ESS 411/511 Geophysical Continuum Mechanics Class #16

Highlights from Class #15 – Andrew Gregovich
Today's highlights on Friday – Madie Mamer

Our text doesn't cover our next topics very thoroughly, so we will use a few other sources, which are posted on the class web site under READING & NOTES. https://courses.washington.edu/ess511/NOTES/notes.shtml

- Stein and Wysession 5.7.2
- Stein and Wysession 5.7.3/4
- Raymond notes on failure

Also see slides about upcoming topics

- Failure and Mohr's circles – slides

Your short CR/NC Pre-class prep writing assignment (1 point) in Canvas

- It will be due in Canvas at the start of class.
- I will send another message when it is posted in Canvas.

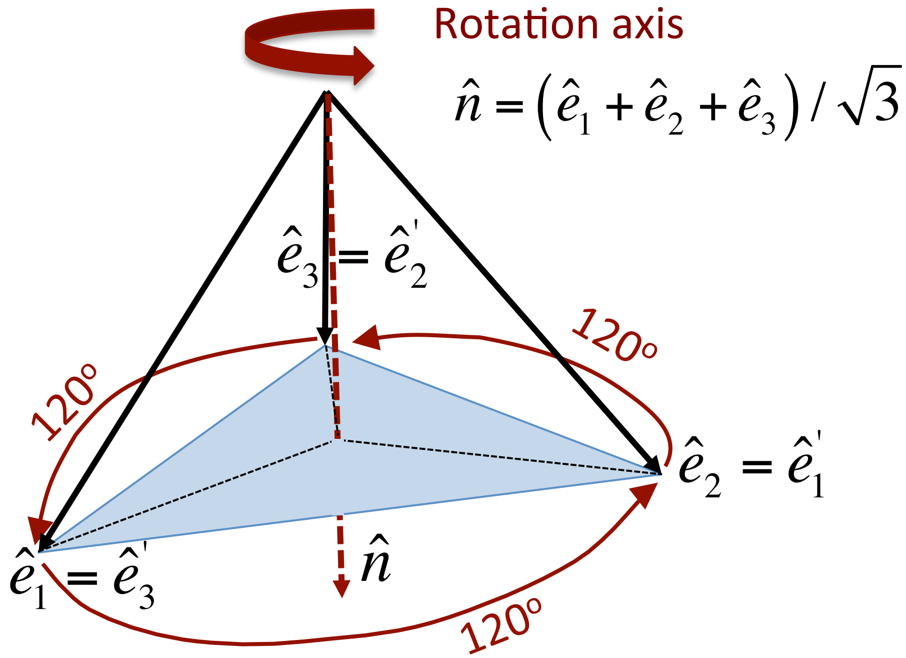
ESS 411/511 Geophysical Continuum Mechanics

Broad Outline for the Quarter

- Continuum mechanics in 1-D
- 1-D models with springs, dashpots, sliding blocks
- Attenuation
- Mathematical tools – vectors, tensors, coordinate changes
- Stress – principal values, Mohr's circles for 3-D stress
- Coulomb failure, pore pressure, crustal strength
- Measuring stress in the Earth
- Strain – Finite strain; infinitesimal strains
- Moments – lithosphere bending; Earthquake moment magnitude
- Conservation laws
- Constitutive relations for elastic and viscous materials
- Elastic waves; kinematic waves

Homework #3 – Question #1

The coordinate axes look like a tripod, and the rotation axis was vertical
Some of you drew a left-handed coordinate system, so the unprimed axes mapped onto the wrong primed axes.
That screwed up your transformation matrix a_{ij} matrix because $a_{ij} = \hat{e}_i^{'} \bullet \hat{e}_j$



After transforming the axes
$T' = A\,T\,A^T$ should still have 9 entries which are just a re-arrangement of the original t_{ij}.
Because T is isotropic, you now have 9 relations equating 2 different elements t_{ij}.
The second rotation gives another 9 relations.

Then the only way to satisfy all those relations is by $\mathbf{T} = \lambda\,\mathbf{I}$

Good news
- even with a left-handed rotation, you should still get the same result ☺

Homework #3 – Question #2

Show that δ_{ij} and ε_{ijk} are unchanged in new coordinate systems

There was a lot of abuse of index notation. For example -

- More than 2 "i" or "j" indices in a term e.g. $a_{ij}\, a_{ij}\, a_{ij}\, \varepsilon_{ijk}$
- det(A) = $a_{ij}\, a_{kl}\, a_{mn}$ LHS is a scalar, but RHS is a 6th order tensor

- A correct expression is $\varepsilon_{qmn}\, \det(A) = \varepsilon_{ijk}\, a_{iq}\, a_{jm}\, a_{kn}$

Homework #3 – Question #3

Determine the principal values of the matrix

$$[K_{ij}] = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 11 & -\sqrt{3} \\ 0 & -\sqrt{3} & 9 \end{bmatrix}$$

and show that the principal axes $Ox_1{}^*x_2{}^*x_3{}^*$ are obtained from $Ox_1x_2x_3$ by a rotation of 60° about the x_1 axis.

- Most people got the 3 eigenvalues (4,8,12) .

There were two approaches to showing that a 60° rotation around x_1 put K_{ij} into its principal coordinates.

1. Find the eigenvectors and show that thei x^*_2 and x^*_3 unit vectors have been rotated 60° from the initial coordinate axes.
2. Find the transformation matrix a_{ij} and then express K'_{ij} in the rotated coordinates : $K'_{ij} = a_{im}a_{jn}K_{mn}$
   - I saw a lot of incorrect transformation matrices, where the off-diagonal elements had the wrong signs, so the rotation went in the wrong direction
   - Remember that $a_{ij} = \hat{e}_i^{'} \bullet \hat{e}_j$
   - If K'_{ij} is going to be in principal coordinates, it had better be in diagonal form, with diagonal elements 4,8,12 (!)

4 Conventions in Stress Polarity

Engineering/Mathematical convention:

Criterion 1: Positive σ_{ii} * signifies extension

Criterion 2: Order $\sigma_I > \sigma_{II} > \sigma_{III}$ (Mase & Mase)

or

$\sigma_I < \sigma_{II} < \sigma_{III}$ (Stein & Wysession)

Geologic/Tectonic/Rock Mechanics convention:

Criterion 1: Positive σ_{ii} * signifies compression

(not a tensor!! Why not?)

Criterion 2: Order $\sigma_I > \sigma_{II} > \sigma_{III}$ (Twiss & Moores)

or

$\sigma_I < \sigma_{II} < \sigma_{III}$ (?)

* No sum implied

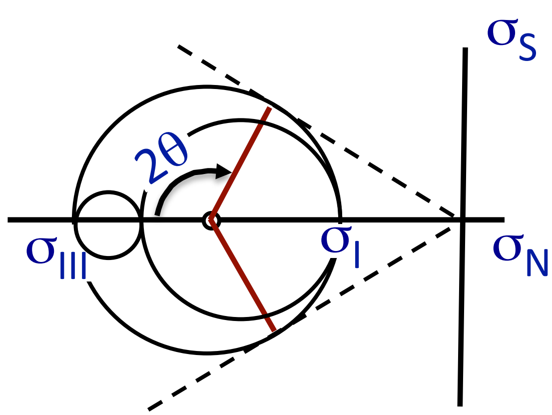
Class-prep questions for today (break-out rooms)

Greatest shear stress is on planes …….. with normal vectors n_i at 45° to σ_{III}. But failure actually happens on planes – – – – at an angle θ>45° between n_i and σ_{III}

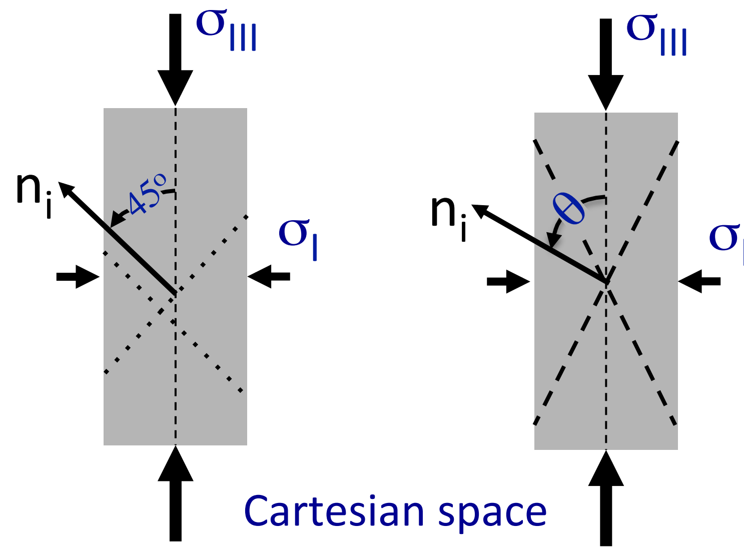
- Why is failure ***not*** on the plane with maximum shear stress? Think of the role of σ_N in preventing slip. Can you relate this to the tangent to the top of the Mohr's circle?
- All surfaces are roughs at some scale. Relate this failure angle to how one rough surface slides over another rough surface.

Failure planes – – – are defined by their normal vectors n_i.

- Why are there 2 conjugate failure planes?
- Relate this to the Mohr's circle.



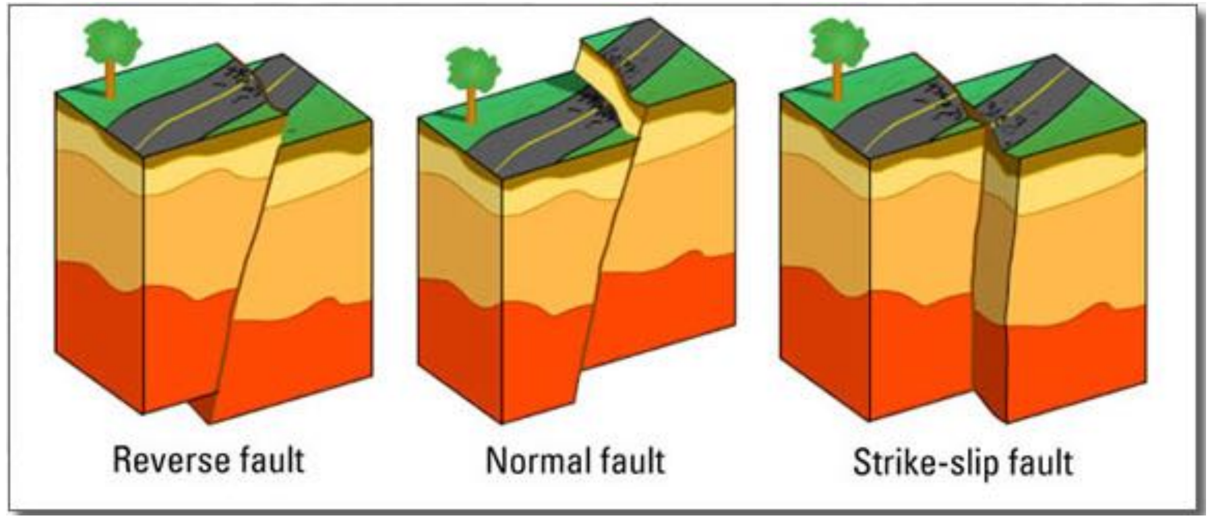
Stress space



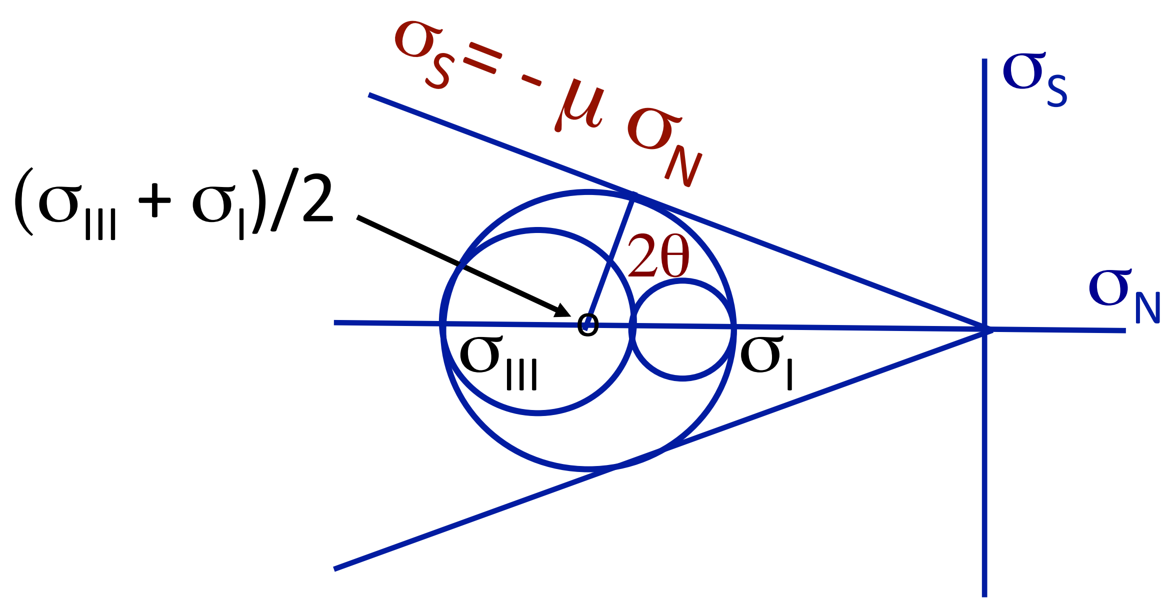
Cartesian space

Types of faults

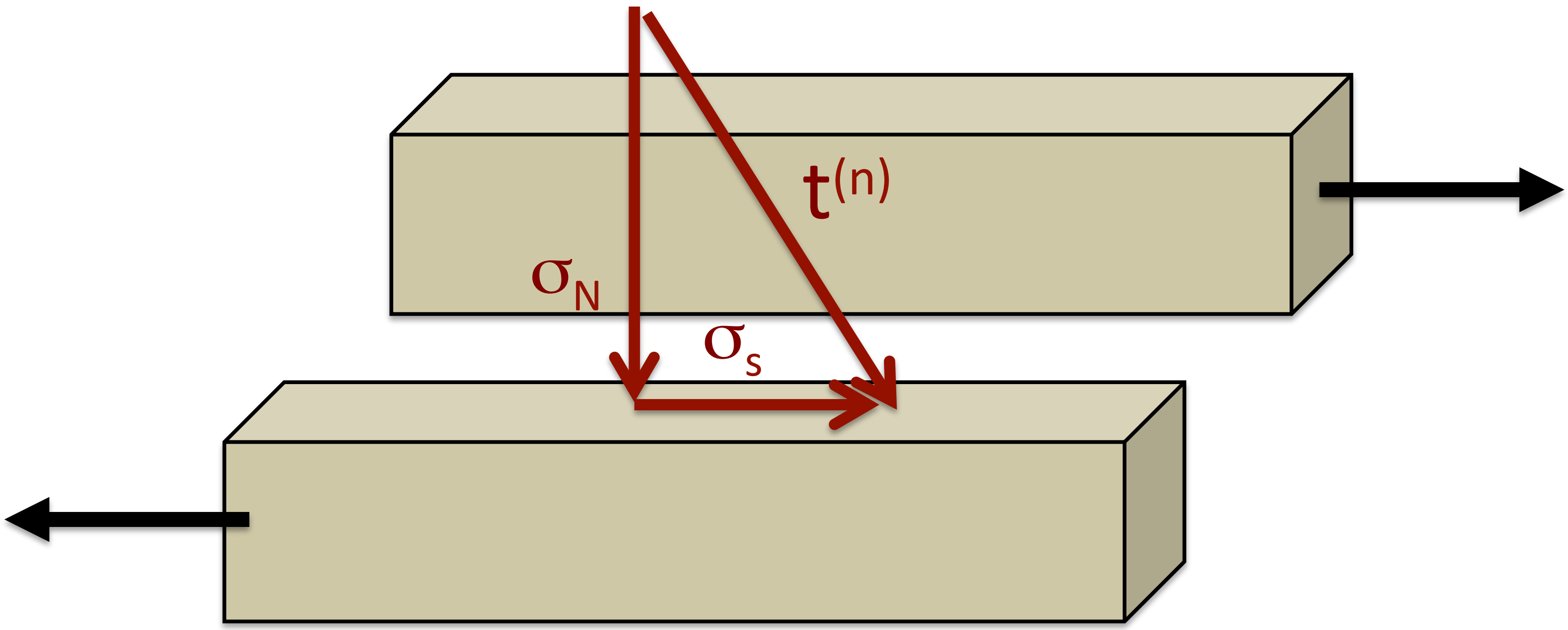
The Earth's surface is traction-free, so one of the principal directions is generally vertical



What are the orientations of the principal axes of stress $\hat{e}_1^*,\ \hat{e}_2^*,\ \hat{e}_3^*$ in each case?

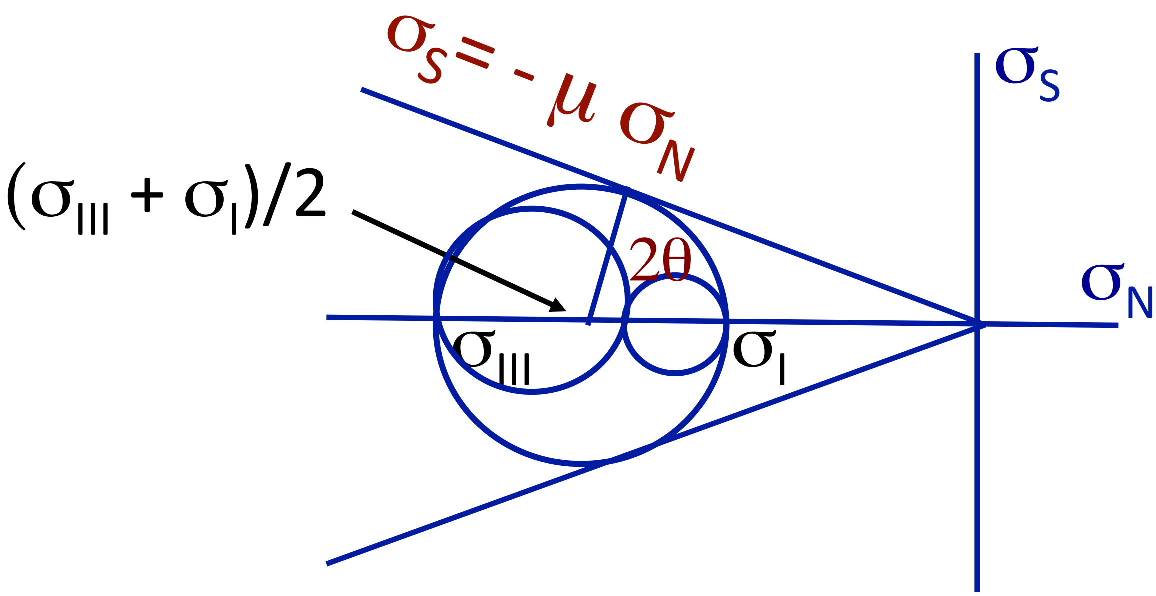


Sliding friction



$\sigma_S = -\mu\,\sigma_N$ μ is ***coefficient of friction*** for sliding on a pre-existing break

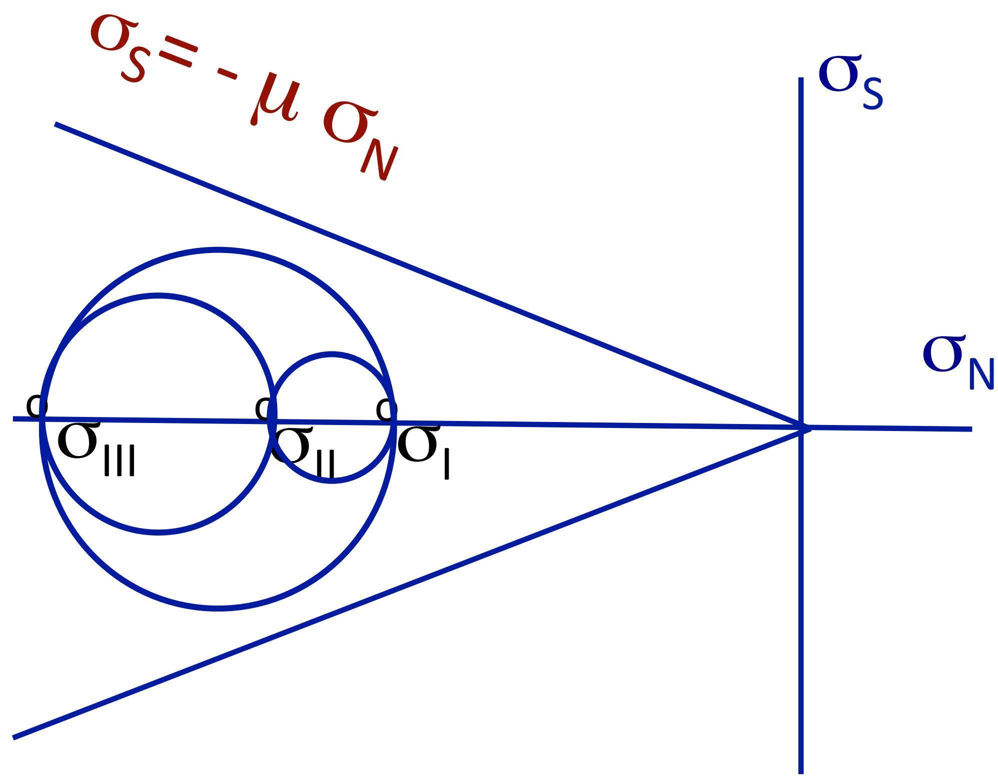
Frictional sliding



$\sigma_S = -\mu\,\sigma_N$ μ is ***coefficient of friction*** for sliding on a pre-existing break

Differential stress

$\sigma_I - \sigma_{III}$

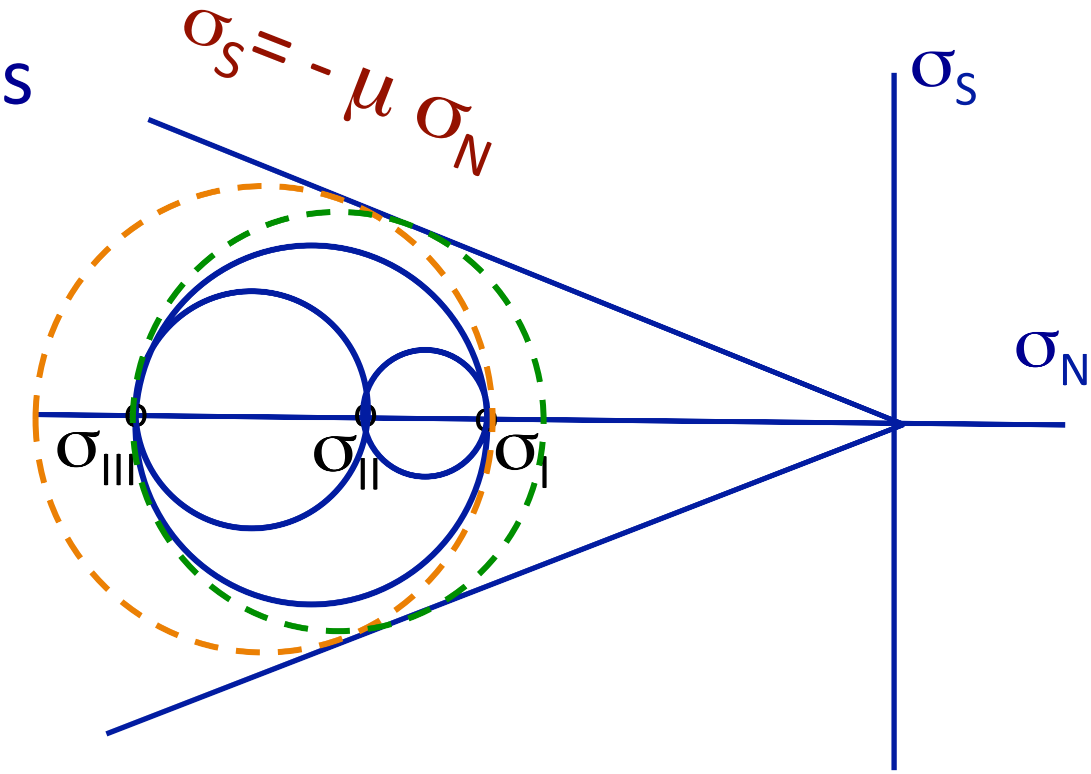


But, if $\sigma_{III} = \sigma_I$, all 3 principal stresses are equal

- What do the 3 Mohr's circle look like?
- Describe this state of stress inside the body.
- Is frictional failure possible, if differential stress is zero?

Differential stress

$\sigma_I - \sigma_{III}$

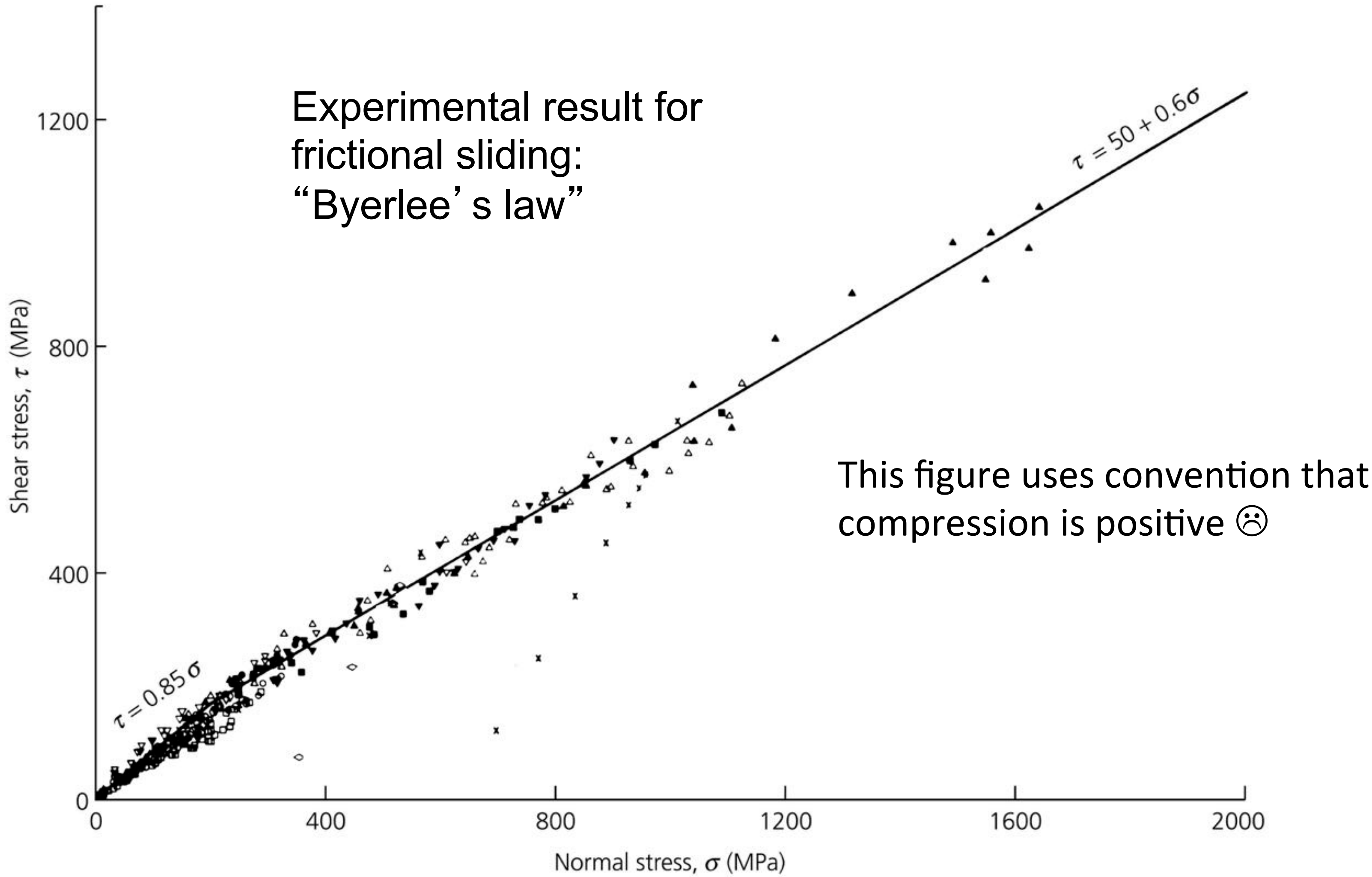


So differential stress is essential in order to have failure

How could we change the stress state in order to cause failure?

- Hold σ_I make σ_{III} more negative (squeeze harder in x_3)
- Hold σ_{III}, make σ_I less negative (don't squeeze as hard in x_1)

Figure 5.7-10: Relation between shear stress and normal stress for frictional sliding.

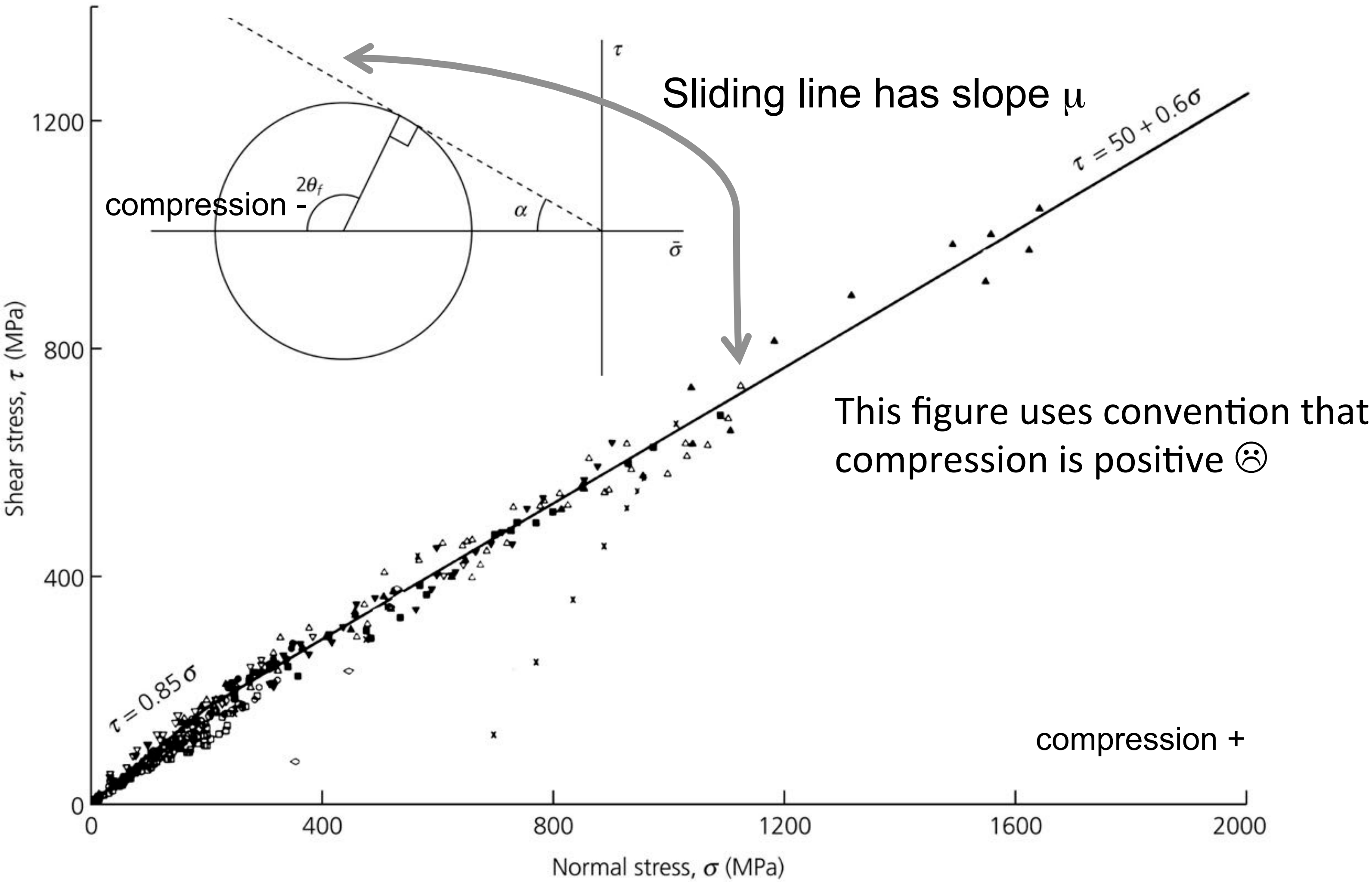


Lab experiments show a linear relation between the maximum shear stress that rocks can support at any given normal stress. This is called Byerlee's Law.

$$\tau \approx -.85\bar{\sigma} \qquad \bar{\sigma} < 200 \text{ MPa}$$

$$\tau \approx 50 - .6\bar{\sigma} \qquad \bar{\sigma} > 200 \text{ MPa}.$$

Figure 5.7-10: Relation between shear stress and normal stress for frictional sliding.



Byerlee's law

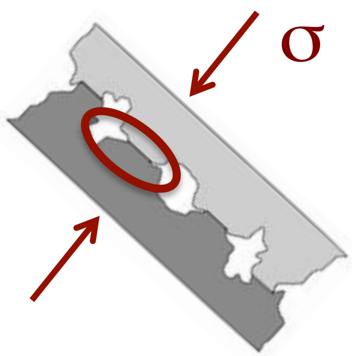
$$\tau \approx -.85\bar{\sigma} \qquad \bar{\sigma} < 200 \text{ MPa}$$
$$\tau \approx 50 - .6\bar{\sigma} \qquad \bar{\sigma} > 200 \text{ MPa.}$$

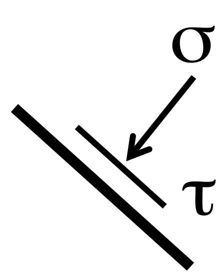
Two regions

Coulomb stress

- Notion of friction:
  - More shear stress τ needed to overcome increase in normal stress σ and cause fault to slip – Byerlee's law is an example
- Coulomb stress
  - $\sigma_S = \tau - \mu (\sigma_N - p)$
  - where μ is intrinsic coefficient of friction, p is pore pressure (***not*** the mean stress $p=-\sigma_{ii}/3$, need to be careful of context)

- Basis is that real area of contact (much smaller than apparent area) is controlled by normal stress
  - deformation of asperities in response to normal stress
  - harder to over-ride asperities at higher normal stress





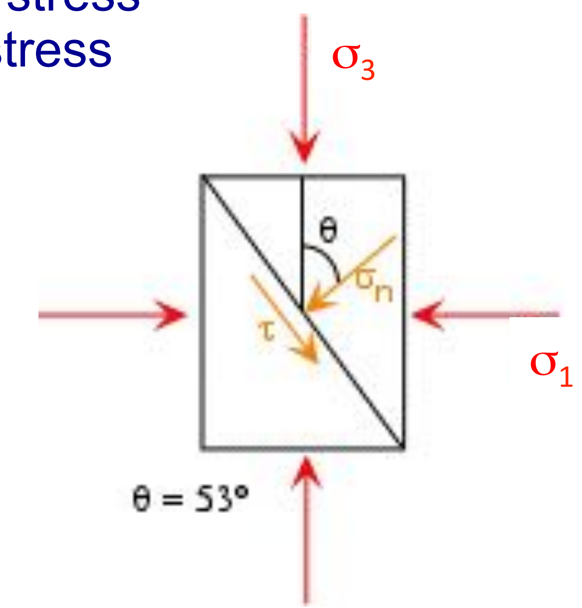
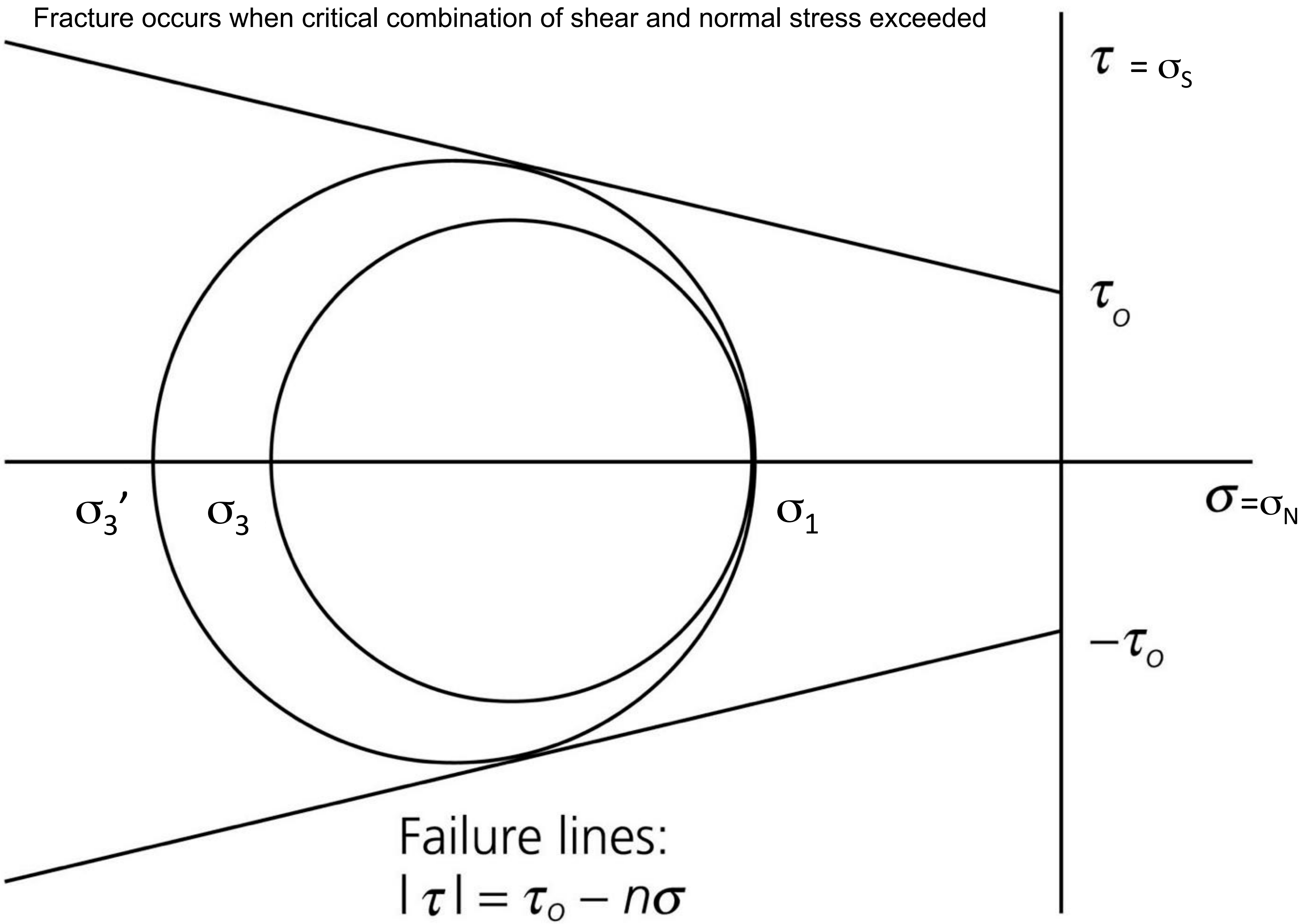
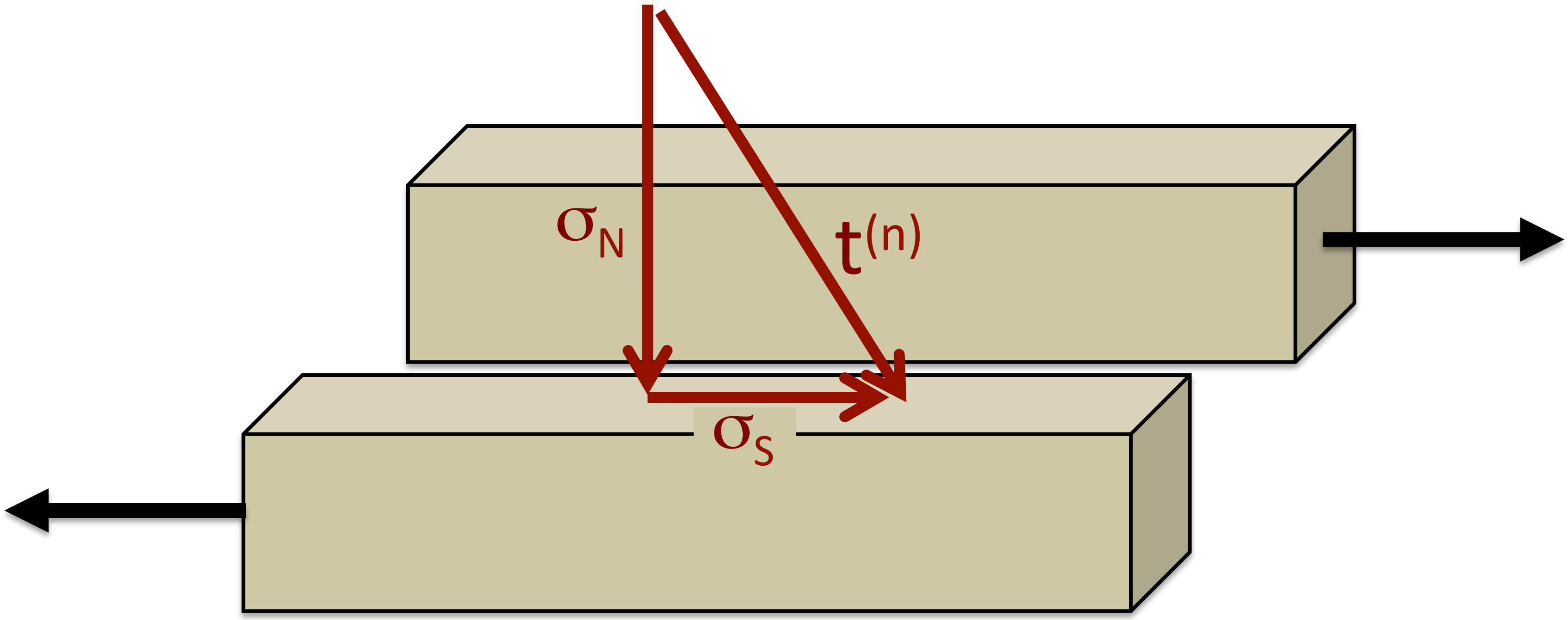


Figure 5.7-6: Definition of the Coulomb-Mohr failure criterion.



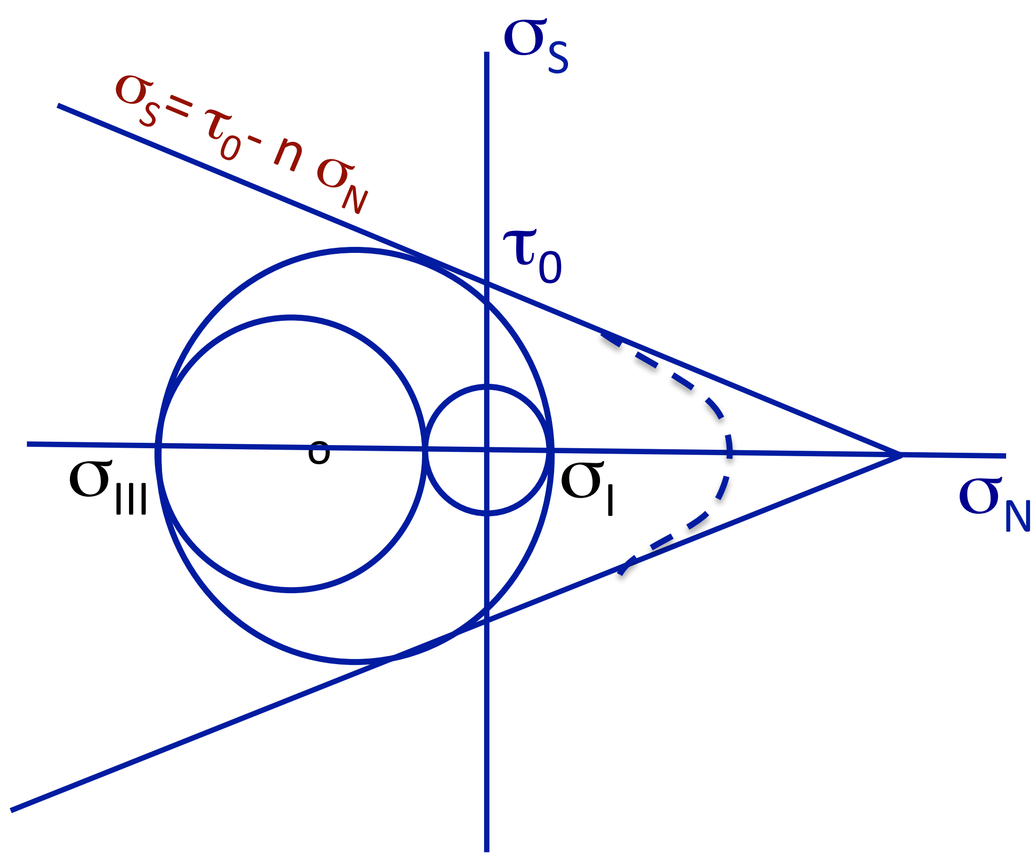
Mohr-Coulomb Fracture



$\sigma_S = \tau_0 - n\,\sigma_N$ n is ***coefficient of internal friction*** for fracture on a new fault surface

τ_0 is cohesion of the material in absence of any confining stress σ_N

Mohr-Coulomb Fracture

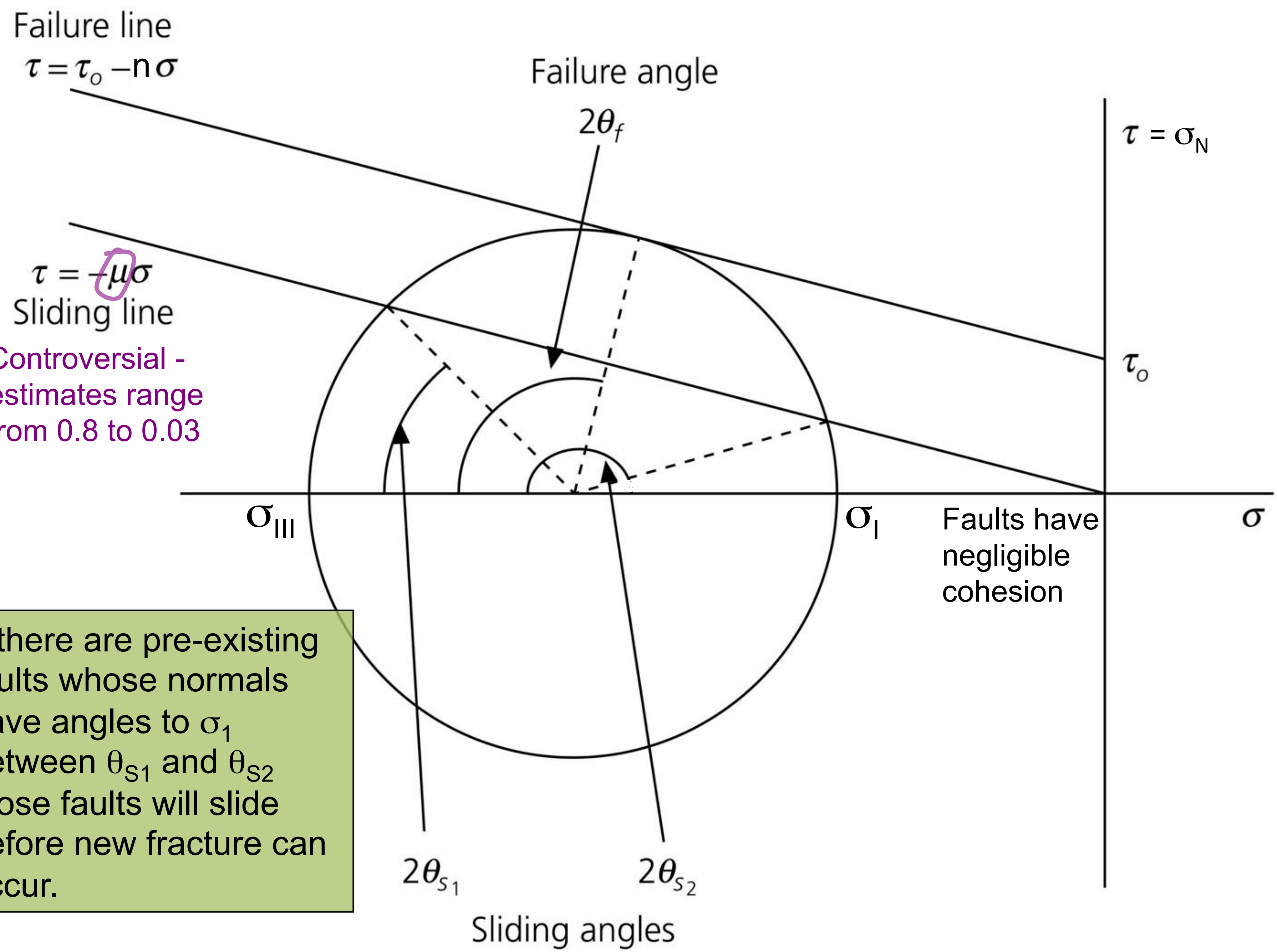


Now we are actually breaking rock …

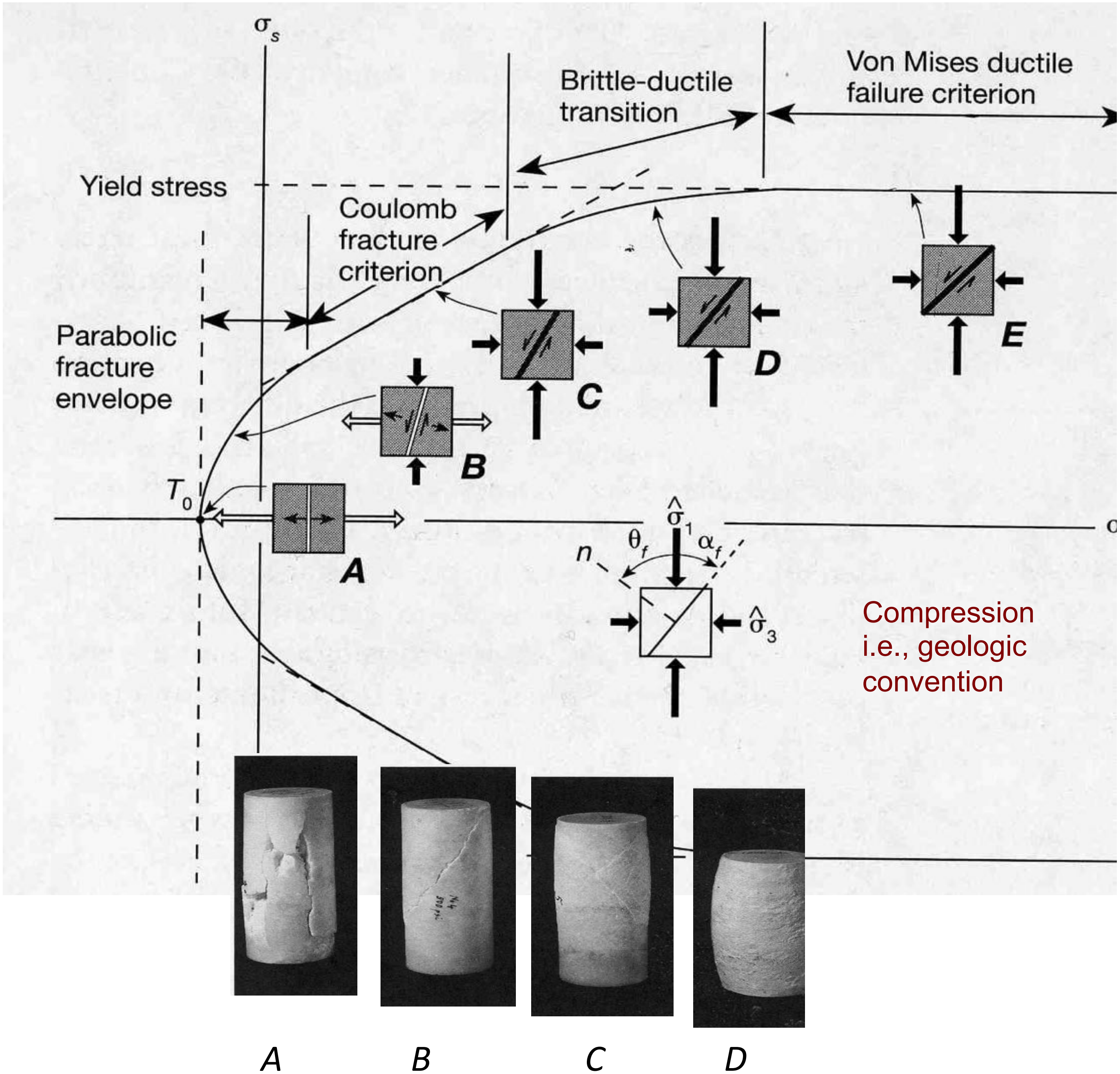
$\sigma_S = \tau_0 - n\,\sigma_N$ n is ***coefficient of internal friction*** for fracture on a new fault surface

τ_0 is cohesion of the material in absence of any confining stress σ_N

Figure 5.7-9: Mohr's circle for sliding on preexisting faults.



If there are pre-existing faults whose normals have angles to σ_1 between θ_{S1} and θ_{S2} those faults will slide before new fracture can occur.



σs
Brittle-ductile transition
Von Mises ductile failure criterion
Yield stress
Coulomb fracture criterion
Parabolic fracture envelope
T0
A
B
C
D
E
σ̂1
σ̂3
n
θf
αf
Compression i.e., geologic convention
A
B
C
D