$1\ mile = 5280\ ft$

a) $1\ \frac{mile}{hr} = 1.46\ ft/sec$

$\frac{1 \times 5280}{3600} =$

PH-122 (3+1) CH's

Total Marks (100 + 50)

Th Pr

Final = 60, Sessional = 40 UNITS

Assignments: 10 / Assessment Test: 20 Mid Term = 20

Deviation:- Diff b/w expected and actual value. $\delta_i = X_i - X$

observation, average value

Errors:-

The amount of deviation in observed values.

Types Of Error's

Non-identifiable → Random Error

Identifiable → Systematic Error → Personal Error, Instrumental Error, Environmental Error.

i) Random Error:- → can be reduced only by taking more number of readings.

Errors occurring in an experimental working due to unknown source (unidentifiable source).

For Example:-

In an experimental working of evaluating Lorentz force, a sudden disruption will change electromagnetism of the system.

ii) Systematic Error:-

Errors occurring due to identifiable sources.

Accuracy => How close is observalve value to actual one.

a) Personal Error:- can be reduced ~~through proper training &~~ by identifying sources & adjust or calibrate instruments

It ~~is~~ occur's due to the person observing or analyzing data.

For Example:-

While measuring length of an object, analyze from directly above of ruler to avoid parallax error.

b) Instrumental Error:- => can be reduced by implementing quality check of instruments and following manufacturer guitlines

It occur's when something is wrong with the instrudment or due to its wear & tear.

For Example:-

Zero Error in measuring instruments or instrument not used correctly.

c) Environmental Error:- => can be ~~maint~~reduced by mantaining controlled environment.

It occur's due to change or variation in laboratory conditions atmospherically.

Standard Error:-

The deviation (change in) from original (OR of observed value) standard value with respect to presetted standard value is called standard error.

$$\text{Error} = \frac{\text{Obs. value} - \text{St. value}}{\text{St. value}}$$

$$\%\ \text{Error} = \left| \frac{\text{Obs. value} - \text{St value}}{\text{St. value}} \right| \times 100$$

To minimize error => Computational calculation Not Manual

⇒ For a fairly large sample of measurement which have a reasonably normal distribution about the mean value $\bar{X}$.

$$\frac{\text{Average Deviation}}{\text{Standard Deviation}} = \frac{d}{\sigma} = 0.80$$

Arithmetic Mean:- average value of certain set of calculations.

$$\bar{X} = \frac{X_1 + X_2 + X_3 + \dots + X_n}{n}$$

$$\bar{X} = \frac{\Sigma X_i}{n} \quad \text{where, } i = 0, 1, 2, \dots, n$$

n = No. of observations.

Standard Deviation:-

The square root of the mean square deviation for an infinite set of measurements

$$\sigma = \sqrt{\frac{(X_1-\bar{X})^2 + (X_2-\bar{X})^2 + \dots (X_n-\bar{X})^2}{n-1}} \leftarrow S$$

OR $\sigma = \sqrt{\frac{S}{n-1}}$

Standard Error:-

If δ (deviation) is given, so

$$SE = \frac{\sigma}{\sqrt{n}} \quad \text{OR} \quad \sigma_n = \sqrt{\frac{S}{n(n-1)}}$$

$S = \Sigma \delta$ or $\delta_1 + \delta_2 + \dots + \delta_n$

Problem #01

Following is the given data set of convex lens of spherometer. Find

i) Standard Error, ii) Probable Error

$\{X = 15.25, 15.42, 15.30, 15.44, 15.8, 16\}$

For $\bar{X}$:-

$$\bar{X} = \frac{15.25 + 15.42 + 15.30 + 15.44 + 15.8 + 16}{6}$$

$$\bar{X} = 15.535$$

Average deviation = $\frac{|x_1-\bar{x}|+|x_2-\bar{x}|+ - - - +|x_n-\bar{x}|}{n}$

can convert linear to non-linear ⇒ Semi-log ⇒ log

For σ :-

$$\sigma = \sqrt{\frac{(15.25-15.535)^2+(15.42-15.535)^2+(15.30-15.535)^2+(15.44-15.535)^2+(15.8-15.535)^2+(16-15.535)^2}{6-1}}$$

$$\sigma = 0.298$$

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{0.298}{\sqrt{6}} = 0.1216$$ ⇒ Std Error

ii) Probable Error :- ⇒ 67.45% (natural constant) of standard error

$$PE = \bar{x} \pm e$$

Const of PE (e) = $0.674 \times SE$

$$e = 0.674 \times 0.1216 = 0.081$$

$PE = 15.535 \pm 0.081$ ⇒ Radius of curvature of convex lens.

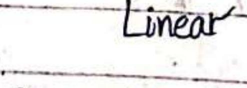
Graphs :- pictorial representation of data set or of any function

Graphs → Linear, Non-linear

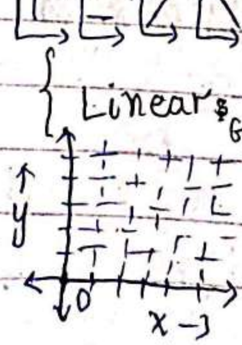
X-Axis ⇒ Independent

Y-Axis ⇒ Dependent axis

$$\overset{\text{Dependent}}{y} = \underset{\text{slope}}{m}\underset{\text{Independent}}{x} + \overset{\text{Intercept}}{c}$$



Linear Graph

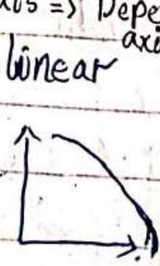


saturation for straight line

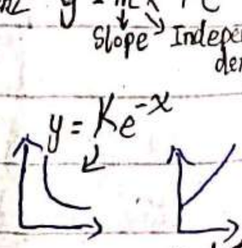
arrow for infinite

decay

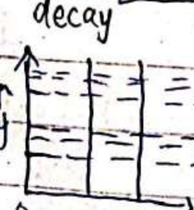
$y = Ke^{x}$



$y = Ke^{-x}$



$y = \cancel{Ke}\ K^{ax}$



Semi-log graph

Log-Log

Q For the given data to the straight line graph and calculate given observations a) Standard Deviation b) Standard Error c) Probable Error d) Final Error

observations:					
Length (cm)	62.005	71.99	85.01	99.99	111.99
Time Period (sec)					

Effects Of Combining Errors:-

⇒ Addition & Substraction:-

Suppose a & b have measured values $a \pm \delta a$ and $b \pm \delta b$ where δa and δb are absolute errors.

Sum:-

$$\underbrace{A \pm \delta A}_{\text{Answer}} = (a \pm \delta a) + (b \pm \delta b) \Rightarrow (a+b) \pm \underbrace{(\delta a + \delta b)}_{\text{absolute uncertanity}}$$

always added

Difference:-

$$A \pm \delta A = (a \pm \delta a) - (b \pm \delta b) \Rightarrow (a-b) \pm (\delta a + \delta b)$$

⇒ Multiplication & Division:-

$$A \pm \delta A = (a \pm \delta a)(b \pm \delta b)$$

In both, relative uncertanity is calculated as.

$$\delta A = \pm \left(\frac{\delta a}{a} \times 100 + \frac{\delta b}{b} \times 100 \right) \%$$

Q Find the charge on capacitor when $C = 2 \pm 0.1F$ & $V = 25 \pm 0.5$ volts

$$\because Q = CV = (2 \pm 0.1)(25 \pm 0.5)$$

$$\frac{\delta Q}{Q} = \pm \left(\frac{0.1}{2} \times 100 + \frac{0.5}{25} \times 100 \right) \Rightarrow \pm 7\%$$

For Error:-

$2 \times 25 = 50$

$$\delta Q = \pm 7\% \times 50 = \pm 3.5\ C$$

Final Answer:-

$$Q = 50 \pm 3.5\ C$$

Range:-

$\Rightarrow 46.5\ C - 53.5\ C$

Graph:-

The pictorial representation of any function

⇒ Dependent-value = y-axis

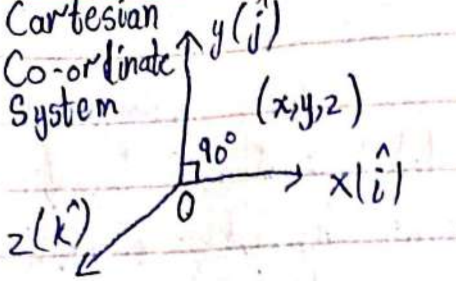
⇒ Independent-value = x-axis

⇒ Define scale and decide frame of reference (mostly origin)

Chapter 02

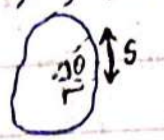
Rectangular OR Vector Analysis

Cartesian Co-ordinate System



Polar co-ordinate

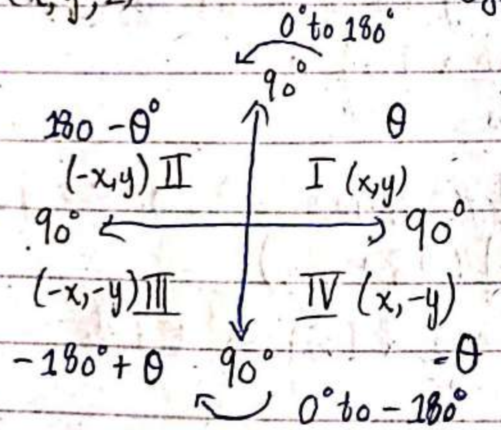
$(r, \theta, S) \Rightarrow \{S = r\theta\}$



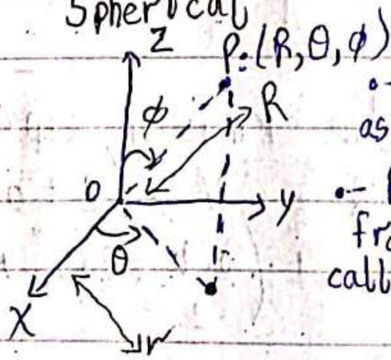
$r \rightarrow$ dist covered from origin

Co-ordinate System

- Cartesian Coordinate System (x, y, z)
- Polar Co-ordinate System
  - Cylindrical (C.S)
  - Spherical (C.S)



Spherical



- Follow same as cartesian
- Draw line from P to origin call it 'R'.

Vector Analysis

- Algebaric (Less precision) $(+, -, *, \div)$
- Calculus
  - Differentiation
  - Integration

Evaluation of a system w.r.t a single variable $\Rightarrow$ ordinary differentiation

" " two or more varibles $\Rightarrow$ Partial differentiation

Partial Differentiation

- Gradient — spread OR of a quantity
- Divergence (Impact of system on surroundings) like charged sphere surroundings
- Curl — Intensity of lines of forces like of solenoid.

$\vec{B}$ $\vec{E}$

In Spherical Co-ordinate System:-

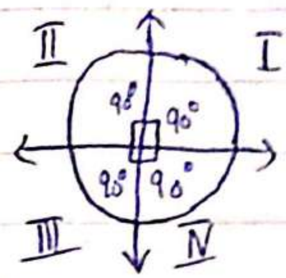
Angle drawn from x-axis => Azimuthal angle (θ)

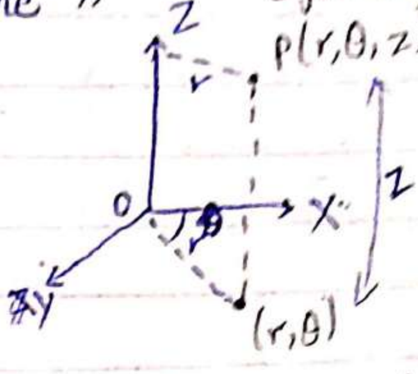
Angle drawn from z-axis => Polar angle (ϕ)

1D-Integration:- line integration

2D- " :- Surface "

3D- " :- Volume "





Cylinderical co-ordinate system

- Move point to xy plane space consider
- Connect from origin to get 'r'.
- Now an angle from x-axis towards 'r' is θ

Chap-1

Numericals:- For the given data of the straight line graph and calculate given observations.

a) Standard Deviation.

b) Standard Error

c) Probable Error

d) Final Error

Observations:-	1	2	3	4	5	6	7	8
Length (cm)	62.0	72.0	85.0	100.0	112.0	120.0	134.0	148.0
Time Period (sec)	1.58	1.71	1.85	2.01	2.12	2.20	2.32	2.44

Observations are taken from simple pendulum,

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$\Rightarrow g = \frac{4\pi^2 L}{T^2} \quad -(i)$$

Putting above readings in eq(i), we get:-

$g_1 = 980.47\ cm/s^2$

$g_2 = 972.07\ cm/s^2$

$g_3 = 980.47\ cm/s^2$

$g_4 = 977.16 \, cm/s^2$

$g_5 = 983.79 \, cm/s^2$

$g_6 = 981.12 \, cm/s^2$

$g_7 = 982.85 \, cm/s^2$

$g_8 = 981.39 \, cm/s^2$

For $\bar{g}$:-

$$\text{Average } \bar{g} = \frac{g_1 + g_2 + g_3 + g_4 + g_5 + g_6 + g_7 + g_8}{8}$$

$$\bar{g} = 979.90 \, cm/s^2$$

Now,

No.	$\delta = g_n - \bar{g}$	δ^2
1-	0.57	0.324
2-	-7.19	51.6
3-	0.57	0.324
4-	-2.74	7.5
5-	3.89	15.13
6-	1.22	1.48
7-	2.9	8.14
8-	1	1

$$\therefore S = \sum \delta^2 = 85.498$$

a) Standard Deviation:-

$$\sigma = \sqrt{\frac{S}{n-1}} = \sqrt{\frac{85.498}{7}} = 3.494 \, cm/s^2$$

b) Standard Error:-

$$\sigma_m = \sqrt{\frac{S}{n(n-1)}} = \sqrt{\frac{85.498}{8(7)}} = 1.235 \ cm/s^2$$

c) Probable Error:-

$$PE = \pm 0.674 \times \sigma_m$$
$$PE = \pm 0.674 \times 1.235$$
$$PE = \pm 0.8334 \ cm/s^2$$

d) Final Error:-

$\therefore$ Value of $g = 979.9 \pm 0.8334 \ cm/s^2$

Chap # 2

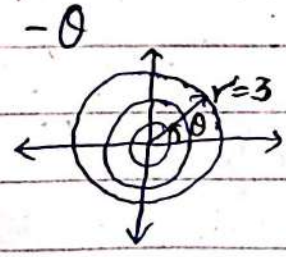
For angle in I and II quadrant, we measure θ anti-clockwise. For angle in III & IV quadrant, we measure θ clockwise.

$I \rightarrow \theta$; $II \rightarrow 180^\circ - \theta$ or $\pi - \theta \Rightarrow \phi - \theta$

phase angle ϕ

$III \rightarrow -180^\circ + \theta$ or $-\pi + \theta \Rightarrow -\phi + \theta$ or $\theta - \phi$

$IV \rightarrow -\theta$



Coordinate System

Cartesian

Polar

Spherical (ρ, θ, ϕ)

Cylendrical (r_c, θ, z)

Co-ordinate system:- Used to find the location of a point with respect to fixed frame of reference (origin)

Conversions:-

1) Cylendrical to Cartesian

$x = r\cos\theta$

$y = r\sin\theta$

$z = z$

2) Cartesian to Cylendrical

$r^2 = x^2 + y^2$ OR $r = \sqrt{x^2 + y^2}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$z = z$

3) Spherical to Cartesian:-

$x = \rho \sin\phi \cos\theta$

$y = \rho \sin\phi \sin\theta$

$z = \rho \cos\phi$

4) Cartesian to Spherical:-

$\rho = \sqrt{x^2 + y^2 + z^2}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$\phi = \cos^{-1}\left(\frac{z}{\rho}\right)$ OR $\phi = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right)$

5) Cylinderical to Spherical:-

ρ OR $R = \sqrt{r^2 + z^2}$

$\phi = \tan^{-1}\left(\frac{r}{z}\right)$

$\theta = \theta$

6) Spherical to Cylinderical:

$r = \rho \sin\phi$

$z = \rho \cos\phi$

$\theta = \theta$

Q Convert $(\overset{r}{2}, \overset{\theta = 240^\circ}{4\pi/3}, \overset{z}{8})$ from cylinderical to cartesian coordinate?

$$\left.\begin{aligned} x &= r\cos\theta = 2\cos\left(4\pi/3\right) = -1 \\ y &= r\sin\theta = 2\sin\left(\frac{4\pi}{3}\right) = -\sqrt{3} \end{aligned}\right\} \text{radians}$$

$z = z \Rightarrow z = 8$

$(2, 4\pi/3, 8) \Rightarrow (-1, -\sqrt{3}, 8) = (x, y, z)$

Q Convert $(\sqrt{3}, 1, 4)$ from cartesian to spherical, cylinderical

$x = \sqrt{3}$, $y = 1$, $z = 4$ units

$$r = \sqrt{(\sqrt{3})^2 + (1)^2} = \sqrt{4} = 2 \text{ units}$$

$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^\circ \text{ or } \frac{\pi}{6} \text{ rad}$$

$$z = 4$$

Cylinderical $\Rightarrow (r, \theta, z) = \left(2, \frac{\pi}{6}, 4\right)$

$$\rho = \sqrt{(\sqrt{3})^2 + (1)^2 + (4)^2} = 2\sqrt{5} \text{ units}$$

$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^\circ \text{ or } \frac{\pi}{6} \text{ rad}$$

$$\phi = \frac{4}{2\sqrt{5}} = \frac{2\sqrt{5}}{5} \text{ units}$$

Spherical $\Rightarrow (\rho, \theta, \phi) = \left(2\sqrt{5}, \frac{\pi}{6}, \frac{2\sqrt{5}}{5}\right)$

Vector Calculus :-

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k} \Rightarrow$ Simple unit vector

1) Del operator :- → used to convert scalar quantity to vector form

↳ on scalar (application)

$\frac{\partial}{\partial x}$ = dabba

$$\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

∇ = nabla

3-D vector analysis is called gradient

- Diffrentiation :- The process used to find the function that outputs rate of change of one variable w.r.t another.

For vectors ⇒ Vector differentiation

$$\frac{d}{dx}(2xyz) \Rightarrow 2\frac{d}{dx}(xyz) \Rightarrow 2(1)\frac{dy}{dx}\cdot\frac{dz}{dx}$$

If added / subs in partial then treated as constants

$$xy + y \Rightarrow \frac{\partial y}{\partial x}\hat{i} + \frac{\partial}{\partial y}y\hat{j} + \frac{\partial y}{\partial z}\hat{k} = \hat{j}$$

$\nabla\phi \Rightarrow$ defines vector field

2) Gradient :-

Increase, spread of a scalar. $\phi \Rightarrow$ a differential scalar field

then gradient $= \nabla\phi$

Q If $\phi = 2xyz$, then gradient $= \nabla\phi$

$$\nabla\phi = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)(2xyz)$$

$$= \frac{\partial}{\partial x}(2xyz)\hat{i} + \frac{\partial}{\partial y}(2xyz)\hat{j} + \frac{\partial}{\partial z}(2xyz)\hat{k}$$

$$= 2yz\frac{\partial}{\partial x}x\hat{i} + 2xz\cdot\frac{\partial}{\partial y}y\hat{j} + 2xy\frac{\partial}{\partial z}z\hat{k}$$

$$\nabla\phi = 2yz\hat{i} + 2xz\hat{j} + 2xy\hat{k}$$

$\Rightarrow$ Gradient of scalar quantity = Vector

$\Rightarrow$ Gradient is used to transform any scalar function (ϕ) to vector form

Q $\nabla(x+y+z)$

$$\nabla = \frac{\partial}{\partial x}(x+y+z)\hat{i} + \frac{\partial}{\partial y}(x+y+z)\hat{j} + \frac{\partial}{\partial z}(x+y+z)\hat{k}$$

$$= \left(\frac{\partial x}{\partial x} + \frac{\partial y}{\partial x} + \frac{\partial z}{\partial x}\right)\hat{i} + \left(\frac{\partial x}{\partial y} + \frac{\partial y}{\partial y} + \frac{\partial z}{\partial y}\right)\hat{j} + \left(\frac{\partial x}{\partial z} + \frac{\partial y}{\partial z} + \frac{\partial z}{\partial z}\right)\hat{k}$$

$$= (1+0+0)\hat{i} + (0+1+0)\hat{j} + (0+0+1)\hat{k}$$

$$= \hat{i} + \hat{j} + \hat{k}$$

Q Find gradient of $(3xz+y)$

$$\nabla(3xz+y)$$

$$\nabla = \frac{\partial}{\partial x}(3xz+y)\hat{i} + \frac{\partial}{\partial y}(3xz+y)\hat{j} + \frac{\partial}{\partial z}(3xy^2+y)\hat{k}$$

⇒ $\nabla\phi$ defines a vector field
⇒ Note that ϕ defines a differentiable scalar field
⇒ Gradient-scalar ($\nabla\phi$) is used to analyze change of scalar quantity into vector form.

$$\nabla\phi = \left[(3z)\frac{\partial x}{\partial x} + \frac{\partial y}{\partial x}\right]\hat{i} + \left[(3)\frac{\partial xy^2}{\partial y} + \frac{\partial y}{\partial y}\right]\hat{j} + \left[(3x)\frac{\partial z}{\partial z} + \frac{\partial y}{\partial z}\right]\hat{k}$$

$$\nabla\phi = 3z\hat{i} + \hat{j} + 3x\hat{k}$$

⇒ In terms of variable; Generalized Solution

Q Find the $\nabla\phi$ (gradient) of $(3x^2y - y^2z^3)$ at $(-1,2,8)$?

$$\nabla\phi = \frac{\partial}{\partial x}(3x^2y - y^2z^3)\hat{i} + \frac{\partial}{\partial y}(3x^2y - y^2z^3)\hat{j} + \frac{\partial}{\partial z}(3x^2y - y^2z^3)\hat{k}$$

$$= \cancel{\left[(6y)\partial x\right.}$$

$$= 3y\left(\frac{\partial x^2}{\partial x}\right)\hat{i} + \left(3x^2\frac{\partial y}{\partial y} - z^3\frac{\partial y^2}{\partial y}\right)\hat{j} + \left(-y^2\frac{\partial z^3}{\partial z}\right)\hat{k}$$

$$= 6xy\hat{i} + (3x^2 - 2yz^3)\hat{j} + (-3y^2z^2)\hat{k}$$

At Pt $(-1,2,8)$:-

☆ Rule:- First differentiate then put values

$$\nabla\phi_{(-1,2,8)} = -12\hat{i} - 2045\hat{j} - 768\hat{k}$$

Q Find direction derivative at $\vec{a} = 2\hat{i} + 3\hat{j} + 1\hat{k}$

For Unit Vector:-

$$\hat{a} = \frac{2}{\sqrt{14}}\hat{i} + \frac{3}{\sqrt{14}}\hat{j} + \frac{1}{\sqrt{14}}\hat{k} \quad \ldots \quad \hat{a} = \frac{\vec{a}}{|\vec{a}|}$$

Directional derivative $= \nabla\phi \cdot \hat{a}$ → dot-product

$\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$

$$(\nabla\phi \cdot \vec{a}) = (-12\hat{i} - 2045\hat{j} - 768\hat{k}) \cdot \left(\frac{2}{\sqrt{14}}\hat{i} + \frac{3}{\sqrt{14}}\hat{j} + \frac{1}{\sqrt{14}}\hat{k}\right)$$

$$= -1851.3 \text{ units}$$

$$(\nabla\phi \cdot \hat{a}) \text{ OR } (\nabla\phi)\cdot\left(\frac{\vec{a}}{|\vec{a}|}\right)$$

$(\nabla \cdot \vec{v})$ Divergence $\Rightarrow$ Spread of the $\vec{v}$

Directional Derivatives:- $\Rightarrow$ used to find the direction of a curve along the given line

Steps:-

- Find Gradient $(\nabla\phi)$ of a scalar, the put values (x, y, z)
- Find $\hat{a}$ of given vector $\vec{a}$.
- Find direction of ~~dire~~ derivative $(\nabla\phi \cdot \hat{a})$

Q Find directional derivative in direction $2\hat{i} + 3\hat{j} - 2\hat{k}$
If $\phi = 2xy + 2y^2z$ at $(0, 1, 2)$

For Gradient $(\nabla\phi)$

$$\nabla\phi = \frac{\partial}{\partial x}(2xy + 2y^2z)\hat{i} + \frac{\partial}{\partial y}(2xy + 2y^2z)\hat{j} + \frac{\partial}{\partial z}(2xy + 2y^2z)$$

$$= 2y\left(\frac{\partial x}{\partial x}\right)\hat{i} + \left(2x\frac{\partial y}{\partial y} + 2z\frac{\partial y^2}{\partial y}\right)\hat{j} + 2y^2\left(\frac{\partial z}{\partial z}\right)\hat{k}$$

$$= 2y\hat{i} + (2x + \overset{4}{\cancel{2}}yz)\hat{j} + 2y^2\hat{k}$$

At Pt $(0, 1, 2)$

$$\nabla\phi = 2\hat{i} + \overset{8}{\cancel{4}}\hat{j} + 2\hat{k}$$

For Directional Derivative:-

$$\hat{a} = \frac{2}{\sqrt{17}}\hat{i} + \frac{3}{\sqrt{17}}\hat{j} - \frac{2}{\sqrt{17}}\hat{k}$$

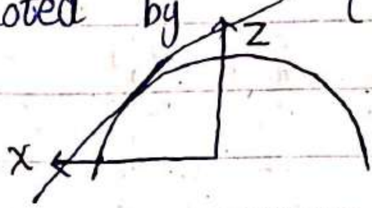
$$(\nabla\phi \cdot \hat{a}) = (2\hat{i} + \overset{8}{\cancel{4}}\hat{j} + 2\hat{k}) \cdot \left(\frac{2}{\sqrt{17}}\hat{i} + \frac{3}{\sqrt{17}}\hat{j} - \frac{2}{\sqrt{17}}\hat{k}\right)$$

$$(\nabla\phi \cdot \hat{a}) = \frac{\overset{24}{\cancel{12}}\sqrt{17}}{17} = \cancel{2.910 \text{ units}}\ 5.821 \text{ units}$$

Directional Derivatives:-

The directional derivative is the rate at which function changes at a point in a particular direction. Its main application is to find out the slope. (In 3D plane)

It is denoted by $(\nabla\phi).\hat{a}$ .... $\hat{a} = \frac{\vec{a}}{|\vec{a}|}$



Divergence:- ⇒ scalar analysis of vector. means answer dealing in scalar but input is vector

How much a quantity diverges. (tells the degree of variation in the amount of quantity).

$$\nabla.\vec{V} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\left(V_1\hat{i} + V_2\hat{j} + V_3\hat{k}\right)$$

$$\nabla.\vec{V} = \left(\frac{\partial V_1}{\partial x}\hat{i}\cdot\hat{i} + \frac{\partial V_2}{\partial y}\hat{j}\cdot\hat{j} + \frac{\partial V_3}{\partial z}\hat{k}\cdot\hat{k}\right)$$

$$\nabla.\vec{V} = \left(\frac{\partial V_1}{\partial x} + \frac{\partial V_2}{\partial y} + \frac{\partial V_3}{\partial z}\right) \text{---} \left(\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1\right)$$

If,

$\vec{\nabla}\cdot\vec{V} = +ve$ = source-field

$\vec{\nabla}\cdot\vec{V} = -ve$ = sink-field

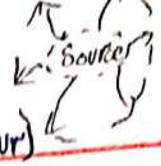
$\vec{\nabla}\cdot\vec{V} = 0$ = $\underbrace{\text{Solenoidal field}}$

↳ It has no sink or source.

Q If $\vec{V} = (x+3y)\hat{i} + (y-2z)\hat{j} + (x+az)\hat{k}$ is solenoidal then find 'a'.

$$\vec{\nabla}.\vec{V} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\cdot\left([x+3y]\hat{i} + (y-2z)\hat{j} + (x+az)\hat{k}\right)$$

→ tries to pull (sink) things in (accepting behaviour)



→ ... have a grip on its surroundings with influence. (rejecting behaviour)

Solenoidal → Have no effect

$$\vec{\nabla}\cdot\vec{V} = \left(\frac{\partial x}{\partial x}\hat{i}\cdot\hat{i}\right) + \left(\frac{\partial y}{\partial y}\hat{j}\cdot\hat{j}\right) + a\left(\frac{\partial z}{\partial z}\hat{k}\cdot\hat{k}\right) = 0$$

$$1 + 1 + a = 0$$

$$a = -2$$

For $a = -2$, the field is solenoidal.

Curl:- ⇒ To determine the behaviour of two quantities moving mutually linearly & perpendicularly

Rotation of a vector. we will use curl.

$$\vec{\nabla}\times\vec{V} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\cdot(V_1\hat{i} + V_2\hat{j} + V_3\hat{k})$$

$$\vec{\nabla}\times\vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_1 & V_2 & V_3 \end{vmatrix}$$

$\vec{\nabla}\times\vec{V} = 0$ (Irrotational)

$\vec{\nabla}\times\vec{V} \neq 0$ Solenoidal OR (Rotational) → means comes to same point after rotation from which is started.

↳ Since not zero shows field is impactful, doing work. Like electron centripetal force is balanced by centrifugal force

$\hat{i}\times\hat{j} = \hat{k}$, $\hat{j}\times\hat{i} = -\hat{k}$

$\hat{k}\times\hat{i} = \hat{j}$, $\hat{i}\times\hat{k} = -\hat{j}$

$\hat{j}\times\hat{k} = \hat{i}$, $\hat{k}\times\hat{j} = -\hat{i}$

Hence;

$$\vec{A}\times\vec{B} = -\vec{B}\times\vec{A} \quad \text{OR} \quad \vec{A}\times\vec{B} \neq \vec{B}\times\vec{A}$$

Q Show that $\vec{A} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$ is Irrotational? If value not given so take 1 or -1.

$$\vec{\nabla} \times \vec{V} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \times \left\{(6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}\right\}$$

$$\vec{\nabla} \times \vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 6xy+z^3 & 3x^2-z & 3xz^2-y \end{vmatrix}$$

If $\vec{V} \times \vec{\nabla}$ so

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6xy+z^3 & 3x^2-z & 3xz^2-y \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{vmatrix}$$

$$= \left\{\frac{\partial}{\partial y}(3xz^2 - y) - \frac{\partial}{\partial z}(3x^2 - z)\right\}\hat{i} - \left\{\frac{\partial}{\partial x}(3xz^2 - y) - \frac{\partial}{\partial z}(6xy + z^3)\right\} - \left\{\frac{\partial}{\partial x}(3x^2 - z) - \frac{\partial}{\partial y}(6xy + z^3)\right\}$$

$$= (-1 + 1)\hat{i} - (3z^2 - 3z^2)\hat{j} + (6x - 6x)\hat{k}$$

$$= 0$$

Hence Irrotational field.

$$\left\{\frac{\partial}{\partial y}(-y) - \frac{\partial}{\partial z}(-z)\right\}\hat{i} - \left\{\frac{\partial}{\partial x}(3xz^2) - \frac{\partial}{\partial z}(z^3)\right\} - \left\{\frac{\partial}{\partial x}(3x^2) - \frac{\partial}{\partial y}(6xy\right.$$

Vector Algebra

i) $\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}|\cos\theta$ (Scalar-Product)

ii) $\vec{A} \times \vec{B} = |\vec{A}||\vec{B}|\sin\theta$ (Vector-Product)

Q If $\vec{P} = 2\hat{i} - \hat{k}$ and $\vec{Q} = 5\hat{j} + 2\hat{k}$ then find:-

i) $|\vec{P}|$ ii) $|\vec{Q}|$ iii) $\vec{P} + \vec{Q}$

iv) $\vec{P} - \vec{Q}$ iv) $|\vec{P} - \vec{Q}|$

i) $|\vec{P}| = \sqrt{(2)^2 + (-1)^2} = \sqrt{5}$ units

ii) $|\vec{Q}| = \sqrt{(5)^2 + (2)^2} = \sqrt{29}$ units

iii) $\vec{P} + \vec{Q} = (2\hat{i} - \hat{k}) + (5\hat{j} + 2\hat{k})$

$= 2\hat{i} + 5\hat{j} - \hat{k}$

iv) $\vec{P} - \vec{Q} = (2\hat{i} - \hat{k}) - (5\hat{j} + 2\hat{k})$

$= 2\hat{i} - 5\hat{j} - 3\hat{k}$

$|\vec{P} - \vec{Q}| = \sqrt{(2)^2 + (-5)^2 + (-3)^2} = \sqrt{38}$ units

Q If $\vec{A} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{B} = 2\hat{i} + 4\hat{j} - \hat{k}$; Find:-

i) $\vec{A} \cdot \vec{B}$ ii) $\vec{A} \times \vec{B}$

iii) Angle b/w $\vec{A}$ and $\vec{B}$.

i) $\vec{A} \cdot \vec{B} = (\hat{i} + 2\hat{j} + 3\hat{k}) \cdot (2\hat{i} + 4\hat{j} - \hat{k})$

$= 2(\hat{i} \cdot \hat{i}) + 8(\hat{j} \cdot \hat{j}) - 3(\hat{k} \cdot \hat{k})$

$= 2(1) + 8(1) - 3(1)$

$\vec{A} \cdot \vec{B} = 7$ units

ii) $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 4 & -1 \end{vmatrix}$

$= (-2 - 12)\hat{i} - (-1 - 6)\hat{j} + (4 - 4)\hat{k}$

$\vec{A} \times \vec{B} = -14\hat{i} + 7\hat{j}$

iii) $\theta = \cos^{-1}\left(\frac{\vec{A} \cdot \vec{B}}{|\vec{A}||\vec{B}|}\right) = \cos^{-1}\left(\frac{7}{\sqrt{14} \cdot \sqrt{21}}\right) = 65.9^\circ$

Q2 If $\vec{A} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{B} = \overset{4}{2\hat{i}} \overset{-2}{-\hat{j}} \overset{+6}{+3\hat{k}}$. Find a unit vector parallel to $\vec{A} - 2\vec{B}$.

First $\vec{A} - 2\vec{B} = -3\hat{i} + 3\hat{j} - 5\hat{k}$

in its direction: $\hat{u} = \frac{\vec{A} - 2\vec{B}}{|\vec{A} - 2\vec{B}|} = \frac{-3}{\sqrt{43}}\hat{i} + \frac{3}{\sqrt{43}}\hat{j} - \frac{5}{\sqrt{43}}\hat{k}$

Q3 If $R = \sin t\,\hat{i} + \cos t\,\hat{j} + t\,\hat{k}$. Calculate:

i) $\frac{dR}{dt}$ ii) $\left|\frac{dR}{dt}\right|$ iii) $\frac{d^2R}{dt^2}$ iv) $\left|\frac{d^2R}{dt^2}\right|$

i) $\Rightarrow \frac{dR}{dt} = \cos t\,\hat{i} - \sin t\,\hat{j} + \hat{k}$

ii) $\Rightarrow \left|\frac{dR}{dt}\right| = \sqrt{(\cos t)^2 + (-\sin t)^2 + (1)^2} = \sqrt{1+1} = \sqrt{2}$ ... $|A| = \sqrt{x^2+y^2+z^2}$

$\sin^2 x + \cos^2 x = 1$

iii) $\Rightarrow \frac{d^2R}{dt^2} = -\sin t\,\hat{i} - \cos t\,\hat{j}$

iv) $\Rightarrow \left|\frac{d^2R}{dt^2}\right| = \sqrt{(-\sin t)^2 + (-\cos t)^2 + (0)^2} = \sqrt{1} = 1$

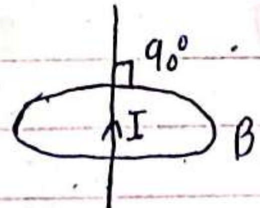
Divergence:- The changes in small part of volume quantity is measured through divergence.

$\nabla \cdot \vec{V}$ (vector) $\Rightarrow$ (Scalar)

$\Rightarrow$ Divergence tells the variation in the amount of quantity.

Curl Vector:-

It is the vector analysis of a vector $\Rightarrow$ Means input, output both vector



(Conducting wire)

$\Rightarrow$ To observe the behaviour of two quantities moving linear and perpendicular respectively; we will use curl of a vector

Summary :-

Gradient

$\phi \rightarrow$ Scalar diffrentiable field

$\nabla . \phi \rightarrow$ Scalar

Divergence

$\vec{V} \Rightarrow$ vector

$\nabla . \vec{V} \Rightarrow$ scalar

Curl

$\vec{V} \Rightarrow$ vector

$\nabla \times \vec{V} \Rightarrow$ vector

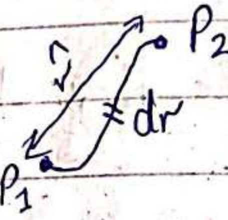
Integration :-

-- Reverse process of diffrentiation or combining different function

Line Integral :- → Example :- Work

↳ (where function is to be integrated along all points on line or on curve)

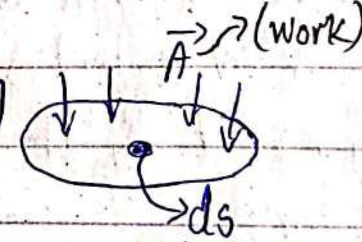
$$I = \int_{P_1}^{P_2} \vec{A} . dr$$



OR $\Rightarrow$ $I = \int A_1 dx + A_2 dy + A_3 dz \Rightarrow$ 3-D Example.

Surface Integral :-

↳ (Function Integration over same surface)



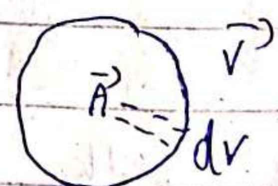
$$I = \iint \vec{A} . ds = \oint A \, ds$$

↳ double integral cause of 2 dimensions

Example :- ~~Work~~, ~~Ete~~ Flux etc

Volume Integral :-

$$I = \iiint \vec{A} . dv = \int_V \vec{A} \, dv$$



V ↳ shows 3-dimensions

Example :- Charge density , mass of cube etc

Numericals:-

Q. Find the work done in moving a particle in a force field given by:-

$$\vec{F} = 3xy\,\hat{i} - 5z\,\hat{j} + 10x\,\hat{k}$$

along the curve, where:-

$x = t^2 + 1$, $y = 2t^2$, $z = t^2$ from $t = 1$ to 2 sec

Diff all w.r.t 't':-

$\frac{dx}{dt} = 2t$, $\frac{dy}{dt} = 4t$, $\frac{dz}{dt} = 2t$

$\Rightarrow dx = 2t.dt$, $dy = 4t.dt$, $dz = 2t.dt$

As we know that:-

$$\text{Work done} = \int_{t_1}^{t_2} \vec{F}\cdot d\vec{r}$$

$$W = \int_1^2 (3xy\,\hat{i} - 5z\,\hat{j} + 10x\,\hat{k})\cdot(2t\,dt\,\hat{i} + 4t\,dt\,\hat{j} + 2t\,dt\,\hat{k})$$

$$W = \int_1^2 \left[3(t^2+1)(2t^2)\hat{i} - 5(t^2)\hat{j} + 10(t^2+1)\hat{k}\right]\cdot(2t\,dt\,\hat{i} + 4t\,dt\,\hat{j} + 2t\,dt\,\hat{k})$$

$\because \hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$

$$W = 6\int_1^2 (t^4 + t^2)\cdot(2t)\,dt - 5\int_1^2 (t^2)(4t)\,dt + 10\int_1^2 (t^2+1)(2t)\,dt$$

$$W = 12\int_1^2 (t^5 + t^3)\,dt - 20\int_1^2 t^3\,dt + 20\int_1^2 (t^3 + t)\,dt$$

$$W = 12\left|\frac{t^6}{6} + \frac{t^4}{4}\right|_1^2 - 20\left|\frac{t^4}{4}\right|_1^2 + 20\left|\frac{t^4}{4} + \frac{t^2}{2}\right|_1^2$$

Applying limits:-

$$W = 12\left[\frac{64}{6} + \frac{16}{4} - \frac{1}{6} - \frac{1}{4}\right] - 20\left[\frac{16}{4} - \frac{1}{4}\right] + 20\left[\frac{16}{4} + \frac{4}{2} - \frac{1}{4} - \frac{1}{2}\right]$$

$$W = 12(\cancel{16.7}\ 14.25) - 20(3.75) + 20(5.25)$$

$$W = \cancel{230.4}\ 201 \text{ Joules}$$

Ans

Q. If $\vec{A} = (3x^2 + 6y)\hat{i} - 14yz\hat{j} + 20x^2z\hat{k}$. Evaluate $\int \vec{A}\, dr$ from $t = 0$ to 1 sec ~~where~~ above following path,

$$x = t \quad , \quad y = t^2 \quad , \quad z = t^3$$

and $r = x\hat{i} + y\hat{j} + z\hat{k}$

$\Rightarrow$ Diff all w.r.t 't'

$$\Rightarrow dx = dt \quad , \quad dy = 2t\,dt \quad , \quad dz = 3t^2 dt$$

$$\because dr = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$$

$$\Rightarrow d\vec{r} = dt\,\hat{i} + 2t\,dt\,\hat{j} + 3t^2\,dt\,\hat{k}$$

Since,

$$\int \vec{A} \cdot d\vec{r} = \int_0^1 \left[(3x^2 + 6y)\hat{i} - 14yz\hat{j} + 20x^2z\hat{k}\right] \cdot (dt\,\hat{i} + 2t\,dt\,\hat{j} + 3t^2\,dt\,\hat{k})$$

$$\int \vec{A} \cdot d\vec{r} = \int_0^1 \left[3(t)^2 + 6(t^2)\,dt - \int_0^1 14(t^2)(t^3)(2t)\,dt + \int_0^1 20(t)^2(t^3)(3t^2\,dt)\right.$$

$$= 9\int_0^1 t^2\,dt - 28\int_0^1 t^6\,dt + 60\int_0^1 t^7\,dt$$

Applying limits

$$\int \vec{A} \cdot d\vec{r} = 9\left|\frac{t^3}{3}\right|_0^1 - 28\left|\frac{t^7}{7}\right|_0^1 + 60\left|\frac{t^8}{8}\right|_0^1$$

$$\int \vec{A} \cdot d\vec{r} = 3 - 4 + \frac{\cancel{30}\ 15}{\cancel{4}\ 2} = \frac{13}{2} = 6.5 \text{ units}$$

Q Find the velocity and acceleration any time (t) sec. The particle is moving along the following curve:-

$x = e^{-t}$, $y = 2\sin 3t$, $z = 2\cos 3t$

At time = t and 0 secs?

i) ∵ Displacement : $S = x + y + z$

$$S = e^{-t}\hat{i} + 2\sin 3t\,\hat{j} + 2\cos 3t\,\hat{k}$$

$$\Rightarrow \frac{dS}{dt} = V = -e^{-t}\hat{i} + 6\cos 3t\,\hat{j} - 6\sin 3t\,\hat{k}$$

$$\Rightarrow \frac{d^2S}{dt^2} = a = e^{-t}\hat{i} - 18\,\overset{\sin}{\cancel{\cos}}\,3t\,\hat{j} - 18\cos 3t\,\hat{k}$$

ii) At $t = 0$ sec?

$$|v|_{t=0} = \sqrt{(-e^{-0})^2 + (6\cos 3(0))^2 + \{(-6\sin 3(0)\}^2} = \sqrt{1+36+0} = \sqrt{37}\ \text{m/s}$$

$$|a|_{t=0} = \sqrt{(e^0)^2 + (-18\sin 3(0))^2 + \{(-18\cos 3(0)\}^2} = \sqrt{1+324} = \sqrt{325}\ \text{m/s}$$
$$= 5\sqrt{13}\ \text{m/s}$$

Q Simply using divergence:- $\vec{V} = 3x^2\hat{i} + (4xy + 2z)\hat{j} + (8z^3 - y)\hat{k}$
Find the nature of field?
At Point (2, 2, 1)

$$\nabla\cdot\vec{V} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\cdot\left\{3x^2\hat{i} + (4xy + 2z)\hat{j} + (8z^3 - y)\hat{k}\right\}$$

$$= \left(\frac{\partial}{\partial x}\cdot 3x^2\right) + \left(\frac{\partial}{\partial y}\right)(4xy + 2z) + \left(\frac{\partial}{\partial z}\right)(8z^3 - y)$$

$$\because\ \hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$$

$$\nabla\cdot\vec{V} = 6x + 4x + 24z^2$$

At Point (2, 2, 1):-

$$\nabla\cdot\vec{V} = 6(2) + 4(2) + 24(1) = 44 \text{ units}$$

Thus the given field is non-solenoidal (source-field)

Chapter 03

Mechanics

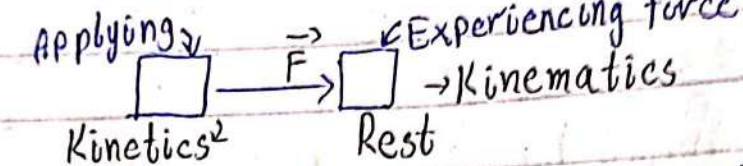
Study of forces (Applicable a/c to Newton laws)

Dynamics: Study of bodies moving as a result of forces (both geometric and cause aspects)

Statistics: study of body at rest or in equilibrium, (within particular frame of reference)

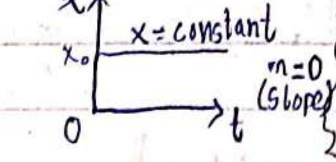
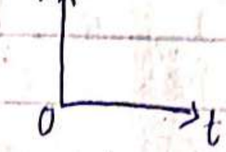
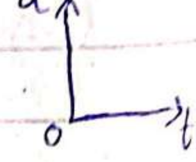
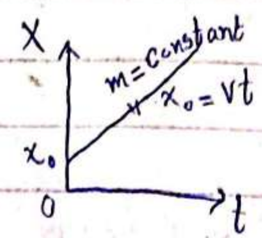
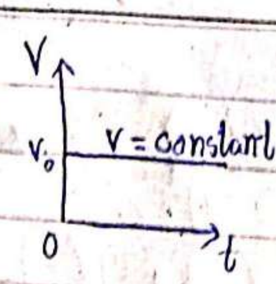
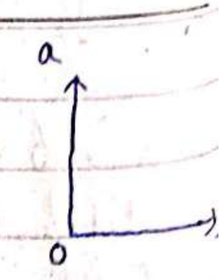
Kinetics: (Study of the possible motions of material body) body applying force

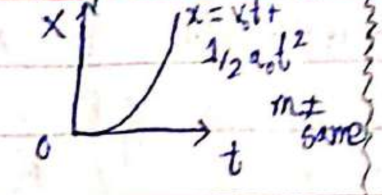
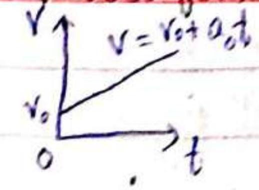
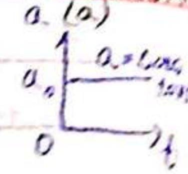
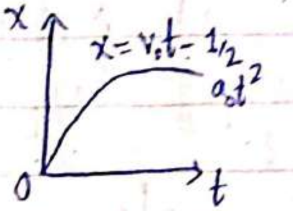
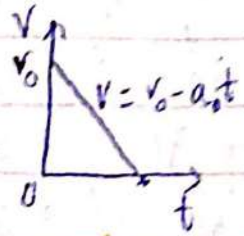
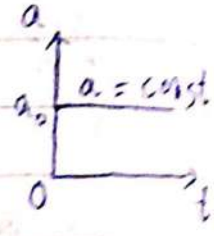
Kinematics: body experiencing force (study & description of force causing motion of material body)



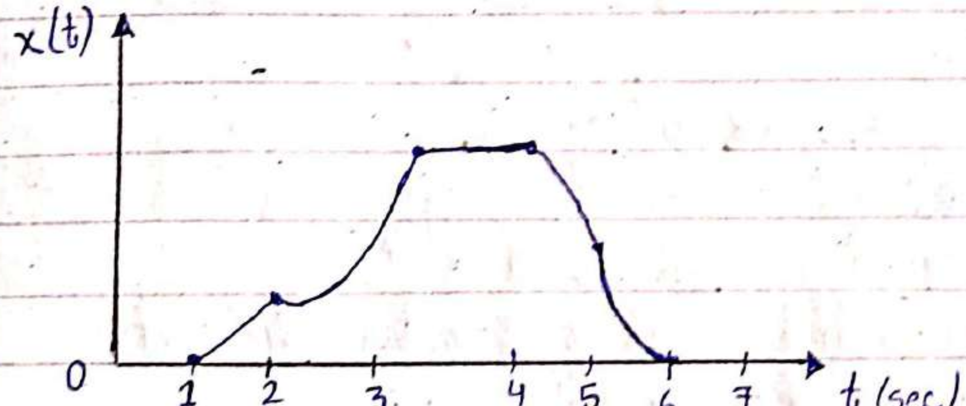
Mechanics:- Mechanics is the study of certain general relations that describe the interactions of material bodies and laws governing them.

Cases Of Motion:-

	Displacement(x)	Velocity (v)	Acceleration(a)
a). At Rest $v=0$			
b) Moving with constant increasing velocity			

	Displacement (x)	Velocity (v)	Acceleration
c) Moving with constant acceleration	 $x = v_0 t + \frac{1}{2} a_0 t^2$ m ± same	 $v = v_0 + a_0 t$	 $a = const$
d) Moving with constant decleration	 $x = v_0 t - \frac{1}{2} a_0 t^2$	 $v = v_0 - a_0 t$	 $a = const$

Q Define the cases of motion for below graph :-



For $t = 0 - 1$	For $t = 1 - 2$	For $t = 2 - 3$
At rest	Constant Velocity	Constant acceleration
For $t = 3 - 4$ Comes to Rest	For $t = 4 - 5$ Constant Decreasing Velocity	For $t = 5 - 6$ Falling object with constant acceleration

Instantaneous Velocity :-

A velocity of an object in motion at a specific point in time or for short interval of time.

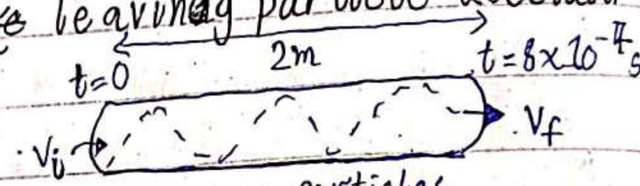
$$v_{ins} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

Same for => $a_{ins} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$

Q An α-particle travels along the inside of an evacuated straight 2 meter long tube forming a part of particle accelator. The α-particle enters the tube at $t=0$ moving at a velocity of 9.5×10^{5} m/s and emerges from the other end of tube at $t = 8 \times 10^{-7}$ sec. Find:-

i) Acceleration of α-particle = ?

ii) ~~Distance~~ Velocity covered ~~before~~ after leaving particle accelator =



Data :-

$S = 2$ meter

$t_i = 0$ sec , $t_f = 8 \times 10^{-7}$ sec

$v_i = 9.5 \times 10^{5}$ m/s

Find:-

a) the accelartion of α-particle = ? a

b) How much ~~distance~~ velocity α-particle will it ~~cover~~ have when it leaves acceleration : a v_f = ?

Sol:-

$$\because S = v_i t + \frac{1}{2} a t^2$$

In particle accelator, distance will be equal to length of tube.

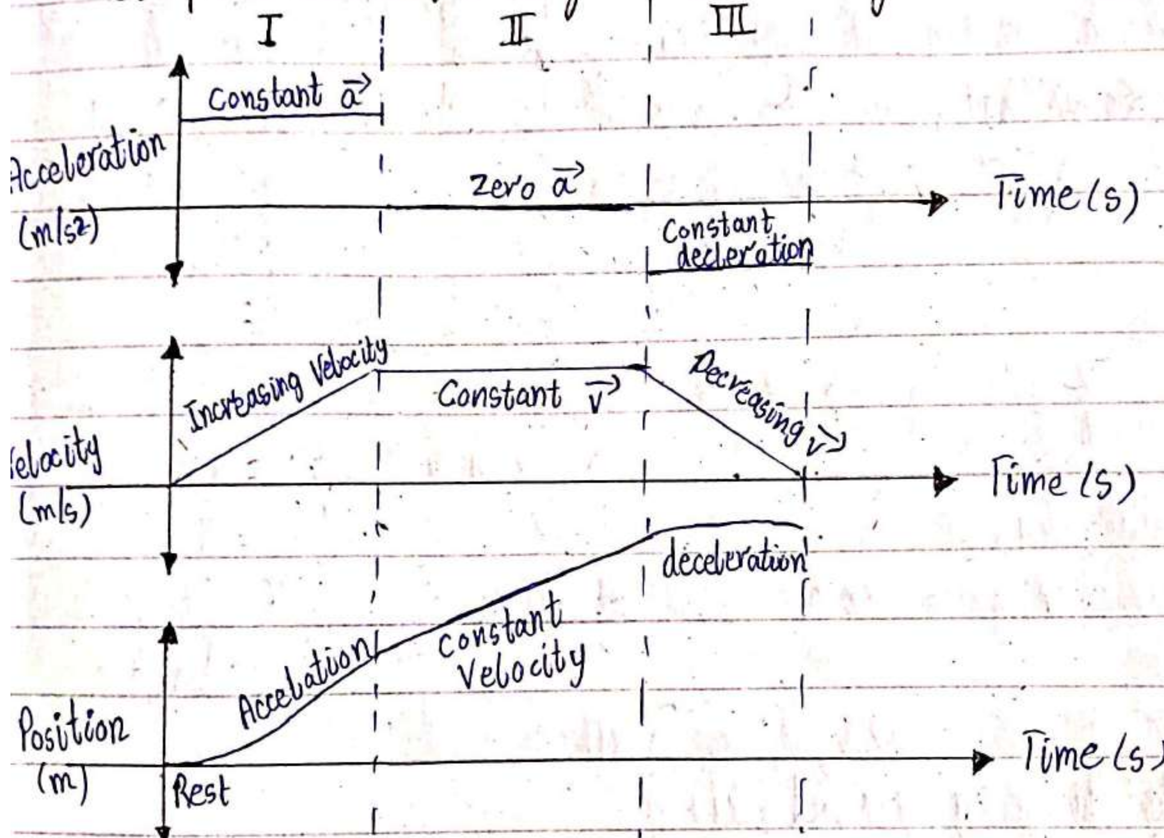
$$\hookrightarrow 2 = (9.5 \times 10^{5})(8 \times 10^{-7}) + \frac{1}{2} a (8 \times 10^{-7})^2$$

$$\frac{(2 - 0.76) \times 2}{(8 \times 10^{-7})^2} = a$$

$$a = 3.875 \times 10^{12} \text{ m/s}^2$$

$$\because v_f = v_i + at$$
$$v_f = 9.5 \times 10^5 + (3.875 \times 10^{12})(8 \times 10^{-7})$$
$$v_f = 4.05 \times 10^6 \text{ m/s}$$

• – Graphs Of Accelerating And Breaking of Car :-



Displacement :- Change in position of a body in specific direction OR The shortest distance between two points.

$$\Delta x = x_2 - x_1$$

=> Rate of change of displacement / position is velocity.

$$\Delta v = \frac{\Delta x}{\Delta t}$$

=> Rate of change of velocity is acceleration.

$$a_{avg} = \frac{\Delta v}{\Delta t}$$

States Of Motion

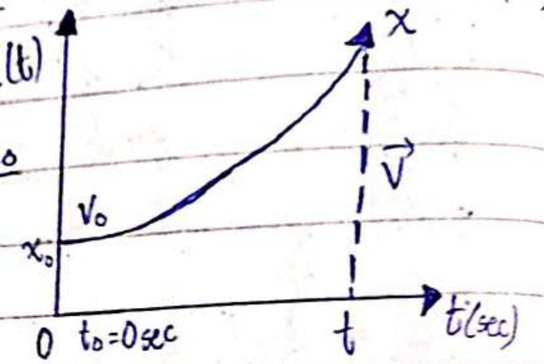
Rest
If force applied;
↳ Motion ($+\vec{a}$)

Motion
By applying force

$+\vec{a}$
($\vec{v}$ increases)

$-\vec{a}$
($\vec{v}$ decrease)
For Retardation, decrease till (max output)

Motion With Constant Acceleration :-



As we know that,

$$\because V_{avg} = \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t - t_0} \Rightarrow \frac{x - x_0}{t}$$

And,

$$a_{avg} = \frac{\Delta V}{\Delta t} = \frac{V - V_0}{t - t_0} \Rightarrow \frac{V - V_0}{t} \Rightarrow v_0 + at = V \quad -(i)$$

Mathematically,

$$V_{avg} = \frac{V_0 + V}{2}$$

$$\Rightarrow \quad V = 2V_{avg} - V_0 \quad -(ii)$$

Substituting eq (ii) into (i)

$$2V_{avg} - V_0 = V_0 + at$$

$$2V_{avg} = 2V_0 + at$$

$$V_{avg} = V_0 + \frac{1}{2}at \quad -(iii)$$

Since,

$$V_{avg} = \frac{x - x_0}{t}$$

So,

$$\Rightarrow \quad \frac{x - x_0}{t} = V_0 + \frac{1}{2}at$$

$$x = x_0 + V_0 t + \frac{1}{2}at^2$$

Summary:- If $x_0 = 0$, so 2^{nd} eq of motion

$\rightarrow x = x_0 + v_0 t + \frac{1}{2} a_x t^2$, $y = y_0 + v_0 t + \frac{1}{2} a_y t^2$

$\rightarrow z = z_0 + v_0 t + \frac{1}{2} a_z t^2$

Equations Of Motions:-

$1^{st} \Rightarrow \quad v_f = v_i + at$

$2^{nd} \Rightarrow \quad x = v_i t + \frac{1}{2} a t^2$

$3^{rd} \Rightarrow \quad 2ax = v_f^2 - v_i^2$

Q A particle moves in any xy-plane in such a way that its coordinates are:-

$x(t) = t^3 - 32t$; $y(t) = 5t^2 + 12t$

Find displacement (position), velocity and acceleration at $t = 3sec$.

Given:-

$\vec{r} = x\hat{i} + y\hat{j}$ (xy-plane)

$t = 3secs$

$v = ?$, $a = ?$, $\vec{r}(t)?$

Solution:-

Position of a particle is given by:-

$\vec{r} = x\hat{i} + y\hat{j}$

$\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j}$

$\vec{r}(t) = (t^3 - 32t)\hat{i} + (5t^2 + 12t)\hat{j}$; At $t = 3sec$ $\vec{r}(t) = -69\hat{i} + 81\hat{j}$ m

Diff w.r.t t:-

$\frac{d\vec{r}}{dt} = (3t^2 - 32)\hat{i} + (10t + 12)\hat{j}$ — (i)

At $t = 3$ secs

$$\frac{d\vec{r}}{dt} = \vec{v} = -5\hat{i} + 42\hat{j} \ m/s$$

Again ~~eqn~~ diff eq (i) by 't' :-

$$\frac{d\vec{v}}{dt} = \vec{a} = 6t\,\hat{i} + 10\,\hat{j}$$

At $t = 3$ sec

$$\vec{a} = 18\hat{i} + 10\hat{j} \ m/s^2$$

Newton's Laws:-

1^{st}:- A body at rest or moving at constant speed in a straight line will continue its state of rest or motion until an external force acts on it. $\vec{F} \rightarrow O$ Then O $\vec{v}$ fr... OR ... until

2^{nd}:-

Momentum:- Quantity of motion contained in a body.

$$p = mv \quad OR \quad \vec{p} \propto \vec{v}$$

"Rate of change of momentum of a body is both equal in magnitude and direction to the force imposed on it. Conversely, if body is not accelerated, there is no net force acting on body."

$$\vec{F} = m\vec{a} \quad OR \quad \vec{F} = \frac{d\vec{p}}{dt} \qquad \xrightarrow{\vec{F}} (m) \xrightarrow{\vec{a}} ; F$$

mass $\propto$ Inertia

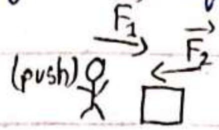
Inertia:- Tendency to resist motion in a body.

3^{rd} :-

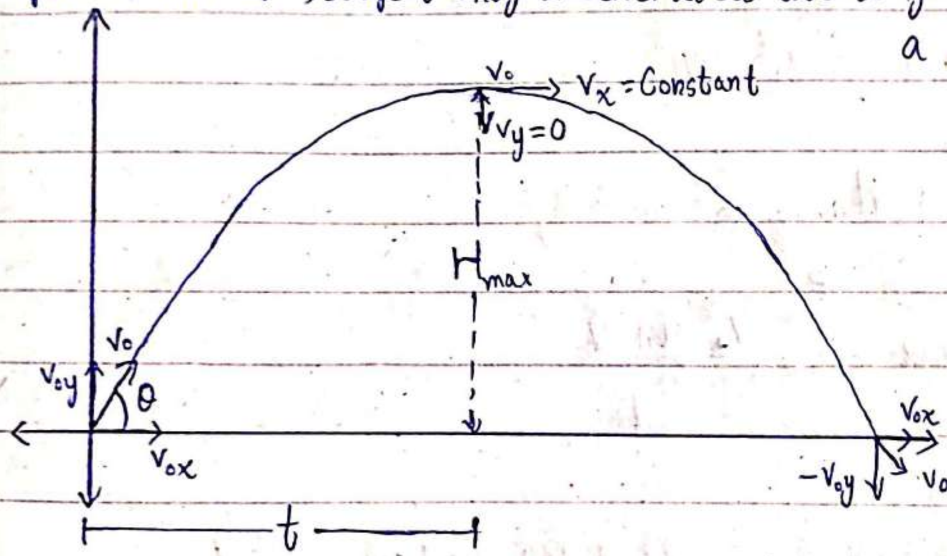
"For every action, there is an equal and opposite reaction."

=) When bodies interact, they apply force equal in magnitude but opposite in direction.

E.g:- Rocket - Propulsion etc



Projectile Motion:- It is the motion of an object thrown / projected into air, subject only accelerated due to gravity and follow a curved path.



Assumptions:-

i) No friction due to air.

ii) Value of g is constant and directed downwards.

iii) Effect due to rotation & or curvature of Earth is negligible.

Formulas:-

i) Time to reach H_{max}:-

A/c to 1^{st} eq of motion:-

$$V_f = V_i + at$$

In projectile motion:-

$\Rightarrow$ Since at max height $v_y = 0$

$$\cancel{v_y}^{\to 0} = v_{oy} + (-g)t$$

$$\cancel{g}\, t = + \frac{v_0 \sin\theta}{g}$$

ii) Total Time:-

$$\because T = 2t \quad \Rightarrow T = \frac{2 v_0 \sin\theta}{g}$$

iii) H_{max} :-

$$\because 2aS = v_f^2 - v_i^2$$

In projectile,

$$\Rightarrow 2(-g) H_{max} = \cancel{v_y^2}^{\to 0} - v_{oy}^2$$

$$\therefore H_{max} = \frac{v_0^2 \sin^2\theta}{2g}$$

iv) Range:-

$\because S = vt$ (as along x-axis, $\vec{a} = 0$, v_{ox} = Consta

In projectile:-

$$\Rightarrow R = v_{ox}\, \cancel{t}\, T$$

$$R = v_0 \cos\theta \left(\frac{2 v_0 \sin\theta}{g} \right)$$

$$\Rightarrow R = \cancel{2 v_0} \frac{v_0^2 \sin 2\theta}{g} \quad \text{---} \sin 2\theta = 2 \sin\theta \cos\theta$$

Q A player kicks a bowl at 36° from the horizontal with an initial speed of 15.5 m/s. Assume the ball moves in vertical plane. Calculate:-

i) $t = ?$, ii) $H_{max} = ?$, iii) $R = ?$, $T = ?$ iv) $R_{max} =$

True velocity

Relative Velocity: – Depends on reference

i) Bodies moving in opposite direction: –

$0 \rightarrow v_1 \quad \leftarrow -v_2$ opposite direction vector so magnitude = –ve

$v_R = v_1 - (-v_2)$

$\Rightarrow v_R = v_1 + v_2$

ii) Same direction: –

$\rightarrow v_1 \quad \rightarrow v_2$

$v_R = v_1 - v_2$

Data:-

$\theta = 36^\circ$

$v_o = 15.5\ m/s$

$t = ?$

$H_{max} = ?$

$R = ?$

$T = ?$

$R_{max} = ?$

Solution:-

i) $t = \dfrac{v_o \sin\theta}{g} = \dfrac{(15.5)\sin 36^\circ}{9.8} = 0.93\ sec$; $T = 2(0.93) = 1.86\ sec$

ii) $H_{max} = \dfrac{v_o^2 \sin^2\theta}{2g} = \dfrac{(15.5)^2 \sin^2 36^\circ}{2 \times 9.8} = 4.23\ m$

iii) $R = \dfrac{v_o^2 \sin 2\theta}{g} = \dfrac{(15.5)^2 \sin(2 \times 36)}{9.8} = 23.32\ m$

iv) $R_{max} = \dfrac{v_o^2}{\cancel{2}g} = \dfrac{(15.5)^2}{\cancel{2\times}9.8} = 24.52\ m$

Application Of Newton Laws:-

• Normal Force:-

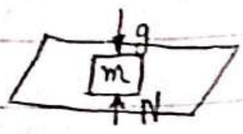
Force exerted by a surface to prevent solid objects to pass through each other.

=> It is a contact force so two surfaces not in contact, can't exert it.

=> Always perpendicular (normal) to the surface

In state of rest ; $N = W = mg$

=> For horizontal surface :- N (+ve), W (–ve)
(when $\theta = 0^\circ$)



For total upward (normal) force :-

$N = W + P \rightarrow$ (push or pull force) only if roped from end

Q A sled of mass $m = 7.5\,kg$ is pulled along a frictionless horizontal surface by a cord. A constant force $P = 21.0\,N$ is applied to the cord. Analyze the motion if

a) the cord is horizontal

b) the cord makes an angle of $\theta = 15^\circ$ with the horizontal.

Data:-

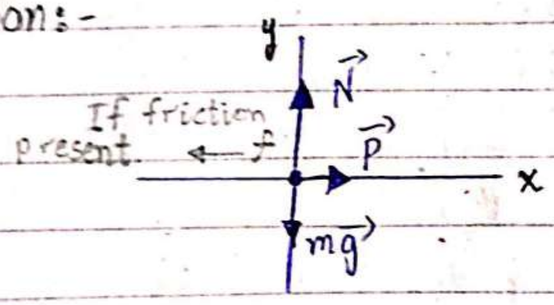
$m = 7.5\,kg$

$P = 21\,N$, $f = 0$ (frictionless)

a) $\theta = 0^\circ$, b) $\theta = 15^\circ$

Solution:-

a)



For x-component:-

$$\Sigma F_x \Rightarrow F_{net} = P$$

$$P = m a_x$$

$$a_x = \frac{21}{7.5} = 2.8\ m/s^2 \text{ (Horizontal acceleration)}$$

For y-component:-

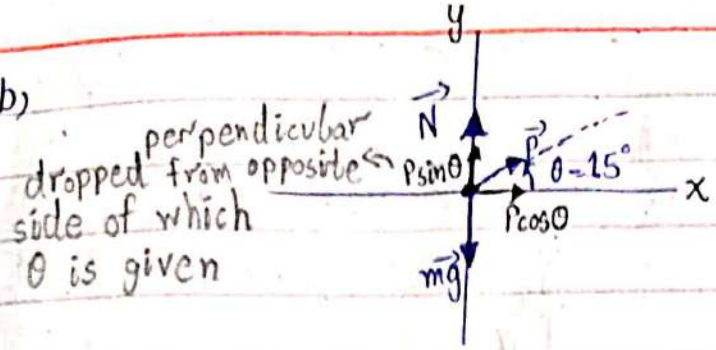
$$\Sigma F_y \rightarrow F_{net} = N - mg$$

$$m a_y = N - mg$$

Since there is no vertical motion, the sled remains on surface and $a_y = 0$. Therefore:-

$$N = mg = (7.5)(9.8) = 73.5\ N$$

ess b)



For y-component:-

$$F_{net} = N + P\sin\theta - mg$$

$$ma_y = N + P\sin\theta - mg$$

For the sled to remain on the surface, $a_y = 0$, Then:-

$$N = mg - P\sin\theta$$

$$N = (7.5)(9.8) - (21)\sin 15^\circ$$

$$N = 68.07 \text{ N}$$

For x-component:-

$$F_{net} = P\cos\theta$$

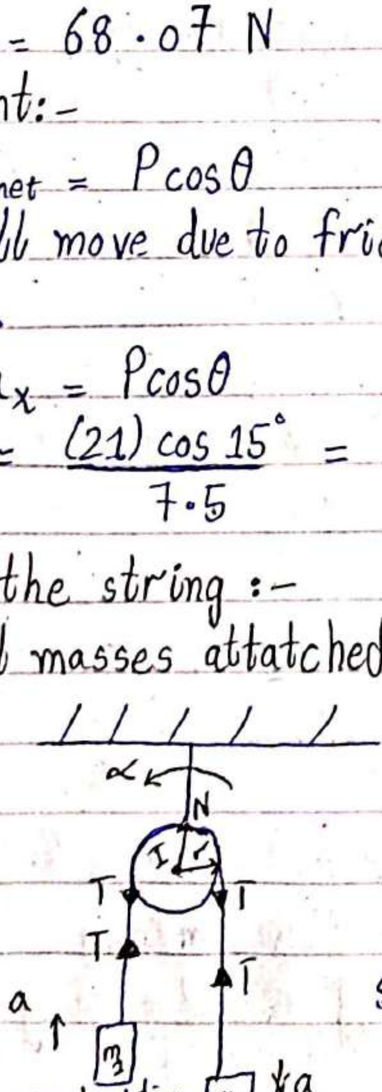
As the sled will move due to frictionless-table, so it will have acceleration.

$$ma_x = P\cos\theta$$

$$a_x = \frac{(21)\cos 15^\circ}{7.5} = 2.7 \text{ m/s}^2$$

• - Tension in the string :- used to demonstrate principles of dynamics and acceleration

Case 01 (Unequal masses attatched vertically to Atwood's machine



For massless pulley:

$$N = T + T = 2T$$

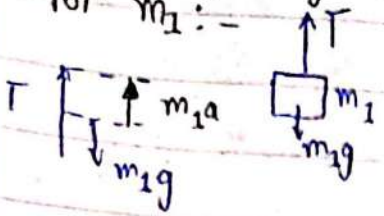
Since same cable used, so:-

$$T_1 = T_2 = T$$

↓ a = acceleration cause $(m_2 > m_1)$

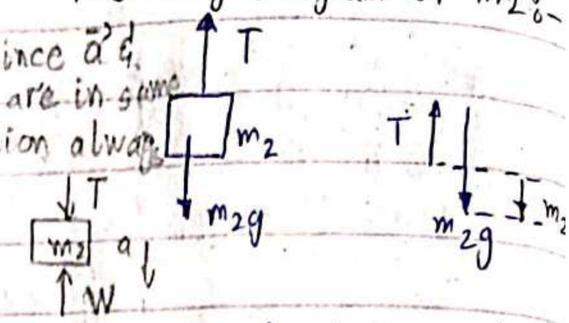
-ve → Left, downward
+ve → Right, upward

Free-body Diagram for m_1 :-



Free-body Diagram of m_2 :-

OR since $\vec{a}$ & $\vec{F}$ are in same direction always so,



For mass m_1 :-

As there is no motion along horizontal to both m_1 & m_2,

$$\sum F_x = 0$$

Along y-axis ⇒ $F_{net} = T - m_1 g$

$$m_1 a = T - m_1 g \quad -(i)$$

For mass m_2 :-

$$\sum F_x = 0$$

→ OR if 'T' made reference so as in f...

F_{net} ↑ $\sum F_y = m_2 a = m_2 g - T \quad -(ii)$ $\qquad T - m_2 g = -m_2 a$

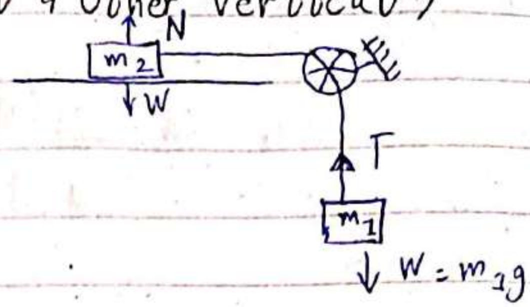
Add eq (i) & (ii)

$$T - m_1 g = m_1 a$$
$$m_2 g - T = m_2 a$$
$$+ \overline{\qquad\qquad\qquad\qquad}$$
$$m_2 g - m_1 g = m_1 a + m_2 a$$
$$\frac{(m_2 - m_1) g}{m_1 + m_2} = a$$

Putting in eq (i)

$$T - m_1 g = m_1 \left[\frac{(m_2 - m_1) g}{m_1 + m_2} \right]$$
$$(T - m_1 g)(m_1 + m_2) = m_1 g (m_2 - m_1)$$
$$m_1 T + m_2 T - \cancel{m_1^2 g} - m_1 m_2 g = m_1 m_2 g - \cancel{m_1^2 g}$$
$$(m_1 + m_2) T = 2 m_1 m_2 g$$
$$\boxed{T = \frac{2 m_1 m_2 g}{m_1 + m_2}}$$

Case-02 (one body is horizontal & other vertical)

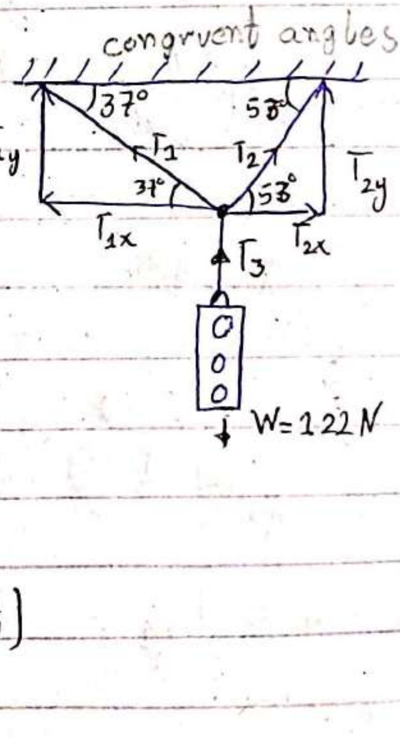


$N = W_2$

$$a = \frac{m_1 g}{m_1+m_2}$$

$$T = \frac{\cancel{2} m_1 m_2 g}{m_1+m_2}$$

Q A traffic light having a weight of 122 N is hanging from a cable attatched to two other cables fastened to a support. The upper cable makes an $\theta_1 = 37°$ and $\theta_2 = 53°$ with the horizontal. The upper cables are not strong as vertical one and will break if tension exceeds 100 N. Will the traffic light remain hanging?



Since both the points (traffic-light & knot) have to be in state of equilibrium prior to breaking so:-

For Traffic Light At Rest:-

Only along vertical;

$$\Sigma F_3 = T_3 - W = 0$$

$$T_3 = W = 122\ N$$

For Whole System:- (Supporting Weight)

Along x-axis:-

$$\Sigma F_x :- T_{2x} - T_{1x} = 0$$

$$T_2 \cos 53° = T_1 \cos 37°$$

$$T_2 = T_1 \frac{\cos 37°}{\cos 53°} \quad \text{— (i)}$$

• – A knot or separation in same string/cable will cause tension of each segment to be different.

Along y-axis:-

$$\Sigma F_y :- \; T_{2y} + T_{1y} - W = 0$$

$$T_2 \overset{\sin}{\cancel{\cos}}\, 53^\circ + T_1 \overset{\sin}{\cancel{\cos}}\, 37^\circ - 122 = 0$$

$$\left[T_1 \frac{\cos 37^\circ}{\cos 53^\circ}\right](\sin 53^\circ) + T_1 \sin 37^\circ = 122$$

$$T_1 \left[\cos 37^\circ . \tan 53^\circ + \sin 37^\circ\right] = 122$$

$$T_1 = \frac{122}{1.66} = 73.4\ N$$

Now eq(i) becomes

$$T_2 = (73.4)\frac{\cos 37^\circ}{\cos 53^\circ}$$

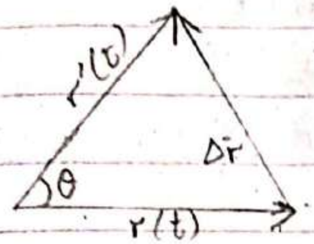
$$T_2 = 97.4\ N$$

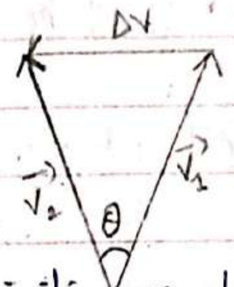
$\Rightarrow$ Since, both the tensions are less than 100N so the cables will not break.

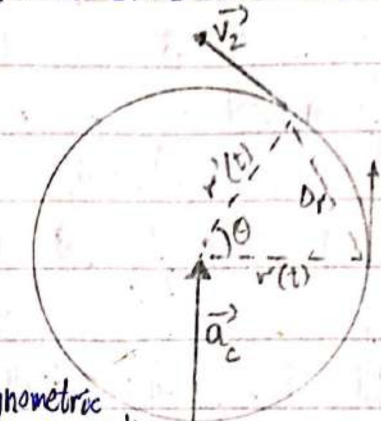
whole force (tension) on above

Uniform Circular Motion:-

A body moving in a circle at a constant speed 'v' is said to be in uniform circular motion.







Both the triangles are similiar as all three sides are equal, using similiar trignometric ratio

$$\frac{\Delta r}{r} = \frac{\Delta v}{v}$$

$$\frac{\Delta v}{\Delta r} = \frac{v}{r}$$

.- Angle b/w r's & v_1, v_2 is equal as two lines & their normals are also equal i.e. velocity is normal to radius.

$|v_1| = |v_2| = v$

Dividing by Δt

$$\frac{\overset{a \to}{\Delta v / \Delta t}}{\underset{v \nearrow}{\Delta r / \Delta t}} = \frac{v}{r} \Rightarrow \boxed{a_c = \frac{v^2}{r}} \rightarrow \text{Centripetal Acceleration}$$

Points:-

For a uniform circular motion:-

- The direction of a_c is towards the centre of circle.
- Direction of ~~linear~~ velocity is tangent to circle.
- Directions of velocity & $\vec{a_c}$ are mutually perependicular.

$\vec{v_1} \perp \vec{a_c}$:- $\theta = 90°$ $\vec{a_c} \rightarrow$ Due to change in direction of velocity.

$\vec{a_T} \rightarrow$ Due to change in speed of velocity.

Frictional Force :-

When an object is moving either on a surface or viscous medium, then due to roughness of surface, there is a resistance in motion, this resistive force is called "frictional force."

•- Friction → A Force that opposes the relative motion between "system in contact." ← necessary condition

Types:-

i) Static Friction:- Friction b/w two systems in contact and stationary relative to each other.

$\mu_s \uparrow$ also humidity $\uparrow$ $f_s = \mu_s N$

$\mu_s \rightarrow$ Co-efficient of static-friction. (Mostly range is $0 \le \mu_s \le 1$ and depends on the nature of surfaces & their roughness).

ii) Limiting Friction:- Maximum static friction when body initiates to move. $f_L = \mu_L N$.

iii) Kinetic Friction:- Friction b/w two system in contact and moving relative to one another.

$$f_K = \mu_K N$$

• $\mu \rightarrow$ Co-efficient of friction is dimensionless & has no unit.

Relations:-

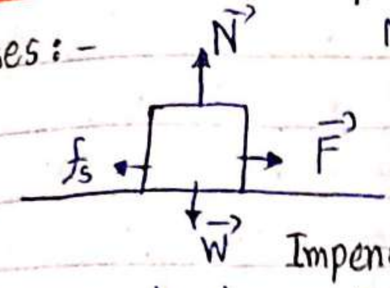
$\mu \downarrow$ $f \downarrow$ Force of motion $\uparrow$

$$\mu_L > \mu_s > \mu_K \rightarrow F f_L > f_s > f_K$$

At Rest
$N = W$
$f_K = 0$

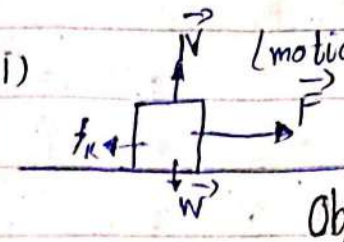
Cases :-

i)



Impending Motion

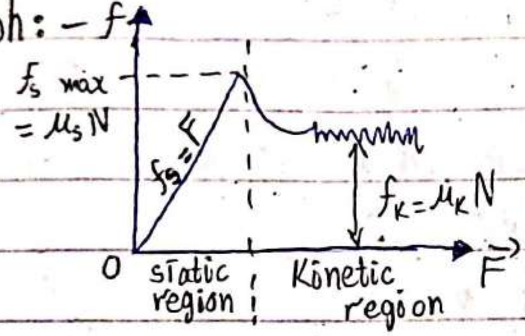
ii)



Object moves

when an object is about to slip from surface, then state of object is impending motion. When static friction reaches its maximum limit, impending motion occurs.

Graph :-



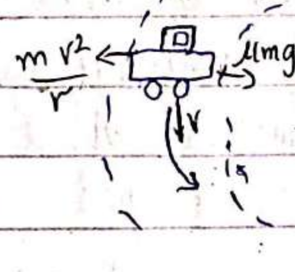
Q A 150 kg car is moving on a flat horizontal nigotiates a curve. If the radius of floor is 35 m & co-efficient of friction b/w tyre and dry pavement (road) is 0.5? Find the max speed a car can have and make a successful turn?

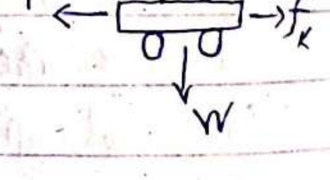
Data :-

$m = 150\ kg$

$r = 35\ m$

$\mu = 0.5$





Solution :-

According to equilibrium condition :-

$\Sigma F_x = f - F = 0$

$f_K = F$ — (i)

ΣF_y :- $N - W = 0$

$N = W = mg$

$N = (150)(9.8) = 1470\ N$

As we know that:-

$$f_k = \mu_k N$$
$$f_k = (0.5)(1470) = 735 \text{ N}$$

Now eq (i) becomes:-

$$f_k = \frac{mv^2}{r} \qquad \text{(body is in circular motion)}$$

$$v = \sqrt{\frac{f_k r}{m}} = \sqrt{\frac{(735)(35)}{150}}$$

$$v = 13.09 \text{ m/s}$$

Q Same question, but car travels on a wet curve at 8 m/s. Find co-efficient of friction?

$$f_k = F$$
$$\mu_k mg = \frac{mv^2}{r} \Rightarrow \mu_k = \frac{v^2}{rg} = \frac{(8)^2}{35 \times 9.8}$$

$$\mu_k = 0.186 \approx 0.19$$

Q A woman at an airport is pulling her 20 kg suitcase at constant speed by an angle above horizontal, with a force of 35 N & force of friction of surface is 20 N.

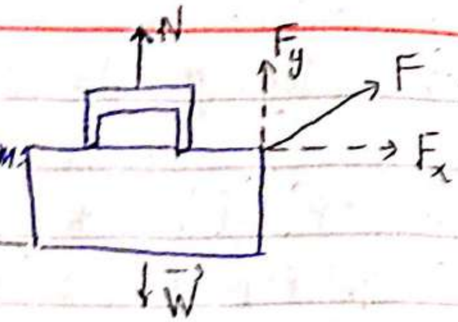
a) Draw a free-body diagram
b) Angle strap makes with horizontal?
c) What normal force ground exerts on suitcase?

Data:-

$m = 20$ kg
$F = 35$ N
$f_k = 20$ N
b) $\theta = ?$ c) $N = ?$

Solution:-

Suppose suit-case is in equilibrium



$\Sigma F_x: \quad F_x - f_k = 0$

$f_k = F\cos\theta$

$\theta = \cos^{-1}\left(\frac{20}{35}\right) = 55.2^\circ$

And,

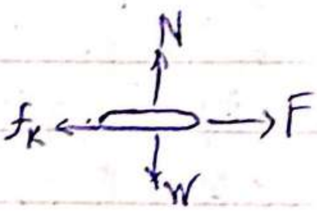
$\Sigma F_y: \quad N + F_y - W = 0$

$N = mg - F\sin\theta = (20)(9.8) - 35\sin(55.2)$

$N = 167.3\ N$

Q. A hockey puck on a frozen pond is moving with a initial speed of 20 m/s. If the puck is to always remain on ice and slides 115 m before turning to rest. Determine co-efficient of friction b/w ice & puck?

Data:-



$v_i = \cancel{11}\ 20\ m/s, \quad v_f = 0\ m/s$

$S = 115\ m$

$\mu_k = ?$

Solution:-

A/c 3^{rd} eq of motion:-

$a = \frac{v_f^2 - v_i^2}{2S} = \frac{(0)^2 - (20)^2}{2(115)} = -1.739\ m/s^2$

$\Sigma F_x: \quad F - f_k = 0$

$ma = \mu_k N$ ---- $\boxed{\text{At Rest } N = W = mg}$

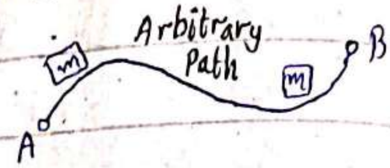
$ma = \mu_k mg$

$\mu_k = \frac{a}{g} = \frac{-1.739}{9.8} = -0.177$

Work, Energy Theorem:-

This law states that:-

" The total work done by the sum of all forces acting on a particle equals to the change in kinetic energy of particles. "



$$\text{Work Done} = \Delta K.E$$

Mathematically:-

$$W = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

Derivation:-

By using line integral,

For Non-Constant Force:-

$$\because \quad W = \int_A^B F_{net} \cdot dx \quad -(i)$$

For F:-

$$F = m a$$

$$F = m \frac{dv}{dt}$$

Applying chain rule:-

$$\Rightarrow F = m \frac{dv}{dt} \times \frac{dx}{dx}$$

$$\Rightarrow F = m \frac{dv}{dx} \times \frac{dx}{dt} \quad \boxed{\frac{dx}{dt} = v}$$

$$\Rightarrow F = m v \frac{dv}{dx} \quad -(ii)$$

Substituting eq (ii) in (i) :-

$$W = \int_A^B m\, v \frac{dv}{dx} \cdot dx$$

$$W = \int_A^B m\, v\, dv$$

$$\Rightarrow W = \left| \frac{m v^2}{2} \right|_A^B \quad \text{OR} \quad W = m \left| \frac{v^2}{2} \right|_i^f$$

Applying limits:-

$$W = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 \quad \text{--- Proved}$$

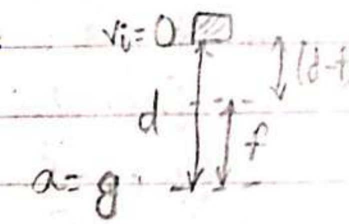
$$W_{net} = \Delta KE$$

Case Study:-

Q A block of mass 'm' falls from a plaza of height 'd'.

i) Calculate the velocity of block at distance 'f' above the ground, and

ii) When it hits the ground? $\rightarrow v = \sqrt{2gd}$



Data:-

$m = m$

$h_i = d$, $h_f = f$

$v_i = 0$

$v_f = ?$, $v_{final} = ?$

Sol:-

A/c to law of conservation of energy:-

Change in KE = Change in PE

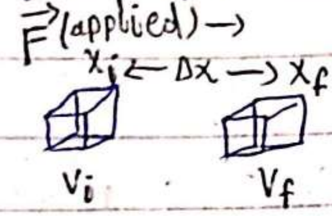
$$\frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = mgd - mgf$$

$$\frac{1}{2} m v_f^2 - 0 = mg(d-f)$$

$$v_f = +\sqrt{2g(h-f)}$$

For Constant Forces:-
($\vec{F}$ that accelerates a body)

Let a body of mass 'm' undergoes a constant force and cover's a distance from x_i to x_f.



$$W_{net} = F_{net} \cdot \Delta x$$
$$W_{net} = ma\,(x_f - x_i) \quad \text{--- (i)}$$

From eq of motion:-

$$\because \; v_f^2 - v_i^2 = 2aS(x_f - x_i) \quad \text{--- (ii)}$$

$$a = \frac{(v_f^2 - v_i^2)}{2S(x_f - x_i)} \text{, put in eq (i)}$$

$$\text{i)} \Rightarrow \; W_{net} = \frac{m(v_f^2 - v_i^2)}{2(x_f - x_i)}(x_f - x_i)$$

$$W_{net} = \frac{mv_f^2}{2} - \frac{mv_i^2}{2}$$

$$W_{net} = KE_f - KE_i$$

$$\boxed{W_{net} = \Delta K.E} \text{ ---- Proved}$$

Q. Calculate the work done of system undergoing force,

$$\vec{F} = (3x^2+6y)\hat{i} + (14yz)\hat{j} + (20xz^2)\hat{k}$$

along path, $x = t$, $y = t^2$, $z = t^3$ from $t = 0$ to 1sec

$$W = \int F \cdot dr \quad -(i)$$

$\because r = x\hat{i} + y\hat{j} + z\hat{k}$ so $dr = dx\hat{i} + dy\hat{j} + dz\hat{k} \quad -(ii)$

$x = t$, $y = t^2$, $z = t^3$

Diff w.r.t t $\Rightarrow \frac{dx}{dt} = 1$, $\xRightarrow{\text{Diff}} \frac{dy}{dt} = 2t$, $\xRightarrow{\text{Diff}} \frac{dz}{dt} = 3t^2$

$dx = dt$, $dy = 2t \cdot dt$, $dz = 3t^2 \cdot dt$

Put these in eq (ii)

$$dr = dt\,\hat{i} + (2t \cdot dt)\hat{j} + (3t^2 \cdot dt)\hat{k}$$

Putting the above in eq (i)

Put x, y & z in $\vec{F}$

$$\vec{F} = [3(t)^2 + 6t^2]\hat{i} + [14(t^2)(t^3)]\hat{j} + [20(t)(t^3)^2]\hat{k}$$

$$\vec{F} = 9t^2\hat{i} + 14t^5\hat{j} + 20t^7\hat{k}$$

~~Now eq (i) becomes~~

$$\cancel{W = \int_0^1 9t^2(dt)}$$

Since Work is dot product so;

$$W = \vec{F} \cdot \vec{dr}$$

$F \cdot dr$

$$W = (9t^2\hat{i} + 14t^5\hat{j} + 20t^7\hat{k}) \cdot (dt\,\hat{i} + 2t \cdot dt\,\hat{j} + 3t^2 \cdot dt\,\hat{k})$$

F.dr

$$W = 9t^2 \cdot dt\,(\hat{i}\cdot\hat{i}) + 14t^5\cdot(2t\,dt)(\hat{j}\cdot\hat{j}) + 20t^7\cdot(3t^2\cdot dt)(\hat{k}\cdot\hat{k})$$

F.dr

$$W = 9t^2\cdot dt + \cancel{14t^5}\,28t^6\,dt + 60t^9\,dt$$

Now eq(i) becomes:-

$$W = \int_0^1 9t^2\cdot dt + 28t^6\,dt + 60\,t^9\,tt$$

$$W = 9\int_0^1 \cancel{9}t^2\,dt + 28\int_0^1 \cancel{28}t^6\,dt + 60\int_0^1 t^9\,dt$$

$$W = 9\left|\frac{t^3}{3}\right|_0^1 + 28\left|\frac{t^7}{7}\right|_0^1 + 60\left|\frac{t^{10}}{10}\right|_0^1$$

Applying limits.

$$W = 9\left[\frac{1^2}{3} - \frac{0^2}{2}\right] + 28\left[\frac{1^7}{7} - \frac{0^7}{7}\right] + 60\left[\frac{1^{10}}{10} - \frac{0^{10}}{10}\right]$$

$$W = \frac{9}{3} + \frac{28}{7} + \frac{60}{10}$$

$$W = \cancel{4.5}\,3 + 4 + 6$$

$$W = 13\ \cancel{14.5}\ \text{Joules}$$

Law Of Conservation Of Mechanical Energy:-

condition — In an isolated system in which only conservative forces act, the total mechanical energy of the system remains conserved.

$$KE_i + PE_i = KE_f + PE_f$$

$$\Delta KE = -\Delta PE$$

$$\Delta KE + \Delta P.E = 0$$

Examples are:-

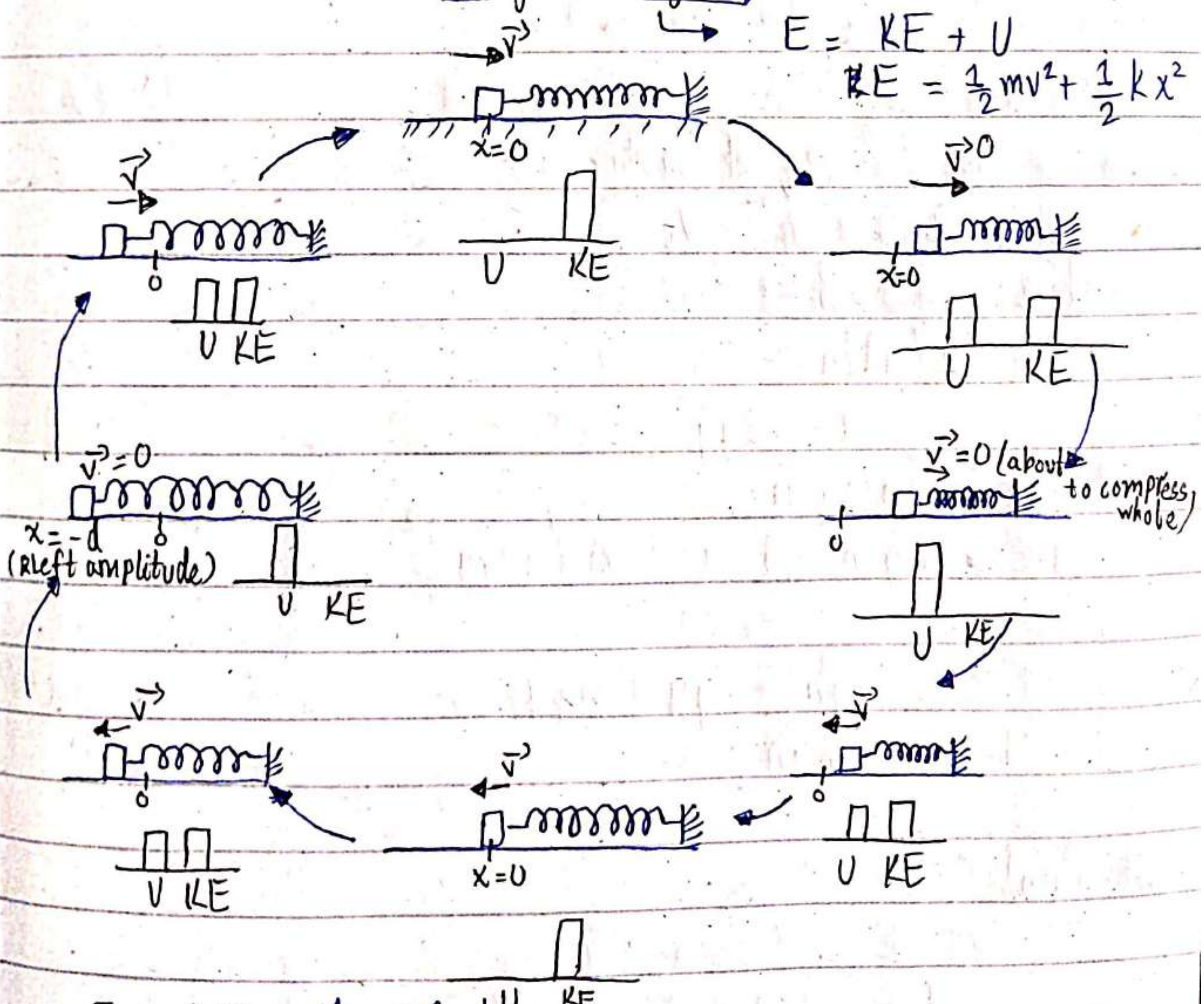
i) Spring Mass System

ii) Gravitational field

Block Attatched To Spring Mass System:-

$E = KE + U$

$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$

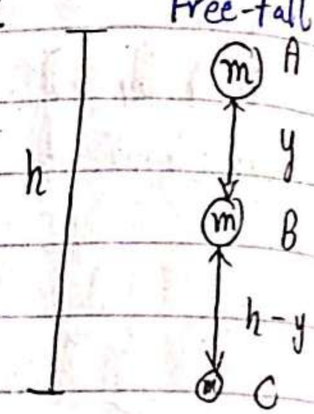


For Spring Mass System;

KE → max (At mean-position) ← min ← PE

min (At extreme position OR Amplitude) ← max

Q Prove that total energy in system remains constant as long as applied force is conservative?



At Point A:-

$$T.E = K.E + P.E$$
$$T.E = mgh + 0$$
$$TE = mgh$$

At Point B:-

$$T.E = P.E + K.E$$
$$T.E = mg(h-y) + \frac{1}{2}mv^2 \quad \text{— (i)}$$

↳ Left height. [h-y]

For v:-

Using 3^{rd} eq of motion:-

$$\because 2aS = v_f^2 - v_i^2$$

Considering $A \rightarrow B$

$$2(g)(y) = v^2 - 0$$
$$v = \sqrt{2gy}$$

Putting 'v' in eq (i)

$$T.E = mg(h-y) + \frac{1}{2}m\left(\sqrt{2gy}\right)^2$$

$$T.E = mgh - mgy + mgy$$
$$T.E = mgh$$

At Point C:-

$$T.E = K.E + P.E$$
$$T.E = \frac{1}{2}mv_f^2 + 0 \quad (v_f = \text{velocity at } C)$$

For v_f :-

Using 3^{rd} eq of motion:-

$\because 2aS = v_f^2 - v_i^2$

Considering $A \to C$:-

$$\Rightarrow 2(g)(h) = v_f^2 - 0$$

$$v_f = \sqrt{2gh}$$

Putting 'v_f' in eq (ii)

$$T.E = \frac{1}{2} m (\sqrt{2gh})^2 = mgh$$

Q Suppose that a neutron travels a distance of 6.2m in a time $t = 160 \times 10^{-6}$ sec. What is its KE in e.V?

Data:-

$S = 6.2\ m$

$t = 160 \times 10^{-6}$ sec

Mass of neutron: $m_{ne} = 1.67 \times 10^{-27}$ kg

K.E (in eV) = ?

Solution:-

$$\because K.E = \frac{1}{2} m v^2$$

$$K.E = \frac{1}{2} m \left(\frac{S}{t}\right)^2 \quad \text{---} \quad v = \frac{S}{t}$$

$$K.E = \frac{1}{2}(1.67 \times 10^{-27})\left(\frac{6.2}{160 \times 10^{-6}}\right)^2$$

$$K.E = 1.25 \times 10^{-19}\ J$$

$$K.E = \frac{1.25 \times 10^{-19}}{1.6 \times 10^{-19}}\ eV \quad \text{---} \quad 1e.V = 1.6 \times 10^{-19}\ J$$

$$K.E = 7.836\ eV$$

Angular Momentum

Linear Motion	Rotational Motion
v	ω
a	α
m	Inertia (I)
F	τ
p	L

$\tau = \vec{r} \times \vec{F}$ OR $rF\sin\theta$

Moment of Force

↳ moment arm

$L = \vec{r} \times \vec{p}$ OR $rp\sin\theta$ --- $p = mv$

For a ~~vr~~ circular motion $\theta = 90^\circ$, so;

$\Rightarrow \boxed{L = mvr}$

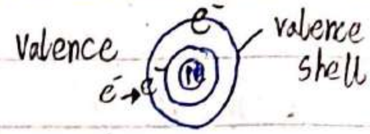
↳ Unit :- J. sec

Chapter 04

Semiconductor Physics

Valence Shell:-

It is the outermost shell of an atom containing weakly bounded e^-. Force of attraction b/w positively charged nucleus & -ve e^- decreases as shell size increases



Energy Band Theory Of Solids:-

There are three bands in solid.

i) Valence Band:-

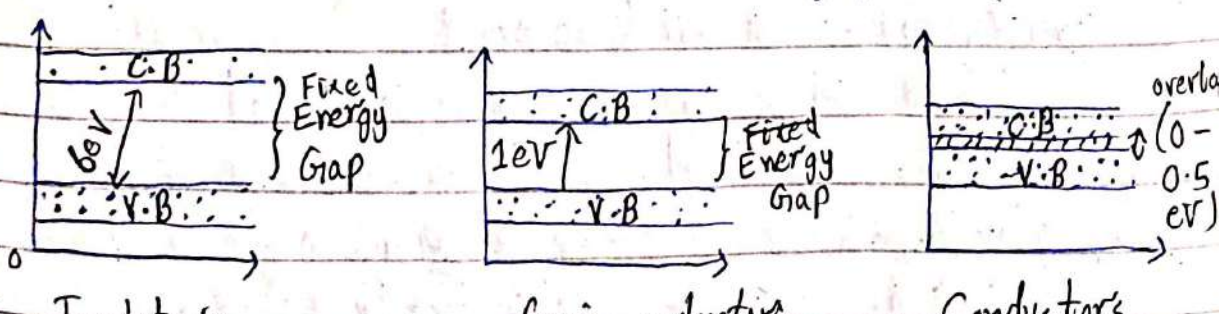
The energy band which is formed by grouping the range of energy levels of the valence electrons or outermost orbit electrons is called as valence band.

ii) Conduction Bond:-

The energy band which is formed by grouping the range of energy levels of free electrons is called as conduction band.

iii) Forbidden Band:-

Energy gap present between the valence and conduction band by separating these two energy bands is called as forbidden band or forbidden gap.



Insulator's Semi-conductors Conductors

=> Free electrons exist in conduction band also called conduction electrons.

Fermi - Energy :- The highest energy level electron can occupy at zero Kelvin

$$E_{Fermi} = \frac{E_{Gap}}{2}$$

i) Insulators :-

Valence band → fully occupied with electrons due to sharing of outer-most orbit e^- to neighbouring atoms

Conductiond band → Empty (no ~~valence~~ free e^-)

Forbidden Gap → Very large in Insulators

⇒ The e^- in valence band can't move as they are locked-up b/w atoms. In order to move, valence e^- to conduction band large amount of external energy (approx equal to forbidden gap) which is practically impossible.

ii) Semi - Conductors :-

Materials which have electrical conductivity b/w that of a conductor and an insulator are semi-conductors

Valence band :-

↳ At low temperature :- valence band completely occupied while conduction band is empty as the electrons does not have enough energy to move into conduction band. Therefore, semi-conductors behave as an insulator at low temperature

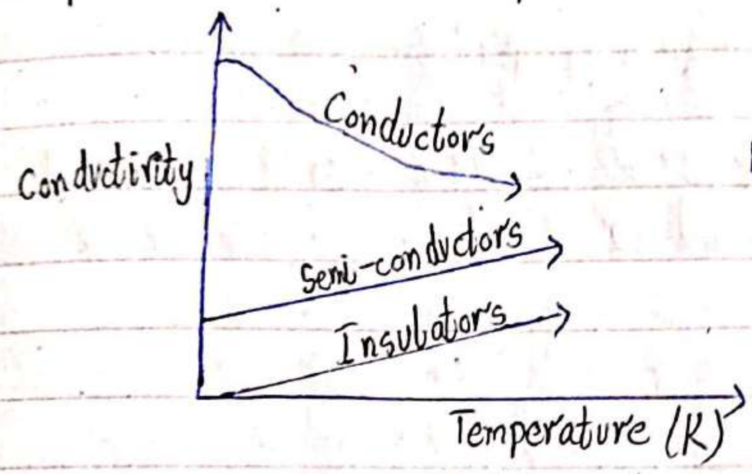
↳ At Room Temperature :- Some e^- gain enough energy in the form of heat to move into conduction band.

↳ Increase In Room Temperature :- Number of valence band e^- moving into conduction band increases with temperature.

This shows that electrical conductivity of semi-conductor's increases with increase in temperature.

Forbidden Gap -> Is very small in semi-conductor's.

Graph Of Affect Of Temperature :-



R↑ K↓ Temp↑
← Conductor's

R↓ K↑ Temp↑
← Semi-conductors

•- At room temp, semi-conductor's mostly behave like insulators

iii) Conductors :-

Forbidden Gap -> In conductor's valence & conduction bands overlap each other.

· At Room Temperature -> conduction band is almost full of e^- whereas valence band is partially occupied

=> When valence band e^- move to conduction band, the become free electrons.

Hole :-

When an electron jumps to conduction band, a vacancy is left in the valence band within the crystal. This vacancy is called hole.

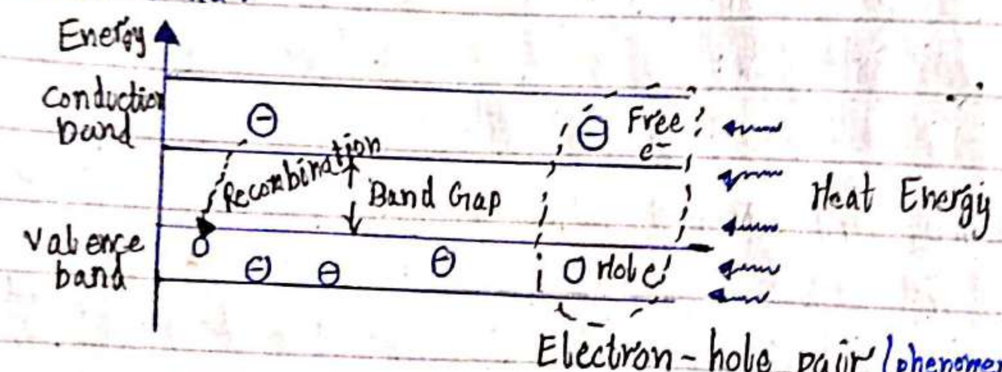
→ Simply called as hole

Electron-hole Pair :-

For every electron raised to the conduction band by external energy, a hole is left in the valence band, ~~erea~~ this process is called electron-hole pair.

Recombination :- → characteristic property of a conductor

This process occurs when a conduction-band electron loses energy and falls back into a hole in the valence band.



Electron-Current :- -ve to +ve terminal
(i)

When a voltage is applied ~~to~~ across a piece of intrinsic silicon, thermally generated free electrons moving randomly in crystal structure are attracted towards positive terminal. This movement of free electrons is called electron-current.

Hole-current :- +ve to -ve terminal
(I)

The current which flows in a circuit due to motion of holes ~~from~~ towards negative end is called hole current (conventional current).

Types Of Semi-Conductors :-
- Intrinsic → Pure (Si, Ge IV-A)
- Extrinsic → Impure
P-type N-type

⇒ Hole & Electron are charge carriers

⇒ As semi-conductors do not conduct current well because of limited free e^- so intrinsic semi-conductor is modified by increasing number of free e^- or holes through impurity to increase its conductivity.

Doping:-

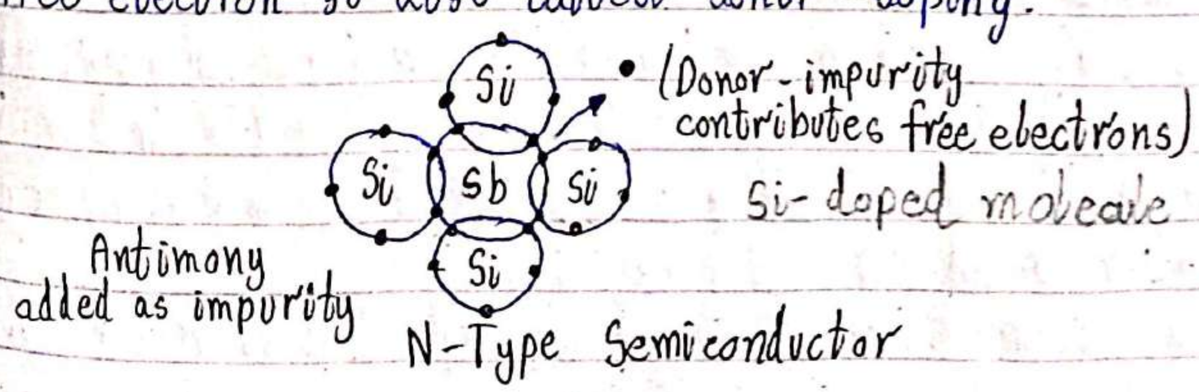
Semi-conductor's conductivity can be increased by the controlled addition of impurity to the intrinsic (pure) semi-conductive materials by a process called doping.

⇒ Doping increases the number of charge carriers (electrons or holes).

Types :-
- N-Type (Pentavalent doping)
  ↳ P, Sb, As
- P-Type (Trivalent doping)
  ↳ B, Al, In (Indium)

i) Pentavalent Doping :-

Doping occurred when Si or Ge are doped with element of group-VA. It has an excess free electron so also called donor-doping.

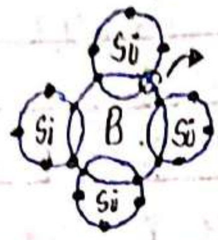


N-Type Semiconductor

ii) Trivalent Doping:-

Doping occured when Si or Ge are doped with a element of group III-A. It lacks a free e^- so known as acceptor doping.

Boron added as an impurity



Acceptor impurity creates a hole

Ratio Of Doping:-

If one has 10 lakh tetravalent atoms, then ~~you~~ one has to add 1 pentavalent or trivalent impurity to achieve high conductivity.

Ratio => $1 : 10^6$

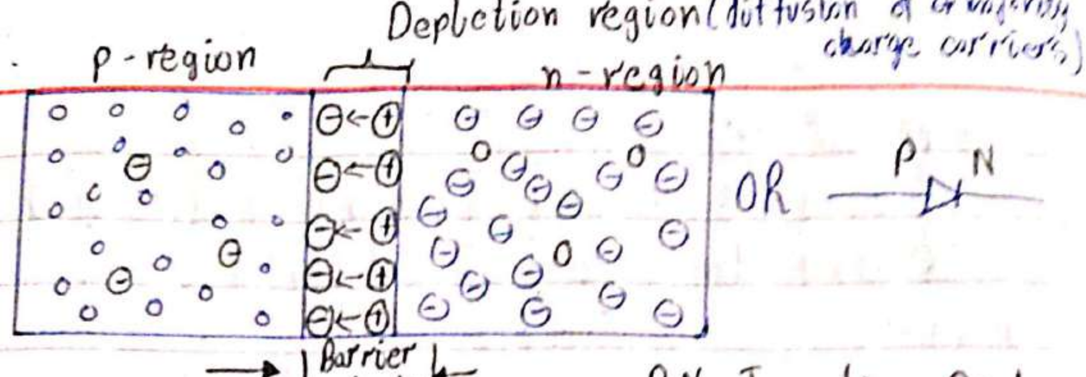
P.N Junction Diode :-

If n-type and p-type semiconductors are brought together, a pn-junction is formed b/w two regions * and a diode is ~~formed~~ created. The p-region has holes as (majority carriers) while n-region has electrons as (majority carriers).

P.N Junction diode is a two-terminal devices and is the basis for diodes, transistors, solar cells, as a switch due to its one-way conduction and other dev

* where free e^- in n-region begin to diffuse across junction and fall into holes of p-region.

↳ (Diffusion continued till barrier potential is reached).



PN-Junction Diode

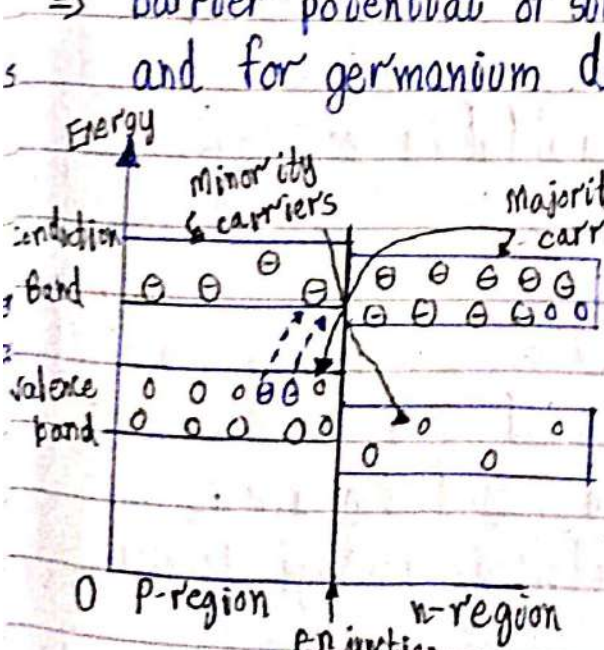
Depletion Region:-

(electrostatic charges are present) It is a region in PN-Junction diode where no mobile charge carriers are present. It acts like a barrier that opposes the flow of further electrons (free) from n-side and holes from p-side.

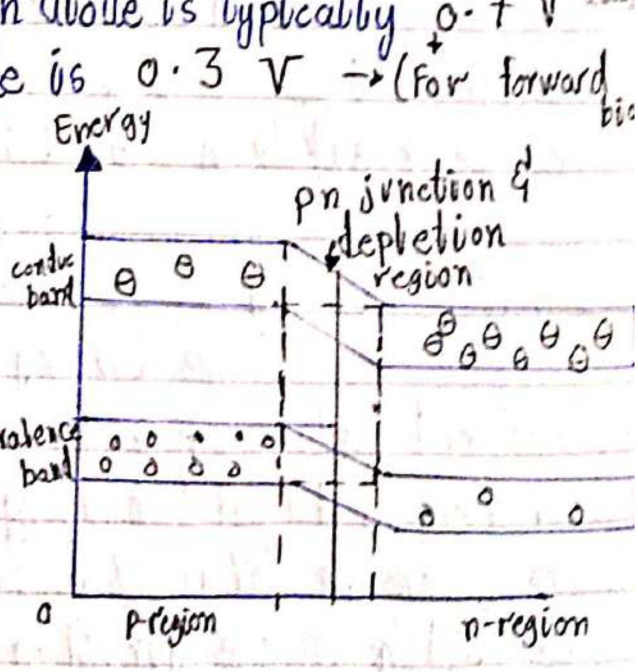
Barrier Potential:-

The potential difference of the electric field across the depletion region is the amount of voltage required to move the electrons or holes through electric field. This potential difference is called barrier potential and is expressed in volts. (to produce current in PN-Junction diode)

=> Barrier potential of silicon diode is typically 0.7 V and for germanium diode is 0.3 V → (For forward bias)



At The Instant Of Junction Formation



At Equilibrium

Biasing:- →OR supply of external dc voltage

The application of electric potential across semi-conductor diode is called biasing.

Types:-

i) Forward Biasing ii) Reverse Biasing

=) There is only current when forward-biased while no current (only leakage one due to thermally generated) minority carriers so neglected.

•- Avalanche occurs in reverse-biased diode if bias voltage equal or exceeds the breakdown voltage.

•- A diode conducts current when forward biased and blocks current when reversed-biased.

•- Reverse breakdown voltage of a diode is typically greater than 50 V.

•- Resistance of forward biased diode is called dynamic or a/c resistance.

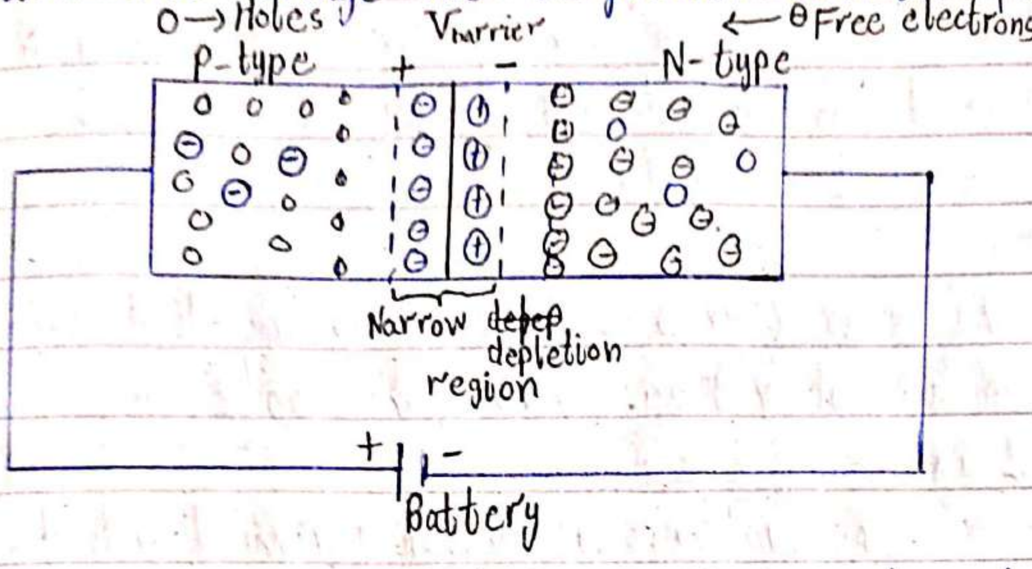
•- Reverse current increases rapidly at reverse breakdown voltage.

•- Reverse breakdown should be avoided in most diodes.

Forward Biasing:-

The biasing in which p-region is connected to +ve end and n-region is connected to -ve terminal of the battery is forward biasing. This condition allows current through PN junction. External voltage decreases the width of the junction.

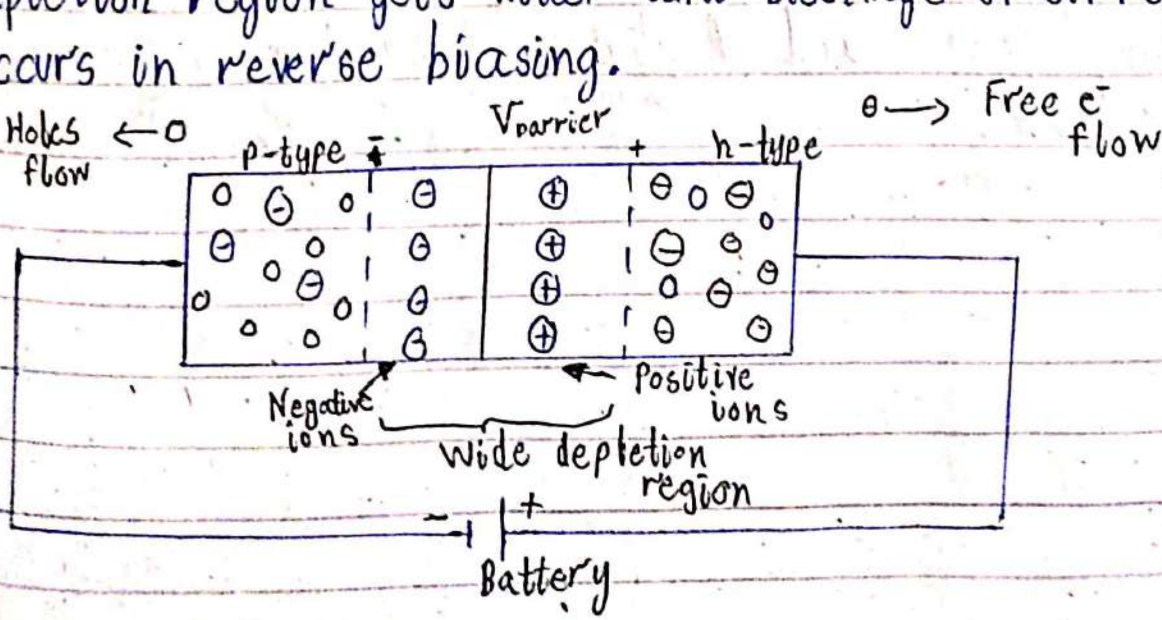
⇒ (Applied) Bias voltage must be greater than the barrier potential.



- Forward bias narrows the depletion region and produces a voltage drop across junction equal to the barrier potential.

Reverse Biasing:-

The biasing in which p-region is connected to -ve terminal and ~~p~~n-region is connected to positive termal of the battery is reverse biasing. External voltage increases the built-in potential barrier as a result depletion region gets wider and blockage of current occurs in reverse biasing.



Graphs are important.

Breakdown - Voltage :- → Usually greater than 50 V

The minimum reverse voltage that makes the diodes conduct apprecabily in reverse is called breakdown voltage.

- The V-I characteristic curve shows the diode current as a function of voltage across the diode.

Knee Voltage :-

The voltage at which forward diode current begins increasing rapidly is known an knee-voltage or cut-in-voltage.

- Knee-Voltage of Ge = 0.3 V
  " " of Si = 0.7 V
- Leakage Current of Ge = $< \overset{50}{\cancel{20}}\,\mu A$
  " " of Si = $< 20\,\mu A$

=> Zener diode is used as reverse diode.
Both 'Zener' and 'Avalanche' breakdown occur in Zener diode.
↳ High reverse voltage at which rapid increase in net current occurs.

Transistors:-

A transistor is a semi-conductor device that regulates current between two terminals, based on the current or voltage at third terminal. Transistor is used for amplification or ~~switching~~ as a switch in electric circuits.

There are two types of transistors:-

i) Bi-Polar Junction Transistor
   - PNP
   - NPN

ii) Field Effect Transistor
   - JFET → Junction Field Effect Transistor
   - MOSFET → Metal Oxide Semiconductor FET

•- Field Effect Transistor:-

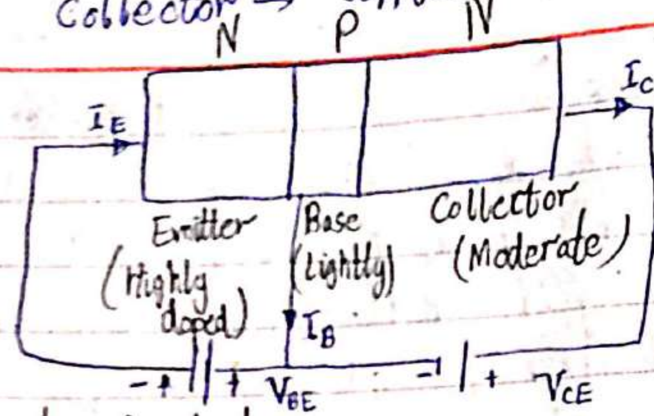
(uni-polar transistor) as involve single carrier-type operation. (one junction)

It is a type of transistor that uses an electric field to control flow of current in a semi-conductor. FET's have three terminals:- source, gate and drain. FET controls the flow of current by the application of voltage to gate, which in turn alter's conductivity between the drain and source.

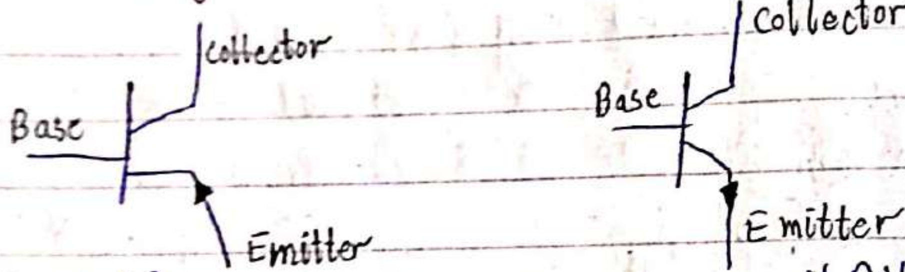
•- Bi-Polar Transistor:-

It is a type of transistor that uses both charge carriers. It allows a small current injected at one of its terminals to control a much larger current flowing between terminals, making device capable of amplification or switching.

Emitter → collect...
Base → use to active transistor
Collector → supplies carriers



Schematic Symbol:-



PNP NPN

(Arrow shows the flow of conventional current.)

I_E → current supply by terminal emittor.
I_B → current from base
I_C → current entering transistor from its collector.

Equations:-

$$V_{BB} = I_B R_B + V_{BE}$$

$$V_{CC} = I_C R_C + V_{CE}$$

value from 20–300

$$I_E = I_B + I_C$$

Current gain →

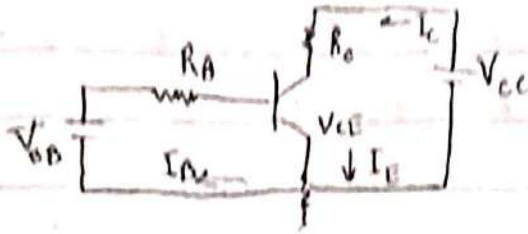
$$\beta = \frac{I_C}{I_B} = \frac{\alpha}{1-\alpha}$$ Don't include the grounded terminal.

Normally from 0.950 – 0.995

$$\alpha = \frac{I_C}{I_E} = \frac{\beta}{1+\beta}$$ Include terminal that is ground

$$V_{CE} = V_{CB} + V_{BE}$$

Q For the given silicon semi-conductor configuration $\beta = 150$ and other values are shown below:-



$R_B = 10\,k\Omega$, $R_c = 100\,\Omega$, $V_{BB} = 5V$, $V_{cc} = 10\,V$

Find:-

V_{BE} , I_B , I_c , I_E , V_{cE} and $V_{cB} = ?$

Solution:-

As the semi conductor is silicon, so:-

$$V_{BE} = 0.7\,V$$

For I_B:-

Using Mesh analysis:-

$$\because V_{BB} = I_B R_B + V_{BE}$$
$$5 = I_B (10 \times 10^3) + 0.7$$
$$I_B = 430\,\mu A$$

For I_c:-

$$\beta = \frac{I_c}{I_B} \Rightarrow I_c = 150 \times 430 \times 10^{-6}$$
$$I_c = 64.5\,mA$$

For I_E:-

$$I_E = I_c + I_B = (64.5 \times 10^{-3}) + (430 \times 10^{-6})$$
$$I_E = 64.9\,mA$$

For V_{CE}:-

$$\because V_{CC} = I_C R_C + V_{CE}$$
$$10 = (64.3 \times 10^{-3})(100) + V_{CE}$$
$$V_{CE} = 3.57\ V$$

For V_{CB}:-

$$\because V_{CE} = V_{CB} + V_{BE}$$
$$V_{CB} = 3.57 - 0.7$$
$$V_{CB} = 2.87\ V$$

Q For a given transistor, value of β is 45 and voltage drop across a collector circuit of $1\,k\Omega$ is 1V. Find out the base current in transistor?

Data:-

$R_C = 1\ k\Omega$

$\beta = 45$

$V_{CC} = 1V$

$I_B = ?$

Solution:-

For I_C:-

Using Ohm's law:-

$$V_{CC} = I_C R_C$$

$$\Rightarrow I_C = \frac{1}{1 \times 10^3} = 1\,mA$$

$$\because \beta = \frac{I_C}{I_B}$$

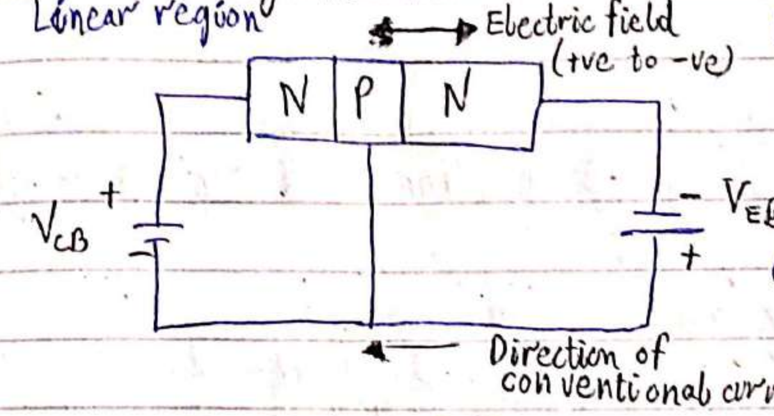
$$I_B = \frac{1 \times 10^{-3}}{45}$$

$$\Rightarrow I_B = 22.2\,\mu A$$

Transistor's Regions Of Operations:-

- → Active
- → False OR Reverse active
- → Cut-off
- → Saturation

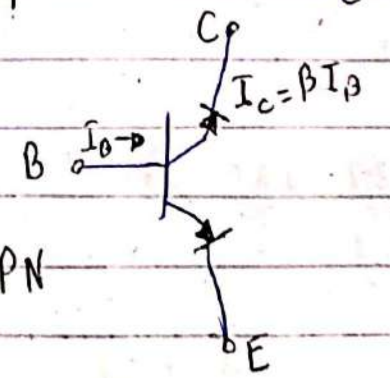
i) Active Region (Mode):-
Linear region



Emitter-Base Junction
↳ Forward Biased

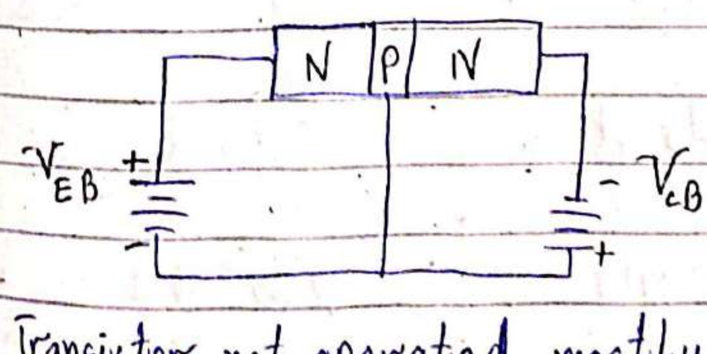
Collector-Base Junction
↳ Reverse Biased

~~Applications:~~ $I_C = \beta I_\beta$



Applications:- Many applications. Better as amplifier

ii) Reverse Active Region:-



Emitter-Base Junction
↳ Reverse Biased

CB Junction
↳ Forward Biased

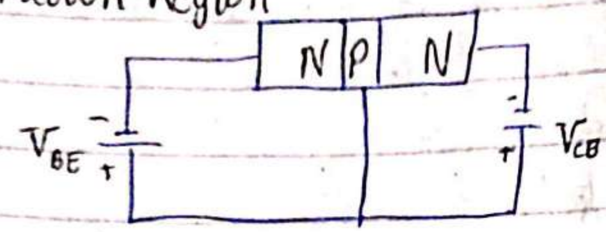
Transistor not operated mostly in this mode as gain is negligible.

For Conventional Current Direction Flow:
- NPN → from collector to emitter
- PNP → from emitter to collector

Applications:-

Rectification / Attenuation (loss of power)

iii) Saturation Region



Both junctions are forward biased.

I_C

$I_B = 0$

So $V_B \approx \infty$

I_{E}

Both $\neq 0$

$I_C = I_B$

Application:- As an open switch

iv) Cut-off Region:-

C

$I_C = 0$

B

$I_B = 0$

So $V_B \approx 0$ as all are 0

$I_E = 0$

E

Both junctions are reverse biased

$I_C = I_B = I_E = 0$

Application:- As a closed switch.

Voltage relationships	NPN Module	PNP Module
$V_E < V_B < V_C$	Active	Reverse
$V_E < V_B > V_C$	Saturation	Cutoff
$V_E > V_B < V_C$	Cutoff	Saturation
$V_E > V_B > V_C$	Reverse	Active

Assignment - 01 (For Mids Preparation)

1 Find if $A = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y\hat{k})$ is irrotational?

$$\vec{\nabla} \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (6xy+z^3) & (3x^2-z) & (3xz^2-y) \end{vmatrix}$$

Don't forget unit vectors

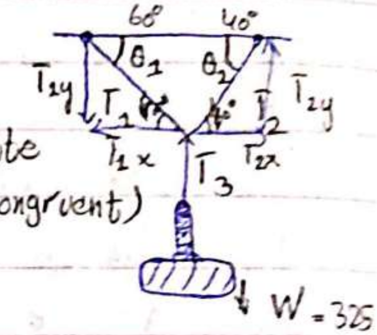
$$\vec{\nabla} \times \vec{A} = \left[\frac{\partial}{\partial y}(3xz^2-y) - \frac{\partial}{\partial z}(3x^2-z)\right]\hat{i} - \left[\frac{\partial}{\partial x}(3xz^2-y) - \frac{\partial}{\partial z}(6xy+z^3)\right]\hat{j} + \left[\frac{\partial}{\partial x}(3x^2-z) - \frac{\partial}{\partial y}(6xy+z^3)\right]\hat{k}$$

$$= [(0-1)-(0-1)]\hat{i} - [(3z^2-0)-(0+3z^2)]\hat{j} - [(6x-0)-(6x+0)]\hat{k}$$

$$\vec{\nabla} \times \vec{A} = 0\hat{i} - 0\hat{j} + 0\hat{k}$$

Hence the field is irrotational.

Q3 A bag of cement weighing 325 N (W) hangs from three wires. Two of which make an angle 60° or 40° with the horizontal respectively. If the system is in equilibrium, calculate tension in wires?



opposite angle (congruent)

As the bag is hanging still so,

$$\cdot \ \Sigma F_y = 0$$

$$T_3 - W = 0$$

$$T_3 = W = 325 \text{ N}$$

Now for whole system:- (Supporting weight)

Along x-axis:-

$$\Sigma F_x : \ T_{2x} - T_{1x} = 0$$

$$T_2 \cos\theta_2 - T_1 \cos\theta_1 = 0$$

$$T_2 = \frac{T_1 \cos 60^\circ}{\cos \cancel{60}\, 40^\circ} \quad \text{— (i)}$$

Along y-axis:-

$$\Sigma F_y : - \ T_{2y} + T_{1y} - W = 0$$

$$T_2 \sin\theta_2 + T_1 \sin\theta_1 - 325 = 0$$

$$\left[\frac{T_1 \cos 60^\circ}{\cos 40^\circ} \right] \sin 40^\circ + T_1 \sin 60^\circ - 325 = 0$$

$$T_1 \left[\frac{\cos 60^\circ \sin 40^\circ}{\cos 40^\circ} \sin 60^\circ \right] = 325$$

$$T_1 = \frac{325}{\cancel{1.5187}\ 1.28558} = 252.81 \text{ N}$$

Putting in eq (i)

$$T_2 = 252.81 \frac{\cos 60^\circ}{\cos 40^\circ}$$

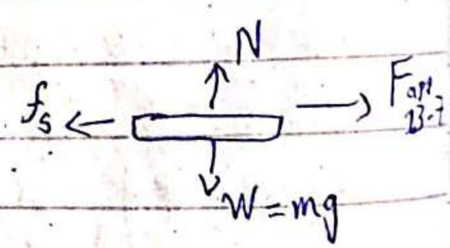
$$T_2 = 165 \text{ N}$$

Q4 A 5.0 kg iron rod is on a level surface where $\mu_s =$ 0.40 and $\mu_k = 0.30$. A 11.35N force is being applied to the rod parallel to surface, if it will remain at rest or begin to slide, when originally:-

a) moving, and the 11.35 N applied force is in the direction of motion,

b) at rest.

Data:-



$m = 5 \text{ kg}$

$\mu_s = 0.4$, $\mu_k = 0.3$

$F_{applied} = 11.35 \text{ N}$

$\theta = 0^\circ$

i) Moving = ? , ii) At Rest = ?

Sol:-

ii) Since force is applied horizontally $\theta = 0^\circ$ so,

$$N = W = mg$$

$$N = (5)(9.8) = 49 \text{ N}$$

Checking limiting friction: ?

$$f_L = \mu_s N = (0.4)(49)$$

If friction & force were in same direction
$a = \frac{F + f_k}{m} = 6.19 \ m/s^2$

$f_L = 19.6 \ N$

So, f_l is more than F_{app} $\therefore f_l > F_{app}$

Hence,

It will remain at rest till applied force is not greater.

$f_k \rightarrow$ Kinetic ~~energy~~ friction of bo

i) For moving body :- $f_k = \mu_k N = (0.30)(49) = 14.7 \ N$ $F < f_k$

If force applied is in direction of motion $f_k \leftarrow \square \rightarrow F$ (greater) (ref force)

so, ~~for deceleration~~ rod will move with const $\vec{a}$:-

$$a = \frac{F_{net}}{m} = \left[\frac{f_k - F}{m}\right] \text{ OR } -\left[\frac{F - f_k}{m}\right]$$

$$a = \cancel{19} \ \frac{F_{app} - f_k}{m} = \frac{11.35 - \cancel{19.6} \ 14.7}{5}$$

minus from F_{app} as it is reference fore

$$a = -\cancel{1.65} \ 0.67 \ m/s^2$$

(Deceleration)

Q5 Prove $\nabla(\vec{F} + \vec{G}) = \nabla \vec{F} + \nabla \vec{G}$

where $\vec{F} = F_x + F_y + F_z$ & $\vec{G} = G_x + G_y + G_z$

Taking L.H.S

$$\nabla(\vec{F} + \vec{G}) = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \cdot (F_x + G_x + F_y + G_y + F_z + G_z)$$

$$\Rightarrow \frac{\partial}{\partial x}(F_x + G_x)\hat{i} + \frac{\partial}{\partial y}(F_y + G_y)\hat{j} + \frac{\partial}{\partial z}(F_z + G_z)\hat{k}$$

$$\Rightarrow \frac{\partial F_x}{\partial x} i + \frac{\partial G_x}{\partial x} i + \frac{\partial F_y}{\partial y} j + \frac{\partial G_y}{\partial y} j + \frac{\partial F_z}{\partial z} k + \frac{\partial G_z}{\partial z} k$$

$$\Rightarrow \left(\frac{\partial}{\partial x} F_x i + \frac{\partial}{\partial y} F_y j + \frac{\partial}{\partial z} F_z k\right) + \left(\frac{\partial}{\partial x} G_x i + \frac{\partial}{\partial y} G_y j + \frac{\partial}{\partial z} G_z k\right)$$

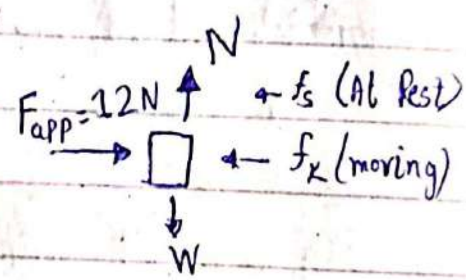
$$\Rightarrow \left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)(F_x+F_y+F_z)+\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)(G_x+G_y+G_z)$$

$$= \nabla F + \nabla G$$

Proved

Q6 On applying 12.0 N force to a 8.00 kg block, if coefficient of static friction b/w block and floor is $\mu_s = 0.700$ and $\mu_k = 0.400$. Will the block begin to slide, or does it remain stationary?

Data:-



$F_{app} = 12\ N$

$\mu_s = 0.7$

$\mu_k = 0.4$

$W = mg = (8)(9.8) = 78.4\ N$

Will block be moving or at rest?

Sol:-

i) Since block is initially at rest, so static friction:-

$$f_s = \mu_s N$$

But according to figure $\Rightarrow N = W$

$$f_s = \mu_s W = (0.7)(78.4)$$

$$f_s = 54.88\ N$$

$\because f_s > F_{app}$

So the block will not begin to slide and will remain at rest.

If body moving initiallly so check with f_K

~~ii) If body is~~ Q2 If $\vec{V} = yz\hat{i} - 3xz^2\hat{j} + 2xyz\hat{k}$ and $\phi(x,y,z) = xyz$. Prove that $(\vec{V} \times \nabla).\phi = \vec{V} \times (\nabla.\phi)$

$$\overset{1^{st}}{\vec{V}} \quad \overset{2^{nd}}{\nabla}$$

Taking L.H.S $= (\vec{V} \times \nabla).\phi$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ yz^2 & -3xz^2 & 2xyz \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{vmatrix} \cdot (xyz)$$

$\leftarrow 1^{st}$ vector

$\leftarrow 2^{nd}$ vector

$$\vec{V} \times \nabla = \left[\frac{\partial}{\partial z}(-3xz^2) - \frac{\partial}{\partial y}(2xyz)\right]\hat{i} - \left[\frac{\partial}{\partial z}(yz^2) - \frac{\partial}{\partial x}(2xyz)\right]\hat{j}$$
$$+ \left[\frac{\partial}{\partial y}(yz^2) - \frac{\partial}{\partial x}(-3xz^2)\right]\hat{k}$$

$$\vec{V} \times \nabla = (-6xz - 2x\cancel{y}z)\hat{i} - (\cancel{y}\, 2yz - 2yz)\hat{j} + (\cancel{2y}z^2 + \overset{3z^2}{\cancel{6xz}})\hat{k}$$

$$\vec{V} \times \nabla = -8xz\,\hat{i} + 0\,\hat{j} \overset{+4z^2}{\cancel{+(2yz + 6xz)}}\hat{k}$$

$$(\vec{V} \times \nabla).\phi = [-8xz\,\hat{i} + \overset{4z^2}{\cancel{(2yz + 6xz)}}\hat{k}].(xyz)$$
$$= -8x^2yz^2\,\hat{i} + \underset{4xyz^3\hat{k}}{\cancel{(2xy^2z^2 + 6x^2yz^2)}}\hat{k}$$

Taking R.H.S

$$\nabla.\phi$$

$$\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right).(xyz)$$

$$\nabla.\phi = yz\hat{i} + xz\hat{j} + xy\hat{k}$$

$$\vec{V} \times (\nabla \cdot \phi) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ yz^2 & -3xz^2 & 2xyz \\ yz & xz & xy \end{vmatrix}$$

$$\vec{V} \times (\nabla \cdot \phi) = \Big[(xy)(-3xz^2) - (2xyz)(xz) \Big] \hat{i} - \Big[(xy)(yz^2) - (2xyz)(yz) \Big] \hat{j} + \Big[(yz^2)(xz) - (-3xz^2)(yz) \Big] \hat{k}$$

$$\vec{V} \times (\nabla \cdot \phi) = (-3x^2yz^2 - 2x^2yz^2)\hat{i} - (xy^2z^2 - 2xy^2z^2)\hat{j} + (xyz^3 + 3xyz^3)\hat{k}$$

$$\vec{V} \times (\nabla \cdot \phi) = -5x^2yz^2\,\hat{i} + xy^2z^2\,\hat{j} + 4xyz^3\,\hat{k}$$

Again doing L.H.S correctly :-

$$(\vec{V} \times \nabla) \cdot \phi = \left[(-3xz^2) \frac{\partial}{\partial z}(xyz) - (2xyz) \frac{\partial (xyz)}{\partial y} \right] \hat{i} - \left[(yz^2) \frac{\partial}{\partial z}(xyz) - (2xyz) \frac{\partial}{\partial x}(xyz) \right] \hat{j} + \left[(yz^2) \frac{\partial}{\partial y}(xyz) - (-3xz^2) \frac{\partial}{\partial x}(xyz) \right] \hat{k}$$

$$(\vec{V} \times \nabla) \cdot \phi = \big[(-3xz^2)(xy) - (2xyz)(xz) \big] \hat{i} - \big[(yz^2)(xy) - (2xyz)(yz) \big] \hat{j} + \big[(yz^2)(xz) - (-3xz^2)(yz) \big] \hat{k}$$

$$(\vec{V} \times \nabla) \cdot \phi = -5x^2yz^2\,\hat{i} + xy^2z^2\,\hat{j} + 4xyz^3\,\hat{k}$$

Hence proved $(\vec{V} \times \nabla) \cdot \phi = \vec{V} \times (\nabla \cdot \phi)$

Mid Imp Questions

Q A 2 kg block of wood in on a level surface where $\mu_s = 0.80$ & $\mu_K = 0.60$. A 13.7 N-force is applied parallel to surface in direction of motion. If the block was originally ~~at rest~~ in motion, analyze its motion?

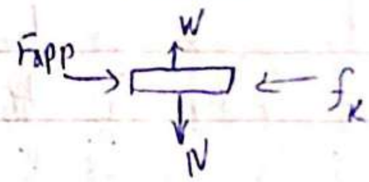
Data:-

$m = 2kg$ $\quad g = 9.8\ m/s^2$

$\mu_K = 0.60$ $\quad \mu_s = 0.80$

$F_{app} = 13.7\ N$ (Initially in motion)

Sol:-



$N = W = mg$

$N = (2) \times (9.8) = 19.6\ N$

For moving body, analyze kinetic friction:-

$f_K = \mu_K N$

$f_K = (0.6)(19.6) = 11.76\ N$

Since $F_{app} > f_K$ so the body will move ~~on~~ with constant acceleration

If force is applied opposite to direction of motion $F_{app} + f_K$

$$a = \frac{F_{app} - f_K}{m}$$

$$a = \frac{13.6 - 11.76}{2} = 0.92\ m/s^2$$

Q A particle travels along the inside of an evacuated straight tube undergoing a Force

$$\vec{F} = e^{-t} + 2\cos 3t + 2\sin 3t$$

Analyze total work done from $t = 0 \rightarrow 1$ sec?

Solution:-

$$x = t \ , \quad y = 1.5\,t^2 \ , \quad z = 3t^2$$

$$dr = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k} \quad \text{—(i)}$$

Differentiating all w.r.t 't'

$$\Rightarrow \frac{dx}{dt} = 1 \ , \quad \frac{dy}{dt} = (1.5)\,2t \ , \quad \frac{dz}{dt} = 6t$$

$$dx = dt \ , \quad dy = 3t\,dt \ , \quad dz = 6t.dt$$

$$W = \int F \cdot dr$$

$$W = \int_0^1 \left(e^{-t} + 2\cos 3t + 2\sin 3t\right) \cdot \left(dt\,\hat{i} + 3t\,dt\,\hat{j} + 6t.dt\,\hat{k}\right)$$

$$= \int_0^1 e^{-t}\,dt + \int_0^1 2\cos 3t\,3t\,dt + \int_0^1 2\sin 3t\,6t\,dt$$

$$W = \int_0^1 e^{-t}\,dt + 6\int_0^1 \cos 3t \cdot t\,dt + 12\int_0^1 \sin 3t \cdot t\,dt \quad \text{—(a)}$$

Integration by parts.

$$\because \int u \cdot \frac{dv}{dx}\,dx = u \cdot v - \int v \cdot \frac{du}{dx}\,dx \quad \text{—(i)}$$

Let,

For $\sin 3t \Rightarrow \frac{dv}{dt} = \sin 3t$

$dv = \sin 3t \cdot dt$

$v = -\frac{\cos 3t}{3}$,

$u = t$, $\frac{dv}{dt} = \cos 3t$

$\frac{du}{dt} = 1$

$dv = \cos 3t \cdot dt$

$du = dt$

$\int dv = \int \cos 3t \cdot dt$

$v = \frac{\sin 3t}{3}$

Now eq (i) becomes: –

$$\int t \cdot \cos 3t \, dt = t \cdot \frac{\sin 3t}{3} - \int \frac{\sin 3t}{3} \cdot (1) \, dt$$

$$= \frac{1}{3} t \cdot \sin 3t - \frac{1}{3}\left[-\frac{\cos 3t}{3} \right] + c$$

$$\int t \cdot \cos 3t \, dt = \frac{1}{3}\left[t \cdot \sin 3t + \frac{\cos 3t}{3} \right] + c$$

For other $\sin 3t$.

$$\int t \cdot \sin 3t = t \cdot -\frac{\cos 3t}{3} - \int -\frac{\cos 3t}{3} (1)$$

$$= -\frac{1}{3} t \cdot \cos 3t + \frac{1}{3} \int \left[\frac{\sin 3t}{3} \right] + c$$

$$\int t \cdot \sin 3t = \frac{1}{3}\left[-t \cdot \cos 3t + \frac{1}{3} \sin 3t \right] + c$$

Now eq (a) becomes.

$$W = \left| \frac{e^{-t}}{-1} + 2\left[t \cdot \sin 3t + \frac{\cos 3t}{3} \right] + 4\left[-t \cdot \cos 3t + \frac{1}{3} \sin 3t \right] \right|_0^1$$

Applying limits:-

$$W = \left[-e^{-1} + 2\left\{ (1)\cdot \sin 3(1) + \frac{\cos 3(1)}{3} \right\} + 4\left\{ -(1)\cos 3(1) + \frac{1}{3}\sin 3(1) \right\} \right] - \left[-e^{-0} + 2\left\{ (0)\cdot \sin 3(0) + \frac{\cos 3(0)}{3} \right\} + 4\left\{ -(0)\cos 3(0) + \frac{1}{3}\sin 3(0) \right\} \right]$$

radian

$$W = \left(\cancel{-e^{-1}} - \frac{1}{e} + \frac{10}{3}\sin(3) - \frac{10}{3}\cos(3) \right) - \left(-1 + \frac{2}{3} \right)$$

~~W =~~ Since calculus only works on radian limits, so:-

$$W = -e^{-1} + \frac{1}{3} + 0.470 \cancel{+} - (-3.299)$$

$$W = 3.734 \text{ Joules}$$