DS-Assignment 02

Question.No:-01

Solution:-

Color $= 12$, Size $= 3$, gender $= 2$

Using "Product Rule" for two events; picking up each colour shirt for each gender and second event is picking size

$$12 \times 3 \times 2 = 72 \text{ different types of shirts.}$$

Question.No:-02

Solution:-

There are 26 possible lowercase alphabets:-

Strings of length 4:-

$$26 \times 26 \times 26 \times 26 = 456,976$$

Strings of length 4 without Letter 'x':-

$$25 \times 25 \times 25 \times 25 = 390,625$$

Strings of letter 4 with atleast one 'x':-

$$456,976 - 390,625 = 66,351 \text{ strings}$$

Question.No:-03

Solution:-

There are 26 possible uppercase letters and 10 digits:-

For Three Letters followed by Three digits:-

$$26^3 \times 10^3 = 17,576,000$$

For Four Letters folled by Two digits:-

$$26^4 \times 10^2 = 45,697,600$$

For overall combinations, using " sum rule."

$$17,576,000 + 45,697,600 = 63,273,600 \text{ ways}$$

Question.No:-04

a) Solution:-

Overall we have to arrange 6 people out of 10 in row. For bride to be present must ; then only need to arrange 5 out of 9 people. Also bride can be in any of the 6 positions

So,

$$6 \times {}^9P_5 = 90,720 \text{ ways}$$

OR $\quad \underset{\substack{\text{bride's} \\ \text{position}}}{6 \times \underset{1}{1}} \times \underset{\text{people arrangement}}{\underline{9 \times 8 \times 7 \times 6 \times 5}}$

b) Solution:-

For both groom and bride to be present; only need arrangement of 4 out of 8 people ; also 6 positions for bride while 5 for groom for arrangement.

So,

$$6 \times 5 \times {}^8P_4 = 50,400 \text{ ways}$$

c) Solution:-

Now since exactly one has to be in picture, so selecting bride and leaving groom, so need to arrange 5 out of 8 people with 6 positions for bride or ~~groom~~ and then multiplying by 2 for vice versa.

$$2 \times 6 \times {}^8P_5 = 80,640 \text{ ways}$$

Question.No:-05

a) Solution:-

There are 20 balls, half of them are red and half blue. For atleast three balls of same color to be picked. Using pigeon-hole principle:-

$$\left\lceil \frac{n}{2} \right\rceil \geq 3 \; ; \; \left\lceil \frac{5}{2} \right\rceil = \lceil 2.5 \rceil = 3$$

So the positive integer which satisfies equation is 5; hence 5 balls must be picked to gurantee that atleast 3 balls are of same color.

b) Solution:-

For atleast 3 blue balls to be picked; looking at worst-case scenario that all first 10 choices may be red balls, so the woman needs to pick 13 balls to gurantee that atleast three are blue.

Question.No:-06

a) Solution:-

The order of bits is not important as we are interested in number of ones, not the number order formed by strings. So using combination. $n = 12, r = 3$

$$^{12}C_3 = 220 \text{ bit strings containing exactly three ones.}$$

b) Solution:-

Since the 12 bit strings atmost (maximum) contains three ones but can be lesser also. So,

$$n = 12 \; , \; r <= 3$$

Overall combinations for each r : -

$$^{12}C_3 + {}^{12}C_2 + {}^{12}C_1 + {}^{12}C_0$$

$$220 + 66 + 12 + 1 = 299 \text{ bit strings containing atmost three ones}$$

c) Solution:-

Since now the bit strings atleast containing (minimum) three ones but can be more so $n = 12, r \geq 3$
Evaluating overall for all r's :-

$$^{12}C_3 + {}^{12}C_4 + {}^{12}C_5 + {}^{12}C_6 + {}^{12}C_7 + {}^{12}C_8 + {}^{12}C_9 + {}^{12}C_{10} +$$

$$^{12}C_{11} + {}^{12}C_{12} = 220 + 495 + 792 + 924 + 792 + 495 + 220$$

$$+ 66 + 12 + 1 = 4017 \text{ bit strings containing atleast three ones.}$$

d) Solution:-

For equal number of 0s' and 1's in a string of length 12, there are six 0's and six 1's :-

$$^{12}C_6 = 924 \text{ bit strings containing equal number of 0's and 1's.}$$

Question.No:-07

Solution:-

To select a committee of six members containing more women than men out of department having 10 men and 15 women; women must be more than 3 while men must be less than 3.

For 4 women and 2 men:-

$$^{15}C_4 \times {}^{10}C_2 = 61,425$$

For 5 women and 1 man:-

$$^{15}C_5 \times {}^{10}C_1 = 30,030$$

For 6 women and no man:-

$$^{15}C_6 \times {}^{10}C_0 = 5005$$

Using sum rule to evaluate total ways for committee selection:-

$$61,425 + 30,030 + 5005 = 96,460 \text{ ways}$$