Chapter 05

Waves & Oscillations

Waves:-

Waves are energy carriers which are used to transmit energy from one place to other in a medium.

OR

Wave is the disturbance or pattern generated in a medium which travels from one location to another.

Types:-

- Electromagnetic Waves
- Mechanical Waves
- Matter Waves (De Broglie Waves)
  ↳ when matter shows wave like behaviour. $\rightarrow \lambda = \frac{h}{p}$

- Mechanical Waves:-

The waves that require medium to transfer from one place to another.

Example:- Sound waves, water waves etc.

Mechanical waves are further divided with respect to direction of propagation:-

a) Longitudinal Waves

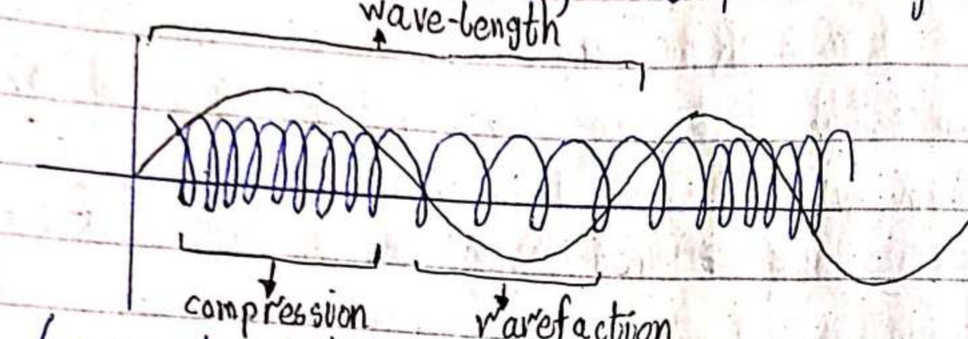
b) Transverse Waves

Amplitude is max distance from wave-axis while crest is displacement.

a) Longitudinal Waves:-

The waves in which direction of motion of particles is ll (parallel) to wave propagation.

Examples:- Sound waves, waves produced by slinky etc



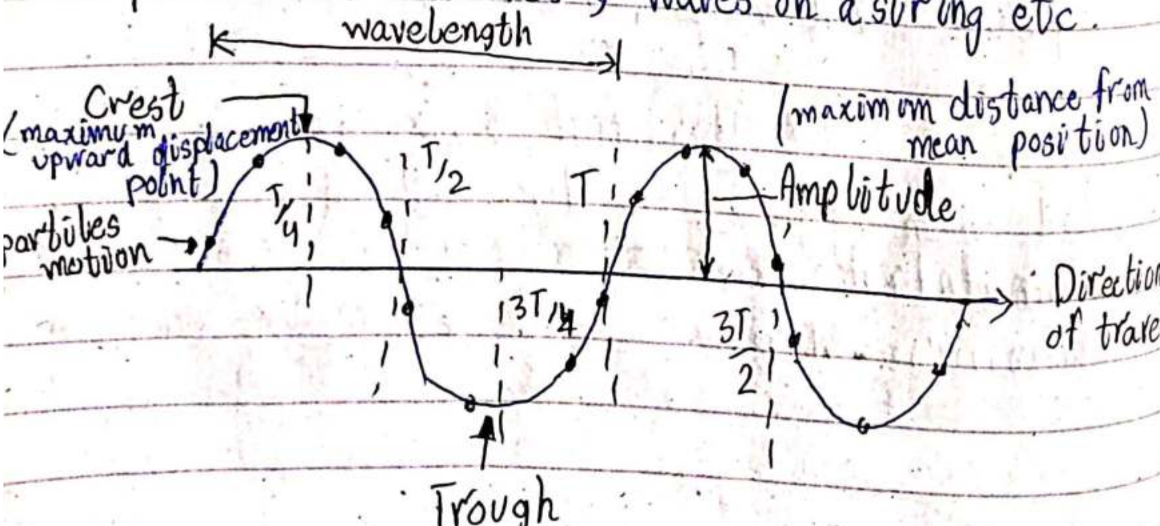
(region where particles are close together)
High Frequency

(are far apart)
Low Frequency

b) Transverse Waves:-

The waves in which direction of motion of particles is ⊥ (perpendicular) to wave propagation.

Examples:- Water waves, waves on a string etc.



(point on wave with minimum or lowest displacement in a cycle) wavelength → The distance between consecutive corresponding points of a wave which are in same phase such as two adjacent crests.

- Electromagnetic Waves :-

Electromagnetic waves are the composition of oscillating electric and magnetic fields. They come in category of transverse waves but require ~~medium for~~ no medium to travel.
Examples :- TV Rays , X-Rays etc

Frequency :-

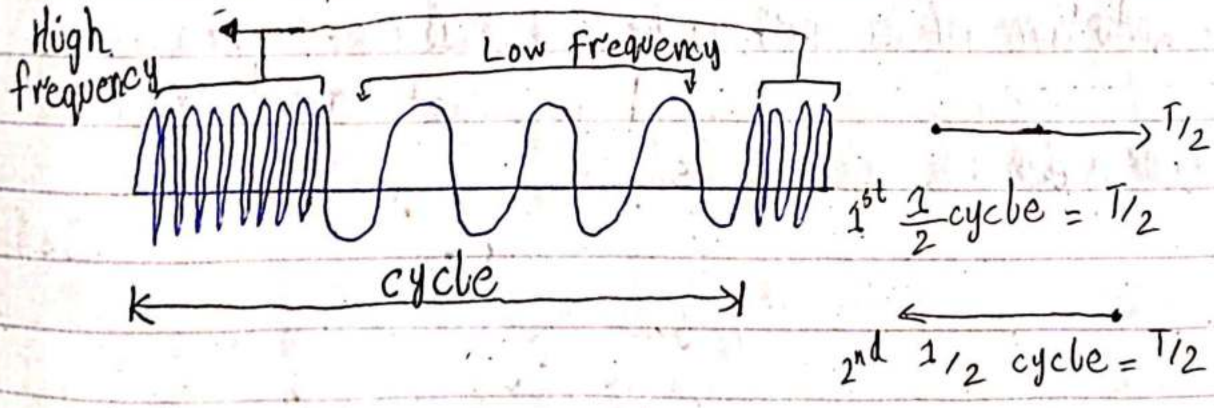
Number of oscillations produced by a body ~~in~~ per second. It is measured in "hertz" or "Hz"

1 Hz = 1 oscillation (vibrations) per second.

Time Period :-

The time required for one complete oscillation or cycle. It is measured in 'second' or 'sec'.

$$T = \frac{1}{f} = \frac{1}{Hz} = sec$$

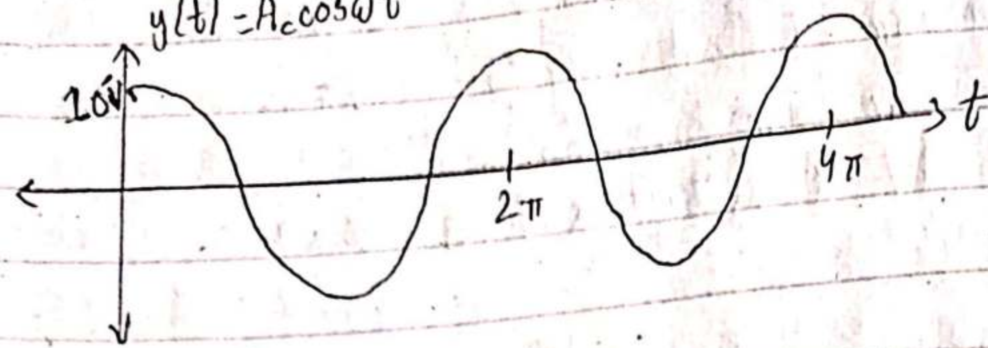


Oscillation :-

Back ~~or~~ and forth or repetitive motion of a body between two or more different points / states.

Q From given wave-form determine amplitude, frequency, time period and angular frequency?

$y(t) = A_c \cos\omega t$

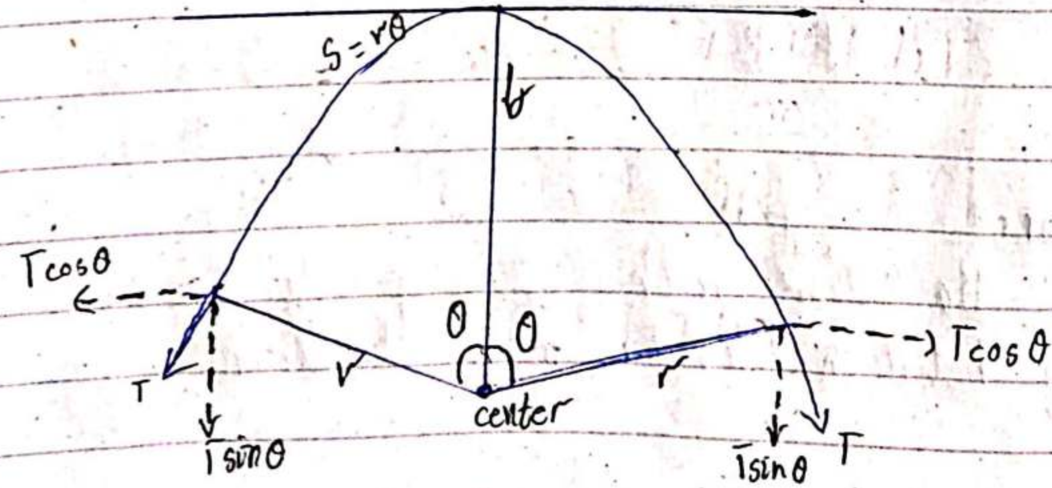


Amplitude = 10 V (Highest point)
Time Period = 2π (where wave repeats)
Frequency = $\frac{1}{T} = \frac{1}{2\pi}$ Hz

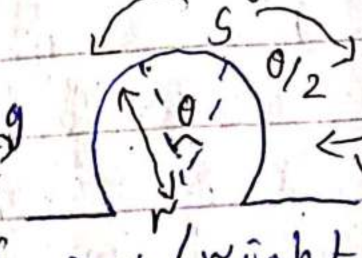
Angular frequency : $\omega = 2\pi f = 1$ rad/sec

Wave Speed In A Stretched String :-
→ Motion of string is vibrational motion.
→ Motion of spring is oscillatory motion.

Derivation:-



Wave Speed in a Stretched String (2nd way):-



If a pulse (wave) is travelling along a stretched string at a velocity $\vec{v}$ from its point of view (right side) and a string is moving to left with velocity 'v' then:- Tension in the rope is 'F'

$$F_c = \frac{mv^2}{r}$$ for wave;

$$F \sin\left(\frac{\theta}{2} + \frac{\theta}{2}\right) = \frac{mv^2}{r}$$ $(\sin\theta \approx \theta)$

$$F\theta = \frac{mv^2}{r}$$

$$F\theta r = mv^2$$ $\because S = r\theta$

$$FS = mv^2$$

$$v^2 = \frac{FS}{m}$$

$$\sqrt{v^2} = \sqrt{\frac{FS}{m}}$$ $\because \mu = \frac{m}{l}$ here $l = S$

$$\frac{1}{\mu} = \frac{S}{m}$$

So,

$$v = \sqrt{\frac{F}{\mu}}$$ but $F = T$ (for wave)

$$v = \sqrt{\frac{T}{\mu}}$$ Proved.

Unit:-

$$v = \sqrt{\frac{N}{kg/m}} = \sqrt{\frac{Kg\,m/s^2}{kg/m}} = \sqrt{m^2/s^2} = m/s$$

Proved

Conclusion:-

The speed of wave on a stretched string is set by the properties of string defined in terms of tension as a force and linear mass density. Theref as a tension of the force increases, the speed of waves also increases and transfer of energy becomes fast.

Angular Frequency:-

Angular displacement of any element of wave per unit time.

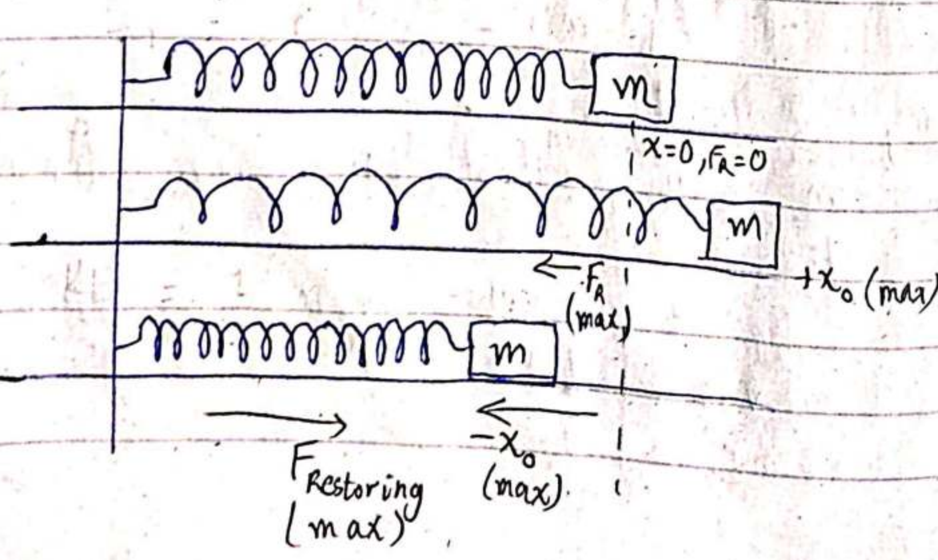
OR

Rate of change of the phase argument of the wave-form.

$$\omega = \frac{\Delta\theta}{\Delta t} \quad OR \quad \omega = \frac{2\pi}{T} = 2\pi f$$

Its SI unit is rad/sec.

Spring Mass System:-



A/c to figure:-

$$F \propto -x$$

$$F = -kx \quad -(i)$$

$$k \propto \frac{1}{\text{spacing}}$$

$$\text{so } k \propto \frac{1}{\text{Length}}$$

Here:-

k = No. of turns OR space b/w spiral turns of spring

For external force:-

$$F = ma \quad -(iii)$$

$$F = m\frac{dv}{dt} = m\frac{d^2x}{dt^2} \quad -(ii)$$

Comparing eq (i) & (ii)

$$m\frac{d^2x}{dt^2} = -kx$$

$$\frac{d^2x}{dt^2} = \left(-\frac{k}{m}\right)x$$

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \quad -(a)$$

Since motion is simple harmonic motion; so,

$$a = -\omega^2 x$$

Eq (iii) becomes:-

$$F = -m\omega^2 x \quad -(iv)$$

Comparing eq (i) and (iv)

$$-m\omega^2 x = -kx$$

$$\omega^2 = \frac{k}{m}$$

$$\Rightarrow \omega = \sqrt{\frac{k}{m}}$$

(This equation describes SHM or free undamped motion.)

Also $\omega = \sqrt{\frac{g}{l}}$

Now eq (a) becomes:-

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

$$\frac{d^2x}{dt^2} + \omega^2 x = 0 \qquad \because \omega^2 = \frac{k}{m}$$

Both these equations are known as:-

2^{nd} order equation of motion for SHO. simple harmonic oscillator

=> Solving these 2^{nd} order eq results in

$$x(t) = A\cos(\omega t + \phi)$$ → It describes the periodic motion of an object undergoing SHM.

Also,

$$\because T = \frac{1}{f}$$

$$\Rightarrow T = \frac{2\pi}{\omega} \qquad \because \omega = 2\pi f \Rightarrow \frac{1}{f} = \frac{2\pi}{\omega}$$

$$\because \omega = \sqrt{\frac{k}{m}}$$

$$\Rightarrow T = 2\pi\sqrt{\frac{m}{k}}$$

And $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

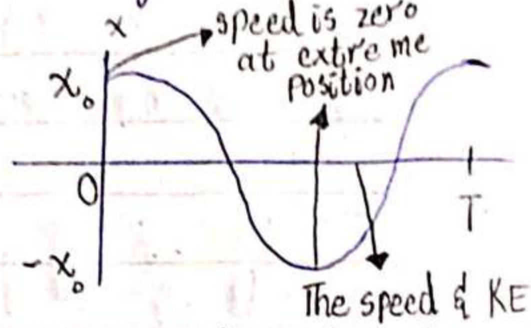
Time Period & frequency for spring mass system.

(Undamped Oscillation)

Simple Harmonic Motion :-

It is defined as a motion in which restoring force is directly proportional to the displacement of body from its mean (or equilibrium) position. The direction of restoring force is always directed towards mean position.

$$a \propto -x$$



Displacement :-

$$x = x_m \cos(\omega t + \phi)$$

↳ considered standard eq by our book.

Here ;

$x_m \rightarrow$ Amplitude

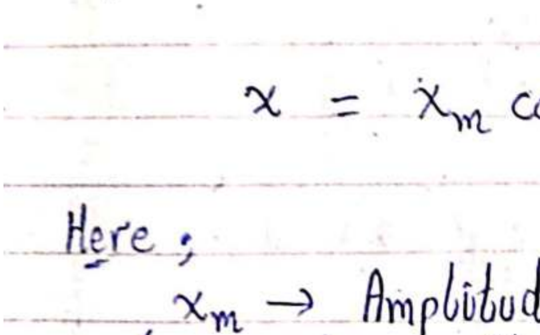
$(\omega t + \phi) \rightarrow$ Phase of the motion.

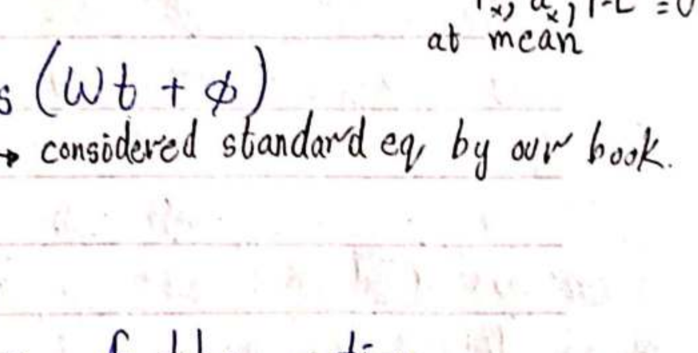
"It is the angle over which function repeats itself"

OR "The angular position of particle at start of motion."

$\phi \rightarrow$ Phase constant.

"It is the shift of function if $x_1 \neq x_2$ at $t = 0$ sec."





If $\phi = -\pi/2 = -90^\circ$ so for example:-

$$x = x_m \cos(\omega t - 90^\circ)$$

$$x = x_m \sin \omega t$$

so displacement will be 0 at $t = 0$ for $\phi = -90^\circ$ But max at $t = 0\,sec$ for $\phi = 0^\circ$

Velocity & Acceleration of SHM:-

$$x = x_m \cos(\omega t + \phi)$$

Differentiating by 't'.

$$\Rightarrow \frac{dx}{dt} = \frac{d}{dt}\left\{x_m \cos(\omega t + \phi)\right\}$$

$$\vec{v} = x_m \cdot \frac{d}{dt} \cos(\omega t + \phi)$$

$$\vec{v} = x_m \cdot -\sin(\omega t + \phi) \cdot \omega \cancel{\frac{dt}{dt}}^{1} + \cancel{\frac{d\phi}{dt}}^{0}$$

$$\vec{v} = \cancel{x_m \omega \sin(\sin} \; -x_m \omega \sin(\omega t + \phi)$$

For max speed $\sin 90^\circ = 1$ so $v_{max} = x_m \omega$

Again differentiating by 't'.

$$\Rightarrow \frac{d\vec{v}}{dt} = \frac{d}{dt}\left\{-x_m \omega \sin(\omega t + \phi)\right\}$$

$$\vec{a} = -x_m \omega^2 \cos(\omega t + \phi)$$

$$\Rightarrow \vec{a} = -\omega^2 \left[x_m \cos(\omega t + \phi)\right] = -\omega^2 \left[x(t)\right]$$

$$\vec{a}(t) \propto -x(t)$$

Conclusion:-

Thus it is proved that in SHM; acceleration is directly proportional to the displacement "x(t)", but opposite in direction ~~as~~ and is related by the square of angular frequency.

Energy In SHM:-

The energy of oscillator at any displacement 'x' from the mean position is partially KE and partially PE but the total energy of simple harmonic oscillator remains constant.

$$E_{Total} = KE + PE$$

OR $$\Delta KE = \Delta P.E$$

Potential Energy Of SHM:-

The work-done in displacing (stretching / compressing) a simple harmonic oscillator will be stored in form of PE. The instantaneous P.E is:-

$$PE = U = \int_0^x F \, dx$$

$$U = \int_0^x \underset{Kx}{\cancel{F}} \, dx$$

$$U = K \int_0^x x \, dx$$

$$U = k \left| \frac{x^2}{2} \right|_0^x$$

$$\Rightarrow U = \frac{1}{2} k x^2 = \frac{k x_m^2}{2} \text{ (For max PE)}$$

$$\text{OR} \quad U = \frac{1}{2} k x_m^2 \cos^2(\omega t + \phi) \quad -(a)$$

$x_m \rightarrow$ Amplitude

Kinetic Energy Of SHO:-

The kinetic energy for SHO can be evaluated as:-

$$KE = \frac{1}{2} m v^2$$

$$KE = \frac{1}{2} m x_m^2 \omega^2 \sin^2(\omega t + \phi) \quad -(b)$$

OR, we know;

$$\omega^2 = \frac{k}{m}$$, maxima of sin-wave ($\sin 90^\circ = 1$)

$$KE = \frac{1}{2} m x^2 \left(\frac{k}{m}\right) (1)^2$$

$$KE = \frac{1}{2} k x_m^2$$

Total Energy Of SHO:-

$$E_T = KE + PE$$

$$E_{Total} = \frac{1}{2} m v^2 + \frac{1}{2} k x^2$$

$$E_{Total} = \frac{1}{2} m \left[-\underset{\hookrightarrow \frac{k}{m} = \omega^2}{x_m \omega} \sin(\omega t + \phi) \right]^2 + \frac{1}{2} k \left[x_m \cos(\omega t + \phi) \right]$$

$$E_{Total} = \frac{1}{2} x_m^2 \, m \left(\frac{k}{m} \right) \sin^2(\omega t + \phi) + \frac{1}{2} x_m^2 k^2 \cos^2(\omega t + \phi)$$

$$E_{Total} = \frac{1}{2} k x_m^2 \left[\sin^2(\omega t + \phi) + \cos^2(\omega t + \phi) \right]$$

---- $\sin^2 x + \cos^2 x = 1$

$$E_{Total} = \frac{1}{2} k x_m^2$$

Hence,

$$\langle E_{Total} \rangle = \langle PE \rangle_{max} = \langle KE \rangle_{max}$$

So for an ideal, SHO total energy of system remains constant.

Velocity of SHO:- (From Total Energy)

$$E_T = KE + PE$$

$$\frac{1}{2} k x_m^2 = \frac{1}{2} m v^2 + \frac{1}{2} k x^2$$

$$\frac{1}{2} k x_m^2 - \frac{1}{2} k x^2 = \frac{1}{2} m v^2$$

$$\frac{1}{2}k\left(x_m^2 - x^2\right) = \frac{1}{2}mv^2$$

$$\frac{k}{m}\left(x_m^2 - x^2\right) = v^2$$

Taking sq. root on both sides:-

$$v_{inst} = \omega\sqrt{x_m^2 - x^2} \quad \because \omega = \sqrt{\frac{k}{m}} \Rightarrow \omega^2 =$$

=> This derivation is similiar to conservation of mechanical energy.

PE w.r.t Newton's law:-

$$U = \int F\, dx$$

$$U = \int m a\, dx$$

$$U = m\int a\, dx$$

$$\because \vec{a} = -x_m\omega^2\cos(\omega t + \phi)$$

$$U = m\int -x_m\omega^2\cos(\omega t + \phi)\, dx$$

$$U = -m\omega^2\int x_m\cos(\omega t + \phi)\, dx$$

$$\because \omega^2 = \frac{k}{m} \text{ and } x = x_m\cos(\omega t + \phi)$$

$$U = -m\left(\frac{k}{m}\right)\int x\,dx$$

$$U = -k\left|\frac{x^2}{2}\right|^{x}$$

$$U = -\frac{1}{2}kx^2$$

Q An object, oscillates along x-axis, its position varies with time. A/c to given equation :-

$$x(t) = 4\cos\left(\pi t + \frac{\pi}{4}\right)$$

a) Determine its amplitude, frequency, and time period of motion.

b) Calculate velocity and acceleration at time $t = 1sec$

a) $$x(t) = x_m\cos(\omega t + \phi)$$

$$x(t) = 4\cos\left(\pi t + \frac{\pi}{4}\right)$$

By comparing :-

Amplitude $= 4m$

Frequency $= \dfrac{\omega}{2\pi} = \dfrac{1}{2}\ Hz$

Time Period : $T = \dfrac{1}{f} = 2\ sec$

b) $$v(t) = -4\pi\sin(\pi t + \pi/4)$$

At $t = 1\ sec$;

$$v(1) = 8.89\ m/s$$

$$\Rightarrow a(t) = -4\pi^2 \cos\left(\pi t + \frac{\pi}{4}\right)$$

a

At $t = 1$ sec

$$a(1) = -27.9\ m/s^2$$

Q If a body of mass 2 kg is attatched to a spring having $k = 4$ N/m placed on a horizontal surface. What will be the time period?

Data:-

$m = 2$ kg $\quad T = ?$

$k = 4$ N/m

Sol:-

For spring mass system;

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$T = 2\pi\sqrt{\frac{2}{4}}$$

$$T = 4.44 \text{ sec}$$

Q A block of mass 'm' is 680 gm is hung to a spring of $k = 65$ N/m. Block is pulled by a distance of $x = 11$ cm from its equilibrium position ($x = 0$ cm) on a frictionless surface and release from rest at $t = 0$ sec?

a) Find ω, f, T.P ?

b) Amplitude of oscillation?

c) Magnitude of max speed and acceleration?

d) Phase constant of motion?

Data:-

$m = 680\ gm = 0.68\ kg$

$k = 65\ N/m$

$x = 11\ cm = 0.11\ m \quad (t = 0\ sec)$

Solution:-

a) $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{65}{\cancel{11}\,0.68}}$

$\omega = 9.78\ rad/sec$

$\because \omega = 2\pi f$

$f = \frac{\omega}{2\pi} = \frac{9.78}{2\pi}$

$f = 1.56\ Hz$

$\because T = \frac{1}{f} = \frac{1}{1.56} = 0.64\ sec$

b) Amplitude w.r.t equilibrium: $x_m = 0.11m$

c) $\because v_{max} = -\omega x = -(9.78)(0.11)$

$v_{max} = 1.08\ m/s$

$\because a_{max} = -\omega^2 x = (-9.78)^2(0.11)$

$a_{max} = -10.52\ m/s^2$

d) For phase constant:-

$$x = x_m \cos$$

$$a = -x_m \omega^2 \cos(\omega t + \phi)$$

For $t = 0$ sec,

We have to detect angle at which amplitude is maximum.

$\because a_{max}$ is at extreme position.

$$a_{max} = -x_m \omega^2 \cos\phi$$

$$\frac{-10.52}{-(0.11)(9.78)^2} = \cos\phi$$

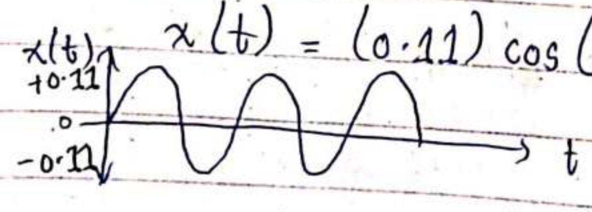
To write ϕ in eq, first convert to radians.

$$\phi = \cancel{\cos^{-1}(0.99) \approx (1)} \quad \text{i.e.} \quad \phi = 0^\circ$$

$$\cos^{-1}(0.997) \approx \cos^{-1}(1)$$

So :-

$$x(t) = (0.11)\cos(9.78t + 0^\circ)$$



Q The block-spring combination of mass '2.43 kg' and $k = 221$ N/m is stretched in +ve x-direction on a distance of 11.6 cm from equilibrium and released? and Time period = 0.6589 sec

a) Total Energy of block?

b) Max speed of block?

c) Magnitude of max acceleration?

d) If block is released at $t = 0$ sec; its position at 0.215 sec

Data:-

$m = 2.43\ kg$

$k = 1\ 221\ N/m$

$x_m = 11.6\ cm = 0.116\ cm$

Sol:-

a) $E_T = \frac{1}{2} k x_m^2 = \frac{1}{2}(221)(0.116)^2$

$E_T = 1.49\ J$

b) The maximum kinetic energy is equal to total energy when ; $U=0$ so,

$E_T = KE$

$v_{max} = \sqrt{\frac{2\ KE_{max}}{m}} = \sqrt{\frac{2(1.49)}{2.43}}$

$V_{max} = 1.11\ m/s$

c) Maximum acceleration occurs at the instant of release when force is max:-

$a_{max} = \frac{F_{max}}{m} = \frac{k x_m}{m}$ ← as released from amplitude

$a_{max} = \frac{(221)(0.116)}{2.43} = 10.6\ m/s^2$

d) As for spring-mass system, time period is independent of amplitude ; so.

$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.6589} = 9.536\ rad/s$

$x = x_m \cos(\omega t + \phi)$ $x = (0.116)\cos(9.536)(0.215)$

$x = -0.0535\ m$ or $-5.35\ cm$

Since maximum displacement is at $t=0$ so $\phi = 0°$ (cosine)

=> To compute phase angle ; (t will be given)
Calculate where was maximum displacement.

=> If max displacement at $t = 0\,sec \rightarrow$ cosine function
If " " at $t = T/4\,sec \rightarrow$ sine function
OR calculate from displacement formula.

$$\cos(\omega t + \phi) = \frac{x(t)}{x_m}$$

$$\cos\phi = \frac{x(t)}{x_m}$$ → If $t = 0$ sec OR use this info if 't' not given

=> Then convert it into radians and use in eqs. not in degrees

Q Suppose the block has mass 2.72×10^5 kg and is designed to oscillate at 10.0 Hz with an amplitude of $x = 20.0$ cm?

a) What is the total mechanical energy of spring mass system?

b) Block's speed as it passes through equilibrium point?

Data:-

$m = 2.72 \times 10^5$ kg

$f = 10$ Hz

$x_m = 20$ cm $= 0.2$m

i) $E_T = ?$

ii) $v_{equilibrium} = ?$

Sol:-

i) $\because$ $E_T = \frac{1}{2} k x_m^2$

$\because k = \omega^2 x^m = 4\pi^2 f^2 m$

$$E_T = \frac{1}{2}(4\pi^2 f^2 m)\, x_m^2$$

$$E_T = 2\pi^2 (10)^2 (2.72 \times 10^5)(0.2)^2$$

$$E_T = 2.15 \times 10^7 \text{ J}$$

ii) For equilibrium posistion ; $x = 0$ m so also $U = 0$

Hence,

$$E_T = KE_{max}$$

$$E_T = \frac{1}{2} m v_{max}^2$$

$$v_{max} = \sqrt{\frac{2E_T}{m}} = \sqrt{\frac{2 \times 2.15 \times 10^7}{2.72 \times 10^5}}$$

$$v_{max} = 12.57 \text{ m/s}$$

Q A certain spring hang vertically with a body of mass M = 1.65 kg is suspended from it, its length increases by 7.33 cm. The spring is then mounted horizontally, and a block of mass m = 2.43 kg is attatched to spring. The block is free to slide on a frictionless surface.

Find,

a) What is the force constant 'k' of spring?

b) How much horizontal force is required to stretch the spring of distance 11.6 cm?

c) If the block is displaced of 11.6 cm and released, with what period will it oscillate?

Data:-

$M_s = 1.65\ kg$ (vertically)

$x_y = 7.33 cm = 0.0733 m$; $x_x = 11.6 cm = 0.116 m$

$m_b = 2.43\ kg$ (horizontally)

Sol:-

a) For force constant of spring 'k' is determined from the force $W = Mg$ necessary to stretch it vertically i.e.

$$\Sigma F_y : F_r - W = 0$$

$$kx = Mg$$

$$k = \frac{Mg}{x} = \frac{(1.65)(9.8)}{(0.0733)}$$

$$k = 221\ N/m$$

b) Magnitude of force needed to stretch the spring is determined by Hook's law:-

$$F = kx = (221)(0.116)$$

$$F = 25.6\ N$$

c) Time period of spring mass system:-

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$T = 2\pi\sqrt{\frac{2.43}{221}} = 0.66\ sec$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.66} = 9.536\ rad/s$$

~~a) Total Energy in~~

Q The block of spring-block combination of 2nd previous problem is stretched from equilibrium in +ve x-direction. At $t = 0$, the displacement of block is $x = +0.0624\,m$ and its velocity is $v = +0.847\,m/s$. Write an equation for $x(t)$ during oscillation?

Data:-

$$m_b = 2.43\,kg, \quad k = 221\,N/m$$
$$\omega = 9.536\,rad/s$$
$$x(t) = ?$$

Sol:-

For generic formula, we must find x_m & ϕ.

$$E_T = KE + U$$

$$E_T = \frac{1}{2} m v_x^2 + \frac{1}{2} k x_x^2$$

$$E_T = \frac{1}{2}(\cancel{221}\ 2.43)(0.847)^2 + \frac{1}{2}(221)(0.0624)^2$$

$$E_T = 1.302\,J$$

$$\because E_T = \frac{1}{2} k x_m^2 \quad so$$

$$x_m = \sqrt{\frac{2E_T}{k}} = \sqrt{\frac{2 \times 1.302}{221}}$$

$x_m = 0.1085 m$

To find the phase constant, using given $t = 0$ sec

$$x(0) = x_m \cos\phi$$

$$\phi = \cos^{-1}\left(\frac{0.0624}{0.1085}\right) = 54.9^\circ \text{ or } 305.1^\circ$$

so possible values of $\cos^{-1}(+0.5751)$ are

so checking by putting in $v(0)$

$$v(0) = -\omega x_m \sin\phi$$

$$\cancel{\sin\phi =} \; v(0) = -(9.536)(0.1085)\sin\phi$$

$$v(0) = -(1.035)\sin\phi$$

$v(0) = -0.847$ m/s , $v(0) = +0.847$ m/s

↳ for $\phi = 54.9^\circ$ ↳ for $\phi = 305.1^\circ$

So $\phi = 305.1^\circ$ is accurate ; therefore converting in radians $305.1^\circ = 5.33$ rad.

We can now write eq as :-

$$x(t) = (0.109\,m)\cos\left[\underset{rad/s}{(9.54)}t + 5.33\,rad\right]$$

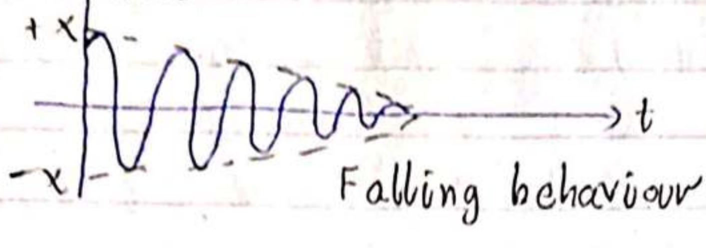
Oscillation with Conditions:-

→ Air resistance

→ Gravity

→ viscous drag

→ Friction



Damped Harmonic Oscillation:-
(Retarding Force)

When the motion of an oscillator is reduced by an external force or by other atmospherical frictional forces, then the motion and its oscillator is said to be damped.

$$F_d \propto v$$

$$F_d = -bv$$

-ve sign indicates damped force opposes direction of motion.

'b' is damping constant

Total net force:-

$$F_{net} = -bv - kx$$

$$ma = -bv - kx$$

$$\frac{d^2x}{dt^2} = -\frac{b}{m}\vec{v} - \frac{k}{m}x$$

$$\frac{d^2x}{dt^2} + \omega^2 x + \frac{b}{m}\vec{v} = 0$$

Here $\frac{b}{m}$ → Damping factor or daming coefficient

Damping $\propto b$ and $\propto \frac{1}{m}$

(Dimensionless) $\frac{b}{\sqrt{Km}} << 1$

=> Simple harmonic motions which persist indefinitely without the loss of amplitude are called free or undamped oscillation.

$T = \frac{2m}{b}$

So Displacement:-

In SHM:-

$x(t) = x_m \cos(\omega t + \phi)$

In Conditions:-

$x(t)_{damp} = x_m e^{-\left(\frac{bt}{2m}\right)} \cos(\omega t + \phi)$

$E(t) = \frac{1}{2} k x_m^2 e^{-}$

where e^{-} shows the falling nature in damped motion

Damping:-

It is defined as the reduction in energy and amplitude of oscillation due to resistive forces acting on oscillating system.

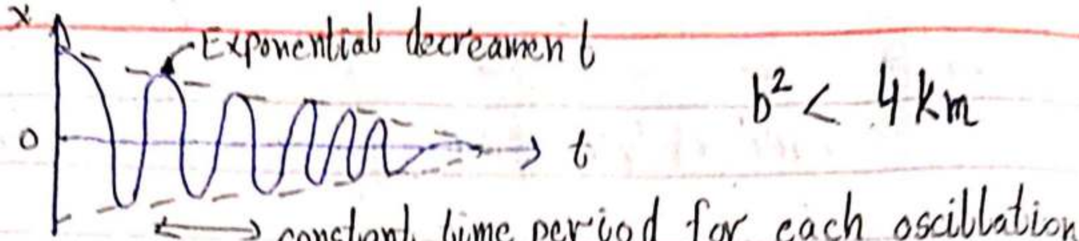
Damping continues until the oscillator comes to rest at the equilibrium position.

Types of Damping

- Undamped (SHO)
- Light or underdamped oscillation
- Heavy or overdamped oscillation
- Critically Damped motion

i) Light Damping:-

Those oscillations in which amplitude gradually decreases with time are said to be lightly damped.



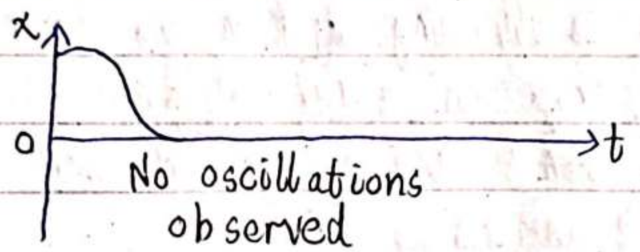
$b^2 < 4km$

Example:-

A simple pendulum in real world decreases its amplitude gradually until it comes to stop.

ii) Critical Damping:-

The oscillation in which oscillator displaced from equilibrium will return to rest at equilibrium position in shortest possible time is said to be critical damped.



$b^2 = 4km$

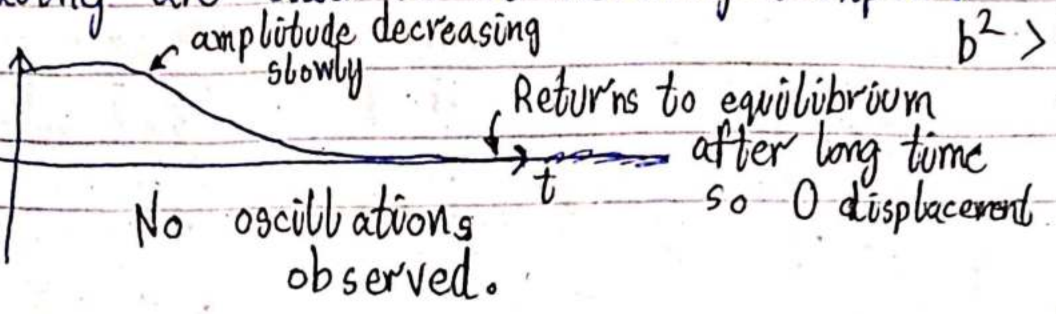
Example:-

car ~~sys~~ suspension system, ~~door magnet~~ etc
prevents car from oscillating when bumped in road.

iii) Heavy Damping:-

The oscillations which take a long time to return to its equilibrium position without oscillating are said to be heavily damped.

$b^2 > 4m$



Examples:-

Door dampers to prevent them slamming shut.

Force Oscillation:-

When a body oscillates by an an external force applied in the direction of motion of body, it is called forced oscillation.

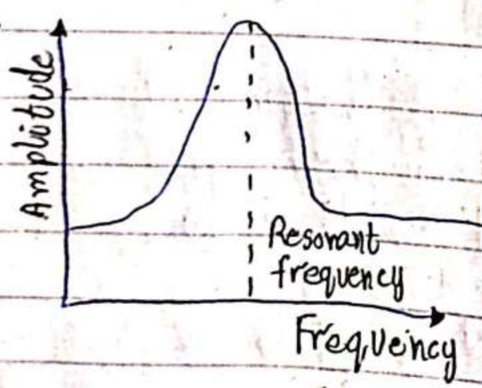
Here, amplitude of oscillation, experiences damping ~~but~~ remains constant due to the external energy supplied to the system.

Resonance:-

When an oscillating system is driven (made to oscillate from an external source) at a frequency which is ~~greater than~~ ~~same at~~ closer or equal its own natural frequency is called ~~dam~~ resonance.

OR

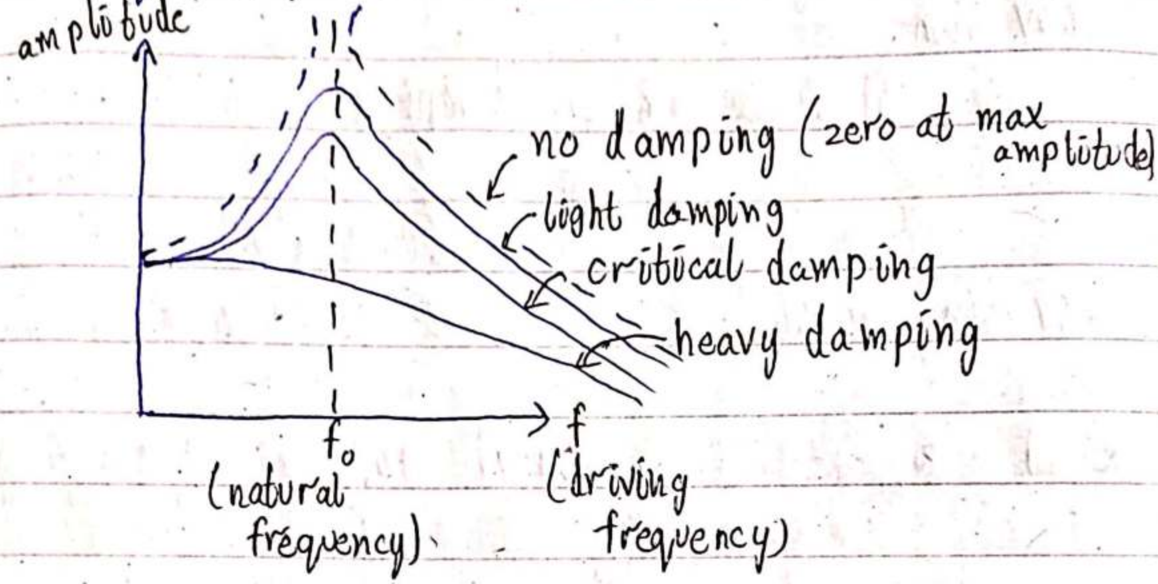
If a force oscillator super imposes the natural frequency of the oscillator is called resonance.



If non-observable peak so
↳ real world resonance like in rejector circuit minimum amplitude is 2 di

Examples :-

Buildings & bridges have been known to resonate with earthquakes, wind.



So; damping $\propto$ 1/Amplitude

$$F_{net} = -kx - bv \overset{+F_0 \cos/\sin(\omega t)}{} = ma$$

$$m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt}$$

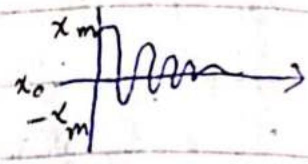
$$\frac{d^2x}{dt^2} + \omega^2 x + \frac{b}{m}\frac{dx}{dt} - \frac{F}{m}\cos(\omega t) = 0$$

$\frac{b}{m}$ → Coefficient of damped oscillation

$\frac{F}{m}\cos(\omega t)$ → Coefficient of forced oscillation

Q Expression for angular frequency 'ω' in case of damped harmonic oscillation?

Derivation:-



Eq. of wave under damping:-

$$x(t) = x_m e^{-\frac{bt}{2m}} \cos(\omega t + \phi) \quad -(i)$$

Derivating w.r.t 't':-

$$\dot{x}(t) = x_m \left\{ -e^{-\frac{bt}{2m}} \sin(\omega t + \phi)(\omega + 0) + e^{-\frac{bt}{2m}} \cos(\omega t + \phi)\left(-bt/2m\right) \right\}$$

$$v(t) = x_m \left\{ -\omega e^{-\frac{bt}{2m}} \sin(\omega t + \phi) - \frac{b}{2m} e^{-\frac{bt}{2m}} \cos(\omega t + \phi) \right\} \quad -(ii)$$

$$v(t) = x_m e^{-\frac{bt}{2m}} \left\{ -\omega \sin(\omega t + \phi) - \frac{b}{2m} \cos(\omega t + \phi) \right\}$$

Derivating w.r.t 't'

$$a(t) = x_m \left[e^{-\frac{bt}{2m}} \left\{ -\omega^2 \cos(\omega t + \phi) + \frac{b\omega}{2m} \sin(\omega t + \phi) \right\} + e^{-\frac{bt}{2m}} \left(\frac{-b}{2m} \right) \left\{ -\omega \sin(\omega t + \phi) - \frac{b}{2m} \cos(\omega t + \phi) \right\} \right]$$

$$a(t) = x_m e^{-\frac{bt}{2m}} \left\{ -\omega^2 \cos(\omega t + \phi) + \frac{b\omega}{2m} \sin(\omega t + \phi) + \frac{b\omega}{2m} \sin(\omega t + \phi) + \frac{b^2}{4m^2} \cos(\omega t + \phi) \right\}$$

$$a(t) = x_m e^{-\frac{bt}{2m}} \left\{ \frac{b\omega}{m} \sin(\omega t+\phi) - \omega^2 \cos(\omega t+\phi) + \frac{b^2}{4m^2} \cos(\omega t+\phi) \right\} \quad \text{— (iii)}$$

According to damped harmonic oscillation:-

$$F_{net} = -kx - bv = ma$$

$$\Rightarrow \quad m\,a(t) = -k\,x(t) - b\,v(t)$$

Substituting values from eq (i), (ii), (iii)

$$\Rightarrow m\left[x_m e^{-\frac{bt}{2m}} \left\{ \frac{b\omega}{m} \sin(\omega t+\phi) - \omega^2 \cos(\omega t+\phi) + \frac{b^2}{4m^2} \cos(\omega t+\phi) \right\} \right]$$

$$= -k\left[x_m e^{-\frac{bt}{2m}} \cos(\omega t+\phi) \right] - b\left[x_m e^{-\frac{bt}{2m}} \left\{ -\omega \sin(\omega t+\phi) - \frac{b}{2m} \cos(\omega t+\phi) \right\} \right]$$

On simplifying:-

$$\Rightarrow m\left\{ \frac{b\omega}{m} \sin(\omega t+\phi) - \omega^2 \cos(\omega t+\phi) + \frac{b^2}{4m^2} \cos(\omega t+\phi) \right\}$$

$$= -k\cos(\omega t+\phi) + b\omega \sin(\omega t+\phi) + \frac{b^2}{2m} \cos(\omega t+\phi)$$

$$\Rightarrow b\omega \sin(\omega t+\phi) - \omega^2 m \cos(\omega t+\phi) + \frac{b^2}{4m} \cos(\omega t+\phi)$$

$$= -k\cos(\omega t+\phi) + b\omega \sin(\omega t+\phi) + \frac{b^2}{2m} \cos(\omega t+\phi)$$

$$\Rightarrow \cos(\omega t+\phi)\left\{\frac{b^2}{4m}-\omega^2 m\right\} = \cos(\omega t+\phi)\left\{\frac{b^2}{2m}-k\right\}$$

$$\Rightarrow \quad \frac{b^2}{4m^2}-\omega^2 m = \frac{b^2}{2m}-k$$

$$\omega^2 m = k-\frac{b^2}{4m}$$

$$\boxed{\omega = \sqrt{\frac{k}{m}-\frac{b^2}{4m^2}}}$$ Derived

Q For a damped oscillation shown in the following figure contains a mass of 0.075 kg. The spring constant of 72 N/m ; damped constant of 90 gm/sec.

a) Find out 'ω'?

b) Time period?

c) Find out the time when the amplitude value becomes $\frac{1}{3}$ of its initial value.

Data:-

$m = 0.075\ kg = 75\ gm$

$b = 90\ gm/s = 0.09\ kg/s$

$k = 72\ N/m$

a) $\omega = ?$

b) $T = ?$

c) t at $\frac{1}{3}\ x(t) = ?$

Solution:-

i) $W = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$

$W = \sqrt{\frac{72}{75} - \frac{(0.09)^2}{4(0.075)^2}}$

$W = 0.77$ rad/sec

ii) $W_d = \frac{2\pi}{T}$

$T = \frac{2\pi}{0.77} = 8.11$ sec

iii) From the given condition

$\frac{1}{3} x(t) = x_m$

At initial $t = 0$ sec

$\frac{1}{3}\left\{ x_m e^{-bt/2m} \cos(wt+\phi)\right\} = x_m e^{\frac{-bt}{2m}} \cos(wt+\phi)$

$\frac{1}{3} e^0 = e^{-bt/2m}$

$t = \frac{-2m \ln(\overset{\text{constant}}{*})}{b}$

Taking ln both sides

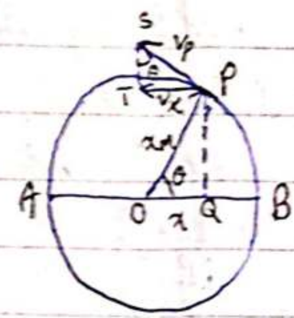
Generic formula for damped oscillation

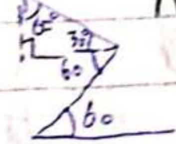
$\ln(e)^{-bt/2m} = \ln\left(\frac{1}{3}\right)$

$t = -\frac{2\ln(1/3)m}{b} = \frac{-2(-1.09)(75)}{90}$

$t = 1.83$ sec.

Formula Of Instantaneous Velocity By Trignometric Ratios:-





Consider a particle moving in a circle of radius 'x_m' with constant angular velocity 'ω'. As it moves along the circumference of the circle, its projection moves along the diameter OAB. The particle has covered angular displacement 'θ' it time 't'. At the instant 'P', particle's projection is at point 'Q'.

Instantaneous Displacement:-

In rt ΔPOQ

$$\cos\theta = \frac{B}{H} = \frac{\overline{OQ}}{\overline{OP}}$$

$$\cos\theta = \frac{x}{x_m}$$

$$x = x_m \cos\theta$$

where,

$\theta \rightarrow$ Angular Displacement $\because \theta = \omega t$ ($S = vt$)

$$x = x_m \cos(\omega t)$$

Here,

$x \rightarrow$ displacement of projection from mean position at time 't'.

$x_m \rightarrow$ Amplitude

Intantaneous Velocity:- The velocity of particle at point 'P' is 'v_p'. The horizontal component of this velocity 'v_x' is equal to the instatancous velocity of projection. So:-

In rt ΔPST:-

$$\sin\theta = \frac{P}{H}$$

$$\sin\theta = \frac{v_x}{v_p}$$

$$v_x = v_p \sin\theta$$

where,

$v_p \rightarrow$ Tanganetial Velocity $(v_p = x_m \omega)$ $v_t = r\omega$

And $\sin\theta = \sqrt{1-\cos^2\theta}$

$$v_x = (x_m \omega)\sqrt{1-\cos^2\theta}$$

but $\cos\theta = \frac{x}{x_m}$

$$v_x = x_m \omega \sqrt{1-\left(\frac{x}{x_m}\right)^2}$$

$$v_x = x_m \omega \sqrt{\frac{x_m^2 - x^2}{x_m^2}}$$

$$v_x = \omega\sqrt{x_m^2 - x^2} \quad \text{OR} \quad v_{inst} = \omega\sqrt{x_m^2 - x^2}$$

Here,

$v_x \rightarrow$ Instantaneous Velocity of projection at a distance 'x' from mean position

Chapter 06

Optics & Lasers

Optics:-

It is the branch of physics which is concerned with light and its behavioural pattern and properties.

Electromagnetic Spectrum:- ($v = c = 3 \times 10^8$ m/s for all)

Full range of electromagnetic radiation, organized by frequency or wavelength.

Range of visible radiation:-

VIBGYOR

(400 nm – 700 nm) λ

shorter wavelength, High Energy ⟵ Low Energy, High wavelength

Light:-

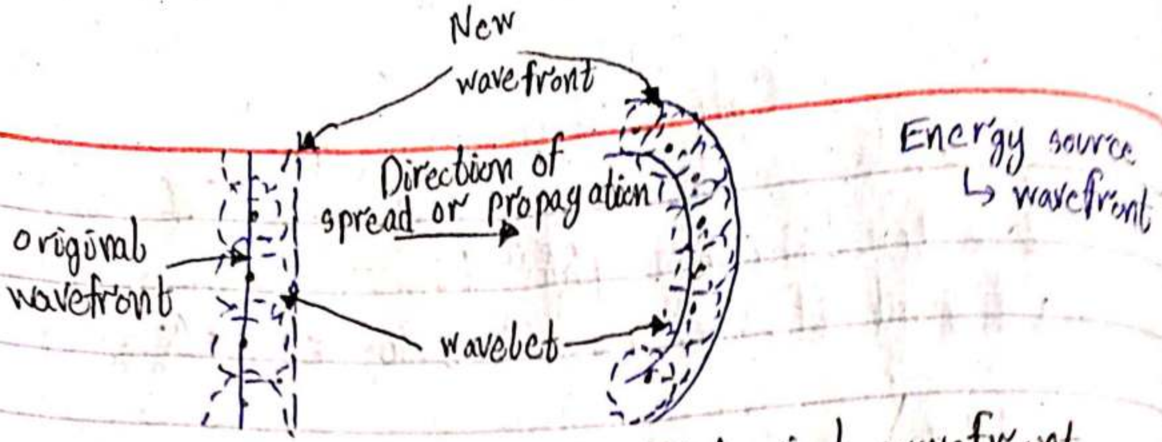
It is an electromagnetic radiation that can be perceived by human eye.

⇒ EM radiations don't need any medium to travel.

Huygen's Principle:-

It states that:-

"Every point on a wavefront acts as a source for secondary ~~wave-fro~~ wavelet. The collection of all the in-phase points on a wavelet form the next wavefront."



Plane wavefront

$E = 0\%$
(backward direction)

Spherical wavefront

$E \neq 0$ (Forward Direction of wave propagation)

Diffraction:-

The bending or speeding of ~~the~~ waves around the obstacle or through the aperture of region is called diffraction.

=> For diffraction, obstacle edge should have dimensions comparable to ~~spec~~ wavelength of light.

Path difference b/w two rays of consecutive slits

$$\overbrace{d \sin\theta} = m\lambda$$

d → Grating Element

$$d = \frac{1}{\text{No. of lines per unit length}}$$

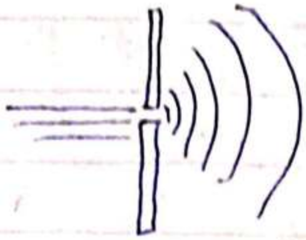
θ → Diffraction Angle

m → Order of diffraction
i.e. 0, ±1, ±2, ---- , λ = wave-length

⇒ Quality of diffraction changes with change in ~~slit.~~ size of slit.

⇒ For ideal diffraction;

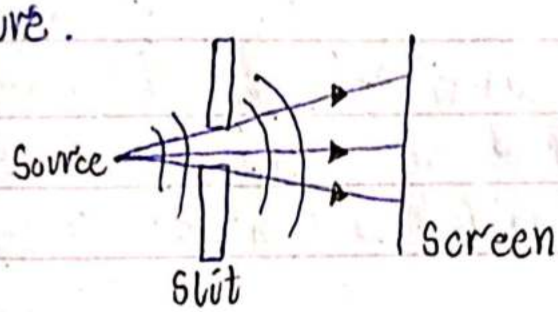
Dimensions Of Slit $\approx$ wavelength of light
(comparable) $\lambda \approx d$



There are two types of diffraction phenomenon:-

i) Fresnel Diffraction:-

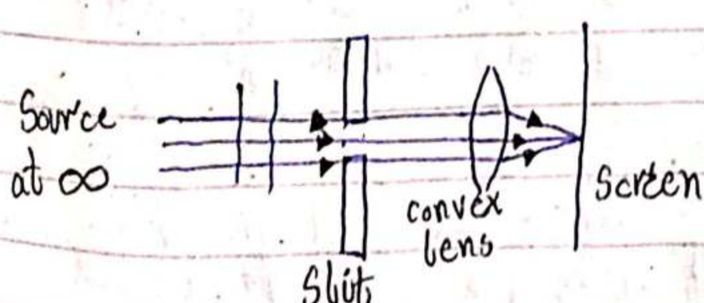
The source of light and screen should be at finite distance from the diffracting aperture.



Fresnel Diffraction

ii) Fraunhöfer Diffraction:-

The source and screen should be at infinite distance from the aperture.



Fraunhöfer Diffraction

Diffraction Of X-Rays:-

X-Rays have wavelength (0.1 nm - 1m) which is shorter than grating element so its diffraction is found by rock-salt crystals (like NaCl, ZnS) using Bragg's law:-

$$\overbrace{2d\sin\theta}^{\text{Path difference}} = m\lambda \quad , \quad d = \frac{a_0}{\sqrt{5}}$$

Here, d → interplanar spacing
a → Unitcell dimension

Interferance:-

When two waves of same frequency, wavelength and amplitude superimpose together, then this phenomenon is called as interference.

⇒ To observe interference pattern:-
i) Light source must be monochromatic → light of single wavelength and frequency
ii) Source must be coherent.
↳ same frequency & constant phase difference.

Constructive Interference:-

When crest of one wave falls on crest of another wave such that amplitude is maximum.

Destructive Interference:-

When crest of one wave falls on trough of another wave such that the amplitude is minimum.

Young's Double Slit Experiment:-

Thomas Young was the first to demonstrate the interferenc in light which verifies its wave nature.

Path difference = $d \sin\theta$

Constructive Interference ~~Conditic~~ Formula:-

$$d\sin\theta = m\lambda \quad , \quad y_m = \frac{mL\lambda}{d}$$

Here;

$m = 0 \rightarrow$ Central Maximum OR Central Bright Fringe

$m = \pm 1, \pm 2$ (-ve for down, +ve for up)

$y_m \rightarrow$ Position of bright fringe from central

Destructive Interference Formula:-

$$d\sin\theta = \left(m + \frac{1}{2}\right)\lambda \quad , \quad y_m = \left(m + \frac{1}{2}\right)\frac{L\lambda}{d}$$

Here, $m = 0 \rightarrow 1^{st}$ dark fringe , $\pm 1, \pm 2$

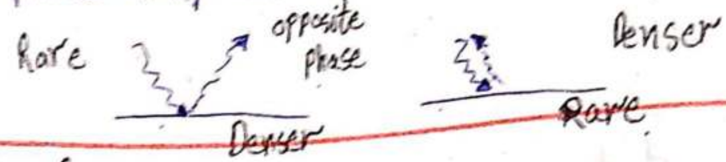
$y_m \rightarrow$ Position of dark fringe from central one.

$d \rightarrow$ Slit separation

$L \rightarrow$ Distance b/w slit and screen

$\theta \rightarrow$ Angle from original direction of beam.

No phase change in refraction only in reflection



Fringe Spacing:-

Distance b/w two consecutive bright or consecutive dark fringes is called fringe spacing.

$$\Delta x = \frac{L\lambda}{d}$$ Same for both contructive & destructive interferences.

Reflection:-

Bouncing back of a light ray from smooth polished (non-absorbing) surface is called reflection.

Refraction:-

Refraction is the bending of light from one medium to another due to difference in densities b/w the two substances.

$$n_1 \sin\angle i = n_2 \sin\angle r$$
OR
$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
Snell's law

$f \rightarrow$ remains same , $v' = \frac{v}{n}$, $\lambda' = \lambda/n$

$\downarrow v = \downarrow\lambda \; f$ (constant)

$$\frac{\sin\angle i}{\sin\angle r} = \frac{n_2}{n_1} = \frac{v_1}{v_2}$$ $\rightarrow$ Ratio of phase velocities

Total Internal Reflection :-

If light passes from optically denser to rarer medium, then at interface instead of refracting, it is completed reflected from the surrounding surfaces back into medium called Total Internal Reflection.

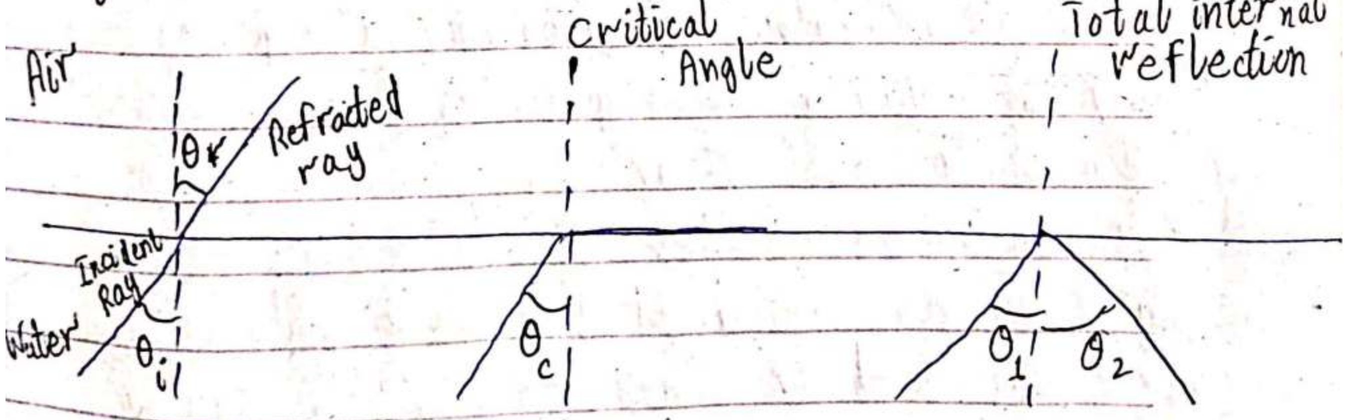
For Total Internal Reflection :-

i) Light ray moves from more denser to rarer medium

ii) Angle of incidence must be greater than critical angle.

Critical Angle :- ⇒ Refracted ray is at 90°.

The angle of incidence for which angle of refraction is 90 degrees is called critical angle. If angle of incidence is more than critical angle, total internal reflection takes place.



$\theta < \theta_c$ (Refraction)　　$\theta_r = 90°$　　$\theta_{Reflection} > \theta_c$

Dispersions:-

The separation of white light into its constituent colours according to wavelength is called dispersion.

Dispersion of grating $\rightarrow D = \dfrac{\Delta\theta}{\Delta\lambda}$ OR $D = \dfrac{m}{d\cos\theta}$

$\Delta\theta \rightarrow$ Angular Separation , $\Delta\lambda \rightarrow$ wavelength interval

Diffraction Grating:-

An optical device consisting of closely spaced slits designed to diffract light into its spectral components. It is used to analyze and separate light into various wavelengths.

Polarization:- → To verify transverse waves

Polarization is a property applying to transverse waves that specifies the geometrical orientation of the oscillations, such that only those light rays in which electric field is parallel to geometry of surface are passed through called as polarized light and the phenomenon is polarization.

=) A light ray w vibrating in more than one plane is known as unpolarized light.

Like used in sunglasses to reduce glare.

Following are three-types of polarization depending on how electric field is oriented:-

- Linear Polarization
- Circular Polarization
- Elliptical Polarization

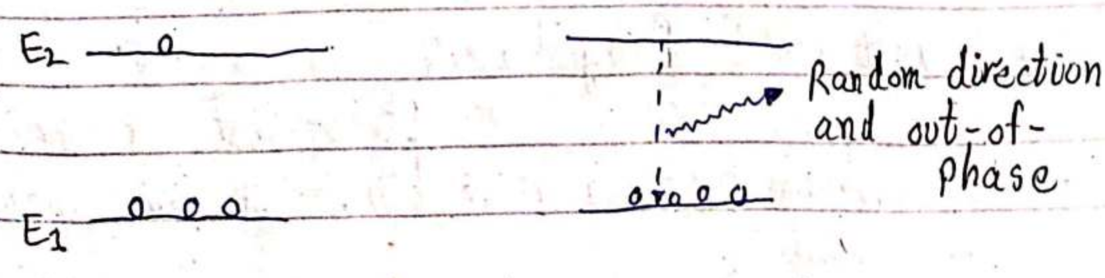
Laser:-

Laser stands for Light Amplification By Stimulated Emission of Radiation. Laser is a coherent and monochromatic light in which all particles (photons) move in one direction and reinforce each other.

Laser Light	Ordinary Light
Monochromatic	Polychromatic
Coherent	incoherent
Uni-directional	Multi-directional
High-intensity	

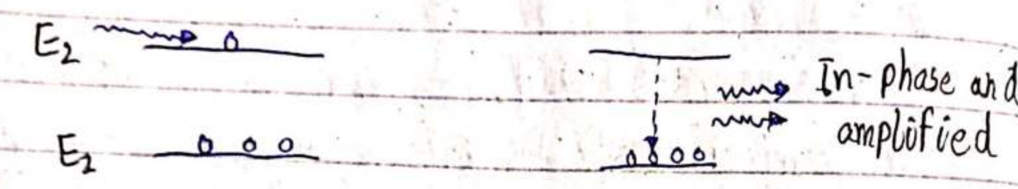
Spontanneous Emission:-

When an atom in an excited state (E_2) decays spontanneously to lower energy level (E_1) releasing energy (as photon) in random direction. This process is called spontanneous emission.



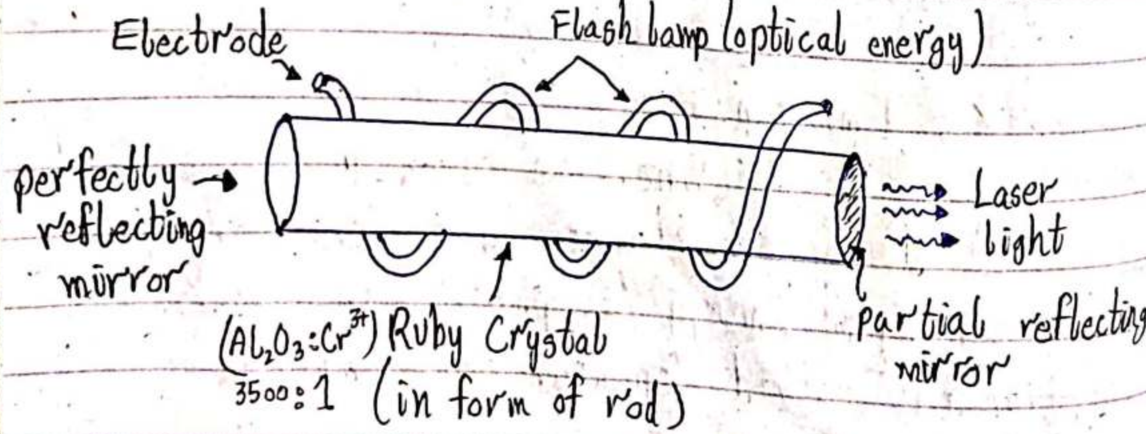
Stimulatted Emission:-

When an incident photon causes an atom in excited state (E_2) to decay to a lower energy level (E_1) emitting a photon whose properties are identical to incident one. This process is called stimulated emission.



Ruby Laser:-

A ruby laser is a solid-state laser that uses syntehetic ruby crystal as its active medium. It was the first successful laser developed by Maiman in 1960. It emits a visible deep red light of wavelength 694.3 nm.



Construction Of Ruby Laser:-

It consists of three important elements: laser medium, the pump source and optical resonator.

i) Active Medium:-

In ruby laser, a single crystal of ruby ($Al_2O_3 : Cr^{+3}$) in form of cylinder act as a laser or active medium.

ii) Pump Source:-

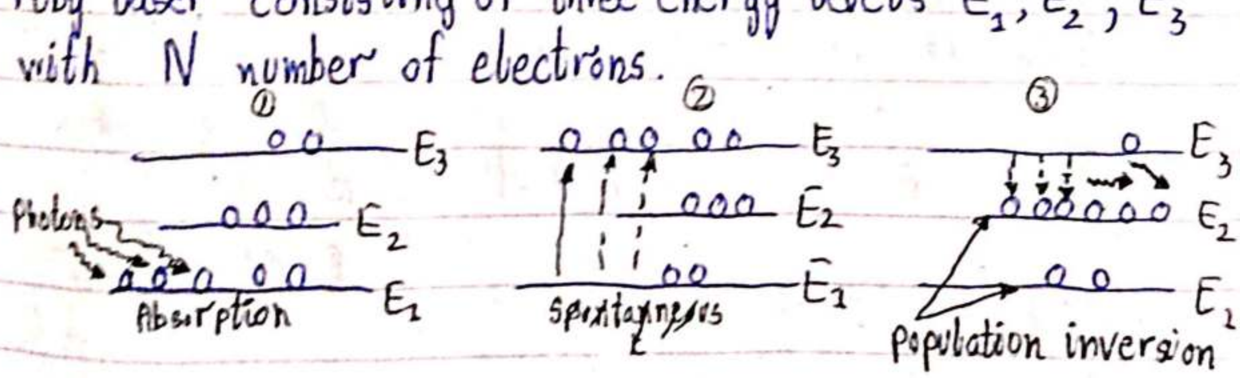
In ruby laser, we use flashtube as the pump source that provides energy to the laser medium (ruby)

iii) Optical Resonator:-

The ends of cylinderical ruby rod are flat and optically coated parallel mirrors. The fully silvered mirror will reflect light completely while partially silvered mirror will reflect most of the light but will allow a small portion through to produce output laser light.

Operation:-

In ruby laser, optical pumping technique is used to supply energy to the laser medium. Consider a ruby laser consisting of three energy levels E_1, E_2, E_3 with N number of electrons.



When light energy is supplied to laser medium, the electrons in ground state (E_1) gains enough energy and jumps into pump state (E_3). As the lifetime of or Laser action will stop when source is disconnected

E_3 is very small (10^{-8} sec) so after a short period, they fall into meta-stable state by spontaneous emission. Since the life-time of metastable state (10^{-3}s) is much greater than pump state and electrons reach to E_2 faster than they leave. This results in increase in no. of electrons in metastable state (E_2) known as population inversion. When the emitted photon interacts with the electron in metastable state, it forcefully makes the electron to fall into ground state (E_1) by stimulated emission, releasing two photons. This photon then stimulates nearby atoms to release more photons resulting in a chain reaction and hence light gain is achieved. The amplified light escapes through the partially reflecting mirror to produce deep red laser light.

90% 10%
Helium-Neon Laser

Type → Gas Laser

Active Medium → Mixture of He and Ne gas (laser or discharge glass tube)

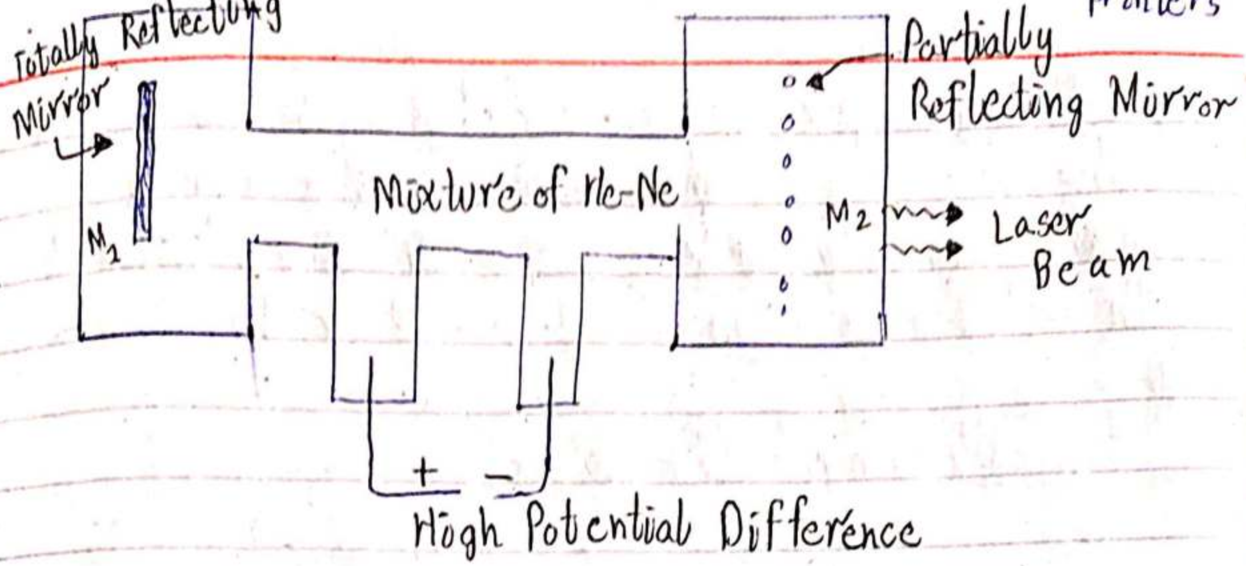
Pump Source → Electrical Pumping (High voltage power supply)

Resonating Cavity

Power Output → He-Ne laser generates power less than 10 mW.

Wavelength of output → Most commonly 632.8 nm

=> Helium - Neon Laser, proved that velocity of light is same in all directions. Lasers used in computer CDs, printers



Q White light is spread out into its spectral components by a diffraction grating. If slit separation is 0.5 mm, at what angle does red light of wavelength 640 nm appear in first order?

Data:-

$m = 1$ (1st order)

$d = 0.5\,mm = 5 \times 10^{-4}\,m$

$\lambda = 640\,nm = 640 \times 10^{-9}\,m$

$\theta = ?$

If 2000 groves/cm

$d = \frac{1 \times 10^{-2}}{2000}\,m$

$d = 5 \times 10^{-6}\,m$

Sol:-

$$d \sin\theta = m\lambda$$

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{1 \times 640 \times 10^{-9}}{5 \times 10^{-4}}\right)$$

$$\theta = 0.073^\circ$$

Q The hydrogen spectrum has a red line at 656 nm and a blue line at 434 nm. What are the angular separations b/w these two spectral lines obtained with a slit separation of 222.22 micro meter?

Data:-

$\lambda_B = 434\ nm = 434 \times 10^{-9} m$

$\lambda_R = 656 \times 10^{-9} m$

$\Delta\theta = ?$

$d = 222.22 \times 10^{-6}\ m$

Sol:-

In 1st order spectrum, diffraction angles are:

$$\theta_{BR} = \sin^{-1}\left(\frac{m\lambda_R}{d}\right) = \sin^{-1}\left(\frac{1 \times 656 \times 10^{-9}}{222.22 \times 10^{-6}}\right)$$

$$\theta_R = 0.169^\circ$$

For blue ray:-

$$\theta_{B_1} = \sin^{-1}\left(\frac{434 \times 10^{-9}}{222.22 \times 10^{-6}}\right) = 0.112^\circ$$

Angular separation in 1st order is:-

$$\Delta\theta = \theta_R - \theta_B = 0.169^\circ - 0.112^\circ$$

$$\Delta\theta = 0.057^\circ$$

In the second-order spectrum;

$$\Delta\theta = \sin^{-1}\left(\frac{2\times\lambda_R}{d}\right) - \sin^{-1}\left(\frac{2\lambda_B}{d}\right) = 0.114^\circ$$

In, third-order spectrum;

$$\Delta\theta = \sin^{-1}\left(\frac{3\lambda_R}{d}\right) - \sin^{-1}\left(\frac{3\lambda_R}{d}\right) = 0.172^\circ$$

Repeat till which order (probably $m = 338$) one of them doesn't appear.

Q A viewing screen is separated from a double slit source by 1.2m. Distance b/w two slits is 0.030mm. and second order bright fringe is 4.5cm from the central fringe. Calculate distance b/w adjacent bright fringes?

Data:-

$y_2 = 4.5\text{ cm} = 4.5\times10^{-2}\text{ m}$ $\quad m = 2$ (2^{nd} order)

$L = 1.2\text{m}$

$d = 0.03\text{ mm} = 0.03\times10^{-3}\text{ m}$

Δx: Fringe Spacing = ?

Solution:-

For bright fringe:-

$$y_m = \frac{mL\lambda}{d}$$

$$\lambda = \frac{y_m d}{mL} = \frac{4.5\times10^{-2}\times0.03\times10^{-3}}{2\times1.2}$$

$$\lambda = 5.625\times10^{-7}\text{ m}$$

$$\therefore \Delta x = \frac{L\lambda}{d} = \frac{1.2\times5.625\times10^{-7}}{0.03\times10^{-3}} = \boxed{\Delta x = 0.0225\text{m}}$$

Chapter 07

Modern And Nuclear Physics

Relative Motion:-

Change in position of a body with respect to fixed reference is called relative motion.

Frame of Reference:-

Coordinate system relative to which observations

- Inertial → (Rest, Uniform) i.e. $a=0$
- Non-inertial → (Accelerated) i.e. a = variable

Theory Of Relativity:-

- Special Theory → Inertial frame
- General Theory → Accelerated OR Non-inertial frame

→ Concept:-

- Speed of light is universal constant i.e. 3×10^{8} m/s
- Energy and mass are interconvertible and same laws for frame of reference at rest or constant speed

i) Mass Increment

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

ii) Length Contraction

$$L = L_0\sqrt{1-\frac{v^2}{c^2}}$$

Lorentz factor

iii) Time Dilation:-

$$t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

iv) Mass - Energy Relation :-

$$E = E_0 + KE$$

$$KE = mc^2 - m_0c^2$$

Wave - Particle Duality :-

A/c to wave theory, light spreads out as a continuous wave, whereas quantum theory suggests it consists of photons, each small enough to be absorbed by electron.

A/c to Planck's law :-

"Energy is emitted or absorbed in discrete amount or in energy packets called quanta or photon. The energy of each photon is directly proportional to the frequency of radiation."

$$E \propto \nu \qquad \because v = \lambda f \text{ or } c = \nu\lambda$$

$$E = h\nu \quad \text{OR} \quad E = hc/\lambda$$

$h \rightarrow$ Planck's constant $= 6.625 \times 10^{-34}$ J.sec

So it can be concluded that :-

Light travels as waves and absorbs and give off energy as particles.

Photoelectric Effect:-

The phenomenon in which ejection of electrons from a metal surface takes place when ~~light~~* falls on it is called photoelectric effect and electrons are called as photo-electrons.

* high frequency radiations (ultraviolet, X-Rays) etc

The photoelectric effect occurs because the electrons at the surface of metal absorb energy from incident radiation ~~and use it to~~ overcome the attractive forces that bind them to the metallic nuclei. A part of this energy is used in making the electron free from metal surface known as work function (ϕ) while the remaining energy is still carried by electron which comes out of the metal.

Hence using law of conservation of energy:-

$$\begin{pmatrix}\text{Incident Photon}\\ \text{Energy}\end{pmatrix} = \begin{pmatrix}\text{Work}\\ \text{Function}\end{pmatrix} + \begin{pmatrix}\text{KE of}\\ \text{electron}\end{pmatrix}$$

$$E = \phi + KE_{max}$$

$$KE_{max} = h\nu - h\nu_0 \qquad \because E = h\nu$$

$$\frac{1}{2} m v_{max}^2 = h\nu - h\nu_0 \qquad \because KE = \frac{1}{2} m v^2 = eV_0$$

$$eV_0 = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$$

Also, $v_{max} = \sqrt{\frac{2eV_0}{m}} \Rightarrow$ From $KE = W$, $\frac{1}{2} m v^2 = eV$

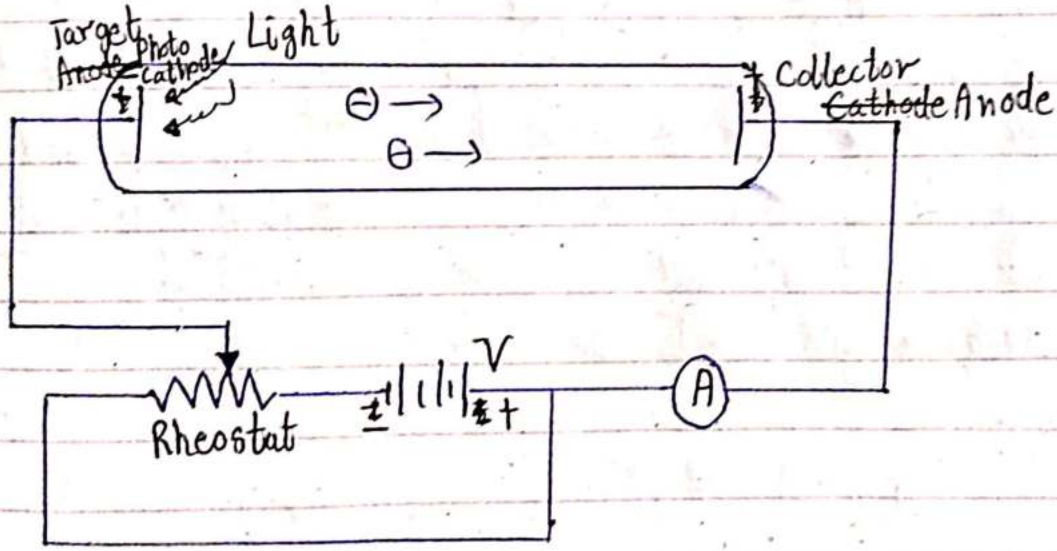
$f_0 \rightarrow$ Cutoff frequency $\neq$ $f_0 \rightarrow$ Threshold Frequency

$\nu_0 \rightarrow$ Threshold Frequency (~~Fre~~ Minimum Frequency of radiation that will produce photoelectric effect)

$\lambda_0 \rightarrow$ Cutoff - or threshold Wavelength (Minimum wavelength of light which causes electrons to be emitted from metal surface)

$\phi \rightarrow$ Work Function (Minimum energy required to eject an electron from metal surface)

$V_0 \rightarrow$ Stopping Potential (The potential required to stop ejection of electron from a metal surface when incident beam of energy greater than work function of metal is directed on it).



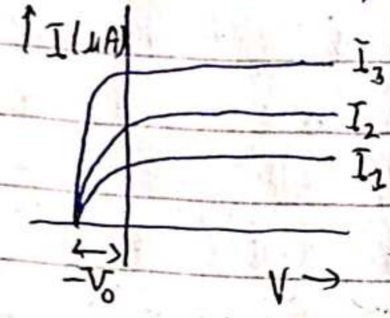
Saturation Current:-

The maximum constant current that can flow when the applied voltage is above certain threshold

Experimental Results:-

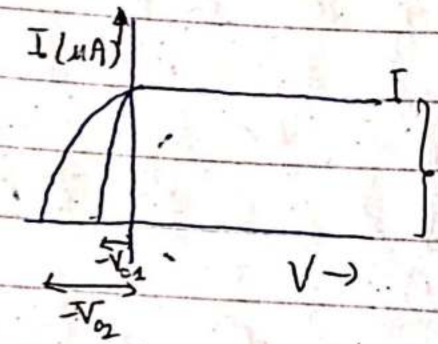
i) Different Intensities, same frequency

Photocurrent vs Voltage



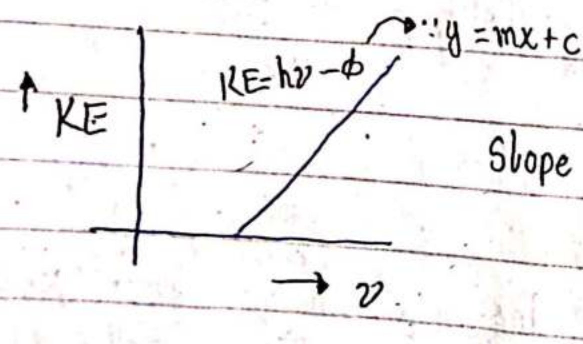
Intensity $\propto$ no. of electrons, Saturation current $\propto$ intensity

ii) Different Frequencies, Same Intensity



frequency $\propto$ velocity of photon (more potential required to stop)

iii) Frequency & KE



Slope of line = h/e (Planck's constant)

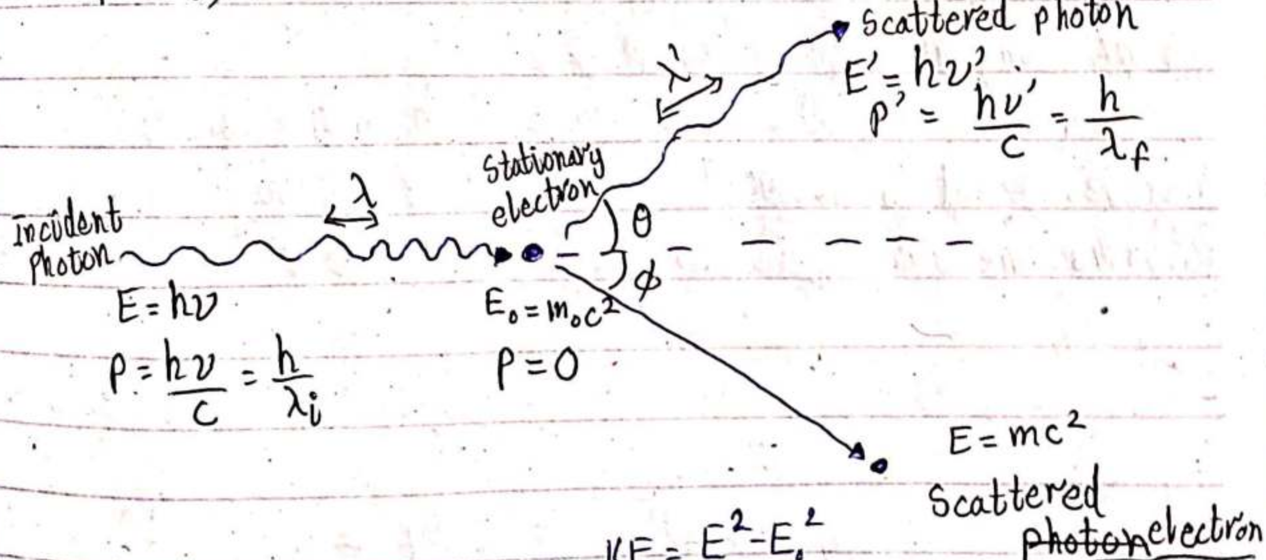
Compton's Effect :- (inelastic scattering) (high energy photons)

~~The phenomenon in which photon~~ When X-Ray photons are scattered by loosely bounded electrons such that there is an increase in wavelength then this phenomenon is called as Compton Effect.

In this collision, incident photon's some energy and momentum is transferred to matter (electron).

Applying energy and momentum conservation laws, we have :-

$$\underbrace{\Delta\lambda}_{\substack{\text{Compton shift}\\(\lambda_f - \lambda_i)}} = \underbrace{\frac{h}{m_0 c}}_{\text{compton wavelength}} (1 - \cos\underbrace{\theta}_{\text{scattering angle of photon}})$$



$KE = E^2 - E_0^2$

$p = mv = \frac{\sqrt{E^2 - (m_0c^2)^2}}{c}$ i.e. $\frac{KE}{c}$

- Compton Effect evidences inelastic collision and incoherent scattering.
- Compton Effect shows particle nature of light.
- The amount by which the wavelength of photon changes is known as Compton shift. $(\Delta\lambda)$.

Extreme Cases Of Compton ~~Shift~~ Scattering:

•- Extreme hit:-

At $\theta = 180°$, photon is scattered directly backwards its original path transferring maximum kinetic energy to the electron.

$$\Delta\lambda = \frac{h}{m_0 c}\left(1-\cos(180°)\right)$$

$$\Delta\lambda = \frac{2h}{m_0 c}$$

$$\Delta\lambda_{max} = 2\,\Delta\lambda_c$$

•- Grazing hit:- To touch while moving past

At $\theta = 0°$, wavelength shift is smallest and scattered photon essentially follows the same path as incident photon.

$$\Delta\lambda = \frac{h}{m_0 c}\left(1-\cos 0°\right)$$

$$\Delta\lambda_{min} = 0 \quad \Rightarrow \lambda_f = \lambda_i$$

Black-body:-

An ideal body that allows all incident radiation to pass into (zero reflectance) and that absorbs internally all the incident radiations (zero transmittance), regardless of its frequency, is called Black Body.

cavity walls are
absorptive
(perhaps painted
black)

A blackbody emitting black-body radiation is also called thermal radiation.

Characteristics

- Independent of the material of black body.
- Only Depends on temperature of cavity.
- Such radiation lie in infrared and visible region only.
- Such radiations are group of different wavelengths at a particular temperature.

Pair Production :- (Only by γ rays)
↳ Energy to matter

$$h\nu = 2m_0c^2 + KE_{e^-} + KE_{e^+}$$

Require $E > 1.02$ MeV to occur.

Pair Annihilation :-
↳ Matter to Energy

$$2m_0c^2 + KE_{e^-} + KE_{e^+} = 2h\nu$$

- Annihilation of matter is reverse of pair production.
- Photoelectric effect is reverse of X-Ray production.

De Broglie Hypothesis :-

Louis de Broglie in 1924

Proposed that :-

"All matter exhibit wave like properties and (relates the observed) wavelength of matter is proportional to its momentum."

A/c to Einstein's mass energy equation:-

$$E = mc^2$$

$$E = mc \cdot c \qquad ; \text{but } p = mv = mc$$

$$E = pc \quad -(i)$$

A/c to Planck's quantum theory of light, we have:-

$$E = h\nu \qquad ; \text{but } \nu = \frac{c}{\lambda} \quad \because v = \lambda f$$

$$E = \frac{hc}{\lambda} \quad -(ii)$$

Comparing eq (i) and (ii),

$$pc = \frac{hc}{\lambda}$$

$$p = \frac{h}{\lambda} \quad \text{OR} \quad \lambda = \frac{h}{p} \qquad \lambda \propto \frac{1}{p}$$

$$\lambda = \frac{h}{mv} \Rightarrow \lambda = \frac{h}{\sqrt{2mKE}} \qquad \because KE = \frac{p^2}{2m}$$

$$\lambda = \frac{h}{\sqrt{2meV}} \qquad ; \text{but } KE = eV$$

⇒ The theoritical prediction of De Broglie hypothesis was experimentally confirmed by Davisson and Germer experiment in 1927.

Heisenberg Uncertanity Principle:-

It states that:-

"It is impossible to measure the position and momentum of a particle simultaneously with great accuracy."

$\Delta x \approx \lambda$

$\Delta p \approx \frac{1}{\lambda}$

$$\Delta x \cdot \Delta p \approx \frac{h}{4\pi}$$

$$\Delta x \cdot \Delta p \approx \frac{\hbar}{2}$$

$$\Delta E \cdot \Delta t \approx \frac{\hbar}{2}$$

Here,

$h \rightarrow$ Planck's constant (6.625×10^{-34} J.sec)

$\hbar \rightarrow$ Reduced Planck's constant OR Dirac's constant (1.05×10^{-34} J.sec)

=) In order to observe position of e^- with less uncertanity, we must use light ~~for~~ of shorter wavelength.

Less uncertain $\downarrow \Delta x \approx \Delta \lambda \downarrow$ so $\uparrow \Delta p \approx \frac{1}{\lambda \downarrow}$

Radioactivity:-
Spontaneous disintegration of nucleus of an atom, in which invisible radiations are emitted from nucleus of atom.

Atomic Nucleus:-
Nucleus is the positively charged center of an atom and contain most of its mass. Nuclei consists of nucleons (neutrons and protons).

$m_p = 1836\, m_e$

Nuclear Radiations:-
Energy particles or rays that are given off from a radioactive element.

Common Source: Radium-226
i) Alpha-Radiation:- Approx Energy: 5 MeV
Positively charged (+2) Helium nuclei, slow moving and less penetrating radiation. They are highly ionzed but least dangerous type of radiation. They can be stopped by a sheet of paper.

Carbon-14
ii) Beta-Radiation:- 0.05 to 1 MeV
Fast moving electrons (-1). They are penetrative but can be stopped by certain metals like aluminium.

Cobalt-60
iii) Gamma Radiations: 1 MeV
Fast moving and can penetrate anything eg: X-Rays, except for concrete or lead. They are neutral radiations of charge (0).

Nuclear Reactions:-

The reactions in which one or more ~~nuclei~~ nuclides are produced from the collision between two atomic nuclei or subatomic particles are called nuclear reactions.

i) Nuclear Fusion:-

A nuclear reaction in which two light nuclei fuse/merge to form a heavy nucleus with the release of energy is called nuclear fusion.

ii) Nuclear Fission:-

The splitting of an atomic nucleus into two lighter nuclei by nuclear reaction or radioactive decay with the release of energy (neutrons, gamma rays) is called nuclear fission.

Example:-

$$^{235}U + {}^{1}_{0}n \longrightarrow {}^{141}Ba + {}^{92}Kr + 3\,{}^{1}_{0}n$$

Radioactive Decay:- ~~An~~

It is the process in which an unstable nuclei decays (loses its energy) by emitting ionizing radiation.

Radio Carbon Dating:-

It is the method for determining the age of an object by using the properties of radio carbon (C^{14}).

$$t = \frac{t_{1/2}}{\ln 2} \ln\left(1 + \frac{N_f}{N_i}\right)$$

$t_{1/2} \rightarrow$ Half-life

Unified Atomic Mass Unit (u):-

- The unified atomic mass unit is one-twelfth of the mass of carbon atom. Formerly was called atomic mass unit (amu).

$$1u = 1\,amu = 1.66 \times 10^{-27}\ kg$$

Mass Defect:-

The difference of mass between the sum of the masses of nucleons and mass of nucleus.

OR

Difference b/w theoritical & actual mass of nucleus.

Mass Defect = Mass of nucleons − Mass of nucleus

$$\Delta m = [Zm_p + (A-Z)m_n] - M$$

Binding Energy:-

Binding energy is The amount of energy required to break a nucleus apart into separated nucleons

$$BE = \Delta m\, c^2 \quad OR \quad \Delta m \times 931.5\ MeV$$

Δm ↳ In kg, SI

Δm ↳ in amu, CGS

SI-unit of Activity is Becquerel

$$\underset{\text{(Curie)}}{1\,ci} = 3.7 \times 10^{10}\ dis/_{sec} \text{ OR } 3.70 \times 10^{10}\ Bq$$

Activity:-

Rate of disintegration of a radioactive nuclei is called activity.

$$\underset{dis/sec}{A} = \frac{\Delta N}{\Delta t} \quad \text{OR} \quad A = \lambda N$$

$\lambda \rightarrow$ Decay constant (sec^{-1})

$$1\,\frac{dis}{sec} = 1\ Bq$$

Half-Life:-

Time required to decay half of the atoms of radioactive element is called half life.

$$N = \frac{N_o}{(2)^n}$$ $n \rightarrow$ no. of half lives

$$T_{1/2} = \frac{\ln(2)}{\lambda}$$

Q-value
+ve $\Rightarrow$ Exothermic
-ve $\rightarrow$ Endothermic

Packing Fraction:-

Mass defect per nucleon. It is measure of stability of nuclei. ~~More~~ Less packing fraction of nuclei, more stable nuclei is. Less

Unit:- $amu/_A$ or $kg/_A$

$$PF = \frac{\Delta m}{A}$$

$A \rightarrow$ nucleon (Mass Number)

Binding Fraction:-

Binding Energy per nucleon.

$$\cancel{BE}\ BF = \frac{BE}{A}$$ Unit: $\frac{MeV}{A}$

Law Of Radioactive Decay:-

It states that:-

"Rate of decay in a radioactive process is directly proportional to number of parent nuclides present at the time of disintegration."

i.e.

$$-\frac{dN}{dt} \propto N$$

Where $-ve$ sign = Decay process
N = No. of parent nuclei.

$$-\frac{dN}{dt} = \lambda N$$

$$\frac{dN}{dt} = -\lambda N \Rightarrow \frac{dN}{N} = -\lambda dt$$

$\lambda \rightarrow$ Decay constant (sec^{-1})

Integrating both sides

$$\int \frac{dN}{N} = \int -\lambda dt$$

$$\int \frac{1}{N} dN = -\lambda \int dt$$

$$\ln(N) = -\lambda t + c \quad -(i)$$

At $t = 0$ sec, $N = N_0$

$$\ln(N_0) = -\lambda(0) + c$$
$$\ln(N_0) = c$$

Now eq.(i) becomes

$$\Rightarrow \ln(N) = -\lambda t + \ln(N_0)$$
$$\ln(N) - \ln(N_0) = -\lambda t$$
$$\ln\left(\frac{N}{N_0}\right) = -\lambda t$$

~~Multiplying by~~ Applying 'e' on both sides

$$e^{\ln(N/N_0)} = e^{-\lambda t}$$
$$\frac{N}{N_0} = e^{-\lambda t}$$
$$N = N_0 e^{-\lambda t}$$

$N \rightarrow$ No. of ~~decayed~~ present nuclei after time 't'

For Half life:-

$$t = T_{1/2} \quad , \quad N = \frac{N_0}{2}$$

$$\Rightarrow \frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}$$

$$\frac{1}{2} = \frac{1}{e^{-\lambda T_{1/2}}} \Rightarrow e^{\lambda T_{1/2}} = 2$$

Applying 'ln' on both sides

$$\ln(e^{\lambda T_{1/2}}) = \ln(2)$$

Paper 5 marks each CLO has 4 questions (1-1-5 ...)
(Free-body diagram must)

$$\lambda T_{1/2} = \ln(2)$$

$$T_{1/2} = \frac{\ln(2)}{\lambda} \quad OR \quad T_{1/2} = \frac{0.693}{\lambda}$$

Radiation Detectors:-

These are instruments or devices used to detect, measure and quantify the presence of ionizing radiations in the environment surrounding.

Three types of detectors:-

i) Gas-filled detectors
ii) Scintillators
iii) Solid state detectors

Q ~~In~~ From activity:-

$$A = \lambda N = -\frac{dN}{dt}$$

$$\Rightarrow A = A_0 e^{-\lambda t}$$

Q ~~Wh~~ In a sample of rock, the ratio of Pb^{206} to U^{238} nuclei is found to be 0.65, with a half life of 4.5×10^9 years. What is the age of rock?

Data:-

$\frac{N_f}{N_i} = 0.65$, $T_{1/2} = 4.5 \times 10^9$ years

$t = ?$

$$e^{\lambda t} = \frac{N_f}{N_{(t)}} + \frac{N_{(t)}}{N_{(t)}} \longleftarrow N_i = (N_i + N_F)\, e^{-\lambda t}$$

$$\ln(e)^{\lambda t} = \ln\left[\frac{N_f}{N_i} + 1\right] \Rightarrow t = \frac{T_{1/2}}{\ln(2)} \ln\left[1 + \frac{N_F}{N_i}\right]$$

Solution:-

1 is added so log remains +ve and 1 for initial quantity

$$t = \frac{T_{1/2}\ \ln}{\ln(2)}\left[1 + \frac{N_F}{N_i}\right] = \frac{4.5\times10^9 \ln[1 + 0.65]}{0.693}$$

$t = \cancel{1.07 \times 10^{16}\ \text{years}}$ ✗ ~~wrong~~ method.

3.25×10^9 years

Also correct as More derived formula

OR $\therefore N(t) = N_0\, e^{-\lambda t}$

Now decay will form $N_0^f = N_{0}^{(new)} + N_f$ $\therefore \lambda = \frac{\ln(2)}{T_{1/2}}$

$N(t) = (N_0^i + N(t))\, e^{-\lambda t}$ → from here formula is derived and 1 is added and 'e' sign is reversed i.e. $t = \frac{T_{1/2}}{\ln(2)} \ln\left[1 + \frac{N_F}{N_i}\right]$

$$\frac{N(t)}{N_0} = e^{-\frac{\ln(2)}{T_{1/2}} t}$$

$$\Rightarrow 1 + 0.65 = e^{+1.54\times10^{-10} \cdot t}$$

Applying 'ln' both sides:-

$$\ln(1.65) = \ln e^{-1.54\times10^{-10} \cdot t}$$

$$\Rightarrow \frac{\ln(1.65)}{+1.54\times10^{-10}} = t$$

$t = \cancel{2.797}\ 3.25 \times 10^9$ years ✓

Q When will 5 mCi of I^{131} ($T_{1/2} = 8.05$ days) and 2 mCi P^{31} ($T_{1/2} = 14.3$ days) have equal activity?

Data:-

$A_I = 5$ mCi or 5×10^{-3} Curie

$\lambda_I = 0.086$ days^{-1} from $\ln(2)/T_{1/2}$

$A_P = 2 \times 10^{-3}$ Ci

$\lambda_P = 0.048$ days^{-1}

$t = ?$

Solution:-

$$1\,Ci = 3.7 \times 10^{10}\ Bq$$

$$A_I = A_P$$

$$A_o\, e^{-\lambda_I t} = A_o\, e^{-\lambda_P t}$$

$$(5\times 10^{-3})\, e^{-(0.086)t} = (2\times 10^{-3})\, e^{-(0.048)t}$$

$$2.5\ e^{-0.086t} = e^{-0.048t}$$

Applying 'ln' both sides

$$\ln(2.5 \cdot e^{-0.086t}) = \ln e^{-0.048t}$$

$$\ln(2.5) + \ln e^{-0.086t} = -0.048t \quad ; \text{ but } \ln(a\cdot b) = \ln(a) + \ln(b)$$

$$t = \frac{\ln(2.5)}{0.086-0.048} = 24.11 \text{ days}$$

Q 1g Thorium emits 4500 α-particles per second. Find $T_{1/2}$ at mass 232.12 amu? disintegrations

Data:-

$A = 4500$ dis/sec

$m = 1g$

$T_{1/2} = ?$

Sol:-

No. of atoms = No. of nuclei

$$N = \frac{\text{mass}}{\text{mol. mass}} \times N_A$$

$$N = \frac{1}{232.12} \times 6.02\times 10^{23}$$

$$N = 2.59\times 10^{21} \text{ nuclei}$$

$1\ amu = 1.66 \times 10^{-24}\ g = 931.5\ MeV/\cancel{c^2}$

$\because A = \lambda N$; but $\lambda = \frac{\ln(2)}{T_{1/2}}$

$$T_{1/2} = \frac{\ln(2)\, N}{A} = \frac{\ln(2) \cdot 2.59 \times 10^{21}}{4500}$$

$$T_{1/2} = 4 \times 10^{17}\ sec$$

Q Calculate Binding Fraction of N^{20}_{20}. Given that, mass of neutron is 1.008665 amu while mass of proton is 1.007276 amu. $N^{20}_{20} = 19.9924$ amu?

Solution:-

$Z = 10$, $A - Z$
$20 - 10 = 10$

For mass defect:-

$$\Delta m = [Z m_p + (A - Z) m_n] - M$$

$$\Delta m = [(10)(1.007276) + (10)(1.008665)] - 19.9924$$

$$\Delta m = 0.16701\ amu$$

$$BE = \Delta m \times 931.5\ MeV$$

$$BF = \frac{0.16701 \times 931.5}{20} \quad \frac{BE}{A}$$

$$BF = 7.78\ MeV/\cancel{amu}$$

Q The Po^{216} nucleus decays to Pb^{212} by emitting an alpha particle i.e. He^4. Given that

$Po^{216} = 216.001905$ amu, $Pb^{212} = 211.99188$ amu

$He^4 = 4.002603$ amu

Find binding energy per nucleon?

Solution:-

$\Delta m = (\text{Mass of reactants}) - (\text{Mass of Products})$

$\Delta m = 216.001915 - (211.991898 + 4.002603)$

$\Delta m = 0.007414$ amu

$$\frac{BE}{A} = \frac{\Delta m \times 931.5}{A} = \frac{0.007414 \times 931.5}{216}$$

$BF = 0.032$ MeV

Q Find radioactive decay of 'N' no. of nuclei disintegrating with λ. Find out its half life if $\lambda = 1.73 \times 10^{-10}\ sec^{-1}$?

Data:-

$N_0 = N$, $N(t) = ?$

$\lambda = 1.73 \times 10^{-10}\ sec^{-1}$

$T_{1/2} = ?$

Sol:-

$\because N(t) = N_0 e^{-\lambda t}$

but given $N_0 = N$

$$N = N e^{-\lambda t}$$

$$1 = e^{-\lambda t}$$

$$e^0 = e^{-\lambda t}$$

Bases equal so comparing powers:

$0 = -\lambda t$; but $\lambda = 1.73 \times 10^{-10}\ sec^{-1}$

$t = 0$ sec

So at $t = 0$ sec, radioactive decay will be equal to 'N' particles

$$T_{1/2} = \frac{\ln(2)}{1.73\times10^{-10}} = 4\times10^{9}\ sec$$

Photoelectric effect:-

Q The min frequency for photoelectric emission in copper is 1.1×10^{15} Hz. Find the max energy of photoelectrons (in eV) when light of frequency 1.5×10^{15} Hz is directed on copper surface?

Data:-

$\nu_0 = 1.1\times10^{15}$ Hz

$\nu = 1.5\times10^{15}$ Hz

$h = 6.63\times10^{-34}$ J-sec

$KE_{max} = ?$

Sol:-

$$KE_{max} = E - \phi$$

$$KE_{max} = h\nu - h\nu_0 = 6.63\times10^{-34}(1.5\times10^{15} - 1.1\times10^{15})$$

$$KE_{max} = 2.652\times10^{-19}\ J$$

; but $1eV = 1.602\times10^{-19}$ J

$$KE_{max} = \frac{2.652\times10^{-19}}{1.602\times10^{-19}}$$

$$KE_{max} = 1.66\ eV$$

Q Find the work function (ϕ) of calcium, if stopping potential of Na is 1.2 volts and cut-off frequency is 5.38×10^{14} Hz?

Data:-

$V_0 = 1.2$ volts

$e = 1.602 \times 10^{-19}$ C

$h = 6.63 \times 10^{-34}$ J·sec

$\nu_0 = 5.38 \times 10^{14}$ Hz

$\phi = ?$

Solution:-

$$\phi = h\nu_0$$

$$\phi = (6.63 \times 10^{-34})(\cancel{1.602 \times 10^{-19}}\ 5.38 \times 10^{14})$$

$$\phi = \frac{3.57 \times 10^{-19}}{1.602 \times 10^{-19}} = 2.23\ eV$$

Compton scattering

Q A beam of X-rays is scattered by a target. At 45° from the beam direction, the scattered X-Rays have a wavelength of 2.2 pico meter. What is the wavelength of X-Rays in direct beam?

Data:-

$\theta = 45°$

λ_f : Scattered photon wavelength $= 2.2 \times 10^{-12}$ m

$h = 6.63 \times 10^{-34}$ J·sec

m_0 : Rest mass of $e^- = 9.109 \times 10^{-31}$ kg

$c = 3 \times 10^8$ m/s

λ_i : Incident photon wavelength = ?

Solution:-

$$\Delta\lambda = \frac{h}{m_o c}(1-\cos\theta)$$

$$\lambda_f - \lambda_i = \frac{h}{m_o c}(1-\cos\theta)$$

$$2.2\times10^{-12} - \left[\frac{6.625\times10^{-34}\,(1-\cos45^\circ)}{9.109\times10^{-31}\times3\times10^{8}}\right] = \lambda_i$$

$$\lambda_i = 1.49\times10^{-12}\ m.$$

Q X-Rays with $\lambda = 71pm$ are scattered from calcite target. The scattered photon is viewed at 30° to incident beam. Calculate the kinetic energy imparted to recoiling electron?

Data:-

$\lambda_i = 71\times10^{-12}\ m$

$\theta = 30^\circ$

$h = 6.63\times10^{-34}\ J\cdot sec \quad c = 3\times10^{8}\ m/s$

$m_o = 9.109\times10^{-31}\ kg$

KE of $e^- = ?$

Solution:-

$$\Delta\lambda = \frac{h}{m_o c}(1-\cos\theta)$$

$$\lambda_f = \frac{6.63\times10^{-34}}{9.109\times10^{-31}\times3\times10^{8}}(1-\cos30^\circ) + 71\times10^{-12}$$

$$\lambda_f = 7.133\times10^{-11}\ m$$

The kinetic energy will be the one supplied by photon as electron is initially at rest, so:-

$$KE = E_{initial} - E_{final}$$

$$KE = \frac{hc}{\lambda_i} - \frac{hc}{\lambda_f}$$

$$KE = (6.63 \times 10^{-34} \times 3 \times 10^{8}) \left[\frac{1}{71 \times 10^{-12}} - \frac{1}{7.133 \times 10^{-11}} \right]$$

$$KE = 1.296 \times 10^{-17} \text{ J} \quad ; \text{ but } 1eV = 1.602 \times 10^{-19} \text{ J}$$

$$KE = \frac{1.296 \times 10^{-17}}{1.602 \times 10^{-19}}$$

$$KE = 80.9 \text{ eV}$$

Q2 Find De-Broglie wavelength of β-particle of mass 9×10^{-31} kg drifting with a speed of 8×10^{3} cm/s and electron whose kinetic energy is 120 eV?

Data:-

a) $m = 9 \times 10^{-31}$ kg $\qquad h = 6.625 \times 10^{-34}$ J·sec

$v = 8 \times 10^{3}$ cm/s $= \frac{8 \times 10^{3}}{1000} = 8$ m/s

$\lambda = ?$

b) $m_0 = 9.109 \times 10^{-31}$ kg $\qquad KE = 120$ eV

$\lambda = ?$

Solution:-

a) For De-Broglie wavelength:-

$$\lambda = \frac{h}{p}$$

$$\lambda = \frac{h}{mv} = \frac{6.625 \times 10^{-34}}{9 \times 10^{-31} \times 8}$$

$$\lambda = 9.201 \times 10^{-5} \, m.$$

b) $KE = 120 \, eV = 120 \, (1.602 \times 10^{-19})$

$KE = 1.9224 \times 10^{-17} \, J$

$$\lambda = \frac{h}{\sqrt{2mKE}}$$

$$\lambda = \frac{6.625 \times 10^{-34}}{\sqrt{2 \times 9.109 \times 10^{-31} \times \cancel{1.602 \times 10^{-19}}}}$$
$$1.9224 \times 10^{-17}$$

$$\lambda = 1.2 \times 10^{-10} \, m$$

$$\lambda = 1.2 \, \mathring{A} \qquad \because 1 \, \mathring{A} = 1 \times 10^{-10} \, m$$

Hence, De Broglie wavelength of electron $\cong$ size of atom.

CHAPTER # 05

(ELECTROSTATIC & MAGNETISM)

CHARGE:~

The intrensic property (property by nature) of any particle is called "charge".

Properties:~

Q → Rest → Electrostatics → F_e ⇒ Columb's law

Q → Moving → Magnetism
- → Constant velocity → current → Ohm's law.
- → Acceleration → EM → Maxwell's law.

"ELECTROSTATICS"

A branch of physics, which deals with electric charges at rest.

"MAGNETISM"

Magnetism is caused by the motion of electric charge. Actually, it is a force that is asserted on a charge causes them to attract or repel.

COLUMB's LAW:~

Fe (attractive)

$q_+ \longrightarrow \longleftarrow q_-$

r^2

STATEMENT :~

"If there were two particles, having electrical charges q_1, q_2, then the force is directly proportional to the product of charges and inversely proportion to the square of distance b/w them"

$\boxed{\text{Electrostatic force doesnot depends on the size of charge.}}$

MATHEMATICALLY:~

$$F \propto q_1 q_2$$

$$F \propto 1/r^2$$

r^2 is due to the path of 2 charges.

$$\Rightarrow Fe = \frac{K q_1 q_2}{r^2}$$

$$\boxed{K = 9 \times 10^9 \ Nm/C^2}$$

where,

$$K = \frac{1}{4\pi \varepsilon_0}$$

$$\boxed{\varepsilon_0 = 8.85 \times 10^{-12} \ F/m}$$

In vector form:~

$$\vec{F} = \left(\frac{Kq_1q_2}{r^2}\right)\hat{r}$$

If the q_1 is applying force on q_2, and it is not responding then the force of q_2 given as:

$$\vec{F}_{21} = \left(\frac{Kq_1q_2}{r^2}\right)\hat{r}_{21}$$

From the above figure:-

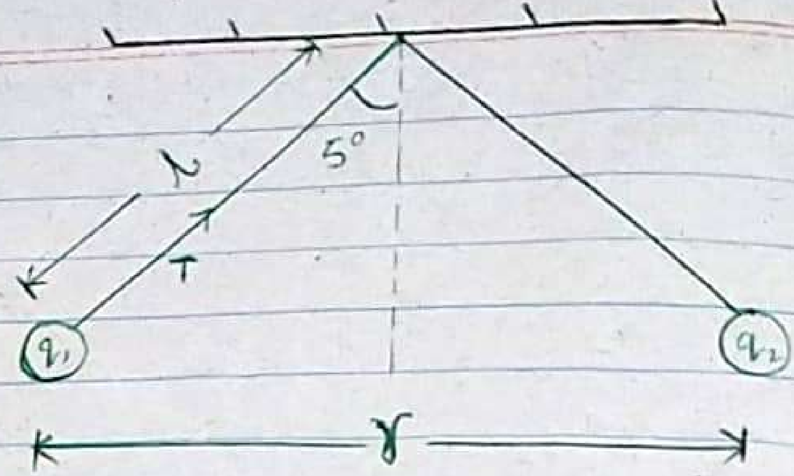
$$r_{12} = -r_{21}$$

and,

$$F_{12} = -F_{21}$$

NUMERICALS:~

Q:~ Two identical small changes spheres each having a mass of 3×10^{-2} kg, in equilibrium state. The lenght of each string $0.15\,m$ and the angle $5°$. Find the magnitude of charge on each sphere?

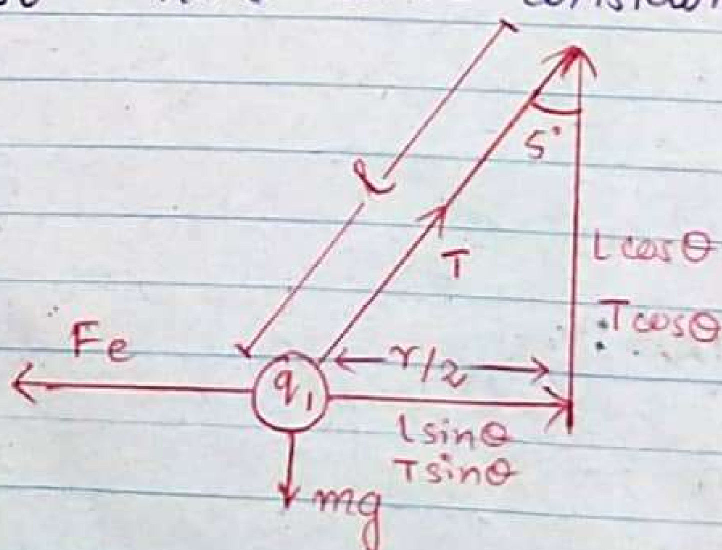


Data:

- $\rightarrow l = 0.15m$
- $\rightarrow m = 3 \times 10^{-2} kg.$
- $\rightarrow \theta = 5^\circ$
- $\rightarrow q = ?$

Solution:

First have some considerations as:



As the figure is in equilibrium:-

$$\Sigma F_x = F_e - T\sin\theta = 0 \quad --- (1)$$

$$\Sigma F_y = mg - T\cos\theta = 0 \quad --- (2)$$

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comparing both equations by division :-

$$\Rightarrow \frac{T\sin\theta}{T\cos\theta} = \frac{F_e}{mg}$$

$$\Rightarrow \tan\theta = \frac{F_e}{mg}$$

$$\Rightarrow F_e = \tan(5^\circ) \times (3\times10^{-2})(9.8)$$

$$\Rightarrow \boxed{F_e = 0.025\,N}$$

By using columb's law :-

$$F_e = \frac{K q_1 q_2}{r^2}$$

$$\Rightarrow q^2 = \frac{F_e r^2}{K} \quad \text{---} \ (a)$$

From the figure:-

$$\frac{r}{2} = L\sin\theta$$

$$\Rightarrow r = 2L\sin\theta = 2(0.15)\sin 5^\circ$$

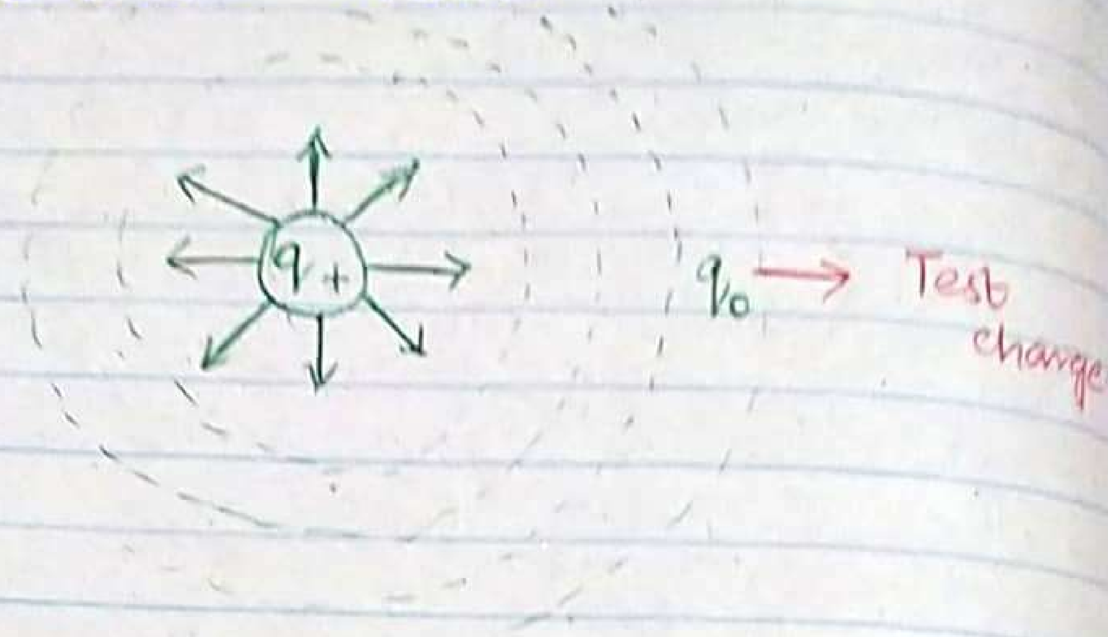
$$\Rightarrow \boxed{r = 0.026\,m}$$

eq --(a) $\Rightarrow$

$$q_2 = \frac{(0.025)(0.026)}{9\times10^9}$$

$\Rightarrow \boxed{q = 43 \times 10^{-9}\, C}$

ELECTRIC FIELD:~

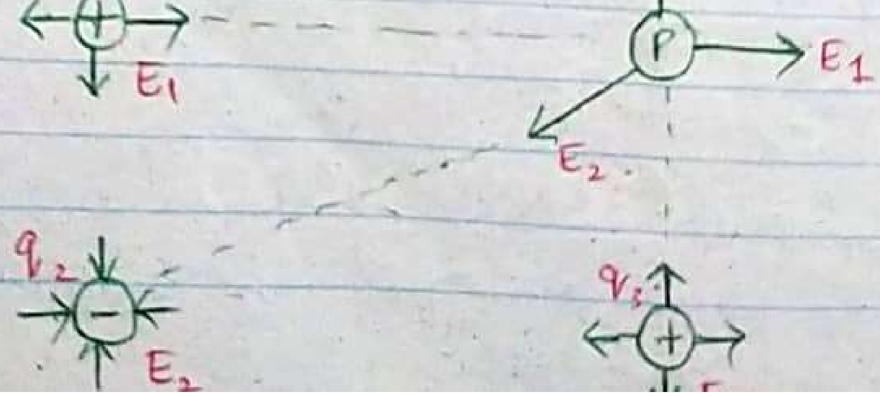


$$E = \frac{F}{q_0} \qquad (\text{Unit} = N/C)$$

SUPERPOSITION PRINCIPAL:~

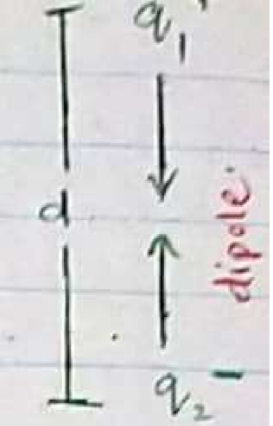
Principal:~

"The pricipal defined the vector sum of electric field due to having multiple charges."

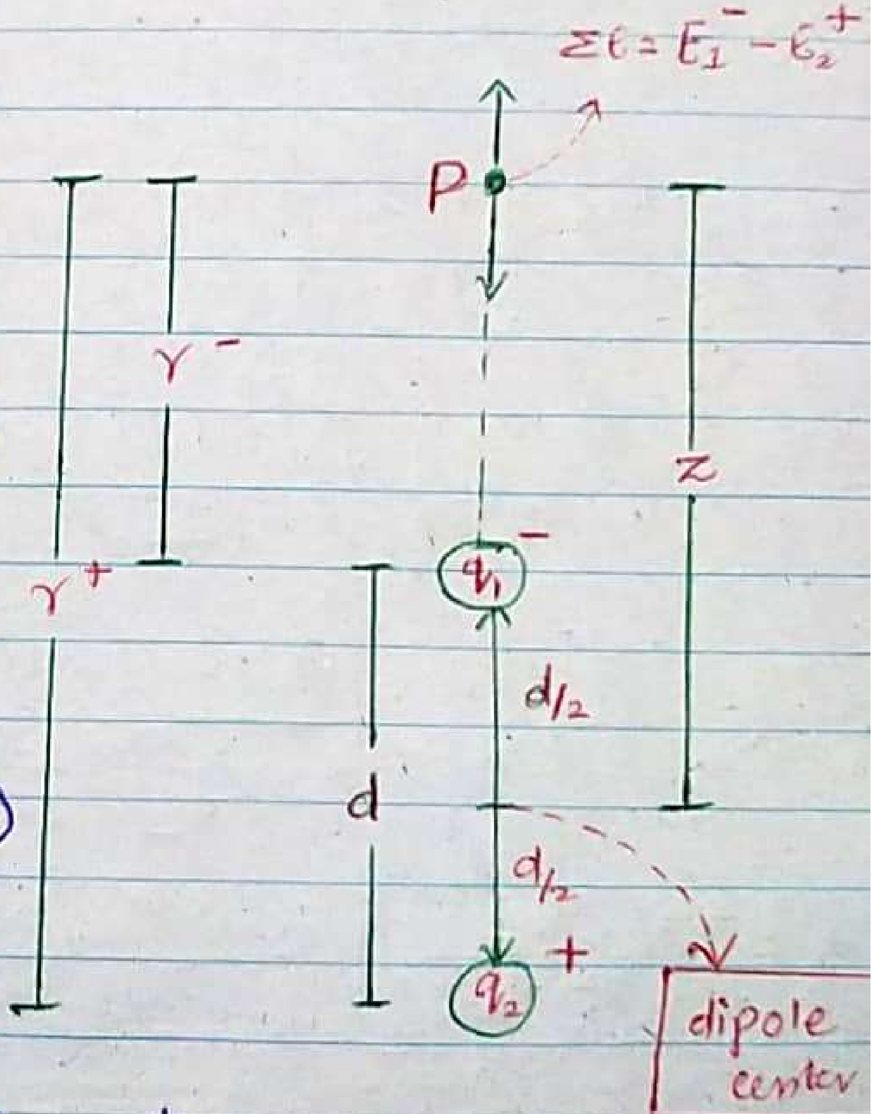


DIPOLE:~

"In order to achieve dipole condition, you have to determine dipole movement"



ELECTRIC FIELD DUE TO DIPOLE:~



From the figure:

$z = r^- + d/2$

$\Rightarrow r^- = z - d/2$ -- ①

And,

$z = r^+ - d/2$

$\Rightarrow r^+ = z + d/2$ -- ②

ectric field on point charge:~

$$\Sigma E_P = E_1^- - E_2^+$$

since,

$$E = \frac{kq}{r^2}.$$

$$\Rightarrow \Sigma E_p = \frac{Kq_1}{(r^-)^2} - \frac{Kq_2}{(r^+)^2}$$

$\because q_1 = q_2 = q$

$$\Rightarrow \Sigma E_p = Kq \left[\frac{1}{(r^-)^2} - \frac{1}{(r^+)^2} \right]$$

From eq -- 1 & 2 :-

$$\Rightarrow \Sigma E_p = Kq \left[\frac{1}{(z - d/2)^2} - \frac{1}{(z + d/2)^2} \right]$$

$$\Rightarrow \Sigma E_p = \frac{Kq}{z^2} \left[\frac{1}{\left(1 - \frac{d}{2z}\right)^2} - \frac{1}{\left(1 + \frac{d}{2z}\right)^2} \right]$$

$$\Rightarrow \Sigma E_p = \frac{Kq}{z^2} \left[\frac{(1 + d/2z)^2 - (1 - d/2z)^2}{(1 - d/2z)^2 (1 + d/2z)^2} \right]$$

$$\Rightarrow \Sigma E_p = \frac{Kq}{z^2} \left[\frac{(1 + d/2z - 1 + d/2z)(1 + d/2z + 1 - d/2z)}{1 - \frac{d^2}{4z^2} + \frac{d^4}{16z^4}} \right]$$

$$\Rightarrow \Sigma E_p = \frac{Kq}{z^2}\left\{\frac{2d/z}{\left(1-\frac{d^2}{4z^2}\right)^2}\right\}$$

$$\Rightarrow \Sigma E_p = \frac{2q \cdot d}{4\pi\varepsilon_0 z^3}\left\{\frac{1}{(1-d^2/4z^2)^2}\right\}$$

$\because P = q \cdot d$

$$\Rightarrow \Sigma E_p = \frac{P}{2\pi\varepsilon_0 z^3}\left\{\frac{1}{(1-d^2/4z^2)^2}\right\}$$

Assume $[z >>>> d]$:~

$$\Rightarrow \Sigma E_p = \frac{P}{2\pi\varepsilon_0 z^3}\left\{\frac{1}{(1-0)^2}\right\}$$

$$\Rightarrow \boxed{E_p = \frac{P}{2\pi\varepsilon_0 z^3}}$$

This is the enpression for the electric field due to dipole.

NUMERICAL :~

Q:~ what is the dipole movemont for a dipole having equal charges with a value of -2C and +2C and sperated with a distance 2cm.

Data:-

→ $q = 2C$ → $d = 2cm = 0.02m$

→ $p = ?$

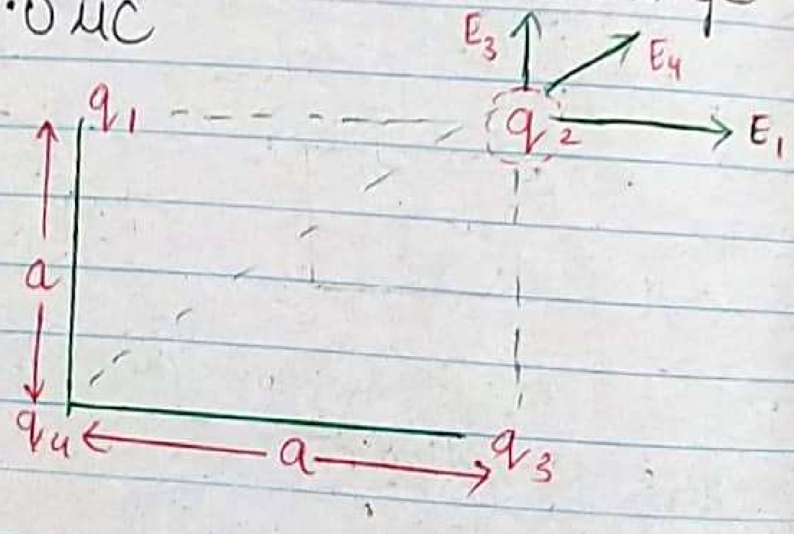
Solution:

∵ $p = q \cdot d = (2)(0.02)$

$\Rightarrow$ $p = 0.04\, C\text{-}m$.

Q:- Find the electric field at the centre of square 2cm on a side as shown below. If one corner is occupied by a charge of value 14 μC and the other charge of value 8.0 μC

→ $m = 1.12kg$



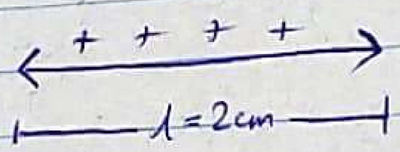
* CHARGE DISTRIBUTION:~

→ Dimensions
→ Continuous / Uniform.

i) LINER CHARGE DENSITY:~

No. of charges per unit lenght

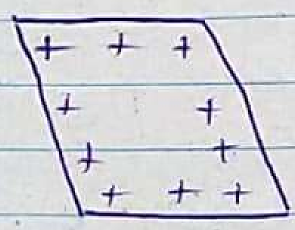
$$\lambda = \frac{q}{l}$$



ii) SURFACE CHARGE DENSITY:~

No. of charges per unit area.

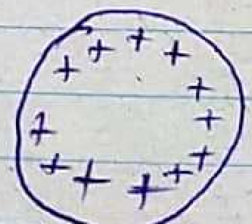
$$\sigma = \frac{q}{A}$$



iii) VOLUME CHARGE DENSITY:~

No. of charges per unit volume.

$$\rho = \frac{q}{V}$$



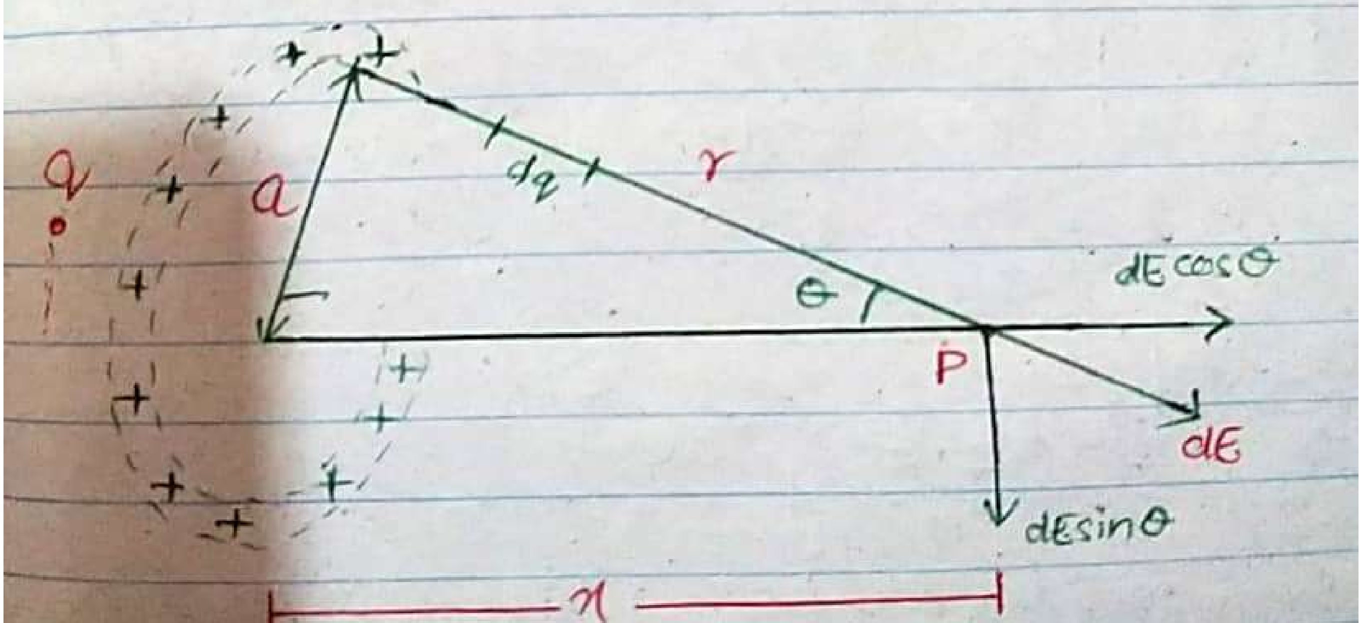
CONTINUOUS CHARGE DISTRIBUTION

The charges are distributed uniformly.

along a line in a straight line or circumference of a circle. In continuous charge distribution infinite number of charges are closely packed and have minor spaces b/w them.

NUMERICALS:~

Q:~ A ring of radius 'a' carries a charge (positive) uniformly distributed as 'Q'. Calculate the electric field due to ring at point 'P' lying a distance 'x' from it's center along the central axis perpendicular to the plane of the ring shown bellow.



Data:-

> ~~dE~~ E_{at} = ?
> Radius = a

-> Perp distance = ?
-> Hypotenuse = r

Solution:

Using triangle formulae:

$$\cos\theta = \frac{x}{r} = \frac{\text{Base}}{\text{Hyp}}$$

By Pythagoras:

$$\therefore (H)^2 = (P)^2 + (B)^2$$

$$\Rightarrow r = \sqrt{x^2 + a^2}$$

By the definition of electric field:-

$$E = \frac{Kq}{r^2}$$

$$\Rightarrow dE_x = \frac{K\,dq}{r^2} \quad \text{.... } (1)$$

There are more then one charge.

$$E = \int dE_x = \int dE \cos\theta$$

So, eq. (1) $\Rightarrow$

$$E = \int dE_x.$$

$$\Rightarrow E = \int \frac{K\,dq}{(\sqrt{a^2 + x^2})^2}$$

$$\Rightarrow E = \frac{K}{x^2+a^2} \int dq.$$

$$\Rightarrow \boxed{E = \frac{Kq}{x^2+a^2}} \text{ — ANSWER.}$$

Q:- A small sphere of mass carries a charge of 19.7 nC, hangs in earth's gravitational field force by a silk thread that makes an angle of 27.4° with a large uniformly charge non-conducting sheet has shown in the figure calculate the uniform charge density. (mass = 1.12kg).

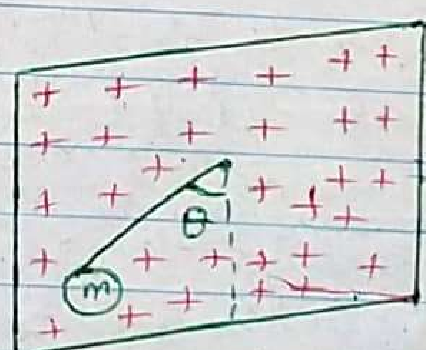
Data:-

$\Rightarrow m = 1.12\ kg$

$\Rightarrow q = 19.7\ nC$

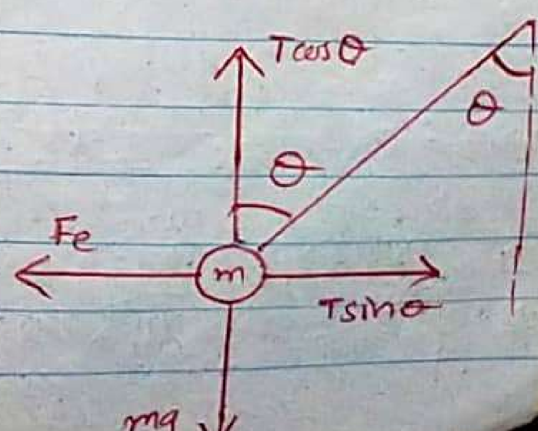
$\Rightarrow \theta = 27.4°$

$\Rightarrow \sigma = ?$



Solution:

As the body is not moving



$\Sigma F_x = F_e - T\sin\theta = 0$

$\Sigma F_y = mg - T\cos\theta = 0$

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On dividing,

$$\frac{T\sin\theta}{T\cos\theta} = \frac{Fe}{mg}$$

$$\Rightarrow \quad \tan\theta = \frac{Fe}{mg}$$

$$\because \quad Fe = Eq$$

$$\Rightarrow \quad \tan\theta = \frac{Eq}{mg}$$

From Gauss's law,

$$E = \frac{\sigma}{2\varepsilon_0}$$

$$\Rightarrow \quad \tan\theta = \frac{\sigma q}{2\varepsilon_0 mg}$$

$$\Rightarrow \quad \sigma = \frac{2\tan\theta \, mg \, \varepsilon_0}{q}$$

$$\Rightarrow \quad \sigma = \frac{2\tan(27.4)(1.12)(9.8)(8.85\times10^{-12})}{19.7\times10^{-9}}$$

$$\Rightarrow \boxed{\sigma = 5.11\times10^{-3}\ C/m^2}$$

* ELECTRIC POTENTIAL ENERGY :-

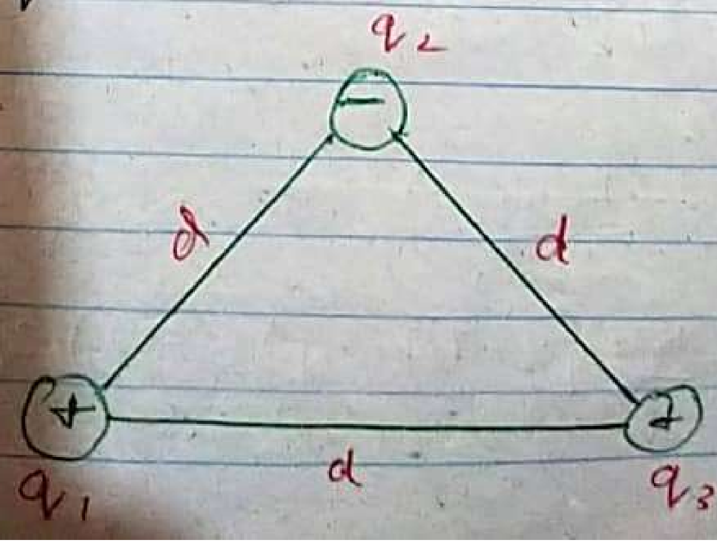
(Assciated with pair of charges).

ΔU (potential energy) = kJ

$$\Delta U = \frac{Kqq_0}{r} = \Delta Vq.$$

$$\Rightarrow \quad \Delta V = \frac{\Delta U}{q} = \frac{W}{q} = \frac{Kq_0}{r}$$

Q:- The following shows 3 charged particles q_1, q_2, q_3 held in a fixed positions by the force that is not shown here. What is the ~~center~~ electric potential energy of this system of charges. Assume the value of displacement b/w the charges 12 cm and $q_1 = +q$ $q_2 = -4q$, $q_3 = +2q$ and the value of q is 150 nC.



Data:-

- $\Delta U_T = ?$
- $q_1 = +q$
- $q_2 = -4q$
- $q_3 = +2q$
- $q = 150\,nC = 150 \times 10^{-9}\,C$
- $d = 12\,cm = 0.12\,m$

Solution:-

$$\because \quad \Delta U_T = \Delta U_1 + \Delta U_2 + \Delta U_3 .$$

since, $\Delta U = \dfrac{Kqq}{d}$

$$\Rightarrow \Delta U_T = \frac{Kq_1q_2}{d} + \frac{Kq_2q_3}{d} + \frac{Kq_3q_1}{d}$$

$$\cancel{\Rightarrow \Delta U_T = \frac{K}{d}(q_1+q_2+q_3)}$$

$$\cancel{\Rightarrow \Delta U_T = \frac{K}{d}(q-4q+2q)}$$

$$\cancel{\Rightarrow \Delta U_T = \frac{Kq}{d} = q}$$

$$\Rightarrow \Delta U_T = \frac{K}{d}\left\{(+q)(-4q)+(-4q)(2q)+(2q)(q)\right.$$

$\Rightarrow \Delta U_T = \dfrac{k}{d}(-10q^2)$

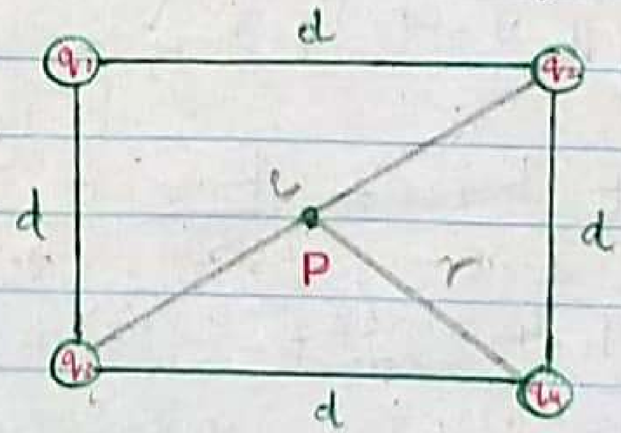
$\Rightarrow \Delta U_T = \dfrac{-10(9\times10^9)(150\times10^{-9})^2}{0.12}$

$\Rightarrow \boxed{\Delta U_T = -0.016\ J = 16\ mJ}$

Q:- What is the electric potential at point P located at the center of the square āl charge particles. The value of $d = 1.3m$ and charge are:

$q_1 = 12nC$ $q_3 = 31nC$

$q_2 = -24nC$ $q_4 = 17nC$.



Data:-

$\Rightarrow q_1 = 12\times10^{-9}C$

$\Rightarrow q_2 = -24\times10^{-9}C$

$\Rightarrow q_3 = 31\times10^{-9}C$

$\Rightarrow q_4 = 17\times10^{-9}C$

$\Rightarrow d = 1.3m$

$\Rightarrow \Delta V =$ electric potential at $P = ?$

Solution:

First we find the lenght of digona 'c'. Using pythagoras:

$$c^2 = d^2 + d^2.$$

$$\Rightarrow \quad c = d\sqrt{2} = 1.3\sqrt{2}$$

$$\Rightarrow \quad \boxed{c = 1.83\,m}$$

Let r be the distance b/w charge &

$$r = \frac{c}{2} = \overline{q_1P} = \overline{q_2P} = \overline{q_3P} = \overline{q_4P}$$

$$\Rightarrow \quad \boxed{r = 0.919\,m}$$

For electric potential at P:-

$$V = V_1 + V_2 + V_3 + V_4$$

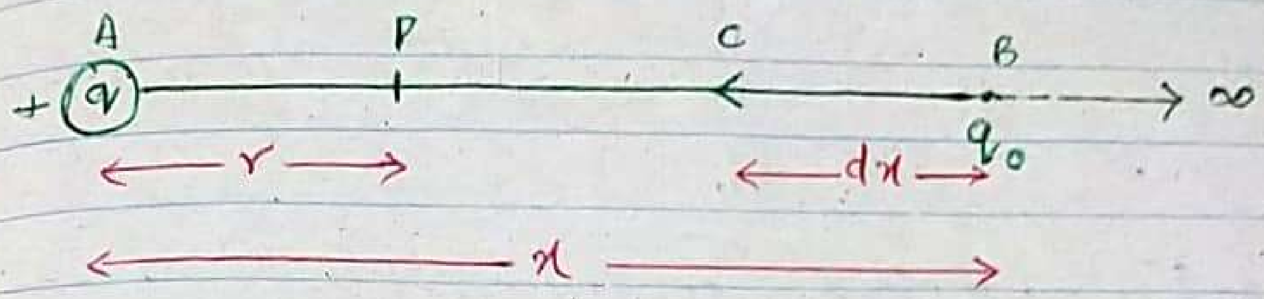
$$\Rightarrow \quad V = \frac{Kq_1}{r} + \frac{Kq_2}{r} + \frac{Kq_3}{r} + \frac{Kq_4}{r}$$

$$\Rightarrow \quad V = \frac{(9\times10^9)}{0.919}\left(12\times10^{-9} - 24\times10^{-9} + 31\times10^{-9} + 17\times10\right.$$

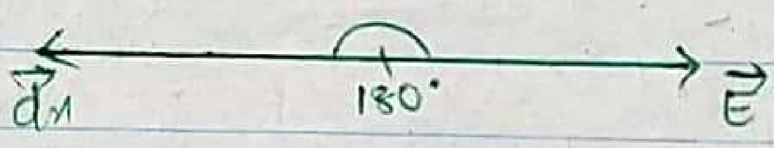
$$\Rightarrow \quad \boxed{V = 352.5 \text{ volts.}}$$

RELATION B/W ELECTRIC FIELD & POTENTIAL :~

$$E = -\frac{dV}{dx}$$



suppose a positive charge +q at point A, relating with point charge q_0 at B (approaching ∞). We have to derived expression for electric field at point P.
Let small distance b/w B - C is $\vec{dx}$



We know that,

$$\text{work} = W = \vec{F} \cdot \vec{x}$$

For small path,

$$dW = \vec{F}\, \vec{dx}$$

$$\because \vec{F} = \vec{E} q_0$$

$$\Rightarrow dW = \vec{E} \cdot \vec{dx}\, q_0$$

$$\Rightarrow dW = q_0 E\, dx \cos(180^\circ)$$

$$\Rightarrow dW = -q_0 E\, dx \quad \text{---} (1)$$

We know that

$$V = \frac{W}{q}$$

$\Rightarrow dV = \frac{dW}{q_0}$

$\Rightarrow dW = dV q_0$ -- (2)

Comparing eq (1) & (2) :-

$\Rightarrow dV q_0 = -q_0 E dx$

$$\Rightarrow \boxed{E = \frac{-dV}{dx}}$$

* ELECTRIC FLUX :-

(The flow of electric charges)

Electric flux depends on:

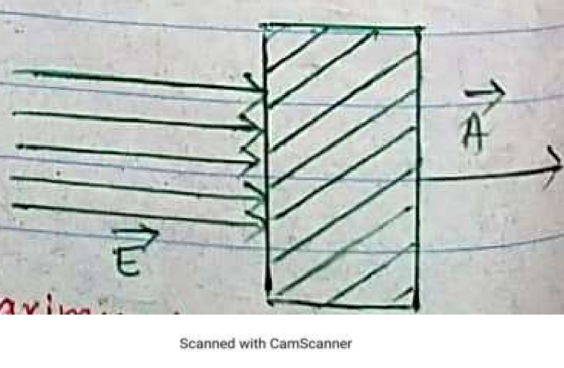
↳ Area vector

↳ Electric field lines → $\vec{E}$ (intensity)

The electric flux is the flow of electric field lines from the certain Area. It is determined by the area vector and electric field intensity. It doesn't depend on the shape of the surface. However, it depends on electric field and nature of medium.

CASE I :-

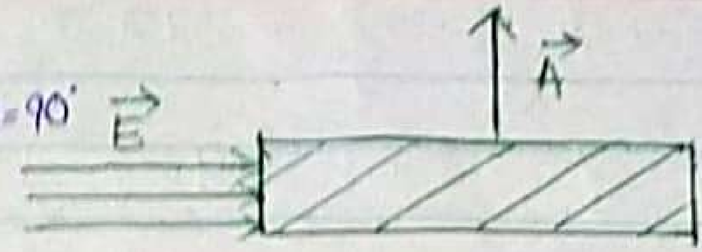
A/c to case I $\theta = 0°$ b/w $\vec{E}$ and $\vec{A}$



$\phi_E = EA$ (maxim...

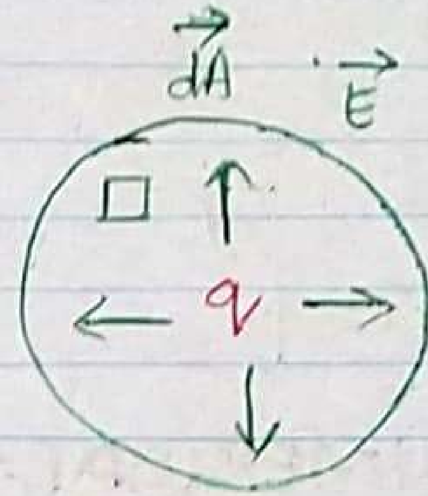
CASE II :~

A/c to case II $\theta = 90°$ b/w $\vec{E}$ & $\vec{A}$



$$\phi_E = 0 \text{ (minimum)}.$$

* GAUSS's LAW:~



$$\phi_T = \oint E\,dA.$$

$$\phi_T = \frac{q_{en}}{\varepsilon_0}$$

NET FLUX:

The net flux through any closed surface surrounding a point charge q is given by:

$$\phi_T = \frac{q_{in}}{\varepsilon_0}$$

It is independent to the shape of that surface.

It is used to determine the electric field of certain area.

DERIVATION:~

From Gauss's law, fig:

$$\phi = \oint \vec{E} \cdot \vec{dA}$$

$\Rightarrow \quad \phi = \oint E \cdot dA \cos\theta \qquad \because \theta = 0°$

$\Rightarrow \quad \phi = E \cdot (4\pi r^2) \qquad \because E = \frac{kq}{r^2}$

$\Rightarrow \quad \phi = \frac{kq}{\cancel{r^2}} (4\pi \cancel{r^2})$

$\Rightarrow \quad \phi = \frac{q}{4\pi\varepsilon_0} (4\pi)$

$\Rightarrow \quad \boxed{\phi_n = \frac{q}{\varepsilon_0}}$

NUMERICAL:-

Q:- The net flux through each phase of the dice has magnitude in unit of $10^3\ Nm^2/C$ equals to the numb of dots N on the face (1-6). The f is inward for N(odd) and outw for N(even). What is the net charge inside the dice.

Data:

ϕ in unit $= 10^3\ Nm^2/C$

N_{even} = flux is outward = +ve

N_{odd} = flux is inward = -ve

$q_n = ?$

Scanned with CamScanner

Solution:

ϕ_{mag}	Dots.
1×10^3	1
2×10^3	2
3×10^3	3
4×10^3	4
5×10^3	5
6×10^3	6.

$$\because \quad \phi_{net} = \phi_{odd} + \phi_{even}.$$

$$\Rightarrow \phi_{net} = (\phi_1 + \phi_3 + \phi_5) + (\phi_2 + \phi_4 + \phi_6)$$

$$\phi_{net} = (-1\times10^3 - 3\times10^3 - 5\times10^3) + (2\times10^3 + 4\times10^3 + 6\times10^3)$$

$$\Rightarrow \phi_{net} = -9\times10^3 + 12\times10^3$$

$$\Rightarrow \phi_{net} = 3\times10^3 \ Nm^2/C.$$

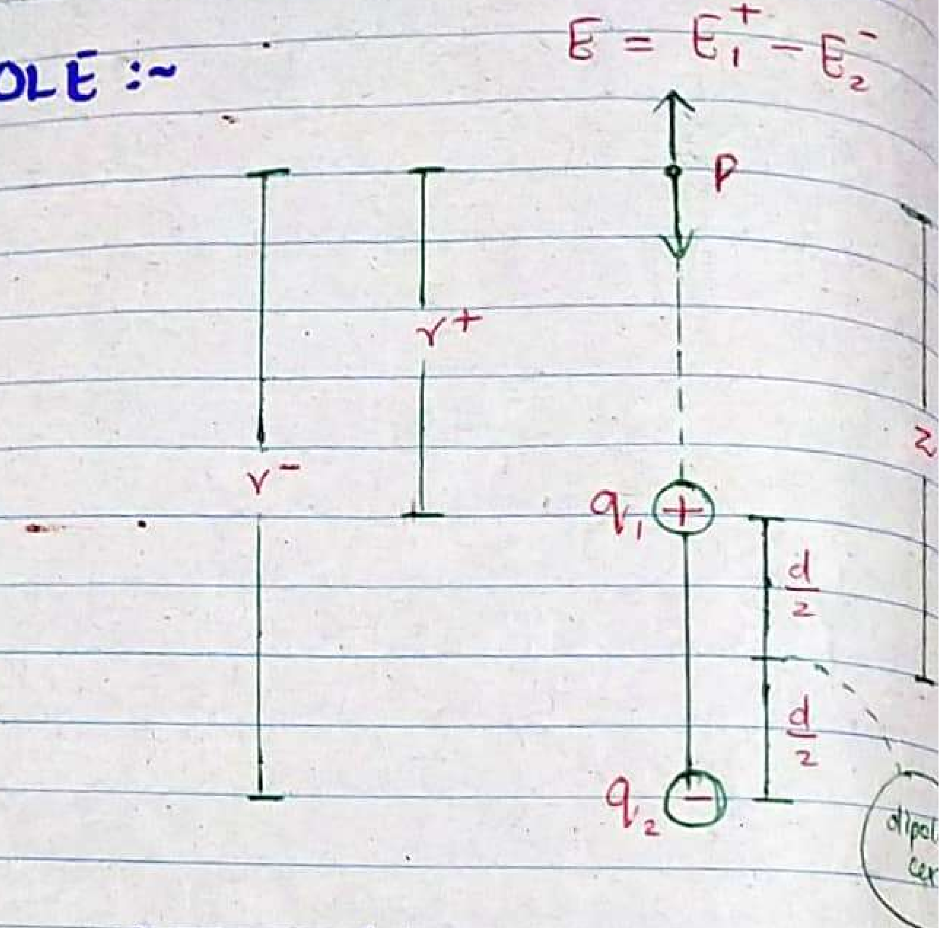
Since,

$$q_n = \phi_{net}\, \epsilon_0.$$

$$\Rightarrow q_n = (3\times10^3)(8.85\times10^{-12})$$

$$\Rightarrow \boxed{q_n = 2.26 \ nC}$$

ELECTRIC POTENTIAL DUE TO DIPOLE :-



Since,

$$V = \frac{Kq}{r}$$

From the above figures

$$r_1^+ = z - d/2$$
$$r_2^- = z + d/2$$

At Point P :-

$$V = V_1 + V_2 .$$

$$\Rightarrow \quad V = \frac{Kq}{r_1^+} - \frac{Kq}{r_2^-}$$

$$\Rightarrow V = kq \left[\frac{1}{\left(z - \frac{d}{2}\right)} - \frac{1}{\left(z + d/2\right)} \right]$$

$$\Rightarrow V = \frac{q}{4\pi\varepsilon_0} \left[\frac{z + \frac{1}{2}d - \left(z - \frac{1}{2}d\right)}{\left(z - \frac{1}{2}d\right)\left(z + \frac{1}{2}d\right)} \right]$$

$$\Rightarrow V = \frac{q}{4\pi\varepsilon_0} \left[\frac{\frac{1}{2}d + \frac{1}{2}d}{(z)^2 - \left(\frac{1}{2}d\right)^2} \right]$$

$$\Rightarrow V = \frac{q}{4\pi\varepsilon_0} \left[\frac{d}{\frac{4z^2 - d^2}{4}} \right]$$

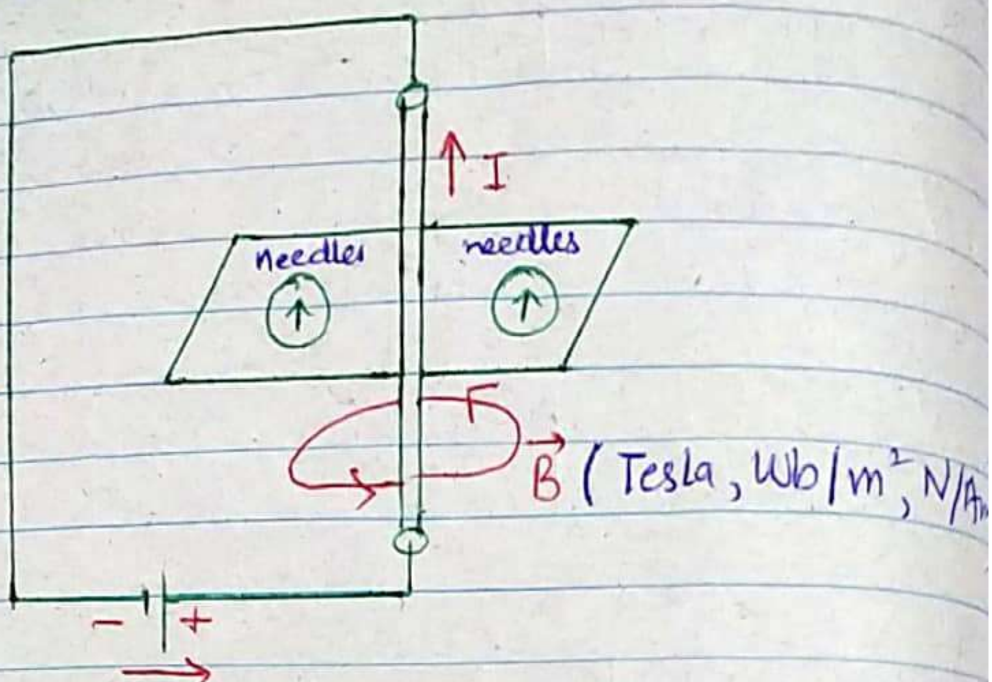
$$\Rightarrow V = \frac{\cancel{4}q \cdot d}{\cancel{4}\pi\varepsilon_0} \left[\frac{1}{4z^2 - d^2} \right]$$

$\because P = q \cdot d$

$$\Rightarrow \boxed{V = \frac{P}{(4z^2 - d^2)\pi\varepsilon_0}}$$

(MAGNETISM)

MAGNETIC FIELD:-



$F = qvB\sin\theta$

NUMERICALS:-

Q:- A uniform magnetic field with magnitude 1.2 mT, is directed vertically upward throughout the volume of laboratory chamber. A proton with K.E 5.3 eVM enter the chamber moving horizontally. What is the magnetic deflecting force act on the proton as it enters the chamber? What will be the affect of acceleration

Data:

→ $B = 1.2\,mT = 1.2\times10^{-3}\,T$

$\therefore F = ?$ (magnetic deflecting force)

$\therefore K.E = 5.3\ MeV = 5.3\times10^{6}\ eV = 8.48\times10^{-13}\ J$

$\therefore a = ?$

$\therefore m_p = 1.67\times10^{-27}\ kg.$

Solution :-

$$\because \quad K.E = \frac{1}{2} m v^2.$$

$$\Rightarrow \quad V = \sqrt{\frac{2K.E}{m}} = \sqrt{\frac{2(8.48\times10^{-13})}{1.67\times10^{-27}}}$$

$$\Rightarrow \quad V = 31.8\times10^{6}\ m/s$$

$$\because \quad F = qVB\sin\theta.$$

$\theta = 90^\circ$ (vertically upward)

$$\Rightarrow \quad F = (1.6\times10^{-19})(31.8\times10^{6})(1.2\times10^{-3})\times\sin 90^\circ$$

$$\Rightarrow \quad F = 6.105\times10^{-15}\ N$$

$$\because \quad F = ma$$

$$\Rightarrow \quad a = \frac{F}{m} = \frac{6.105\times10^{-15}}{1.67\times10^{-27}}$$

$$\Rightarrow \quad \boxed{a = 3.65\times10^{12}\ m/s^2}$$

Q:- A particle of charge $+7.5\,\mu C$ is a speed of $32.5\,m/s$ enters a uniform ~~mag~~ magnetic field whose magnetude is $0.5\,T$. For each given cases find out the magnetude and direction of force on particle.

(a) $\theta = 30^\circ$ (c) $\theta = 150^\circ$

(b) $\theta = 90^\circ$

Data:

$\Rightarrow q = +7.5\,\mu C$

$\Rightarrow v = 32.5\,m/s$

$\Rightarrow B = 0.5\,T$

$\Rightarrow F = ?$

Solution:

$\because \quad F = qBv\sin\theta$

PART (a)

$\Rightarrow F = (7.5)(32.5)(0.5)\,\mu \sin 30^\circ$

$\Rightarrow F = 6.09 \times 10^{-5}\,N.$

PART (b)

$\Rightarrow F = (7.5 \times 10^{-6})(32.5)(0.5)\sin 90^\circ$

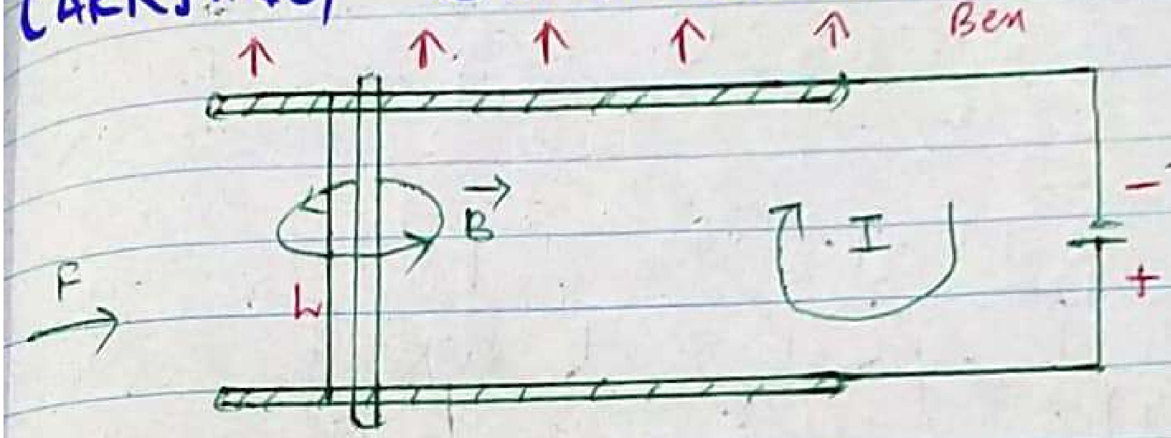
$\Rightarrow F = 1.21 \times 10^{-4}\,N.$

PART (c)

$\Rightarrow F = (7.5\times10^{-4})(32.5)(0.5)\sin 150^\circ$

$\Rightarrow F = 6.09\times10^{-5}\,N.$

MAGNETIC FIELD ON CURRENT CARRYING CONDUCTOR:-



From the above figure,

$F \propto I$

$F \propto L$

$F \propto B$

$F \propto \sin\theta$

On combining,

$F \propto IBL\sin\theta$

$\Rightarrow \boxed{F = ILB\sin\theta}$

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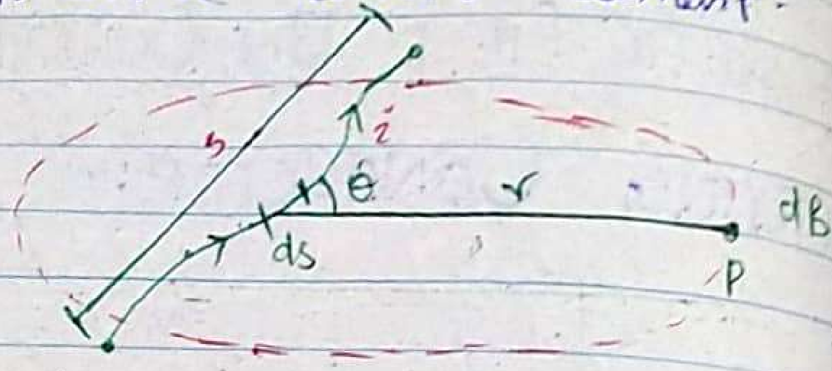
BIOT & SAVART LAW:~

It is used to define the magnetic field accross small portion current in a conductor, the small portion is called current element.



The field due to ds be dB. The element of current depends on ds (i·ds). The dB around ds is ⊥ maximum

$$dB \propto i$$
$$dB \propto ds$$
$$dB \propto \sin\theta$$
$$dB \propto \frac{1}{r^2}$$

on combining,

$$dB = \frac{i\,ds\sin\theta}{r^2}$$

$$\Rightarrow dB = \frac{\mu_0}{4\pi} \cdot \frac{i\, ds \sin\theta}{r^2}$$

$\because \frac{\mu_0}{4\pi} = \text{constant}.$

$$\Rightarrow dB = \frac{\mu_0 i\, ds \sin\theta}{4\pi r^2} \times \frac{r}{r}$$

In vector,

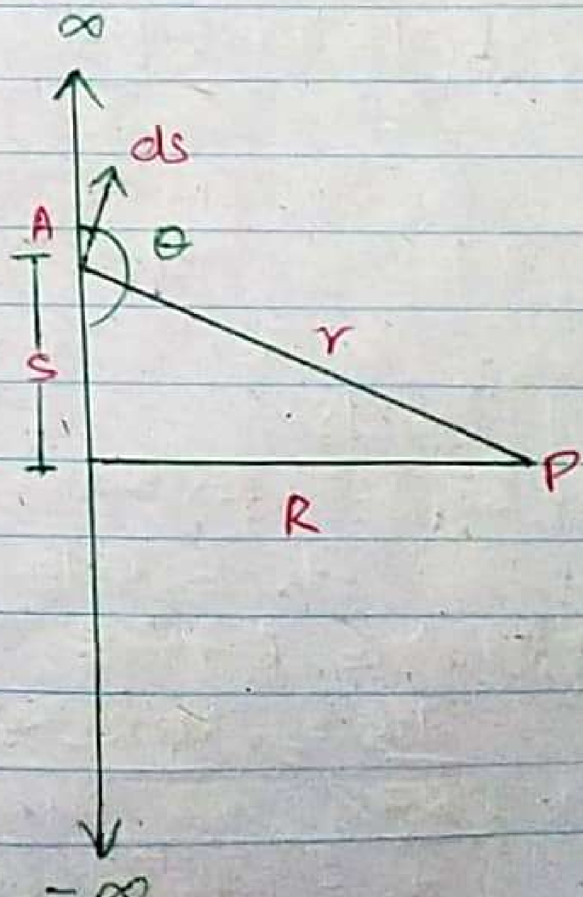
$$\Rightarrow \vec{dB} = \frac{\mu_0 i\, ds \sin\theta}{4\pi r^3}\, \hat{r}$$

$\because \vec{r} = r \cdot \hat{r}$

$$\Rightarrow \vec{dB} = \frac{\mu_0 i\,(\vec{r} \times \vec{ds})}{4\pi r^3}$$

APPLICATIONS:-

(i) MAGNETIC FIELD DUE TO THE CURRENT IN A LONG WIRE:-



Using Biot & Savart,

$$\vec{dB} = \frac{\mu_0 i\, \vec{ds} \times \vec{r}}{4\pi r^3}$$

$$\Rightarrow \vec{dB} = \frac{\mu_0 i\, \cancel{r}\, ds \sin\theta}{4\pi r^{\cancel{3}}}$$

$$\Rightarrow \vec{dB} = \frac{\mu_0 i\, ds \sin\theta}{4\pi r^2}$$

In ΔABD is right triangle, using pythagoras:

$$r = \sqrt{R^2 + s^2}$$

$$\sin\theta = \frac{R}{r}$$

eq - (1) =>

$$dB = \frac{\mu_0 i \, ds \left(\frac{R}{r}\right)}{4\pi (R^2 + s^2)}$$

$$\Rightarrow dB = \frac{\mu_0 i \, ds \, R}{4\pi (R^2 + s^2)^{3/2}}$$

Applying integral on both sides:-

$$\Rightarrow B = \frac{\mu_0 i R}{4\pi} \int_{-\infty}^{+\infty} \frac{ds}{(R^2 + s^2)^{3/2}} \quad \text{--- (2)}$$

$$\int_{-\infty}^{+\infty} \frac{ds}{(R^2 + s^2)^{3/2}}$$

$$\because \int_{-\infty}^{\infty} = 2\int_{0}^{\infty}$$

Let; $s = R\tan\theta$

$$\frac{ds}{d\theta} = R\sec^2\theta$$

$$\boxed{ds = R\sec^2\theta \, d\theta}$$

$$(R^2+S^2)^{3/2} = \left(R^2 + R^2\tan^2\theta\right)^{3/2}$$

$$\Rightarrow \boxed{(R^2+S^2)^{3/2} = R^3\sec^3\theta}$$

For :- $\int_{-\infty}^{+\infty}$

$\therefore 0 = R\tan\theta$ OR $\infty = R\tan\theta$

$\Rightarrow \boxed{\theta = 0}$ $\Rightarrow \boxed{\theta = \frac{\pi}{4}}$

Now,

$$\int_0^{\pi/4} \frac{R\sec^2\theta\, d\theta}{R^3\sec^3\theta}$$

$$\Rightarrow \frac{2}{R^2}\int_0^{\pi/4} \frac{d\theta}{\sec\theta}$$

$$\Rightarrow \frac{2}{R^2}\int_0^{\pi/4} \cos\theta\, d\theta$$

$$\Rightarrow \frac{2}{R^2}\left[\sin\theta\right]_0^{\pi/4}$$

$$\Rightarrow \frac{2}{R^2}(1-0)$$

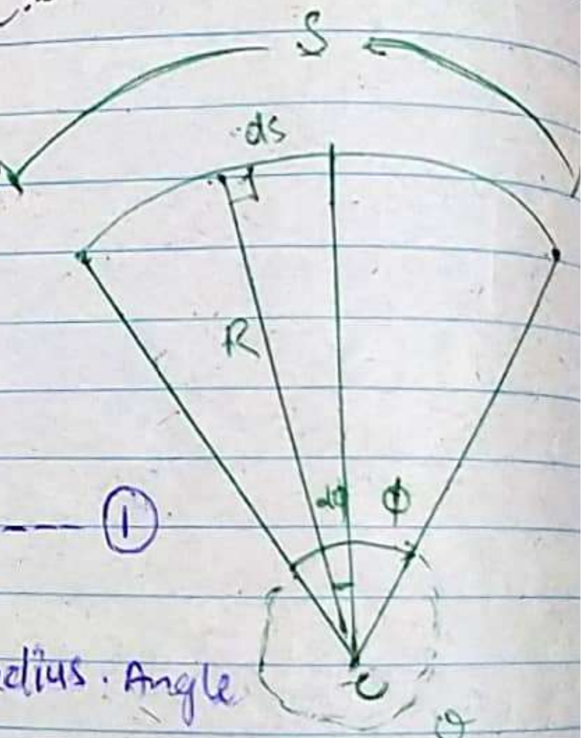
$$\Rightarrow \boxed{\frac{2}{R^2}}$$

eq (2) =>

$$B = \frac{\mu_0 i R}{4\pi}\left(\frac{2}{R^2}\right)$$

$$\Rightarrow \boxed{B = \frac{\mu_0 i}{2\pi R}}$$

(ii) MAGNETIC FIELD DUE TO A CURRENT IN CIRCULAR ARC:-



A/c to Biot & Savart,

$$dB = \frac{\mu_0 i \vec{r} \times \vec{ds}}{4\pi r^3}$$

$$\Rightarrow dB = \frac{\mu_0 i R \, ds}{4\pi R^3} \quad \text{--- (1)}$$

$\because$ Arc lenght = radius . Angle

$$\Rightarrow s = r\phi$$

For considered portion :-

$$ds = R d\phi$$

eq (1) =>

$$dB = \frac{\mu_0 i R (R d\phi)}{4\pi R^3}$$

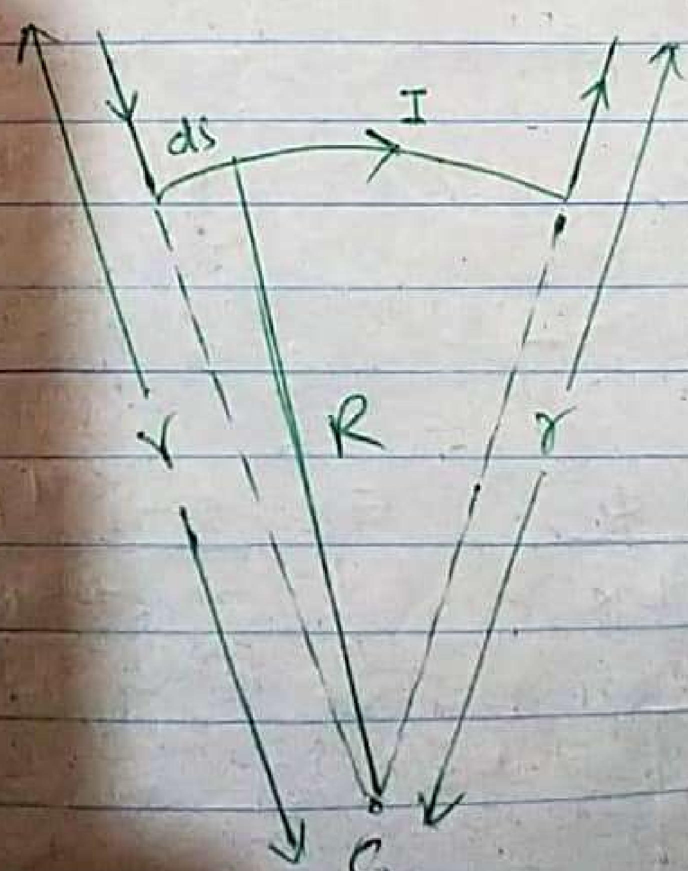
$$\Rightarrow dB = \frac{\mu_0 i \, d\phi}{4\pi R_1}$$

Taking integral:-

$$\Rightarrow \boxed{B = \frac{\mu_0 i \phi}{4\pi R}}$$

NUMERICAL:-

Q:- The two wires in the fig carries the current i and consist of circular arc of radius R. The given central angle is $\frac{\pi}{2}$ radian and the two straight sections whose extensions intersects the centre 'C' of the arc. What magnet field does the current produce at point C.



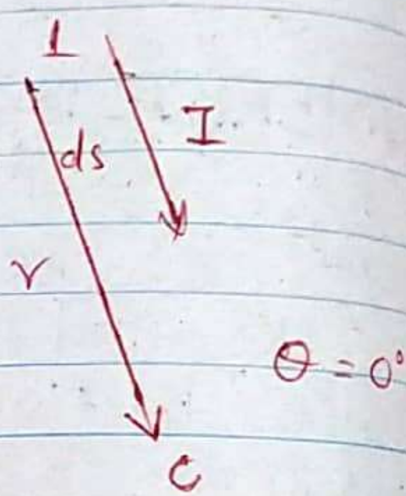
Solution:

Given that,

$$\phi = \frac{\pi}{2}$$

$$dB = \frac{\mu_0 i \, \vec{r} \times \vec{ds}}{4\pi r^3}$$

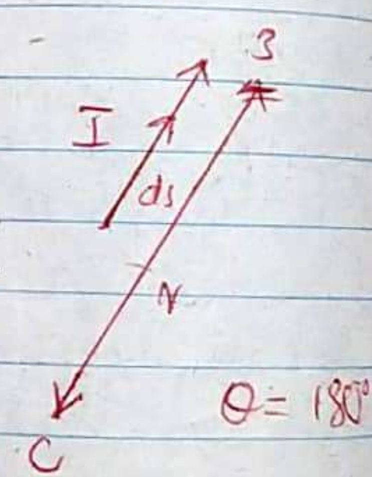
FOR SECTION 1:-



$$dB_1 = \frac{\mu_0 i \, r ds \sin 0^\circ}{4\pi r^3}$$

$$\Rightarrow \boxed{dB_1 = 0}$$

FOR SECTION 3:-



$$dB_3 = \frac{\mu_0 I \, r ds \sin 180^\circ}{4\pi r^3}$$

$$\Rightarrow \boxed{dB_3 = 0}$$

FOR SECTION 2:-

$$B_2 = \frac{\mu_0 i \phi}{4\pi R} = \frac{\mu_0 i (\pi/2)}{4\pi R}$$

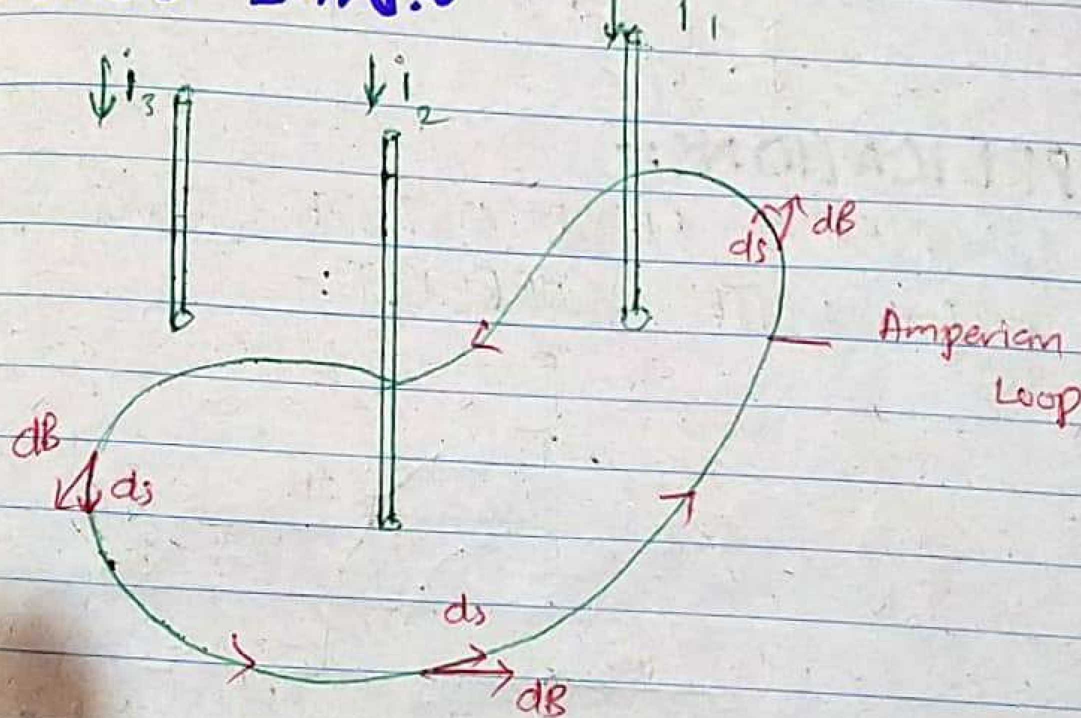
$$\Rightarrow \boxed{B_2 = \frac{\mu_0 i}{8R}}$$

for magnetic field at point C:-

$$B_c = B_1 + B_2 + B_3$$

$$\boxed{B_C = \frac{\mu_0 i}{8R}}$$

AMPERE'S LAW:~



from the above figure:

$$\oint B \cdot ds = \mu_0 i_{enclosed}$$

Since,

the current enclosed,

$$i_{encl} = i_1 - i_2$$

$$\Rightarrow \oint \vec{B} \cdot d\vec{B} = \mu_0 (i_1 - i_2)$$

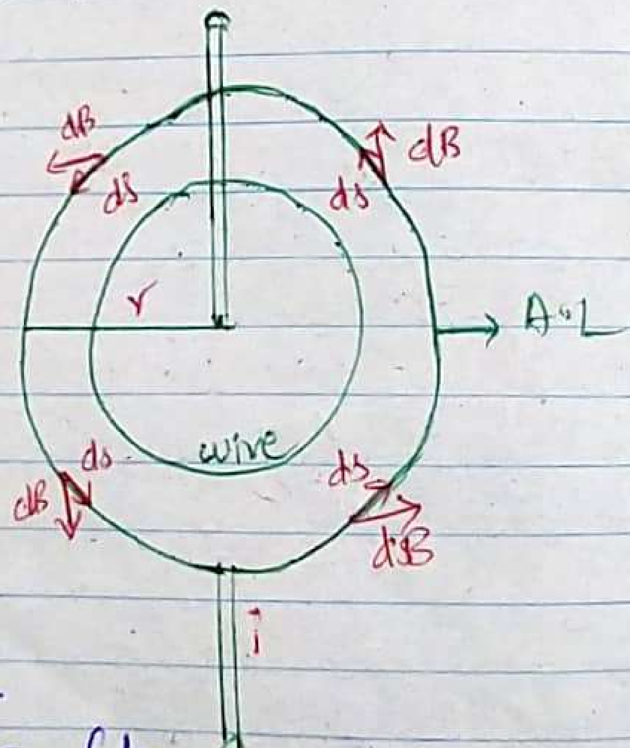
STATEMENT:-

For any enlosed loop path the sum of the lenght element "ds" times the magnetic field in the direction of lenght element is equals to the permiabiliby of the vacum times the current enclosed in the loop.

$$\oint B \cdot ds = \mu_0 i_{encl}$$

APPLICATIONS:-

① MAGNETIC FIELD OUTSIDE LONG STRAIG WIRE WITH CURRENT:-



We know that:-

$$s = 2\pi r = \int ds$$

From Ampere's law,

$$\int \vec{B} \cdot \vec{ds} = \mu_0 i_{enc}$$

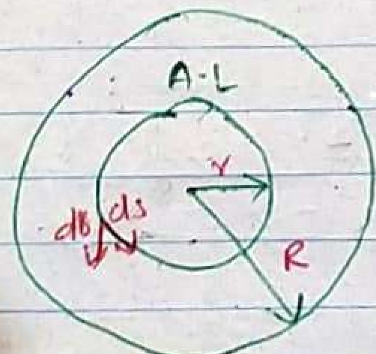
$$\Rightarrow \int B\,ds\cos 0^\circ = \mu_0 i_{enc}$$

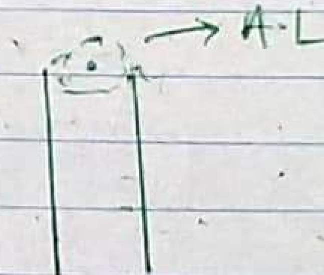
$$\Rightarrow B\int ds = \mu_0 i_{enc}$$

$$\Rightarrow B(2\pi r) = \mu_0 i_{enc}$$

$$\Rightarrow \boxed{B = \frac{\mu_0 i_{enc}}{2\pi r}}$$

(ii) MAGNETIC FIELD INSIDE A LONG SRAIGHT WIRE :-





From Ampere's law :-

$$\oint \vec{B}\cdot\vec{ds} = \mu_0 i_{enc}$$

$$\Rightarrow B(2\pi r) = \mu_0 i_{enc} \quad \text{--- (1)}$$

By using area ratio formula:-

$$\frac{i_{enc}}{i} = \frac{\pi r^2}{\pi R^2}$$

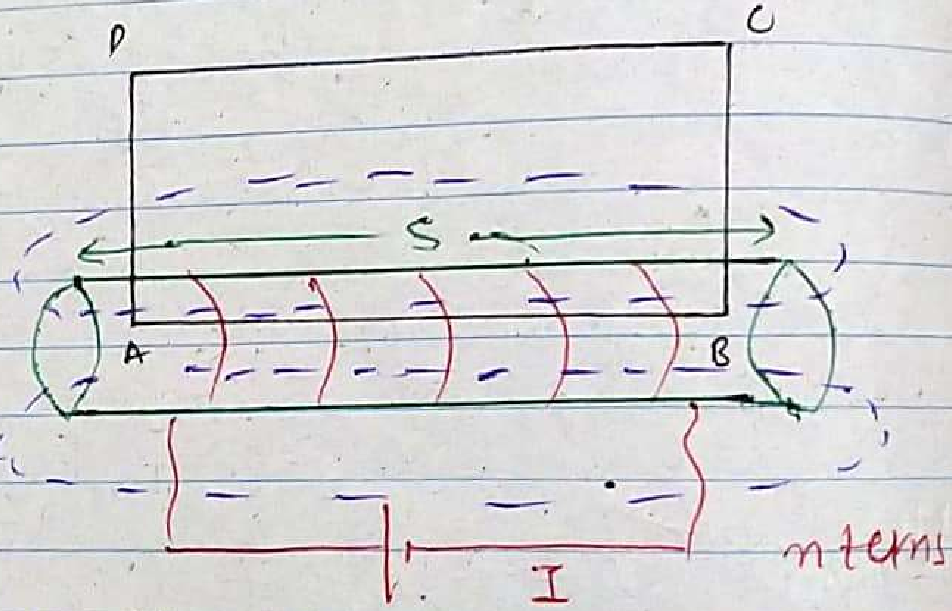
$$\Rightarrow \quad i_{enc} = \left(\frac{r^2}{R^2}\right) i$$

eq (1) $\Rightarrow$

$$B(2\pi \cancel{r}) = \mu_0 \, i \left(\frac{r^{\cancel{2}}}{R^2}\right)$$

$$\Rightarrow \quad \boxed{B = \frac{\mu_0 i r}{2\pi R}}$$

"SOLENOID"



From Ampere's law;

$$\oint B \cdot ds = \mu_0 i_{enc}$$

From figure

$$\Rightarrow \int_A^B B\,ds + \int_B^C B\,ds + \int_C^D B\,ds + \int_D^A B\,ds = \mu_0 i$$

$\vec{AB}$ → Field is strng. at $\theta = 0$.
$\vec{BC}$ → field is $\perp$
$\vec{CD}$ → Field is too week
$\vec{DA}$ → field is $\perp$.

$\Rightarrow \int_A^B B \cdot ds = \mu_0 i_{enc}$ — (1)

Since
$N = ns$ (total turns)

So, $i_{enc} = NI = nsI$

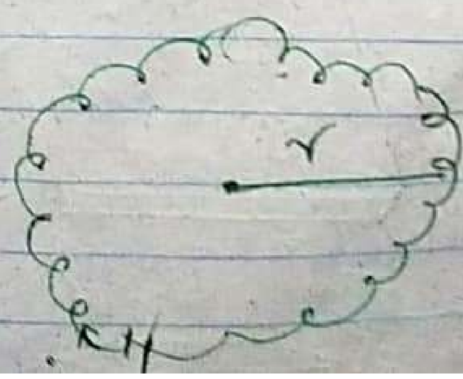
Eq (1) ⇒

$\Rightarrow B \int_A^B ds = \mu_0 (NI)$

$\Rightarrow B(s) = \mu_0 nsI$

$\Rightarrow \boxed{B = \mu_0 n I}$

"TOROID"



No. of turns = N

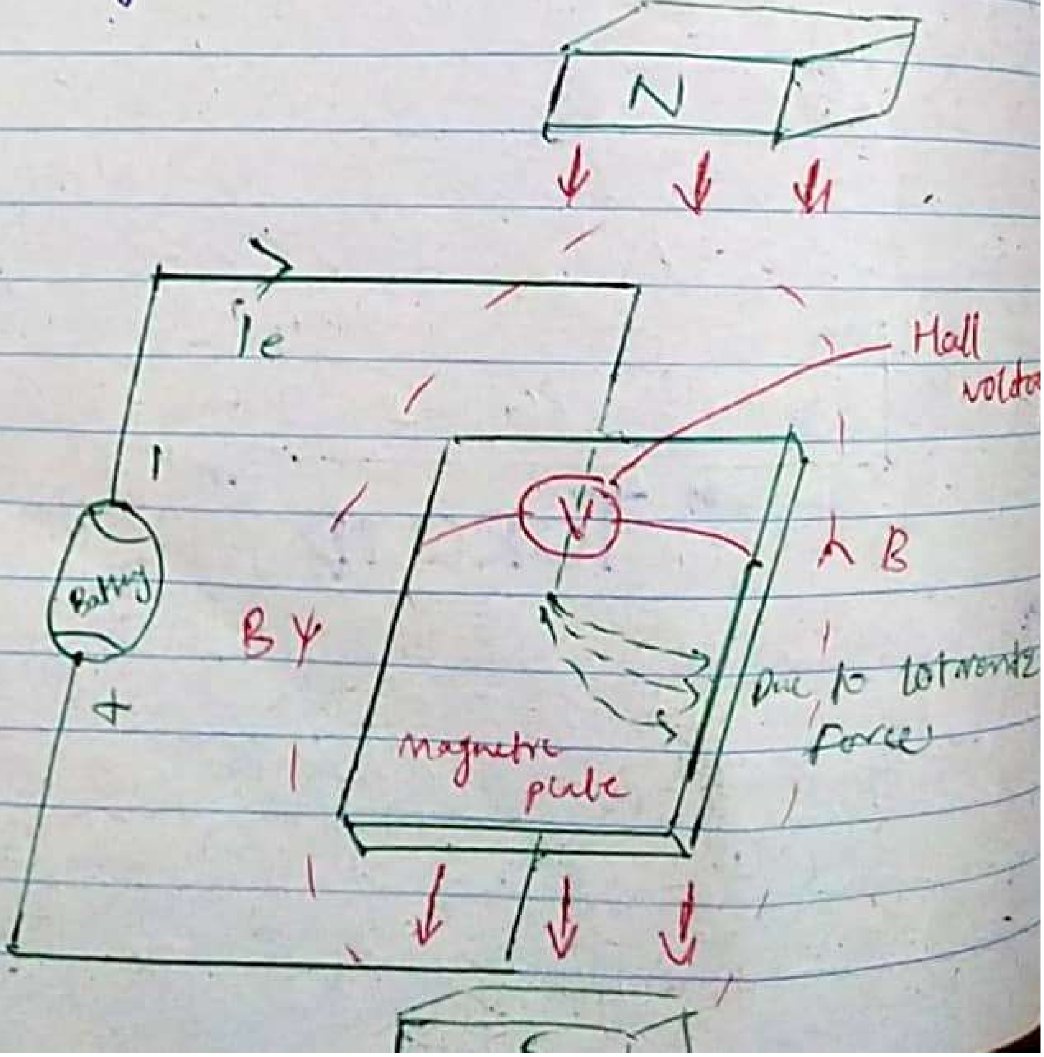
$$\Rightarrow \oint B \cdot dl = \mu_0 I_{enc}$$

$$\Rightarrow B(2\pi r) = \mu_0 (IN)$$

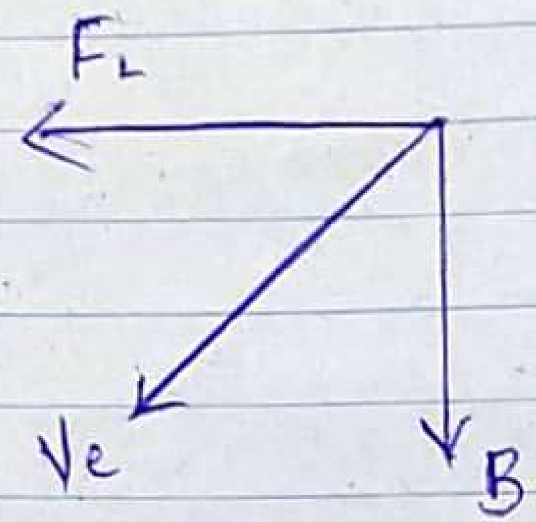
$$\Rightarrow \boxed{B = \frac{\mu_0 N I}{2\pi r}}$$

HALL EFFECT:~

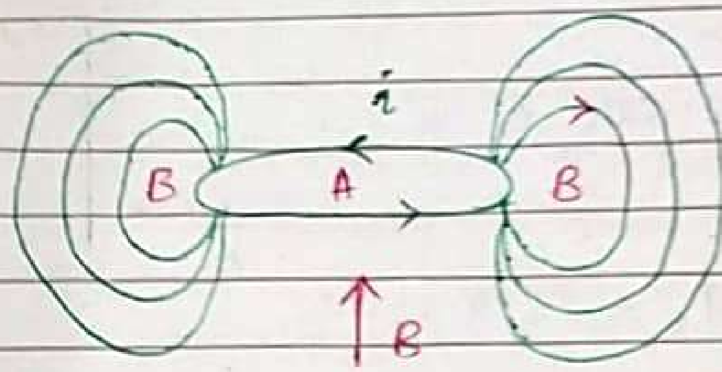
It is the production of voltage difference accross an electrical conductor (metallic plate), transvers to an electric current in the conductor & an applied magnetic field ⊥ to current.



FLEMING LEFT HAND RULE:



* MAGNETIC DIPOLE OF A CURRENT CARRYING LOOP:~



$M \propto A$
$M \propto i$] from the figure

On combining,

$$M \propto Ai$$

$\Rightarrow \quad M = KAi \qquad K = 1$ (No. of coils)

$$\Rightarrow \quad \boxed{M = Ai}$$

Unit : Am^2

* MAGNETIC FLUX:~

In electromagnetism, the magnetic flux through a surface is the surface integral of the normal component of the magnetic field B over that

Date

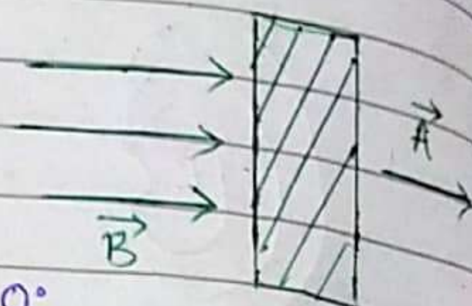
surface -

$$\phi_B = \vec{B} \cdot \vec{A} = BA \cos\theta$$

S.I Unit : Weber..

CASE I :-

A/c to this case $\theta = 0°$
b/w $\vec{B}$ and $\vec{A}$

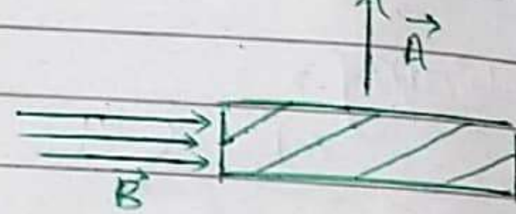


$$\phi_B = B \cdot A \cos 0°$$

$$\phi_B = B \cdot A \cdot$$ (Maximum)

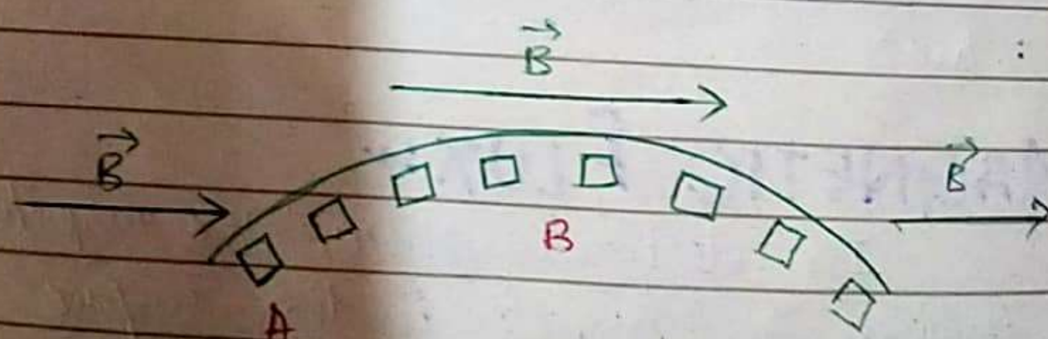
CASE II :-

Here $\theta = 90°$



$$\phi_B = 0 \cdot$$ (Minimum)

CASE III :-



No.

Date

* MAGNETIC MATERIAL :-

Properties	Diamagnetic	Paramagnetic	Feromagnetic
Effect of magnet	slightly repelled	slightly attracted	Strongly attracted
Magnetized Direction.	opposite to the external magnetic field	Same direction as the external magnetic field.	Same direction as the enteral magnetic field.
Movement of non-uniform field	high to low	Low to high	Low to high
Pairing	All paired	Some Unpaired	Some unpaired.

No.

CHAPT

(SEMICOND

ENERGY LEV

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BANDS IN

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(i) VALENCE B

It

The electron,

strongly bond

distribution of charges

Imp for final:-

Chap 1

Errors (All)

Chap 2

Divergence

Curl, Directional Derivative

Chap 3

Graphs (s/t, v/t, a/t)

Application of Newton laws

(All string Cases)

Frictional forces

Work-Energy For Const, Non-const

Cons. of Energy → Waves Energy

Chap 4

v_{inst}, a, ω, Energy

Cha $\langle E_T \rangle = \langle KE_{max} \rangle = \langle PE_{max} \rangle$

Verification of cons of energy

Damping,

Forced Oscill, Resonance

Chap 5

PN Junction

VI - curve

Transistors with types

Free-body Diagram

Paper in Sequential Order

Chap 6

Young Slit, Diff (all Numericals wise)

Huygen's Principle

Fresnel Huygen Correction

Laser with Energy Level Diagram.

Chap 7 $\approx (10^{-7}$ to 10^{-8} sec)

Wave Particle Duality

Photo, Comp., De Brog, Uncern with Graphs

(with Numericals)

Half-life Derivation

BE, Mass Defect, Radioactive Law

For Decay Find ($N(t)$) else for Activity (A), Radioactivity

Chap 8

All in Pdf

ElectroStatics, Dynamics

Quantization, Uniform, Continuous Charge distribution

Gauss's law Application

For Num, Derivations

i) Free-body Diagram

ii) Data, Formula

iii) Unit must write

Data in $days^{-1}$ will so ans be in days.

ALL CLO 4 Qs.

$3\times4 = 12 \times 5 = 60$

Theory

min → 1 page

max → 1.5 to 2 pages