SEAT NO. CT-23025

NED UNIVERSITY OF ENGINEERING & TECHNOLOGY FIRST YEAR (Computer Science) SPRING SEMESTER EXAMINATIONS 2024

Time: 3 Hours

Batch 2023

Dated: 24-JUL-24 Max Marks: 60

Differential & Integral Calculus - MT-171

Instructions: Attempt all questions. All questions carry equal marks.

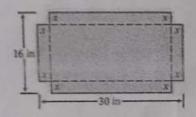
Question 1

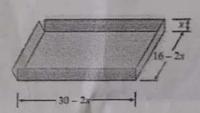
CLO 1

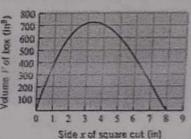
12 Marks

- (a) An open box is to be made from a 16-inch by 30-inch piece of card-board by cutting out squares of equal size from the four corners and bending up the sides.
 - Let V be the volume of the box that results when the squares have sides of length x. Define a formula for V as a function of x.
 - (ii) Identify the domain of V(x).
 - (iii) Use the graph of V given in figure to estimate the range of V(x).
 - (iv) Describe in words what the graph tells you about volume.

(4)







- (b) Discuss the continuity of the function $f(x) = \begin{cases} 0, & x \le -3 \\ \frac{x-3}{x^2-9}, & -3 < x < 3 \\ \frac{1}{6}, & x \ge 3 \end{cases}$ at x = -3 and x = 3. (4)
- (c) Identify the asymptotes (vertical, horizontal, and/or slant) for the function $g(x) = \frac{2x^2 x 3}{x 2}$. (4)

Question 2

CLO 1

12 Marks

(a) Simplify $\frac{(\sqrt{3}+i)^3(-1+i)^4}{(1-\sqrt{3}i)^6}$.

(4)

(b) List the all four 4^{th} roots of 1+i.

(4)

(c) Use De Moivre's Theorem to express $\sin \theta$ in the power series of $\sin \theta$.

(4)

Question 3

CLO 2

12 Marks

(a) Approximate the value of $\sin 46^\circ$ using Taylor Series about $x = \frac{\pi}{4}$.

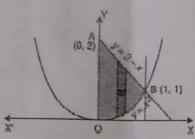
(4)

- (b) An international airline has a regulation that each passenger can carry a suitcase having the sum of its width, length and height less than or equal to 135 cm. Select the dimensions of the suitcase of maximum volume that a passenger may carry under this regulation. (4)
- (c) Engineers use the radius of curvature in road design to determine the appropriate curvature of roads and highways. To ensures safe and efficient navigation for vehicles, particularly at higher speeds on a parabolic curved $y^2 = 8x$, choose the points on which radius of curvature $\rho = \frac{125}{16}$.

(4)

(a) Use Beta integral to evaluate $\int_0^1 x^4 (1 - \sqrt{x})^5 dx$.

- (4)
- (b) Derive Reduction Formula for $I_n = \int \sin^n x \, dx$ and hence use it to compute $\int_0^{\pi/2} \sin^4 x \, dx$. (4)
- (c) Apply double integration to find the area of a lamina bounded by x = 0, $y = x^2$ and x + y = 2 shown as shaded region in the given figure.



Question 5

CLO 2

12 Marks

- (a) Compute the directional derivative of the scalar function $\phi = x^2 y^2 + 2z^2$ at the point P(1, 2, 3) in the direction of the line PQ where Q is the point (5, 0, 4).
- (b) Select the values of constants a and b such that the vector field \vec{A} is irrotational. $\vec{A} = (2xy + 3yz)\hat{\imath} + (x^2 + axz 4z^2)\hat{\jmath} + (3xy + byz)\hat{k}$ (4)
- (c) Find the work done to move a particle in the xy-plane from O (0,0) to A(1,4) with a variable force $\vec{F} = 2x^2y \ i + 3xyj$
 - (i) Along the curve $y = 4x^2$.
 - (ii) Along a straight line joining O and A.