac{1}{2}\gamma_{13} \\ \frac{1}{2}\gamma_{12} & \epsilon_{22} & \frac{1}{2}\gamma_{23} \\ \frac{1}{2}\gamma_{13} & \frac{1}{2}\gamma_{23} & \epsilon_{33} \end{bmatrix}$$

Magma is on the move at Mt Baker ski area

- You are a surveyor with a "total station" (theodolite and EDM) to measure angles and distances.
- Stress τ_{ij} is related to strain ε_{ij} by $\tau_{ij} = k\, \varepsilon_{ij}$ (k is an elastic-strength parameter)
- you have 3 benchmarks (survey stations) arranged as a right-angled triangle.



Assignment

(1) Understanding the strain tensor

- Physical meaning of the diagonal entries?
- Physical meaning of the off-diagonal entries?

(2) The strain tensor and the stress tensor

You have 1 month to determine the strain rate and stress in the ground from multiple surveys using your 3 survey benchmarks and the mountain peak.

- What measurements will you plan to make over the one-month period, to derive the strain tensor from your data?
- How will you calculate the stress tensor?
- How will you determine the orientation of a potential fissure?

Small strain entries

$$(dx)^2 - (dX)^2 = 2\epsilon_{ij} dX_i dX_j$$

$$\frac{dx - dX}{dX} \cdot \frac{dx + dX}{dX} = 2\epsilon_{ij} \frac{dX_i}{dX} \frac{dX_j}{dX}$$

$$dX_i/dX = N_i$$

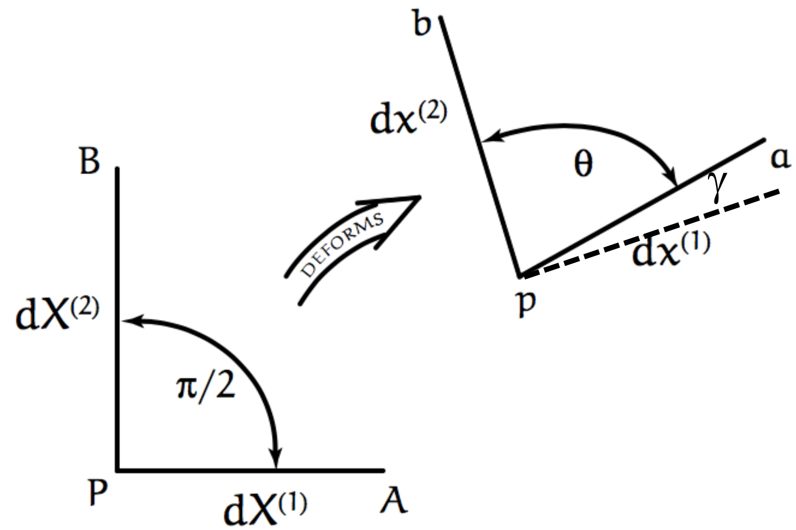
a unit vector in the direction of d**X**

$$(dx + dX)/dX \approx 2$$

$$\frac{dx - dX}{dX} = \epsilon_{ij} N_i N_j$$

change in length per unit original length for the element in the direction of **N**, called longitudinal strain.

If $N = \hat{I}_1$, then $\hat{\mathbf{I}}_1 \cdot \boldsymbol{\epsilon} \cdot \hat{\mathbf{I}}_1 = \epsilon_{11}$



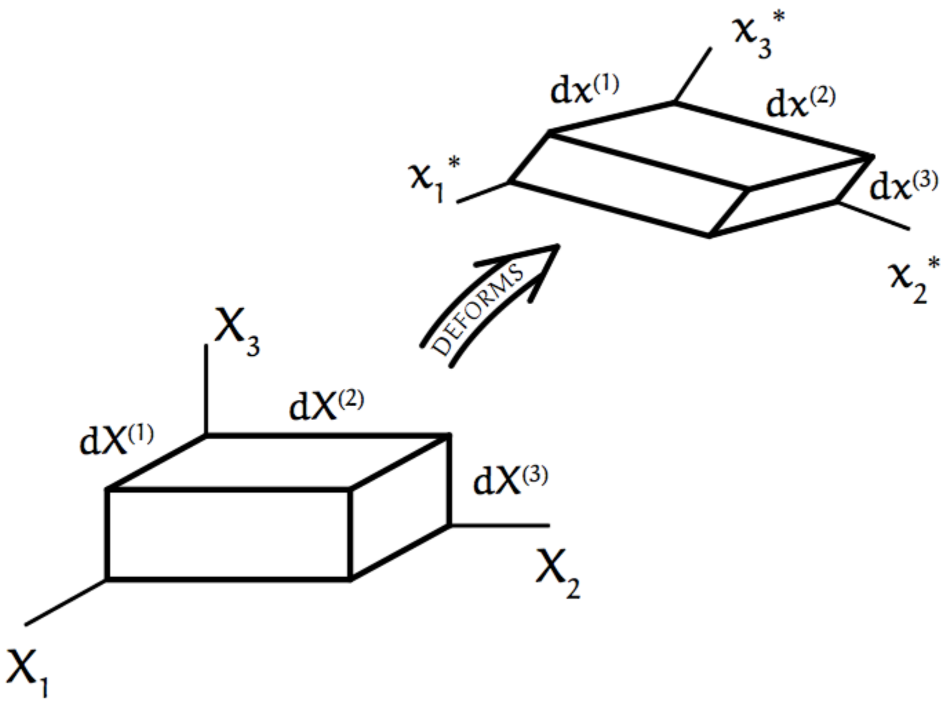
$$\cos\theta = \cos\left(\frac{\pi}{2} - \gamma\right) = \sin\gamma \approx \gamma$$

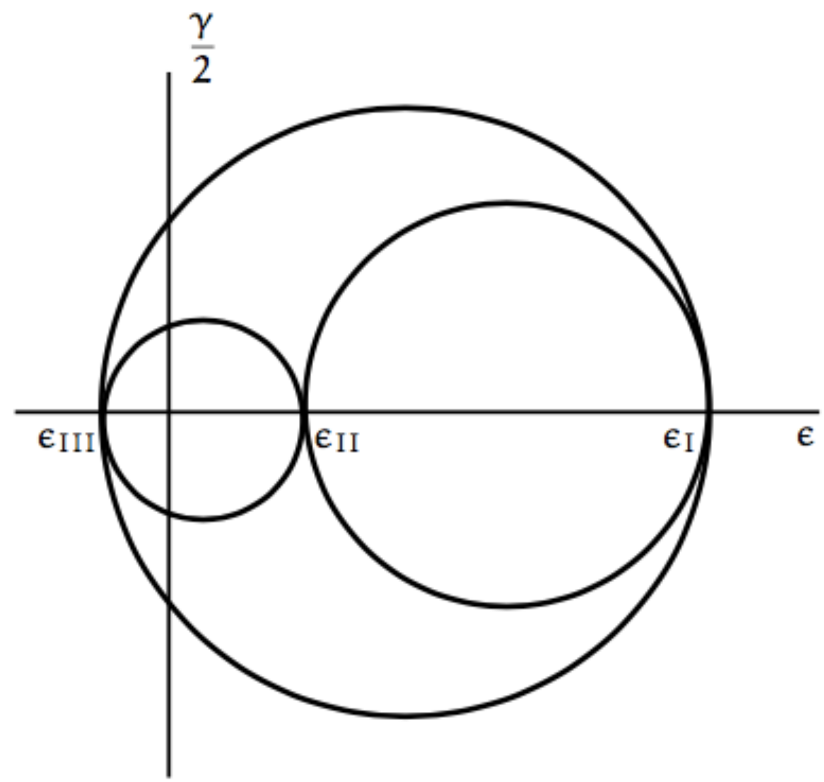
$$\mathbf{dx}^{(1)} \approx \mathbf{dX}^{(1)} \text{ and } \mathbf{dx}^{(2)} \approx \mathbf{dX}^{(2)}$$

$$\gamma \approx \cos\theta = \frac{\mathbf{dX}^{(1)}}{dx^{(1)}} \cdot 2\boldsymbol{\epsilon} \cdot \frac{\mathbf{dX}^{(2)}}{dx^{(2)}} \approx \hat{\mathbf{N}}_{(1)} \cdot 2\boldsymbol{\epsilon} \cdot \hat{\mathbf{N}}_{(2)}$$

If we set $\hat{\mathbf{N}}_{(1)} = \hat{\mathbf{I}}_1$ and $\hat{\mathbf{N}}_{(2)} = \hat{\mathbf{I}}_2$

$$\gamma_{12} = 2\,[1, 0, 0] \begin{bmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{12} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{13} & \epsilon_{23} & \epsilon_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = 2\epsilon_{12}$$


x_3^*
$dx^{(1)}$
$dx^{(2)}$
x_1^*
$dx^{(3)}$
x_2^*
DEFORMS
X_3
$dX^{(2)}$
$dX^{(1)}$
$dX^{(3)}$
X_2
X_1


$\frac{\gamma}{2}$
ϵ_{III}
ϵ_{II}
ϵ_{I}
ϵ

Deviator strain

Just like deviatoric stress …

$$\eta_{ij} = \epsilon_{ij} - \frac{1}{3}\delta_{ij}\epsilon_{kk} = \epsilon_{ij} - \delta_{ij}\epsilon_M \ ,$$

and in matrix form

$$\begin{bmatrix} \eta_{11} & \eta_{12} & \eta_{13} \\ \eta_{21} & \eta_{22} & \eta_{23} \\ \eta_{31} & \eta_{32} & \eta_{33} \end{bmatrix} = \begin{bmatrix} \epsilon_{11} - \epsilon_M & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{12} & \epsilon_{22} - \epsilon_M & \epsilon_{23} \\ \epsilon_{13} & \epsilon_{23} & \epsilon_{33} - \epsilon_{33} \end{bmatrix}$$

A measure for strain $(\mathrm{d}x)^2 - (\mathrm{d}X)^2$

$$
\begin{aligned}
(\mathrm{d}x)^2 - (\mathrm{d}X)^2 &= (x_{i,A}\,\mathrm{d}X_A)(x_{i,B}\,\mathrm{d}X_B) - \delta_{AB}\,\mathrm{d}X_A\,\mathrm{d}X_B \\
&= (x_{i,A}x_{i,B} - \delta_{AB})\,\mathrm{d}X_A\,\mathrm{d}X_B \\
&= (C_{AB} - \delta_{AB})\,\mathrm{d}X_A\,\mathrm{d}X_B
\end{aligned}
$$

Green's deformation tensor

$$C_{AB} = x_{i,A}x_{i,B} \qquad \text{or} \qquad \mathbf{C} = \mathbf{F}^{\mathsf{T}} \cdot \mathbf{F}$$

Lagrangian finite strain tensor

$$2E_{AB} = C_{AB} - \delta_{AB} \qquad \text{or} \qquad 2\mathbf{E} = \mathbf{C} - \mathbf{I}$$

A measure for strain $(\mathrm{d}x)^2 - (\mathrm{d}X)^2$

$$
\begin{aligned}
(\mathrm{d}x)^2 - (\mathrm{d}X)^2 &= \delta_{ij}\,\mathrm{d}x_i\,\mathrm{d}x_j - (X_{A,i}\,\mathrm{d}x_i)(X_{A,j}\,\mathrm{d}x_j) \\
&= (\delta_{ij} - X_{A,i}X_{A,j})\,\mathrm{d}x_i\,\mathrm{d}x_j \\
&= (\delta_{ij} - c_{ij})\,\mathrm{d}x_i\,\mathrm{d}x_j
\end{aligned}
$$

Cauchy deformation tensor

$$c_{ij} = X_{A,i}X_{A,j} \qquad \text{or} \qquad \mathbf{c} = (\mathbf{F}^{-1})^{T} \cdot (\mathbf{F}^{-1})$$

Eulerian finite strain tensor

$$2e_{ij} = (\delta_{ij} - c_{ij}) \qquad \text{or} \qquad 2\boldsymbol{e} = (\mathbf{I} - \mathbf{c})$$

Strain tensors in terms of displacements $u_i = (x_i - X_i)$

Lagrangian view

$$u_A(x_i) = x_A - X_A(x_i) \quad \text{so} \quad x_A = u_A(x_i) + X_i(x_i)$$

$$2E_{AB} = x_{i,A}x_{i,B} - \delta_{AB} = (u_{i,A} + \delta_{iA})(u_{i,B} + \delta_{iB}) - \delta_{AB}$$

$$2E_{AB} = u_{A,B} + u_{B,A} + u_{i,A}u_{i,B}$$

(Let's check it out in breakout rooms)

Eulerianian view

$$u_A(x_i) = x_A - X_A(x_i) \quad \text{so} \quad X_i(x_i) = x_A - u_A(x_i)$$

$$2e_{ij} = \delta_{ij} - X_{A,i}X_{A,j} = \delta_{ij} - (\delta_{Ai} - u_{A,i})(\delta_{Aj} - u_{A,j})$$

$$2e_{ij} = u_{i,j} + u_{j,i} - u_{A,i}u_{A,j}$$

(Let's check it out in breakout rooms)