

# C.O.B.E.N (Hard Version)

Input file: standard input  
Output file: standard output  
Time limit: 10 seconds  
Memory limit: 512 megabytes

This is the hard version of the problem. The only difference between the two versions is the time limit, constraints on  $n$ , and that ties are permitted. You can make hacks only if both versions of the problem are solved.

Among the Mogger enters the **Moniforces** into a multiplayer Nim tournament. However, they don't know the optimal strategy and thus play no better than random.

You are given  $n$  players with their current scores  $p_1, p_2, \dots, p_n$  and  $n$  non-negative bonuses  $s_1, s_2, \dots, s_n$ . In the next round, each player will randomly receive one of the bonuses. The player with the lowest total score after the round will be eliminated. **In the event of a tie for the lowest total score among  $k$  players, one of them is eliminated uniformly at random, with each having a probability of  $\frac{1}{k}$ .**

Your task is to calculate, for each player  $i$ , the exact probability of being eliminated after the next round, returned modulo 998244353.

## Input

The first line contains  $t$  ( $1 \leq t \leq 20$ ), the number of test cases.

Then,  $t$  test cases follow and for each test case:

The first line contains one integer,  $n$  ( $1 \leq n \leq 150$ )

The second line contains  $n$  space separated integers, where  $p_i$  ( $0 \leq p_i \leq 10^9$ ) represents the current score of player  $i$ .

The third line contains  $n$  space separated integers, where  $s_i$  ( $0 \leq s_i \leq 10^9$ ) represents the score bonus associated with  $i^{\text{th}}$  place.

It is guaranteed that the sum of  $n$  over all test cases will not exceed 150.

## Output

A single line containing  $n$  integers, space separated, where the  $i^{\text{th}}$  integer represents the probability mod 998244353 that player  $i$  is eliminated.

It can be shown that the expected value has the form  $\frac{P}{Q}$ , where  $P$  and  $Q$  are non-negative coprime integers, and  $Q \neq 0$ . Report the value of  $P \cdot Q^{-1} \pmod{998244353}$ .

## Example

standard input	standard output
3	166374059 332748118 499122177
3	0 58230921 58230921 144190851 737591661 0
60 40 20	249561089 748683265
100 50 25	
6	
7495 4929 5117 6657 5763 7521	
5767 4037 2833 2307 1730 1442	
2	
3 4	
5 4	

## Note

We have 3 players with current scores  $p = [60, 40, 20]$  and bonus options  $s = [100, 50, 25]$ . There are  $3! = 6$  ways to assign the 3 bonuses, and in each assignment, the player with the lowest total ( $p_i + s_{\pi(i)}$ ) is eliminated. Enumerating:

1.  $[160, 90, 45] \rightarrow$  Player 3 is eliminated.
2.  $[160, 65, 70] \rightarrow$  Player 2 is eliminated.
3.  $[110, 140, 45] \rightarrow$  Player 3 is eliminated.
4.  $[110, 65, 120] \rightarrow$  Player 2 is eliminated.
5.  $[85, 140, 45] \rightarrow$  Player 3 is eliminated.
6.  $[85, 90, 120] \rightarrow$  Player 1 is eliminated.

**Player 1** is eliminated in 1 of 6 permutations ( $\frac{1}{6}$ ).

**Player 2** is eliminated in 2 of 6 permutations ( $\frac{1}{3}$ ).

**Player 3** is eliminated in 3 of 6 permutations ( $\frac{1}{2}$ ).

A tie is possible for the last test case; if we allocate the score bonus 5 to player 1 and the score bonus 4 to player 2, both end with a total score of 8. Therefore, we begin the procedure and uniformly pick one of them at random, which means they each have a  $\frac{1}{2}$  chance of being eliminated if those score bonuses are allocated in that manner. The probabilities of elimination over all permutations are  $\frac{3}{4}$  for player 1 and  $\frac{1}{4}$  for player 2.