

# Navigation of autonomous vehicles

Kristijan Maček

Autonomous Systems Lab, EPF Lausanne

Department of Control and Computer  
Engineering in Automation, FER Zagreb

# Contents

---

- Introduction
- Lane detection vision module
- Off- and on-line path planning
- Obstacle avoidance
- Path following
- Conclusions

# Introduction - Navigation of autonomous vehicles

---

- Perception and sensing: extracting relevant information from the environment
- Map building and localization: environment information segmentation and determining of ego-position
- Motion planning: determining a feasible path from a start to a goal configuration
- Motion control: determining a sequence of motion commands for a given path

# Lane detection vision module

- “**SPARC**” - “Secure Propulsion Using Advanced Redundant Control”
- Final objective: a driver support system that would assist (warning mode) and control (correction mode) the driver in a vehicle
- Primary target: determine position of the ego vehicle on the road
- Basic approaches:
  - Specifically developed roads with beacons
  - Vision system in the vehicle performing road boundary detection



SECURE PROPULSION USING  
ADVANCED REDUNDANT CONTROL

# Vehicle setup

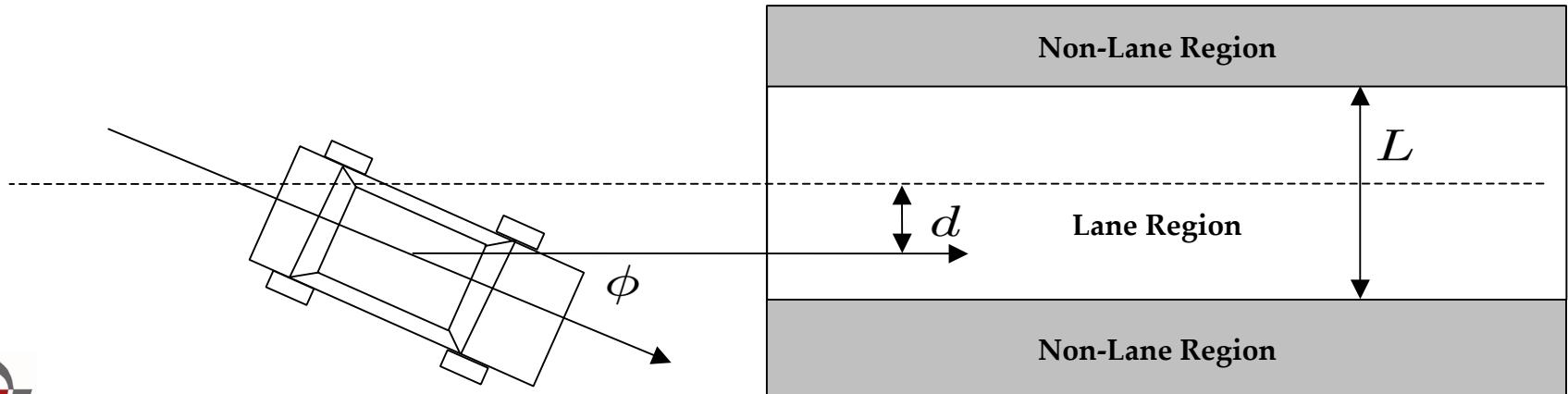
- Single digital camera positioned centrally
- Laser sensor for obstacle detection
- Firewire connected standard laptop with GenoM  
([Fleury *et al.*, 1997])



# System state description

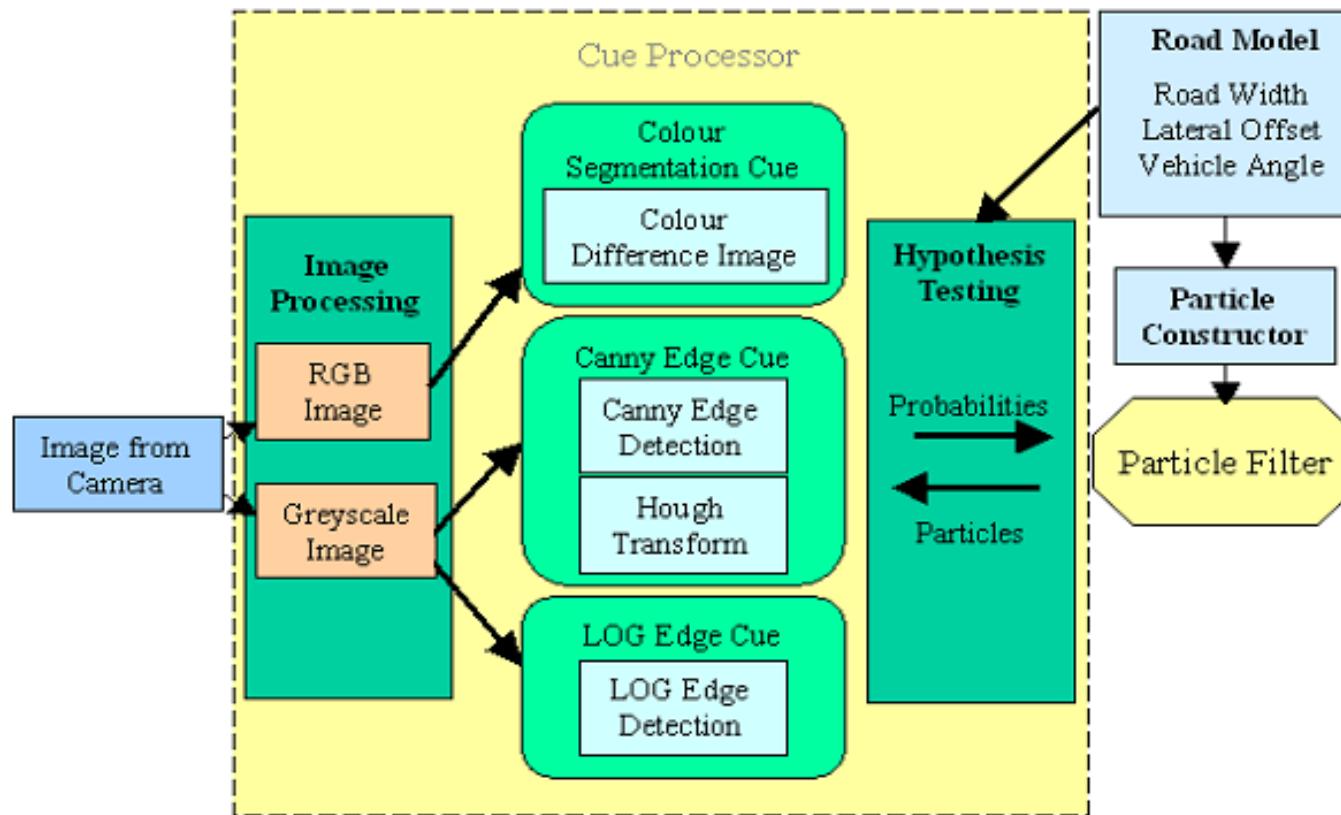
- Straight-line road model is assumed which is valid for small curvature roads (projective transformation preserves lines)
- Relative vehicle to road position consisting in three components: road width, yaw and offset:

$$X_d = \{x^{(i)} | x^{(i)} = (L^{(i)}, \phi^{(i)}, d^{(i)}) , i = 1, \dots, N\}$$



# Vision module scheme

- Three basic components: **image processing**, **cue testing** ([Zelinsky *et al.*, 2003]), **particle filtering**.



# Particle filtering

---

- Also known as Condensation or Monte Carlo algorithm (*[Isard et al., 1996]*)
- Based on Bayesian reasoning under Markov assumption
- Recursive Bayesian update equation:

$$Bel(x_t) = \eta P(o_t|x_t) \int P(x_t|x_{t-1}, a_{t-1}) Bel(x_{t-1}) dx_{t-1}$$

$Bel(x_t)$  - current belief about system state

$P(o_t|x_t)$  - sensor model

$P(x_t|x_{t-1}, a_{t-1})$  - action model

# Particle representation

---

- Idea: representing the continuous belief  $Bel(x_t)$  by a set of  $N$  weighted samples – **particles**:

$$Bel(x_t) \approx \{x_t^{(i)}, \omega^{(i)}\}_{i=1,\dots,N}$$

$x_t^{(i)}$  – sample state, a pose

$\omega^{(i)}$  – importance factor (probability measure)

- Recursive discrete belief update:

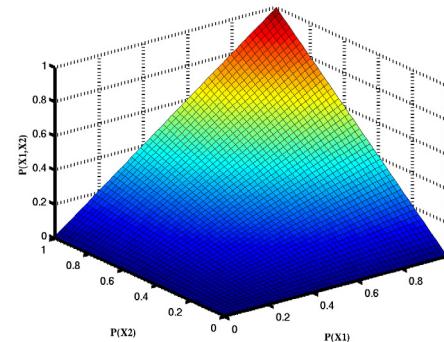
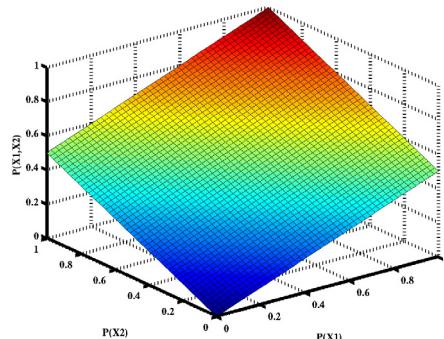
$$Bel(x_t) = \eta P(o_t|x_t) \sum_{i=1}^N P(x_t|x_{t-1}, a_{t-1}) Bel(x_{t-1}) dx_{t-1}$$

# Cue processing - Inference

- A  $j$ -th cue (image filter) performs testing on the whole particle set  $\{x_t^{(i)}, \omega^{(i)}\}$  to obtain  $j$ -th conditional probability distribution  $P(o_t^{(j)}|x_t)$ .
- Total a posteriori pdf for the sensor model:

$$P(o_t|x_t) = \prod_{j=1}^M P(o_t^{(j)}|x_t)$$

- Probabilistic inference - summation vs. product



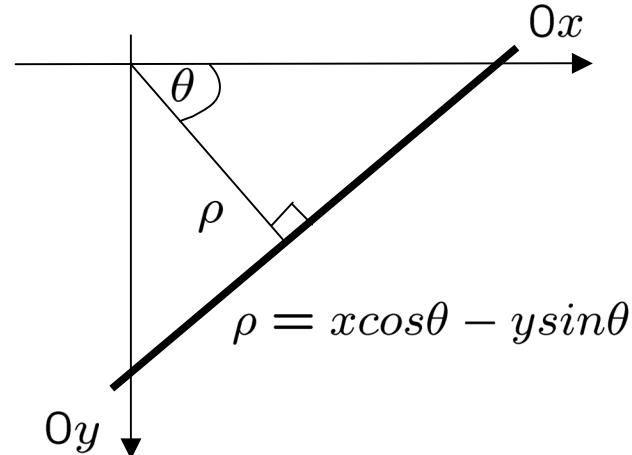
# Cue testing

- Cues test different information on state of the vehicle relative to the road
- RGB input image for color cue processing
- Smoothed grayscale image for edge cue processing



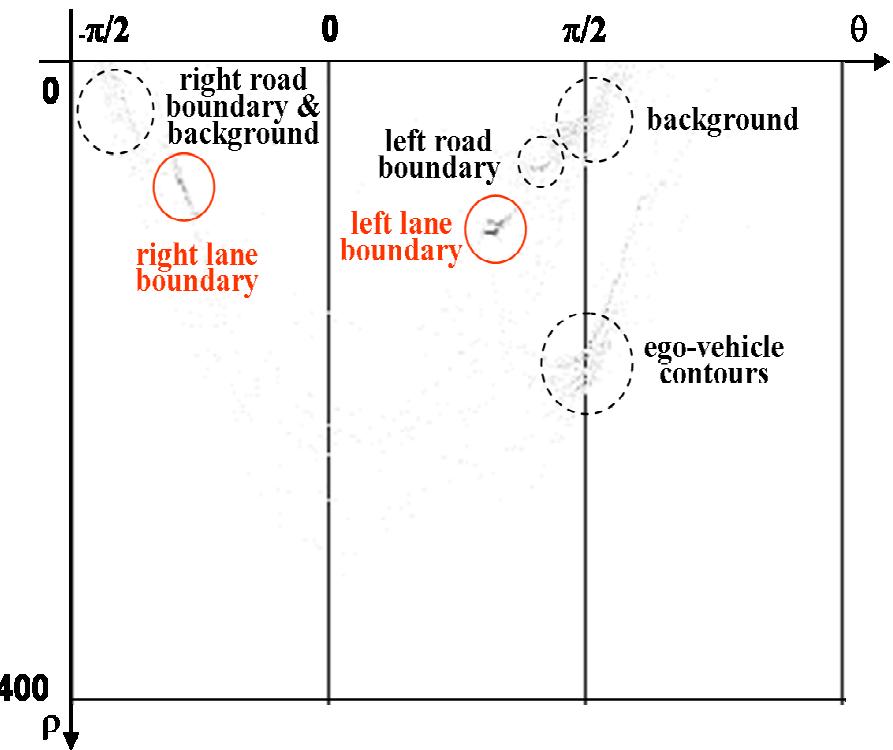
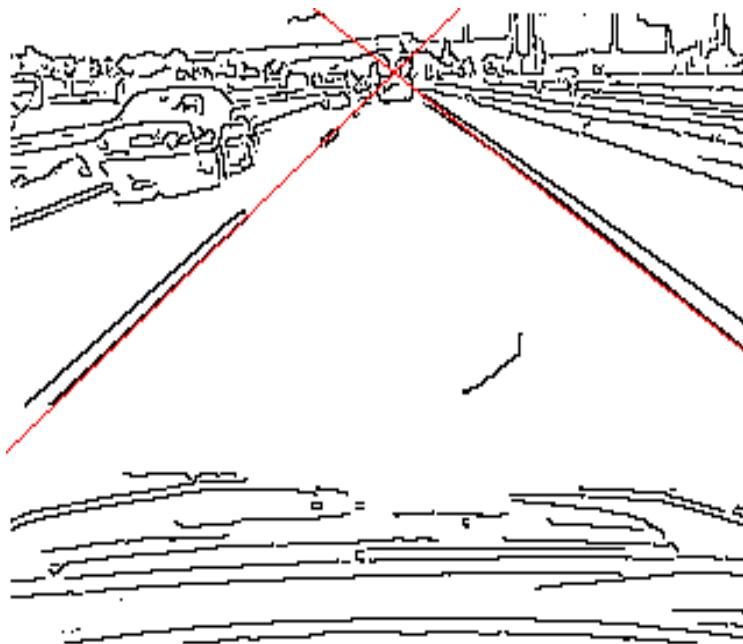
# Canny - Hough filter cue (1)

- Canny edge filter:
  - First order adaptive threshold filter
  - Result: single pixel connected edges
- Hough transform:
  - A transform that accumulates pixels lying along a common line (or a generalized curve)
  - The line transform:



# Canny - Hough filter cue (2)

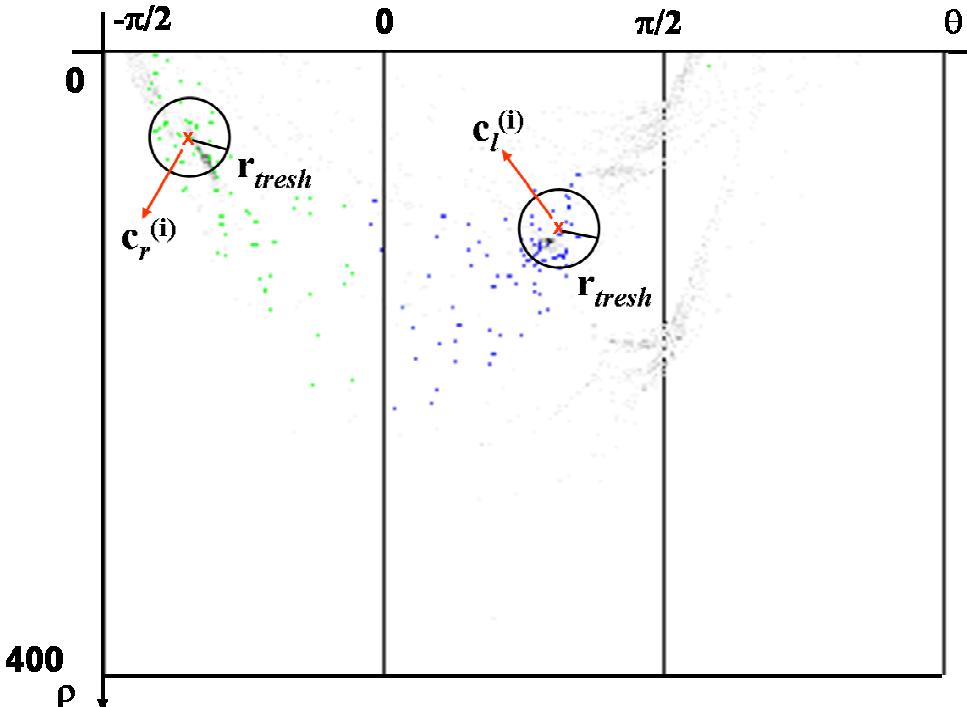
- A Canny edge pixel map and the resulting Hough transform array:



- Lane boundaries:  $(\theta_l, \rho_l) = \{59^\circ, 105\}$ ,  $(\theta_r, \rho_r) = \{-51^\circ, 78\}$ .

# Canny - Hough filter cue (3)

- Each particle's state  $x^{(i)}$  defined by left and right lane edge Hough centers  $c_r^{(i)} = \{\theta_{cr}^{(i)}, \rho_{cr}^i\}$ ,  $c_l^{(i)} = \{\theta_{cl}^{(i)}, \rho_{cl}^i\}$ .
- Testing of each lane edge against a circular cluster region of Hough edge map:



# Canny - Hough filter cue (4)

---

- The choice of transformation origin determines sensitivity of Hough transform to gradient direction error of the Sobel derivative operator
- A single centrally placed transformation origin not satisfactory
- Solution: **two** Hough transforms for each particle – upper left L and right R image corner origin (3 quadrant transform) ([Macek *et al.*, 2004])
- Final  $i$ -th particle probability:

$$\omega_{Canny}^{(i)} = \omega_L^i \omega_R^i$$

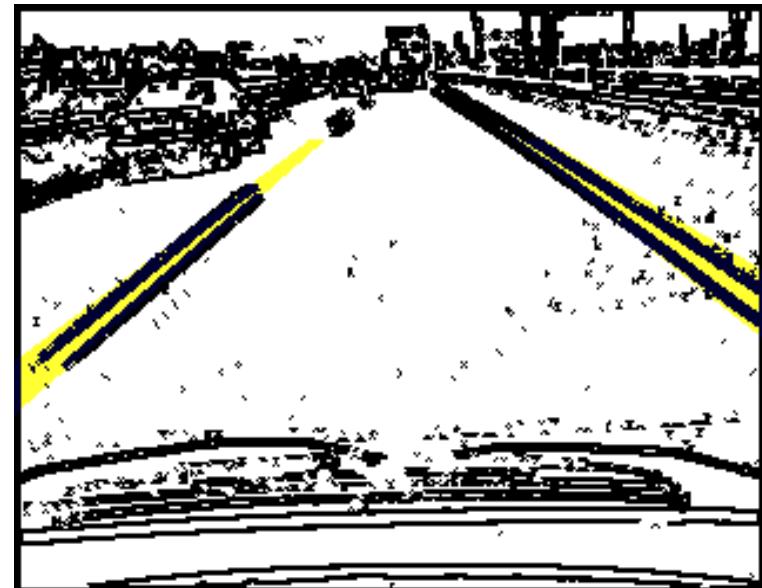
# LoG filter cue

- Laplacian-Gaussian kernel edge filter with fixed thresholding
- Result: thick edge boundaries suitable for direct perspective mask testing in image space
- Total edge pixel hit within a boundary:

$$\delta_{l,r}^{(i)} = \sum_{k=1}^{N_{l,r}} I_{l,r}$$

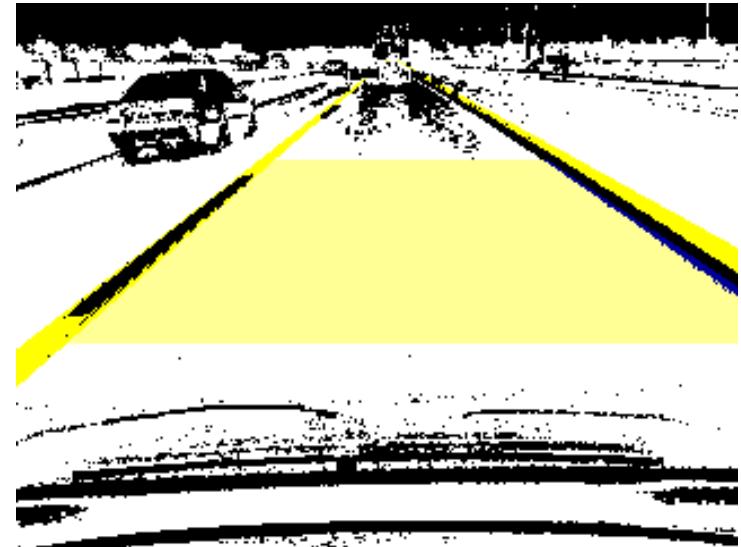
- Total  $i$ -th particle probability:

$$\omega_{LoG}^{(i)} = \left( \frac{\delta_l^{(i)} - \delta_{l_{min}}}{\delta_{l_{max}} - \delta_{l_{min}}} + p_0 \right) \left( \frac{\delta_r^{(i)} - \delta_{r_{min}}}{\delta_{r_{max}} - \delta_{r_{min}}} + p_0 \right)$$



# Color segmentation filter cue

- RGB color difference map (delta map)



- Road and non-road pixel count:

$$\delta_{l,r,c}^{(i)} = \sum_{k=1}^{N_{l,r,c}} I_{l,r,c}$$

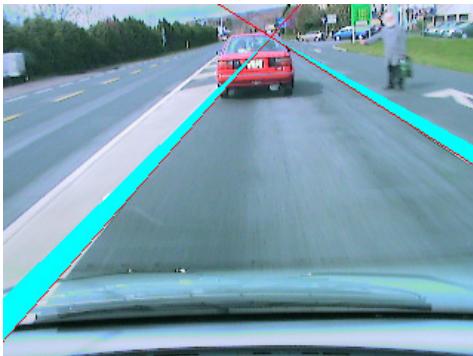
- Testing of left and right lane non-road colour boundary stripes and central road colour area:

$$\omega_{Color}^{(i)} = \left( \frac{\delta_l^{(i)} - \delta_{l_{min}}}{\delta_{l_{max}} - \delta_{l_{min}}} + p_0 \right) \left( \frac{\delta_c^{(i)} - \delta_{c_{min}}}{\delta_{c_{max}} - \delta_{c_{min}}} + p_0 \right) \left( \frac{\delta_r^{(i)} - \delta_{r_{min}}}{\delta_{r_{max}} - \delta_{r_{min}}} + p_0 \right)$$

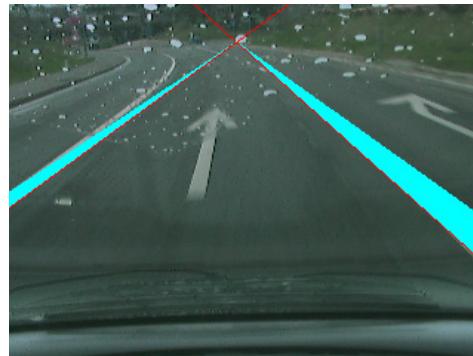
# Results - Magistral road

- Various road conditions tested

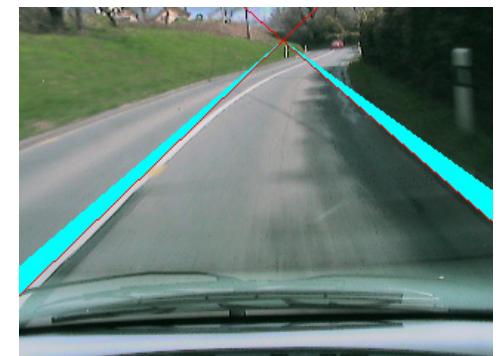
Inner city traffic



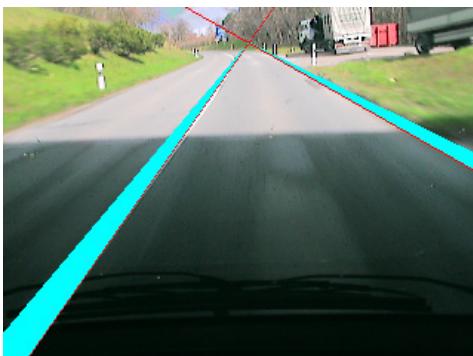
Ground signs



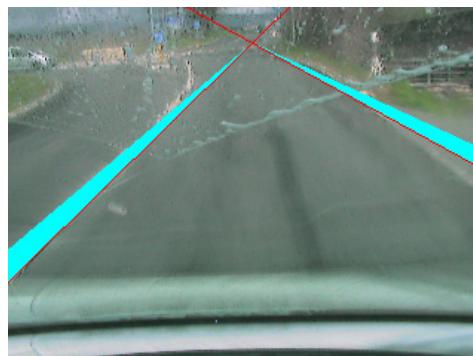
Country-side lane



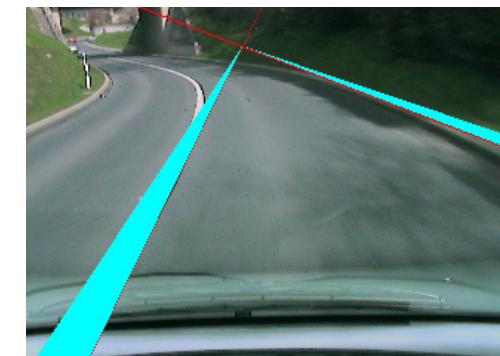
Tunnel exit



Obscured windscreen

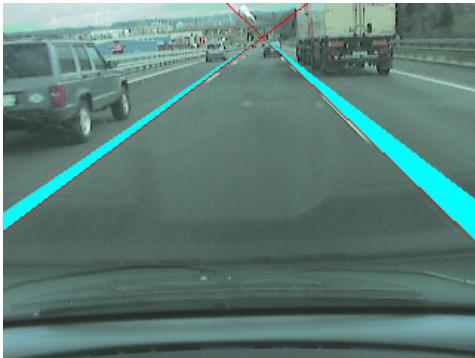


High curvature

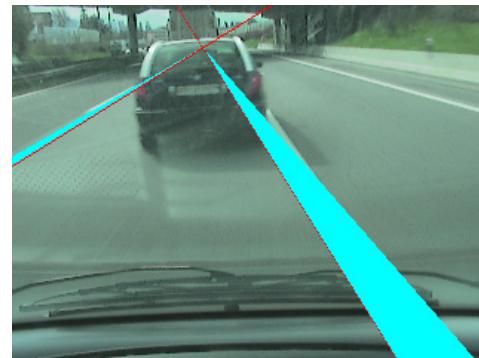


# Results - Highway

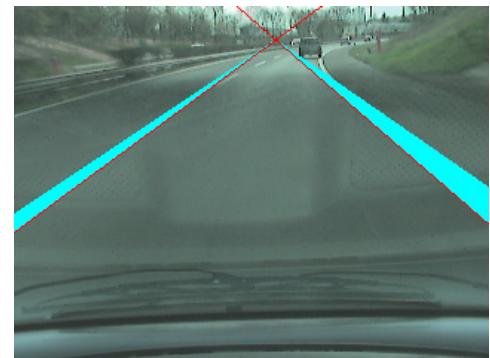
Multilane traffic



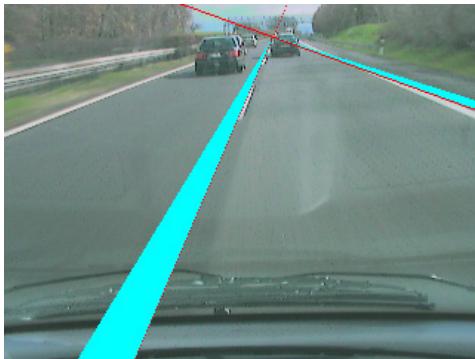
Car occluding



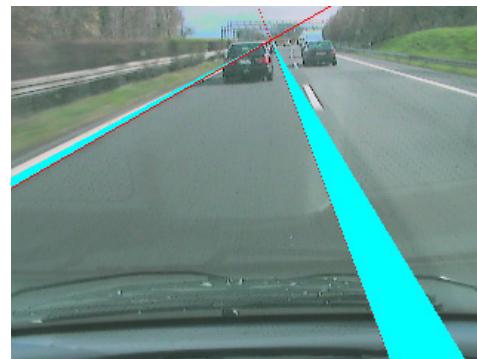
High curvature



Lane changing (1)

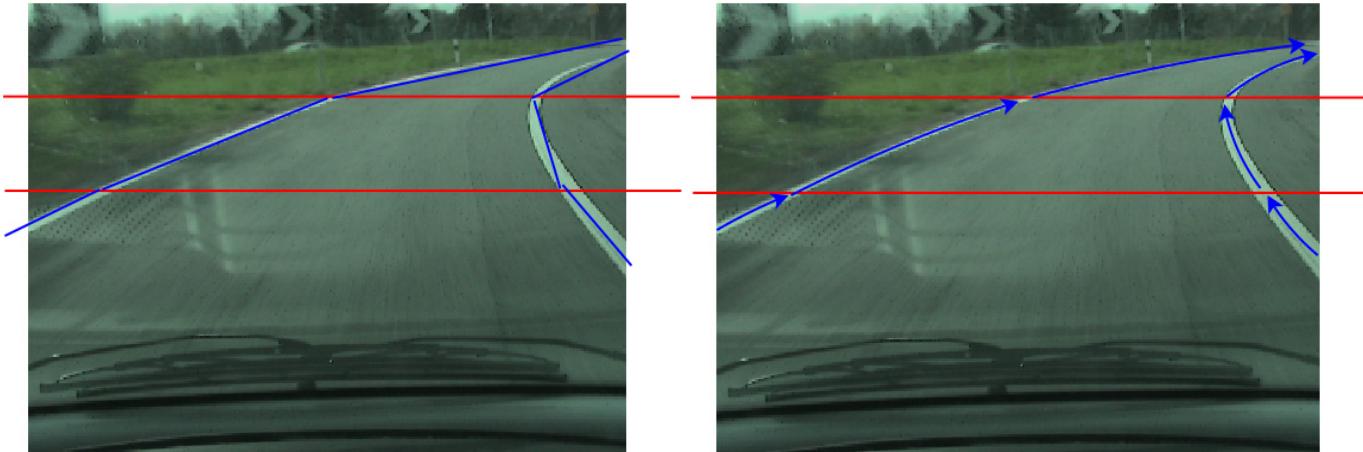


Lane changing (2)



# Extensions

- Cue processing flexible for adding different information (also non-image based)
- Extension to a curved model based on a polyline fit



- Introduction
- Lane detection vision module
- Off- and on-line path planning
- Obstacle avoidance
- Path following
- Conclusions

# Off-line path planning

---

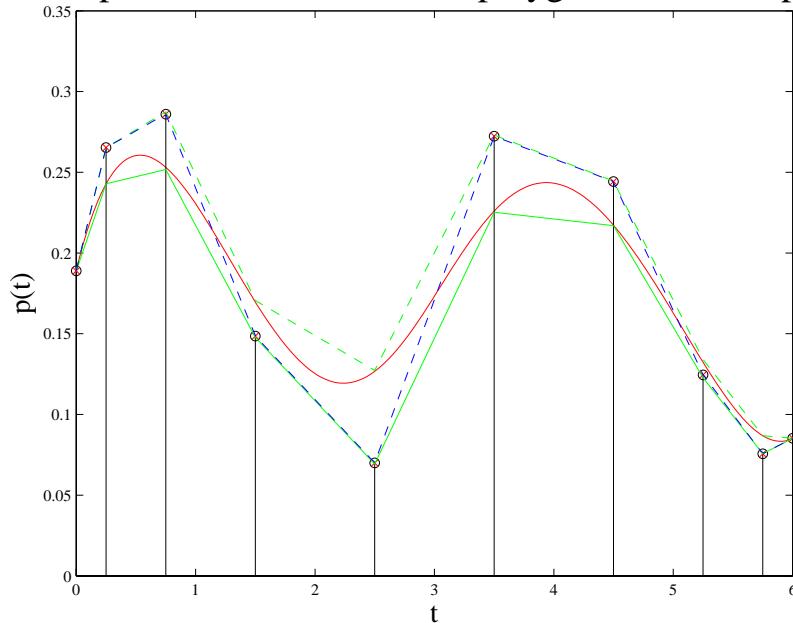
- Smooth paths with different optimality criteria possible
- B-spline curve representation:
  - Smoothness to any order can be acquired
  - Local controllability of the curve
  - Convex hull property of B-spline curve within its control polygon
- B-spline curve described by control point vector  
 $\vec{b} = [b_0, \dots, b_m]$  and basis functions  $N_j^d$  of degree  $d$  on the knot vector  $t_0 \leq t_1 \dots \leq t_{m+d+1}$ :

$$p(t) = \sum_{j=0}^m b_j N_j^d(t)$$

# Control polygon and envelopes

- Control polygon  $l(t)$  - a piecewise linear interpolant at control points  $\vec{b}$
- If the knot vector is fixed, tight linear envelope bounds  $e_{low}(t), e_{high}(t)$  can be defined ([Lutterkort, 2000]):

B-spline curve with control polygon and envelopes

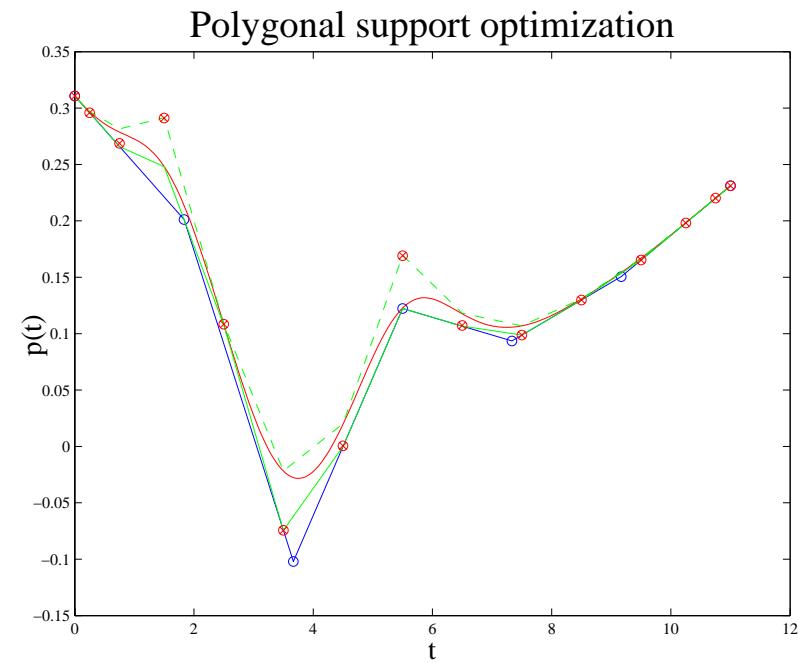


# Support problem optimization

- Environment is a polygonal chain  $c$  - structured environment
- SUPPORT problem: given a polygonal chain  $c$  find a B-spline curve that fits as close as possible to  $c$
- Linear optimization problem:

$$\min_{\vec{b}} \sum_{j=0}^m (b_j - c_j)$$

subj. to:  $\left\{ \begin{array}{l} c \leq e_{low} \end{array} \right\}$



# Channel problem optimization

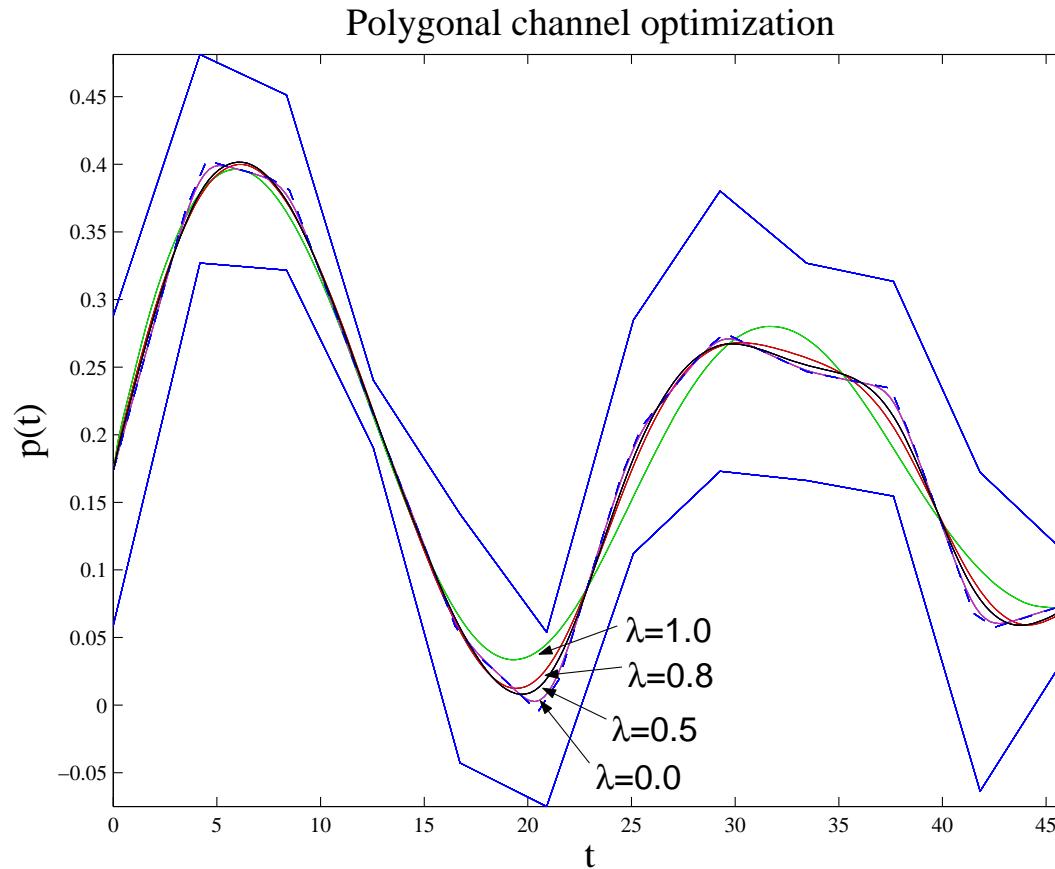
- CHANNEL optimization problem: given a polygonal chain  $c_{low}, c_{high}$  find a B-spline curve that minimizes the **curvature change** and maximizes the **safety distance**
- Nonlinear optimization problem:

$$\min_{\vec{b}} \left\{ \int_{t_0}^{t_{m+d+1}} \dot{K}(s) ds + \sum_{j=0}^m (b_j - c_j) \right\}$$

subj. to:

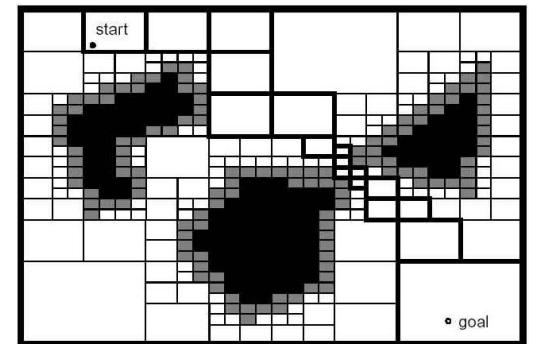
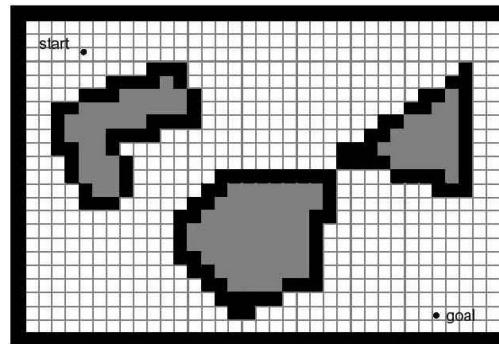
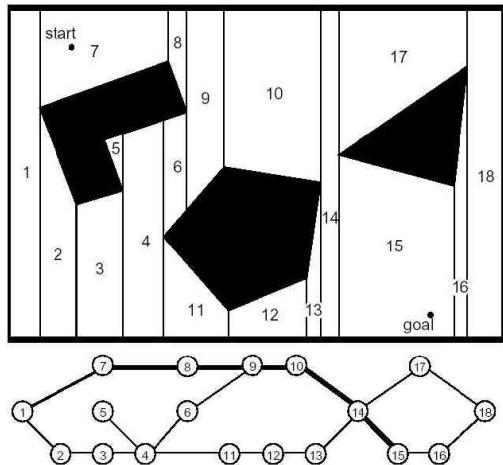
$$\left\{ \begin{array}{l} c_{low} \leq e_{low} \\ e_{high} \leq c_{high} \end{array} \right\}$$

# Smoothness and safety optimization



# Path planning - Cell decomposition

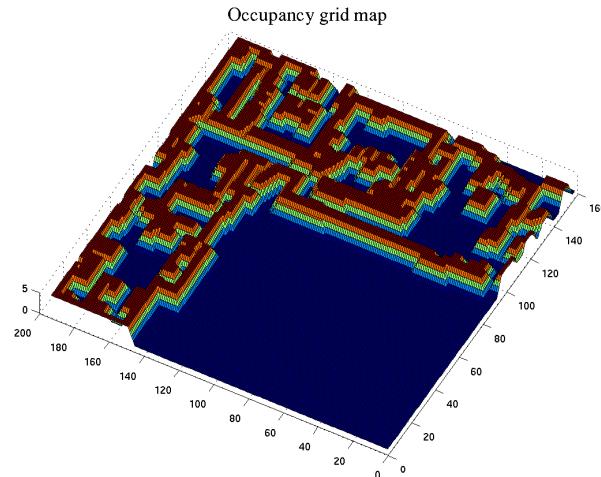
- Cell decomposition path planning:
  - Exact cell decomposition
  - Approximate cell decomposition
  - Adaptive cell decomposition



[courtesy R.Siegwart et al.]

# Occupancy grid based graph search

- Occupancy grid map with cost mask expansion:



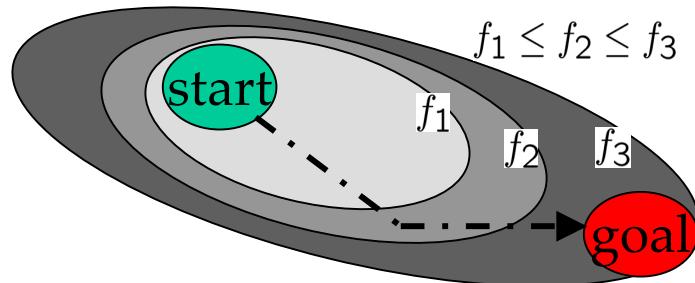
- Graph search algorithms implemented for finding the path between the start and a goal configuration
  - A\* graph search ([Dijkstra, 1959])
  - D\* graph search ([Stentz, 1994])

# A\* path search algorithm

- Uses node expansion from the start to goal position
- Each node (grid cell) is assigned a cost function:

$$f(n) = g(n) + h(n)$$

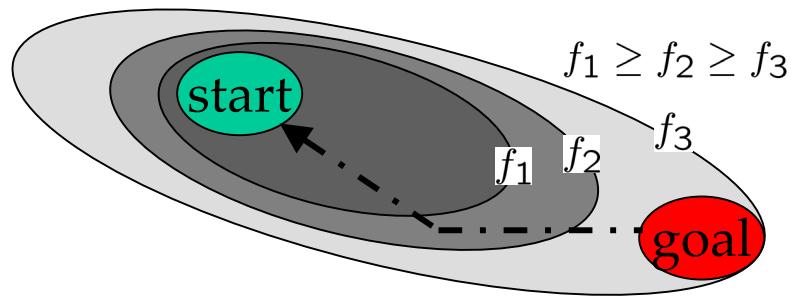
- Optimality and completeness guaranteed



- Major drawback: computational inefficiency in dynamic environments

# D\* path search algorithm (1)

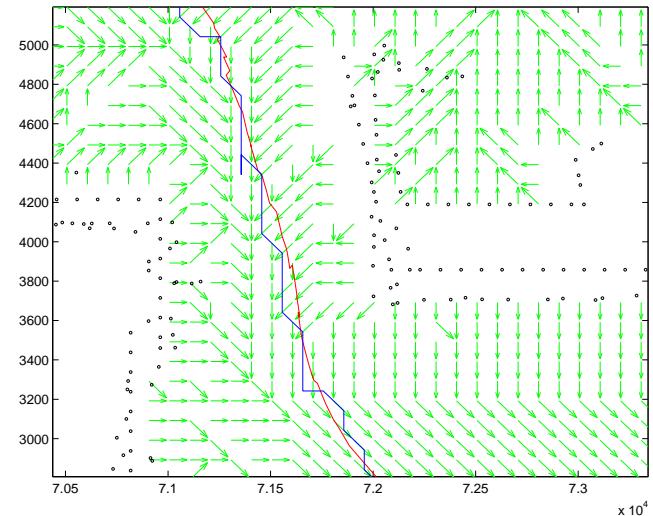
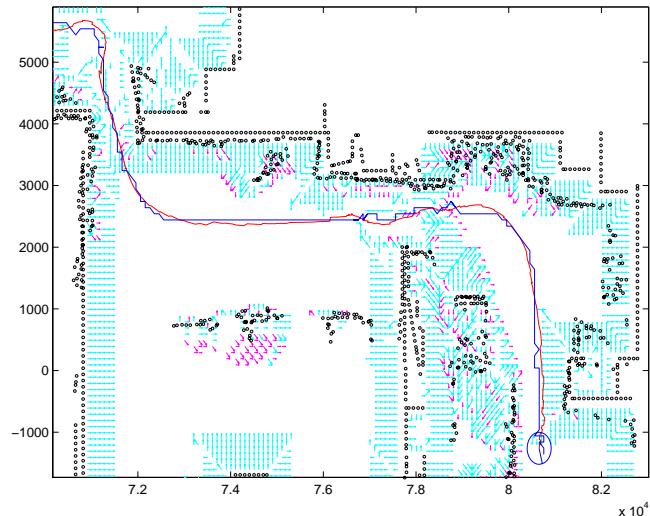
- Node expansion from goal to start position in an incremental manner by maintaining **optimal wave front**
- Reuses search information from previous steps



- Suitable for path search in **dynamic** environments by maintaining a list of RAISE and LOWER states

# D\* path search algorithm (2)

- Path represented as a list of pointers

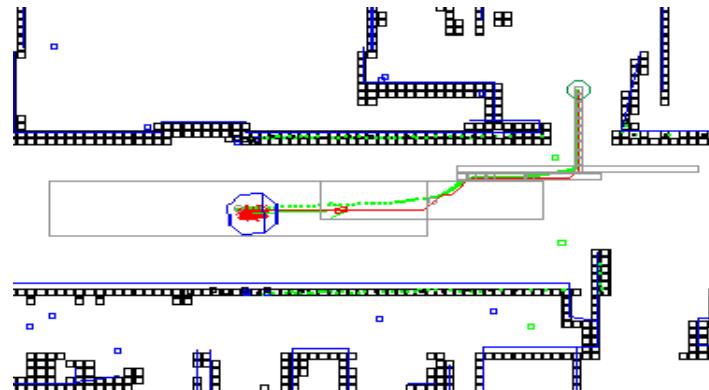


# Free-space bubbles

- A **coarse representation** of the global geometric path composed of connected occupancy grid cells
- A **rectangular bubble** formed as free space around the current robot configuration  $\vec{q}$  and the nearest obstacle ([Quinlan *et al.*, 1993]):

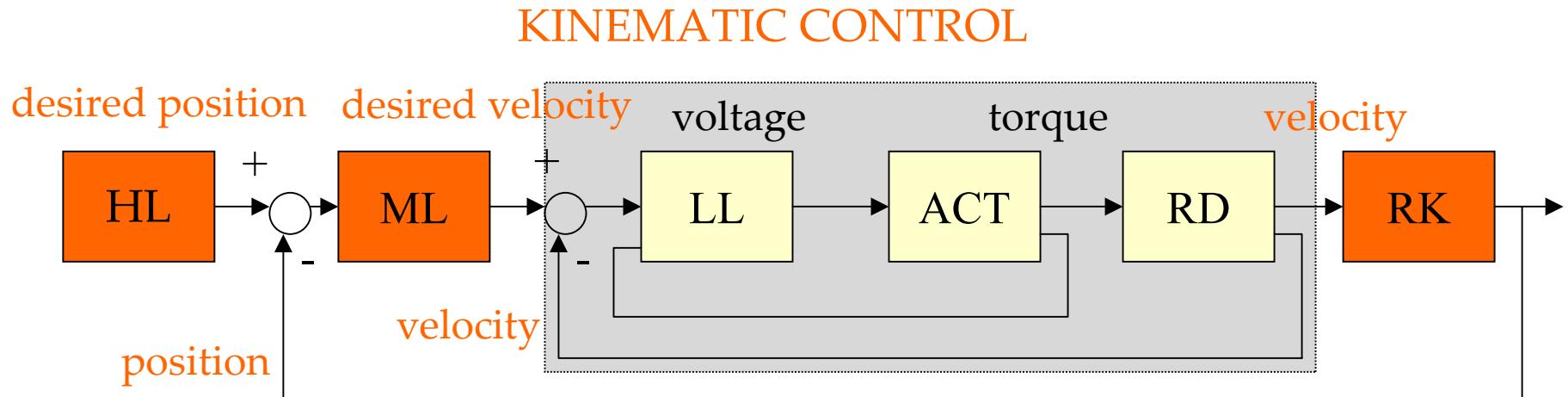
$$\beta(\vec{q}) = \{\vec{p} : |q_x - p_x| < d_x, |q_y - p_y| < d_y\}$$

- Spline smoothing determined by free-space bubble centers



- Introduction
- Lane detection vision module
- Off- and on-line path planning
- **Obstacle avoidance**
- Path following
- Conclusions

# Motion control functional diagram



HL – high level control

ML – medium level control

LL – low level control

RK – robot kinematics

RD – robot dynamics

# Obstacle avoidance -Velocity space based approaches

---

- Velocity space based approaches:
  - Curvature -Velocity method (*[Simmons, 1996]*)
  - Lane - Curvature method (*[Simmons, 1998]*)
- Dynamic Window approach (*[Fox et al., 1997]*)
- Key feature: obstacle avoidance problem formulated in velocity space - kinematic and dynamic robot constraints are taken directly into account
- Geometric local obstacle configuration compared to possible robot trajectories

# General motion equations (1)

---

- General robot motion equations given initial state,  $x_{t_0}$ ,  $y_{t_0}$ ,  $\theta_{t_0}$  and velocities  $v(t)$  and  $\omega(t)$  in interval  $t \in [t_0, t_n]$ :

$$x(t_n) = x(t_0) + \int_{t_0}^{t_n} v(t) \cos \theta(t) dt$$

$$y(t_n) = y(t_0) + \int_{t_0}^{t_n} v(t) \sin \theta(t) dt$$

$$\theta(t_n) = \theta(t_0) + \int_{t_0}^{t_n} \omega(t) dt$$

# General motion equations (2)

- Between two arbitrary points in time  $t_0$  and  $t_n$  robot can be controlled only by limited number of acceleration commands  $\dot{v}_i$  and  $\dot{\omega}_i$  for  $i = 1, 2, \dots, n$  that are assumed **piecewise constant** in  $[t_i, t_{i+1}]$ .
- Piecewise independent trajectory generation:

$$x(t_n) = x(t_0) + \sum_{i=0}^{n-1} \int_{t_i}^{t_{i+1}} \left( v(t_i) + \dot{v}_i \Delta_t^i \right) \cdot \\ \cdot \cos \left( \theta_{t_i} + \omega(t_i) \Delta_t^i + \frac{1}{2} \dot{\omega}_i (\Delta_t^i)^2 \right) dt$$

# Piecewise circular arc trajectories (1)

- Approximation by sequence of **circular arcs**:

$$x(t_n) = x(t_0) + \sum_{i=0}^{n-1} \left( F_x^i(t_{i+1}) \right)$$

where:

$$F_x^i(t) = \begin{cases} \frac{v_i}{\omega_i} (\sin \theta(t_i) - \sin (\theta(t_i) + \omega_i (t - t_i))) & : \omega_i \neq 0 \\ v_i \cos(\theta(t_i)) t & : \omega_i = 0 \end{cases}$$

- Center of rotation:

$$M_x^i = -\frac{v_i}{\omega_i} \sin \theta(t_i)$$

# Piecewise circular arc trajectories (2)

- Robot trajectory in interval  $[t_i, t_{i+1}]$  :

$$(F_x^i - M_x^i)^2 + (F_y^i - M_y^i)^2 = \left(\frac{v_i}{\omega_i}\right)^2$$

where  $v_i$  and  $\omega_i$  are constant.

- Simulation of robot trajectory for a fixed control sequence:

$$v_1 = v_2 = \dots = v_n \quad \& \quad \omega_1 = \omega_2 = \dots = \omega_n$$

reduces to a single circular arc.

# Velocity search space for Dynamic Window approach

- Generating robot trajectory to a goal position in  $[t_0, t_n]$  is a 2-dimensional search of velocity vectors  $(v_i, \omega_i)$  in each control interval – **one step optimization.**
- Circular trajectory constraints
- Possible velocity space – max. velocity constraints :

$$V_s = \{v, \omega | v \leq v_{max} \wedge \omega \leq \omega_{max}\}$$

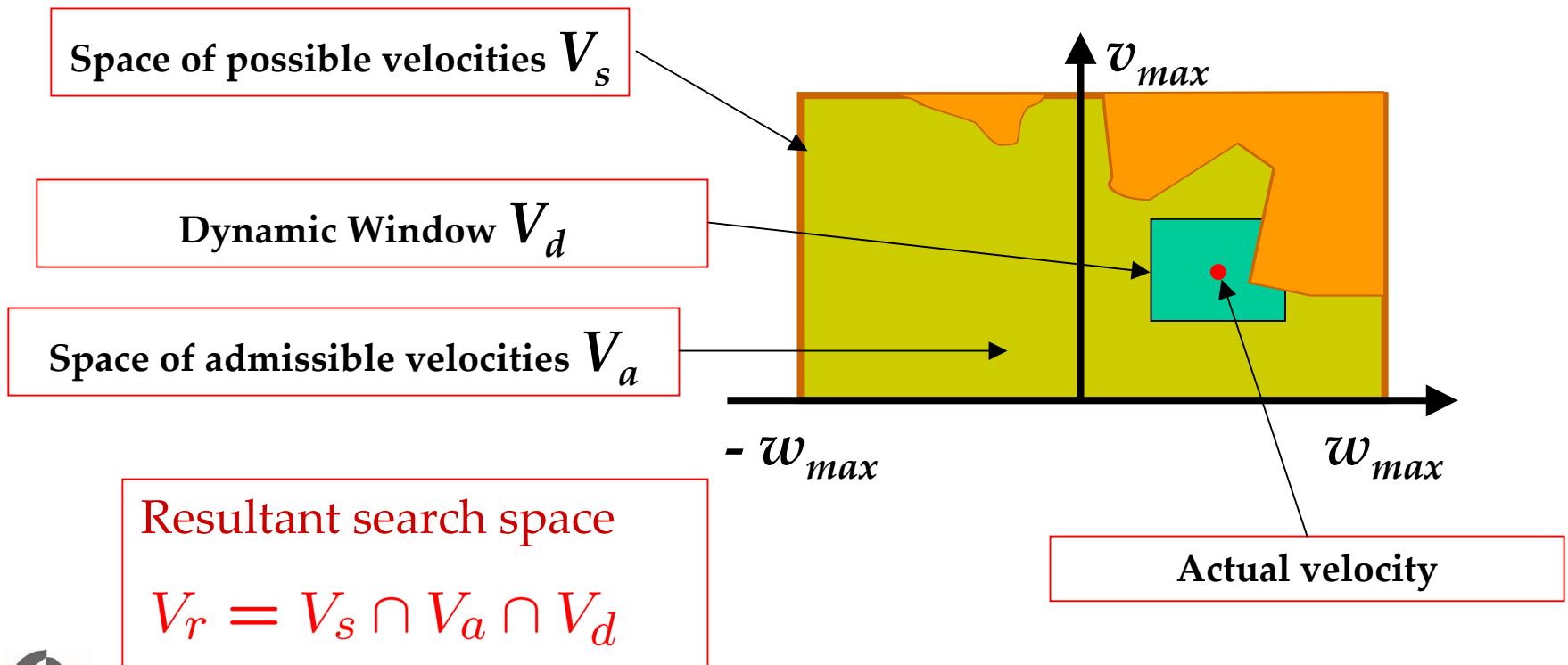
- Admissible velocity space – obstacle constraints:

$$V_a = \left\{ v, \omega \leq \sqrt{2\rho_{min}(v, \omega)\dot{v}_b} \wedge \sqrt{2\rho_{min}(v, \omega)\dot{\omega}_b} \right\}$$

# Resultant search space

- Dynamic window – acceleration constraints:

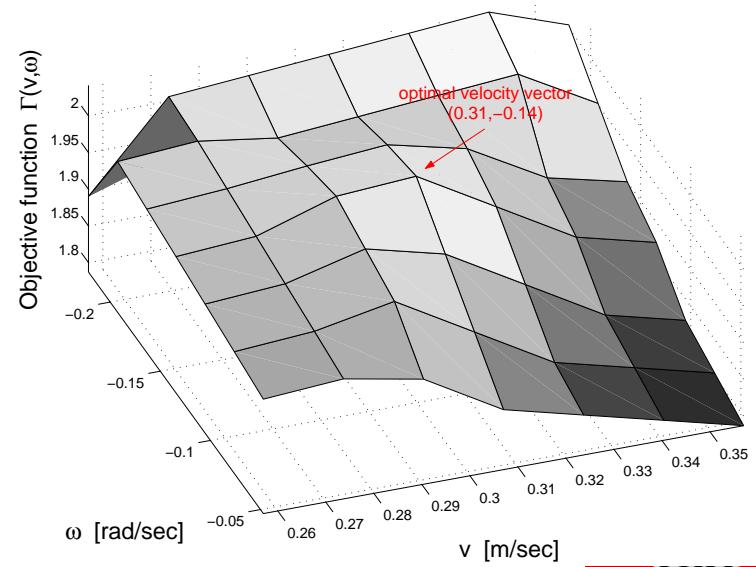
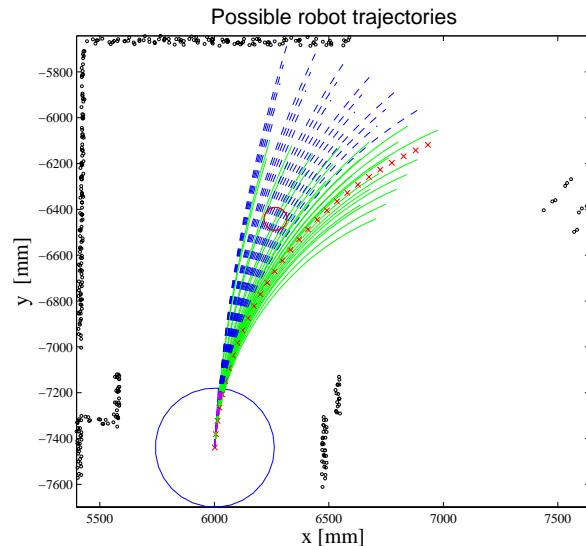
$$V_d = \{(v, \omega) | v \in [v_a - \dot{v}t, v_a + \dot{v}t] \wedge \omega \in [\omega_a - \dot{\omega}t, \omega_a + \dot{\omega}t]\}$$



# Global objective

- From a discrete set of velocity vectors  $(v_k^i, \omega_k^i)$  within resultant search space  $V_r$ , the  $(v_{optim}^i, \omega_{optim}^i)$  is chosen that **maximizes objective function**:

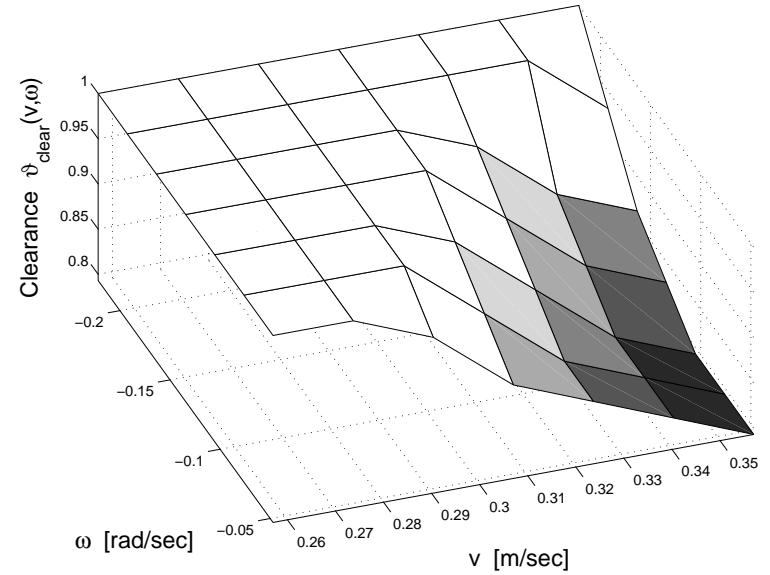
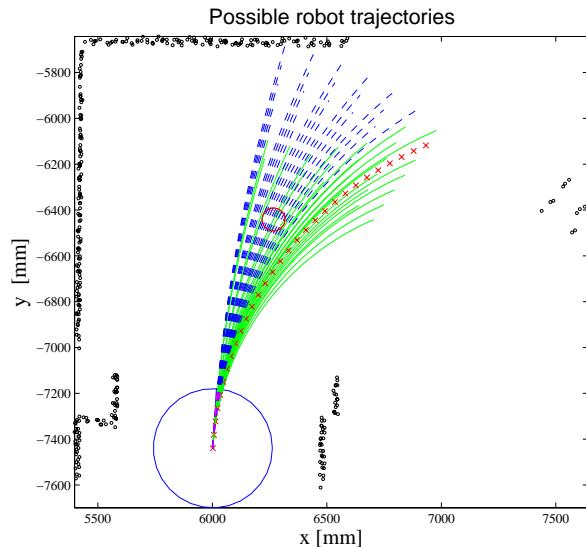
$$\Gamma(v, \omega) = \lambda_{clear}\vartheta_{clear} + \lambda_{head}\vartheta_{head} + \lambda_{vel}\vartheta_{vel}$$



# Clearance objective

- Time to collision with obstacles based measure:

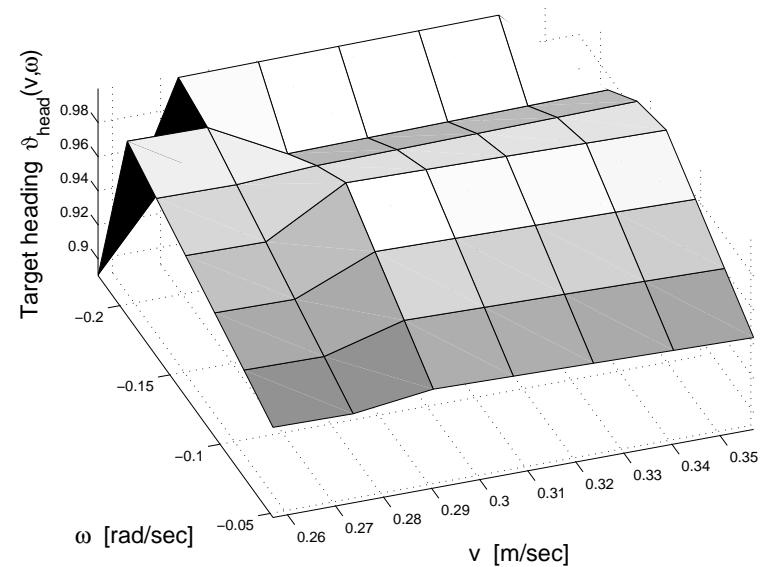
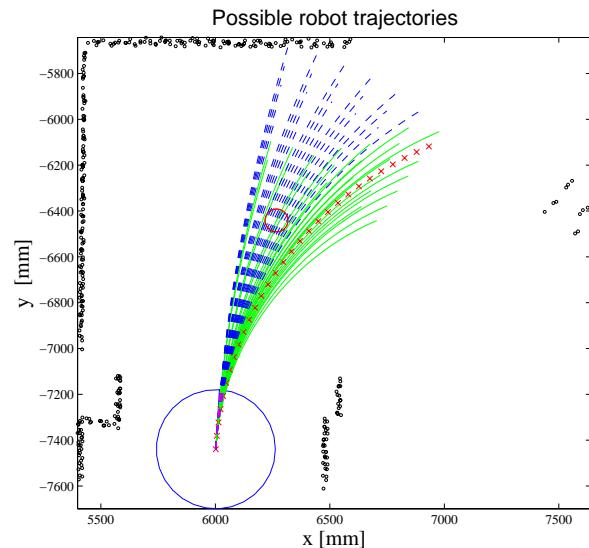
$$\vartheta_{clear}(v, \omega) = \begin{cases} 0 & : t_{col}(v, \omega) \leq T_b(v, \omega) \\ \frac{t_{col}(v, \omega) - T_b(v, \omega)}{T_{max} - T_b(v, \omega)} & : T_b(v, \omega) < t_{col}(v, \omega) < T_{max} \\ 1 & : t_{col}(v, \omega) > T_{max} \end{cases}$$



# Target heading objective

- Based on **heading difference** between the current goal and predicted robot orientation in next step:

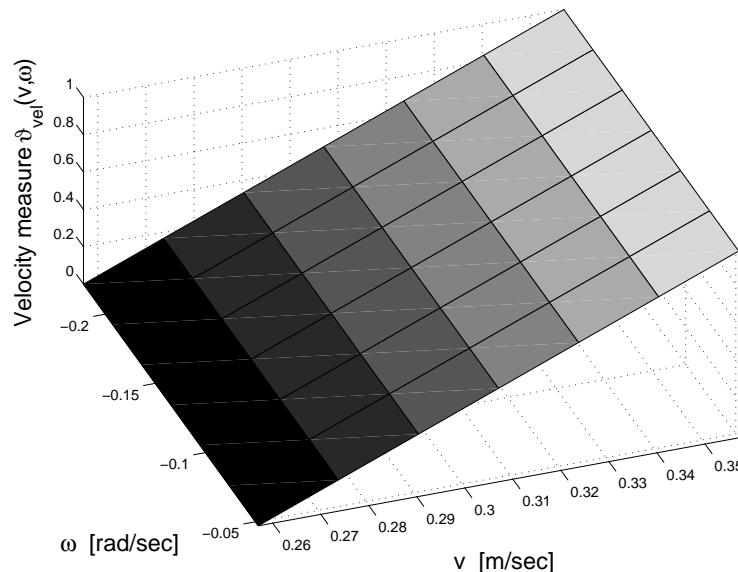
$$\vartheta_{head}(v, \omega) = 1 - \frac{|\theta_t(v, \omega) - \psi_p(v, \omega)|}{\pi}$$



# Translational velocity objective

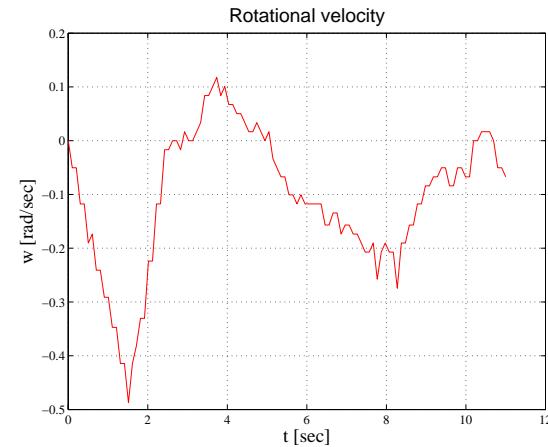
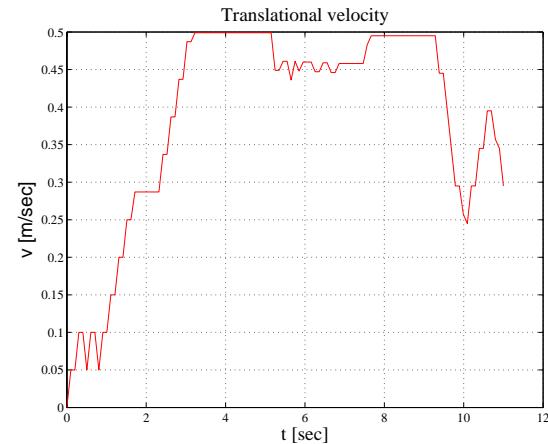
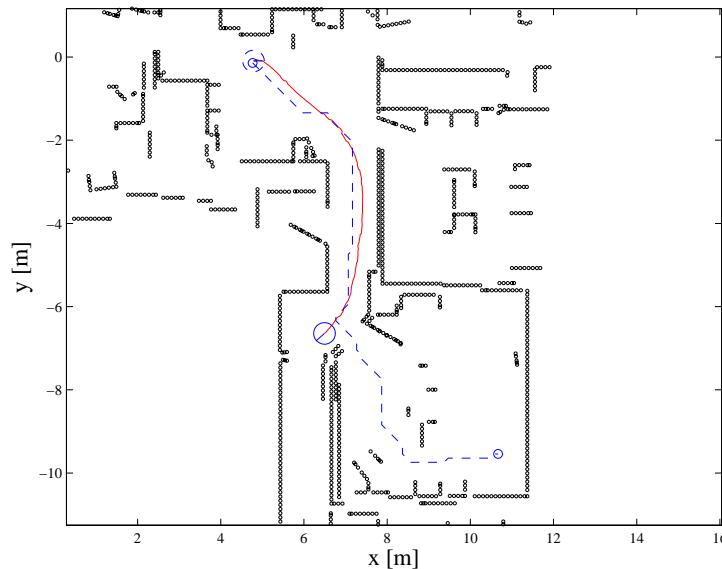
- Higher translational velocities within the current dynamic window preferred:

$$\vartheta_{vel}(v, \omega) = \frac{v - v_{d_{min}}}{v_{d_{max}} - v_{d_{min}}}$$



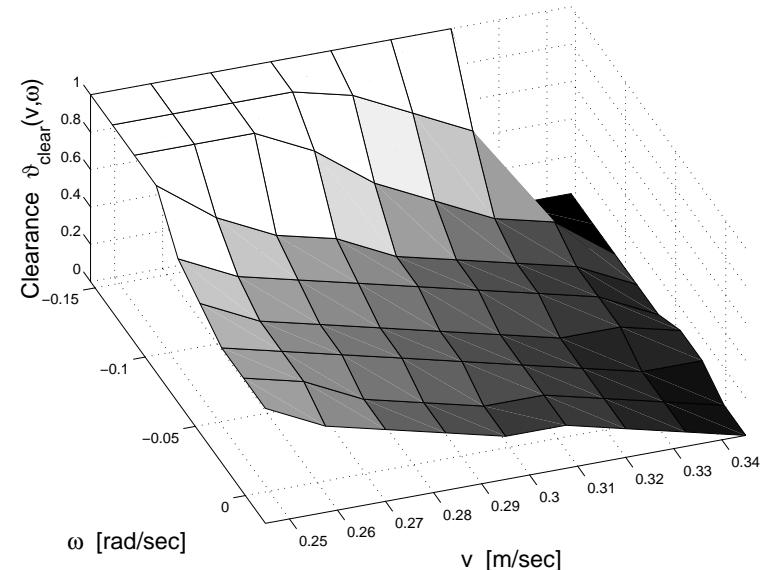
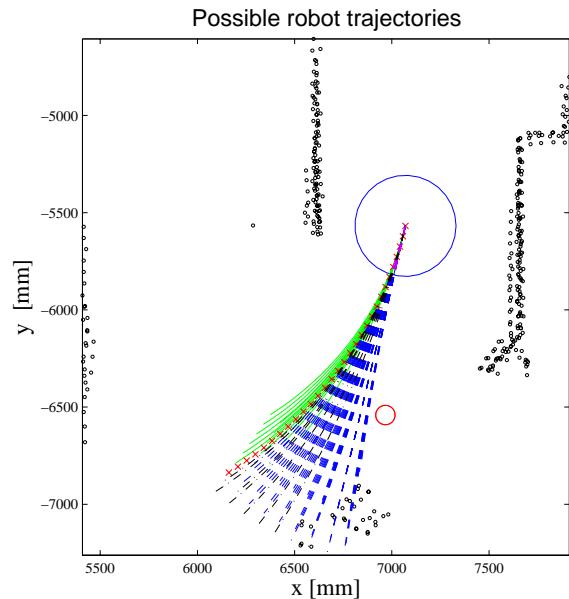
# Sensitivity to different objectives (1)

- Overshoot situation – clearance vs. heading and velocity objectives.

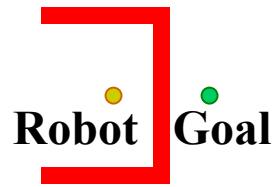


# Sensitivity to different objectives (2)

- Snapshot of a doorway entering phase



- Local minima problem:



- Introduction
- Lane detection vision module
- Off- and on-line path planning
- Obstacle avoidance
- Path following
- Conclusions

# Path following - Introduction

---

- Path following: kinematic model based motion control
- Path tracking: dynamic model based motion control
- Reference point based approaches: reference point determines the effective “line-of-sight” of the robot along the geometric path
- Two distinct approaches:
  - Global integration of local obstacle avoidance techniques - **Global path integrated Dynamic Window**
  - Inherent global path following techniques - **Modified virtual vehicle approach**

# Global path integrated Dynamic Window approach - Introduction

---

- **Aim:** reduce sensitivity to opposing objectives in original Dynamic Window approach (path clearance vs. target heading and translational velocity measure)
- **Solving local minima problem** - global space connectivity achieved using a path generated by a graph search planner on occupancy grid map
- Initial global geometric path is line segmented with multiples of  $45^\circ$  - not suitable for path following

# Related approaches

---

- Global Dynamic Window (*[Brock et al., 2000]*)

$$\Gamma(v, \omega) = \lambda_{clear} \vartheta_{clear} + \lambda_{vel} \vartheta_{vel} + \lambda_{nf1} n_{f1} + \lambda_{\Delta nf1} \Delta n_{f1}$$

- parameter tuning difficult
- translational velocity objective not related to obstacle configuration

- Reduced Dynamic Window (*[Arras et al., 2002]*)

$$\Gamma(v, \omega) = \lambda_{clear} \vartheta_{clear} + \lambda_{head} \vartheta_{head}$$

- only a coarse set of predefined linear velocities allowed
- polygonal shapes taken into account

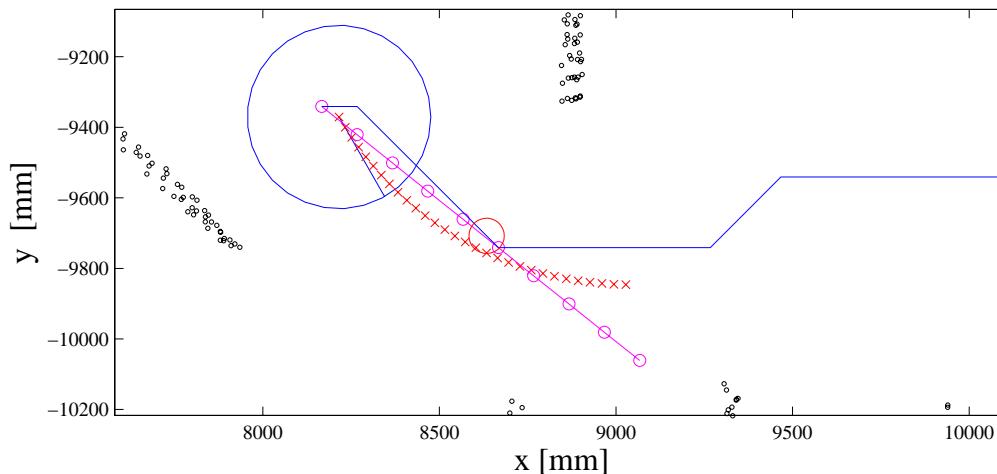
# Global connectivity - effective path

---

- Effective path introduced for alignment comparison between the DW generated trajectories and the global geometric path ([Macek *et al.*, 2003])
- Represents the global path in the local vicinity of the robot
- It is a straight line connecting the current robot position and the reference point on the global path.
- Its length determines the current translational velocity reference  $v_{ref}$ .
- Its orientation determines the current rotational velocity reference  $\omega_{ref}$ .

# Effective path vs. global path

- Its length and orientation change dynamically according to the current robot state and local path configuration (reference point)



- Path orientation change: related to inflection points on the global path
- Dynamic robot constraints: limit the effective path length

# Path alignment measure

- Among all possible DW generated trajectories the optimal one is determined that is best aligned with the effective path
- Let  $k$ -th DW trajectory be represented by  $N_t$  points  $\{(x_{t_k}(i), y_{t_k}(i)), i = 1, 2, \dots, N_t\}$  and the effective path by  $\{(x_p(j), y_p(j)), j = 1, 2, \dots, N_p\}$  of  $N_p$  points and  $d_{tk}(i, j)$  distance between two points on each curve
- Path alignment measure:

$$\vartheta_{path}(k) = \frac{\sum_{i=1}^{N_t} \sum_{j=1}^{N_p} j d_{t_k}(i, j) - D_{min}}{D_{max} - D_{min}}$$

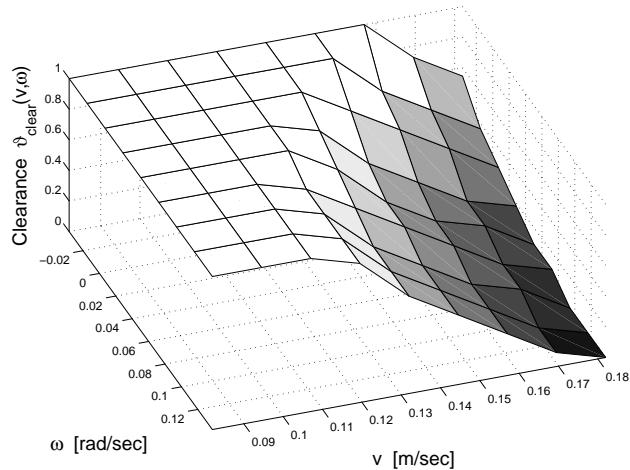
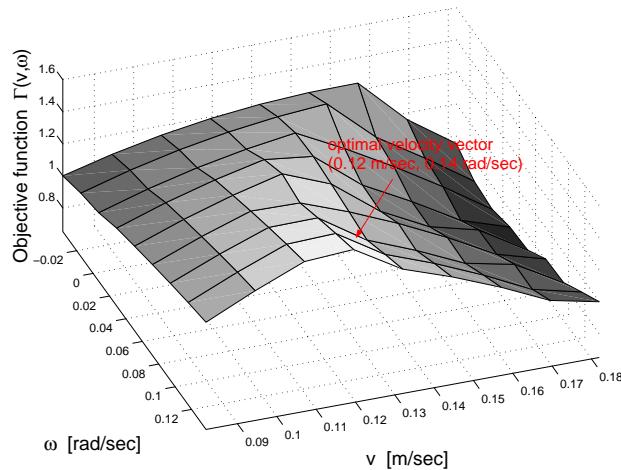
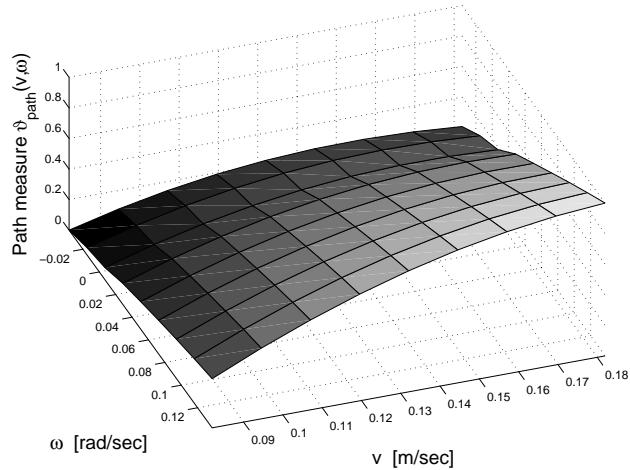
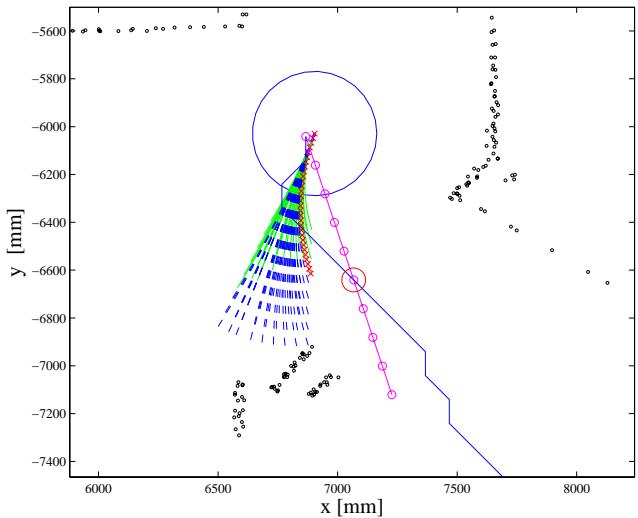
# Global objective

---

- Global path related objectives of heading  $\vartheta_{head}$  and translational velocity  $\vartheta_{vel}$  integrated in a single measure  $\vartheta_{path}$  :

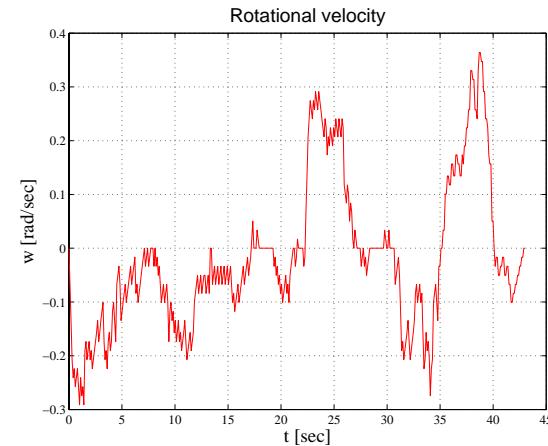
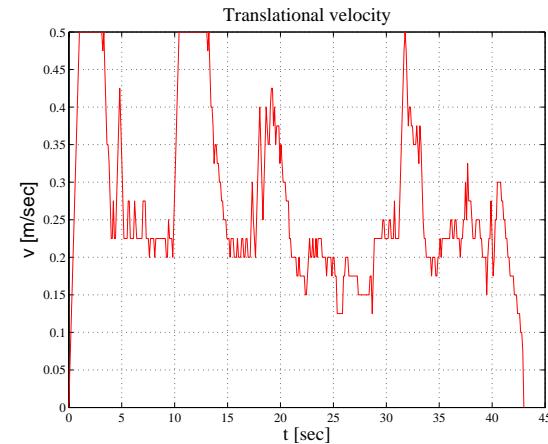
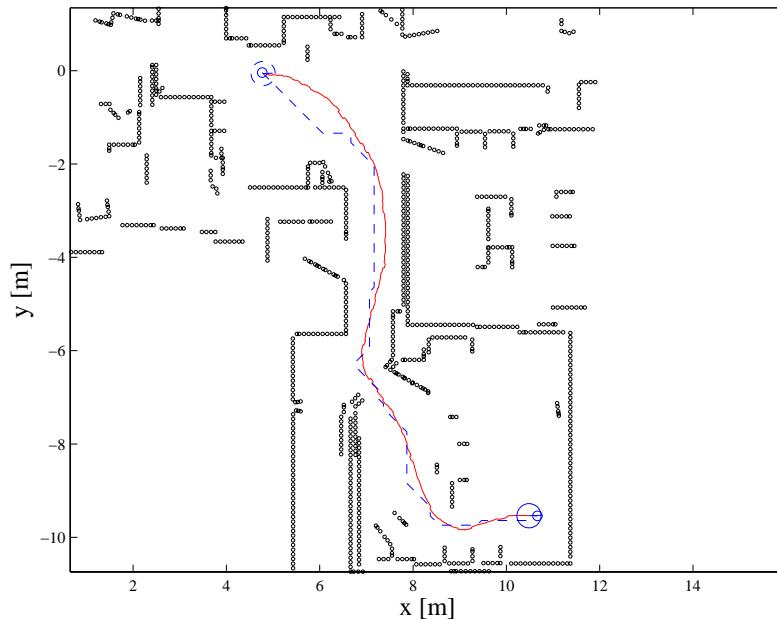
$$\Gamma(v, \omega) = \lambda_{clear}\vartheta_{clear} + \lambda_{path}\vartheta_{path}$$

# A corridor environment run (1)



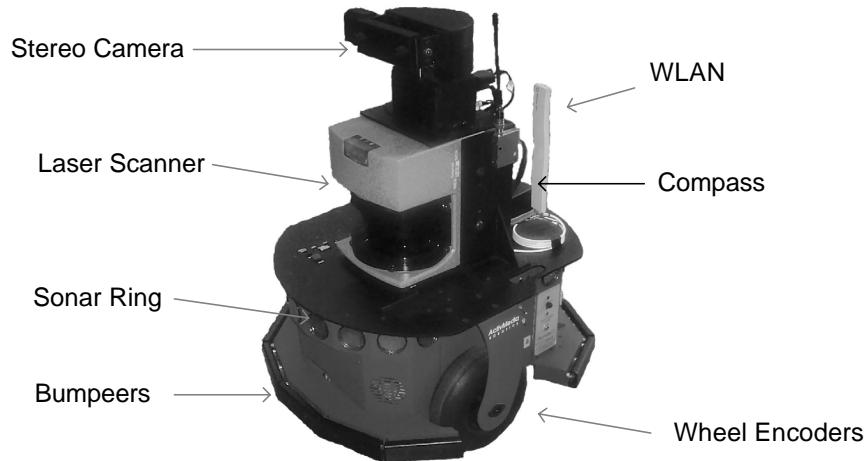
# A corridor environment run (2)

- Successful doorway entry maneuver



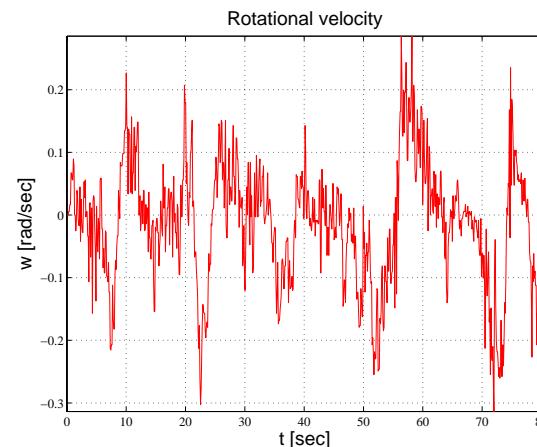
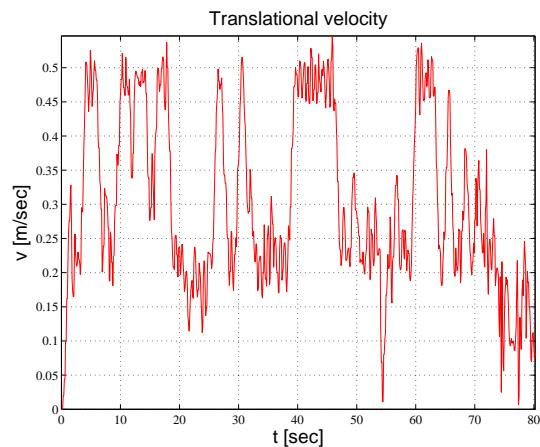
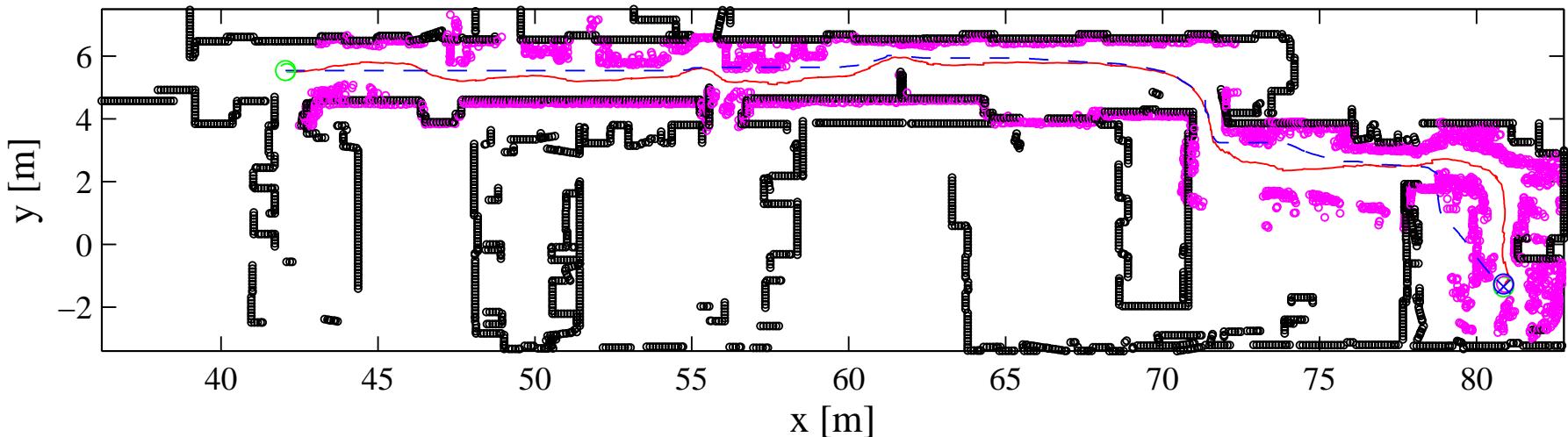
# Experimental results – Robot setup

- A Pioneer 2 ActiveMedia Robotics robot platform



- A proprietary server-client software architecture
- An UDP socketing communication implemented for off-board computation

# Experimental results – Global path integrated Dynamic Window (1)



# Virtual vehicle approach

---

- Control algorithm statement – finding a longitudinal control  $v(t)$  and lateral control  $\omega(t)$  to follow a smooth path  $c(s)$  described by curve parameter  $s$
- Desired **reference point** coordinates on the curve  $c(s)$ :

$$x_d = p(s)$$

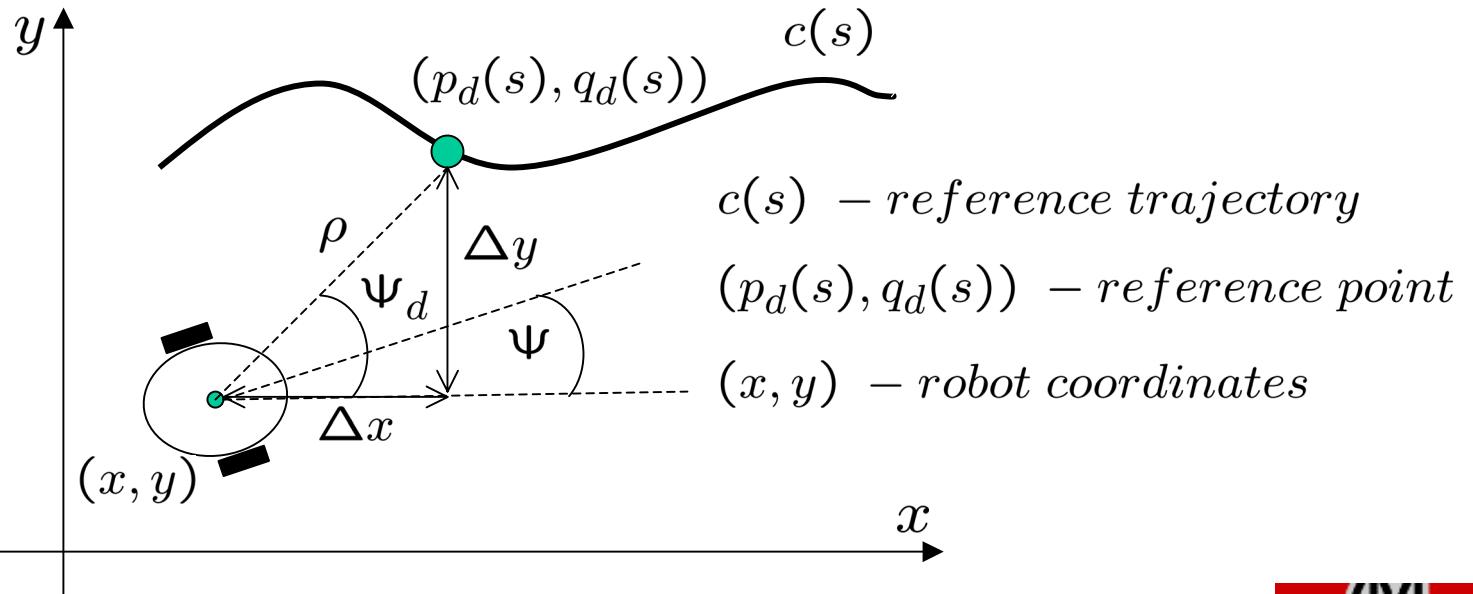
$$y_d = q(s), (0 \leq s \leq s_f)$$

where  $\|c'(s)\| = \sqrt{p'^2 + q'^2} \neq 0, \forall s \in [0, s_f]$  (valid for unit speed curves).

# Control objective

- Control objective: keep longitudinal and lateral difference within specific bounds:

$$\lim_{t \rightarrow \infty} \sup \rho_t \leq d_\rho$$
$$\lim_{t \rightarrow \infty} \sup \| \psi - \psi_d \| \leq d_\psi$$



# Reference point dynamics

---

- Reference point acts as a **virtual vehicle** whose dynamics is governed by the tracking error feedback (*[Egerstedt et al., 2003]*):

$$\dot{s} = \frac{c_o e^{-\alpha\rho} v_n}{\sqrt{p'^2(s) + q'^2(s)}}$$

- Steady state conditions:

$$\rho(t) = d_\rho, \quad v(t) = v_n, \quad \Psi(t) = \Psi_d$$

- Ensures a globally stable path following with exponential tracking error decay (both positional and angular)

# Control law

---

- Both translational and lateral control are **proportional regulators**:

$$\omega_{ref} = k\Delta\psi + \dot{\psi}_d$$

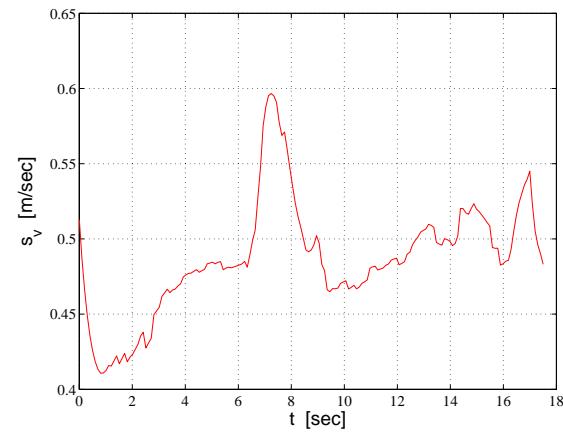
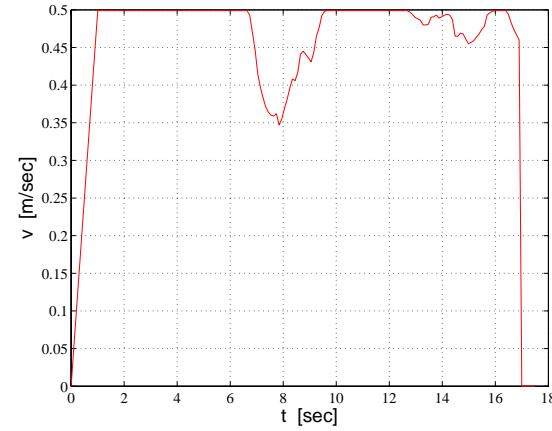
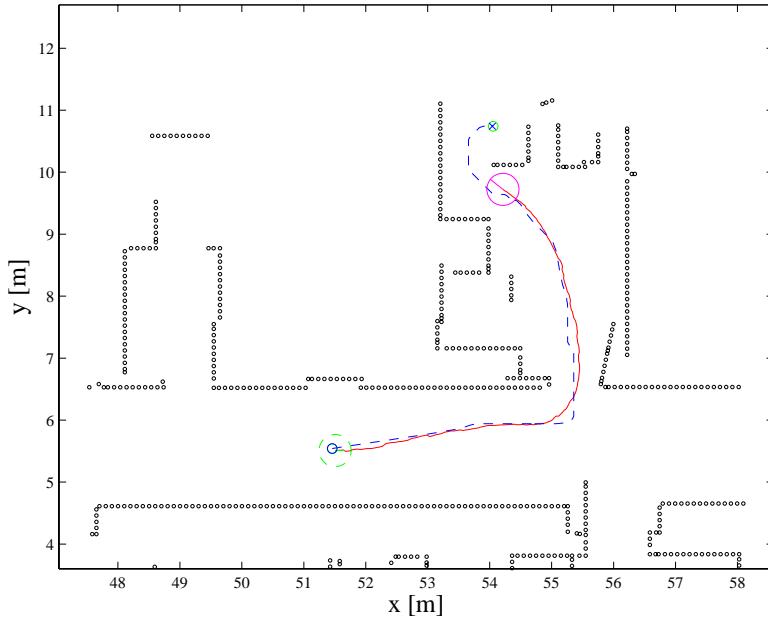
$$v_{ref} = \gamma\rho\cos(\Delta\psi)$$

where  $\Delta\psi = \psi_d - \psi$ .

- It is assumed that the actuator level controllers are implemented – higher level velocity control that is **model independent**.

# Off-line planned path run

- Nominal translational velocity maintained



# Major drawbacks

---

- Essentially an off-line path following approach
- Dynamics of the reference point dependent only on tracking error feedback - local path configuration taken into account only at the instantaneous reference point position.

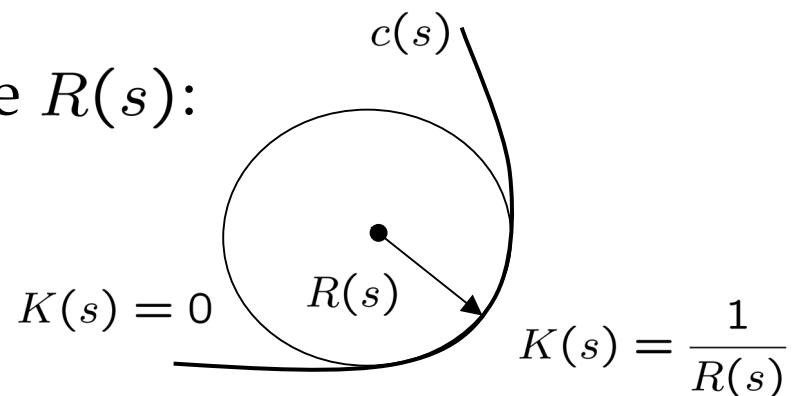
# Modified virtual vehicle approach

- Keypoint: **local curve configuration** between the robot and reference point position included in the reference point dynamics
- Curvature  $\kappa(s)$  describes how much a curve  $c(s)$  bends locally:

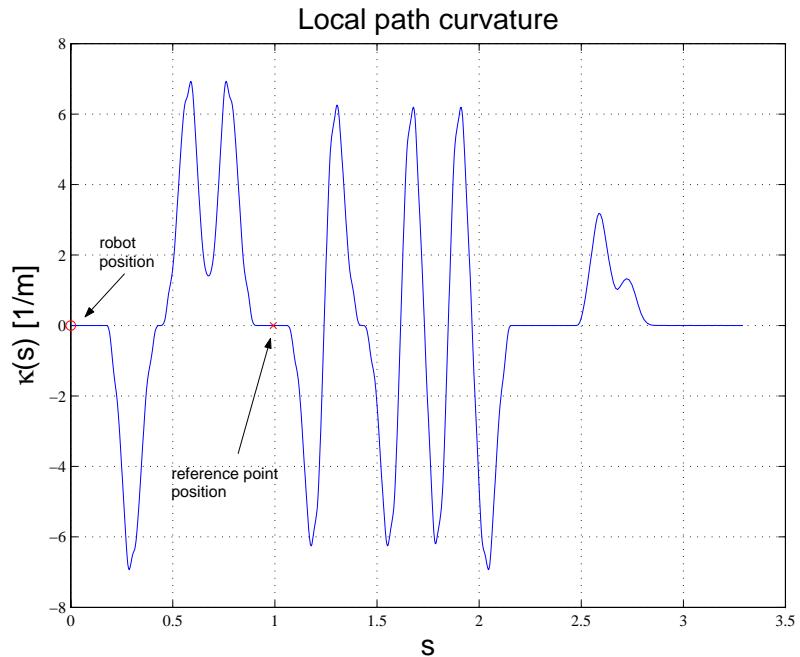
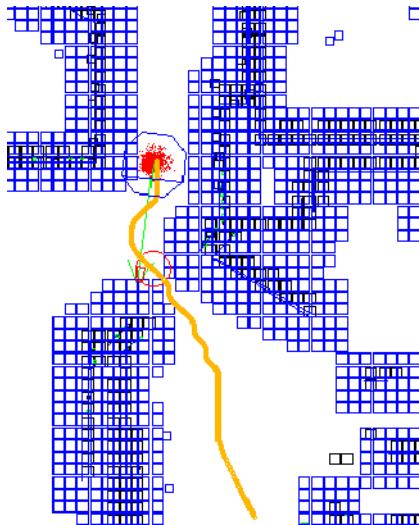
$$\kappa(s) = \frac{p'q'' - p''q'}{(p'^2 + q'^2)^{\frac{3}{2}}}$$

- Relation to the osculating circle  $R(s)$ :

$$\kappa(s) = \frac{1}{R(s)}$$



# Curvature along the global path



- Inverse model tracking: current robot trajectory curvature  $\kappa_r = \frac{\omega}{v}$  equals the path curvature  $\kappa_c$ .

# Curvature effort

---

- For a smooth velocity transition the change in curvature must be considered - **curve smoothness**
- **Curvature effort** measure describes the local path curvature changes between the robot and reference point:

$$\chi(\rho) = \frac{\sum_{i=1}^{N_\rho} \|\Delta\kappa_i\| \Delta s_i}{\rho}, \quad \forall \rho > 0$$

with  $\{(x_c(s), y_c(s)) : s = (1, 2, \dots, N_\rho)\}$  a collection of dense waypoints along the reference curve

- Differences:  $\Delta\kappa_i = \kappa_i - \kappa_{i-1}$ ,  $\Delta s_i = s_i - s_{i-1}$ .

# Modified reference point dynamics

---

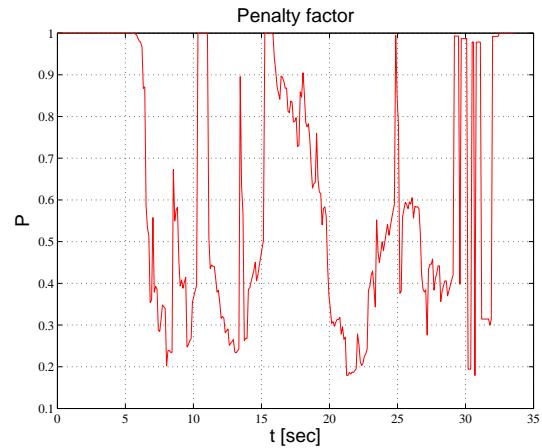
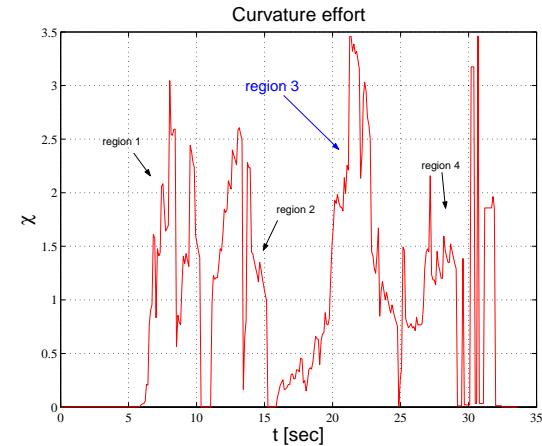
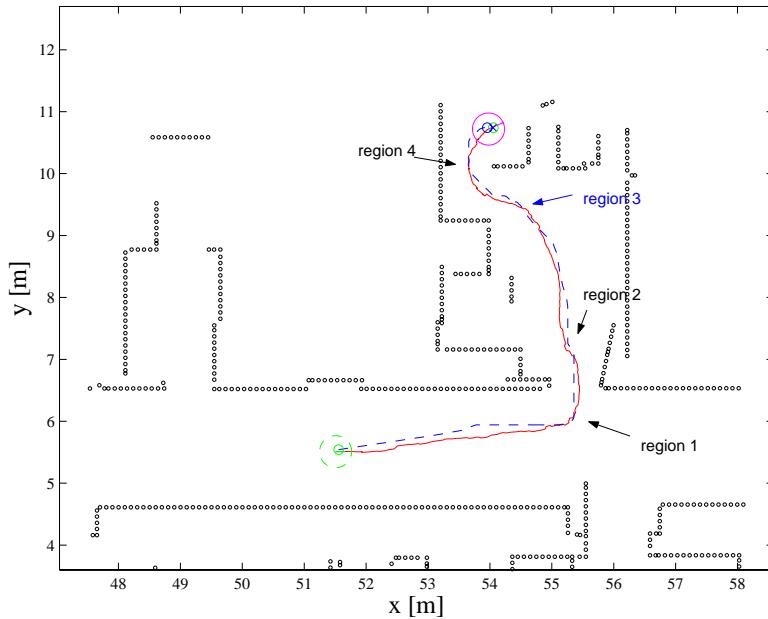
- When significant curvature changes are encountered along the local path, robot must slow down to follow the path at a safe margin
- Translational velocity of the robot is directly influenced by **modified reference point dynamics**:

$$\dot{s} = \frac{c_o e^{-\alpha\rho} v_n}{\sqrt{p'^2(s) + q'^2(s)}} \left[ 1 - \frac{2}{\pi} \text{atan}(\zeta \chi(\rho)) \right]$$

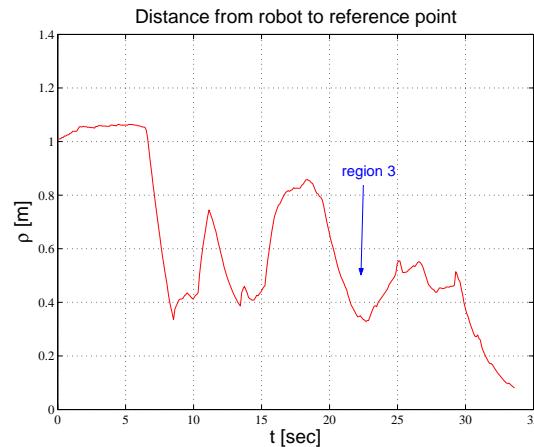
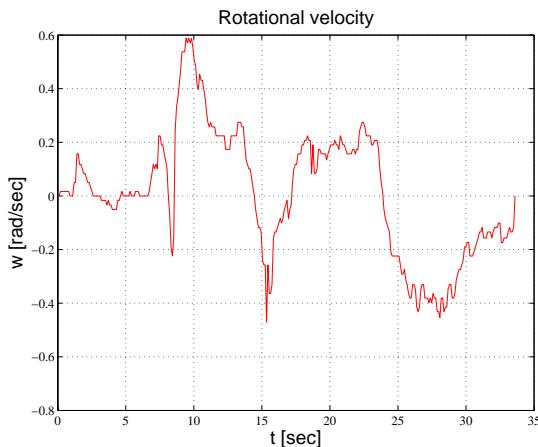
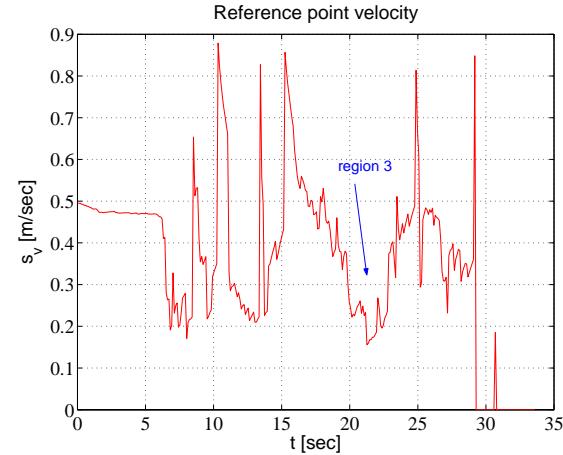
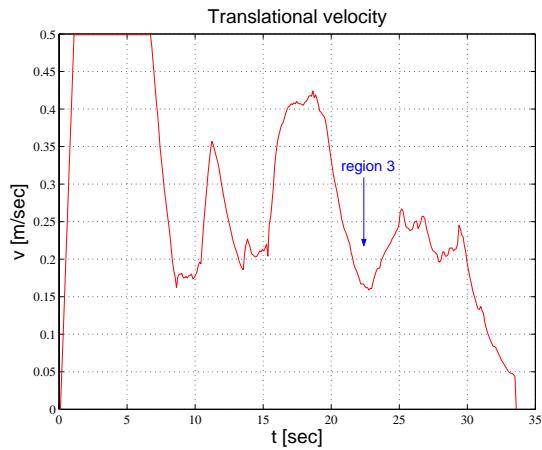
- Penalty factor  $P(\rho) \in [0, 1)$  :

$$P(\rho) = 1 - \frac{2}{\pi} \text{atan}(\zeta \chi(\rho))$$

# A modified virtual vehicle run (1)



# A modified virtual vehicle run (2)



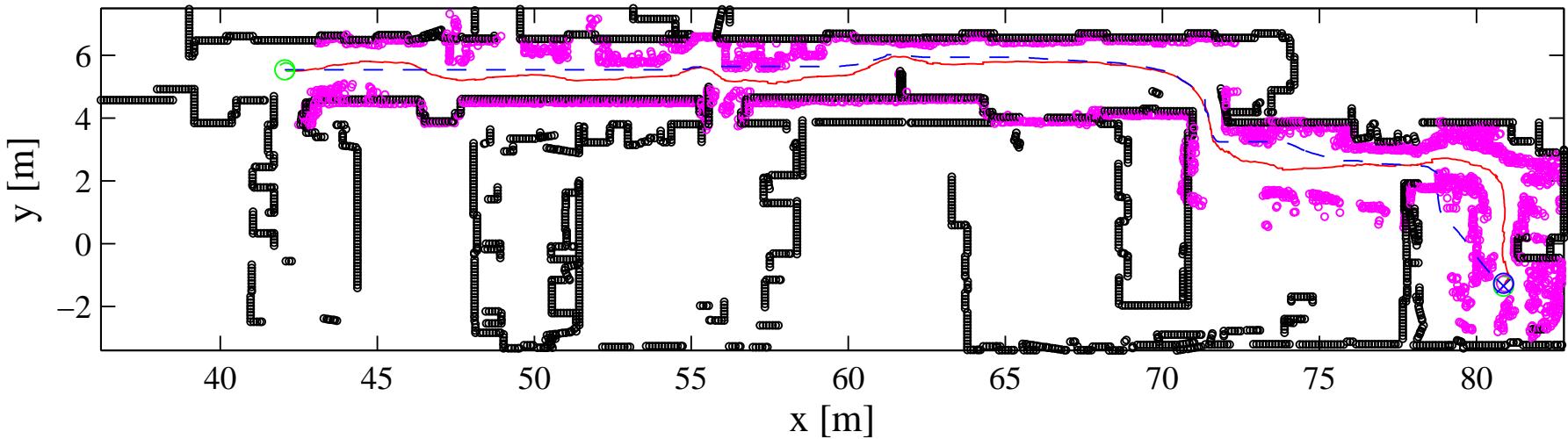
# Incremental path build-up

---

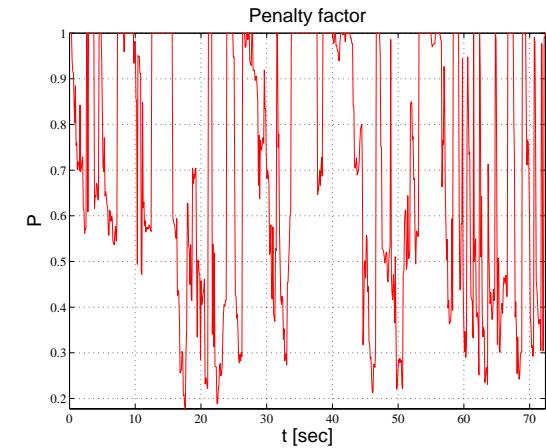
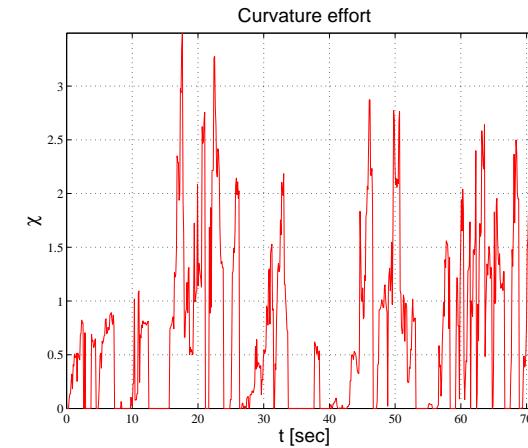
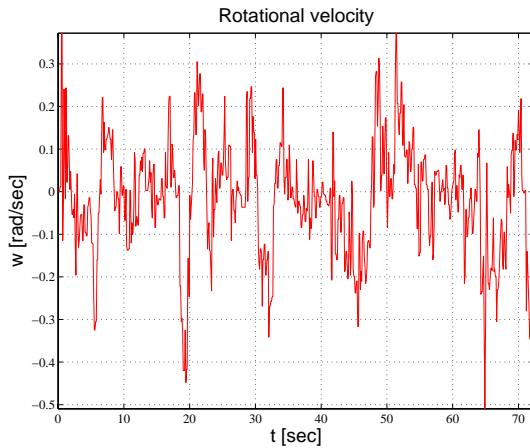
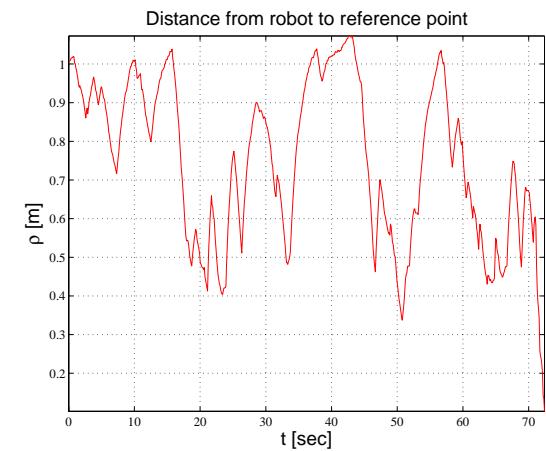
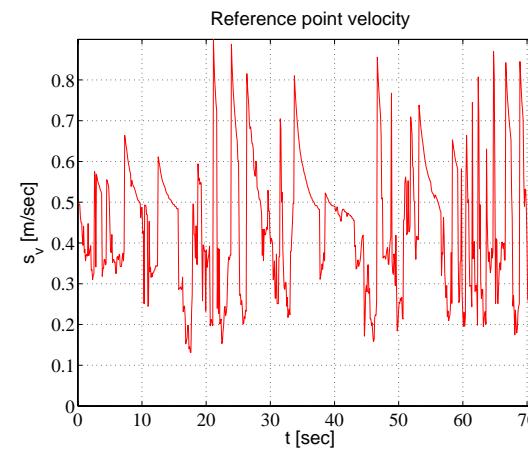
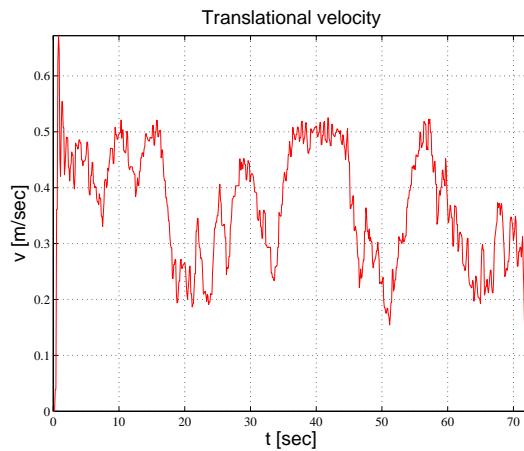
- On-line replanning based on local subgoal points
- Local curve characteristics and form changes (curve parameter values different) between two successive steps - artificial reference point oscillations could occur
- Solution - one step simulation of both robot and reference point motion
- Localization problem

# Experimental results – Modified Virtual Vehicle (1)

- Results comparable to commonly used Gradient navigation method (*[Konolige et al., 2003]*):



# Experimental results – Modified Virtual Vehicle (2)



# Conclusions

---

- Lane detection vision module for driver assistance based on a probabilistic framework
- Off-line path planning based on curvature change and safety distance optimality criteria
- On-line path planning based on graph search A\* and D\* algorithms on an occupancy grid map
- Global path integration method with Dynamic Window obstacle avoider for motion control
- Modified virtual vehicle approach to motion control with on-line global smooth path replanning scheme

---

# THANK YOU!