

ESS 411/511 Geophysical Continuum Mechanics Class #17

Highlights from Class #16 – Madie Mamer
Today's highlights on Monday – Abigail Thienes

Our text doesn't cover our next topics very thoroughly, so we will use a few other sources, which are posted on the class web site under READING & NOTES. <https://courses.washington.edu/ess511/NOTES/notes.shtml>

- Stein and Wyss session 5.7.2
- Stein and Wyss session 5.7.3/4
- Raymond notes on failure

Also see slides about upcoming topics

- Failure and Mohr's circles – slides

Your short CR/NC Pre-class prep writing assignment (1 point) in Canvas

- It will be due in Canvas at the start of class.
- I will send another message when it is posted in Canvas.

ESS 411/511 Geophysical Continuum Mechanics

Broad Outline for the Quarter

- Continuum mechanics in 1-D
- 1-D models with springs, dashpots, sliding blocks
- Attenuation
- Mathematical tools – vectors, tensors, coordinate changes
- Stress – principal values, Mohr's circles for 3-D stress
- Coulomb failure, pore pressure, crustal strength
- Measuring stress in the Earth
- Strain – Finite strain; infinitesimal strains
- Moments – lithosphere bending; Earthquake moment magnitude
- Conservation laws
- Constitutive relations for elastic and viscous materials
- Elastic waves; kinematic waves

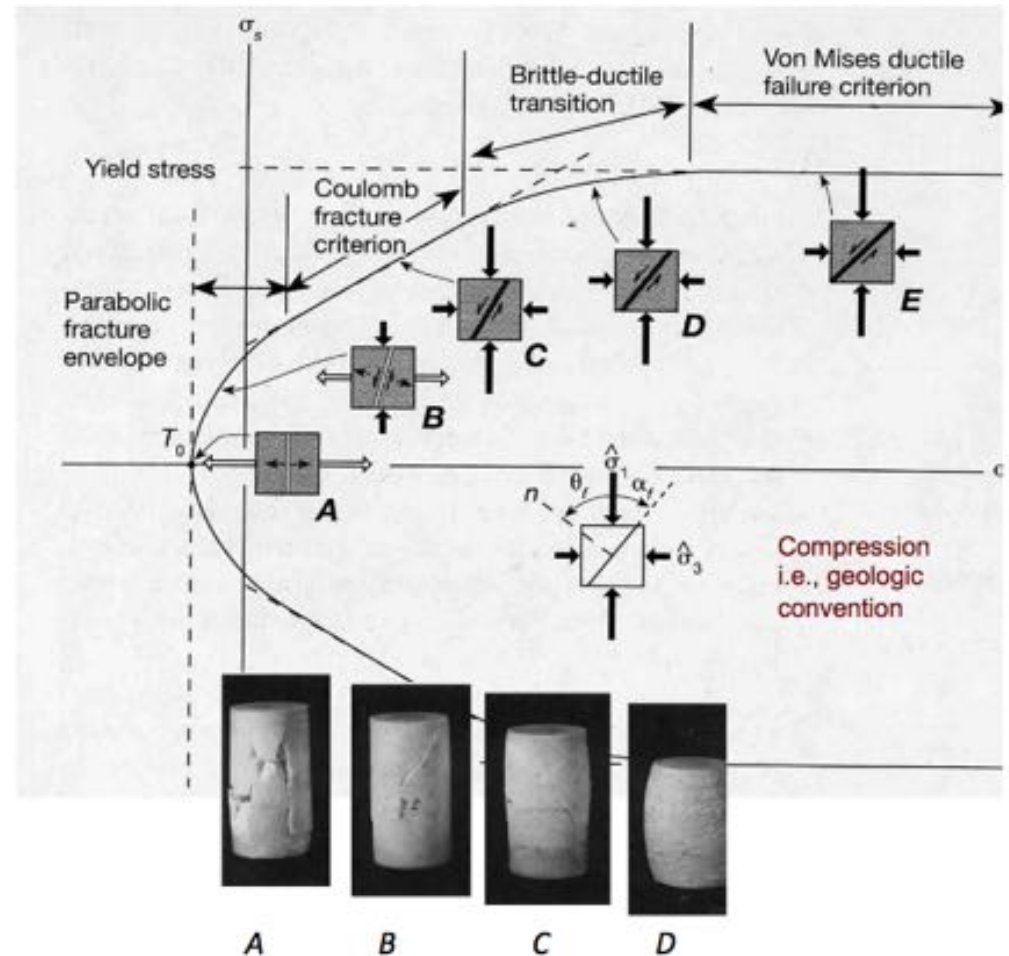
ESS 511 60-second Project updates

- Andrew
- Barrett
- Maleen
- Zoe
- Madeleine

Class-prep questions for today (break-out rooms)

Style of Failure under Various Normal Stresses σ_N

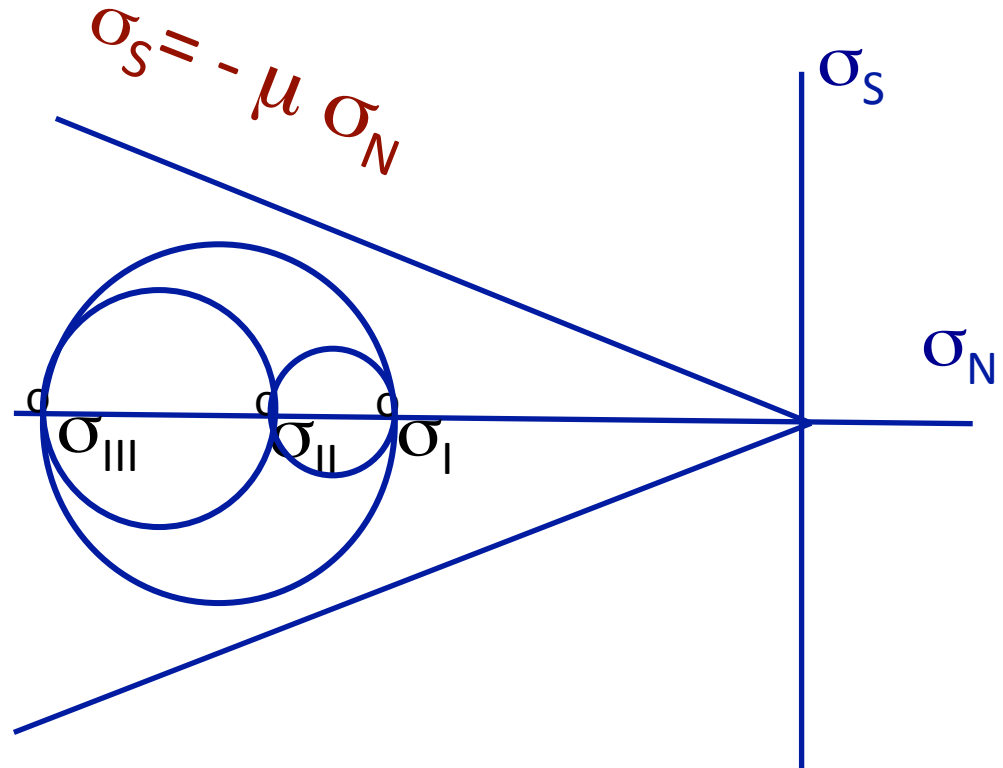
The figure shows the failure envelope and failure modes in stress space, based on experiments on rocks subjected to a range of normal stresses σ_N . Note that these authors used the convention that compression is positive (yuck ...)



- Describe in words what is happening in this generalization of the failure envelopes that we have discussed in class.
- In a sentence or two for each, describe characteristics of the failure mode in each of the 5 stress regimes A, B, C, D, and E. The regime names, the angles of the failure planes, and the visual states of the samples after the experiments ended may be helpful.

Differential stress

$$\sigma_I - \sigma_{III}$$



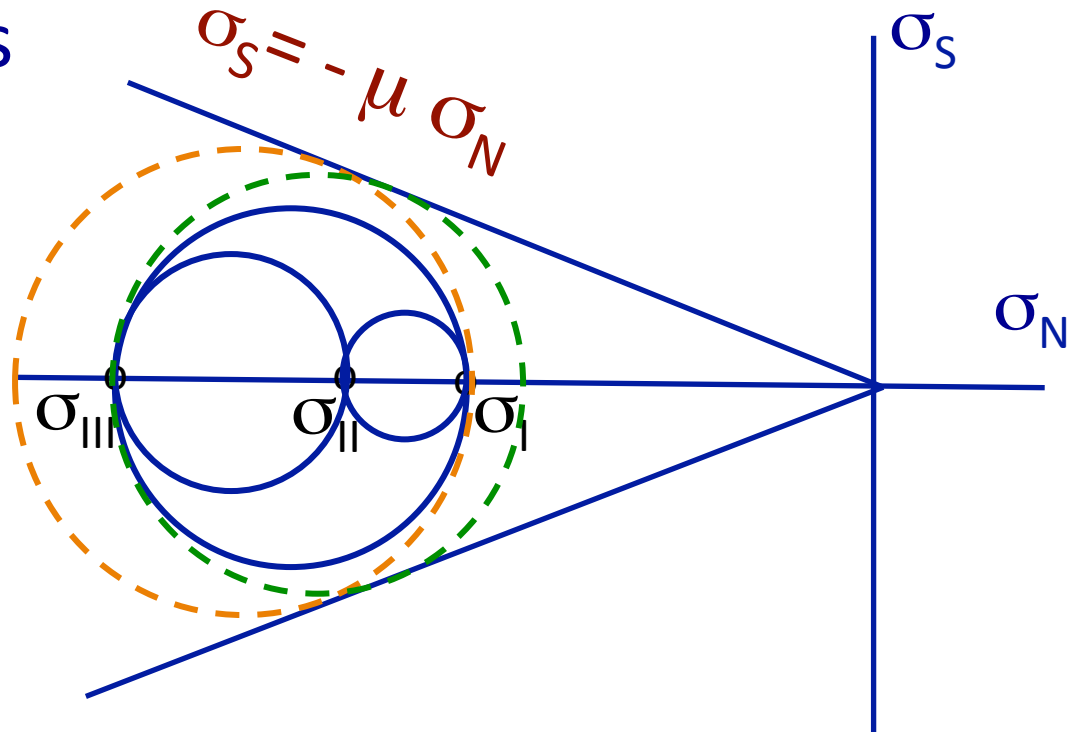
But, if $\sigma_{III} = \sigma_I$, all 3 principal stresses are equal

- What do the 3 Mohr's circle look like?
- Describe this state of stress inside the body.
- Is frictional failure possible, if differential stress is zero?

Differential stress

$$\sigma_I - \sigma_{III}$$

So differential stress is essential in order to have failure



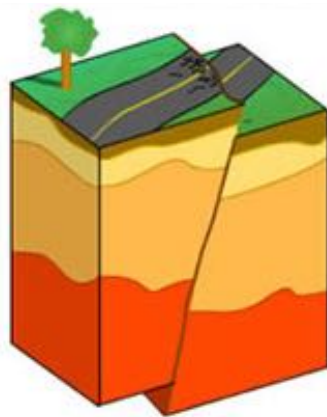
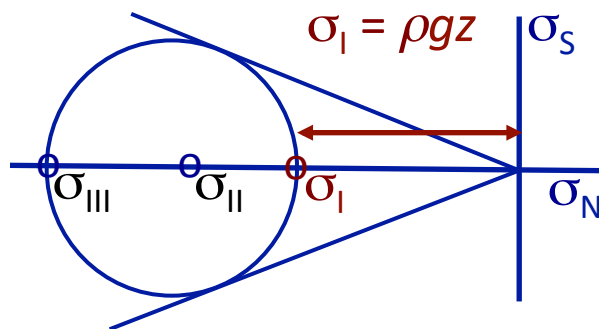
How could we change the stress state in order to cause failure?

- Hold σ_I make σ_{III} more negative (squeeze harder in x_3)
- Hold σ_{III} , make σ_I less negative (don't squeeze as hard in x_1)

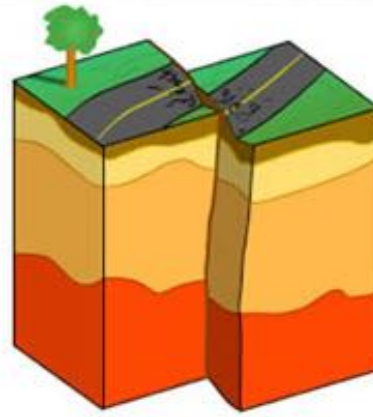
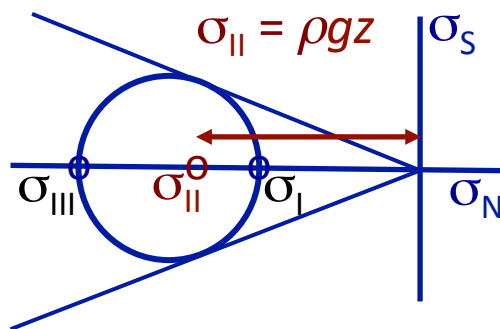
Types of faults

The Earth's surface is traction-free, so one of the principal directions is generally vertical

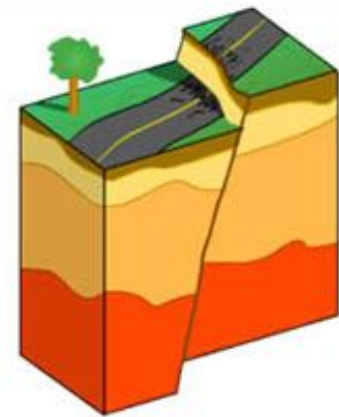
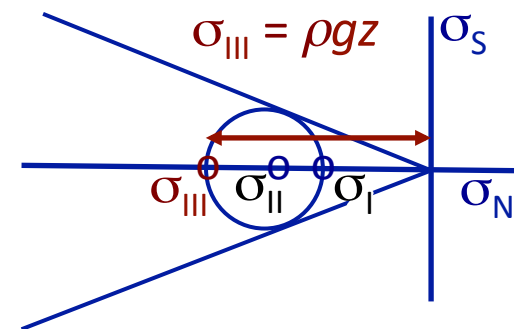
- What are the orientations of the principal axes of stress \hat{e}_1^* , \hat{e}_2^* , \hat{e}_3^* in each case?



Reverse fault

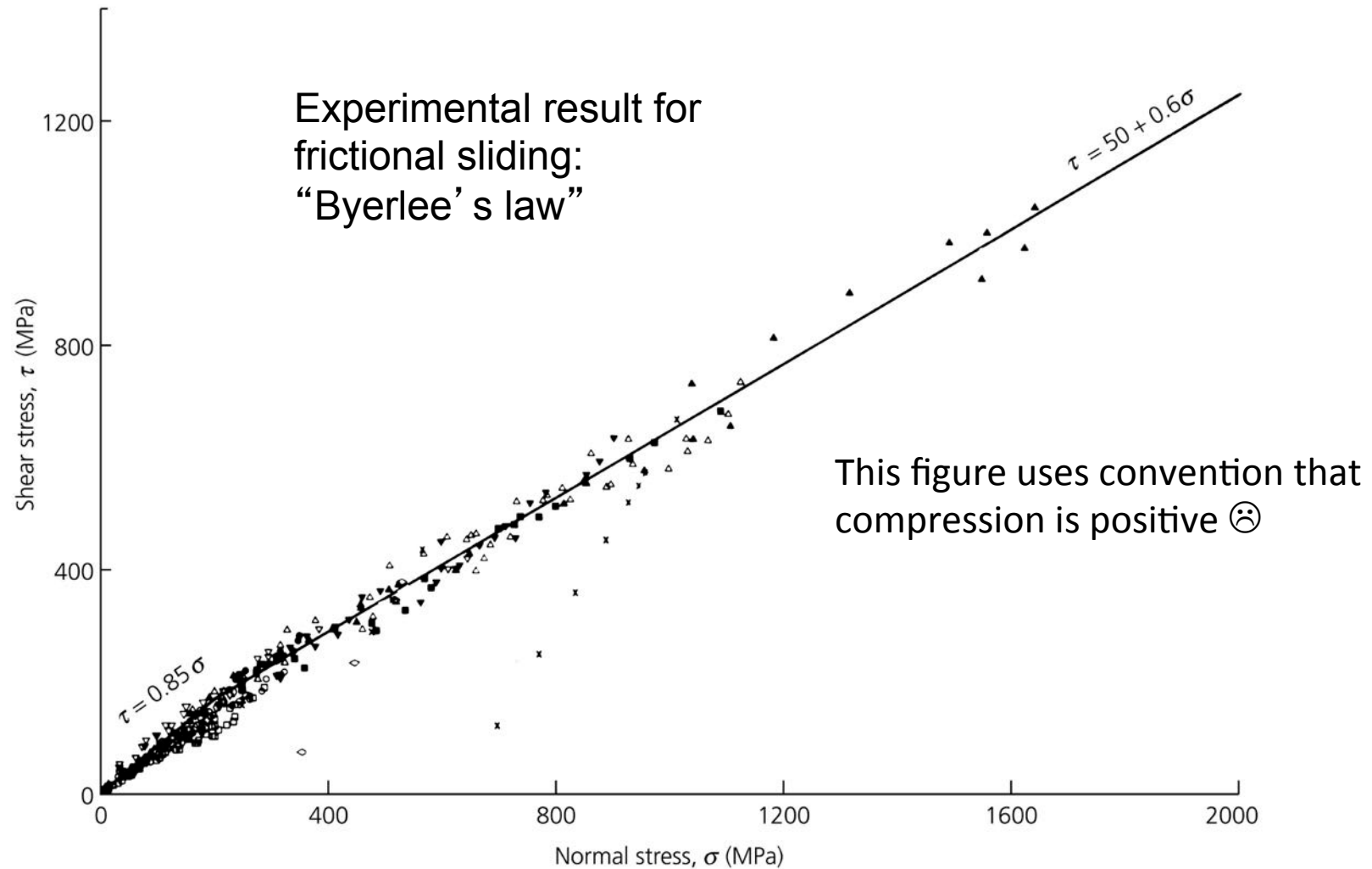


Strike-slip fault



Normal fault

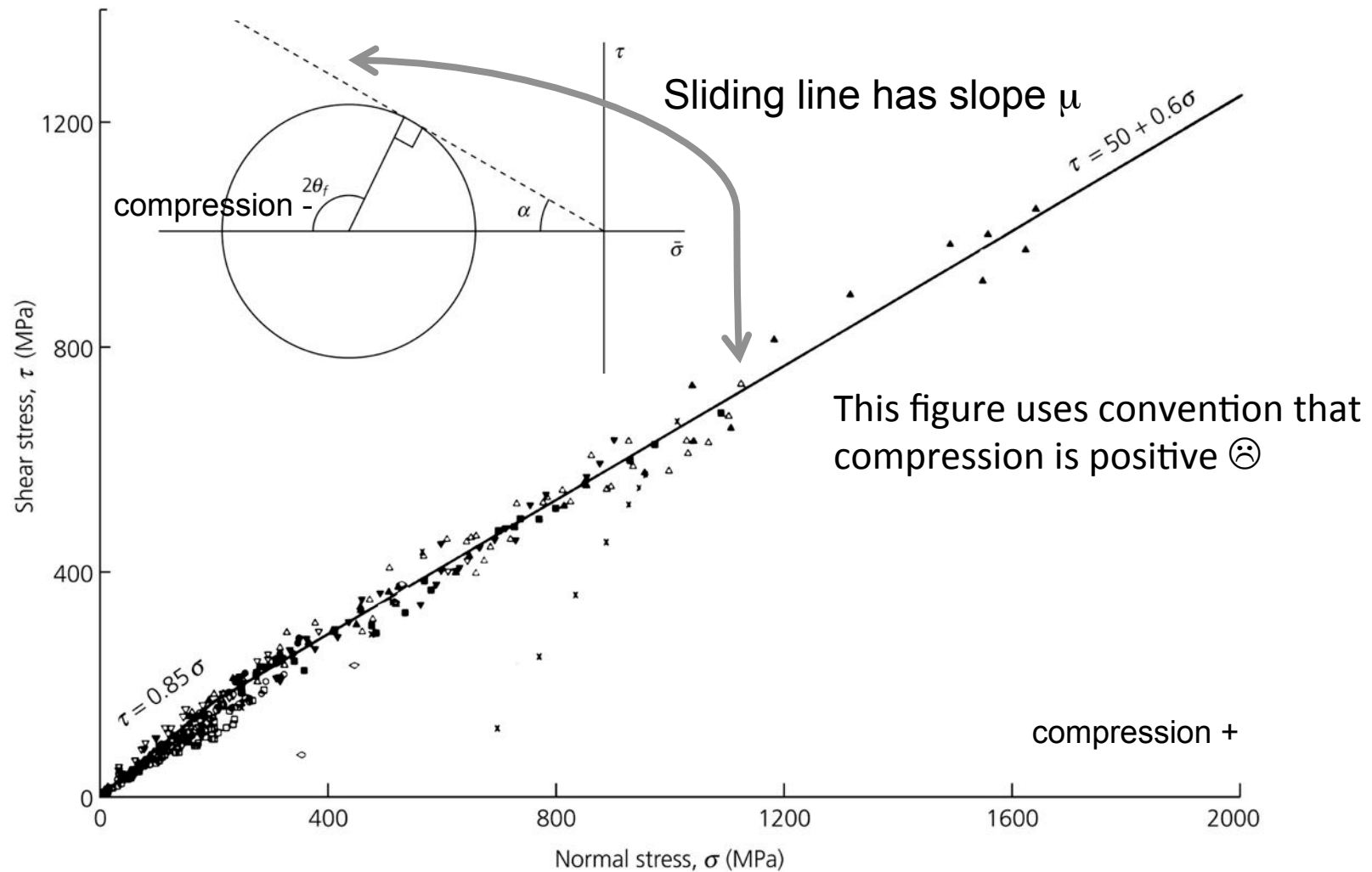
Figure 5.7-10: Relation between shear stress and normal stress for frictional sliding.



Lab experiments show a linear relation between the maximum shear stress that rocks can support at any given normal stress. This is called Byerlee's Law.

$$\begin{aligned}\tau &\approx -.85\bar{\sigma} & \bar{\sigma} < 200 \text{ MPa} \\ \tau &\approx 50 - .6\bar{\sigma} & \bar{\sigma} > 200 \text{ MPa.}\end{aligned}$$

Figure 5.7-10: Relation between shear stress and normal stress for frictional sliding.



Byerlee's law

$$\begin{aligned} \tau &\approx -.85\bar{\sigma} & \bar{\sigma} < 200 \text{ MPa} \\ \tau &\approx 50 - .6\bar{\sigma} & \bar{\sigma} > 200 \text{ MPa.} \end{aligned}$$

Two regions

Coulomb stress and rock fracture

- Notion of friction:
 - More shear stress τ is needed to overcome increase in normal stress σ and cause a fault to slip – Byerlee's law is an example
- Coulomb stress
 - $\sigma_s = \tau - \mu (\sigma_N - p)$
 - where μ is intrinsic coefficient of friction, p is pore pressure
(**not** the mean stress $p = -\sigma_{ij}/3$, need to be careful of context)
- Basis is that real area of contact (much smaller than apparent area) is controlled by normal stress
 - deformation of asperities in response to normal stress increases contact area
 - harder to over-ride asperities at higher normal stress

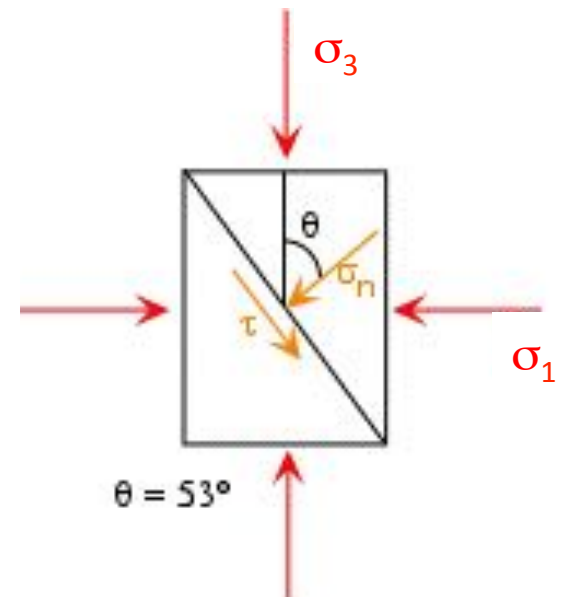
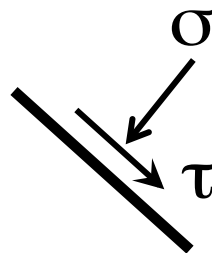
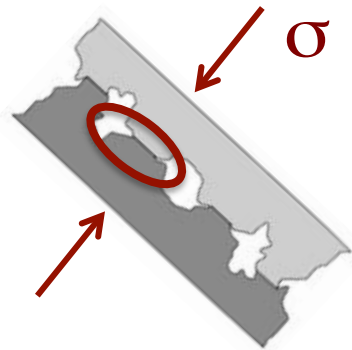
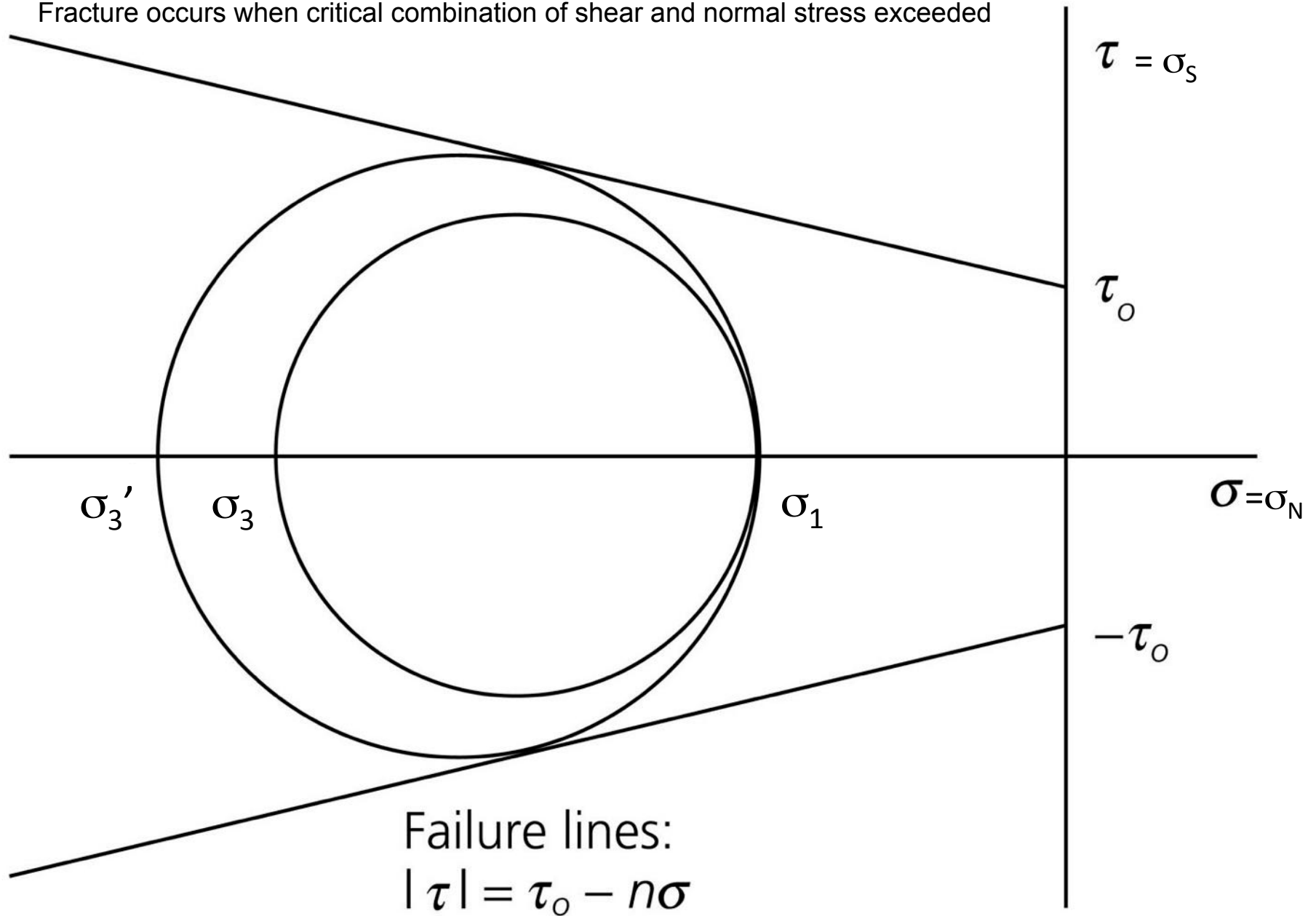
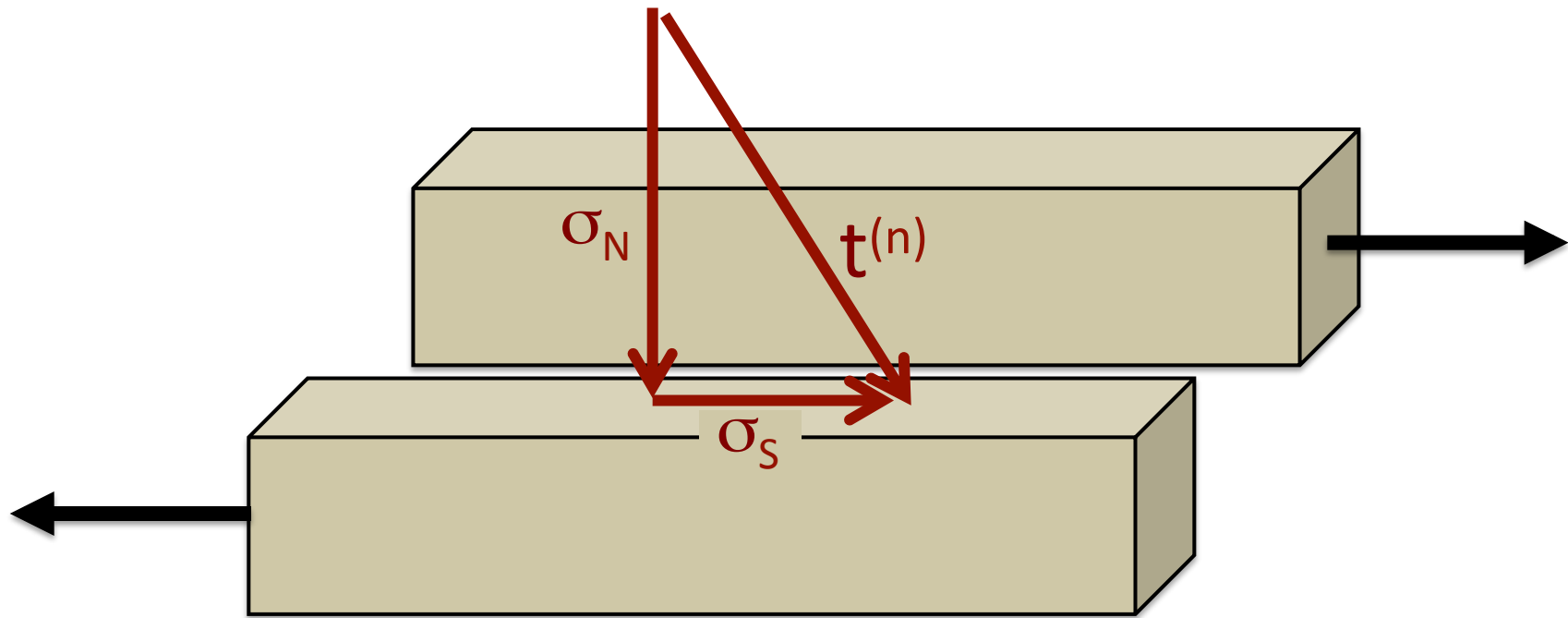


Figure 5.7-6: Definition of the Coulomb-Mohr failure criterion.

Fracture occurs when critical combination of shear and normal stress exceeded



Mohr-Coulomb Fracture

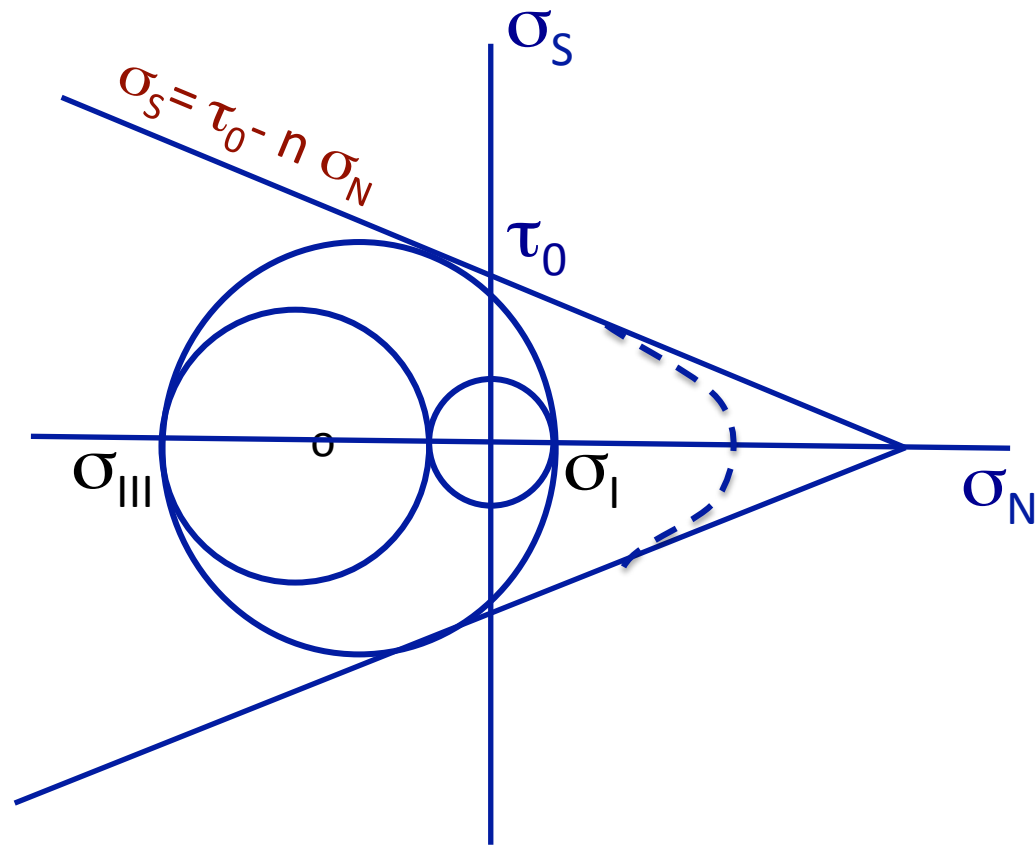


$\sigma_S = \tau_0 - \eta \sigma_N$ η is *coefficient of internal friction* for fracture on a new fault surface

τ_0 is cohesion of the material in absence of any confining stress σ_N

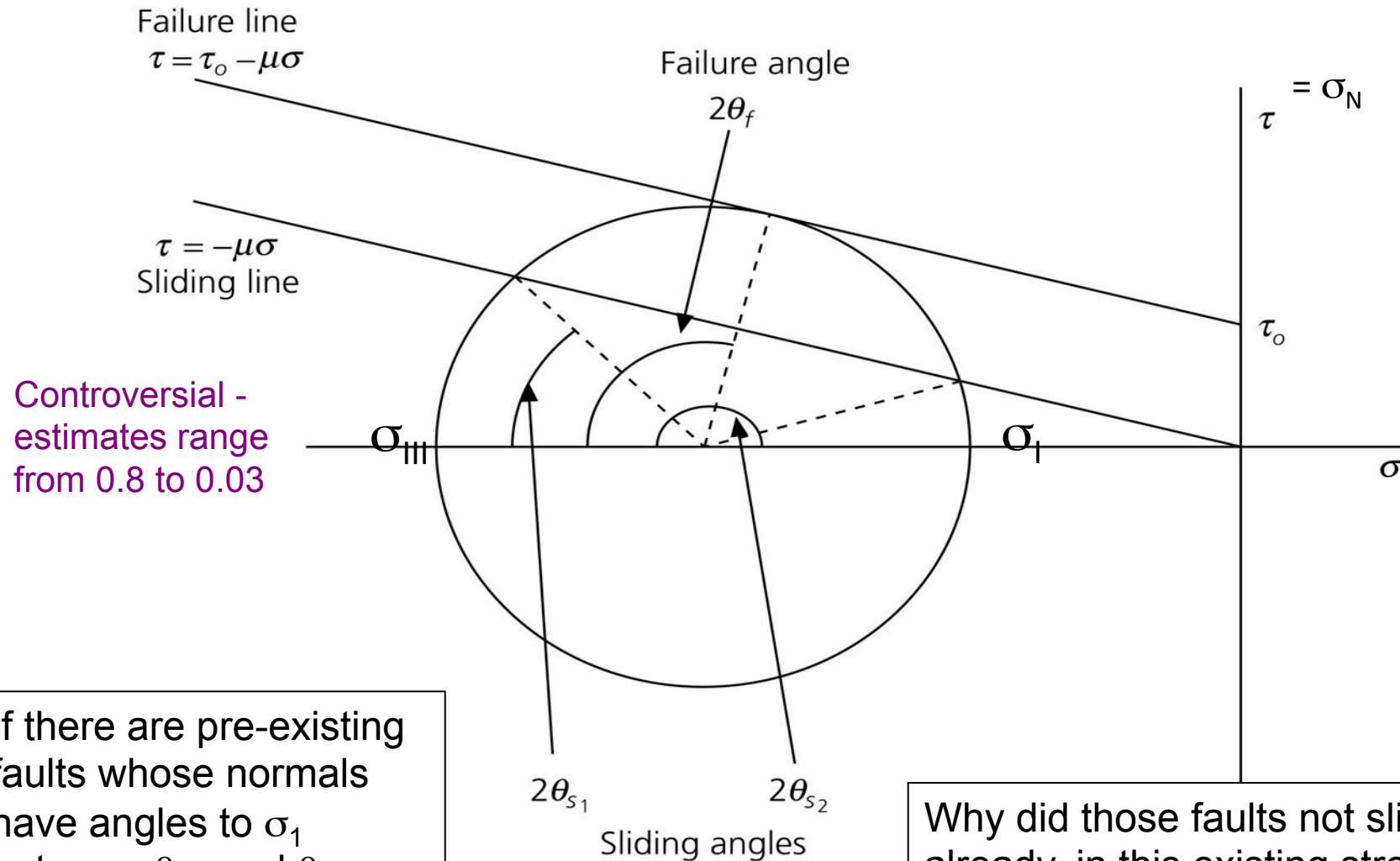
Mohr-Coulomb Fracture

Now we are actually
breaking rock ...



$\sigma_S = \tau_0 - n \sigma_N$ n is **coefficient of internal friction** for fracture on a new fault surface
 τ_0 is cohesion of the material in absence of any confining stress σ_N

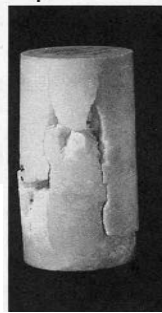
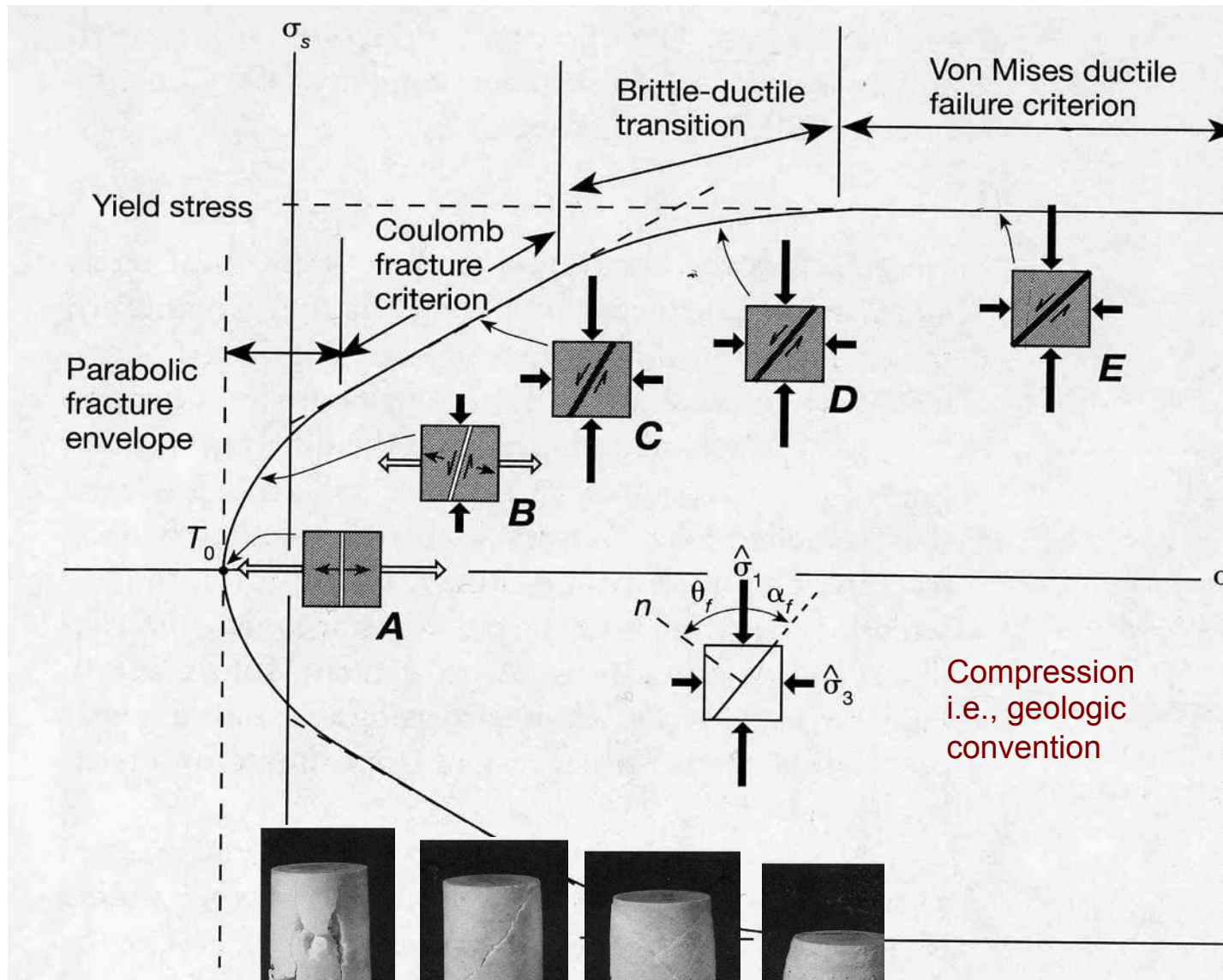
Figure 5.7-9: Mohr's circle for sliding on preexisting faults.



If there are pre-existing faults whose normals have angles to σ_1 between θ_{s1} and θ_{s2} those faults will slide before new fracture can occur.

Why did those faults not slide already, in this existing stress field?

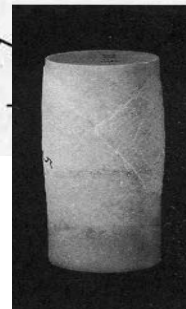
- A recent earthquake could have just raised the differential stress ($\sigma_I - \sigma_{III}$)



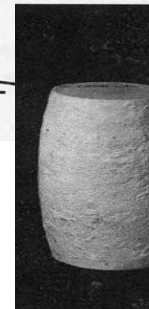
A



B



C



D

Depths and Overburden stress

$$P = \rho g z$$

$$\rho = 2700 \text{ kg m}^{-3}$$

$$g \sim 10 \text{ m s}^{-2}$$

$$z = 10 \text{ km}$$

Estimate overburden load at 10 km depth

Pore pressure p

Fluids in rock pores and cracks is a lubricant

Failure when pore fluid is present with pore pressure p

$$\sigma_S = -\mu(\sigma_N + p)$$

remember that p is positive, but compressive stress is negative

- Pore pressure reduces the clamping effect of σ_N
- Pressure jacks apart the locking asperities on faults
- Can lead to failure

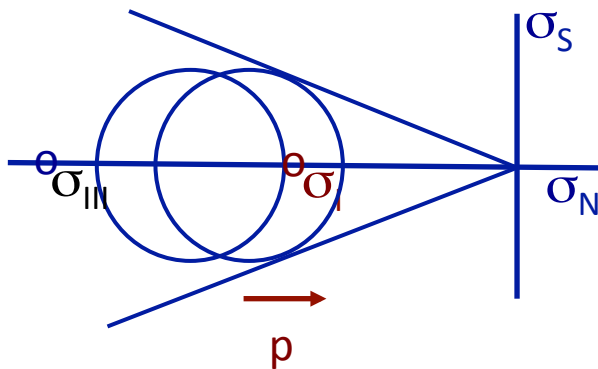
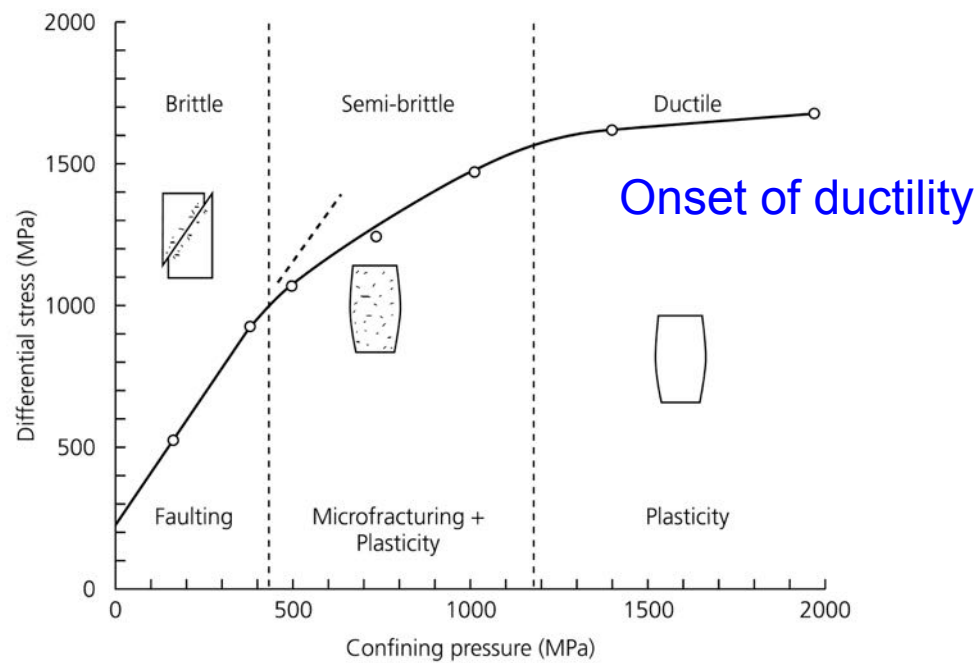
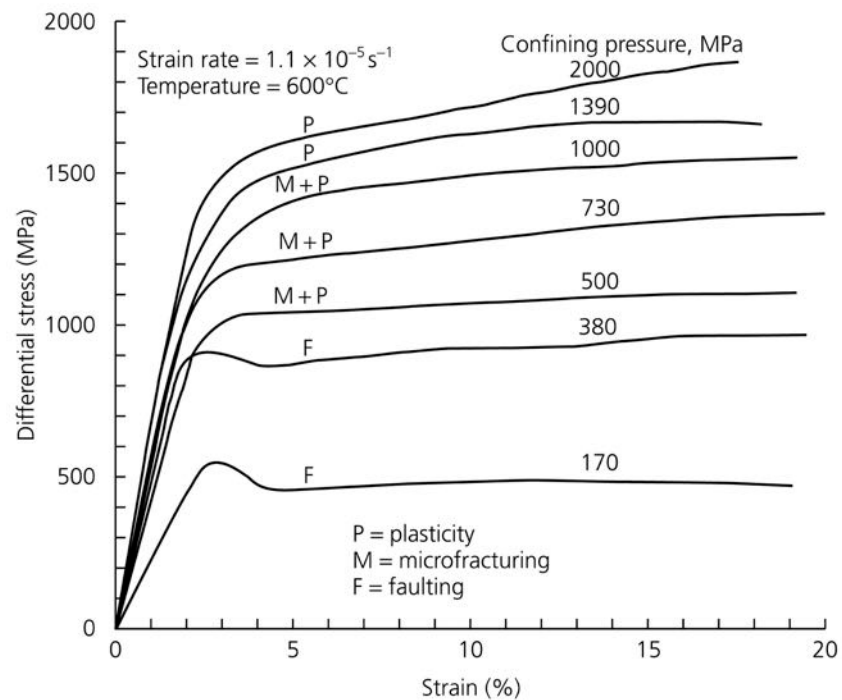
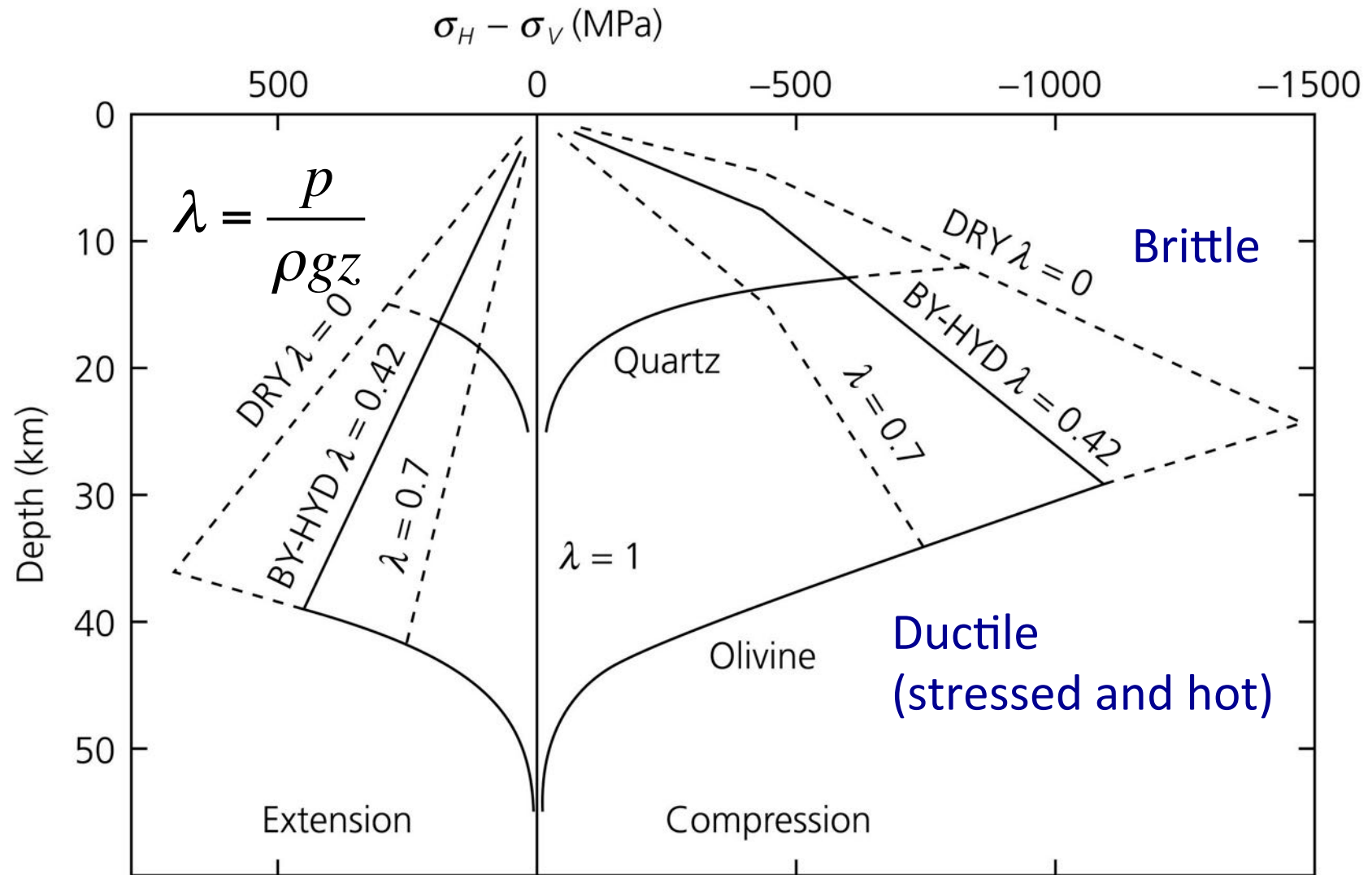


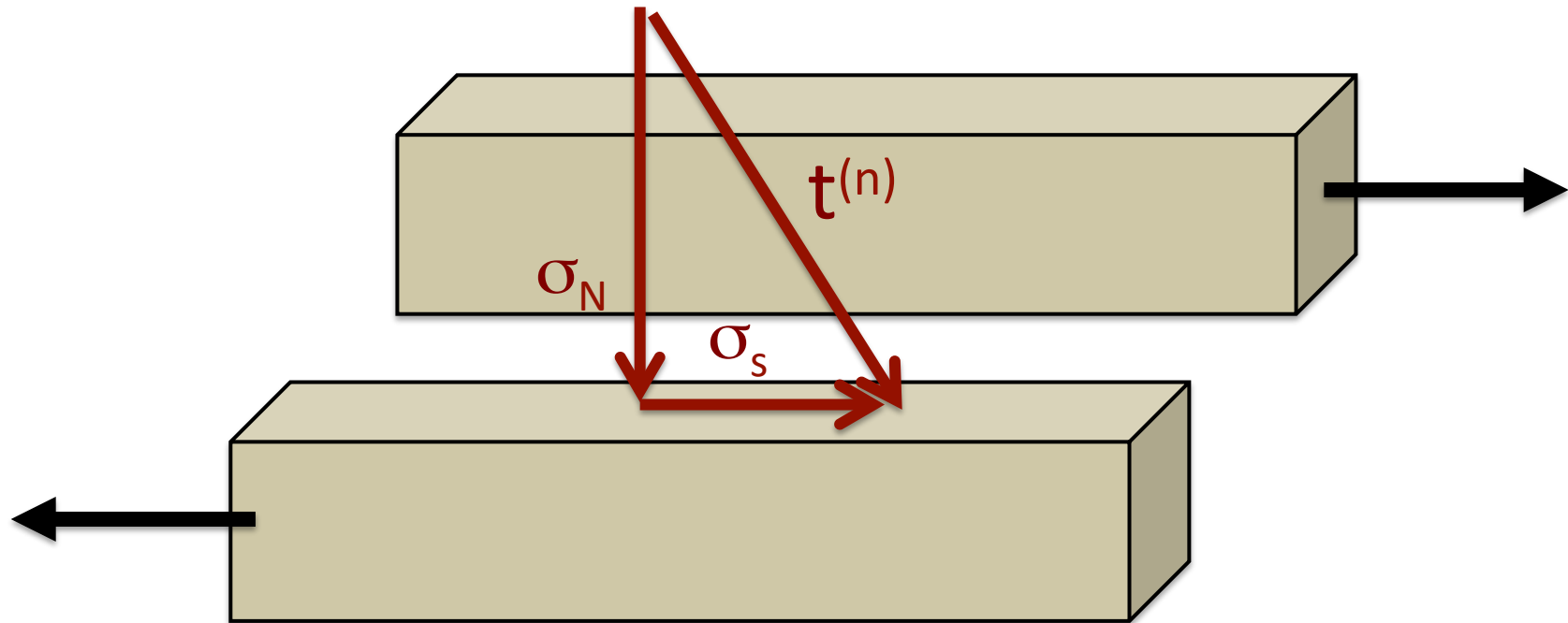
Figure 5.7-3: Rheology of rocks subjected to large compressive stresses.



Strength of the Lithosphere

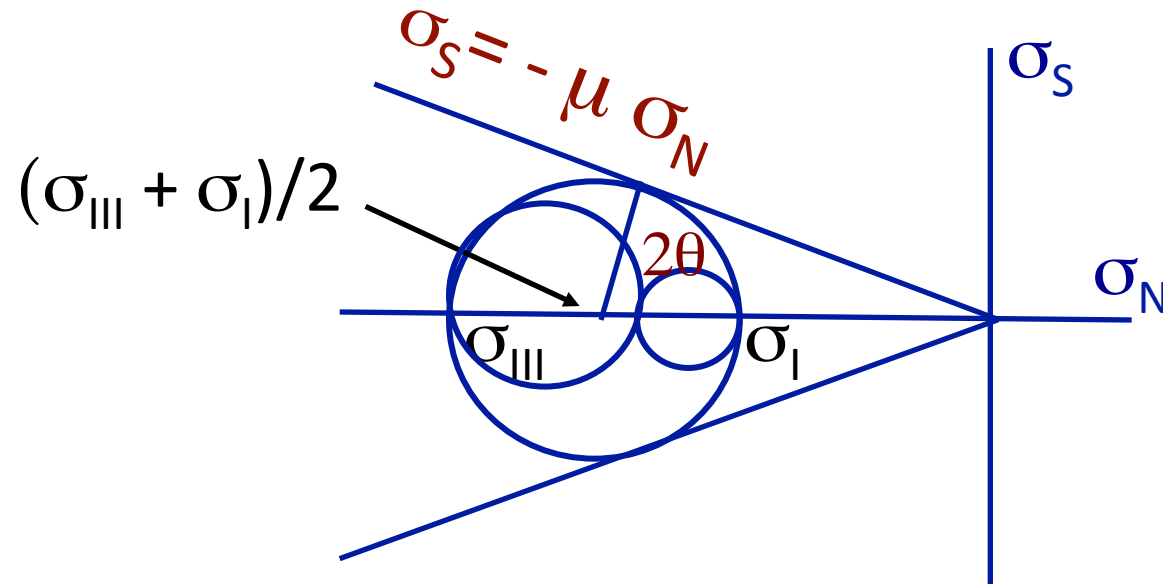


Sliding friction



$$\sigma_s = -\mu \sigma_N \quad \mu \text{ is } \textit{coefficient of friction} \text{ for sliding on a pre-existing break}$$

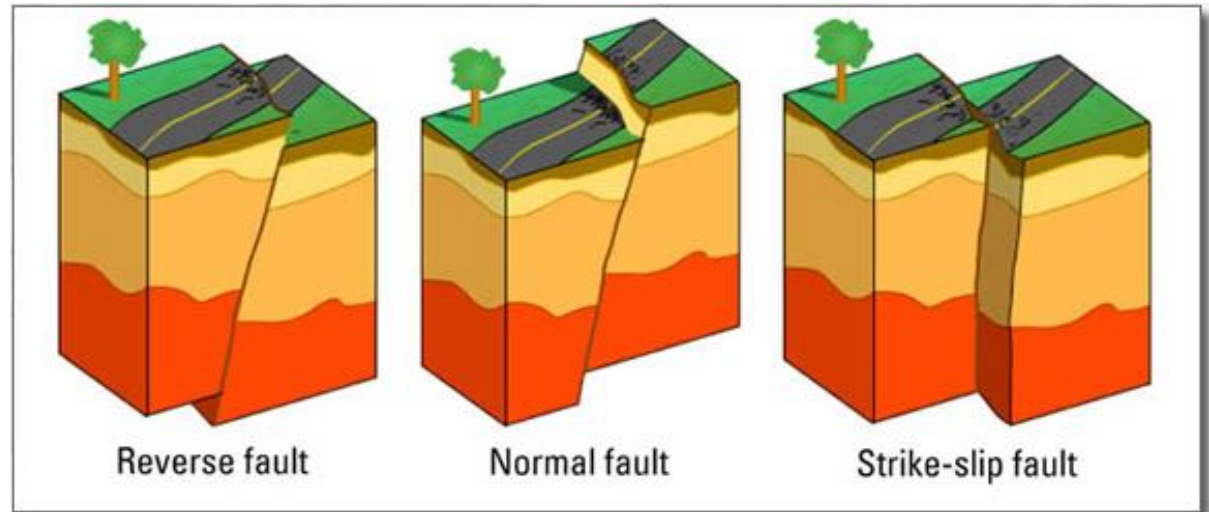
Frictional sliding



$\sigma_S = -\mu \sigma_N$ μ is **coefficient of friction** for sliding on a pre-existing break

Types of faults

The Earth's surface is traction-free, so one of the principal directions is generally vertical



What are the orientations of the principal axes of stress \hat{e}_1^* , \hat{e}_2^* , \hat{e}_3^* in each case?

