1 Introduction to Sonar

History of Sonar

"If you cause your ship to stop, and place the head of a long tube in the water and place the outer extremity to your ear you will hear ships at a great distance from you"

— Leonardo da Vinci, 1490.

1. "...ship to stop" $\longrightarrow$ reduce self-noise
2. "...tube in water" $\longrightarrow$ transducer
3. "...to your ear" $\longrightarrow$ receiver
4. "...will hear ships" $\longrightarrow$ detection
5. "...at a great distance" $\longrightarrow$ low attenuation

1687 Issac Newton: first theoretical prediction of sound speed

$$\frac{\partial \rho}{\partial p} \approx \frac{1}{c^2}$$

1847 Culladon and Sturm:

- first accurate measurement of speed of sound in water
- light flash/underwater bell

1900 Submarine Signal Company: first commerical application.

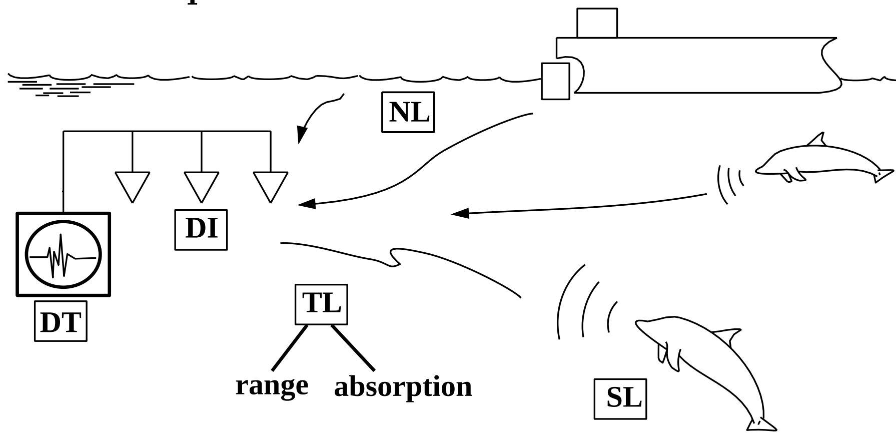
- range from ship to lighthouse
- simulataneous sounding of underwater bell and foghorn

1914 Fessendon: first active sonar system (detect iceberg 2 miles)

World War I: experiments.

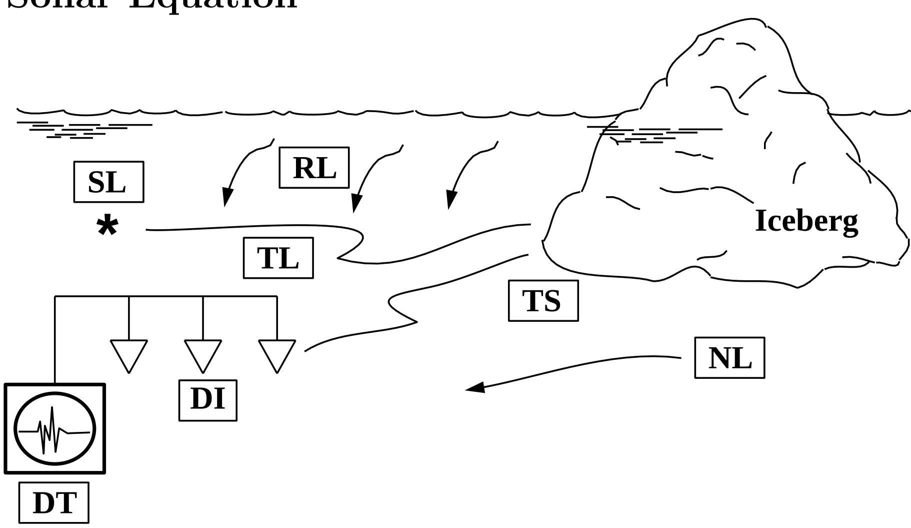
- passive sonar operational equipment
- active sonar experiments

Passive Sonar Equation



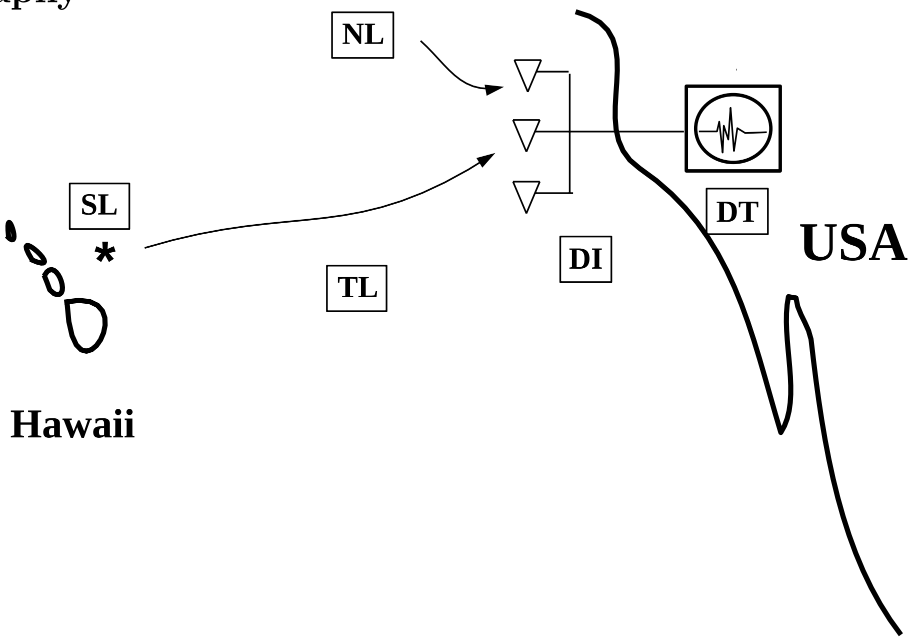
$$SL - TL - (NL - DI) = DT$$

Active Sonar Equation



$$SL - 2TL + TS - \left(\begin{array}{c} NL - DI \\ RL \end{array} \right) = DT$$

Tomography



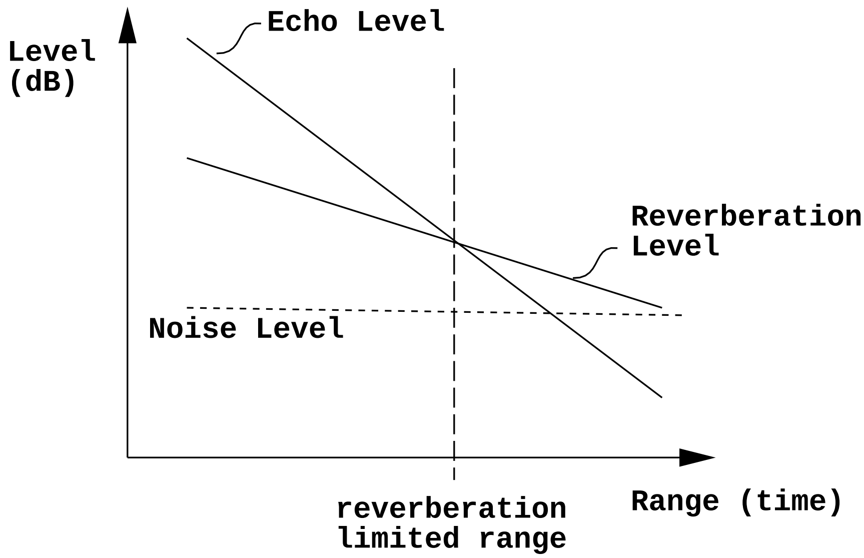
$$SL - TL - (NL - DI) = DT$$

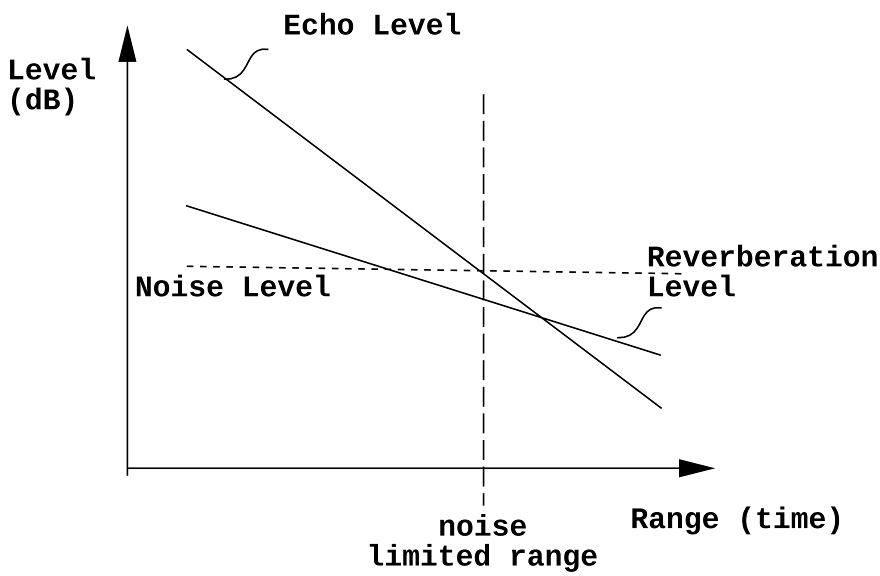
Parameter definitions:

- SL = Source Level
- TL = Transmission Loss
- NL = Noise Level
- DI = Directivity Index
- RL = Reverberation Level
- TS = Target Strength
- DT = Detection Threshold

Reverberation vs. Noise-limited Range

- Active Sonar
- Range independent noise vs. range dependent reverberation
- Define *Echo Level*: EL = SL − 2TL + TS





Definition of sound intensity

dB = decibel (after Alexander Graham Bell)

For acoustics:

$$\mathrm{dB} = 10 \log_{10} \frac{I}{I_{\mathrm{ref}}}$$

2 Arrays

Summary of array formulas

Source Level

- $\mathrm{SL} = 10\log\frac{I}{I_{ref}} = 10\log\frac{p^2}{p_{ref}^2}$ (general)
- $\mathrm{SL} = 171 + 10\log\mathcal{P}$ (omni)
- $\mathrm{SL} = 171 + 10\log\mathcal{P} + \mathrm{DI}$ (directional)

Directivity Index

- $\mathrm{DI} = 10\log(\frac{I_D}{I_O})$ (general)

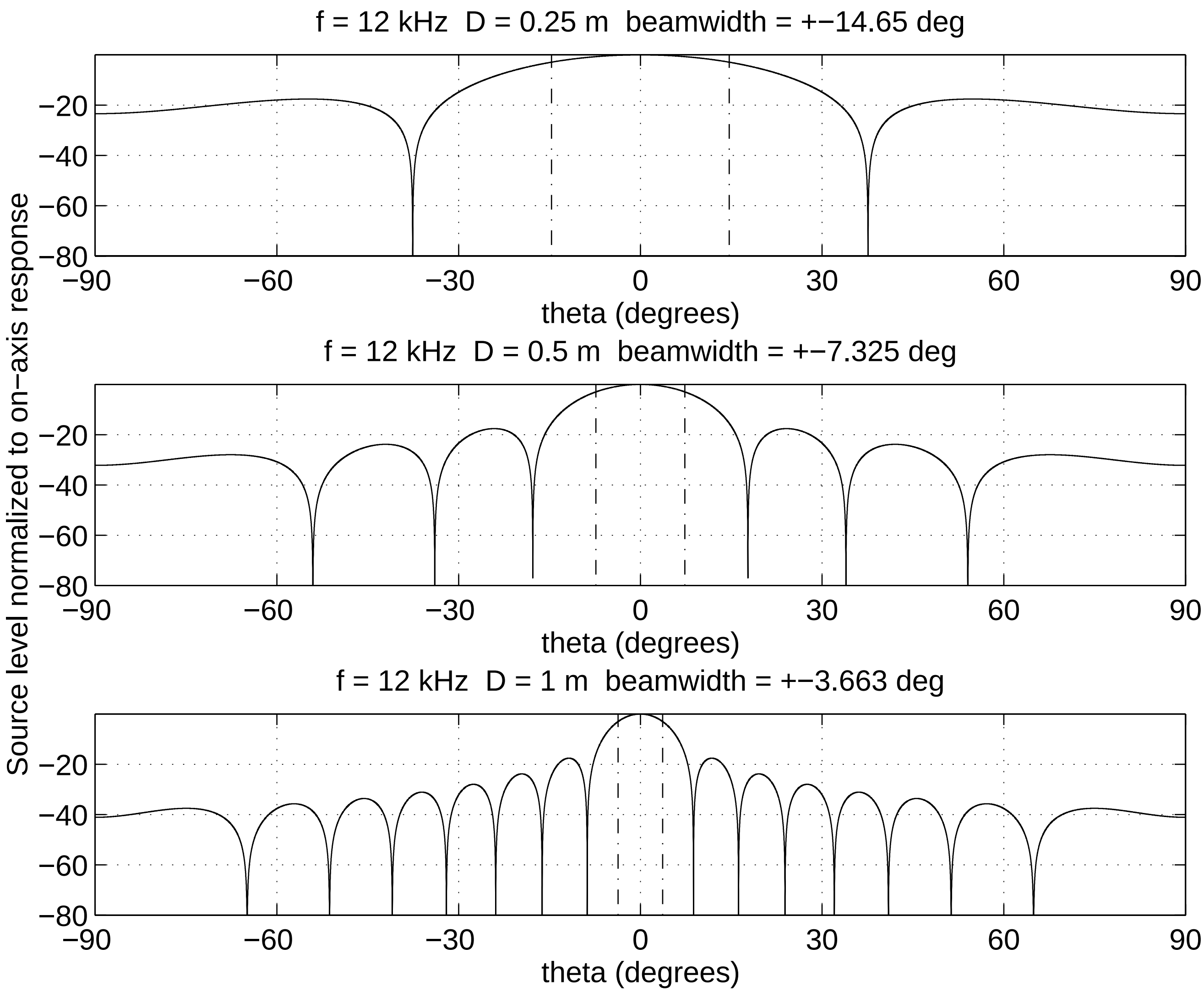
  I_D = directional intensity (measured at center of beam)

  I_O = omnidirectional intensity
  (same source power radiated equally in all directions)

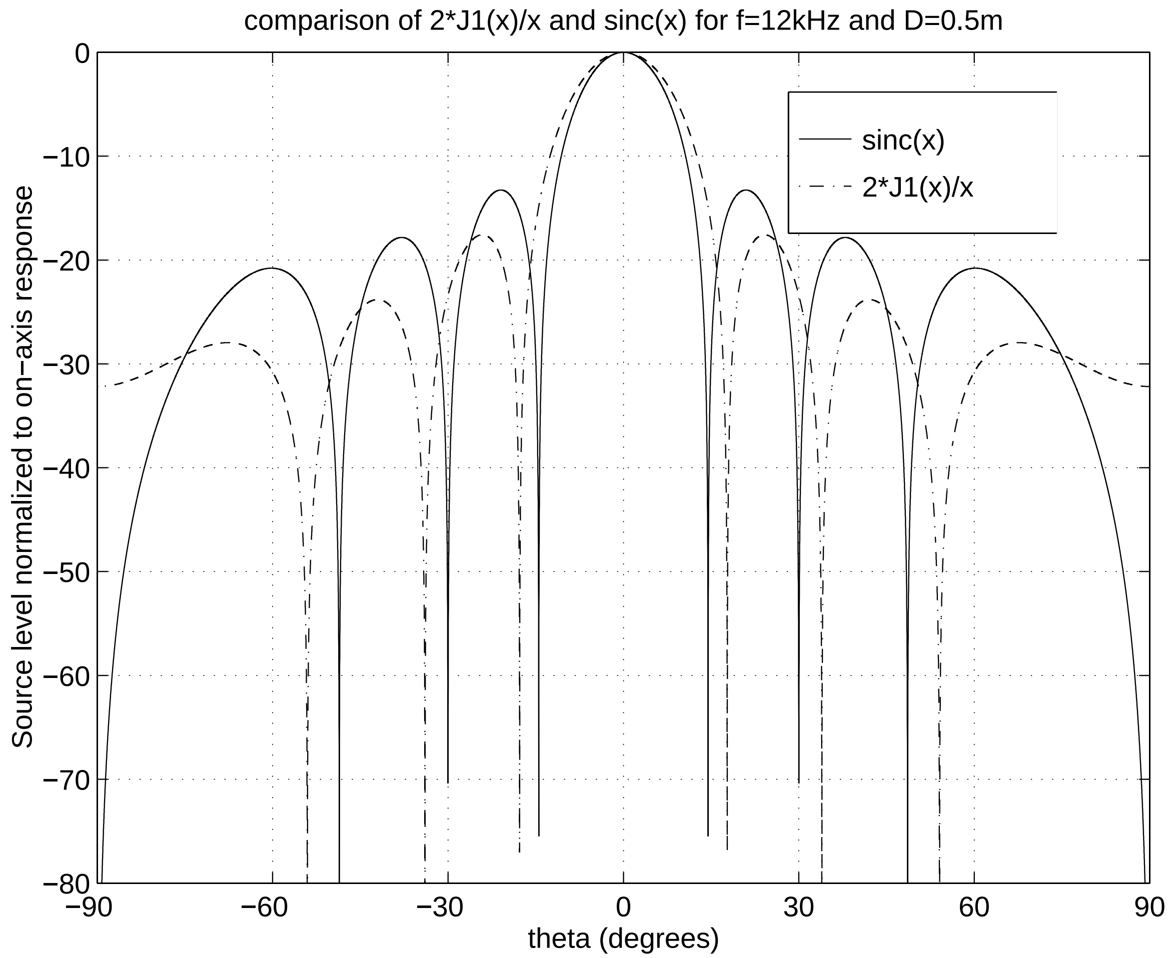
- $\mathrm{DI} = 10\log(\frac{2L}{\lambda})$ (line array)
- $\mathrm{DI} = 10\log((\frac{\pi D}{\lambda})^2) = 20\log(\frac{\pi D}{\lambda})$ (disc array)
- $\mathrm{DI} = 10\log(\frac{4\pi L_x L_y}{\lambda^2})$ (rect. array)

3-dB Beamwidth θ_{3dB}

- $\theta_{3dB} = \pm\frac{25.3\lambda}{L}$ deg. (line array)
- $\theta_{3dB} = \pm\frac{29.5\lambda}{D}$ deg. (disc array)
- $\theta_{3dB} = \pm\frac{25.3\lambda}{L_x}, \pm\frac{25.3\lambda}{L_y}$ deg. (rect. array)



This figure shows the beam pattern for a circular transducer for D/λ equal to 2, 4, and 8. Note that the beampattern gets narrower as the diameter is increased.



This figure compares the response of a line array and a circular disc transducer. For the line array, the beam pattern is:

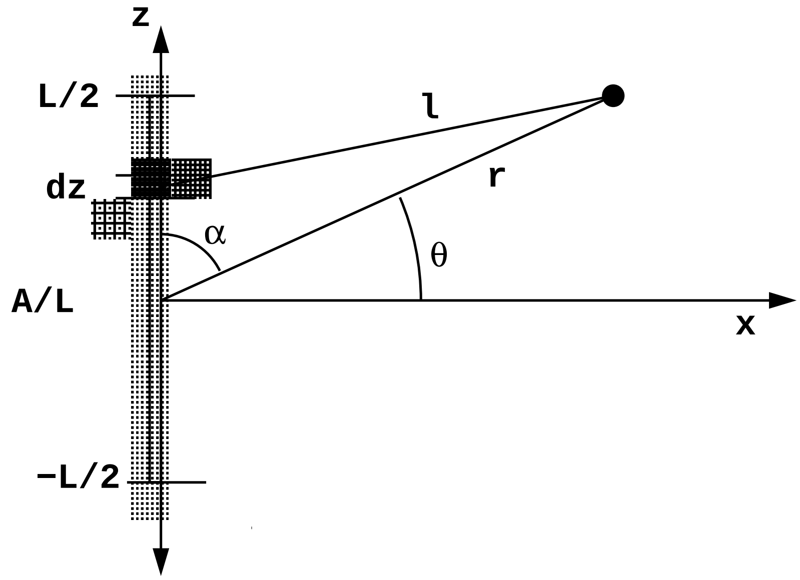
$$b(\theta) = \left[\frac{\sin(\frac{1}{2}kL\sin\theta)}{\frac{1}{2}kL\sin\theta}\right]^2$$

whereas for the disc array, the beam pattern is

$$b(\theta) = \left[\frac{2J_1(\frac{1}{2}kD\sin\theta)}{\frac{1}{2}kD\sin\theta}\right]^2$$

where $J_1(x)$ is the Bessel function of the first kind. For the line array, the height of the first side-lobe is 13 dB less than the peak of the main lobe. For the disc, the height of the first side-lobe is 17 dB less than the peak of the main lobe.

Line Array



Problem geometry

Our goal is to compute the acoustic field at the point (r, θ) in the *far field* of a uniform line array of intensity A/L. First, let's find an expression for l in terms of r and θ. From the law of cosines, we can write:

$$l^2 = r^2 + z^2 - 2rz\cos\alpha.$$

If we factor out r^2 from the left hand side, and substitute $\sin\theta$ for $\cos\alpha$, we get:

$$l^2 = r^2\left[1 - \frac{2z}{r}\sin\theta + \frac{z^2}{r^2}\right]$$

and take the square root of each side we get:

$$l = r\left[1 - \frac{2z}{r}\sin\theta + \frac{z^2}{r^2}\right]^{\frac{1}{2}}$$

We can simplify the square root making use of the fact:

$$(1+x)^p = 1 + px + \frac{p(p-1)}{2!} + \frac{p(p-1)(p-2)}{3!} + \cdots$$

and keeping only the first term for $p = \frac{1}{2}$:

$$(1+x)^{\frac{1}{2}} \cong 1 + \frac{1}{2}x$$

Applying this to the above expression yields:

$$l \cong r\left[1 + \frac{1}{2}\left(-\frac{2z}{r}\sin\theta + \frac{z^2}{r^2}\right)\right]$$

Finally, making the assuming that $z << r$, we can drop the term $\frac{z^2}{r^2}$ to get

$$l \cong r - z\sin\theta$$

Field calculation

For an element of length dz at position z, the amplitude at the field position (r, θ) is:

$$dp = \frac{A}{L}\frac{1}{l}e^{-i(kl-\omega t)}dz$$

We obtain the total pressure at the field point (r, θ) due to the line array by integrating:

$$p = \frac{A}{L}\int_{-L/2}^{L/2}\frac{1}{l}e^{-i(kl-\omega t)}dz$$

but $l \cong r - z\sin\theta$, so we can write:

$$p = \frac{A}{L}e^{-i(kr-\omega t)}\int_{-L/2}^{L/2}\frac{1}{r - z\sin\theta}e^{ikz\sin\theta}dz$$

Since we are assuming we are in the far field, $r >> z\sin\theta$, so we can replace $\frac{1}{r-z\sin\theta}$ with $\frac{1}{r}$ and move it outside the integral:

$$p = \frac{A}{rL}e^{-i(kr-\omega t)}\int_{-L/2}^{L/2}e^{ikz\sin\theta}dz$$

Next, we evaluate the integral:

$$p = \frac{A}{rL}e^{-i(kr-\omega t)}\left[\frac{e^{ikz\sin\theta}}{ik\sin\theta}\right]_{-L/2}^{L/2}$$

$$p = \frac{A}{rL}e^{-i(kr-\omega t)}\left[\frac{e^{\frac{1}{2}ikL\sin\theta} - e^{-\frac{1}{2}ikL\sin\theta}}{ik\sin\theta}\right]$$

Next move the term $\frac{1}{L}$ into the square brackets:

$$p = \frac{A}{r}e^{-i(kr-\omega t)}\left[\frac{e^{\frac{1}{2}ikL\sin\theta} - e^{-\frac{1}{2}ikL\sin\theta}}{ikL\sin\theta}\right]$$

and, using the fact that $\sin(x) = \frac{e^{ix}-e^{-ix}}{2i}$, we can write:

$$p = \frac{A}{r}e^{-i(kr-\omega t)}\left[\frac{\sin(\frac{1}{2}kL\sin\theta)}{\frac{1}{2}kL\sin\theta}\right]$$

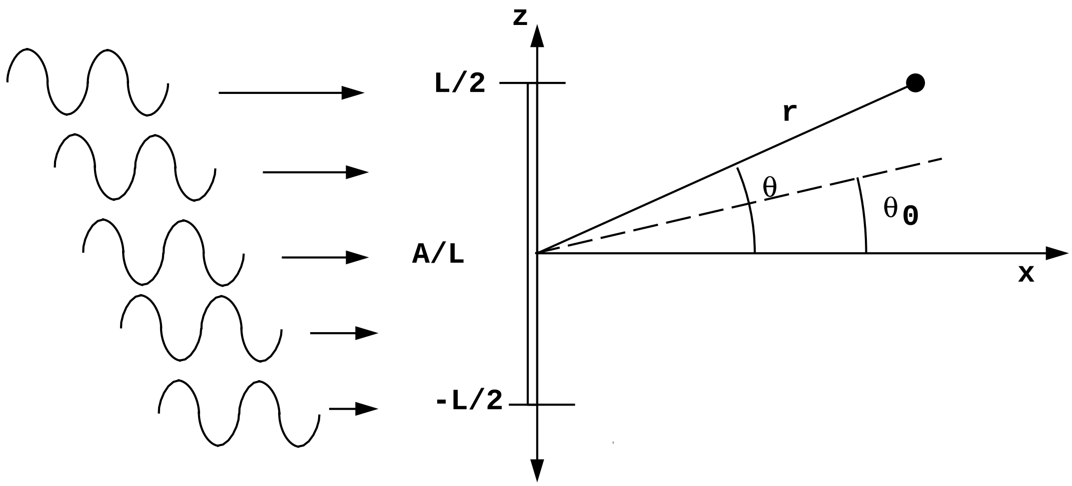
which is the pressure at (r, θ) due to the line array. The square of the term in brackets is defined as the *beam pattern* $b(\theta)$ of the array:

$$b(\theta) = \left[\frac{\sin(\frac{1}{2}kL\sin\theta)}{\frac{1}{2}kL\sin\theta}\right]^2$$

Steered Line Array

Recall importance of phase:

- Spatial phase: $kz \sin\theta = \frac{2\pi}{\lambda} z \sin\theta$
- Temporal phase: $\omega t = \frac{2\pi}{T} \ \ ; T = \frac{1}{f}$



$k \sin\theta$ = vertical wavenumber
$k \sin\theta_0$ = vertical wavenumber reference

To make a steered line array, we apply a linear phase shift $-zk \sin\theta_0$ to the excitation of the array:

$$dp = \frac{A/L}{r} e^{iz(k \sin\theta - k \sin\theta_0)} e^{i\omega t} dz \tag{1}$$

We can write

$$zk \sin\theta_0 = \omega z \frac{\sin\theta_0}{c}$$

$$zk \sin\theta_0 = \omega T_0(z) \ ; \ T_0(z) = \frac{z \sin\theta_0}{c}$$

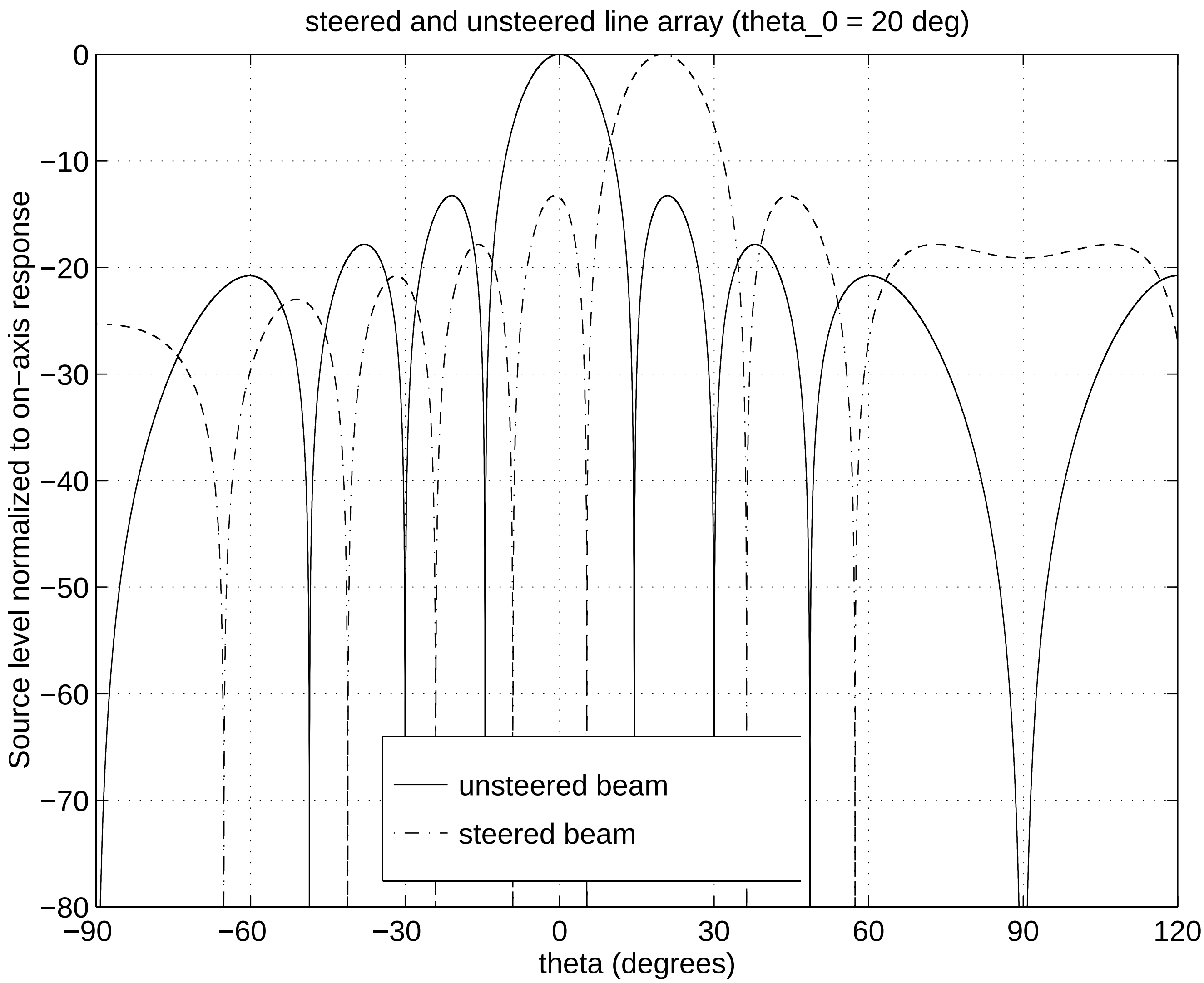
The phase term is equivalent to a *time delay* $T_0(z)$ that varies with position along the line array. We can re-write the phase term as follows.

$$e^{iz(k \sin\theta - k \sin\theta_0)} e^{i\omega t} = e^{ikz \sin\theta} e^{-i\omega(t + T_0(z))}$$

integrating Equation 1 yields:

$$p = \frac{A}{r} e^{-i(kr - \omega t)} \left[\frac{\sin(\frac{kL}{2}[\sin\theta - \sin\theta_0])}{(\frac{kL}{2}[\sin\theta - \sin\theta_0])} \right]$$

The resulting beam pattern is a shifted version of the beampattern of the unsteered line array. The center of the main lobe of the response occurs at $\theta = \theta_0$ instead of $\theta = 0$.

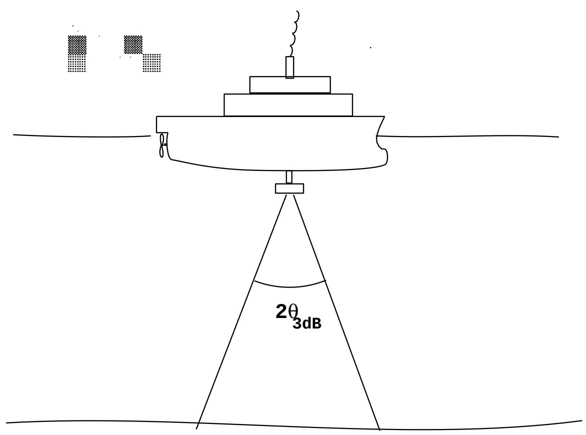


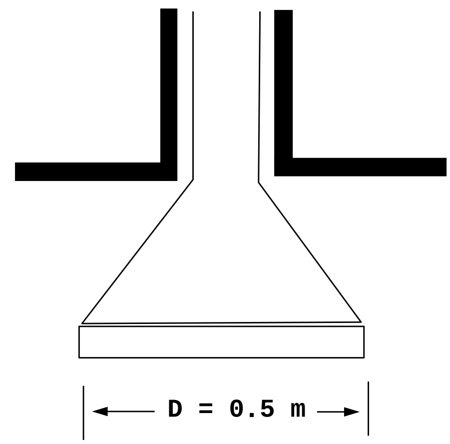
This plot shows the steered line array beam pattern

$$b(\theta) = \left[\frac{\sin(\frac{kL}{2}[\sin\theta - \sin\theta_0])}{(\frac{kL}{2}[\sin\theta - \sin\theta_0])} \right]^2$$

for $\theta_0 = 0$ and $\theta_0 = 20$ degrees.

Example 1: Acoustic Bathymetry





Given:

- $f = 12$ kHz
- Baffled disc transducer
- $D = 0.5$ m
- Acoustic power $\mathcal{P} = 2.4$ W

Compute:

- $\lambda =$ ______________
- DI = ______________
- SL = ______________
- $\theta_{3dB} =$ ______________

Spatial resolution, ϵ

$$\epsilon = 2d \tan \theta_{3dB}$$

$$= d \ \cdot \ \_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Depth resolution, δ

$$T_F = \frac{2d}{c} \text{ (earliest arrival time)}$$

$$T_L = \frac{2r}{c} \text{ (latest arrival time)}$$

$$\delta = (T_L - T_F) \cdot c/2$$

$$= d\left(\frac{1}{\cos \theta_{3dB}} - 1\right)$$

$$= d \cdot \_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

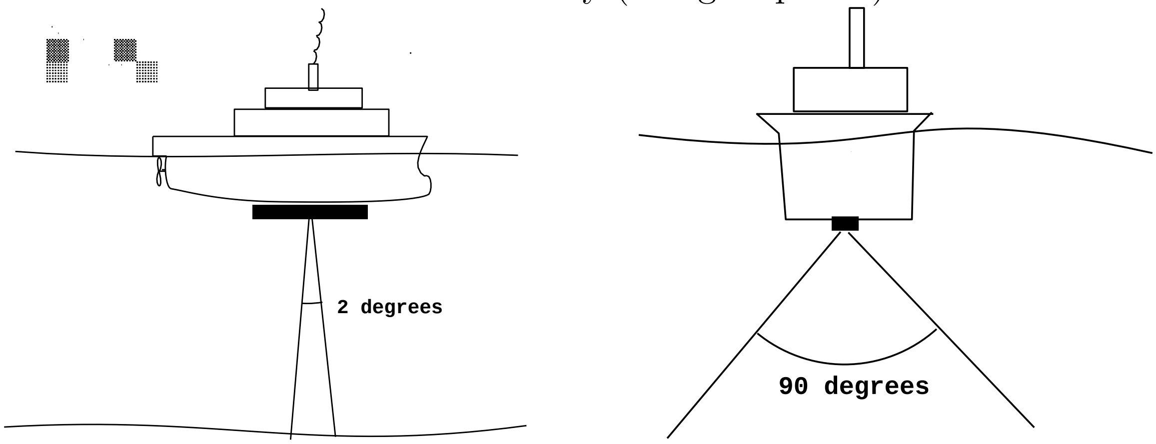
For $d = 2$ km

$\epsilon =$ ______________

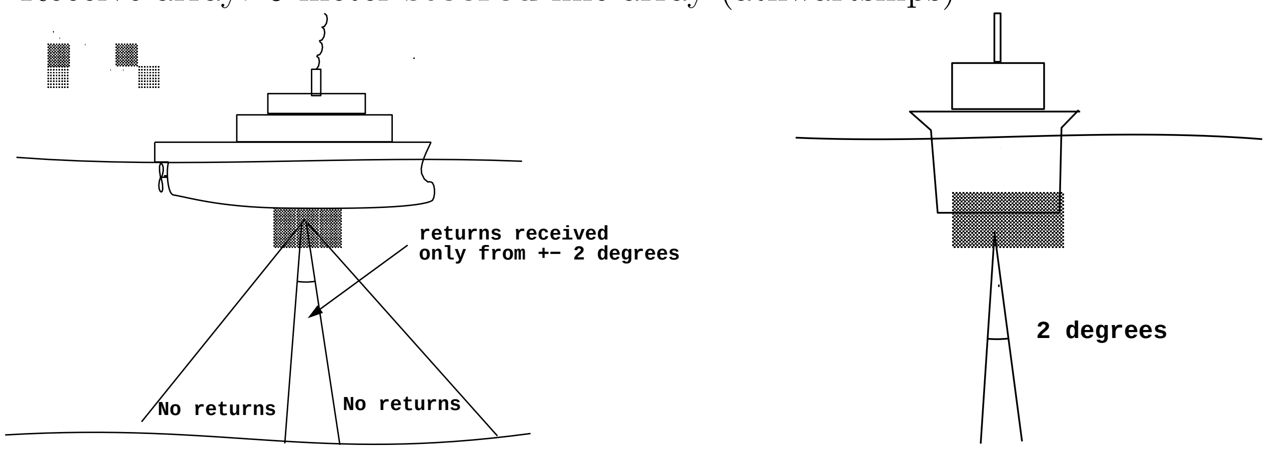
$\delta =$ ______________

Example 2: SeaBeam Swath Bathymetry

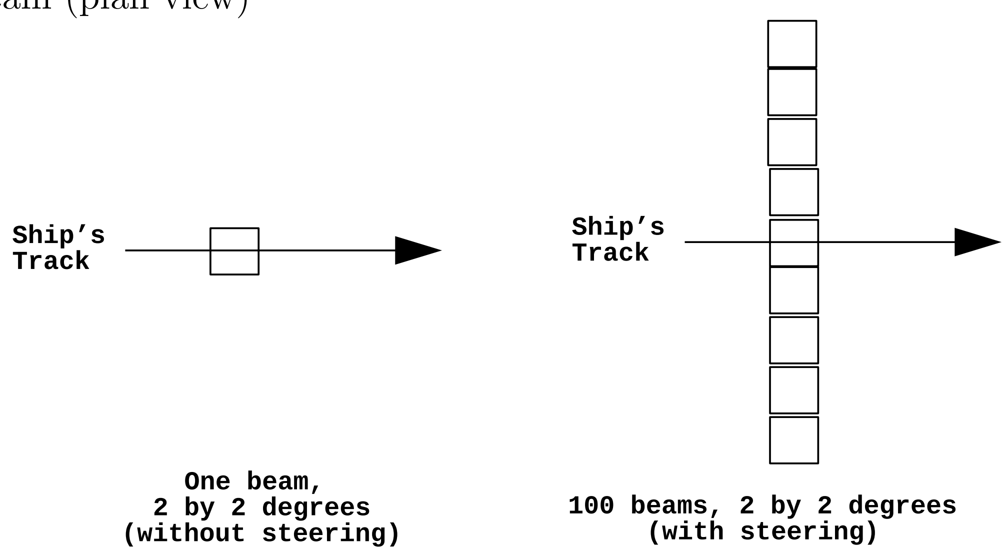
Transmit: 5 meter unsteered line array (along ship axis)



Receive array: 5 meter **steered** line array (athwartships)



Net beam (plan view)

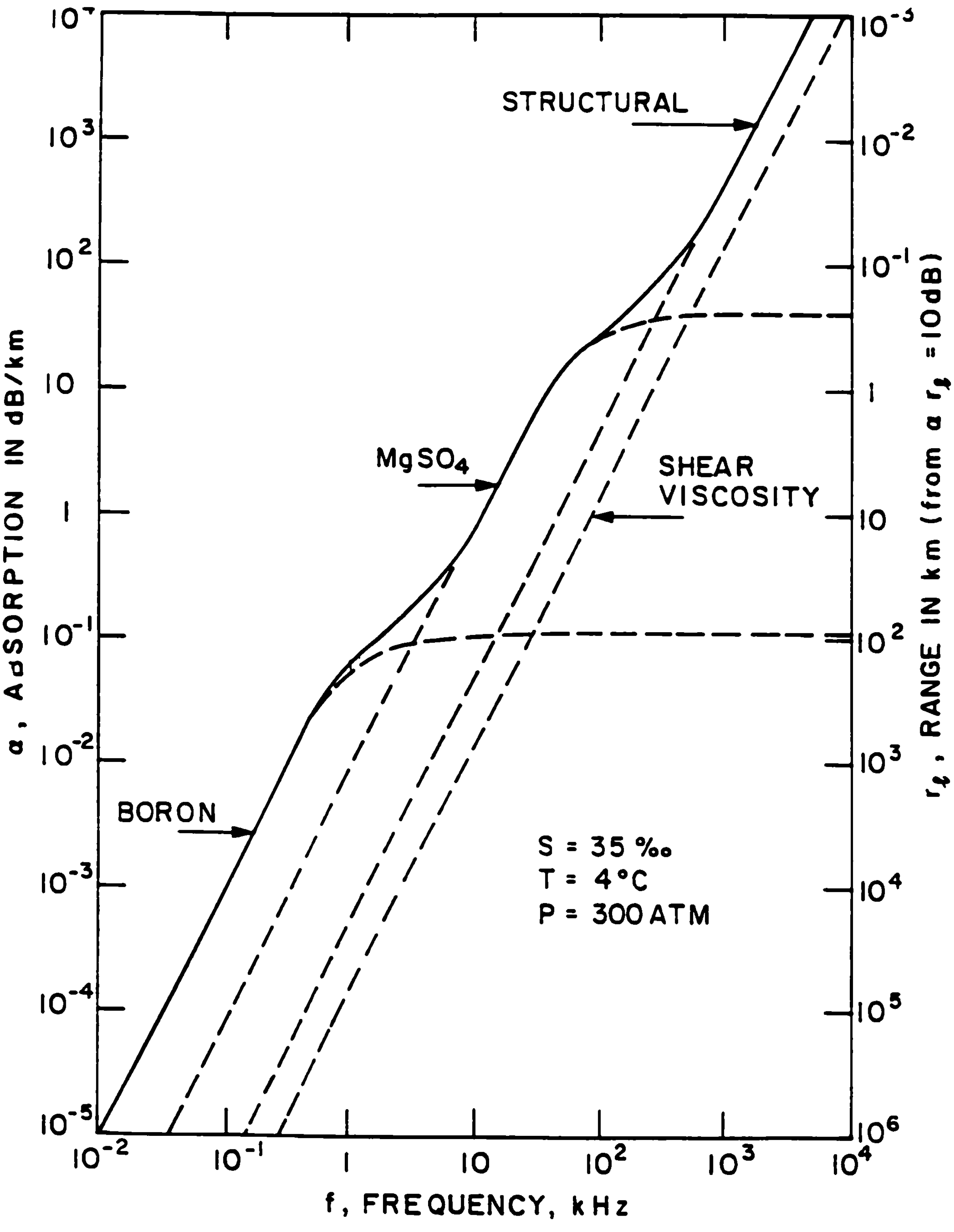


3 Propagation Part I: spreading and absorption

Table of values for absorption coefficent (alpha)

13.00 Fall, 1999 Acoustics: Table of attentuation coefficients

frequency [Hz]	alpha [dB/km]	frequency [Hz]	alpha [dB/km]
1	0.003	50000	15.9
10	0.003	60000	19.8
100	0.004	70000	23.2
200	0.007	80000	26.2
300	0.012	90000	28.9
400	0.018	100000 (100 kHz)	31.2
500	0.026	200000	47.4
600	0.033	300000	63.1
700	0.041	400000	83.1
800	0.048	500000	108
900	0.056	600000	139
1000 (1kHz)	0.063	700000	174
2000	0.12	800000	216
3000	0.18	900000	264
4000	0.26	1000000 (1 MHz)	315
5000	0.35	2000000	1140
6000	0.46	3000000	2520
7000	0.59	4000000	4440
8000	0.73	5000000	6920
9000	0.90	6000000	9940
10000 (10 kHz)	1.08	7000000	13520
20000	3.78	8000000	17640
30000	7.55	9000000	22320
40000	11.8	10000000 (10 MHz)	27540



Absorption of sound in sea water (from 13.851 class notes).

Absorption of sound in sea water

Relaxation mechanism

(conversion of acoustic energy to heat)

Four mechanisms

- shear viscosity ($\tau \approx 10^{-12}$ sec)
- structural viscosity ($\tau \approx 10^{-12}$ sec)
- magnesium sulfate — $MgSO_4$ ($\tau \approx 10^{-6}$ sec) [1.35 ppt]
- boric acid ($\tau \approx 10^{-4}$ sec) [4.6 ppm]

Relaxation time, τ

- if $\omega\tau << 1$, then little loss.
- if $\omega\tau \approx 1$ or greater, then generating heat (driving the fluid too fast).

The attenuation coefficient α depends on temperature, salinity, pressure and pH. The following formula for α in dB/km applies at T=4° C, S=35 ppt, pH = 8.0, and depth = 1000 m. (Urick page 108).

$$\alpha \approx 3.0 \times 10^{-3} + \frac{0.1f^2}{1+f^2} + \frac{44f^2}{4100+f^2} + 2.75 \times 10^{-4} f^2$$

Solving for range given transition loss TL

The equation

$$20 \log r + 10^{-3} \alpha r = TL$$

cannot be solved analytically. If absorption and spreading losses are comparable in magnitude, you have three options:

- In a "back-of-the-envelope" sonar design, one can obtain an initial estimate for the range by first ignoring absorption, then plugging in numbers with absorption until you get an answer that is "close enough".
- For a more systematic procedure, one can do Newton-Raphson iteration (either by hand or with a little computer program), using the range without absorption as the initial guess.
- Another good strategy (and a good way to check your results) is to make a plot of TL vs. range with a computer program (e.g., Matlab).

Newton-Raphson method: (Numerical recipes in C, page 362)

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

To solve for range given TL, we have:

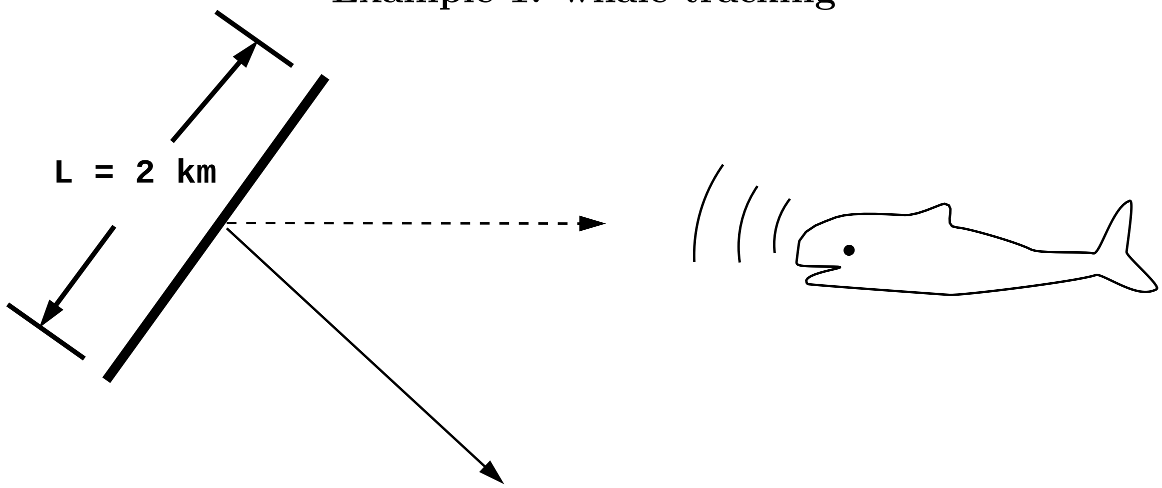
$$f(x_i) = TL - 20 \log x_i - 10^{-3} \alpha x_i$$

$$f'(x_i) = -\frac{20}{r} - 10^{-3} \alpha$$

$\alpha =$ ________ TL = ________ $x_0 =$ ________

i	x_i	20 $\log x_i$	$0.001 \alpha x_i$	$f(x_i)$	$f'(x_i)$	$f(x_i)/f'(x_i)$	$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$
0							
1							
2							
3							
4							

Example 1: whale tracking



Passive sonar equation: ________________________________

Given:

- $f_0 = 250$ Hz
- $\mathcal{P} = 1$ Watt (omni)
- line array: $L = 2$ km
- DT = 15 dB
- NL = 70 dB

Question: How far away can we hear the whale?

TL = ____________________ = ____________

$\lambda =$ ____________

DI = ____________

SL = ____________

$\mathrm{TL} = 20 \log r + \alpha * r * 10^{-3} =$ ____________

$\alpha =$ ____________

$R_t = \frac{8680}{\alpha} =$ ____________

$r =$ ________ (w/o absorption) $r =$ ________ (with absorption)

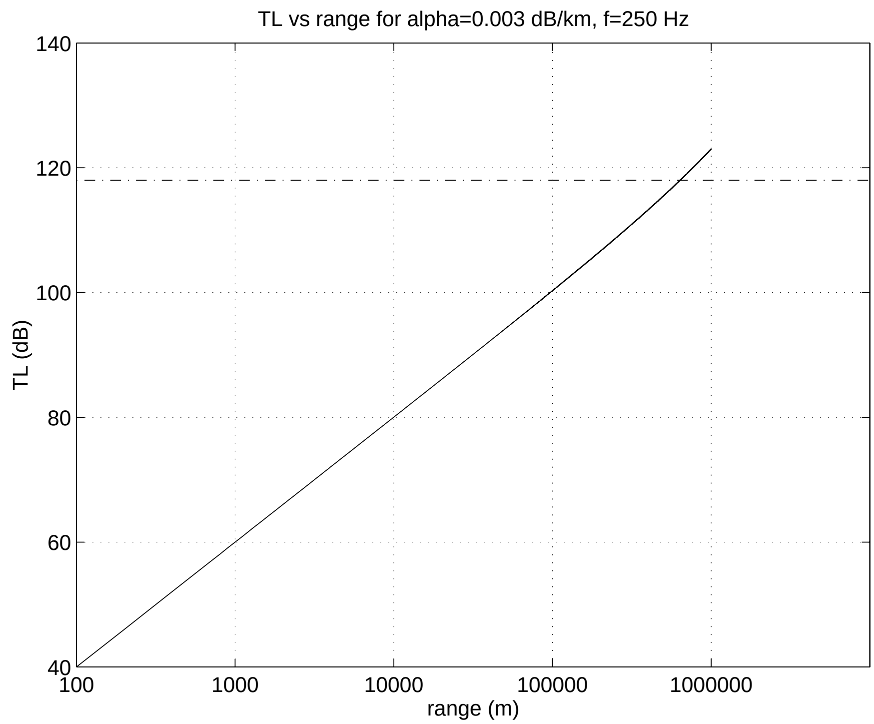


Figure 1: TL vs. range for whale tracking example (f=250 Hz, $\alpha = 0.003$ dB/km).

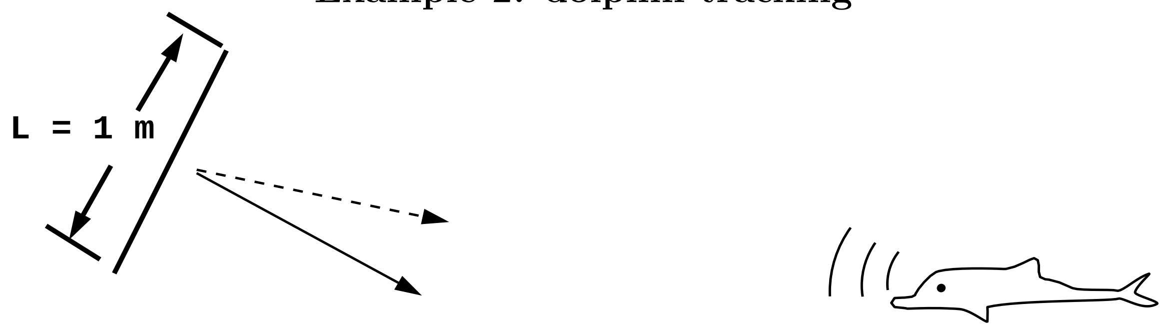
$$f(x_i) = TL - 20\log x_i - 10^{-3}\alpha x_i = \underline{\hspace{5cm}}$$

$$f'(x_i) = -\frac{20}{r} - 10^{-3}\alpha = \underline{\hspace{5cm}}$$

$\alpha =$ ________ TL = ________ $x_0 =$ ________

i	x_i	20 log x_i	$0.001\alpha x_i$	$f(x_i)$	$f'(x_i)$	$f(x_i)/f'(x_i)$	$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$
0	500,000	114	1.5	-1.5	-3.7×10^{-5}	40540	459500
1	459,500	13.2	1.38	-0.6	-4.05×10^{-5}	14805	444694
2	445,000	112.96	1.34	-0.3			
3							
4							

Example 2: dolphin tracking



Passive sonar equation: ________________________________

Given:

- $f_0 = 125$ kHz
- SL 220 dB re 1 μ Pa at 1 meter
- line array: $L = 1$ m
- DT = 15 dB
- NL = 70 dB

Question: How far away can we hear (detect) the dolphin?

TL = ____________________ = ____________

$\lambda =$ ____________

DI = ____________

$\mathrm{TL} = 20 \log r + \alpha * r * 10^{-3} =$ ____________

$\alpha =$ ____________

$R_t = \frac{8680}{\alpha} =$ ____________

$r =$ ________ (w/o absorption) $r =$ ________ (with absorption)

$r =$ ________ (w/o spreading)

$$f(x_i) = TL - 20 \log x_i - 10^{-3} \alpha x_i = \text{\_\_\_\_\_\_\_\_\_\_\_\_}$$

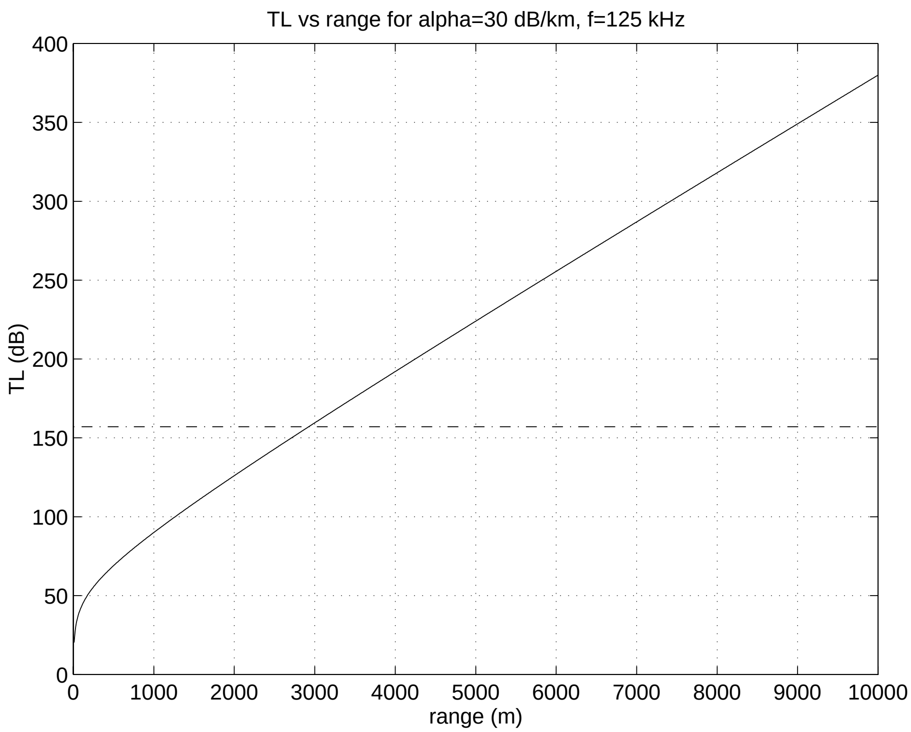


Figure 2: TL vs. range for dolphin tracking example (f=125 kHz, $\alpha = 30$ dB/km).

$$f'(x_i) = -\frac{20}{r} - 10^{-3}\alpha = \underline{\hspace{4cm}}$$

$\alpha =$ ________ TL = ________ $x_0 =$ ________

i	x_i	$20 \log x_i$	$0.001\alpha x_i$	$f(x_i)$	$f'(x_i)$	$f(x_i)/f'(x_i)$	$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$
0	5000	74	150	-67	-0.034	1970	3030
1	3030	69.6	90.9	-3.5	-0.037	95	2935
2	2935	69.35	88.05	-0.4	-0.368	10.9	2925
3	2925						
4							

4 Propagation Part II: refraction

In general, the sound speed c is determined by a complex relationship between salinity, temperature, and pressure:

$$c = f(S, T, D)$$

Medwin's formula is a useful approximation for c in seawater:

$$c = 1449.2 + 4.6T - 0.055T^2 + 0.00029T^3$$
$$+(1.34 - 0.010T)(S - 35) + 0.016D$$

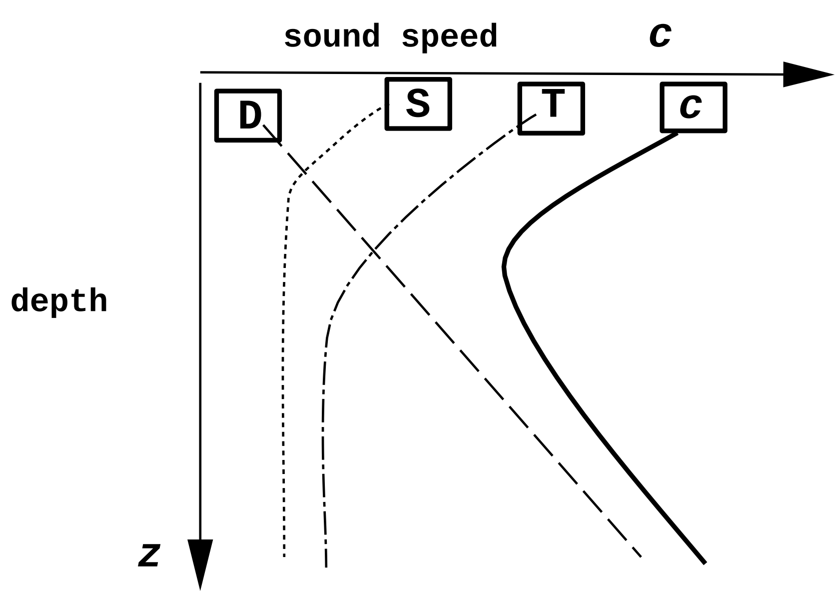
where S is the salinity in parts per thousand (ppt), T is the temperature in degrees Celsius, and D is the depth in meters. (See Ogilvie, Appendix A.)

Partial derivatives:

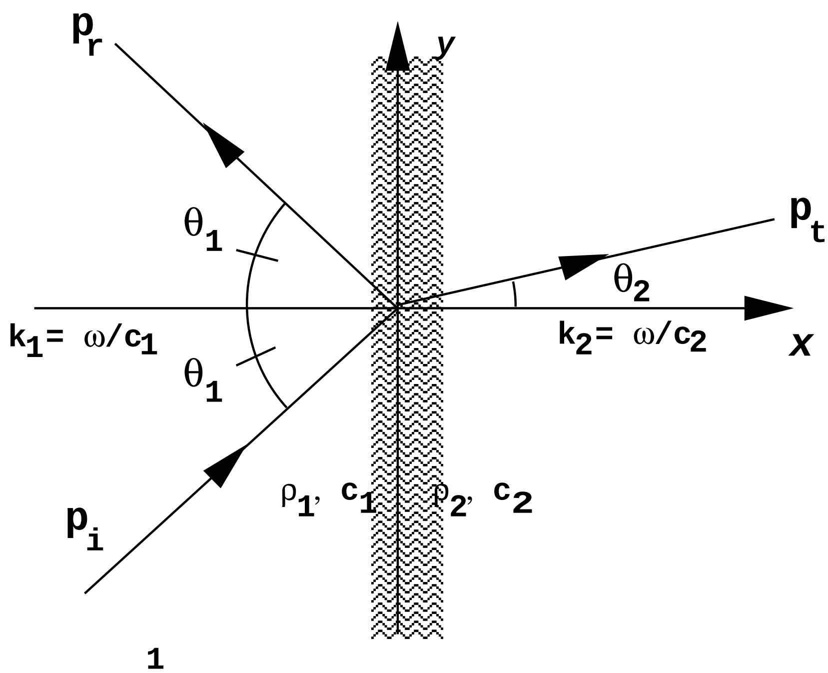
$$\frac{\partial c}{\partial T} = 4.6 \text{ m/sec/C}^\circ \qquad \frac{\partial c}{\partial S} = 1.34 \text{ m/sec/ppt} \qquad \frac{\partial c}{\partial D} = 0.016 \text{ m/sec/m}$$

For example:

- $\Delta T = 25^\circ \Longrightarrow \Delta c = 115$ m/sec
- $\Delta S = 5$ ppt $\Longrightarrow \Delta c = 6.5$ m/sec
- $\Delta D = 6000$ m $\Longrightarrow \Delta c = 96$ m/sec



Sound across an interface



$$p_1 = p_i + p_r = Ie^{-i(k_1 x \cos\theta_1 + k_1 y \sin\theta_1)} + Re^{-i(-k_1 x \cos\theta_1 + k_1 y \sin\theta_1)}$$

$$p_2 = p_t = Te^{-i(k_2 x \cos\theta_2 + k_2 y \sin\theta_2)}$$

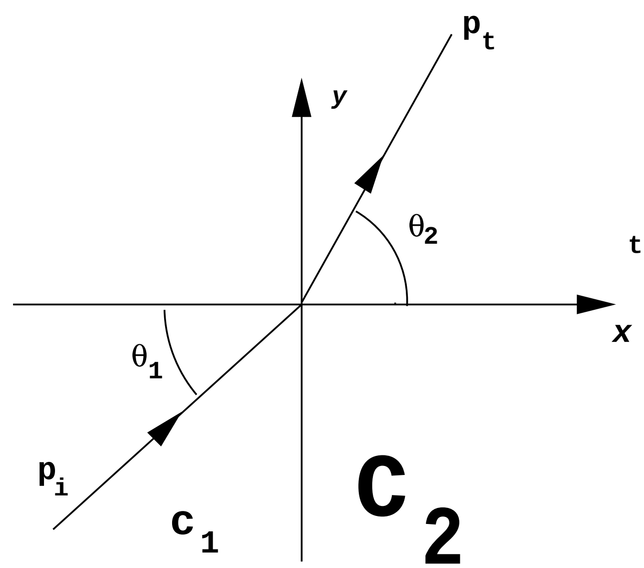
At $x = 0$, we require that $p_1 = p_2$ (continuity of pressure)

$$(I + R)e^{-ik_1 y \sin\theta_1} = Te^{-ik_2 y \sin\theta_2}$$

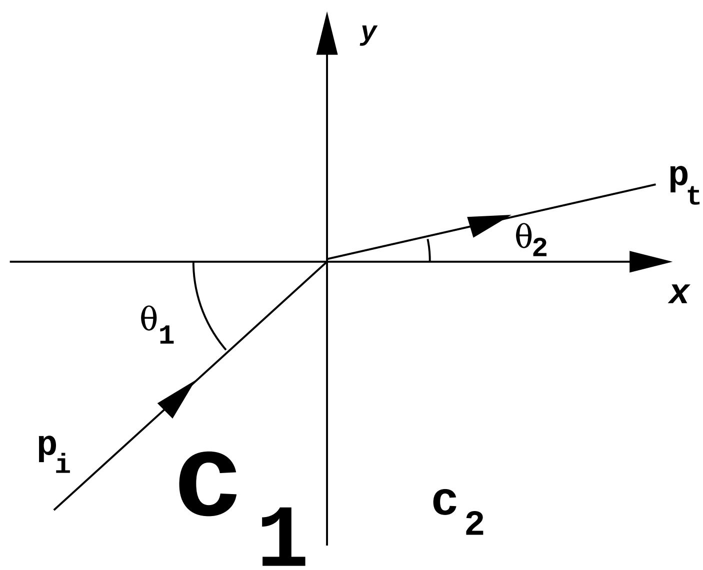
Match the phase to get **Snell's law**:

$$\frac{\sin\theta_1}{c_1} = \frac{\sin\theta_2}{c_2}$$

If $c_2 > c_1$, then $\theta_2 > \theta_1$

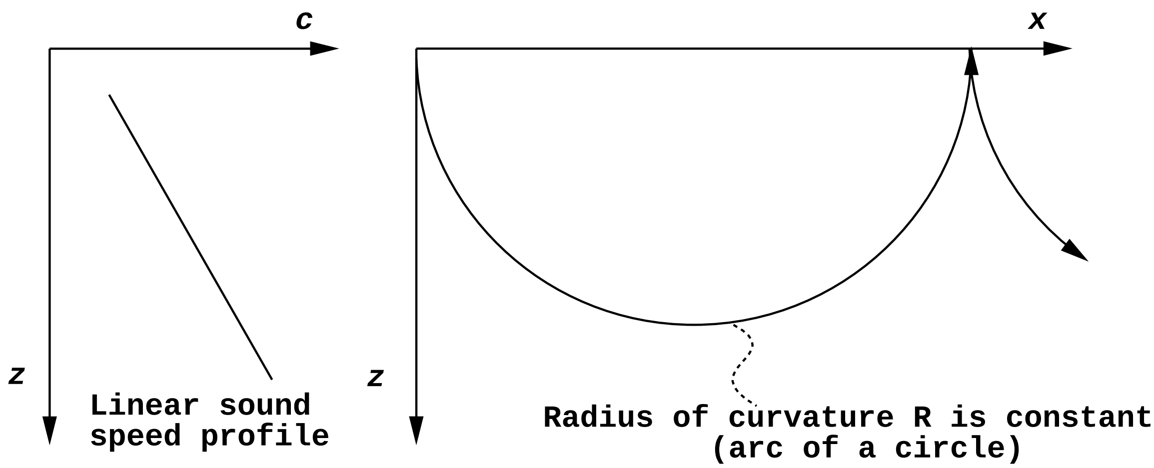


If $c_2 < c_1$, then $\theta_2 < \theta_1$



Sound bends *towards* region with *low* velocity

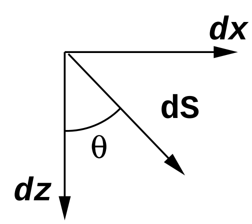
Sound bends *away* from region with *high* velocity



Goal: prove that the radius of curvature R is constant for a linear sound speed gradient.

Use the following:

1. Snell's law: $\frac{\sin\theta}{c} = \text{constant} = \sigma$ or $c = \frac{\sin\theta}{\sigma}$
   (σ is the horizontal slowness or ray parameter)
2. radius of curvature: $R = \frac{dS}{d\theta}$
3. gradient: $g = \frac{dc}{dz}$
4. $dz = dS\cos\theta$



First use Equation 4 in Equation 2:

$$R = \frac{dS}{d\theta} = \frac{dz}{d\theta} \cdot \frac{1}{\cos\theta}$$

$$R = \frac{dz}{dc}\frac{dc}{d\theta} \cdot \frac{1}{\cos\theta}$$

Then use Equation 3 for $\frac{dz}{dc}$:

$$R = \frac{1}{g\cos\theta} \cdot \frac{dc}{d\theta}$$

but from Equation 1 we can write:

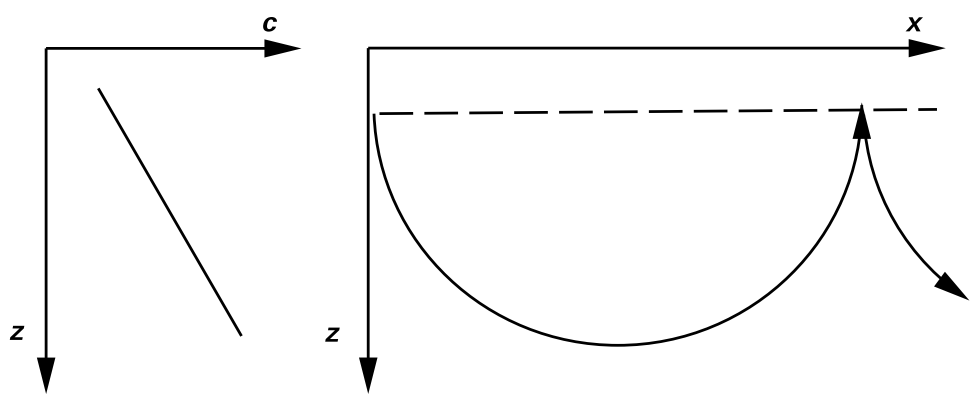
$$\frac{dc}{d\theta} = \frac{\cos\theta}{\sigma}$$

and so we can write:

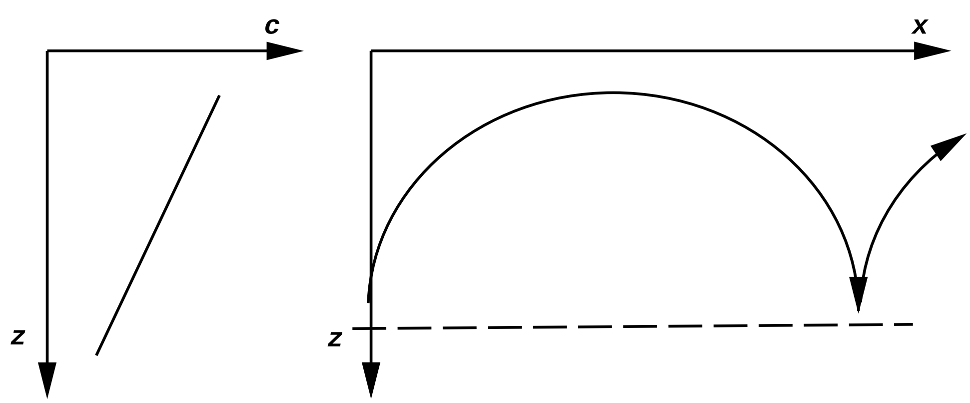
$$R = \frac{1}{g\sigma}$$

Hence, for linear sound speed gradient, radius of curvature is constant.

$$\Longrightarrow \textbf{ray paths are arcs of a circle}$$



For gradient $g > 0$, upward refraction occurs.

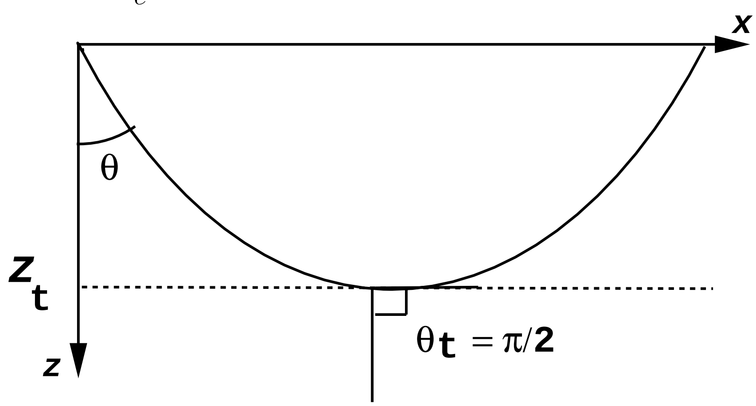


For gradient $g < 0$, downward refraction occurs.

Other forms for R

Since $\frac{\sin\theta}{c} = \sigma =$ constant for a ray, we can choose any known value.

For example, choose $\frac{\sin\theta}{c}$ at the turning point $z = z_t$.



$$\text{At } \theta = \frac{\pi}{2} \Longrightarrow \frac{\sin\theta(z_t)}{c(z_t)} = \frac{1}{c(z_t)} = \sigma$$

Then:

$$R = \frac{c(z_t)}{g}$$

But

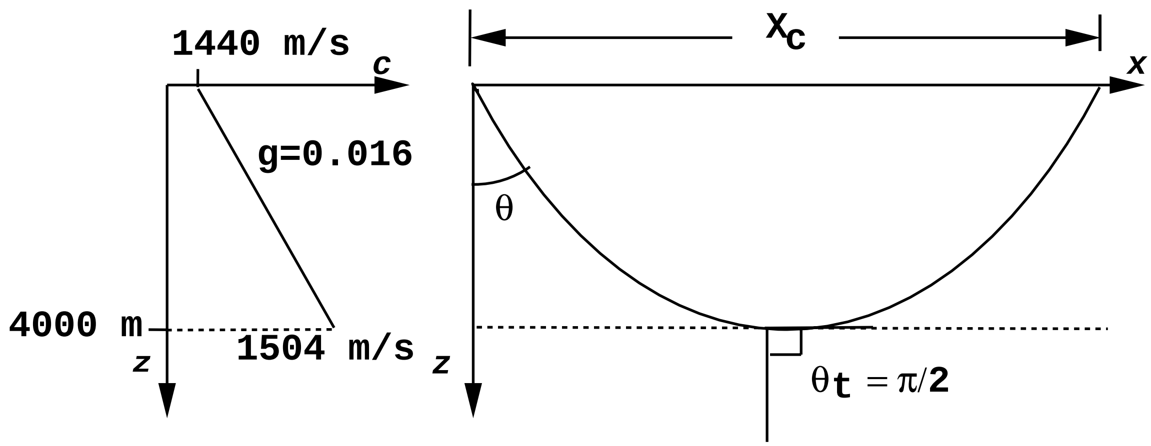
$$c(z_t) = c(z_0) + g(z_t - z_0)$$

So we can write:

$$R = \frac{c(z_0)}{g} + (z_t - z_0)$$

for z_0 any given depth.

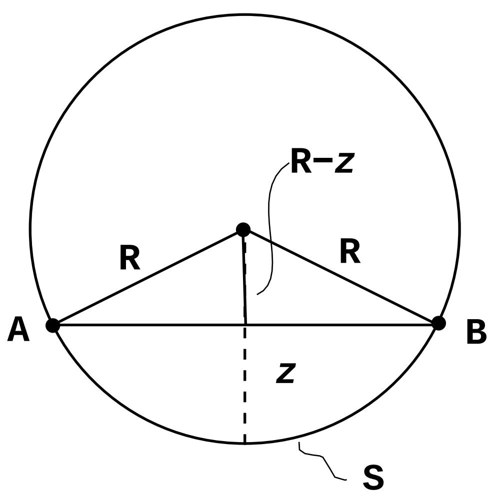
Example 1: Arctic propagation



At $z_t = 4000$ m, $\theta_t = \pi/2$, $c(z_t) = 1504$ m/s.

Then radius of curvature $R = \frac{c(z_t)}{g} =$ ________________

What is range to first turning point, X_c?



$$AB = 2\sqrt{2Rz - z^2}$$

$$S = 2R\cos^{-1}(\frac{R-z}{R})$$

Distance X_c is the chord:

$$X_c = 2\sqrt{2Rz - z^2} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Arc length S is the distance traveled by the ray:

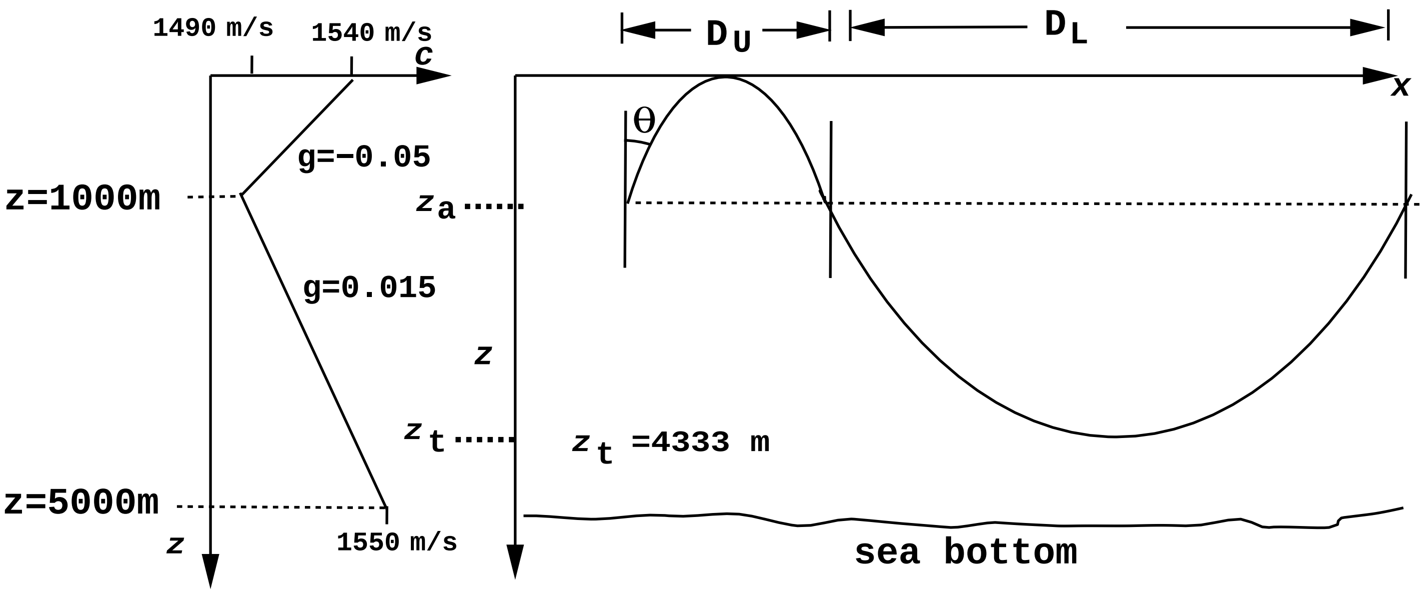
$$S = 2R\cos^{-1}\left(\frac{R-z}{R}\right) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

What is the launch angle $\theta(z_0)$ for this ray?

$$\frac{\sin(\theta(z_0))}{c(z_0)} = \frac{1}{c(z_t)}$$

$$\theta(z_0) = \sin^{-1}\left(\frac{c(z_0)}{c(z_t)}\right) = \underline{\hspace{6cm}}$$

Example 2: Bi-linear duct



Upper:

$$R_U = \frac{c(z_t)}{g} = \underline{\hspace{6cm}}$$

$$D_U = 2\sqrt{2R_U z_a - z_a^2} = \underline{\hspace{6cm}}$$

Lower:

$$R_l = \frac{c(z_t)}{g} = \underline{\hspace{6cm}}$$

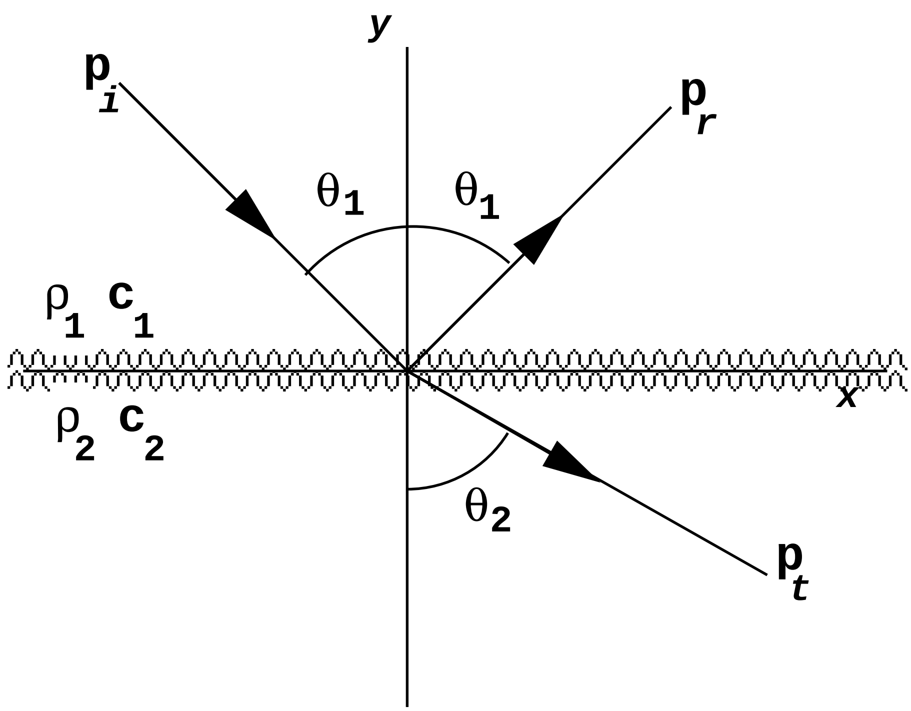
$$z_t = R_L - \frac{c(z_a)}{g} + z_a = \underline{\hspace{6cm}}$$

$$D_L = 2\sqrt{2R_L(z_t - z_a) - (z_t - z_a)^2} = \underline{\hspace{6cm}}$$

5 Reflection and target strength

Interface reflection

1



$$p_i = Ie^{i\omega t}e^{-i(k_1 x \sin\theta_1 - k_1 y \cos\theta_1)}$$

$$p_r = Re^{i\omega t}e^{-i(k_1 x \sin\theta_1 + k_1 y \cos\theta_1)}$$

$$p_t = Te^{i\omega t}e^{-i(k_2 x \sin\theta_2 - k_2 y \cos\theta_2)}$$

Boundary condition 1: continuity of pressure:

$$@\ \ y = 0 \qquad p_1 = p_i + p_r = p_t = p_2$$

Boundary condition 2: continuity of normal particle velocity:

$$@\ \ y = 0 \qquad v_1 = v_i + v_r = v_t = v_2$$

Momentum equation

$$\frac{\partial v}{\partial t} = -\frac{1}{\rho} \cdot \frac{\partial p}{\partial y}$$

$$\text{but} \quad \frac{\partial v}{\partial t} = i\omega v \quad (\text{since } v \propto e^{i\omega t})$$

$$\Longrightarrow v = \frac{i}{\omega\rho}\frac{\partial p}{\partial y}$$

Continuity of pressure:

$$p_i \left.\right|_{y=0} + p_r \left.\right|_{y=0} = p_t \left.\right|_{y=0}$$

$$(I + R)e^{-ik_1 x \sin\theta_1} = Te^{-ik_2 x \sin\theta_2}$$

$$(I + R)e^{-i\omega x \frac{\sin\theta_1}{c_1}} = Te^{-i\omega x \frac{\sin\theta_2}{c_2}}$$

And so using Snell's law, we can write:

$$I + R = T \tag{2}$$

Continuity of normal velocity:

$$v_i = \frac{i}{\omega\rho_1} \cdot ik_1 \cos\theta_1 p_i = -\frac{\cos\theta_1}{\rho_1 c_1} p_i$$

$$v_r = \frac{\cos\theta_1}{\rho_1 c_1} p_r$$
$$v_t = -\frac{\cos\theta_2}{\rho_2 c_2} p_t$$
$$v_i \big|_{y=0} + v_r \big|_{y=0} = v_t \big|_{y=0}$$

$$\rho_2 c_2 \cos(\theta_1)(I - R) = \rho_1 c_1 \cos(\theta_2) T \tag{3}$$

We can solve Equations 1 and 2 to get the reflection and transmission coeficients $\mathcal{R}$ and $\mathcal{T}$:

$$\mathcal{R} = \frac{R}{I} = \frac{\rho_2 c_2 \cos\theta_1 - \rho_1 c_1 \cos\theta_2}{\rho_2 c_2 \cos\theta_1 + \rho_1 c_1 \cos\theta_2}$$

$$\mathcal{T} = \frac{T}{I} = \frac{2\rho_2 c_2 \cos\theta_1}{\rho_2 c_2 \cos\theta_1 + \rho_1 c_1 \cos\theta_2}$$

Recall that given θ_1, we can compute θ_2 with Snell's law:

$$\frac{\sin\theta_1}{c_1} = \frac{\sin\theta_2}{c_2}$$

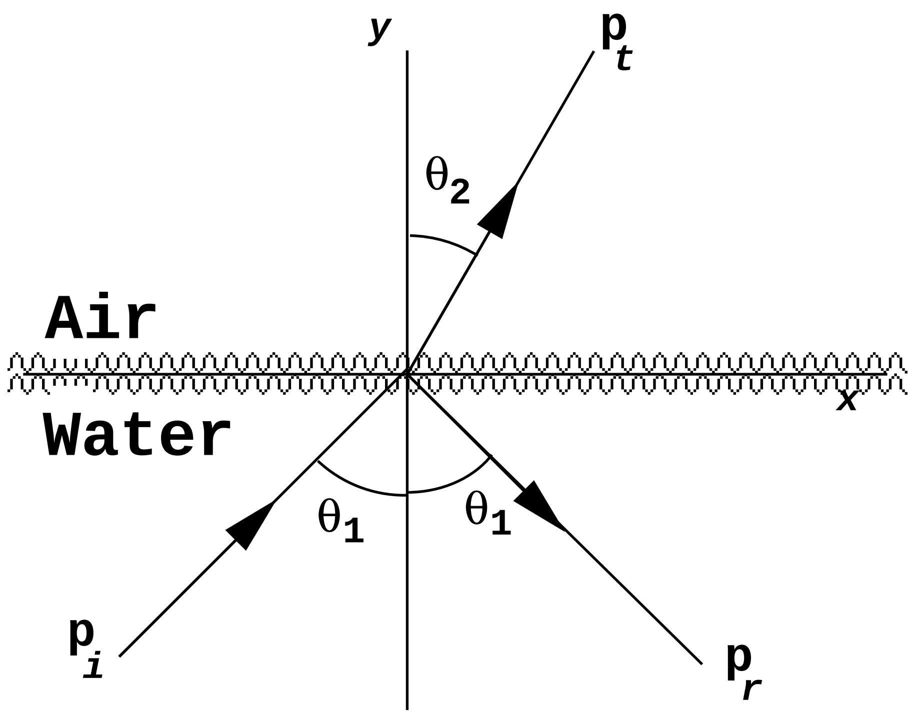
Special case: $\theta = 0$ (normal incidence)

$$\mathcal{R} = \frac{\rho_2 c_2 - \rho_1 c_1}{\rho_2 c_2 + \rho_1 c_1} \equiv \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

$$\mathcal{T} = \frac{2\rho_2 c_2}{\rho_2 c_2 + \rho_1 c_1} \equiv \frac{2Z_2}{Z_2 + Z_1}$$

where $Z = \rho c$ is defined as the *acoustic impedance*.

Example 1: source in water

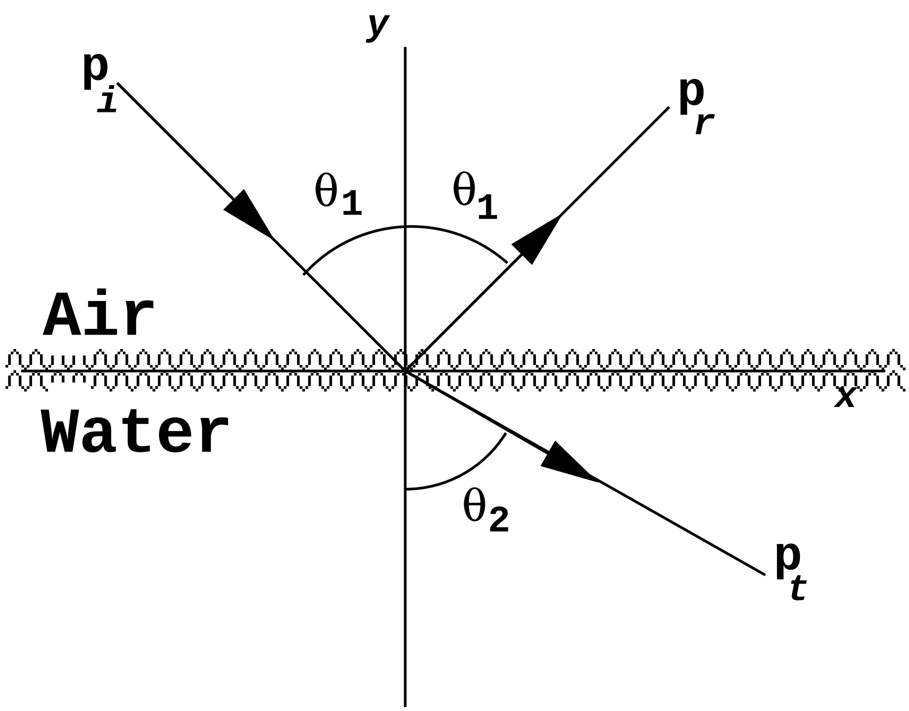


$\rho_a = 1.2$ kg/m^3, $c_a = 340$ m/sec $\Longrightarrow Z_a = 408$ kg/m^{2}s
$\rho_w = 1000$ kg/m^3, $c_w = 1500$ m/sec $\Longrightarrow Z_w = 1.5 \times 10^6$ kg/m^{2}s

$$\mathcal{R} = \frac{408\cos\theta_1 - 1.5\times 10^6\cos\theta_2}{408\cos\theta_1 + 1.5\times 10^6\cos\theta_2} \approx -1$$

$$\mathcal{T} = 1 + \mathcal{R} \approx 0 \quad \text{(no sound in air)}$$

Example 2: source in air



$$\mathcal{R} = \frac{1.5 \times 10^6 \cos\theta_1 - 408 \cos\theta_2}{1.5 \times 10^6 \cos\theta_1 + 408 \cos\theta_2} \approx +1$$

$$\mathcal{T} = 1 + \mathcal{R} \approx 2 \quad \text{(double sound in water!)}$$

Does this satisfy your intuition?

Consider intensity:

$$I_a = \frac{p_a^2}{\rho_a c_a} = \frac{p_a^2}{408}$$

$$I_w = \frac{p_w^2}{\rho_w c_w} = \frac{4p_a^2}{\rho_w c_w} = \frac{4p_a^2}{1.5 \times 10^6} = \frac{p_a^2}{3.75 \times 10^5}$$

$$\Longrightarrow I_w \approx \frac{1}{1000} I_a$$

Does this satisfy your intuition?

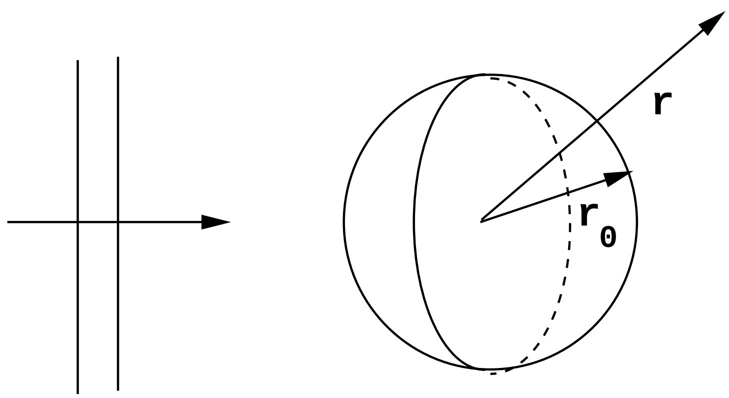
Target Strength

Assumptions:

- large targets (relative to wavelength)
- plane wave source
  - no angular variation in beam at target
  - curvature of wavefront is zero

Example: rigid or soft sphere

$$\text{TS} = 10 \log \frac{I_{\text{scat}}}{I_{\text{inc}}} \Big|_{r=r_{\text{ref}}}$$



$$I_{\text{inc}} = \frac{p^2}{\rho c}$$

$$\mathcal{P}_{\text{inc}} = \pi r_0^2 I_{\text{inc}}$$

Assume $\mathcal{P}_{\text{scat}} = \mathcal{P}_{\text{inc}}$ (omnidirectional scattering)

$$I_{\text{scat}} = \frac{\mathcal{P}_{\text{scat}}}{4\pi r^2} = \frac{\pi r_0^2 I_{\text{inc}}}{4\pi r^2}$$

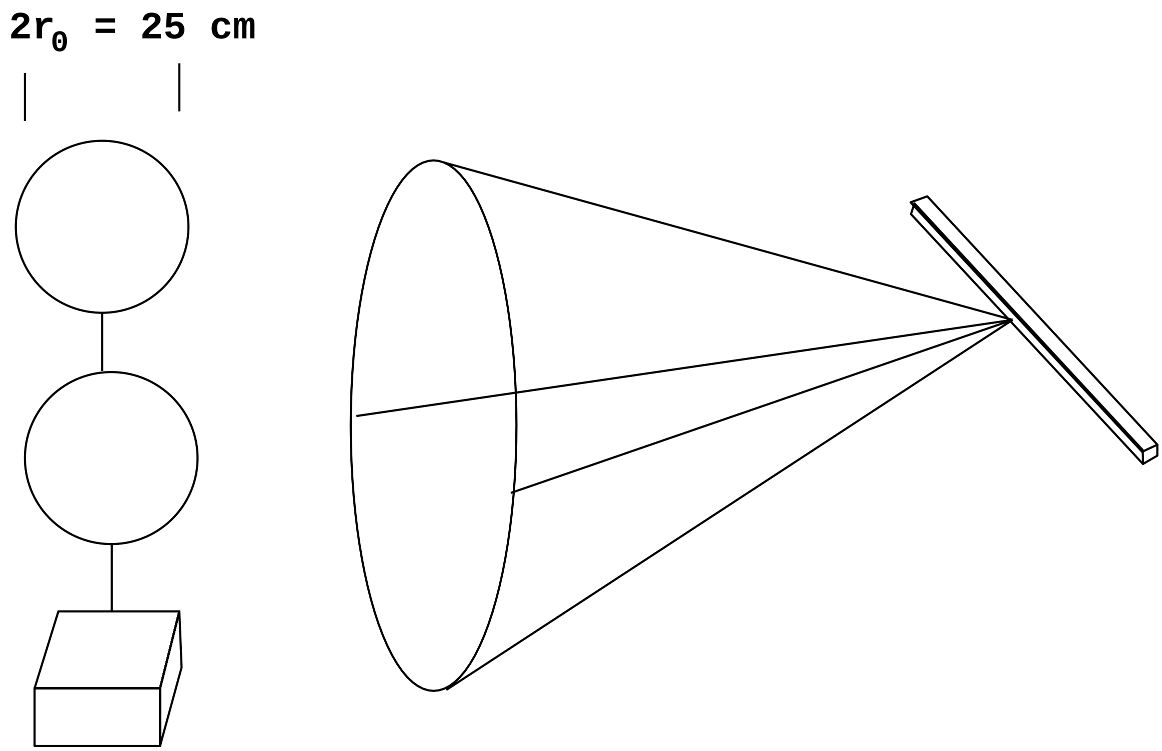
For $r = r_{\text{ref}} = 1$ meter:

$$\text{TS} = 10 \log \frac{r_0^2}{4} \qquad \text{(Assuming } r_0 >> \lambda\text{)}$$

If $r_0 = 2$ meters, then TS = 0 dB

6 Design problem: tracking neutrally buoyant floats

Sonar design problem



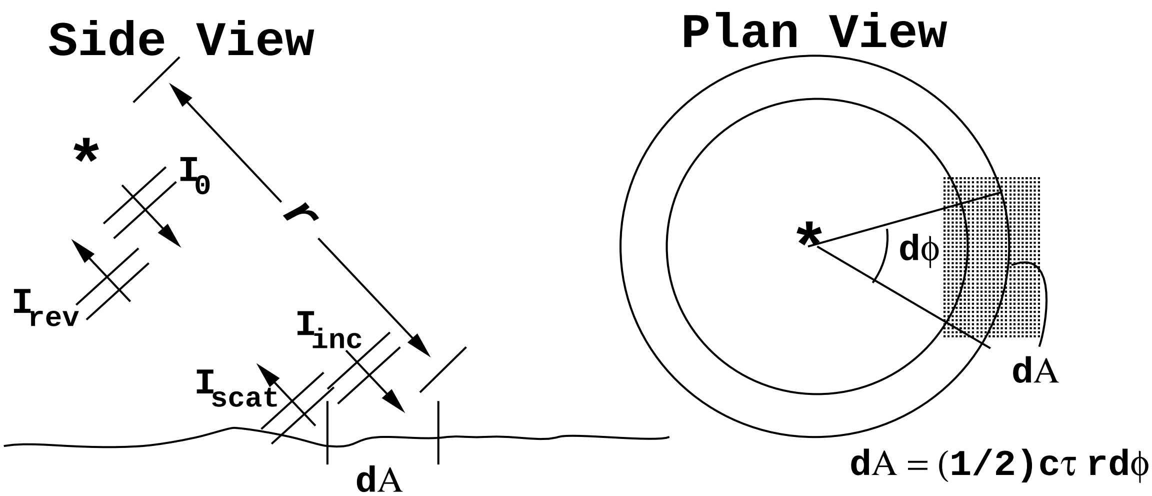
Require:

- Track to $\pm$ 3° bearing
- Range error: $\delta \pm 10$ meters
- Maximum range: R = 10 km
- Active sonar with DT = 15 dB
- sonar and float at sound channel axis
- baffled line array (source and receiver)
- Noise from sea surface waves (design for Sea State 6)

receiver DI:	$\mathrm{DI}_R =$	______________
pulse length:	$\tau =$	______________
array length:	$L =$	______________
source level:	$SL =$	______________
noise level:	$NL =$	______________
transmission loss:	$TL =$	______________
wavelength:	$\lambda =$	______________
source DI:	$\mathrm{DI}_T =$	______________
time-of-flight:	$T =$	______________
ping interval:	$T_p =$	______________
frequency:	$f =$	______________
target strength:	$TS =$	______________
range resolution:	$\delta =$	______________
acoustic power:	$\mathcal{P} =$	______________
average acoustic power:	$\overline{\mathcal{P}} =$	______________

7 Reverberation

Surface reverberation



$$I_{\text{rev}} = \int_A I_0 \cdot b(\theta,\phi) \cdot \frac{1}{r^2} \cdot \frac{I_{\text{scat}}}{I_{\text{inc}}} \cdot \frac{1}{r^2} \cdot b'(\theta,\phi) dA$$

Define the ratio $\frac{I_{\text{scat}}}{I_{\text{inc}}}$ as s_s

$$I_{\text{rev}} = \frac{I_0}{r^4} s_s \int_A b(\theta,\phi) b'(\theta,\phi) dA$$

From figure:

$$dA = \frac{c\tau}{2} r d\phi$$

consider only beampatterns of the form:

$$b(0,\phi) = b'(0,\phi) = \begin{cases} 1\ , & |\phi| \le \Phi/2 \\ 0\ , & |\phi| > \Phi/2 \end{cases}$$

$$I_{\text{rev}} = \frac{I_0}{r^4} s_s \frac{c\tau}{2} r\Phi$$

Define A as the insonified area $\frac{c\tau}{2} r \Phi$. Take logs to define the reverberation level RL_s:

$$\mathrm{RL}_s = \mathrm{SL} - 40 \log r + \mathcal{S}_s + 10 \log A \qquad \text{(No absorption)}$$

Example: 100 kHz side-scan sonar

- f = 100 kHz (α = 30 db/km)
- $\theta_{3dB-H} = 0.5°$
- $\theta_{3dB-V} = 30°$
- SL = 201 dB re 1 μ Pa @ 1 meter (rectangular array)
- DT = 15 dB
- NL = 35 dB
- $\tau = 0.1 \times 10^{-3}$ sec

Sonar equation:

$$\mathrm{SE} = \mathrm{SL} - 2\mathrm{TL} - (\mathrm{NL} - \mathrm{DI}_r) + \begin{pmatrix} \mathcal{S}_s + 10\log \mathrm{A} \\ \mathrm{TS} \end{pmatrix} - \mathrm{DT}$$

How much signal excess for rock bottom vs clay bottom at 300 meter range?

- λ = ______________
- L_x = ______________
- L_y = ______________
- DI_R = ______________
- 2 TL = ______________
- 10 log A = ______________

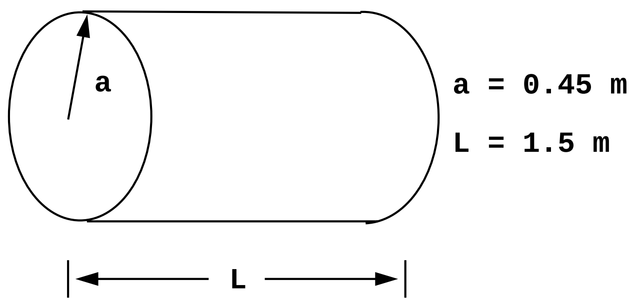
Rock: $\mathcal{S}_s$ = -20 dB $\Longrightarrow$ SE = ______________

Clay: $\mathcal{S}_s$ = -40 dB $\Longrightarrow$ SE = ______________

Compare the **echo level** of a steel drum to the **reverberation level** of rocks and clay at 300 m.

- EL = SL - 2TL + TS
- RL = SL - 2TL + $\mathcal{S}_S$ + 10 log A

Drum is finite cylinder $\Longrightarrow$ TS = 10 log $\frac{aL^2}{2\lambda}$ = ____________



EL = ____________

$\text{RL}|_{\text{rocks}}$ = ____________

$\text{RL}|_{\text{clay}}$ = ____________

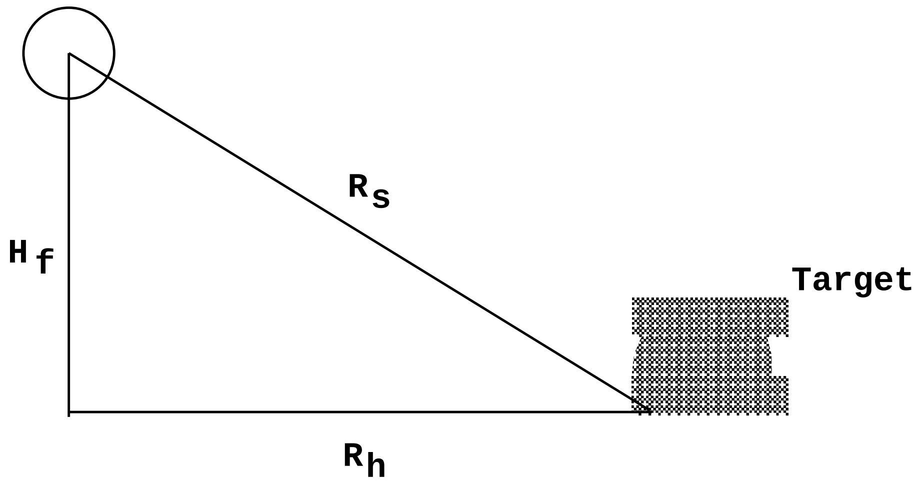
What if drum is in a boulder field with boulders of diameter 2 meters? (assume spherical boulders)

$\text{TS}|_{\text{boulder}} = 10 \log \frac{r^2}{4}$ = ____________

$\text{EL}|_{\text{drum}} - \text{EL}|_{\text{boulder}}$ = ____________

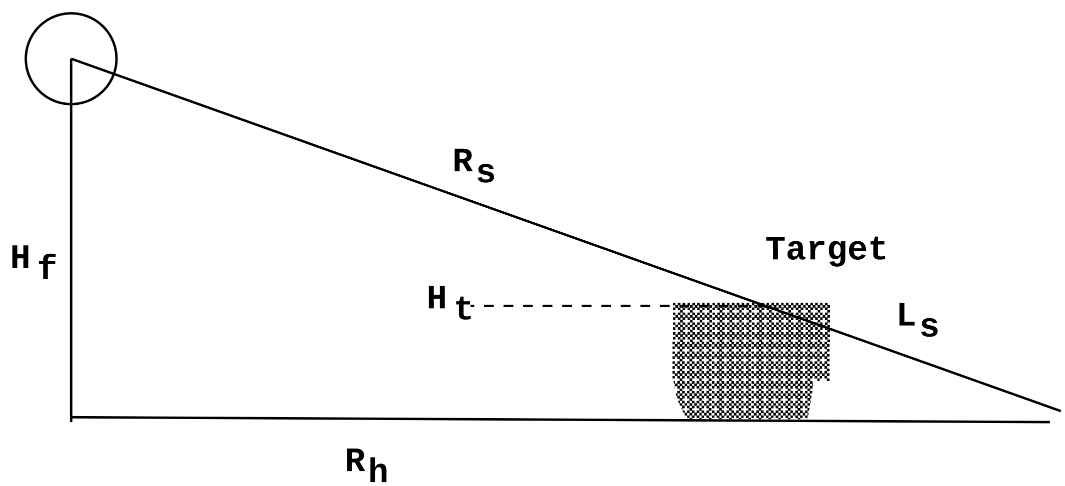
Side scan sonar analysis

Slant range correction:

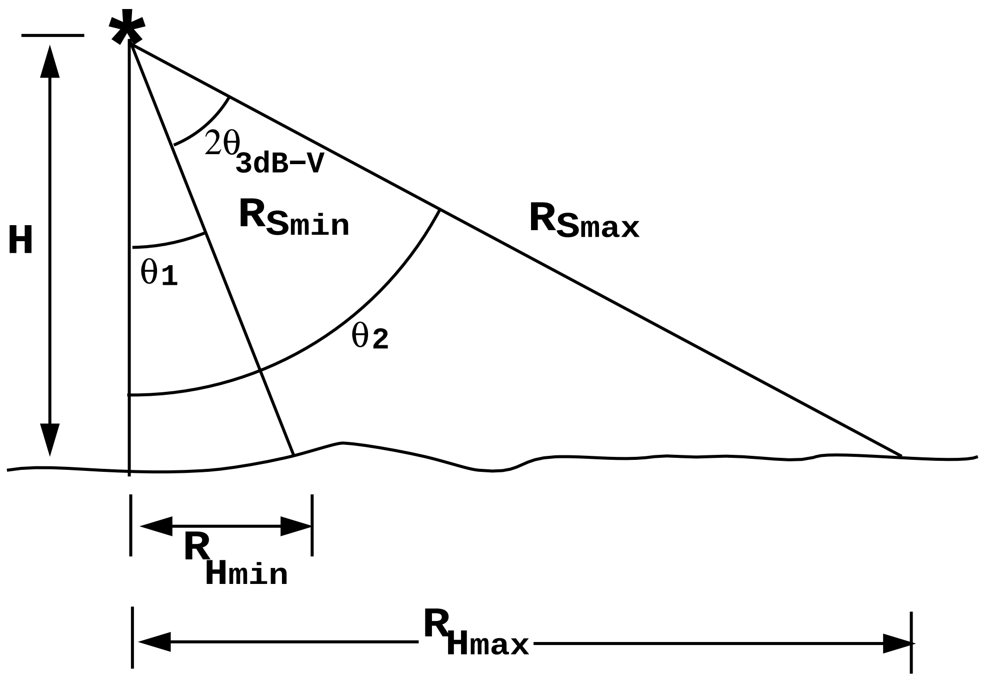


By Pythagorus: $R_h = \sqrt{R_s^2 - H_f^2}$

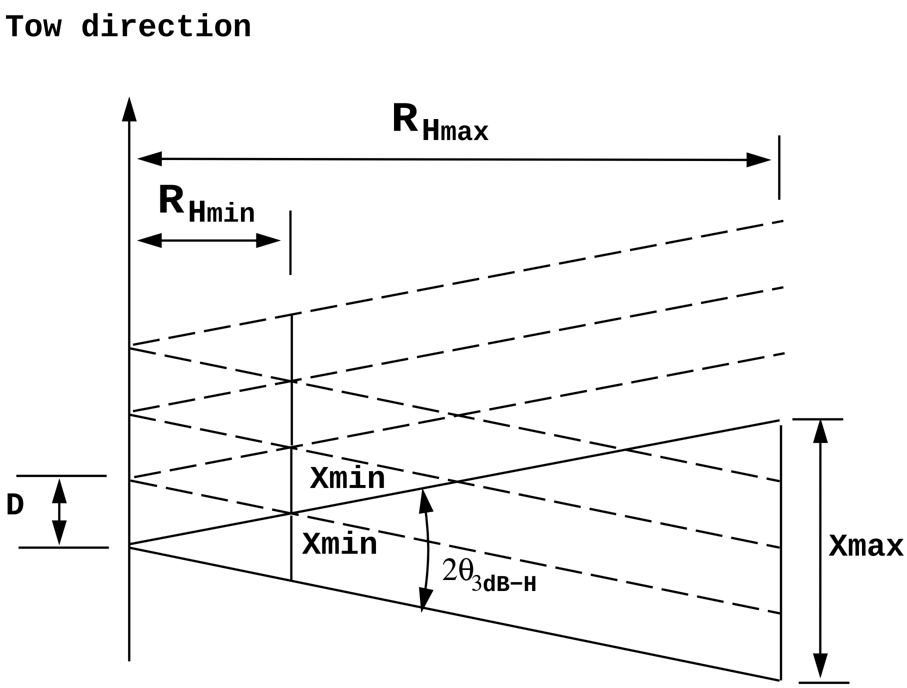
Object height:



By similar triangles: $H_t = \dfrac{L_s H_f}{R_s + L_s}$



Side view of sidescan geometry.



Plan view of sidescan geometry.

Summary of variable definitions for side-scan sonar.

- θ_1 is the angle from the vertical to the near edge of the fan beam in the vertical plane.
- θ_2 is the angle from the vertical to the far edge of the fan beam in the vertical plane.
$$\theta_2 = \theta_1 + 2 * \theta_{\text{3dB-V}}$$
- R_{Smax} is the maximum slant range.
$$R_{\text{Smax}} = H/\cos(\theta_2)$$
- R_{Smin} is the minimum slant range.
$$R_{\text{Smin}} = H/\cos(\theta_1)$$
- R_{Hmax} is the maximum horizontal range.
$$R_{\text{Hmax}} = H\tan(\theta_2)$$
- R_{Hmin} is the minimum horizontal range.
$$R_{\text{Hmin}} = H\tan(\theta_1)$$
- X_{max} is the along-track resolution at maximum range.
$$X_{\text{max}} = 2R_{\text{Hmax}} * \tan\theta_{\text{3dB-H}}$$
- X_{min} is the along-track resolution at minimum range
$$X_{\text{min}} = 2R_{\text{Hmin}} * \tan\theta_{\text{3dB-H}}$$
- T_f is the time-of-flight, which is the time required for the sonar ping to travel to maximum range and back.
$$T_f = 2 * R_{\text{Smax}}/c$$
- T_p is the pulse repetition interval, which is the time between pings. To avoid overlap of echos from one ping to the next:
$$T_p \geq T_f$$
- v is the velocity of the towfish.
- D is the distance traveled by the sonar towfish during one ping cycle. To avoid gaps in the sonar coverage:
$$D = v * T_p \leq X_{\text{min}}$$

8 Noise

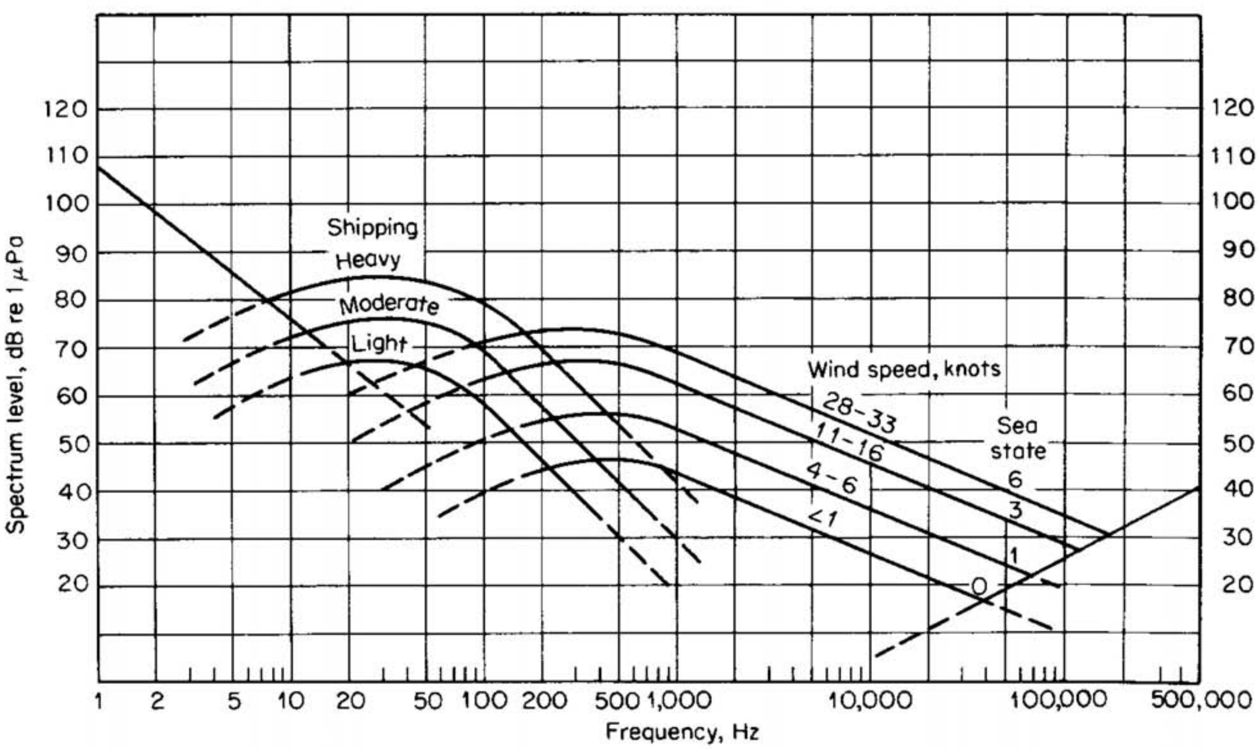
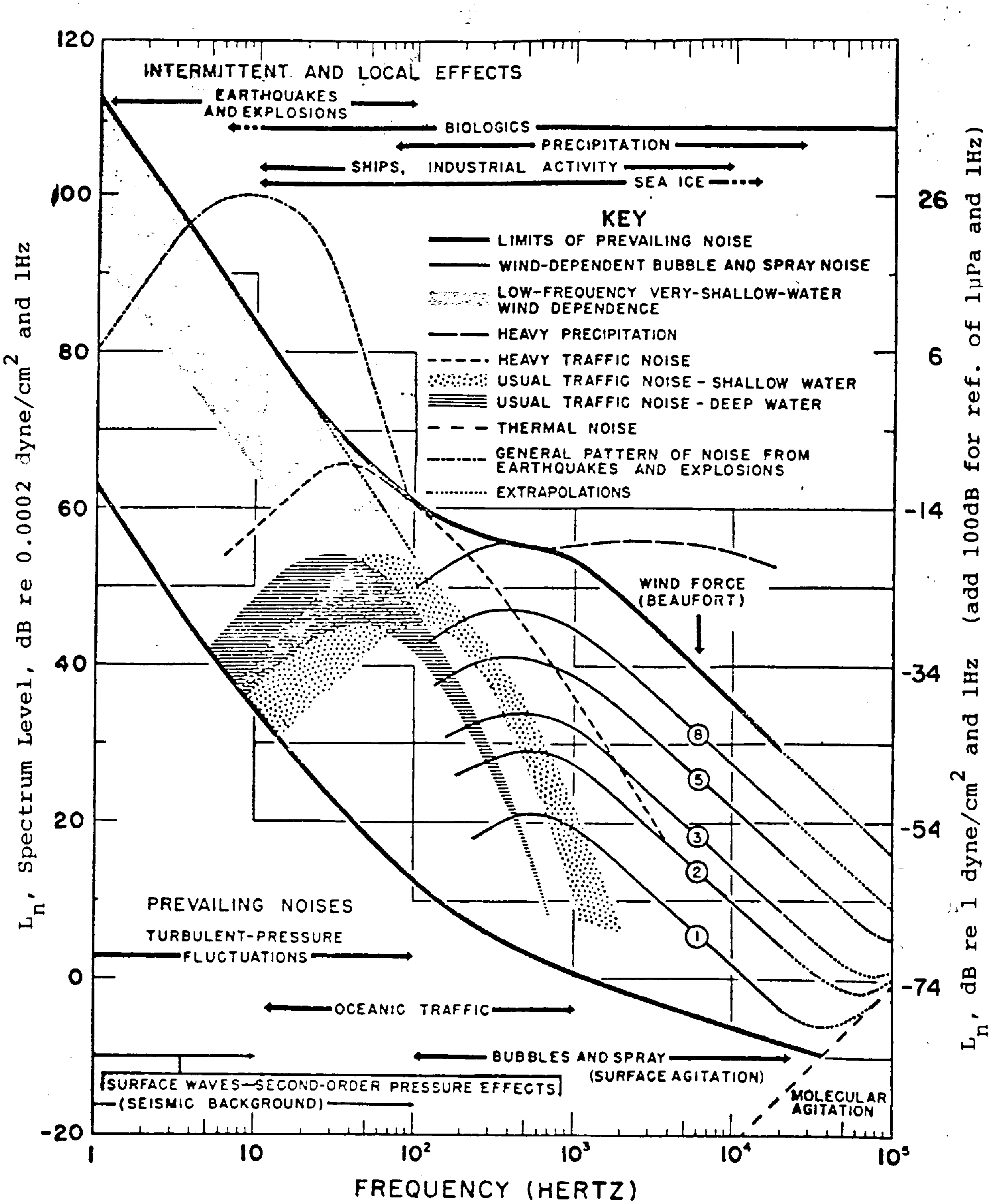


fig. 7.5 *Average deep-water ambient-noise spectra.*

Noise levels in the deep ocean (from Urick).



Sources of noise in the ocean (from Urick).

Five bands of noise:

I. $f < 1$ Hz: hydrostatic, seismic

II. $1 < f < 20$ Hz: oceanic turbulence

III. $20 < f < 500$ Hz: shipping

IV. 500 Hz $< f <$ 50 KHz: surface waves

V. 50 kHz $< f$: thermal noise

Band I: $f < 1$ Hz

- Tides $\quad f \approx 2$ cycles/day

$$p = \rho g H \approx 10^4 \cdot H \quad \text{Pa}$$

$$\text{noise level: NL } = 200\text{dB re } 1\ \mu\text{Pa} - 20 \log H$$

$$\text{example: 1 meter tide} \Longrightarrow \text{NL} = 200 \text{ dB re } 1\ \mu\text{Pa}$$

- microseisms $\quad f \approx \frac{1}{7}$ Hz

  On land, displacements are

$$\eta \approx 10^{-6}\text{meters}$$

  Assume harmonic motion

$$\eta \propto e^{i\omega t} \Longrightarrow v = \frac{d\eta}{dt} = i\omega\eta$$

  Noise power due to microseisms

$$p = \rho c v = i\omega\rho c\eta = i2\pi f \rho c\eta$$

$$|p| = 2\pi f \rho c\eta = 1.4 Pa \Longrightarrow \text{ NL} = 123 \text{ dB re } 1\ \mu\text{Pa}$$

Band II: Oceanic turbulence $1 \text{ Hz} < f < 20 \text{ Hz}$

Possible mechanisms:

- hydrophone self-noise (spurious)
- internal waves
- upwelling

Band III: Shipping $20 \text{ Hz} < f < 500 \text{ Hz}$

Example: $\approx$ 1100 ships in the North Atlantic

assume 25 Watts acoustic power each

$$\text{SL} = 171 + 10 \log \mathcal{P} = 215 \text{ dB re } 1\ \mu\text{Pa at } 1 \text{ meter}$$

Mechanisms

- Internal machinery noise (strong)
- Propeller cavitation (strong)
- Turbulence from wake (weak)

Band IV: surface waves $500 \text{ Hz} < f < 50 \text{ KHz}$

- Observations show NL is a function of local wind speed (sea state)
- Possible mechanisms:
  - breaking waves (only at high sea state)
  - wind flow noise (turbulence)
  - cavitation (100-1000 Hz)
  - long period waves

$$\omega = \sqrt{kg}$$
$$c_p = \frac{\omega}{k}$$
$$c_p^2 = \frac{g\lambda}{2\pi}$$

if $\lambda \approx 2000$ km, then $c_p \approx 1500$ m/sec $\Longrightarrow$ radiate noise!

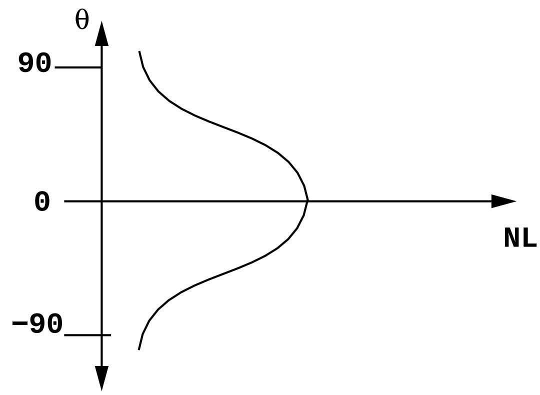
Band V: Thermal noise $50 \text{ KHz} < f$

$$\text{NL} = -15 + 20 \log f$$

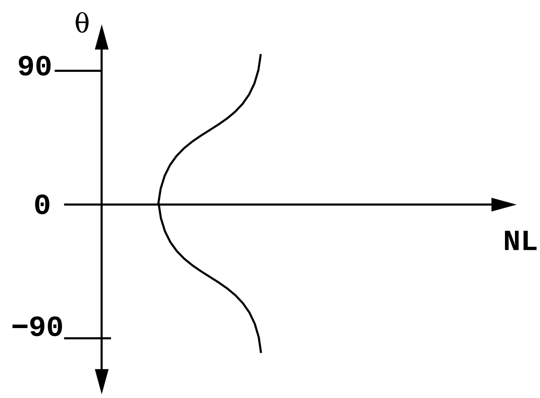
Directionality of noise

Vertical

- Low frequency
  - distant shipping dominates
  - low attenuation at horizontal



- High frequency
  - sea surface noise
  - local wind speed dominates
  - high attenuation at horizontal



Horizontal

- Low frequency: highest in direction of shipping centers
- High frequency: omnidriectional

9 Design Principles

Summary of important design principles

1. Required maximum range determines maximum frequency

$$\text{Dyer's rule}: \quad \alpha \times r \times 0.001 = 10 \text{ dB} \quad [\text{for r in meters}]$$

2. Required angular resolution determines array size

- $\theta_{3dB} = \pm\frac{25.3\lambda}{L}$ deg. (line array)
- $\theta_{3dB} = \pm\frac{29.5\lambda}{D}$ deg. (disc array)
- $\theta_{3dB} = \pm\frac{25.3\lambda}{L_x}, \pm\frac{25.3\lambda}{L_y}$ deg. (rec. array)

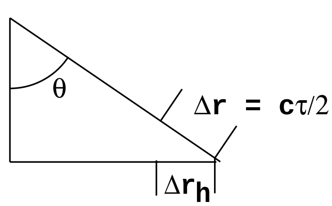
3. Required range resolution determines maximum pulse length

$$\Delta r = \frac{c\tau}{2}$$

4. In a side-scan sonar, the horizontal (across-track) range resolution Δr_h is determined by the beam geometry:

$$\Delta r_h = \Delta r \sin\theta$$

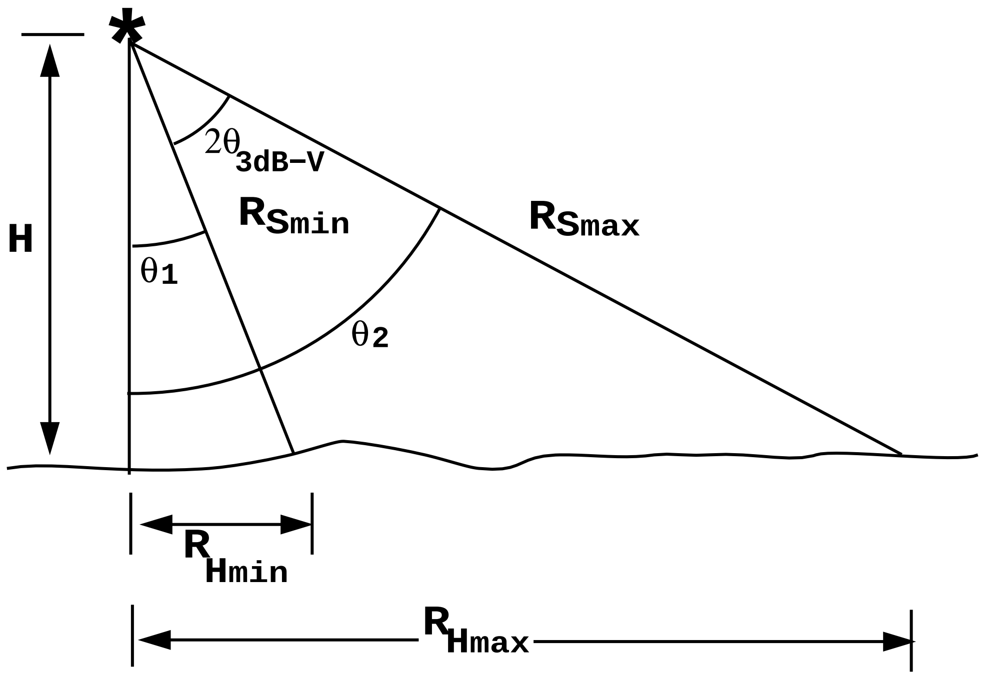
where θ is the angle between a ray drawn at the maximum range and the vertical.



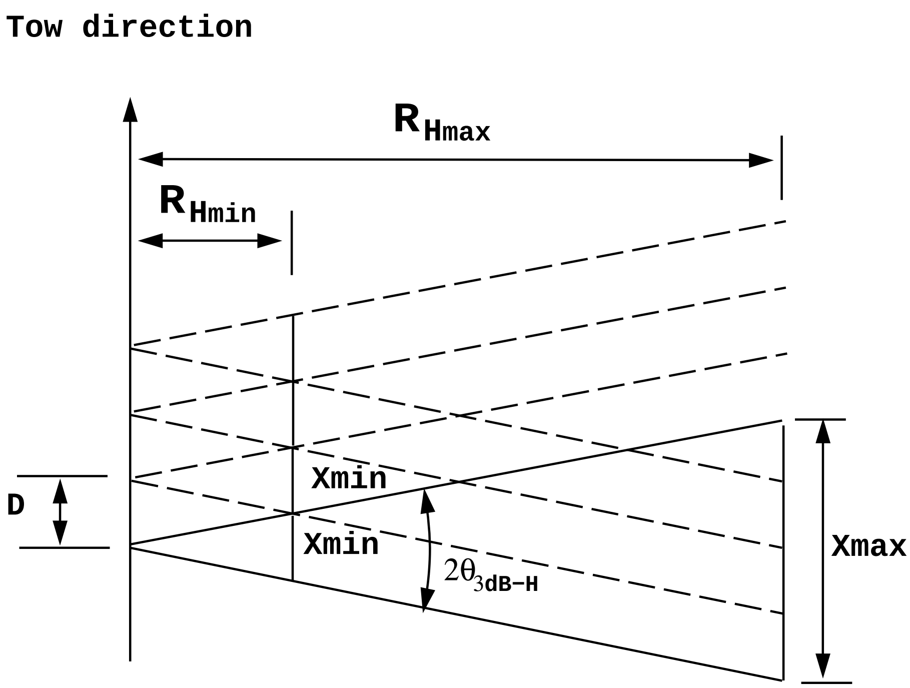
5. Characteristics of the transducer determine minimum pulse length.

- For a narrowband transducer, you must have at least 10 to 15 cycles of the carrier frequency.

10 Sidescan sonar



Side view of sidescan geometry.



Plan view of sidescan geometry.

Summary of variable definitions for side-scan sonar.

- θ_1 is the angle from the vertical to the near edge of the fan beam in the vertical plane.
- θ_2 is the angle from the vertical to the far edge of the fan beam in the vertical plane.

$$\theta_2 = \theta_1 + 2 * \theta_{\text{3dB-V}}$$

- R_{Smax} is the maximum slant range.

$$R_{\text{Smax}} = H/\cos(\theta_2)$$

- R_{Smin} is the minimum slant range.

$$R_{\text{Smin}} = H/\cos(\theta_1)$$

- R_{Hmax} is the maximum horizontal range.

$$R_{\text{Hmax}} = H\tan(\theta_2)$$

- R_{Hmin} is the minimum horizontal range.

$$R_{\text{Hmin}} = H\tan(\theta_1)$$

- X_{max} is the along-track resolution at maximum range.

$$X_{\text{max}} = 2R_{\text{Hmax}} * \tan\theta_{\text{3dB-H}}$$

- X_{min} is the along-track resolution at minimum range

$$X_{\text{min}} = 2R_{\text{Hmin}} * \tan\theta_{\text{3dB-H}}$$

- T_f is the time-of-flight, which is the time required for the sonar ping to travel to maximum range and back.

$$T_f = 2 * R_{\text{Smax}}/c$$

- T_p is the pulse repetition interval, which is the time between pings. To avoid overlap of echos from one ping to the next:

$$T_p \geq T_f$$

- v is the velocity of the towfish.
- D is the distance traveled by the sonar towfish during one ping cycle. To avoid gaps in the sonar coverage:

$$D = v * T_p \leq X_{\text{min}}$$

11 Summary of important formulae

Source Level

- $\mathrm{SL} = 171 + 10\log\mathcal{P} + \mathrm{DI}$

Directivity Index

- $\mathrm{DI} = 10\log(\frac{2L}{\lambda})$ (line array)
- $\mathrm{DI} = 20\log(\frac{\pi D}{\lambda})$ (disc array)
- $\mathrm{DI} = 10\log(\frac{4\pi L_x L_y}{\lambda^2})$ (rectangular array)

3-dB Beamwidth θ_{3dB}

- $\theta_{3dB} = \pm\frac{25.3\lambda}{L}$ deg. (line array)
- $\theta_{3dB} = \pm\frac{29.5\lambda}{D}$ deg. (disc array)
- $\theta_{3dB} = \pm\frac{25.3\lambda}{L_x}, \pm\frac{25.3\lambda}{L_y}$ deg. (rectangular array)

Transmission loss, spherical spreading and absorption:

- $\mathrm{TL} = 20\log r + 10^{-3}\alpha r$

Transmission loss, cylindrical spreading and absorption:

- $\mathrm{TL} = 10\log r + 10^{-3}\alpha r$

Target strength of a sphere (r_0 = radius, assumes $r_0 >> \lambda$):

- $\mathrm{TS} = 10\log\frac{r_0^2}{4}$

Target strength of a cylinder: (at normal incidence, a = radius, L = length)

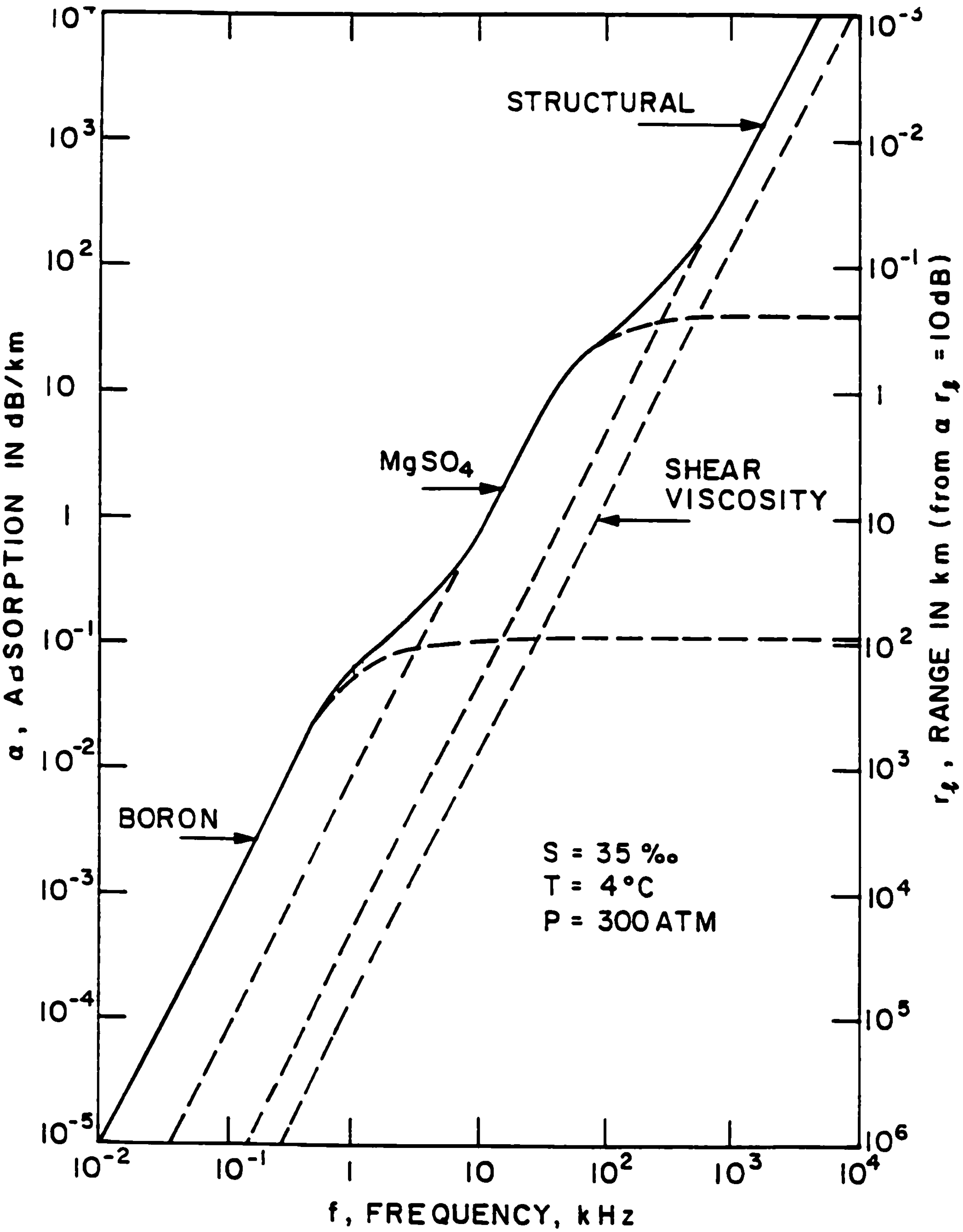
- $\mathrm{TS} = 10\log\frac{aL^2}{2\lambda}$

Rule-of-thumb for picking frequency given maximum range

- $\alpha \times r \times 0.001 = 10$ dB [for r in meters]

Range resolution

- $\Delta r = \frac{c\tau}{2}$



Absorption of sound in sea water (from Prof. Dyer's 13.851 class notes).

Table of values for absorption coefficent (alpha)

13.00 Fall, 1999 Acoustics: Table of attentuation coefficients

frequency [Hz]	alpha [dB/km]	frequency [Hz]	alpha [dB/km]
1	0.003	50000	15.9
10	0.003	60000	19.8
100	0.004	70000	23.2
200	0.007	80000	26.2
300	0.012	90000	28.9
400	0.018	100000 (100 kHz)	31.2
500	0.026	200000	47.4
600	0.033	300000	63.1
700	0.041	400000	83.1
800	0.048	500000	108
900	0.056	600000	139
1000 (1kHz)	0.063	700000	174
2000	0.12	800000	216
3000	0.18	900000	264
4000	0.26	1000000 (1 MHz)	315
5000	0.35	2000000	1140
6000	0.46	3000000	2520
7000	0.59	4000000	4440
8000	0.73	5000000	6920
9000	0.90	6000000	9940
10000 (10 kHz)	1.08	7000000	13520
20000	3.78	8000000	17640
30000	7.55	9000000	22320
40000	11.8	10000000 (10 MHz)	27540

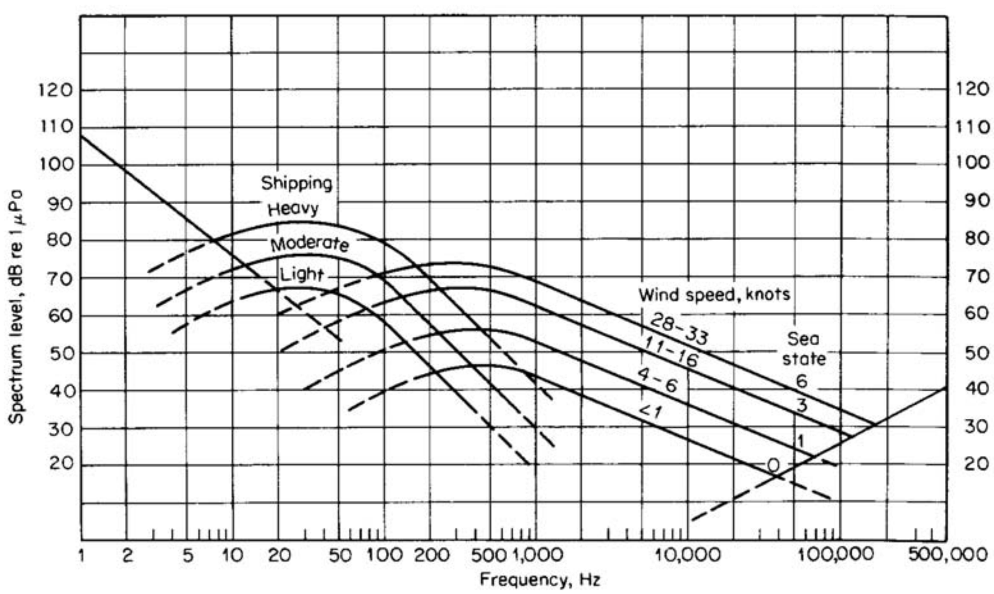


fig. 7.5 *Average deep-water ambient-noise spectra.*

Noise levels in the deep ocean (from Urick).