Network Science HS24

Assignment 4

Blockchain & Distributed Ledger Technologies Group

FOR STUDIES PURPOSES ONLY

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1 Random Graph Models (6 points)

1. (1 point) Consider Erdős-Rényi $G(N, p)$ model. Generate Erdős-Rényi graphs with $N = 100$ nodes for different edge creation probabilities $p \in [0, 1]$ and plot the probability that a node belongs to the Largest Connected Component N_{LCC}/N as a function of p and mark with a vertical line the critical probability $p_C = 1/N$.

   Hint: You can use `Networkx` *function* `nx.erdos_renyi_graph` *to produce the graphs. Anyway, writing your own function will be judged positively.*

   Hint: To plot the probability N_G/N you need to average your results by generating many (approx. 100) graphs for each value of p. Use logarithmic spacing for the values of p.

2. (1 point) Generate ER graphs with $N = 100$ nodes for different edge creation probabilities $p \in [0, 1]$ and plot the average clustering coefficient $\langle c \rangle$ and the Average Path Length $\langle \mathbf{d} \rangle$ as a function of p. Provide an interpretation of the result in light of what we discussed during the lecture.

3. (1 point) Write a function to generate a one-dimensional lattice with periodic boundary conditions and coordination number k, defined as we saw in the class and assignment 1. The function should take in input two parameters:

   - N: the number of nodes of the lattice,
   - k: the coordination number,

   and it should output a network (the data structure to represent the network is left to your choice, in the hint a suggestion is provided). You have to implement the algorithm yourself and cannot rely on modules' functions. Finally, produce a visualisation of a network generated using such function, with $N = 21$ and $k = 5$. *Hint: We suggest you to return the network as a* `networkx.Graph` *object because it is going to make your life easier in the later points, but ultimately it is your decision. Hint: In* `Networkx` *you can use the function* `networkx.circular_layout()` *for the layout.*

4. (1 point) Compute the Average Path Length $\langle \mathbf{d} \rangle$ of a sequence of one-dimensional periodic lattices where $k = 10$ is fixed and on the number of nodes N ranges between 50 and 1000. Then plot $\langle \mathbf{d} \rangle$ as a function of N. What is the relation between the $\langle \mathbf{d} \rangle$ and N? Comment the plot.

   Hint: In order to calculate the average distance (also known as the average shortest path length) you can use the `networkx` *function:*

   ```
   networkx.average_shortest_path_length()
   ```

5. (1 point) Implement the original Watts-Strogatz Watts and Strogatz [1998] model, following the example we saw in the slides. It should take in input three parameters:

   - N: the number of nodes of the network,
   - k: the coordination number of the 1-d original lattice with periodic condition,
   - p: the rewiring probability,

   and it should output a network (the data structure to represent the network is left to your choice, in the hint a useful suggestion is provided). Implement the algorithm yourself and do not rely on `networkx`/third parties module functions. Finally, plot the network generated using such a function, for $N = 21, p = 0.1, k = 5$, using a circular layout.

Hint: We suggest you return the network as a `networkx.Graph` *object because it is going to make your life easier in the later points, but ultimately it is your decision*.

6. (1 point) Compute and plot toegheter $\langle \mathbf{d} \rangle$ and c, the global clustering coefficient, on a sample of networks generated from the Watts-Strogatz model (using the python function you wrote in the previous point), as a function of $p \in [0, 1]$ (the rewiring parameter), keeping $N = 1000$ and $k = 10$ fixed. Normalize the value using the clustering and Average Path Distance of a one-dimensional lattice with parameters $N = 1000$ and $k = 10$, to obtain a plot similar to the one in Watts and Strogatz [1998].

   Hint: Remember you need to compute $\langle \mathbf{d} \rangle$ and c as an average over a sample of networks (approx. 10 or 20 should be enough) for each choice of the parameters. The goal is to recreate the famous "small-world" plot from Watts and Strogatz [1998]. Because connectivity is not ensured, make sure to check for unconnected graph outliers.

   Hint: Use a logarithmic spacing for p.

   Hint: You can use `networkx` *functions for this exercise, such as the function to compute the average shortest path length we saw before or the networkx function to compute the clustering coefficient.*
   Hint: Allocate time in advance for this last item, computations may take some time.

References

Watts, D. J. and Strogatz, S. H. [1998]. Collective dynamics of ‘small-world’networks, *nature* **393**(6684): 440–442.