Anisotropic Acoustic Waves In Rarefied Nematic Liquid Crystals

P. E. Farrell *, U. Zerbinati *

* *Mathematical Institute*
University of Oxford

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Why Rarefied Nematic Liquid Crystal ?

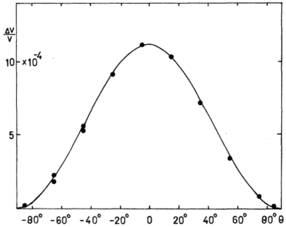

FIG. 2. Angular dependence of sound velocity. $T=21°C$, $\nu=10$ MHz, and $H=5$ kOe. θ is the angle between the field direction and propagation direction. Solid line is $12.5\times10^{-4}\cos^2\theta$.

Figure: It was observed in [MLS72] that acoustic waves travel in NLC faster in the direction parallel to the nematic director.

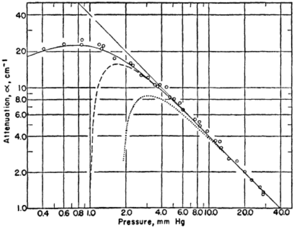

FIG. 1. Attenuation of sound at 1 Mc/sec. in helium. Circles—experimental results. Heavy full line—exact hydrodynamic. Light full line—first approximation, hydrodynamic and Burnett. Dashed line—second approximation, hydrodynamic. Dotted line—second approximation, Burnett.

Figure: It was observed in [Gre49] that first order theory better fit experimental data on acoustic attenuation at low pressure.

Curtiss Collision Operator

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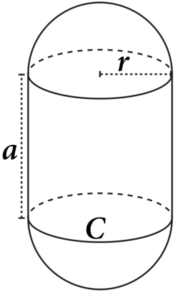
Curtis in his seminal paper [Cur56] proposed a kinetic theory for spherocylindrical molecules as an idealisation of polyatomic gas.

- His considered a larger configuration space made by **position**, **velocity**, **Euler's angles** for describing the orientation of each molecules and the **angular velocity** with respect to a fixed coordinate system.
- Molecules would interact by **excluded volume**, which give rise to **short range interactions** hence the **nematic ordering**.



This led Curtiss to formulate the following **Boltzmann** type equation,

$$\partial_t f + \nabla_{\boldsymbol{r}} \cdot (\boldsymbol{v} f) + \nabla_{\boldsymbol{\alpha}} \cdot (\dot{\boldsymbol{\alpha}} f) = C[f, f] \tag{1}$$

where $f(\boldsymbol{r}, \boldsymbol{v}, \boldsymbol{\alpha}, \boldsymbol{\omega})$ is the usual first reduced distribution function and $C[f, f]$ is the collision operator defined as

$$C[f, f] = -\int\int\int\int (f_1' f' - f_1 f)(\boldsymbol{k} \cdot \boldsymbol{g}) S(\boldsymbol{k}) d\boldsymbol{k} d\boldsymbol{v}_1 d\boldsymbol{\alpha}_1 d\boldsymbol{\omega}_1 \tag{2}$$

with $\S(\boldsymbol{k}) d\boldsymbol{k}$ being the surface element of the excluded volume and $\boldsymbol{g} = \boldsymbol{v} - \boldsymbol{v}_1$. Here with out loss of generality the equation is stated in **absence of external force** and **torque**.

References

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- M. E. Mullen, B. Lüthi, and M. J. Stephen, *Sound velocity in a nematic liquid crystal*, Phys. Rev. Lett. **28** (1972), 799–801.