Probabilistic Robotics	Recursive state estimation (exercises)

1. Suppose we live at a place where days are either sunny, cloudy, or rainy. The weather transition function is a Markov chain with the following transition table:

		Tomorrow will be …		
		Sunny	Cloudy	Rainy
Today it's…	Sunny	0.8	0.2	0.0
	Cloudy	0.4	0.4	0.2
	Rainy	0.2	0.6	0.2

   a) Suppose Day 1 is a sunny day. What is the probability of the following sequence of days: Day2 = cloudy, Day3 = cloudy, Day4 = rainy ?
   b) Write a simulator that can randomly generate sequences of "weathers" from this state transition function.
   c) Use your simulator to determine the stationary distribution of this Markov chain. The stationary distribution measures the probability that a random day will be sunny, cloud, or rainy.
   d) Can you devise a closed-form solution to calculating the stationary distribution based on the state transition matrix above ?
   e) What is the entropy of the stationary distribution ?
   f) Using Bayes rule, compute the probability table of yesterday's weather given today's weather. It is okay to provide the probabilities numerically, and it is also okay to rely on results from previous questions in this exercise.
   g) Suppose we added seasons to our model. The state transition function above would only apply to the Summer, whereas different ones would apply to Winter, Spring, and Fall. Would this violate the Markov property of this process? Explain your answer.

2. Suppose that we cannot observe the weather directly, but instead rely on a sensor. The problem is that our sensor is noisy. Its measurements are governed by the following measurement model:

		Our sensor tells us …		
		Sunny	Cloudy	Rainy
The actual weather is …	Sunny	0.6	0.4	0.0
	Cloudy	0.3	0.7	0.0
	Rainy	0.0	0.0	1.0

   a) Suppose Day 1 is sunny (this is known for a fact), and in the subsequent four days our sensor observes cloudy, cloudy, rainy, sunny. What is the probability that Day 5 is indeed sunny as predicted by our sensor ?
   b) Once again, suppose Day 1 is known to be sunny. At Days 2 through 4, the sensor measures sunny, sunny, rainy. For each of the Days 2 through 4, what is the most likely weather on that day ? Answer the question in two ways: one in which only the data

available to the day in question is used, and one in hindsight, where data from future days is also available.

c) Consider the same situation (Day 1 is sunny, the measurements for Days 2, 3, and 4 are sunny, sunny, rainy). What is the most likely sequence of weather for Days 2 through 4? What is the probability of this most likely sequence ?

3. In this exercise we will apply Bayes rule to Gaussians. Suppose we are a mobile robot who lives on a long straight road. Our location x will simply be the position along this road. Now suppose that initially, we believe to be at location $x_{init} = 1000m$, but we happen to know that this estimate is uncertain. Based on this uncertainty, we model our initial belief by a Gaussian with variance $\sigma_{init}^2 = 900m^2$.

   To find out more about our location, we query a GPS receiver. The GPS tells us our location is $z_{GPS} = 1100m$. This GPS receiver is known to have an error variance of $\backslash sigma_{init}^2 = 100m^2$.

   a) Write the probability density functions of the prior $p(x)$ and the measurement $p(z|x)$.
   b) Using Bayes rule, what is the posterior $p(x|z)$? Can you prove it to be Gaussian?
   c) How likely was the measurement $x_{GPS} = 1100m$ given our prior, and knowledge of the error probability of our GPS receiver?