Question.No:-01

a) $\{0, 3, 6, 9, 12\}$

$\{x \mid x = 3k$ where k is a non-negative integer $\wedge k < 5.\}$

b) $\{-3, -2, -1, 0, 1, 2, 3\}$

$\{x \in Z \mid -3 \leq x \leq 3\}$

Question.No:-02

a) Set of People who speak English with Australian accent are a subset of people of who speak English. (i.e. $B \subseteq A$)

b) Set of citrus fruits are subset of set of all fruits. (i.e. $B \subseteq A$)

c) There could be some students in set of students studying discrete maths but not data structures and also vice versa. Therefore ($A \not\subseteq B$ or $B \not\subseteq A$) neither is a subset of other.

Question.No:-03

a) $\phi \in \{\phi\}$

True

b) $\phi \in \{\phi, \{\phi\}\}$

True

c) $\{\phi\} \in \{\phi\}$

False as the set containing empty set can not be an element of itself.

d) $\{\phi\} \in \{\{\phi\}\}$

True

e) $\{\phi\} \subset \{\phi, \{\phi\}\}$

True as for proper subset atleast one element of 1st set should not be member of other.

f) $\{\{\phi\}\} \subset \{\phi, \{\phi\}\}$

True

g) $\{\{\phi\}\} \subset \{\{\phi\}, \{\phi\}\}$

False, as for ~~subset or~~ proper-subset, atleast one element of 1'st set should be different but here both sets contain same element. but these two can be subsets of other. (like $\{\{\phi\}\} \subseteq \{\{\phi\}, \{\phi\}\}$

Question.No:-04

Given:-

$A - B = \{1, 5, 7, 8\}$

$B - A = \{2, 10\}$

$A \cap B = \{3, 6, 9\}$

Find:- $A = ?$, $B = ?$

Sol:-

According to sets formula, we know that :-

$A = (A - B) \cup (A \cap B)$, $B = (B - A) \cup (B \cap A)$ (same as $(A \cap B)$)

$A = \{1, 5, 7, 8\} \cup \{3, 6, 9\} = \{1, 3, 5, 6, 7, 8, 9\}$

$B = \{2, 10\} \cup \{3, 6, 9\} = \{2, 3, 6, 9, 10\}$

Question.No:-05

a) Since function consists of all pairs of positive integers, so its maximum will also be a positive integer.

$$\text{Domain} = \{(x, y) \mid x, y \in Z^+\}$$

$$\text{Range} = \{ x \mid x \in Z^+ \}$$

b) Since the function provides the count of block "11" appeared in the bit string (binary number) so ~~count~~ count of "11" can not be less than 0.

$$\text{Domain} = \{ x \mid x \in \{0^m, 1^n\} : m, n \in \mathbb{N} \}$$

(Set of all string bits)

$$\text{Range} = \{0, 1, 2, 3, \dots\} \text{ or } \{ x \mid x \in Z^+ \}$$

Question.No:-06

a) $(-2)^n$ Since we have simplified n^{th} term; so simply substitute 'n'.

$$a_0 = (-2)^0 = 1$$

$$a_1 = (-2)^1 = -2$$

$$a_2 = (-2)^2 = 4$$

$$a_3 = (-2)^3 = -8$$

b) $7 + 4^n$

$$a_0 = 7 + 4^0 = 7 + 1 = 8$$

$$a_1 = 7 + 4^1 = 7 + 4 = 11$$

$$a_2 = 7 + 4^2 = 7 + 16 = 23$$

$$a_3 = 7 + 4^3 = 7 + 64 = 71$$

Question.No:-07

a) $a_n = -2a_{n-1}$, $a_0 = -1$

$$a_1 = -2a_{1-1} = -2a_0 = -2(-1) = 2$$

$$a_2 = -2a_1 = -2(2) = -4$$

$a_3 = -2a_2 = -2(-4) = 8$

$a_4 = -2a_3 = -2(8) = -16$

$a_5 = -2a_4 = -2(-16) = +32$

$a_6 = -2a_5 = -2(+32) = -64$

So the first six terms of given recurrence relations are:-

$a_1, a_2, ----, a_6 = 2, -4, 8, -16, 32, -64$

b) $a_n = a_{n-1} - a_{n-2}$, $a_0 = 2$ and $a_1 = -1$

$a_2 = a_{2-1} - a_{2-2} = a_1 - a_0 = -1 - 2 = -3$ ~~$2-(-1) = 3$~~

$a_3 = a_2 - a_1 = -3 - (-1) = -2$

$a_4 = a_3 - a_2 = -2 - (-3) = 1$

$a_5 = a_4 - a_3 = 1 - (-2) = 3$

$a_6 = a_5 - a_4 = 3 - 1 = 2$

$a_7 = a_6 - a_7 = 2 - 3 = -1$

So the fix six terms of given recurrence relation are:-

$a_2, a_3, ----, a_7 = -3, -2, 1, 3, 2, -1$

Question. No:-08

a) Converse:-

If I will stay at home, then it snows tonight.

Contrapositive:-

If I will not stay at home, then it does not snow ~~rain~~ tonight.

Inverse:-

If it does not snow tonight then I will not stay at home.

b) Converse:-

It is a sunny summer day whenever I go to beach.

Contrapositive:-

It is not a sunny summer day whenever I do not go to beach.

Inverse:-

I do not go to beach whenever it is not a sunny summer day.

c) Converse:-
If I sleep until noon then it is necessary I stay up late.

Contrapositive:-
If I do not sleep until noon then it is necessary I did not stay up late.

Inverse:-
If I did not stay up late then it is necessary I do not sleep until noon.

Question.No:-09

a) $p \rightarrow \neg p$

p	$\neg p$	$p \rightarrow \neg p$
T	F	F
F	T	T

b) $p \leftrightarrow \neg p$

p	$\neg p$	$p \leftrightarrow \neg p$
T	F	F
F	T	F

c) $p \oplus (p \vee q)$

p	q	$p \vee q$	$p \oplus (p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	T
F	F	F	F

Question No:- 10

Let us suppose that:-

p = "System is being upgraded."

q = "User can access file system."

r = "User can save new files."

The following given propositional cases can be represented in symbols as:-

i) $p \rightarrow \neg q$

ii) $q \rightarrow r$

iii) $\neg r \rightarrow \neg q$

p	q	r	$\neg q$	$\neg r$	$p \rightarrow \neg q$	$q \rightarrow r$	$\neg r \rightarrow \neg q$
T	T	T	F	F	F	T	T
T	T	F	F	T	F	F	F
T	F	T	T	F	T	T	T
T	F	F	T	T	T	T	T
F	T	T	F	F	T	T	T
F	T	F	F	T	T	F	F
F	F	T	T	F	T	T	T
F	F	F	T	T	T	T	T

So all the propositional cases are True at various inputs like when

p = True , q = False , r = True

p = True , q = False , r = False

p = False , q = True , r = True

p = False , q = False , r = True

p = False , q = False , r = False

which specifies that the system specifications are consistent.