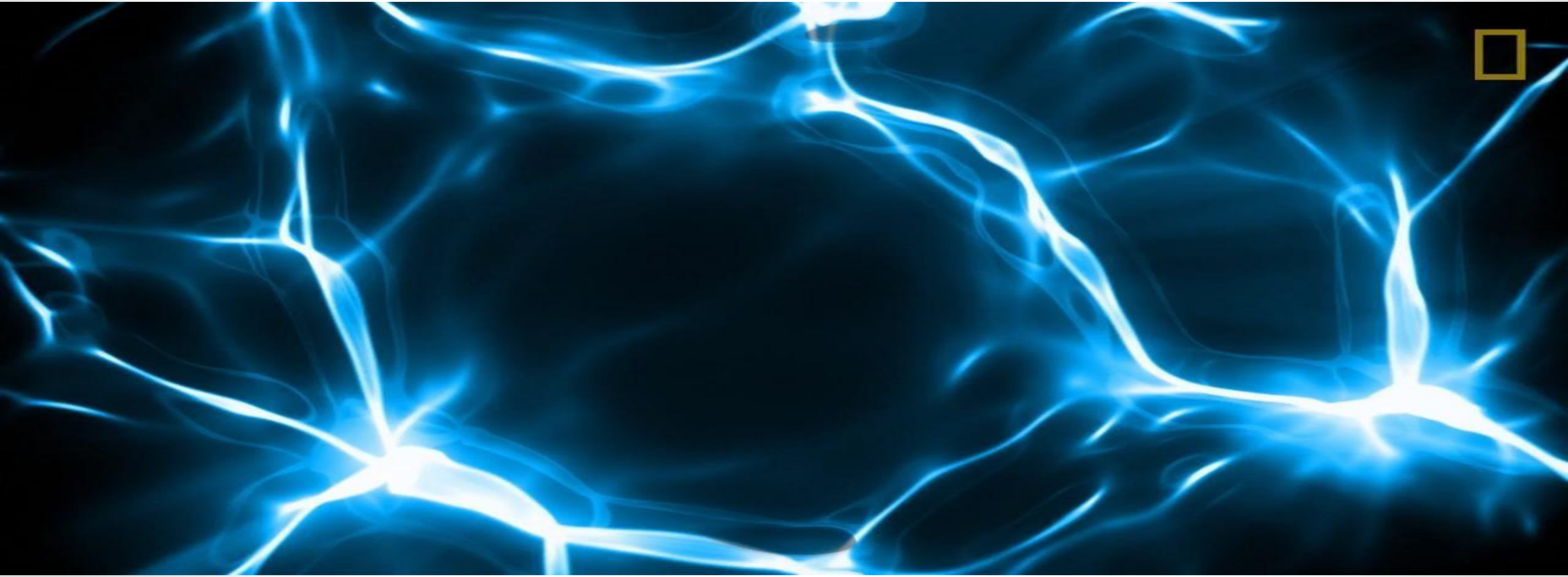
ELECTRICITY AND MAGNETISM



Electrostatics and Electrodynamics:

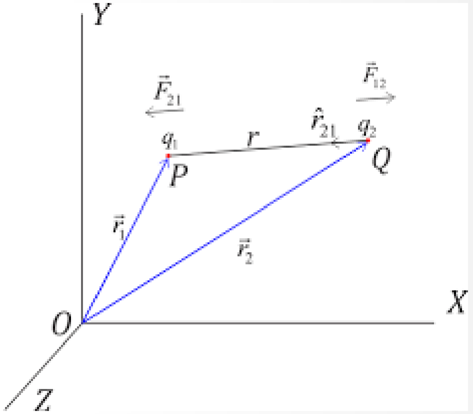
- It is the study of electromagnetic phenomenon when electric charges are at rest (static electricity). It involves electric charges, the forces between them and their behavior.
- It is the study of electromagnetic phenomenon when electric charges are in motion (dynamic electricity).

CONSERVATION OF CHARGE:

"Electrons can never be created nor be destroyed, but are simply transferred from one material to another."

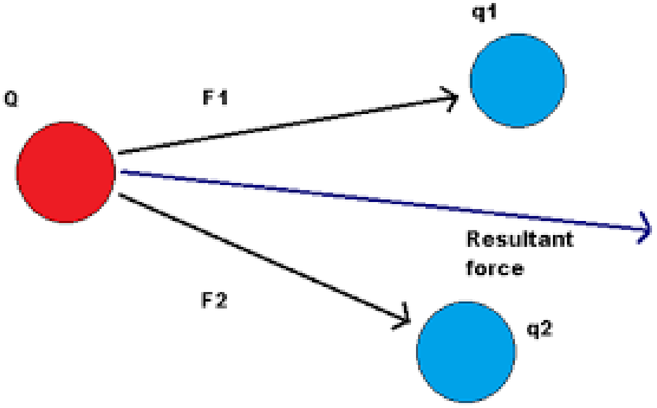
Coulomb's Law

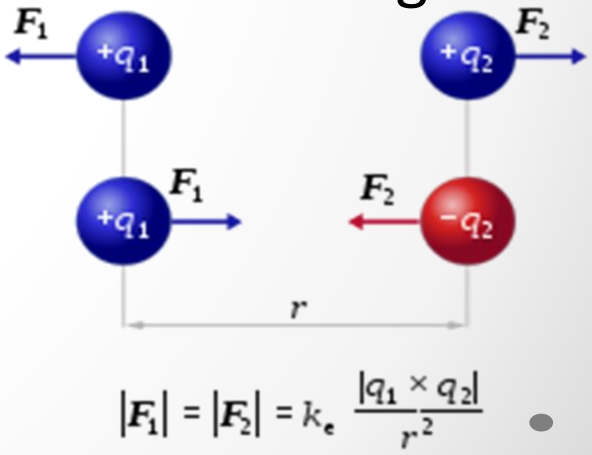
- "If there are two particles having electrical charges q and qo , then the force will directly proportional to production of charges and inversely proportional to the square of the distance between the charges"



- $F = kqq_0/r^2$ (magnitude of force)
- $F_{12} = \frac{k\, q1\, q2}{r_{12}{}^{\wedge}2}\, \hat{r}_{12}$ (vector from of Coulomb's law)
- For multiple charges net force is found by vector addition of all forces through super-position principle.

$$\vec{F} = \Sigma_{i=1}^{n}\, \vec{F_i}$$

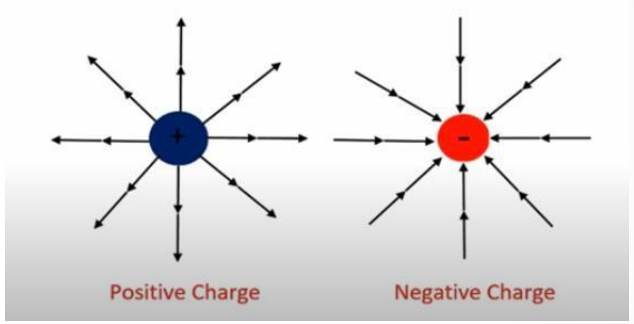




Electric Field

- Force exerted per unit positive unit test charge

$$E = \frac{F}{q_0}$$



- Similar to force due to multiple charges, electric field due to multiple charges is also given by super-position principle.

$$\vec{E} = \sum_{i=1}^{n} \overrightarrow{E_i}$$

Continuous Charge Distribution

- Charges are quantized (discrete in nature);
- So what is continuous charge distribution?

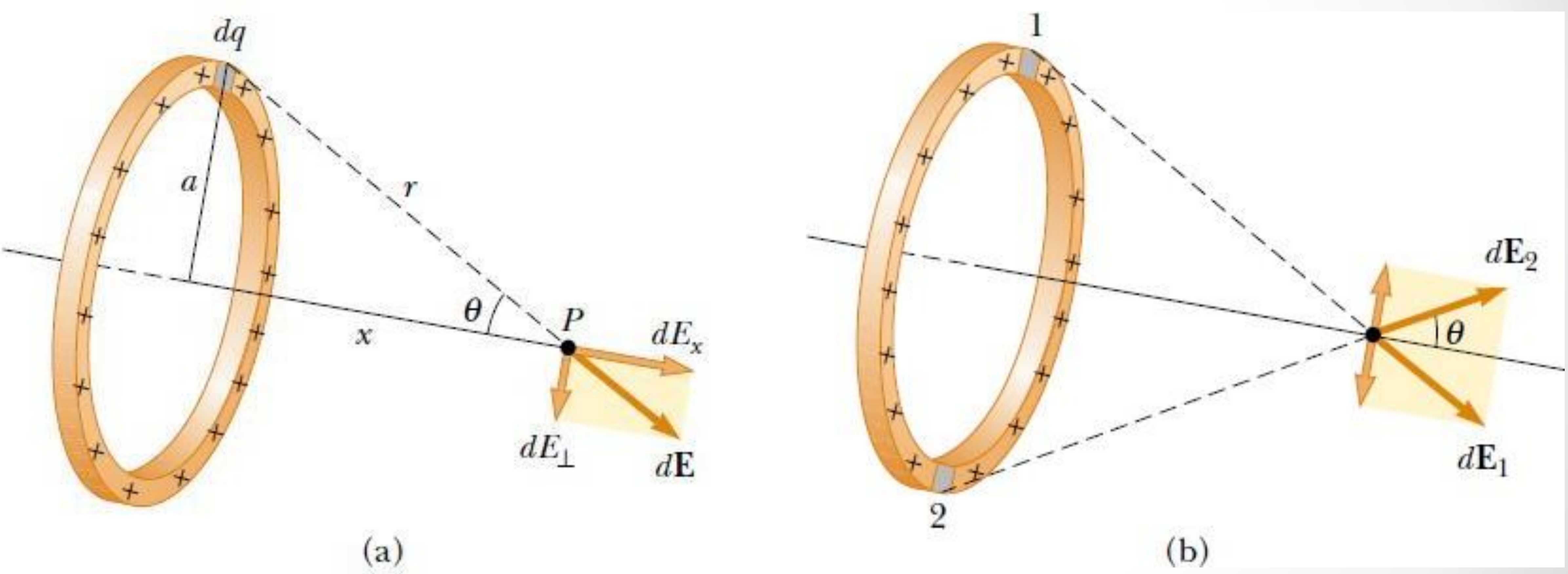
"*An assembly of charges that has a high density i.e the charges are so closely packed that it is not possible to identify each charge in isolation or individually*".

- Charge density = Electric charge per unit length
- Charge density can be of :
  - ➢ linear charge density (λ),
  - ➢ surface charge density (σ), and
  - ➢ volume charge density (ρ).

Linear Charge Distribution

- Linear charge density is defined as $\lambda = \frac{q}{l}$
- $\frac{q}{l} = \frac{\mathrm{dq}}{\mathrm{dl}}$ (Uniform charge distribution).
- $\frac{q}{l} \neq \frac{\mathrm{dq}}{\mathrm{dl}}$ (non-uniform charge distribution).
- Electric field due to an element of distribution $dE = \frac{d}{r^2 q}$
- Electric field due to entire distribution $E = \int dE = \int \mathrm{k} \frac{(\lambda dl)}{r^2}$ $\leftarrow$ $\lambda = \frac{dq}{dl}$

Example



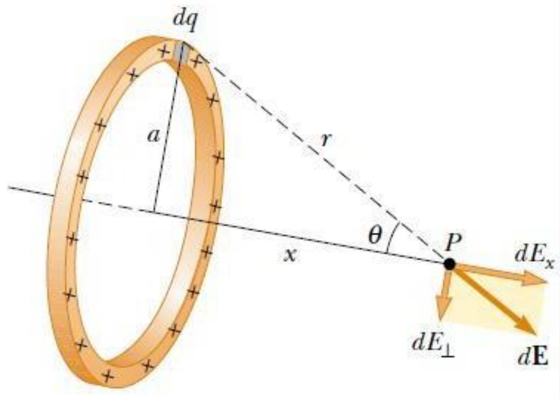
Solution

$$dE = k_e \frac{dq}{r^2}$$

But according to figure only x- components add up to give the electric field at point P.

$$dE_x = dE \cos\theta$$

From figure $r = (x^2 + a^2)^{1/2}$ and $\cos\theta = x/r$



(a)

$$dE_x = dE\cos\theta = \left(k_e \frac{dq}{r^2}\right)\frac{x}{r} = \frac{k_e x}{(x^2+a^2)^{3/2}}\, dq$$

$$E = k\left(\frac{\lambda dl}{r^2}\right) = dE$$

$$E_x = \int \frac{k_e x}{(x^2+a^2)^{3/2}}\, dq = \frac{k_e x}{(x^2+a^2)^{3/2}} \int dq \quad = \quad \frac{k_e x}{(x^2+a^2)^{3/2}}\, Q$$

Gauss' Law

- We know that flux through a surface is given by:

$$\mathbf{E} \cdot \Delta \mathbf{A}_i = E \Delta A_i$$

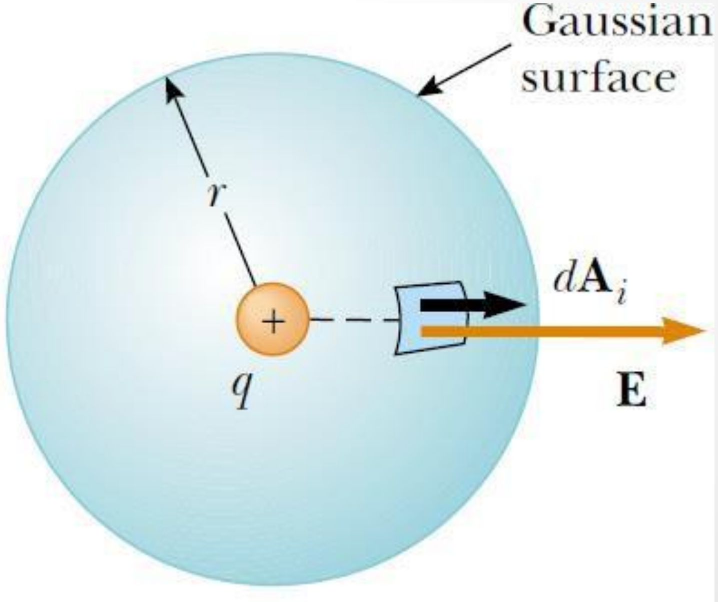
$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \oint E\, dA = E \oint dA$$

But,

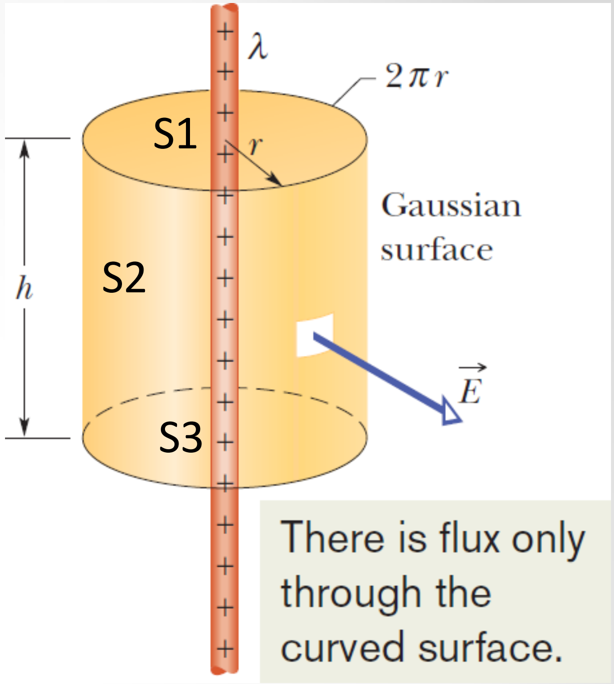
$$\oint dA = A = 4\pi r^2$$

$$\Phi_E = \frac{k_e q}{r^2}(4\pi r^2) = 4\pi k_e q$$ ke = 1/4πε0

$$\Phi_E = \frac{q}{\epsilon_0}$$



Application of Gauss's law



- Find electric field due to a line of charge by using Gauss's law
- We first consider a Gaussian Surface that
- surrounds the line of charge which has a linear charge density λ
- The Gaussian surface considered is cylindrical in shape and hence is made up of three surface S1, S2 and S3

Application of Gauss's law

- Applying Gauss's law

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{\text{in}}}{\epsilon_0}$$

- The integral will be broken down into three parts over S1, S2 and S3. But dot product is $EA \cos\theta$ non-zero only over S2. Therefore;

$$\Phi = EA \cos\theta = E(2\pi rh)\cos 0 = E(2\pi rh)$$

$$\varepsilon_0 \Phi = q_{\text{enc}},$$

$$\varepsilon_0 E(2\pi rh) = \lambda h$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Problem

- The net flux through each face of a dice has magnitude in units of $10^3 Nm^2/C$ equal to number 'N' of dots on the face (1-6). The flux is inward for 'N' odd and outward for 'N' even. What is the net charge inside the dice.

Solution

- Flux directed inward is taken as negative

$$\varphi_{odd} = -\varphi_1 - \varphi_3 - \varphi_5$$

$$\varphi_{odd} = -1X10^3 - 3\mathrm{X}10^3 - 5\mathrm{X}10^3 = -9\mathrm{X}10^3\mathrm{Nm}^2/\mathrm{C}$$

- Flux directed outward is taken as positive.

$$\varphi_{even} = \varphi_2 + \varphi_4 + \varphi_6 = 2X10^3 + 4\mathrm{X}10^3 + 6\mathrm{X}10^3$$

$$\varphi_{even} = 12\mathrm{X}10^3\mathrm{Nm}^2/\mathrm{C}$$

- Total flux;

$$\varphi = \varphi_{odd} + \varphi_{even} = 3X10^3 Nm^2/C$$

$$q = \varphi\epsilon_0 = 2.66\mathrm{X}10^{-8}\mathrm{C}$$

Magnetic Field

"The field that passes through space (vector field) and generated a magnetic force".

- A magnetic field is a vector field that describes the magnetic influence of electric charges in relative motion and magnetized materials. A charge that is moving parallel to a current of other charges experiences a force perpendicular to its own velocity.

$$B = F_B/(q \times v)$$

θ = angle between direction of velocity and magnetic field.

q =charge of the particle.

$F_B = qv \times B$ (this shows direction of force by using right hand rule)

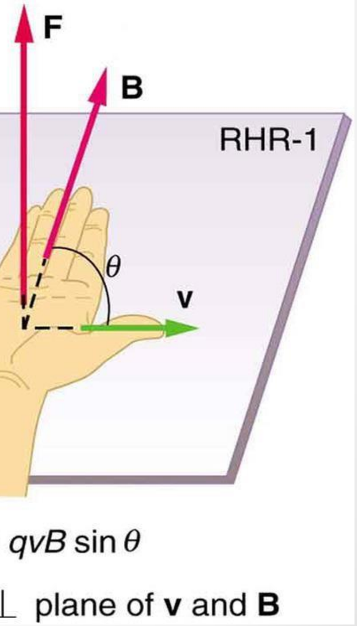
$F_B = qvB \sin\theta$ (this shows magnitude of force)

Unit:

1 Tesla =1newton/$\boxed{\text{(coulomb/second)}}$(meter)= 1N/A.m

Ampere

- $F_B = q\, v \times B$



Right hand rule

Biot-savart Law

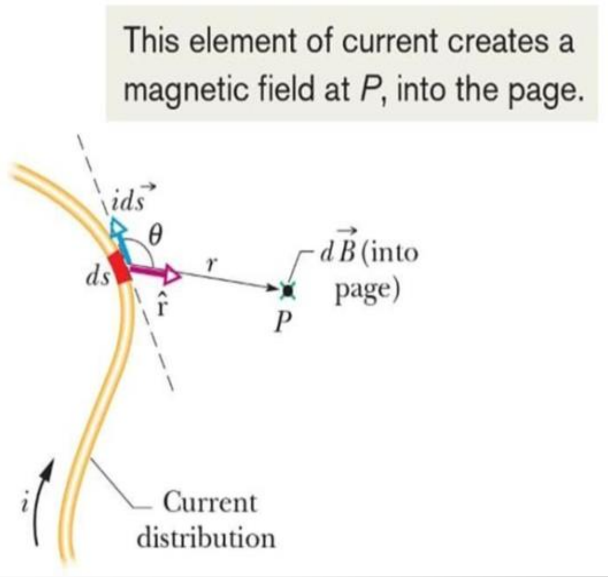
- The magnetic field set up by a current-carrying conductor can be found from the Biot–Savart law.

- This law asserts that the contribution to the field produced by a current-length element at a point P located a distance r from the current element is

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{s} \times \hat{\mathrm{r}}}{r^2} \qquad \text{(Biot–Savart law)}$$

$\mu_{0=}$ permiability of free space

$= 4\pi \times 10e\text{-}7$ h/m



Here r is a unit vector that points from the element toward P.

Ampere’s law

- Ampere's Law states that,

“For any closed loop path, the sum of the length elements times the magnetic field in the direction of the length element is equal to the permeability times the electric current enclosed in the loop".

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc} \quad \text{(Ampere's law).}$$

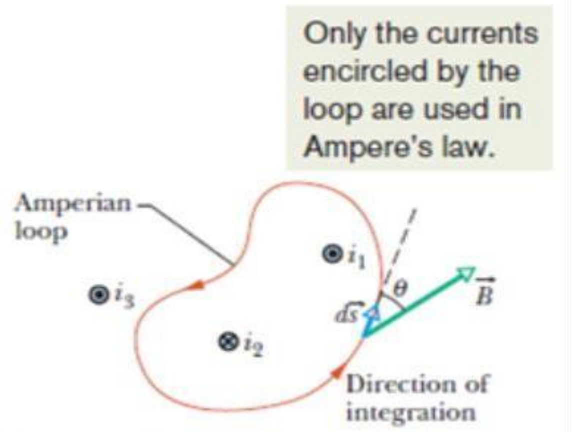


Figure Ampere’s law applied to an arbitrary Amperian loop that encircles two long straight wires but excludes a third wire. Note the directions of the currents.

- The line integral in this equation is evaluated around a closed loop called an Amperian loop.
- The current i on the right side is the net current encircled by the loop.

Magnetic Field Outside a Long Straight Wire with Current

A long straight wire that carries current '*i*' directly out of the page.

$$\oint \vec{B} \cdot d\vec{s} = \oint B \cos\theta \, ds = \mu_0 i_{enc}.$$

$$\oint \vec{B} \cdot d\vec{s} = \oint B \cos\theta \, ds = B \oint ds = B(2\pi r).$$

- Our right-hand rule gives us a plus sign for the current of Fig..The equation is now

$$B(2\pi r) = \mu_0 i$$

$$B = \frac{\mu_0 i}{2\pi r} \quad \text{(outside straight wire).}$$

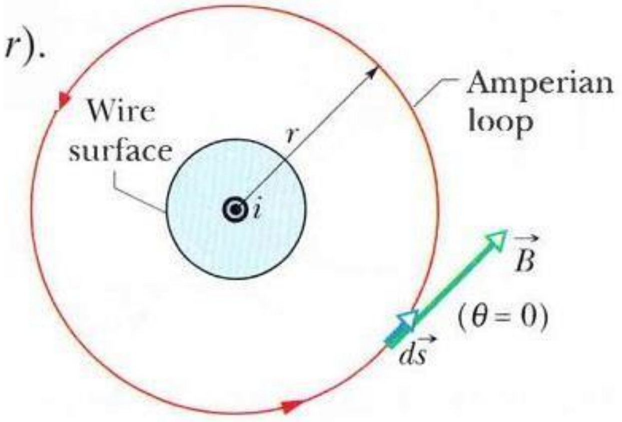


FIG. 29-14 Using Ampere's law to find the magnetic field that a current *i* produces outside a long straight wire of circular cross section. The Amperian loop is a concentric circle that lies outside the wire.

Magnetic Field Inside a Long Straight Wire with Current

- Cross section of a long straight wire of radius R with a uniformly distributed current i directly out of the page.
- Because the current is uniformly distributed over a cross section of the wire, the magnetic field produced by the current must be cylindrically symmetrical. Here r<R so

$$\oint \vec{B} \cdot d\vec{s} = B \oint ds = B(2\pi r).$$

- i_{enc} encircled by the loop is proportional to the area encircled by the loop; (current is uniformly distributed)

$$i_{\mathrm{enc}} = i \frac{\pi r^2}{\pi R^2}.$$

Only the current encircled by the loop is used in Ampere's law.

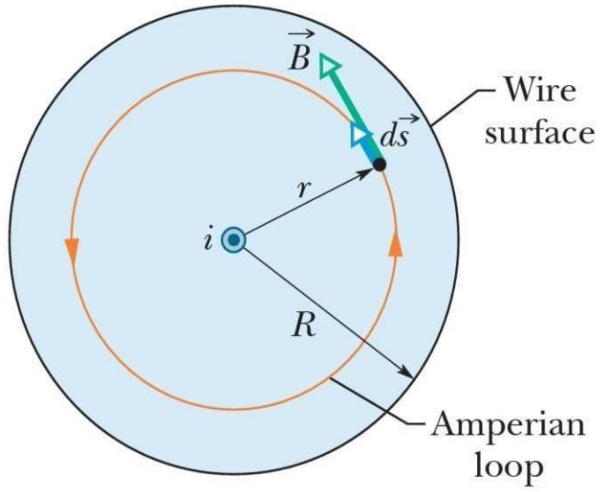
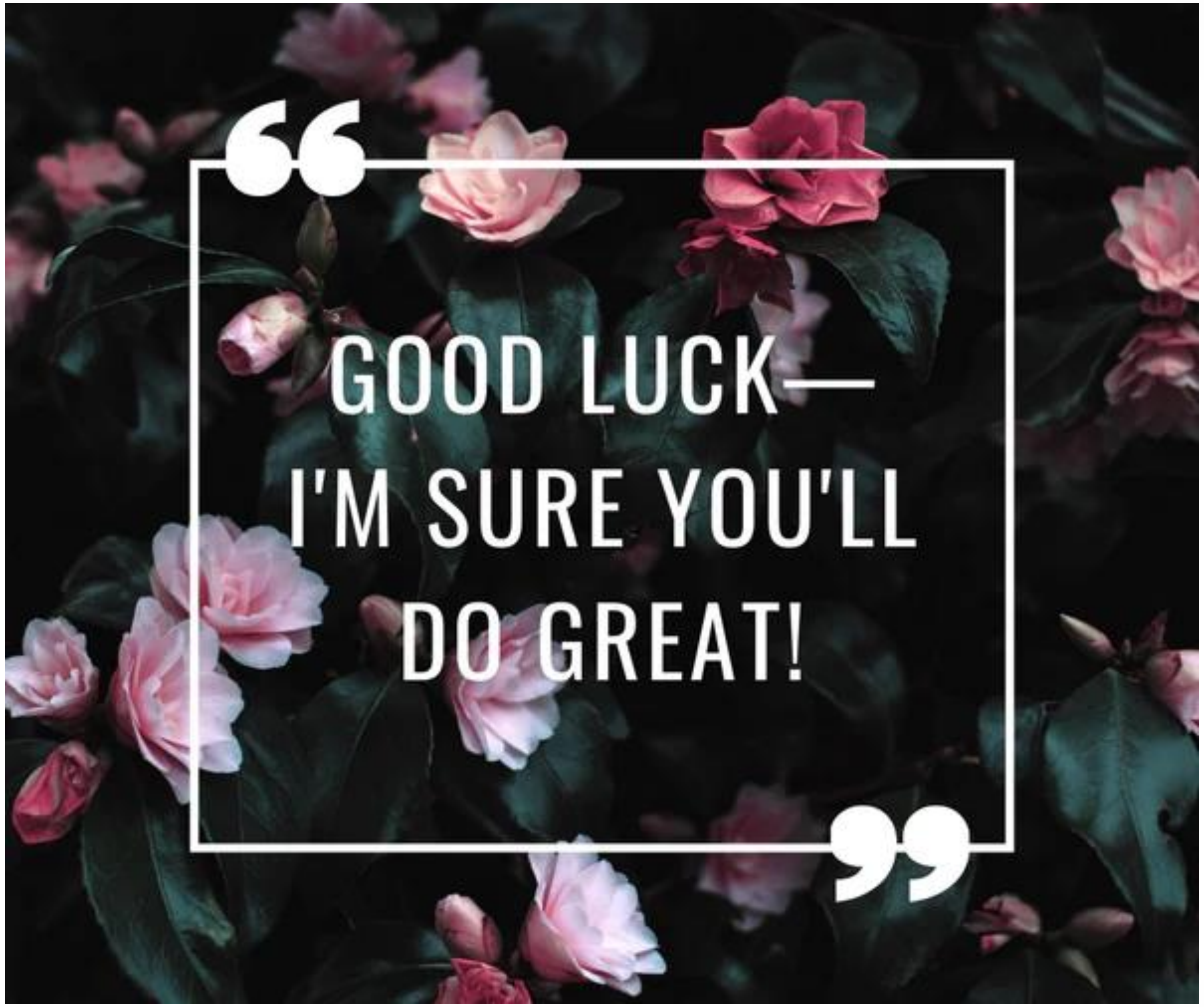


Figure Using Ampere's law to find the magnetic field that a current *i* produces inside a long straight wire of circular cross section. The current is uniformly distributed over the cross section of the wire and emerges from the page. An Amperian loop is drawn inside the wire.

- Our right-hand rule tells us that i_{enc} gets a plus sign. Then Ampere's law gives us

$$B(2\pi r) = \mu_0 i \frac{\pi r^2}{\pi R^2}$$

$$B = \left(\frac{\mu_0 i}{2\pi R^2}\right) r \qquad \text{(inside straight wire).}$$


GOOD LUCK—
I'M SURE YOU'LL
DO GREAT!