ESS 411/511 Geophysical Continuum Mechanics Class #10

Highlights from Class #9 – Chloe Mcburney
Today's highlights on Friday – Brandon Lomahoynaya

For Friday class
- Please read Mase, Smelser, and Mase, Ch 3 through Section 3.6

Your short CR/NC Pre-class prep writing assignment (1 point) in Canvas
- It will be due in Canvas at the start of class.
- I will send another message when it is posted in Canvas.

ESS 411/511 Geophysical Continuum Mechanics

Broad Outline for the Quarter

- Continuum mechanics in 1-D
- 1-D models with springs, dashpots, sliding blocks
- Attenuation
- Mathematical tools – vectors, tensors, coordinate changes
- Stress – principal values, Mohr's circles for 3-D stress
- Coulomb failure, pore pressure, crustal strength
- Measuring stress in the Earth
- Strain – Finite strain; infinitesimal strains
- Moments – lithosphere bending; Earthquake moment magnitude
- Conservation laws
- Constitutive relations for elastic and viscous materials
- Elastic waves; kinematic waves

Problem Sets

Problem Set #2 due in Canvas today

Problem Set #3 in Problem session tomorrow

ESS 511 Term Projects

For those of you taking this class as ESS 511, a reminder that in class on Friday, I will ask each of you for a 60-second outline of your ideas so far about your term topic.

Warm-up (break-out rooms)

Vectors and tensors get more interesting when they vary with time t and with position $\boldsymbol{x}$ inside a body.

Explain in words what is meant by:

- $\boldsymbol{v}(\boldsymbol{x},t)$ $\sigma_{ij}(\boldsymbol{x},t)$
- v_i $v_{i,i}$ $v_{i,j}$ $\varepsilon_{ijk}v_{k,j}$
- σ_{ij} $\sigma_{ii,k}$ $\sigma_{ij,k}$ $\sigma_{ij,kk}$ $\sigma_{ij,kl}$
- $x_{i,j} = \delta_{ij}$

Class- prep questions (break-out rooms)

- Choosing eigenvectors.

Derivatives of tensors

A tensor can vary smoothly in space and time, so it has derivatives.

$\dfrac{\partial v_j}{\partial t}$ = rate at which velocity vector is changing at a point.

$\nabla \vec{v} = v_{i,j}$ = rate at which velocity vector is changing at a point.
There are 2 indices, so this is a 3x3 array.

$$\begin{bmatrix} \dfrac{\partial v_1}{\partial x_1} & \dfrac{\partial v_1}{\partial x_2} & \dfrac{\partial v_1}{\partial x_3} \\ \dfrac{\partial v_2}{\partial x_1} & \dfrac{\partial v_2}{\partial x_2} & \dfrac{\partial v_2}{\partial x_3} \\ \dfrac{\partial v_3}{\partial x_1} & \dfrac{\partial v_3}{\partial x_2} & \dfrac{\partial v_3}{\partial x_3} \end{bmatrix}$$

Elements show how each vector component varies in each spatial direction.

Divergence

A tensor can vary smoothly in space and time, so it has derivatives.

Scalar - a scalar doesn't have divergence ☹

Vector - $\nabla \bullet \vec{v} = v_{i,i}$

Tensor $\nabla \bullet T = t_{ij,i}$

$$\begin{bmatrix} t_{11,1} + t_{21,2} + t_{31,3} \\ t_{12,1} + t_{22,2} + t_{32,3} \\ t_{13,1} + t_{23,2} + t_{33,3} \end{bmatrix}$$

Each row is the divergence of the corresponding column of ***T***

Gradient

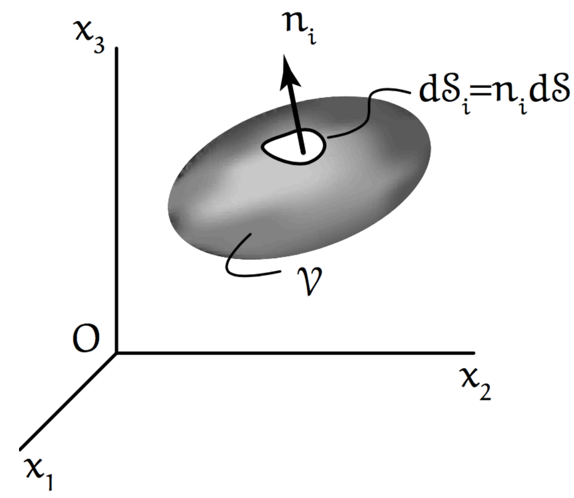
A tensor can vary smoothly in space and time, so it has derivatives.

Scalar $\dfrac{\partial \phi}{\partial x_j} = \phi_{,j}$

Vector $\nabla \vec{v} = \dfrac{\partial v_i}{\partial x_j} = v_{i,j}$

Tensor etc …

Integral theorems



We may want to know what is going on inside a body but have access only to its surface (or vice versa)

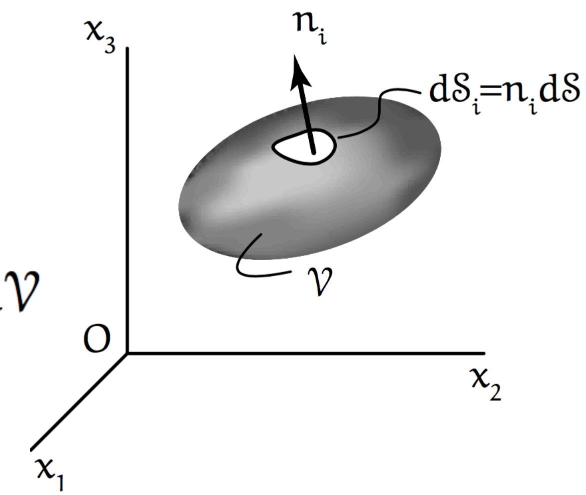
A volume V has surface S.

- Each small patch dS on the surface is defined by its normal vector n_i.

Divergence theorem $$\int_S t_{ij\ldots k} n_q \, dS = \int_V t_{ij\ldots k,q} \, dV$$

The total amount of $t_{ij\ldots k}$ directed out across S is the same as the total amount of spreading (divergence) everywhere inside V.

Special cases



Divergence theorem $\displaystyle\int_{\mathcal{S}} t_{ij\ldots k} n_q \, d\mathcal{S} = \int_{\mathcal{V}} t_{ij\ldots k,q} \, d\mathcal{V}$

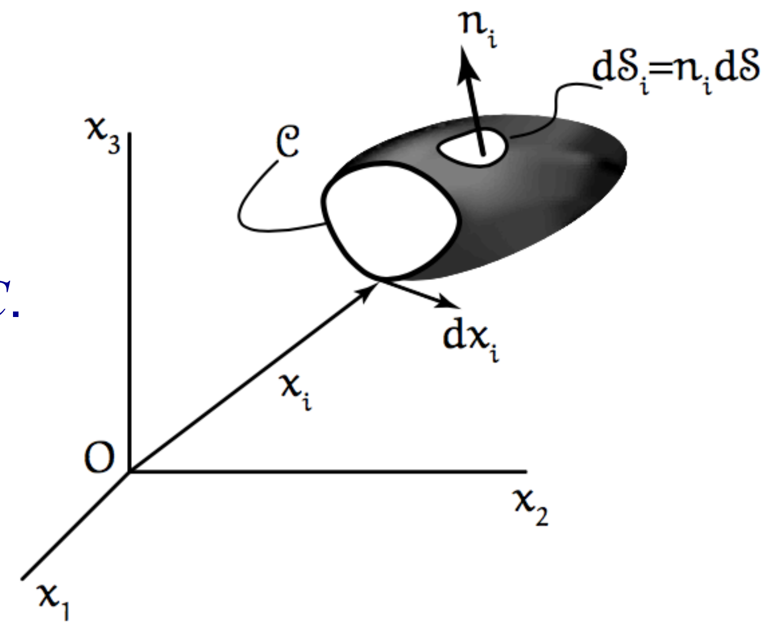
$$\int_{\mathcal{S}} v_q n_q \, d\mathcal{S} = \int_{\mathcal{V}} v_{q,q} \, d\mathcal{V} \quad \text{or} \quad \int_{\mathcal{S}} \boldsymbol{v} \cdot \hat{\mathbf{n}} \, d\mathcal{S} = \int_{\mathcal{V}} \operatorname{div} \boldsymbol{v} \, d\mathcal{V}$$

If density ρ is uniform, the total amount of "stuff" flowing out across S with velocity *v* (the flux across S) is the same as the total amount of spreading (divergence) of that "stuff" everywhere inside V.

Stokes theorem

C is the perimeter of a cap on an open surface.

- d***x*** is the tangent to the perimeter C.
- ***v*** is the material velocity.



$$\int_{\mathcal{S}} \varepsilon_{ijk} n_i v_{k,j}\, d\mathcal{S} = \int_{\mathcal{C}} v_k\, dx_k \quad \text{or} \quad \int_{\mathcal{S}} \hat{\mathbf{n}} \cdot (\boldsymbol{\nabla} \times \boldsymbol{v})\, d\mathcal{S} = \int_{\mathcal{C}} \boldsymbol{v} \cdot d\mathbf{x}$$

If density ρ is uniform, the total circulation of "stuff" (curl) within the cap ("churning") is equal to the net flow along the perimeter C ("the racetrack").

($\varepsilon_{ijk} v_{j,k}$ is curl of ***v***)

Definition of a tensor

In any rectangular coordinate system, a tensor is defined by 9 components that transform according to the rule

$$R'_{ij\ldots k} = a_{iq} a_{jm} \cdots a_{kn} R_{qm\ldots n}$$

And where the basis vectors are related by

$$\hat{e}'_i = a_{ij} \hat{e}_j$$

Forces in a continuum

Body forces b_i force per volume
Surface forces f_i force per area
(on exterior or interior surfaces)

Newton's second law F = ma

$$\int_V \rho(\vec{x}) b_i dV + \int_S t_i^{(\hat{n})} dS = \frac{d}{dt} \int_V \rho v_i dV$$