Network Science HS24

Assignment 1

Blockchain & Distributed Ledger Technologies Group

FOR STUDIES PURPOSES ONLY

UZH Blockchain Center
Faculty of Business, Economics and Informatics
University of Zurich
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One-dimensional Lattice (6 points)

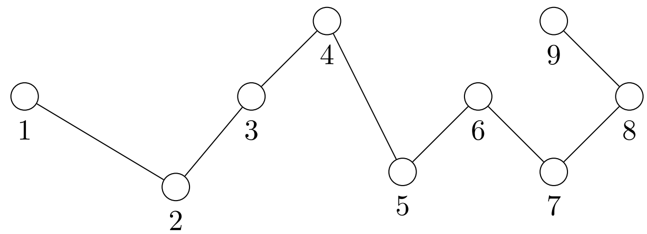

Figure 1: A simple network.

1. (1 point) Consider the network $G = (V, E)$ depicted in Figure 1. Compute the Diameter d_{max} of graph G, defined as the maximum distance between any pair of vertices $i, j \in V$:

$$d_{max} = \max_{i,j \in V} d_{ij}$$

Where d_{ij}, the distance between two nodes, is the number of links on the shortest path between two vertices.

Figure 2: An open chain C of length n.

2. (1 point) Consider a chain, defined as follows:

Definition 1. *A chain $C = (V, E)$ of length n is an undirected one-dimensional lattice with no boundary periodic condition and coordination number $k = 1$, where $V = \{1, \ldots, n+1\}$ and $E = \{(i, i+1), \quad i = 1, \ldots, n\}$. The length of the chain corresponds to the number of edges in the chain.*

Two examples of chains are provided in Figures 1,2. Write down the explicit expression of the distance d_{ij} between any two nodes i, j of a chain C of length $n-1$. Write down clearly all the passages and your reasoning.

3. (1 point) Use the result from the previous question to calculate the explicit expression of the Average Distance (or Average Path Length) for a chain C of length $n-1$. Remember the Average Distance is defined as:

$$\langle \mathbf{d} \rangle = \frac{1}{n(n-1)} \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij}. \tag{1}$$

4. (1 point) Consider the network in Figure 3 and calculate the Average Distance (as defined in eq. 1).

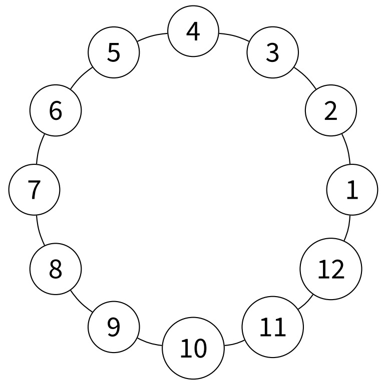

Figure 3: This network looks symmetric.

5. (1 point) Consider a ring, defined as follows:

   Definition 2. *A ring $R = (V, E)$ of size n is an undirected one-dimensional lattice with periodic condition and coordination number $k = 1$, where $V = \{1, \ldots, n\}$ and $E = \{(1,2), (2,3), (3,4), \ldots, (n-1, n), (n, 1)\}$.*

   You can now appreciate that the graph is figure 3 is a ring of size 12. Write down the explicit expression of the Average Distance on a generic ring of size n when n is an odd number. Write down clearly all the passages and your reasoning.

   Hint: take into account the symmetry of the problem. Start by finding the formula for L_i the average distance from the generic node i:

$$L_i = \frac{1}{n-1} \sum_j d_{ij}.$$

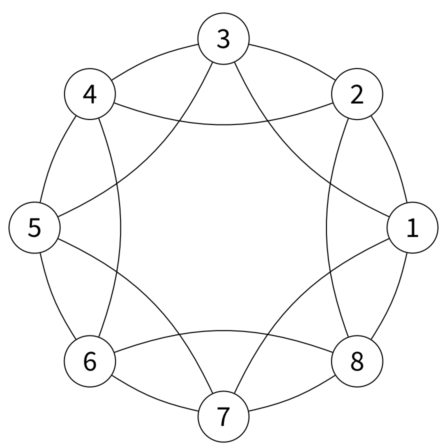

Figure 4: A 1-d lattice with periodic conditions of size $n = 8$ and $k = 2$.

6. (1 point) Consider a generic 1-d lattice with coordination number k and number of nodes n, in the special case where n is odd and $\frac{n-1}{2k}$ is an integer (meaning $n - 1$ is divisible by $2k$). Write down the explicit expression of the Average Distance for such a network. Write down clearly all the passages and your reasoning.