ESS 411/511 Geophysical Continuum Mechanics Class #21

Highlights from Class #20 – Brandon Lomahoynaya
Today's highlights on Friday – ____________________

For Friday, please read MSM Chapter 4.1 through 4.6
Kinematics of Deformation and Motion
The Problem Set this week is about kinematics and strain.

We will get back to stress and moments (torques) later. When we do, keep in mind these websites about measuring stress in the Earth:

https://www.eoas.ubc.ca/courses/eosc433/lecture-material/L7-InSituStress.pdf

http://www.hydrofrac.com/hfo_home.html

Also see slides on class web site about Measuring Stress in the Earth
https://courses.washington.edu/ess511/NOTES/SLIDE_SHOWS/PDF/stress_class_show_2017_all.pdf

Mid-term course evaluation

Due to major changes resulting from Covid-19, the UW has set up an option for mid-term course evaluations in addition to the traditional end-of-Quarter evaluations.
I hope will all be able to give me feedback. Thanks!

This evaluation will close on Nov 20, (Friday) at 11:59pm.

ESS 411 A & ESS 511 A - Geophysical Continuum Mechanics

https://uw.iasystem.org/survey/230856
(This evaluation link is only for the courses listed above.
It can only be accessed by students enrolled in the courses.)

Problem Set #4

- I’m working on it ☺

Mid-term

- I’m working on it next …

ESS 411/511 Geophysical Continuum Mechanics

Broad Outline for the Quarter

- Continuum mechanics in 1-D
- 1-D models with springs, dashpots, sliding blocks
- Attenuation
- Mathematical tools – vectors, tensors, coordinate changes
- Stress – principal values, Mohr's circles for 3-D stress
- Coulomb failure, pore pressure, crustal strength
- Measuring stress in the Earth
- Strain – Finite strain; infinitesimal strains
- Moments – lithosphere bending; Earthquake moment magnitude
- Conservation laws
- Constitutive relations for elastic and viscous materials
- Elastic waves; kinematic waves

Kinematics

- Description without reference to forces

Concept of particle in a continuum

- Just an infinitesimal point in the material, labeled with a vector field *X*

Displacement

- Vector mapping of an object from initial *X* to final configuration *x*

Deformation

- Change of shape described by a displacement field

Rigid-body rotation and translation

- No deformation, but displacement can differ from point to point

Strain or distortion

- Elongation or shear

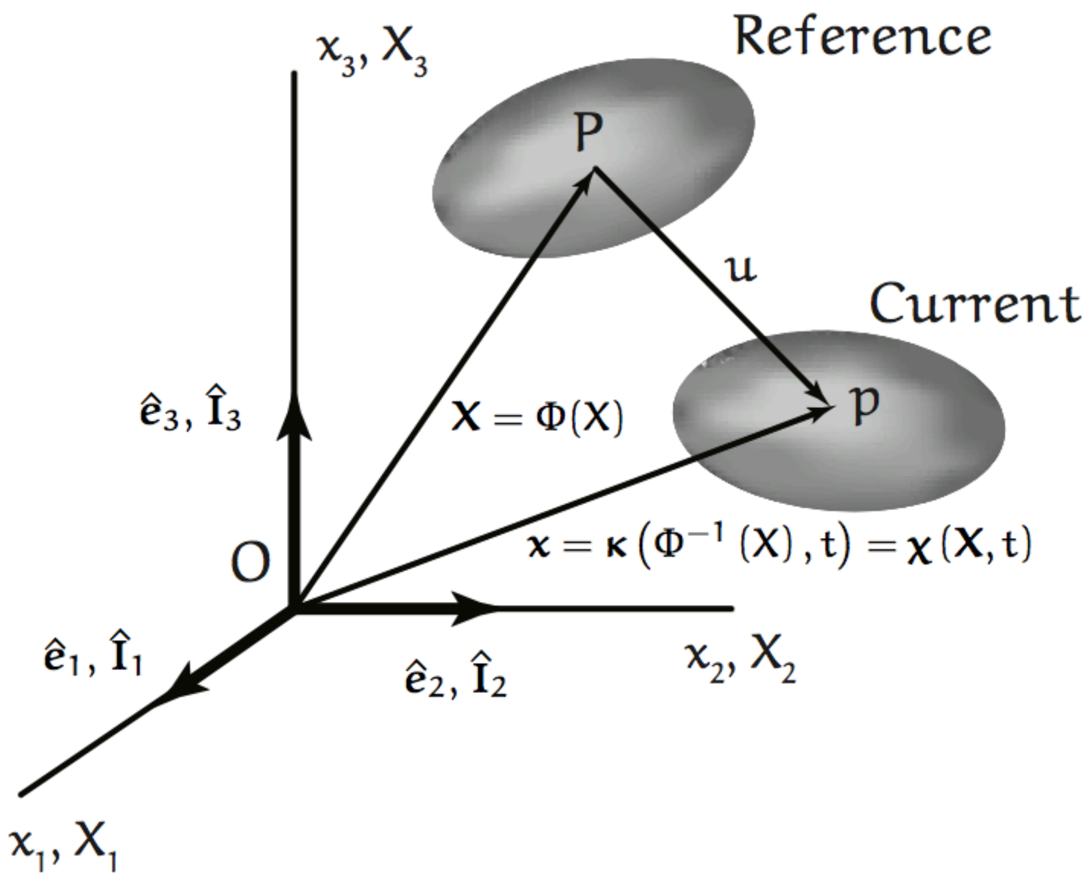
Homogeneous deformation

- Initially straight material lines stay straight

Finite strain

- Material lines can become curved

Initial and Final Configurations



Material and Spatial Coordinates

Position

$$\mathbf{X} = \mathbf{\Phi}(X) \qquad \mathbf{X} = X_A \hat{\mathbf{I}}_A$$

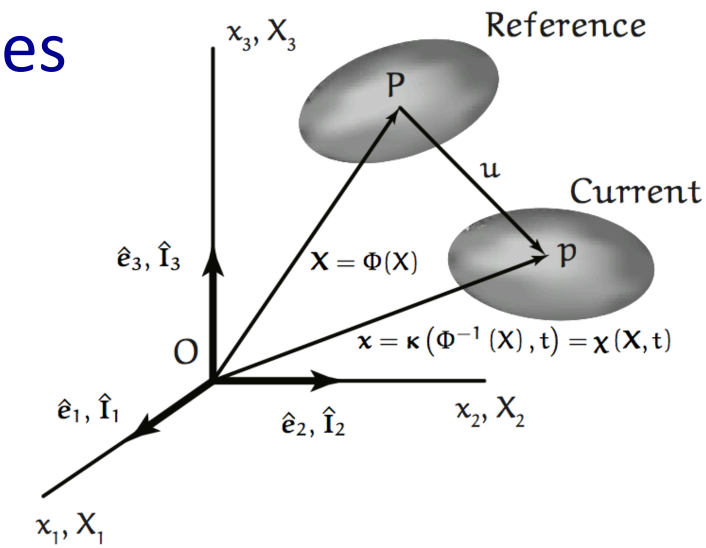
$$\boldsymbol{x} = \boldsymbol{\kappa}\left[\Phi^{-1}(\mathbf{X}), t\right] = \boldsymbol{\chi}(\mathbf{X}, t)$$

$$\boldsymbol{x} = x_i \hat{\boldsymbol{e}}_i$$

$$x_i = x_i(X_A, t) \qquad \text{or} \qquad \boldsymbol{x} = \boldsymbol{x}(\mathbf{X}, t)$$

$$\boldsymbol{x} = \boldsymbol{\chi}(\mathbf{X}, 0) = \mathbf{X}$$

$$\boldsymbol{x}^P = \boldsymbol{\chi}(\mathbf{X}^P, t)$$



Velocity and Acceleration

$$\boldsymbol{v}^P = \frac{d\boldsymbol{x}^P}{dt} = \dot{\boldsymbol{\chi}}^P = \left(\frac{\partial \boldsymbol{\chi}}{\partial t}\right)_{\mathbf{X}=\mathbf{X}^P}$$

$$\boldsymbol{v} = \dot{\boldsymbol{x}} = \frac{d\boldsymbol{x}}{dt} = \frac{\partial \boldsymbol{\chi}(\mathbf{X}, t)}{\partial t} = \frac{\partial \boldsymbol{x}(\mathbf{X}, t)}{\partial t}$$

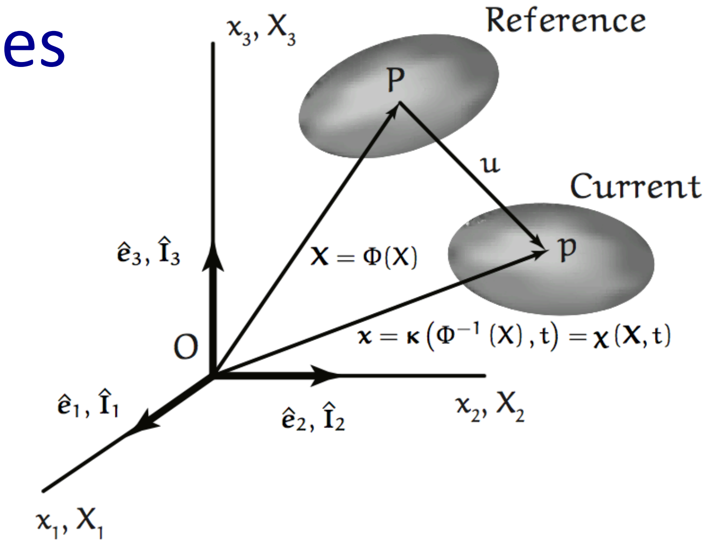
$$\boldsymbol{a} = \dot{\boldsymbol{v}} = \ddot{\boldsymbol{x}} = \frac{d^2\boldsymbol{x}}{dt^2} = \frac{\partial^2 \boldsymbol{\chi}(\mathbf{X}, t)}{\partial t^2}$$

Material and Spatial Coordinates

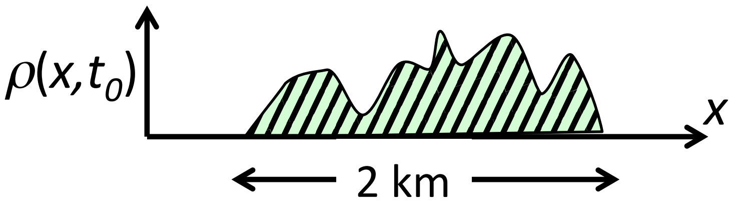
Reversible mapping or strain

$$J = \left| \frac{\partial \chi_i}{\partial X_A} \right| \neq 0$$

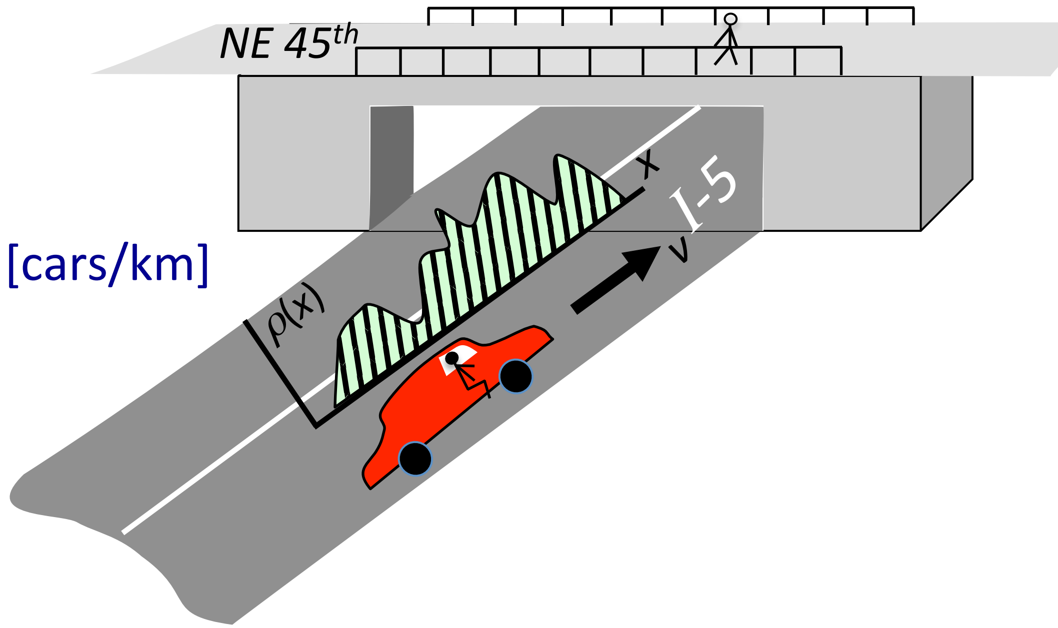
$$J = \left| \frac{\partial x_i}{\partial X_A} \right|$$



Class-prep for Wednesday
Two ESS411/511 students observe I-5 traffic



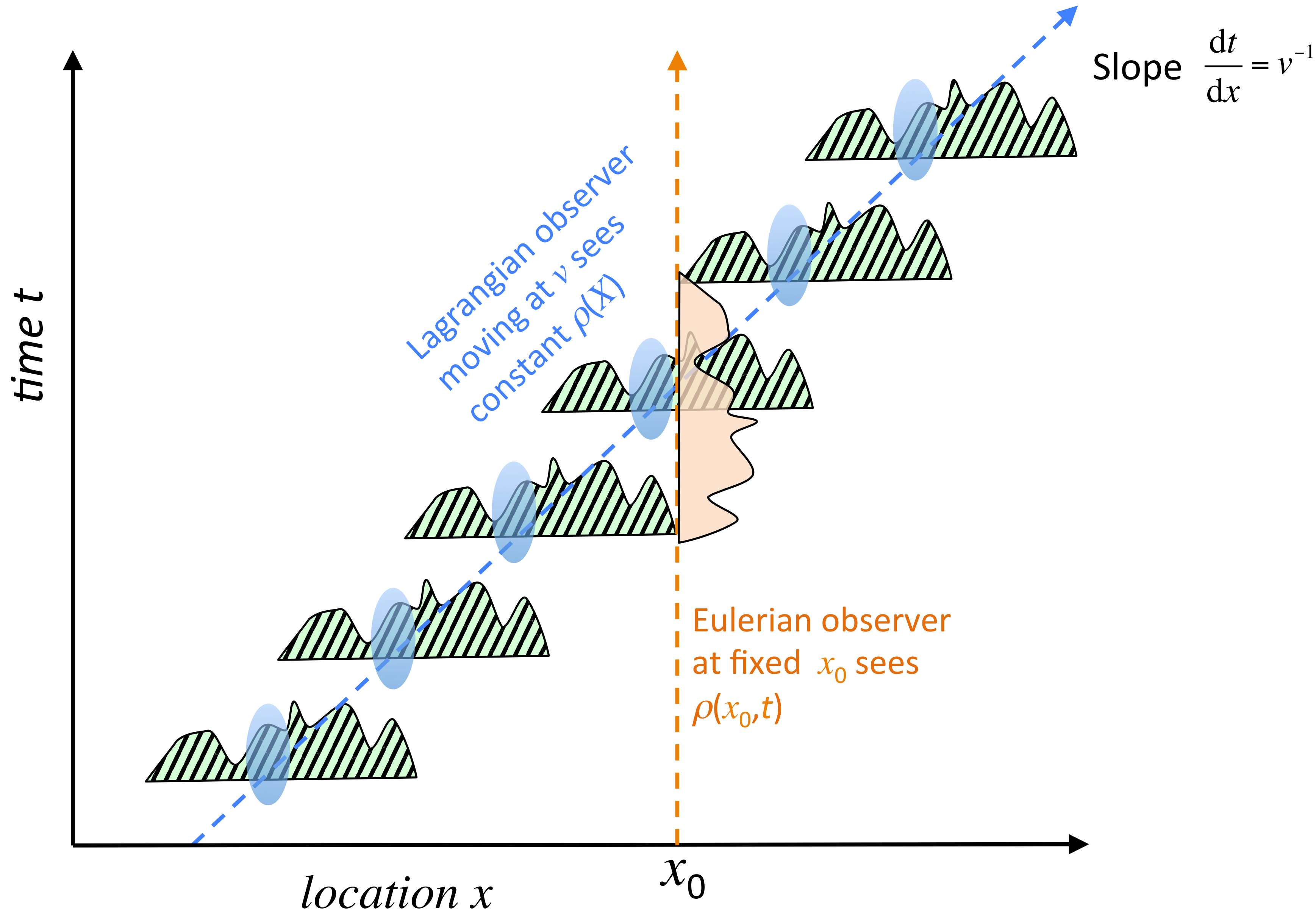
Units of traffic density: $[\rho(x,t_0)]$ = [cars/km]



The only cars on *I-5* are in the 2-km-long $\rho(x,t)$ packet. All the cars are moving at the same speed v, so the density packet maintains its shape as it moves along *I-5.*

$$\rho(x,t) = = \rho_0 f(x-vt)$$

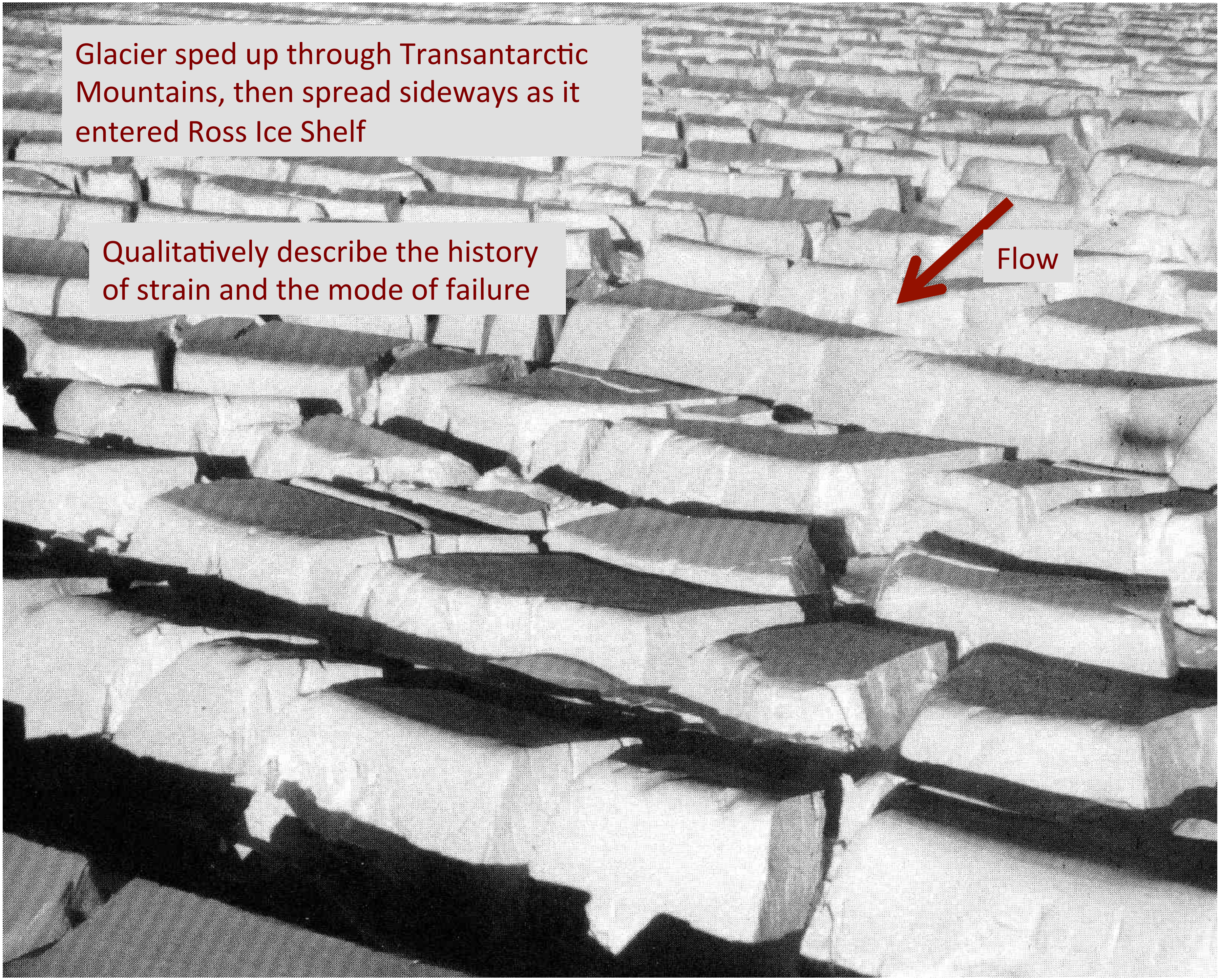
- Describe in words the history of traffic density as seen by the observer on the overpass.
- Describe the history of traffic density as seen by the observer in the car.
- Sketch out $\rho(x,t)$ on a graph with x horizontal, and t on the vertical axis. e.g., you can show the density profile $\rho(x,t_j)$ at several different times t_j. Then add the paths of the 2 observers through x and t.



Slope $\frac{\mathrm{d}t}{\mathrm{d}x} = v^{-1}$
Lagrangian observer moving at v sees constant $\rho(X)$
time t
Eulerian observer at fixed x_0 sees $\rho(x_0,t)$
location x
x_0

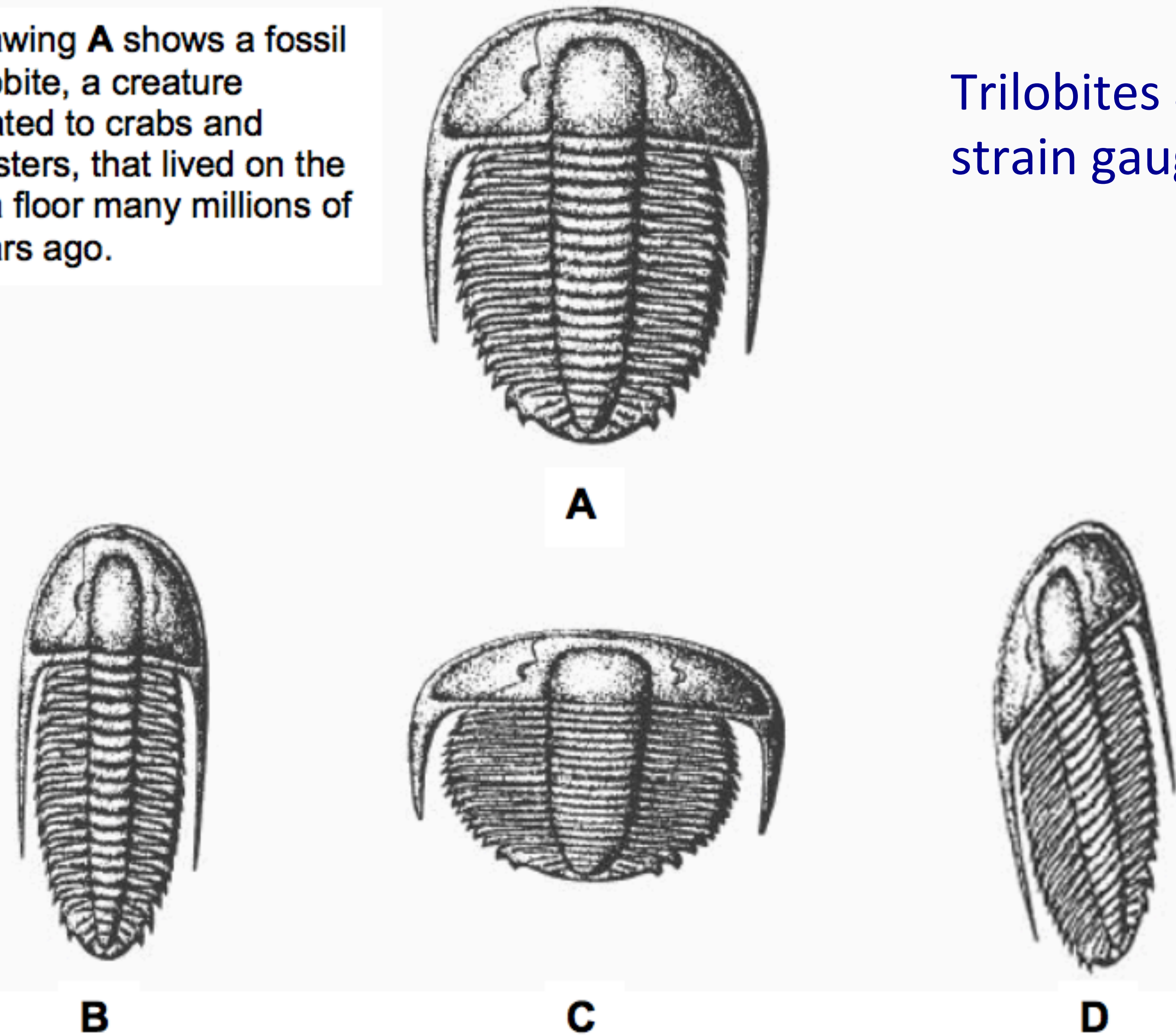
Translation, Rotation, and Strain

ESS 411/511

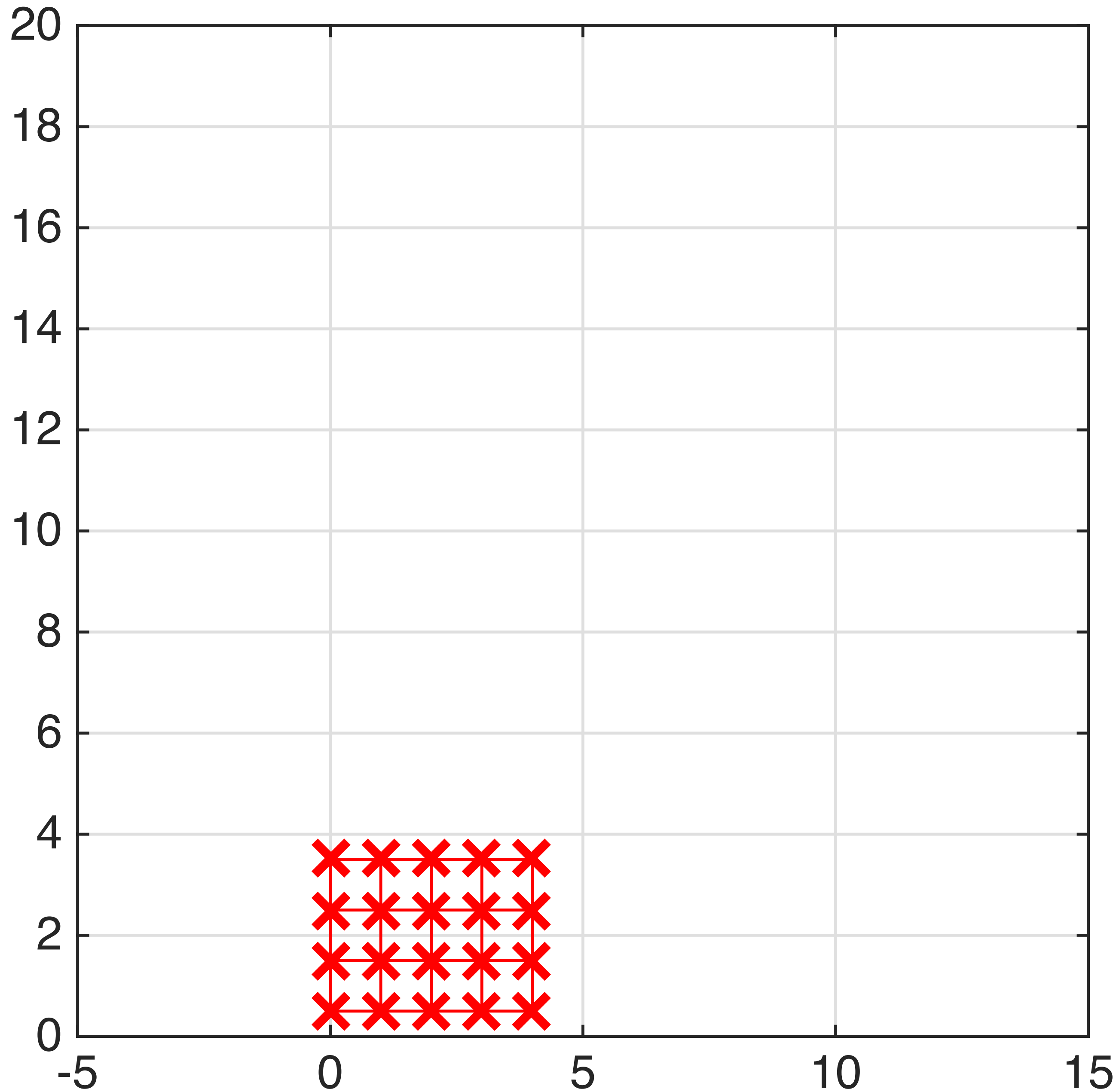

Glacier sped up through Transantarctic Mountains, then spread sideways as it entered Ross Ice Shelf
Qualitatively describe the history of strain and the mode of failure
Flow

Trilobites as strain gauges

Drawing **A** shows a fossil trilobite, a creature related to crabs and lobsters, that lived on the sea floor many millions of years ago.



Drawings **B, C** and **D** are of fossils of the *same species* of trilobite that were found in rocks that have been squashed and folded.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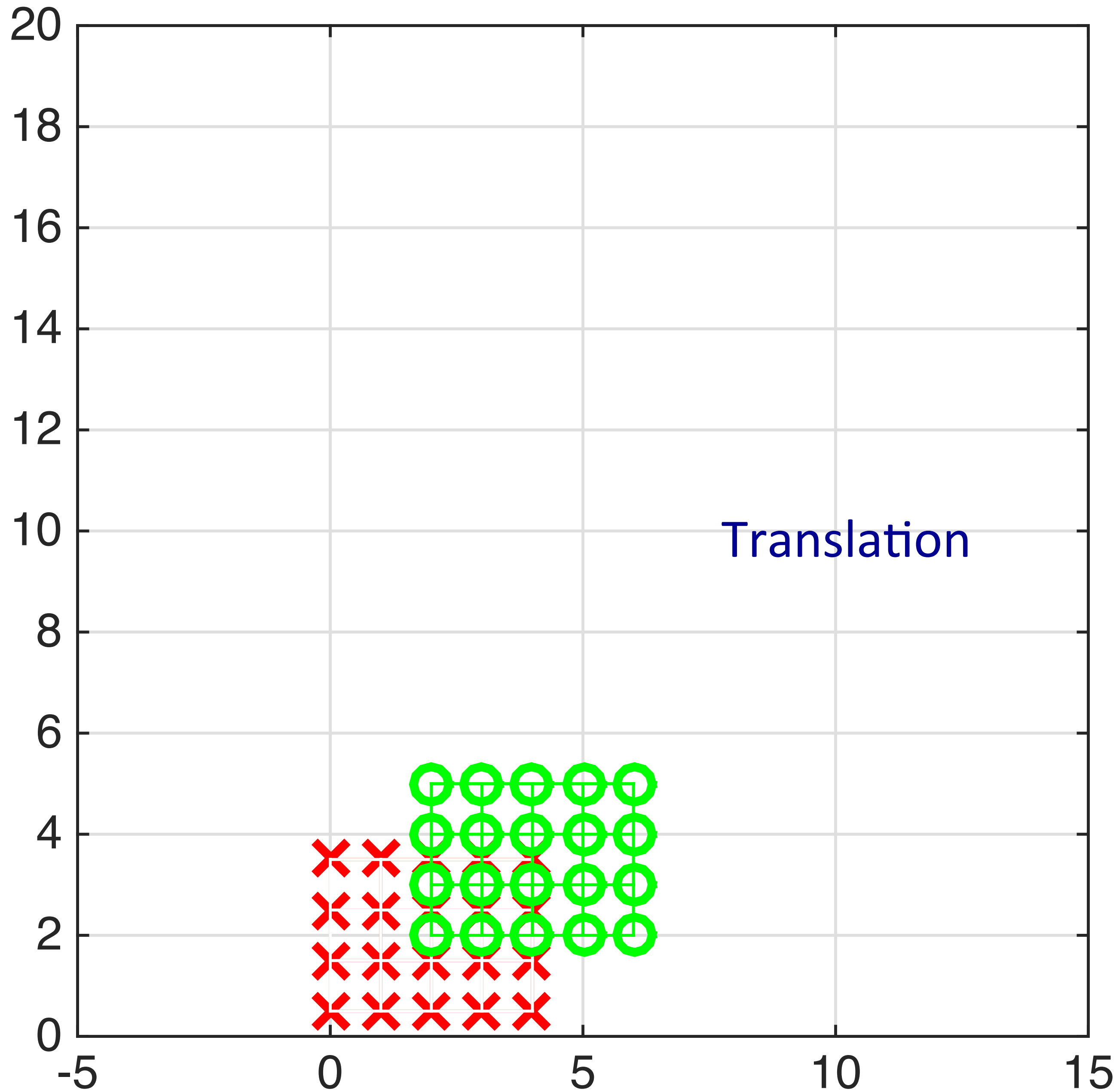
What happened?



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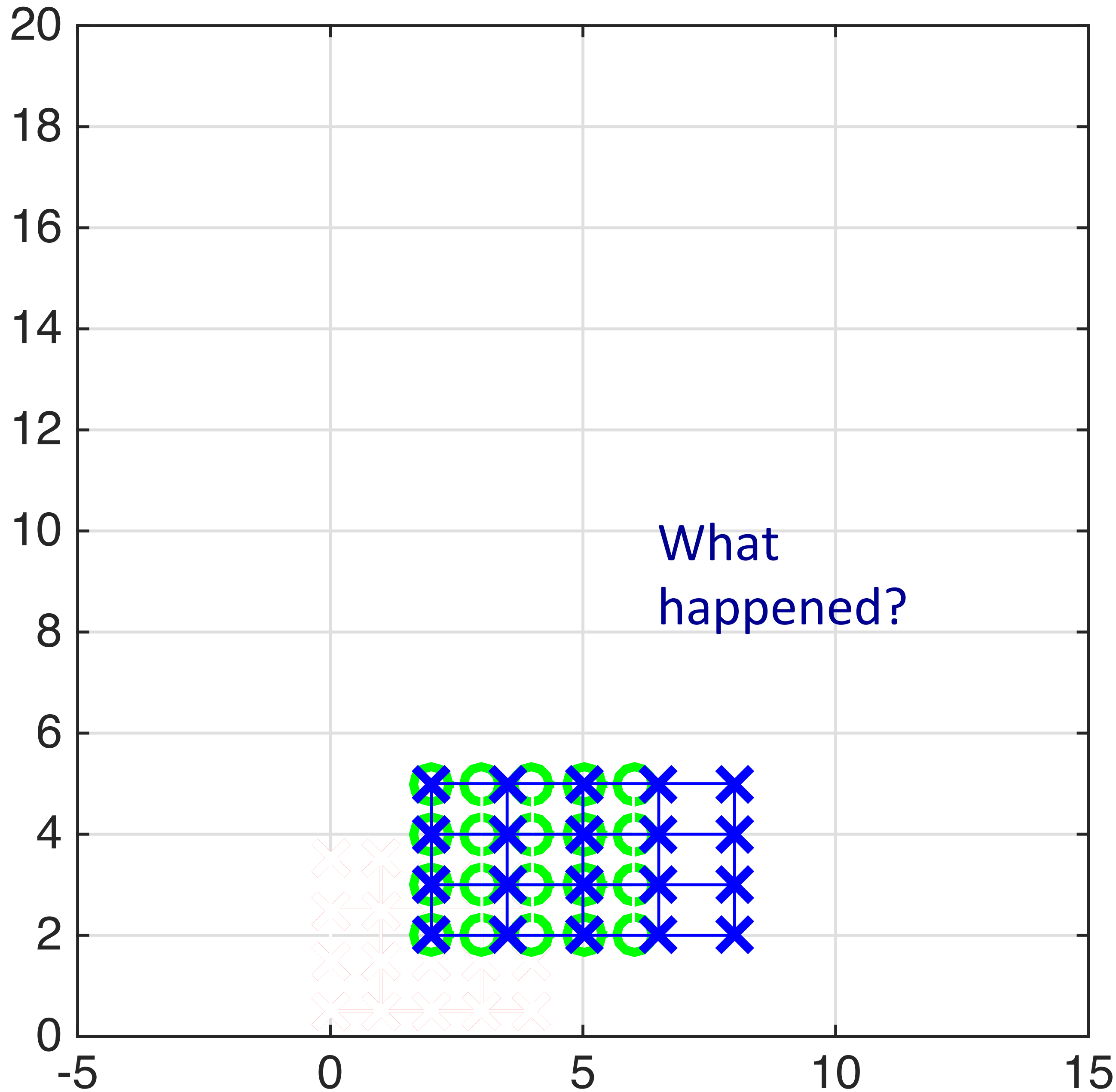
Translation



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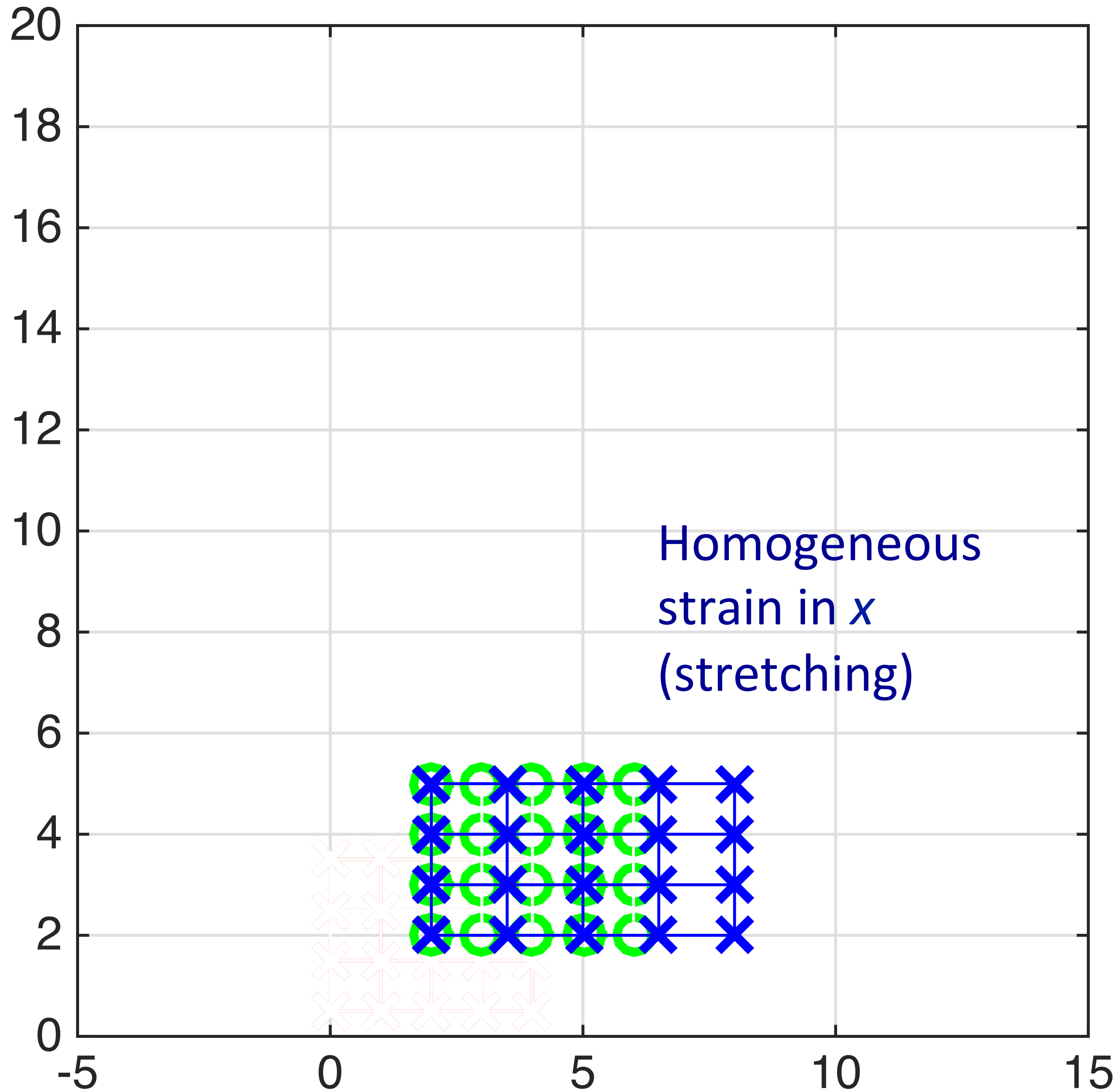

Homogeneous strain in x (stretching)
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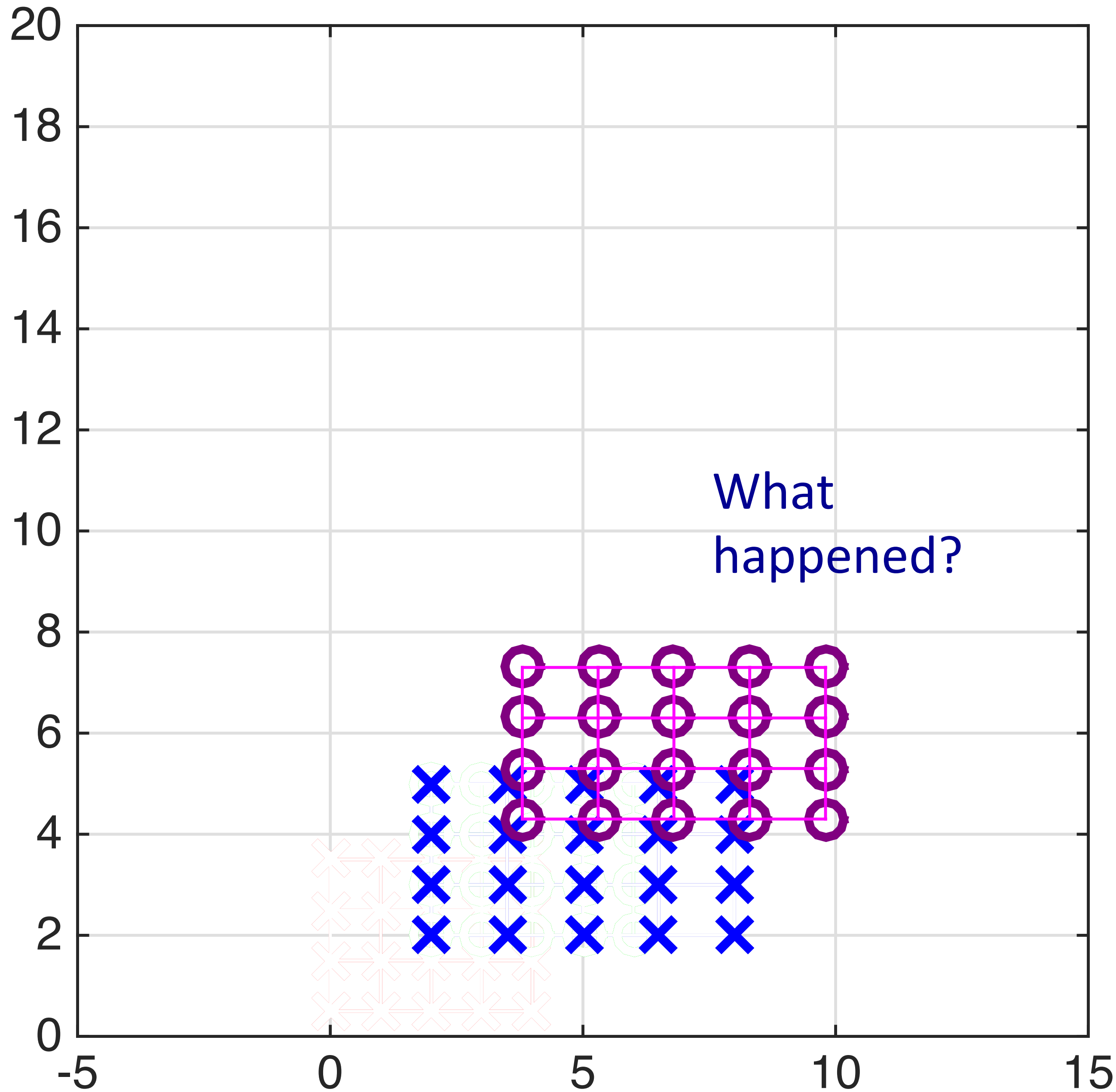

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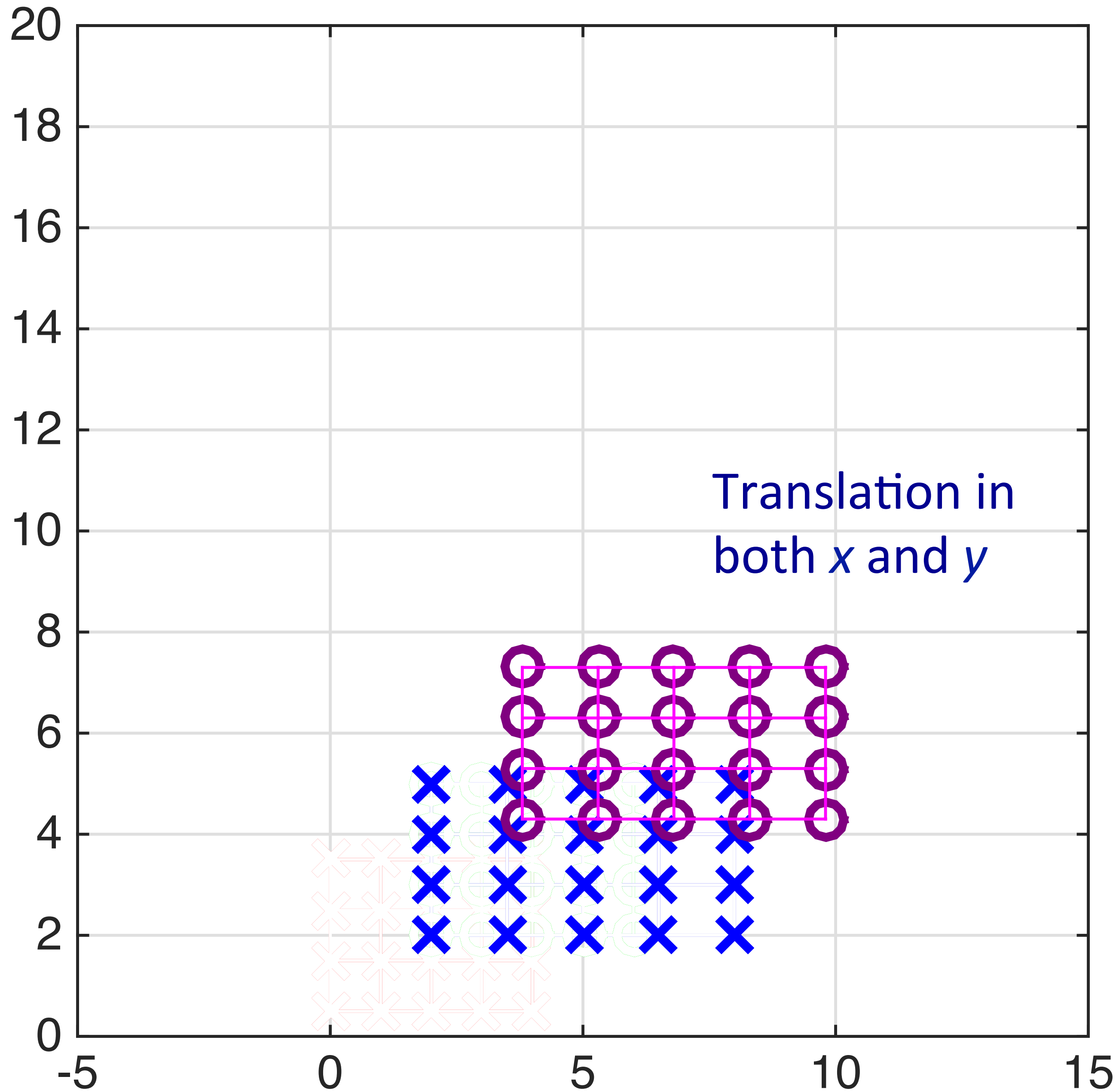
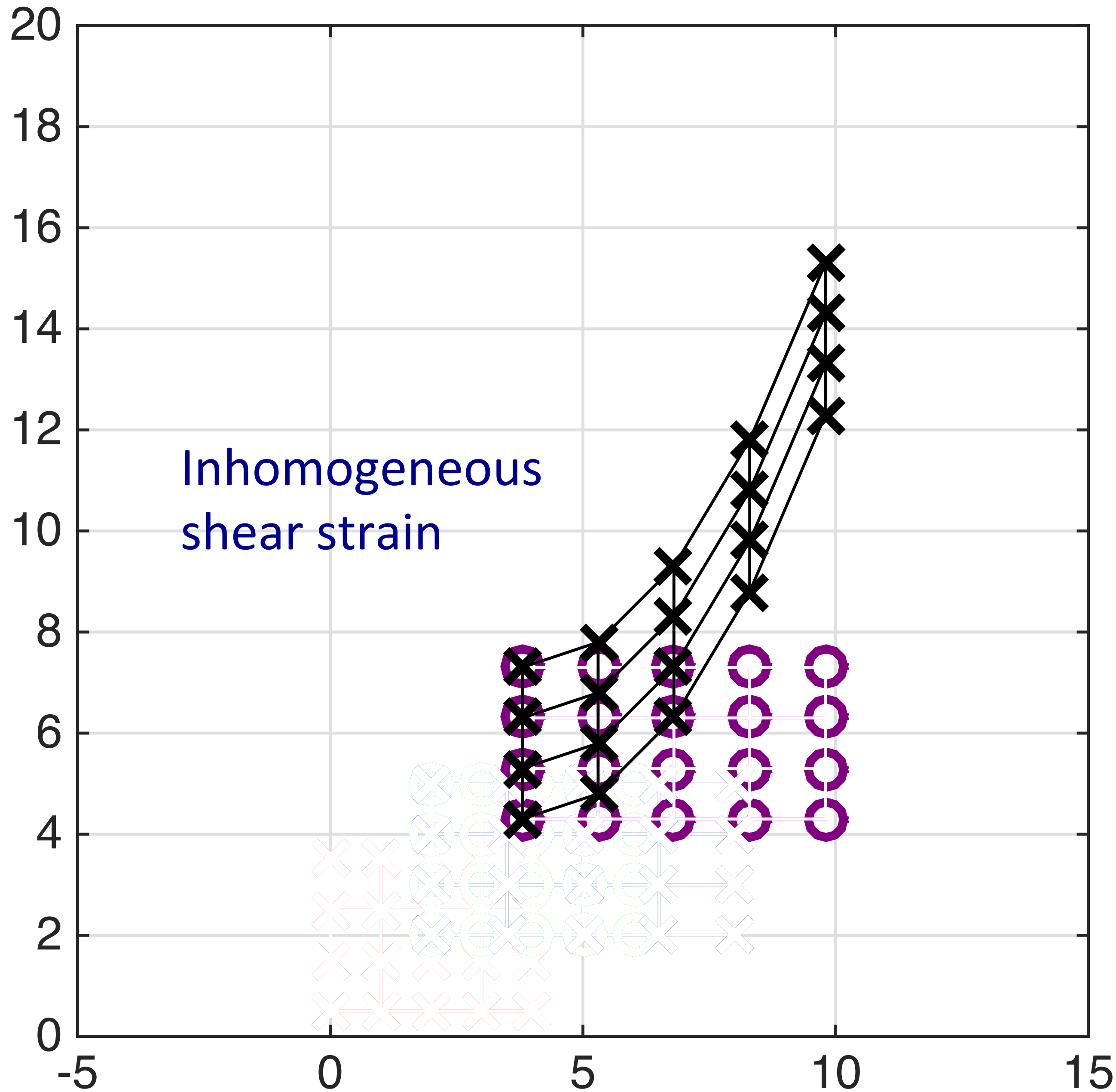
What
happened?


Translation in
both *x* and *y*



Inhomogeneous
shear strain
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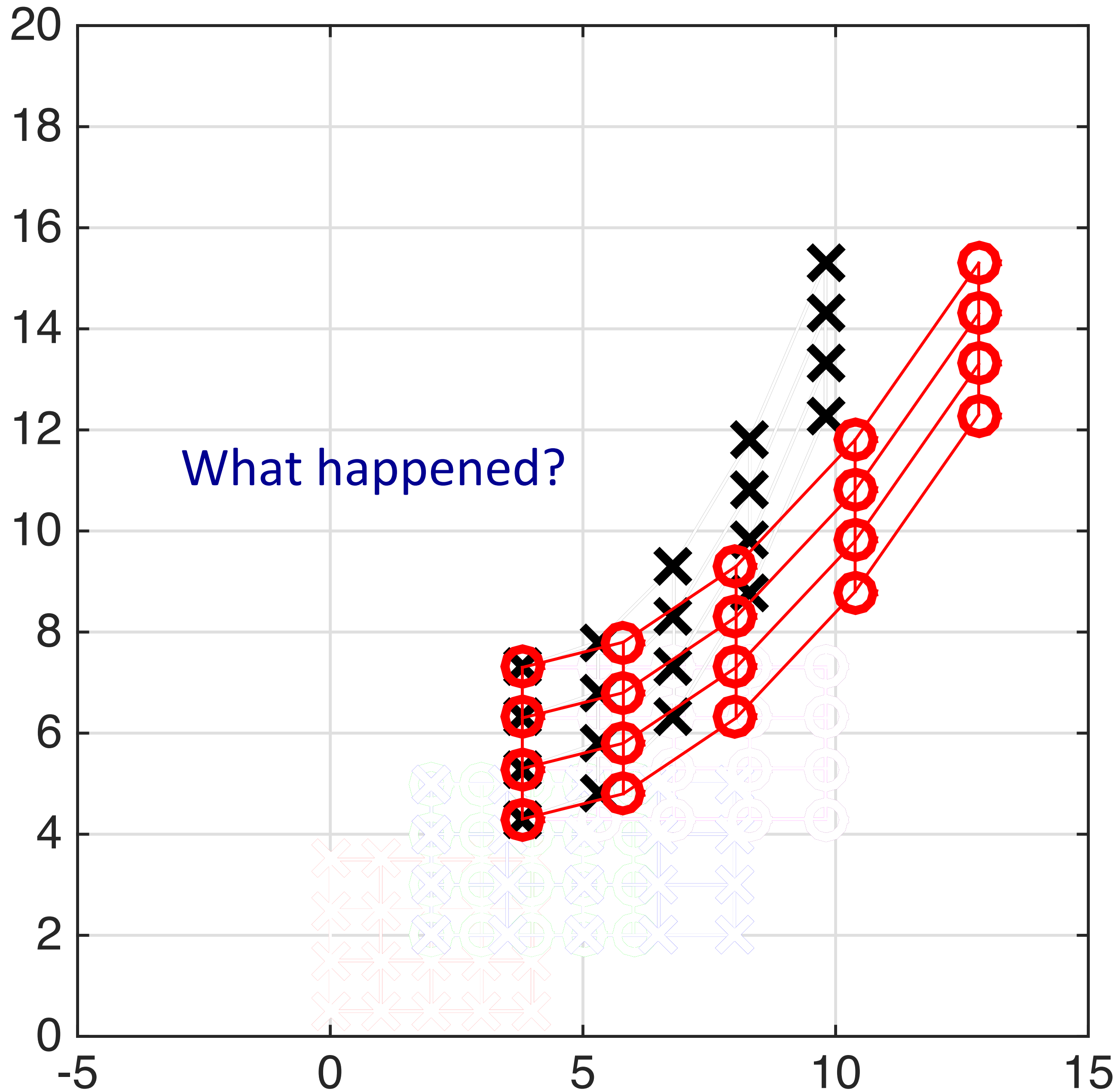
What happened?
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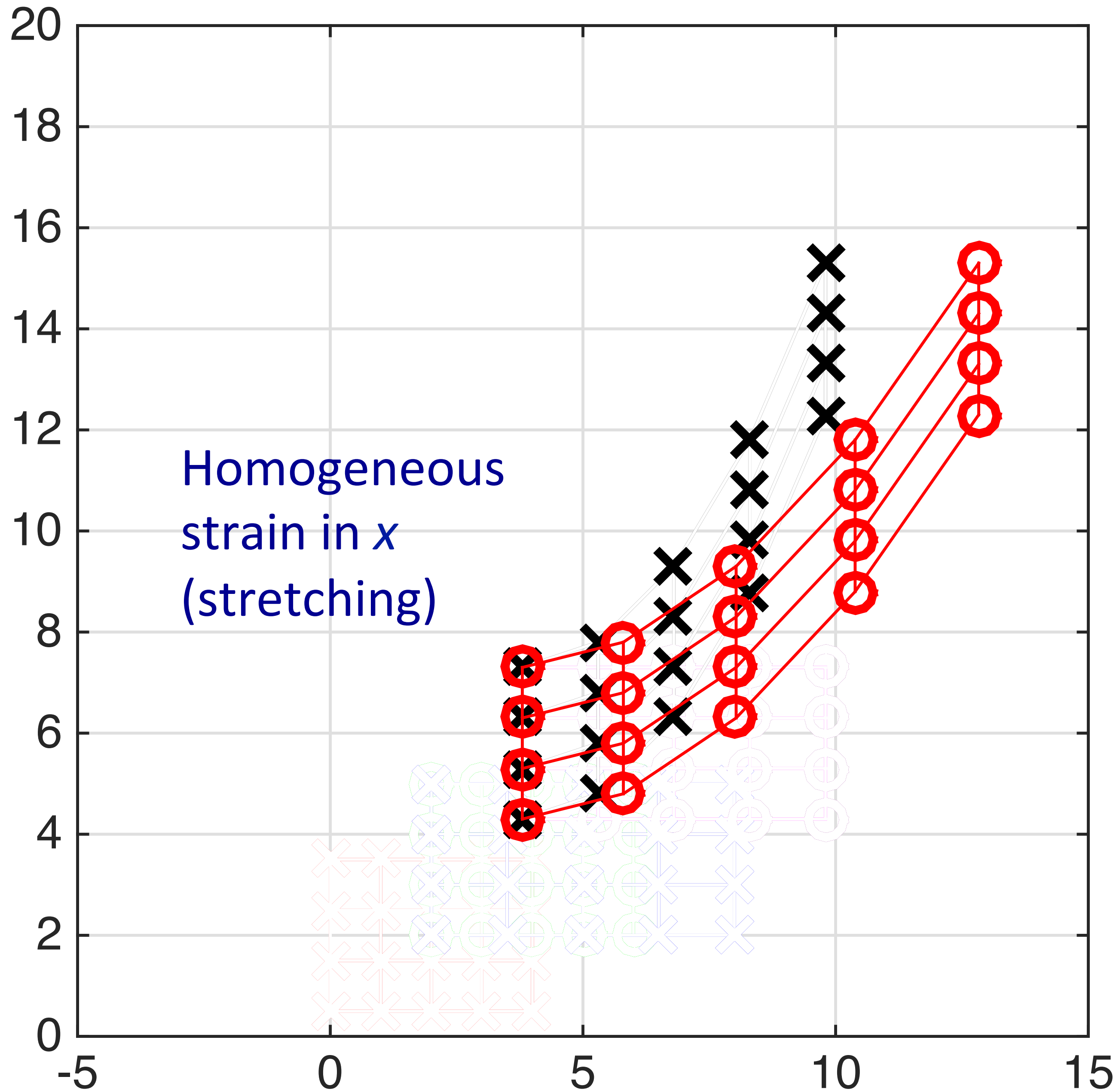

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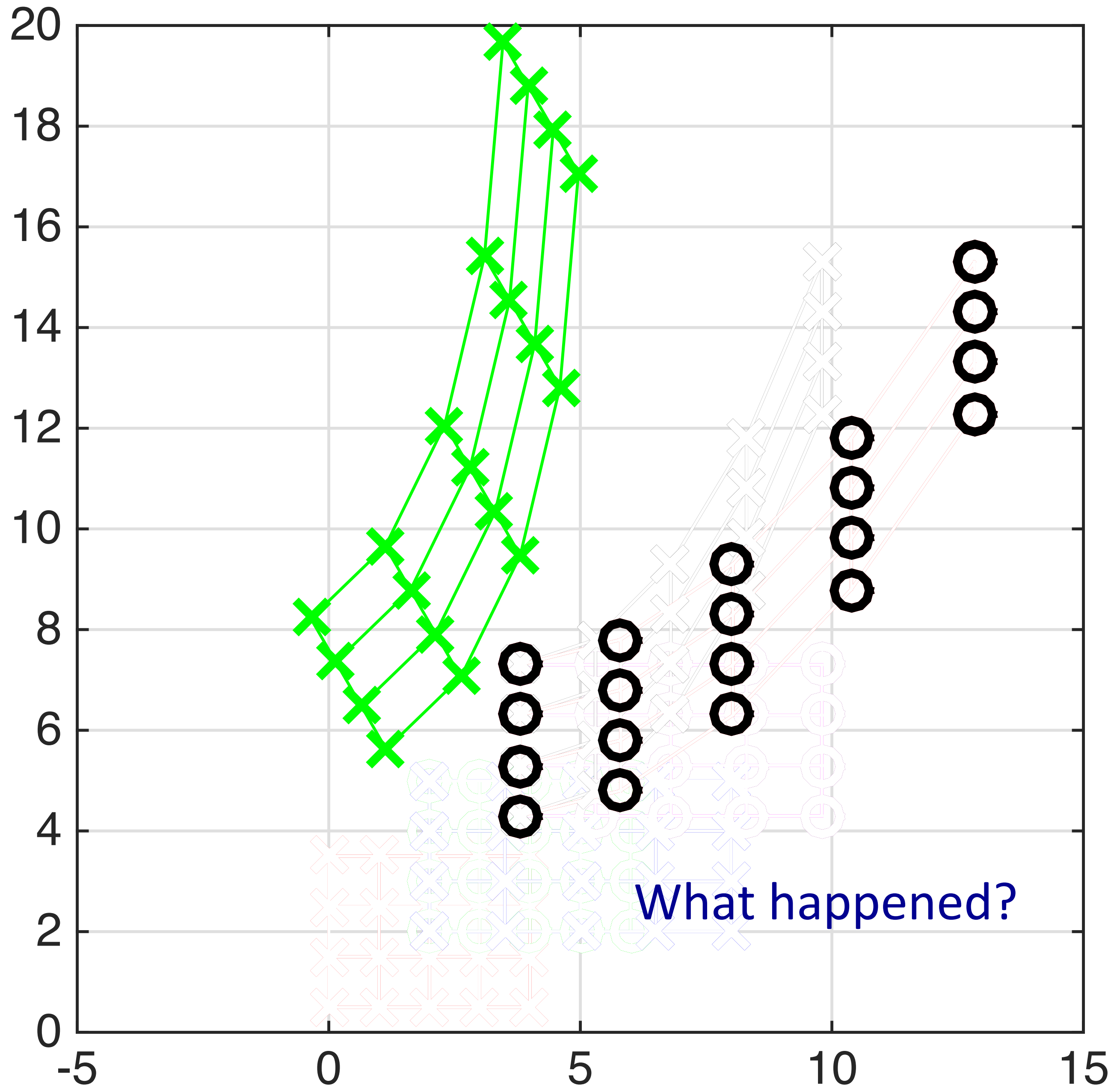
Homogeneous
strain in x
(stretching)



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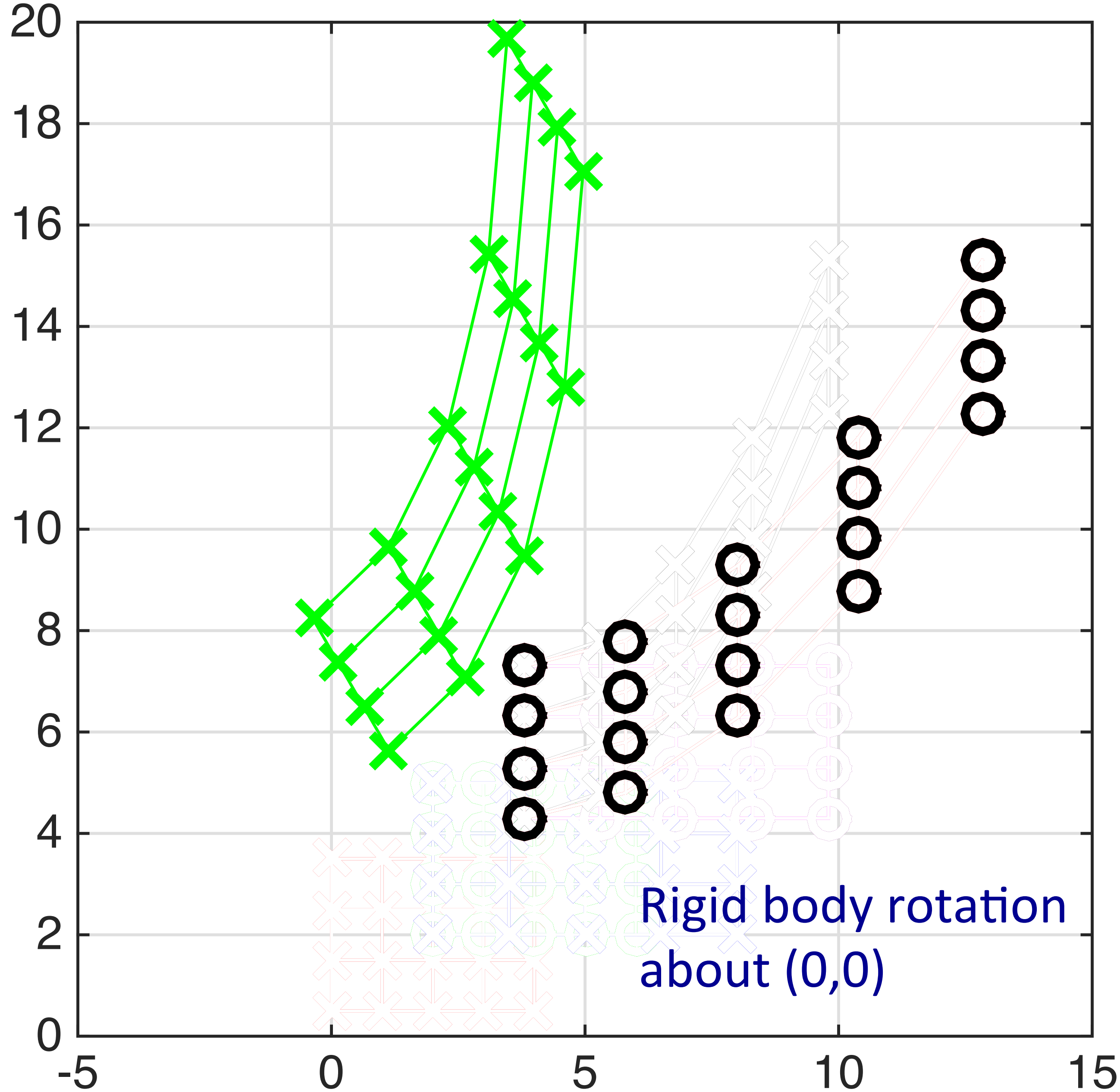
What happened?



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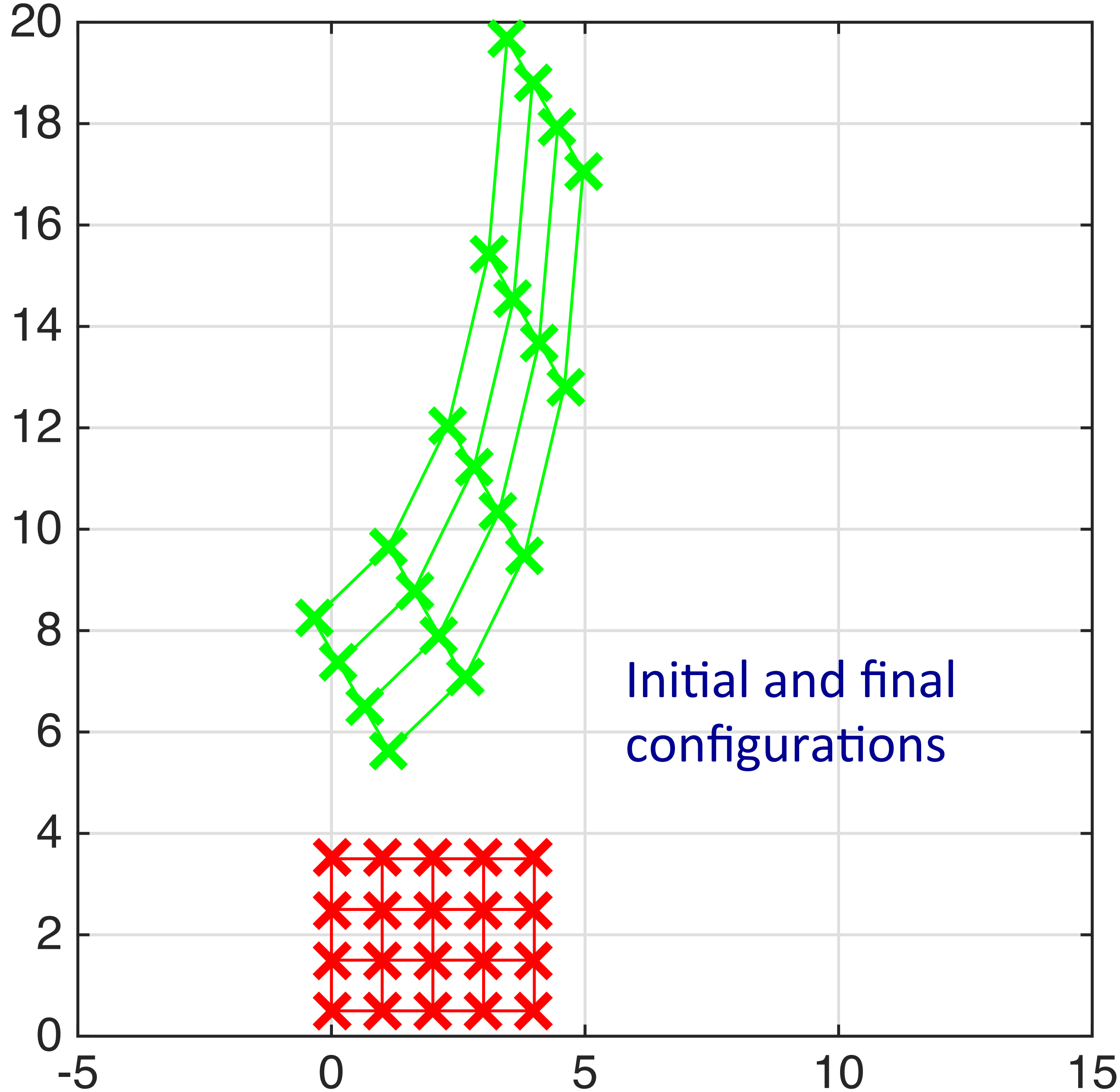
Rigid body rotation
about (0,0)



Initial and final
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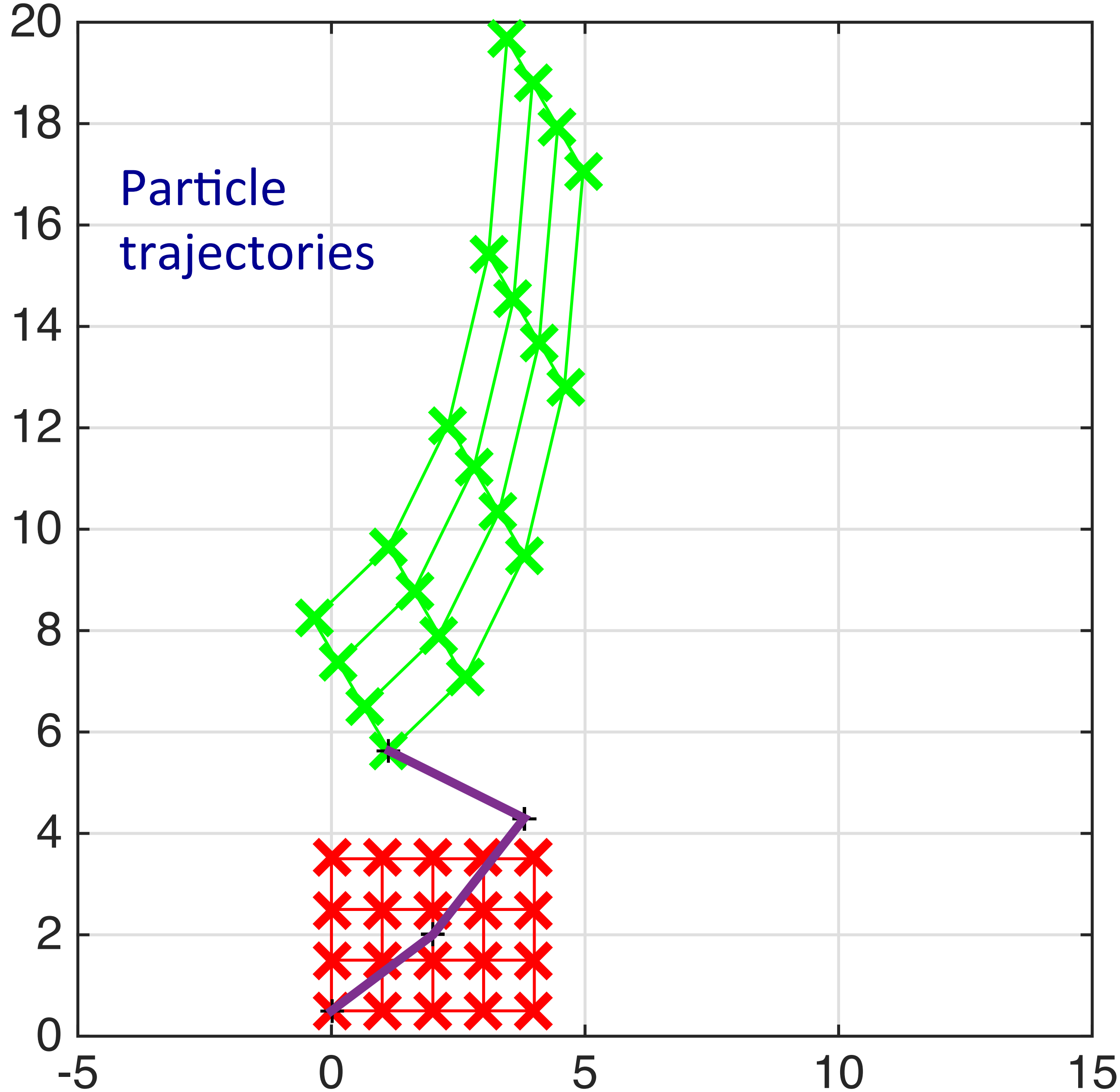

Particle
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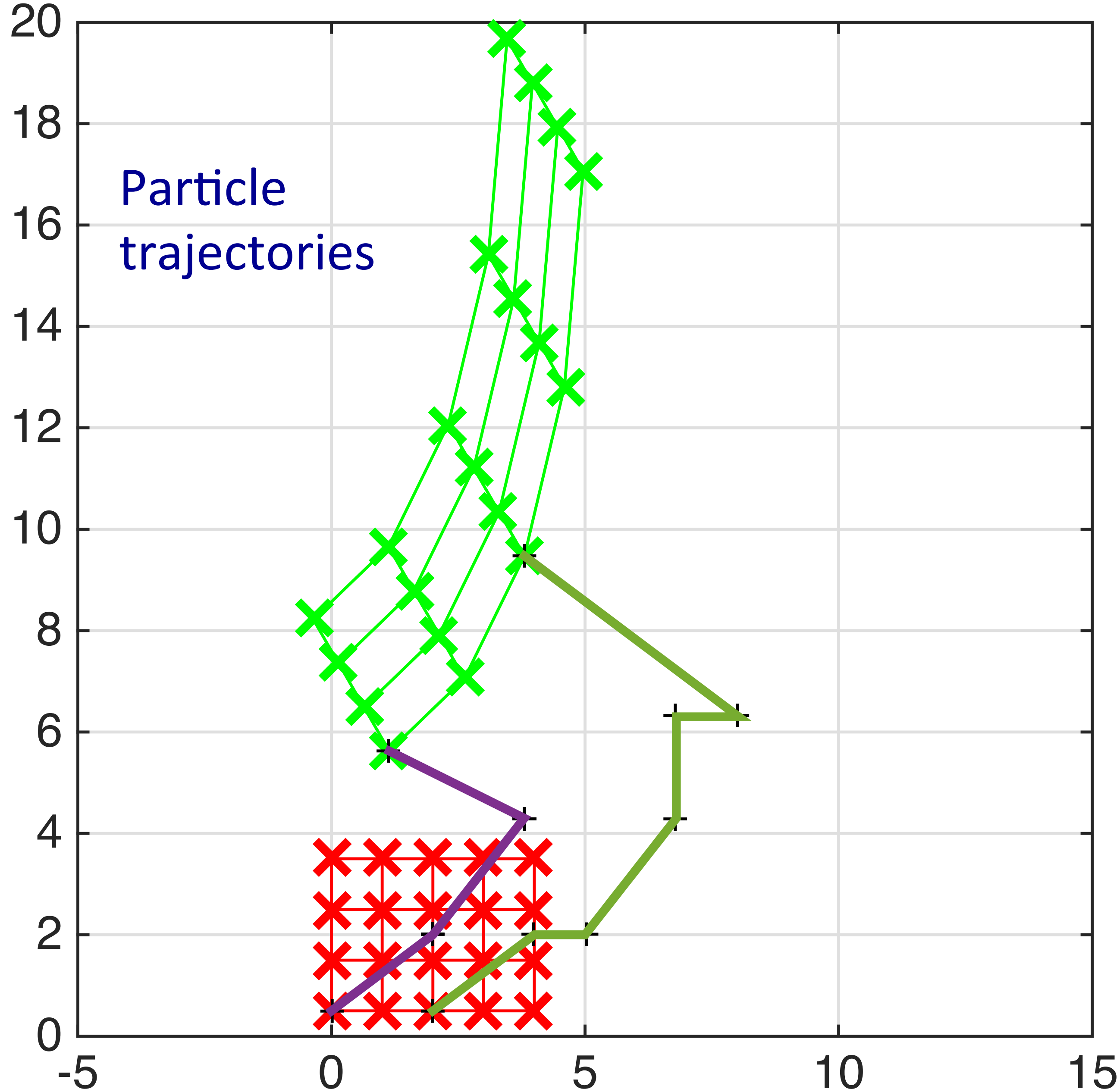
Particle
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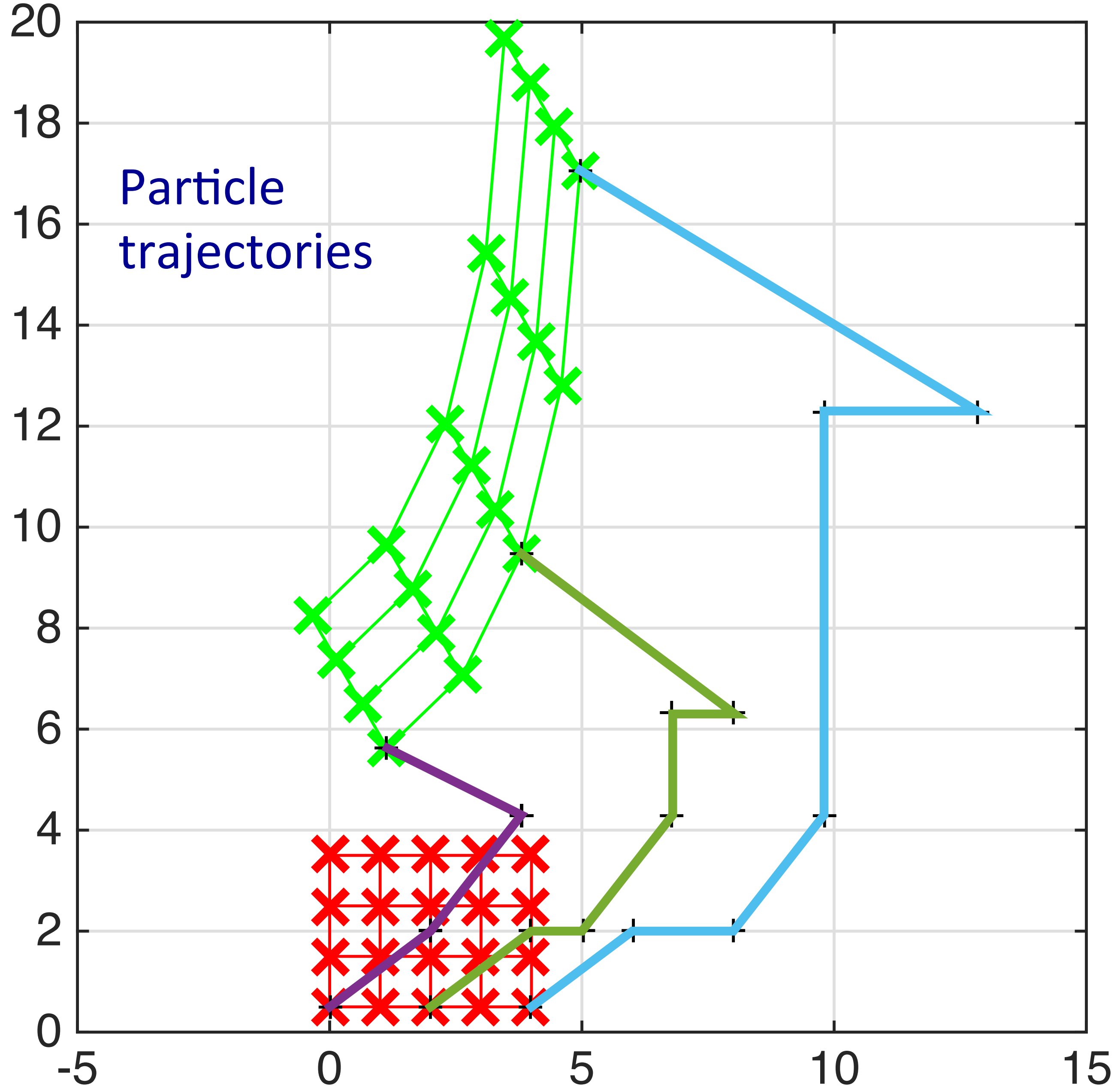

Particle trajectories
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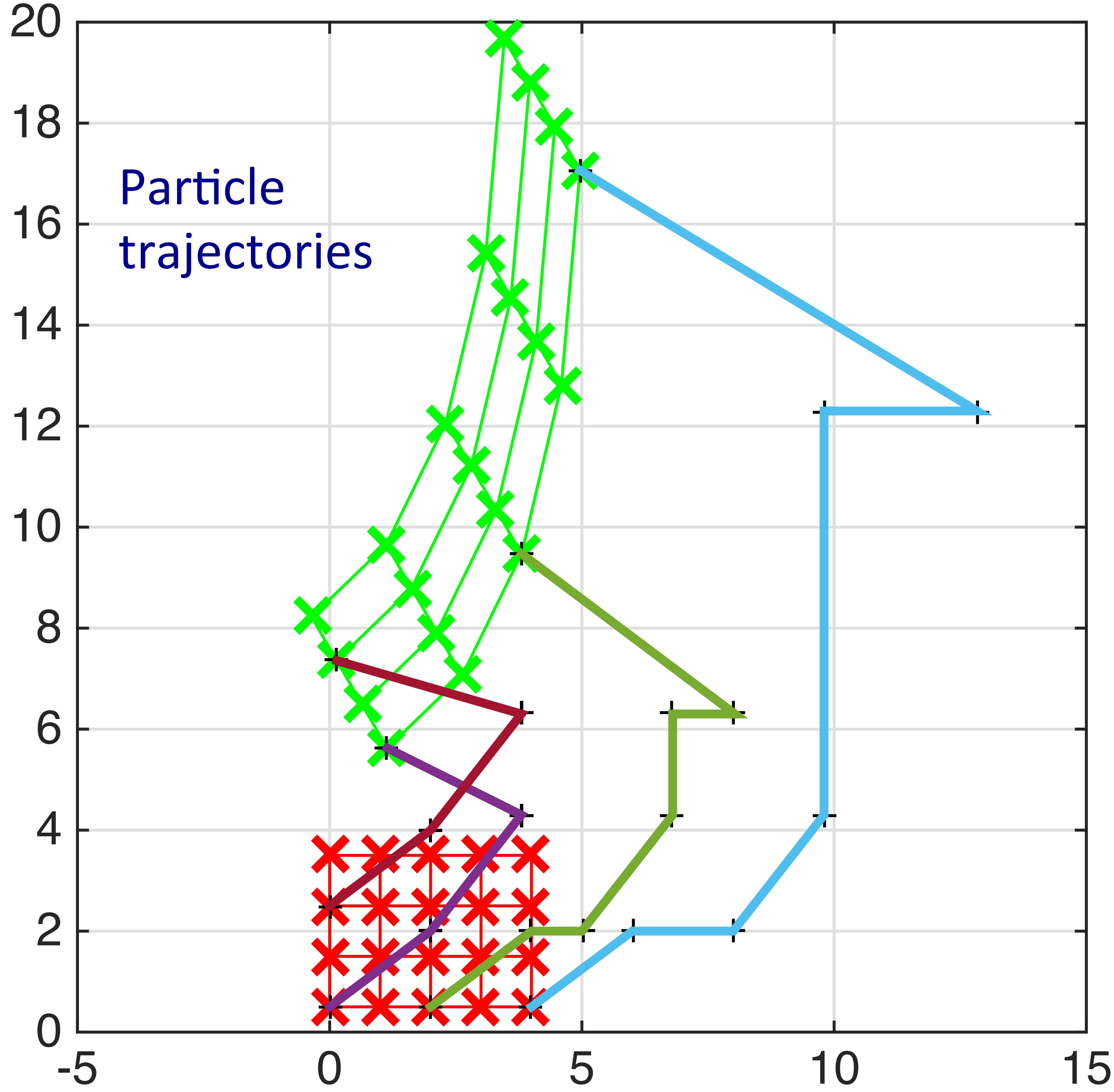
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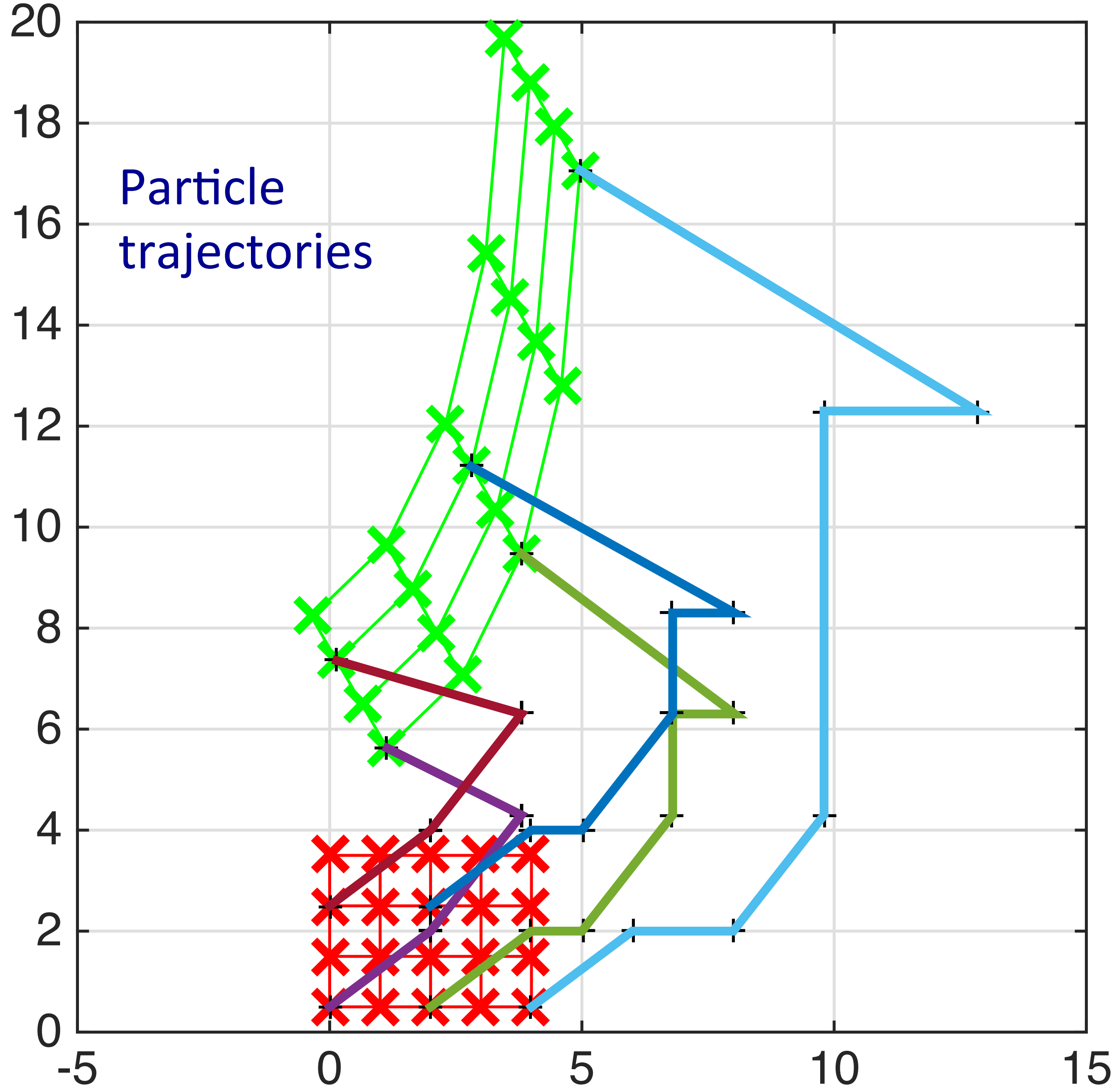
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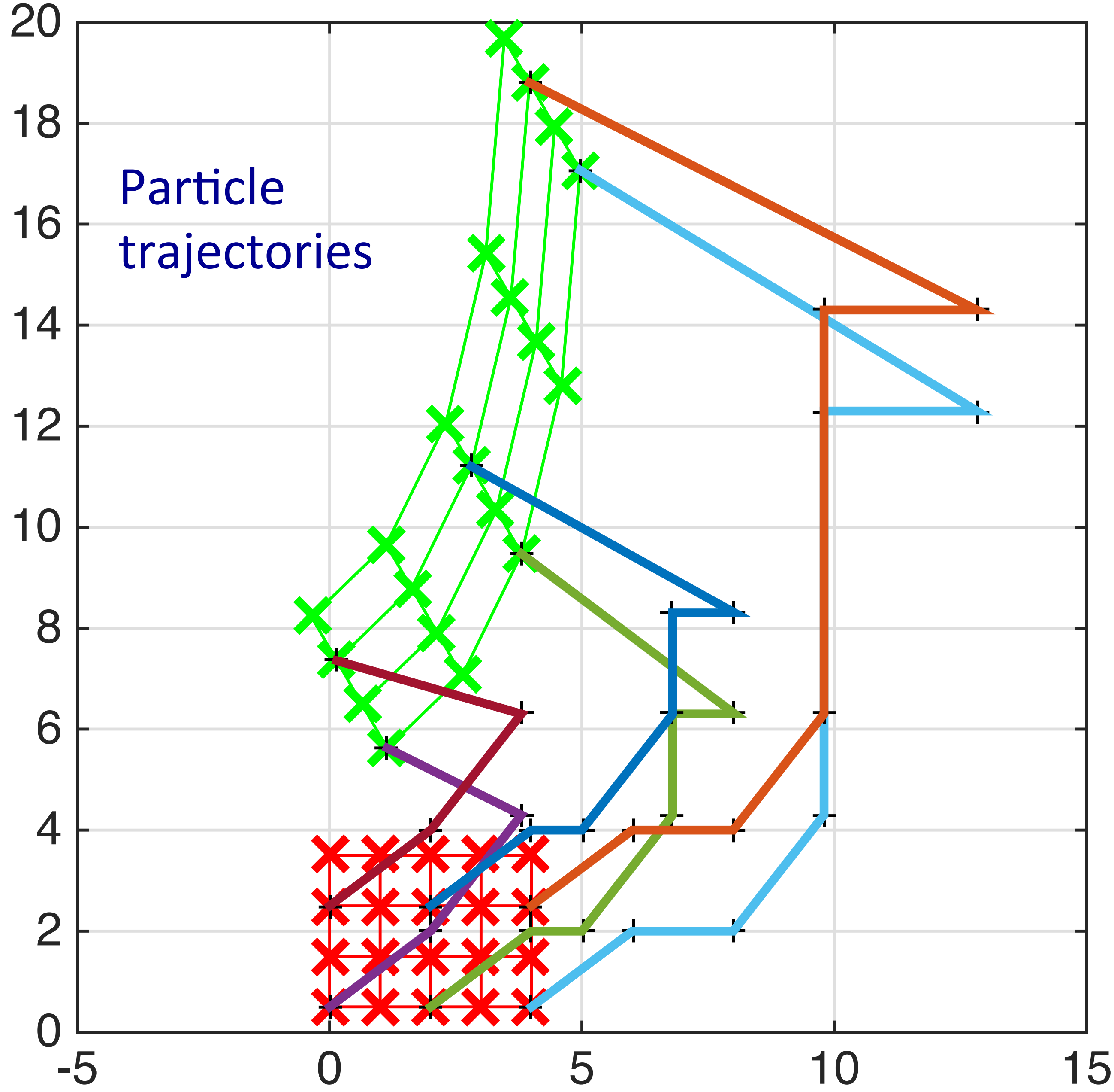

Particle
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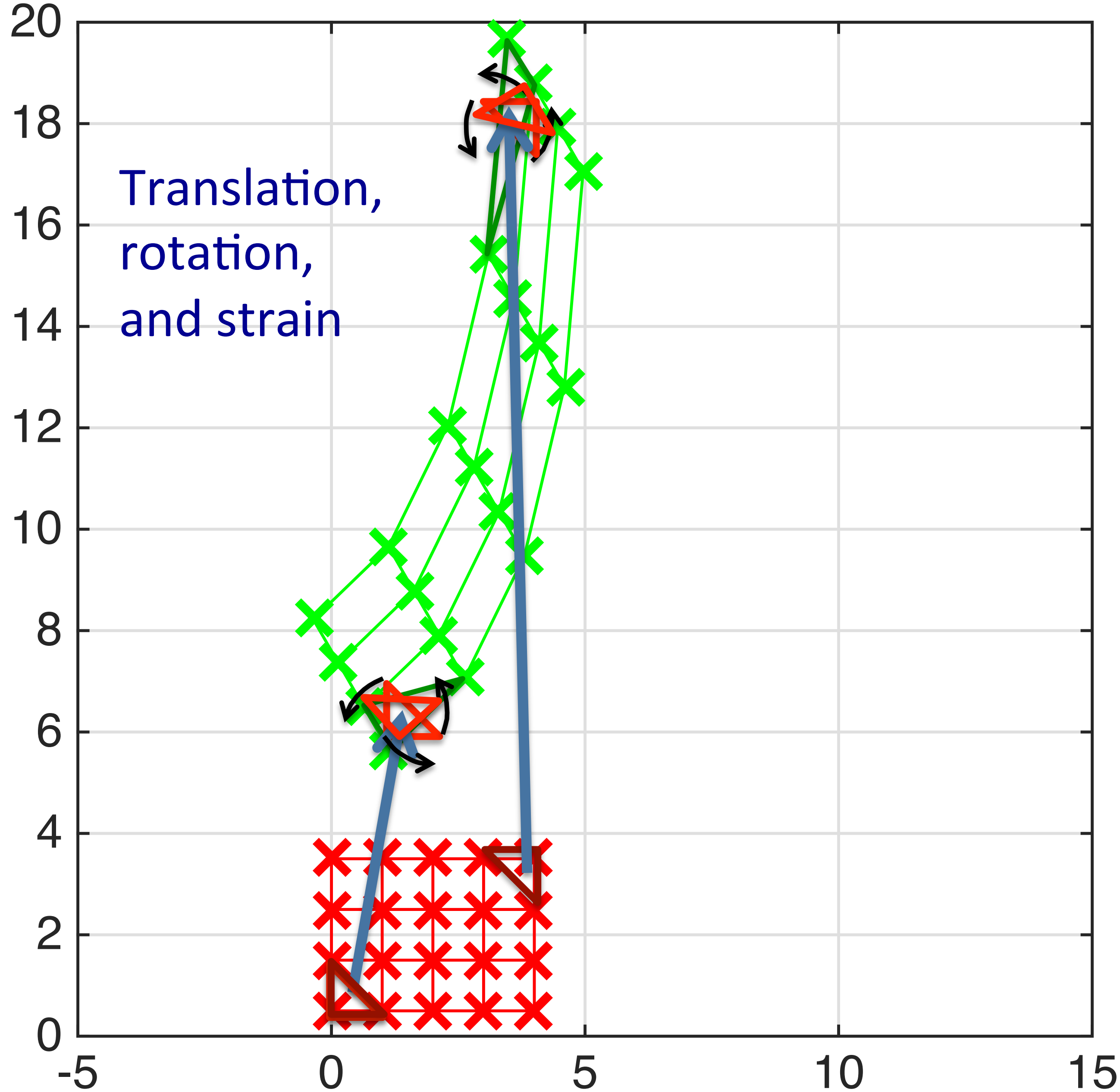

Translation,
rotation,
and strain
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